Overview of Neutrino Flavor Models

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NTN Workshop on Neutrino Non-Standard Interactions, Washington University - St. Louis, May 28, 2019

Where Do We Stand?

Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni, Schwetz, 1811.05487

- Latest 3 neutrino global analysis:

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$
$\sin^2\theta_{23}$	$0.582^{+0.015}_{-0.019}$	$0.428 \to 0.624$	$0.582^{+0.015}_{-0.018}$	$0.433 \to 0.623$
$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	$40.9 \to 52.2$	$49.7^{+0.9}_{-1.0}$	$41.2 \to 52.1$
$\sin^2\theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	$0.02044 \to 0.02437$	$0.02263^{+0.00065}_{-0.00066}$	$0.02067 \to 0.02461$
$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	$8.22 \to 8.98$	$8.65^{+0.12}_{-0.13}$	$8.27 \to 9.03$
$\delta_{CP}/^\circ$	217^{+40}_{-28}	$135 \to 366$	280^{+25}_{-28}	$196 \to 351$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.525^{+0.033}_{-0.031}$	$+2.431 \to +2.622$	$-2.512^{+0.034}_{-0.031}$	$-2.606 \to -2.413$

➡ evidence of $\theta_{13} \neq 0$

➡ hints of $\theta_{23} \neq \pi/4$

➡ **expectation of Dirac CP phase δ**

Recent T2K result ➪ $\delta \simeq -\pi/2$, consistent with global fit best fit value

Open Questions - Neutrino Properties

☞ Majorana vs Dirac?

☞ CP violation in lepton sector?

☞ Absolute mass scale of neutrinos?

☞ Mass ordering: sign of (Δm_{13}^2)?

☞ Precision: $\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, $\theta_{23} = \pi/4$?

☞ Sterile neutrino(s)?

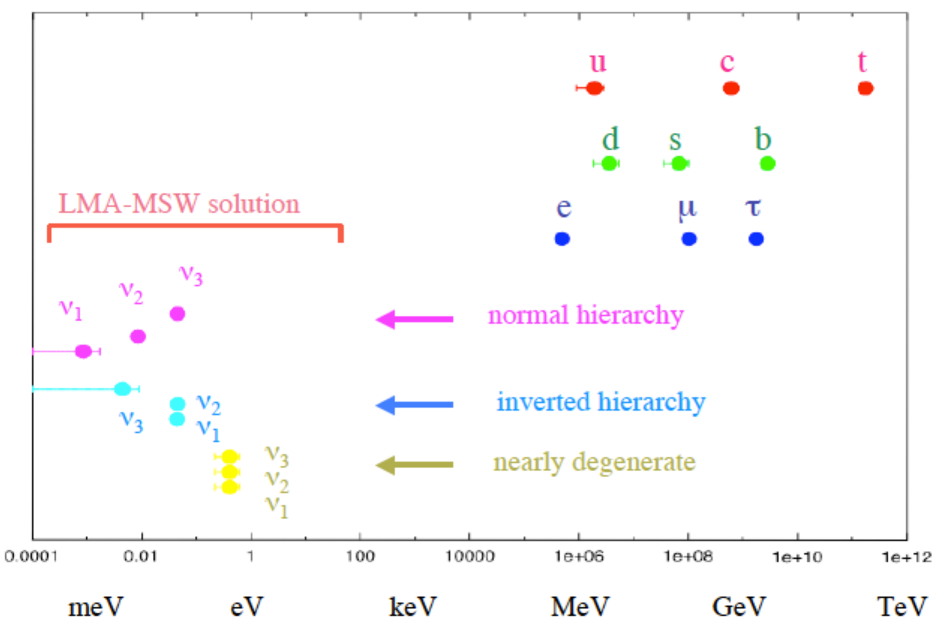
a suite of current and upcoming experiments to address these puzzles

some can only be answered by oscillation experiments

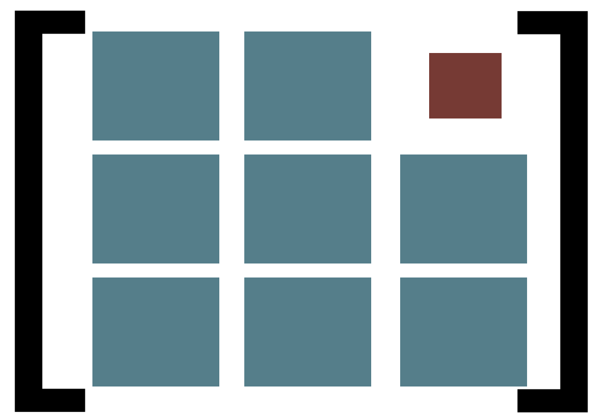
Open Questions - Theoretical

☞ Smallness of neutrino mass:

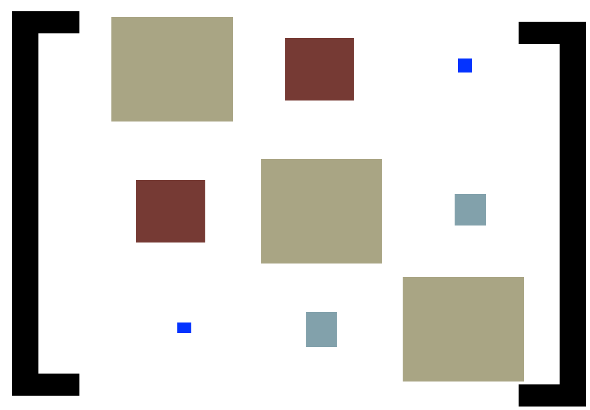
$$m_\nu \ll m_{e,\,u,\,d}$$



☞ Flavor structure:



leptonic mixing



quark mixing

Open Questions - Theoretical

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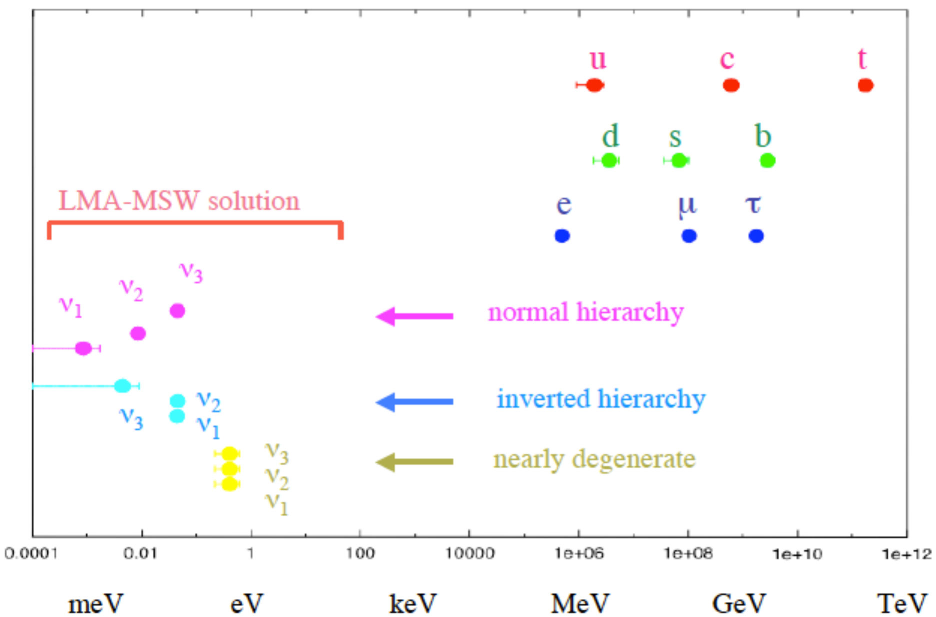
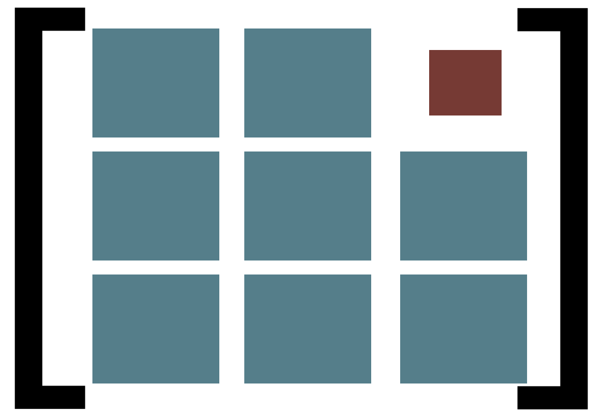
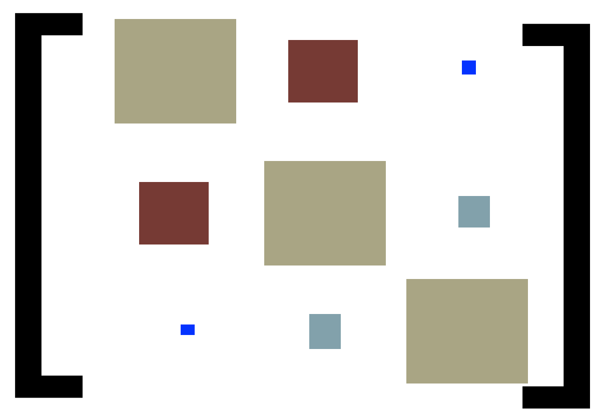


Fermion mass and hierarchy problem ➟ Many free parameters in the Yukawa sector of SM

☞ Flavor structure:



leptonic mixing



quark mixing

Smallness of neutrino masses

What is the operator for neutrino mass generation?

- Majorana vs Dirac
- scale of the operator
- suppression mechanism

Neutrino Mass beyond the SM

- SM: effective low energy theory

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{\mathcal{O}_{5D}}{M} + \frac{\mathcal{O}_{6D}}{M^2} + \dots$$

→ new physics effects

- only one dim-5 operator: most sensitive to high scale physics

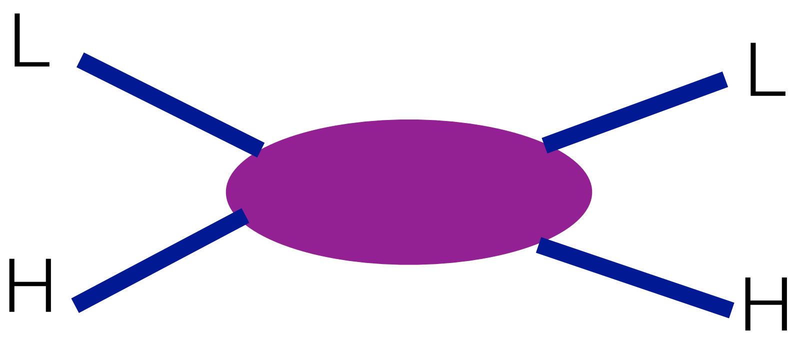
$$\frac{\lambda_{ij}}{M} H H L_i L_j \quad \Rightarrow \quad m_\nu = \lambda_{ij} \frac{v^2}{M}$$

Weinberg, 1979

- $m_\nu \sim (\Delta m^2_{\rm atm})^{1/2} \sim 0.1$ eV with $v \sim 100$ GeV, $\lambda \sim O(1) \Rightarrow M \sim 10^{14}$ GeV
- Lepton number violation **ΔL = 2** ⇨ Majorana fermions

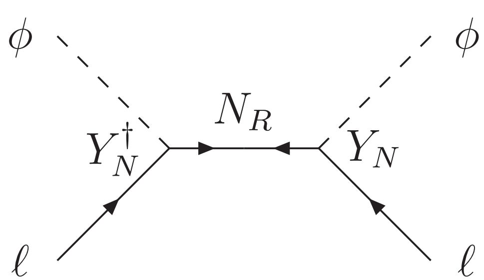
↕ GUT scale

Neutrino Mass beyond the SM



3 possible portals

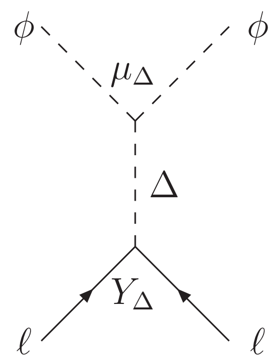
Type-I seesaw



N_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,1,0)

Minkowski, 1977; Yanagida, 1979; Glashow, 1979;
Gell-mann, Ramond, Slansky,1979;
Mohapatra, Senjanovic, 1979;

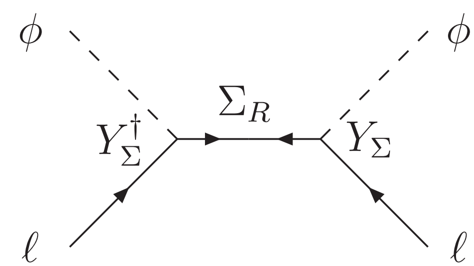
Type-II seesaw



Δ: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,3,2)

Lazarides, 1980; Mohapatra, Senjanovic, 1980

Type-III seesaw



$$\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$$

Σ_R: $SU(3)_c$ x $SU(2)_w$ x $U(1)_Y$ ~(1,3,0)

Foot, Lew, He, Joshi, 1989; Ma, 1998

Why are neutrinos light? (Type-I) Seesaw Mechanism

- Adding the right-handed neutrinos:

$$\begin{pmatrix} \nu_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

$$m_\nu \sim m_{light} \sim \frac{m_D^2}{M_R} << m_D$$

$$m_{heavy} \sim M_R$$

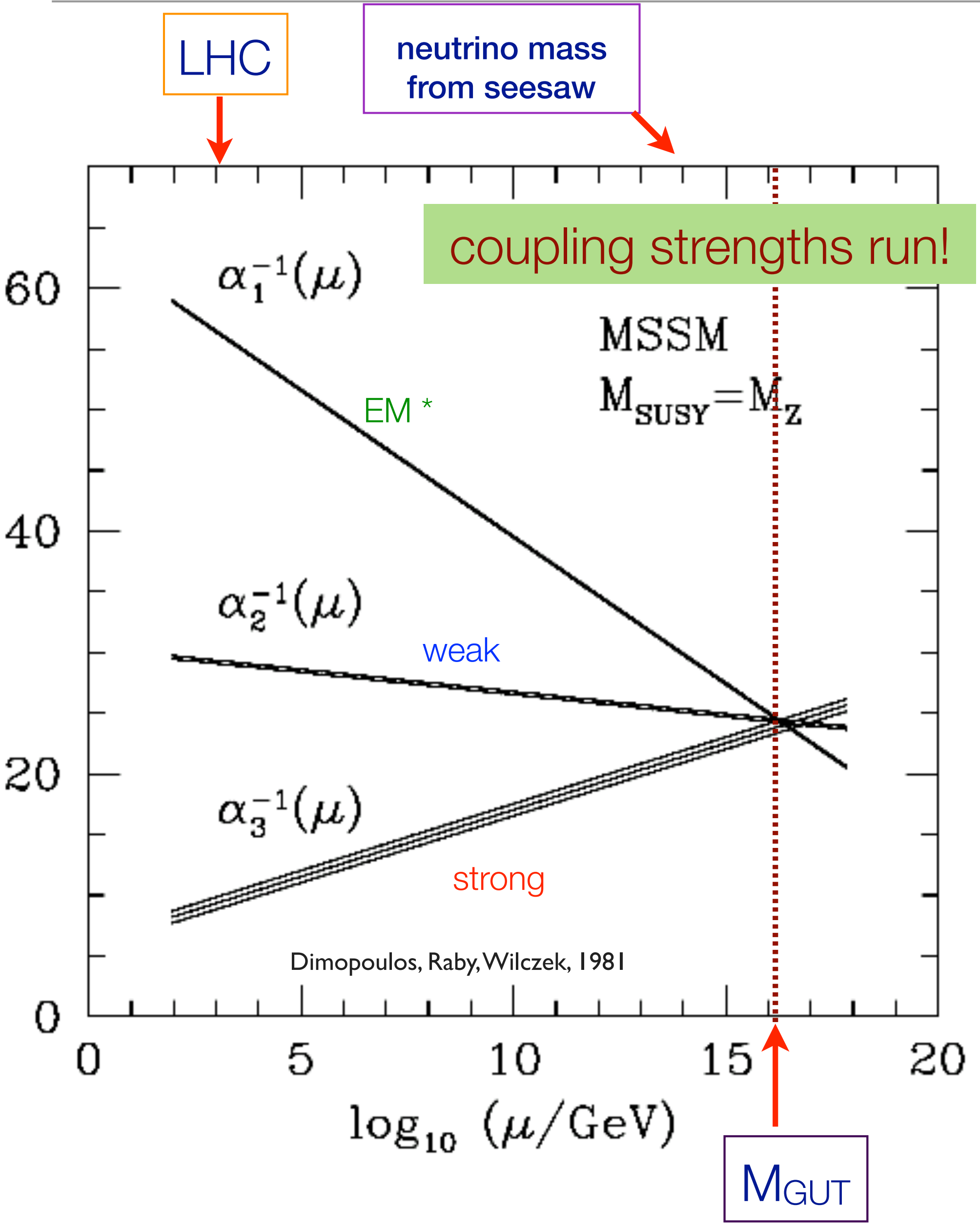
For $m_{\nu_3} \sim \sqrt{\Delta m^2_{atm}}$

If $m_D \sim m_t \sim 180 \;\; GeV$

$\Rightarrow$ $M_R \sim 10^{15} \; GeV \;$ (GUT !!)

Minkowski, 1977; Yanagida, 1979; Gell-Mann, Ramond, Slansky, 1979; Mohapatra, Senjanovic, 1981

Grand Unification Naturally Accommodates Seesaw



- origin of the heavy scale ⇒ $U(1)_{B-L}$
- exotic mediators ⇒ predicted in many GUT theories, e.g. SO(10)

$$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$$

$$+ (3^*, 1, -2/3) \sim (u^c \;\; u^c \;\; u^c)$$

$$+ (3^*, 1, 1/3) \sim (d^c \;\; d^c \;\; d^c)$$

$$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$$

$$+ (1, 1, 1) \sim e^c$$

$$+ (1, 1, 0) \sim \nu^c$$

Fritzsch, Minkowski, 1975

Low Scale Seesaws

Talk by Tao Han

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.1$ eV with $v \sim 100$ GeV, $\lambda \sim 10^{-6}$
$\Rightarrow M \sim 10^2$ GeV

- New particles:
  - Type I seesaw: generally decouple from collider experiments
  - Type II seesaw: $\Delta^{++} \to e^+ e^+,\ \mu^+ \mu^+,\ \tau^+ \tau^+$ Franceschino, Hambye, Strumia,2008
  - Type III seesaw: observable displaced vertex
  - inverse seesaw: non-unitarity effects
  - radiative mass generation: model dependent - singly/doubly charged SU(2) singlet, even colored scalars in loops
- New interactions:
  - LR symmetric model: W_R
  - R parity violation: $\tan^2 \theta_{\rm atm} \simeq \frac{BR(\tilde{\chi}_1^0 \to \mu^\pm W^\mp)}{BR(\tilde{\chi}_1^0 \to \tau^\pm W^\mp)}$ Mukhopadhyaya, Roy, Vissani, 1998
  - .....

What if neutrinos are Dirac?

Naturally Light Dirac Neutrinos from ~~SUSY~~

- MSSM: many attractive features (solving gauge hierarchy problem, gauge unification)
  - Dirac neutrino mass from Kähler potential   Arkani-Hamed, Hall, Murayama, Tucker-Smith, Weiner (2001)

$$K \supset k_{LH_u\bar{\nu}} \frac{X^\dagger}{M_P^2} L\, H_u\, \bar{\nu} + \mathrm{h.c.} \quad \Longleftrightarrow \quad Y_\nu \sim \frac{m_{3/2}}{M_P} \sim \frac{\mu}{M_P}$$

<X>: SUSY breaking VEV

  - However, it has several problems
    - mu problem: $\mu \ll M_{pl}$
      - Giudice-Masiero mechanism   Giudice, Masiero (1988)
        - absence of mu term in superpotential
        - effective mu term (non-perturbatively) from Kähler potential

$$K \supset k_{H_u H_d} \frac{X^\dagger}{M_P} H_u\, H_d + \mathrm{h.c.} \quad \Longleftrightarrow \quad \mathscr{W}_{\mathrm{eff}} \sim \frac{F_X}{M_P} H_u\, H_d \;=:\; \mu_{\mathrm{eff}}\, H_u\, H_d$$

    - proton decay through dim-4, dim-5 operators
      - dim-4 operators: forbidden by imposing R-parity
      - dim-5 operators: severe experimental constraints on the models
  - no symmetry reason for the absence of holomorphic mu term/Dirac neutrino mass

Neutrino Mass and the μ Term

- Requiring Symmetries
  - to forbid mu term
  - be anomaly-free → **R Symmetries**
  - be consistent with SU(5)
- continuous R symmetries not available → **Discrete R Symmetries**

  A.H. Chamseddine, H.K. Dreiner (1996)

- Search Abelian discrete R symmetries, $\mathbb{Z}_M^R$, that satisfy

  K.S. Babu, I. Gogoladze, K. Wang (2002)

  - Majorana neutrino case for q_θ = integer:
    - anomaly freedom (allowing Green-Schwarz) →
    - mu term forbidden perturbatively
    - consistent with SU(5)
    - usual Yukawa allowed
    - Weinberg operators allowed

- five viable symmetries found;
- one unique solution consistent with SO(10) ➜ Z4 R-symmetry

H.M. Lee, S. Raby M. Ratz, G.G. Ross, R. Schieren, K. Schmidt-Hoberg, P.K. Vaudrevange, (2011);

M.-C. C., M. Ratz, Ch. Staudt, P. K. Vaudrevange (2012)

Dirac Neutrino Mass and the μ Term

- Search Abelian discrete R symmetries, $\mathbb{Z}_M^R$, that satisfy

M.-C. C., M. Ratz, Ch. Staudt, P. K. Vaudrevange (2012)

  - Dirac neutrino case for q_θ = integer:
    - anomaly freedom (a la Green-Schwarz)
    - forbidding mu term perturbatively ⇒ classes of models found
    - consistent with SU(5)
    - allowing usual Yukawa
    - Weinberg operators forbidden perturbatively
  - an example: $\mathbb{Z}_8^R$ symmetry
    - at non-perturbative level

$$\mathscr{W}_{\text{eff}} \sim m_{3/2}\, H_u\, H_d + \frac{m_{3/2}}{M_{\text{P}}}\, L\, H_u\, \bar{\nu} + \frac{m_{3/2}}{M_{\text{P}}^2}\, Q\, Q\, Q\, L$$

    - Δ L = 2 operators forbidden ⇒ no neutrinoless double beta decay
    - ΔL = 4 operators allowed ⇒ new LNV processes

Dirac Neutrinos and SUSY Breaking

- Symmetry realization in MSSM: discrete R symmetries, $\mathbb{Z}_M^R$

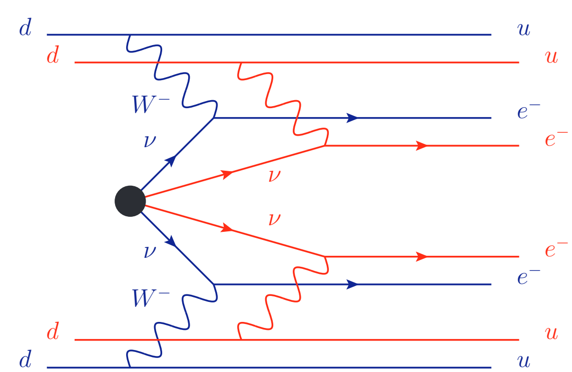
M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)

  - **Dirac neutrinos**, with naturally small masses
  - Δ L = 2 operators forbidden to all orders ⇒ no neutrinoless double beta decay
  - **New signature: lepton number violation ΔL = 4 operators, $(\nu_R)^4$, allowed ⇒ new LNV processes, e.g.**

M.-C. C., M. Ratz, C. Staudt, P. Vaudrevange (2012)

    - neutrinoless quadruple beta decay

Heeck, Rodejohann (2013)



- mu term is naturally small
- dangerous proton decay operators forbidden/suppressed
- can also give dynamical generation of RPV operators with size predicted

M.-C. C., M. Ratz, V. Takhistov (2015)

Flavor structure

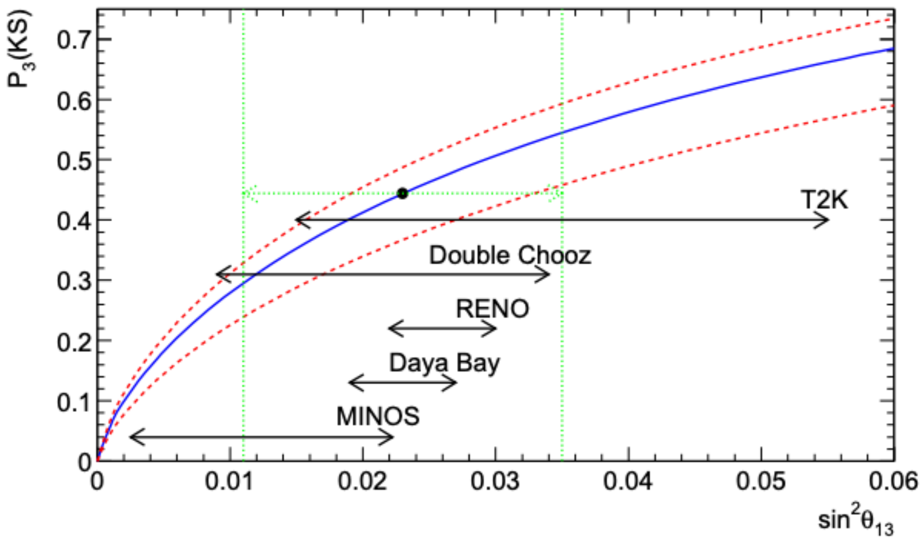
anarchy vs symmetry

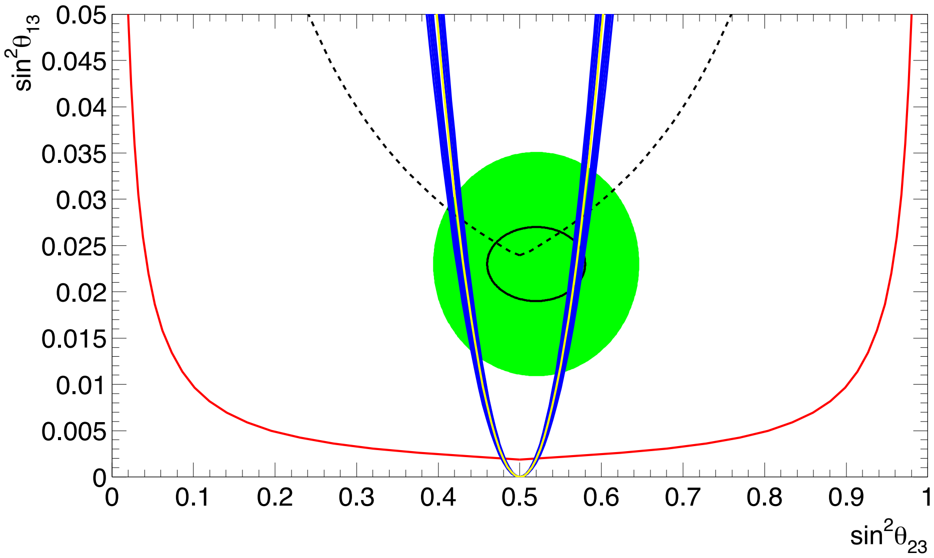
Anarchy

Hall, Murayama, Weiner (2000);
de Gouvea, Murayama (2003)

- there are no parametrically small numbers
- large mixing angle, near mass degeneracy statistically preferred

de Gouvea, Murayama (2012)



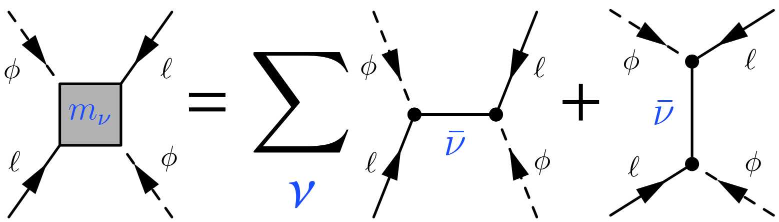


- UV theory prediction can resemble anarchy
  - warped extra dimensions
  - heterotic string theory

Expectations from Heterotic String Theories

- heterotic string models: O(100) RH neutrinos

Buchmüller, Hamaguchi, Lebedev, Ramos-Sánchez, Ratz (2007)

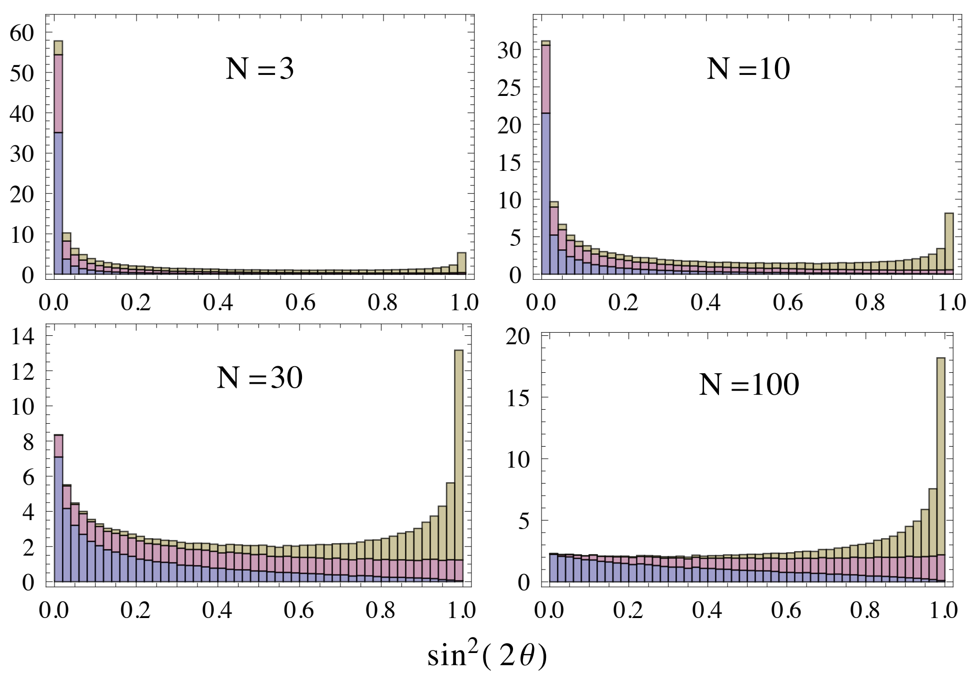


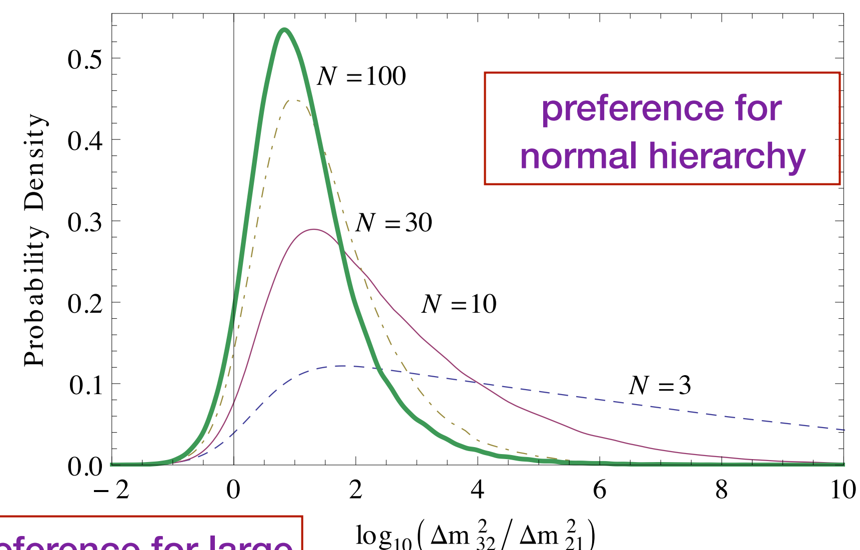
$$m_\nu \sim \frac{v^2}{M_*}$$

$M_* \sim \frac{M_{GUT}}{10...100}$

- statistical expectations with large N (= # of RH neutrinos)

Feldstein, Klemm (2012)





preference for normal hierarchy

preference for large mixing angles

Symmetry Relations

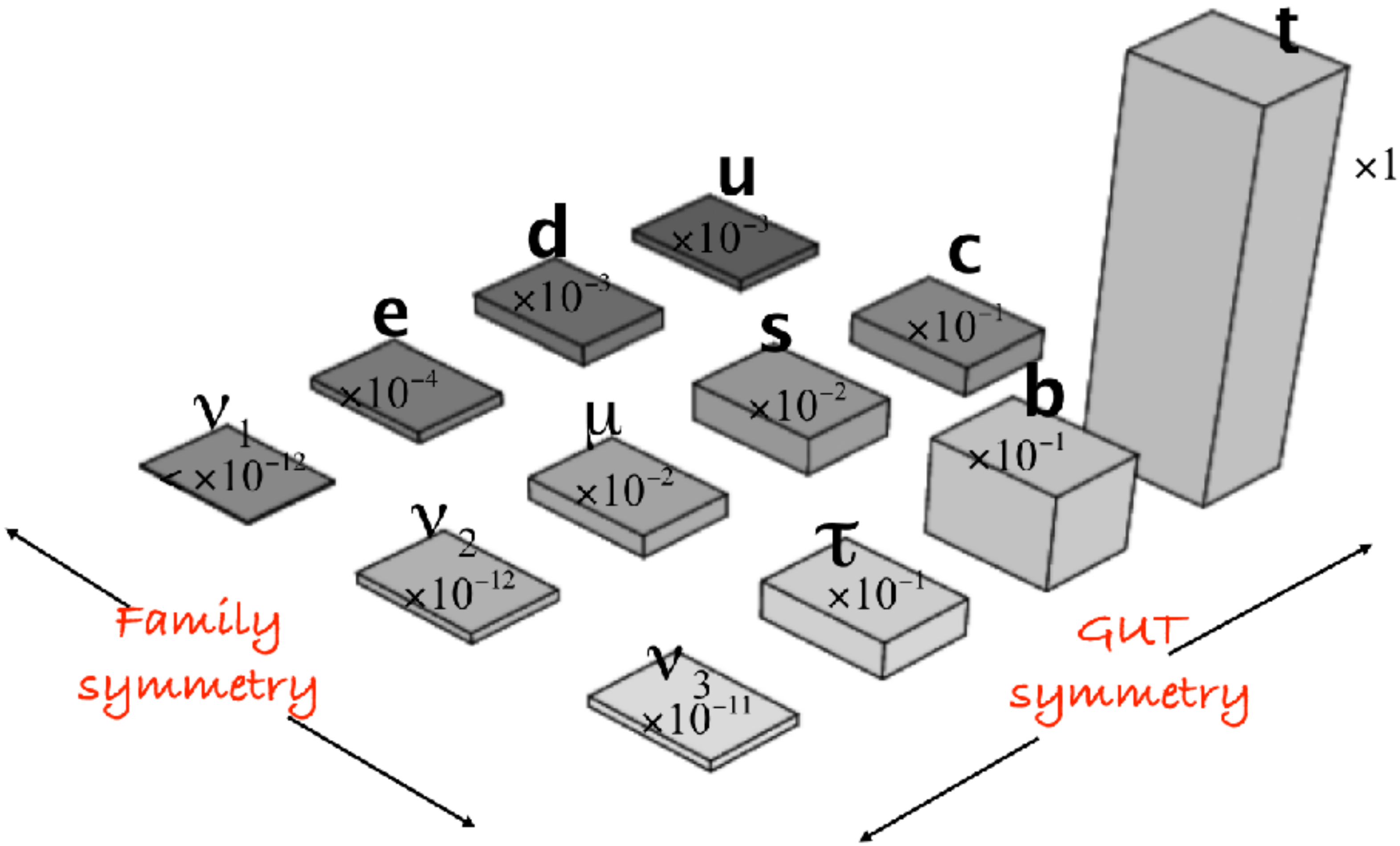
Grand Unified Theories: GUT symmetry

Quarks ⬌ Leptons

Family Symmetry:

e-family ⬌ muon-family ⬌ tau-family

Mass Spectrum of Elementary Particles



Symmetry Relations

Symmetry ⇒ relations among parameters

⇒ reduction in number of fundamental parameters

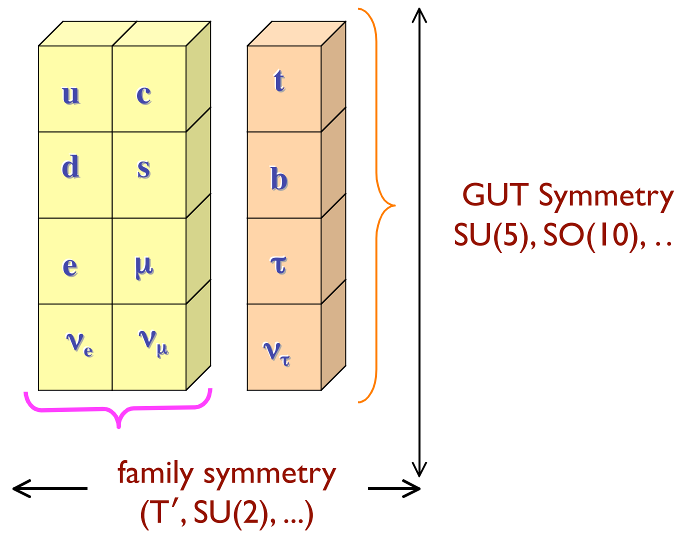
Symmetry Relations

Symmetry ⇒ relations among parameters
⇒ reduction in number of fundamental parameters

Symmetry ⇒ experimentally testable correlations among physical observables

Origin of Flavor Mixing and Mass Hierarchies

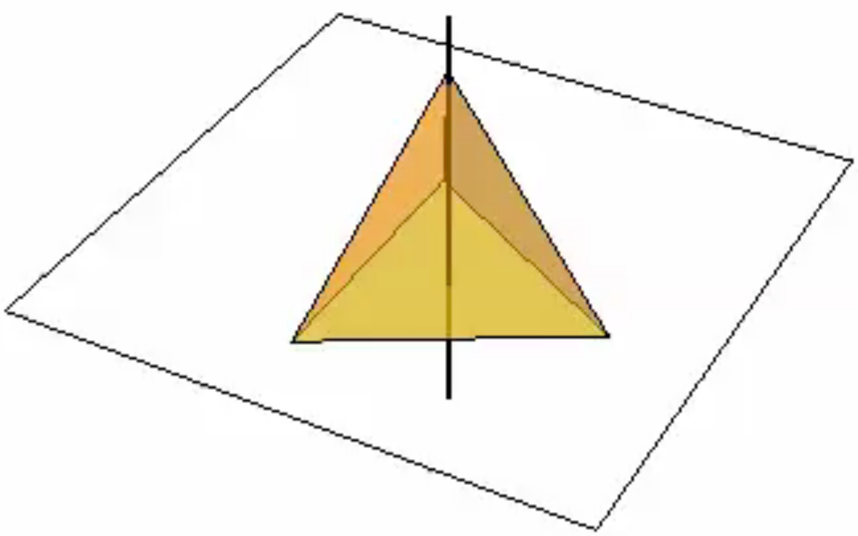
- several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T' (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
- Extra dimensional origin
- Modular symmetry



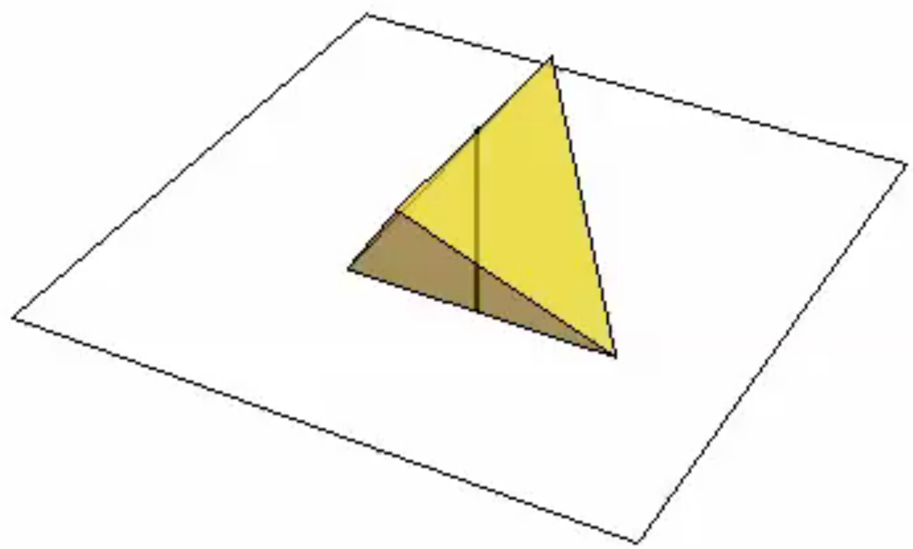
Example: Tetrahedral Group A_4

- Smallest group giving rise to tri-bimaximal neutrino mixing: **tetrahedral group A_4**

T: (1234) → (2314)



S: (1234) →(4321)



Tri-bimaximal Neutrino Mixing

- Latest Global Fit (3σ)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ $[\theta^{lep}_{23} \sim 49.7°]$

Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni, Schwetz, 1811.05487

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ $[\theta^{lep}_{12} \sim 33.8°]$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ $[\theta^{lep}_{13} \sim 8.61°]$

- Tri-bimaximal Mixing Pattern   Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\text{atm, TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\text{TBM}} = 1/3$$

$$\sin\theta_{13,\text{TBM}} = 0.$$

- Leading Order: TBM (from symmetry) + higher order corrections/contributions
- More importantly, corrections to the kinetic terms   Leurer, Nir, Seiberg ('93); Dudas, Pokorski, Savoy ('95)
  - small for quarks
  - sizable in discrete symmetry models for leptons   M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

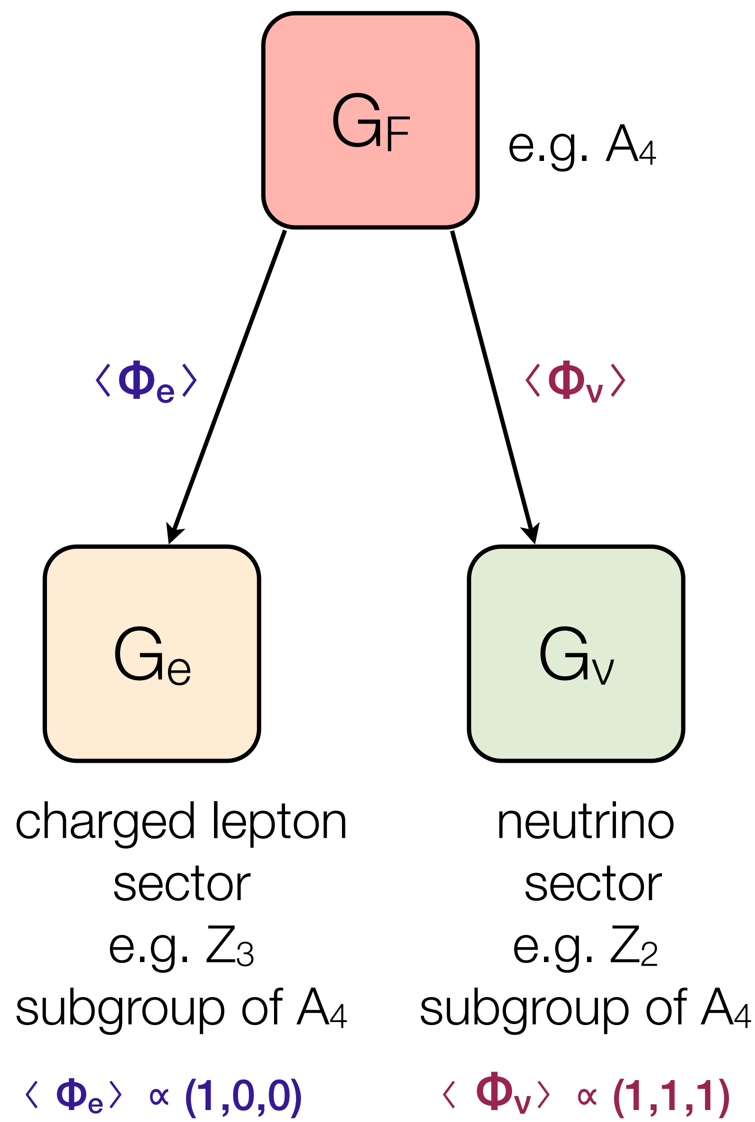
2 free parameters

relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

General Structure



Example: SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T´: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- Symmetries ⇒ **correlations among quark and lepton mixing parameters**

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

← CG's of SU(5) & T´

no free parameters!

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

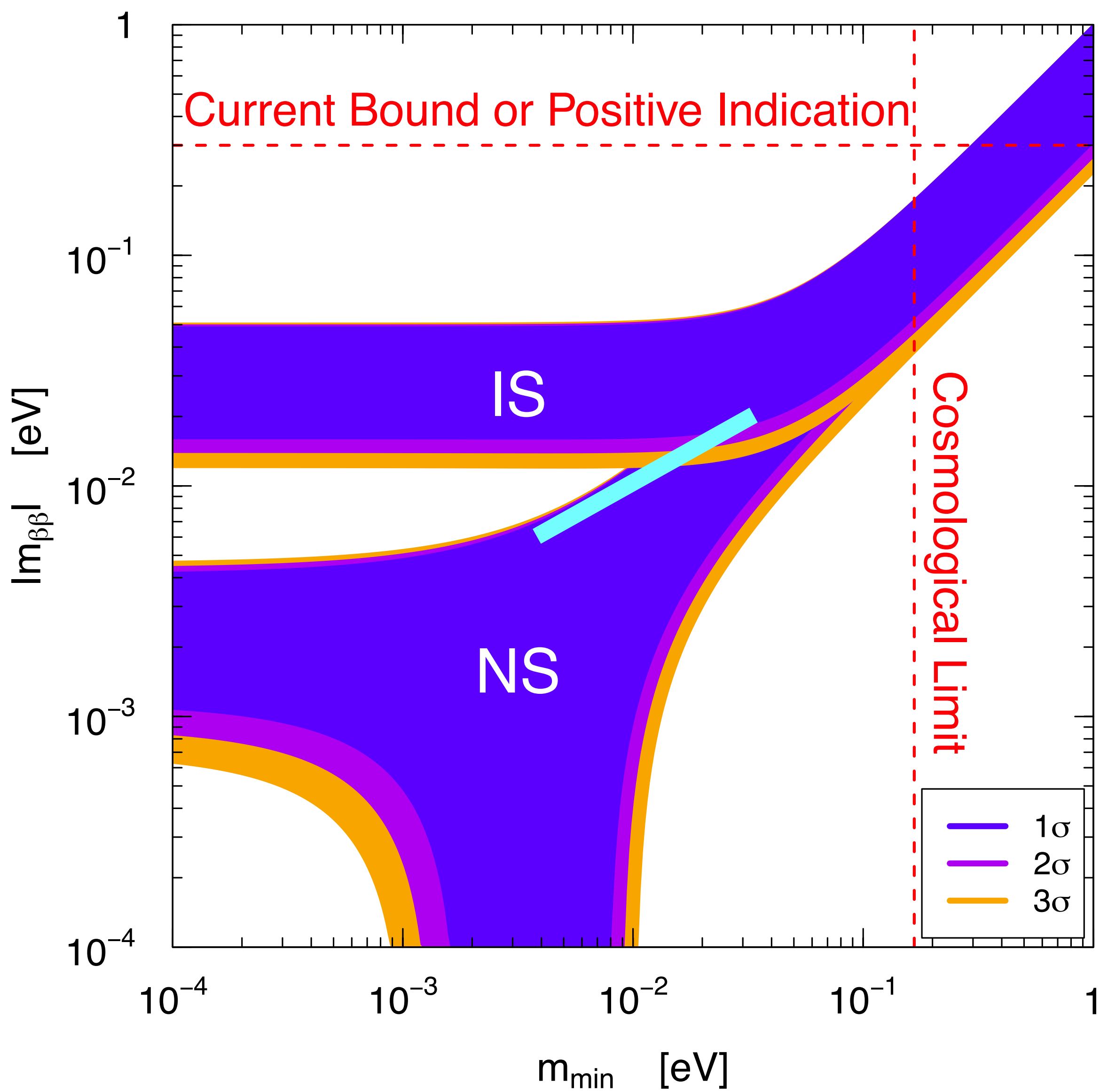
neutrino solar mixing

1/2

quark Cabibbo mixing

leptonic CP phase

Neutrinoless Double Beta Decay



our model prediction

sum rule among masses
⇒ small predicted region

[Plot taken from C. Giunti, LIONeutrino2012]

Symmetry Relations

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	41.2°	35.1° - 52.6°
θ^e_{12}	33.6°	30.6° - 36.8°
θ^e_{13}	8.9°	7.5° -10.2°

- QLC-I $\theta_C + \theta_{sol} \cong 45^o$ Raidal, '04; Smirnov, Minakata, '04

  (BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$ ☜ **slight inconsistent**

- QLC-II $\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_C / 2) * \cos\delta_e$ Ferrandis, Pakvasa; Dutta, Mimura; M.-C.C., Mahanthappa

  (TBM) $\theta^e_{13} \cong \theta_C / 3\sqrt{2}$ ☜ **Too small**

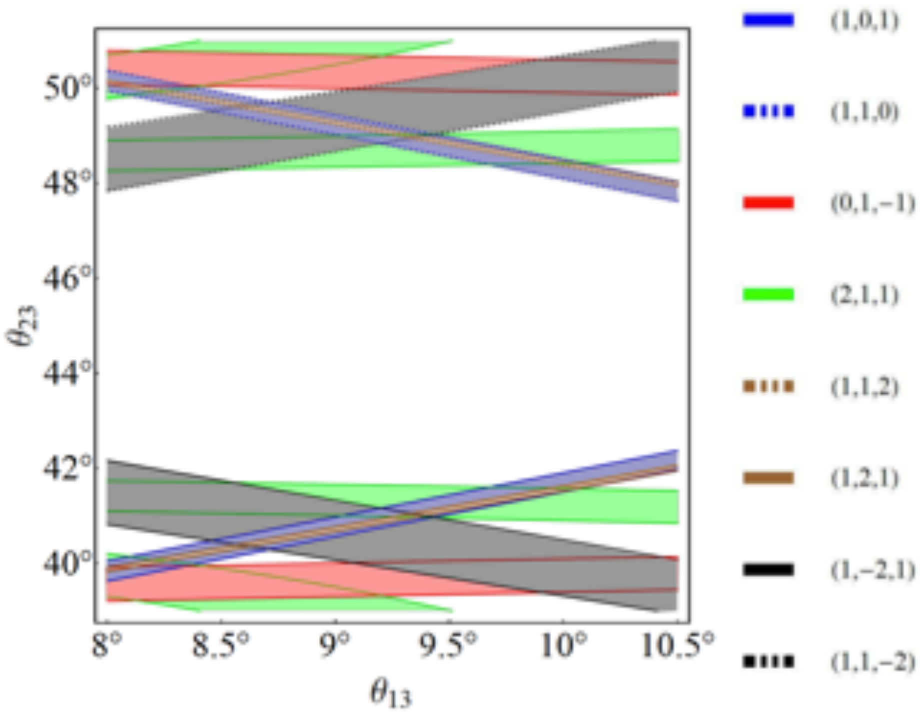
- testing symmetry relations: a *more* robust way to distinguish different classes of models

measuring leptonic mixing parameters to the precision of those in quark sector

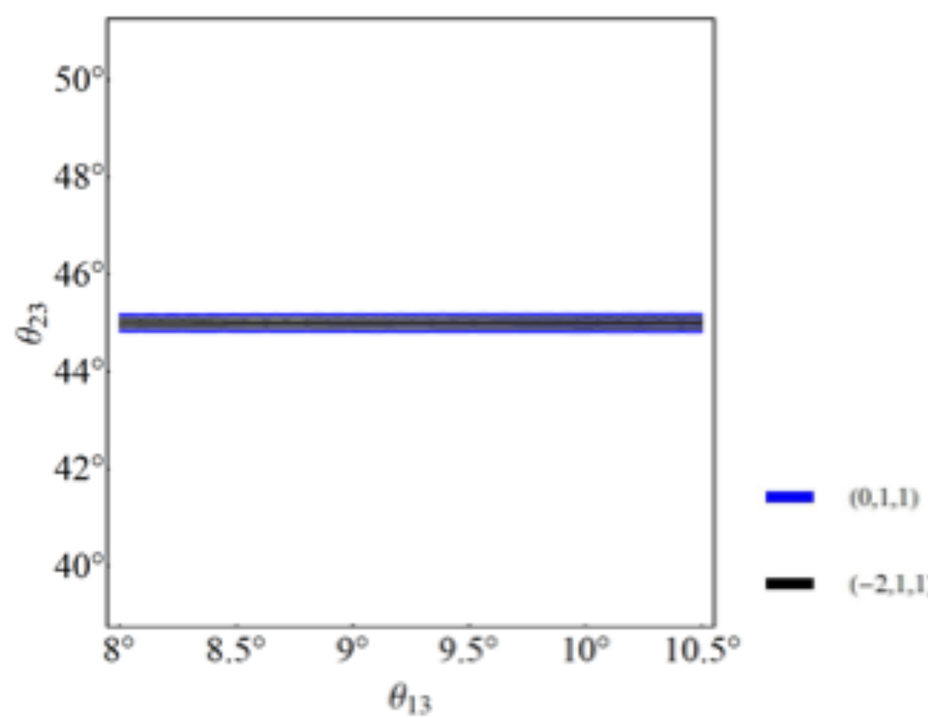
“Large” Deviations from TBM in A_4

M.-C.C, J. Huang, J. O’Bryan, A. Wijangco, F. Yu, (2012)

- Different A4 breaking patterns:



normal



inverted

deviations correlated

non-maximal θ_{23} ⇨ normal hierarchy

mass ordering ⇨ symmetry breaking patterns

CP Violation

CP Violation in Nature

- ~~CP~~ so far only observed in flavor sector
- ➥ it appears natural to seek connection between flavor physics & ~~CP~~
- flavor structure may be explained by (non–Abelian discrete) flavor symmetries

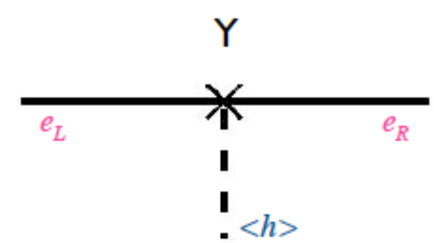
non–Abelian discrete (flavor) symmetry $G \leftrightarrow$ ~~CP~~

Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

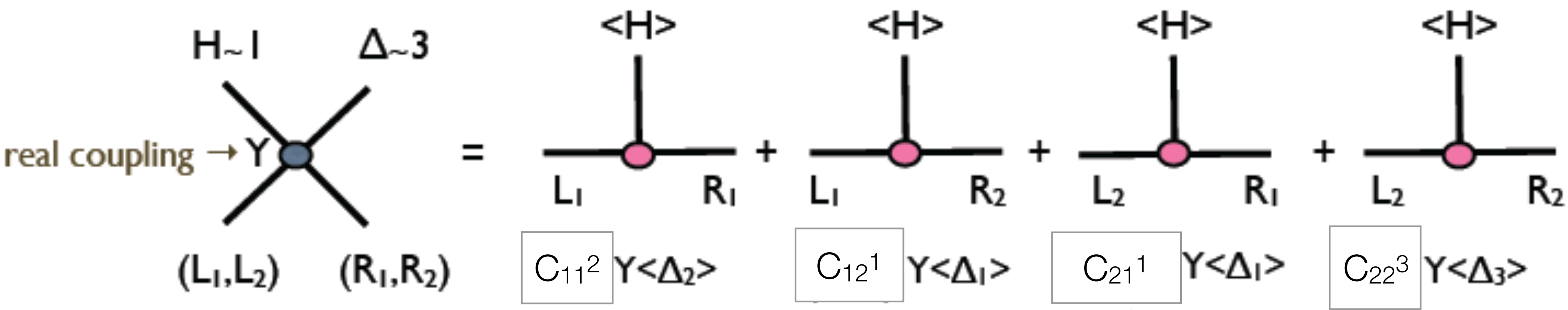
CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass hierarchy)

Group Theoretical Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

Basic idea

Discrete symmetry ***G***



- if Z_3 symmetric $\Rightarrow$ $\langle \Delta_1 \rangle = \langle \Delta_2 \rangle = \langle \Delta_3 \rangle \equiv \langle \Delta \rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$C_{ij}{}^{k}$: complex CG coefficients of ***G***

$$\begin{pmatrix} L_1 & L_2 \end{pmatrix} \quad M = \begin{pmatrix} C_{11}{}^{2} & C_{21}{}^{1} \\ C_{12}{}^{1} & C_{22}{}^{3} \end{pmatrix} Y \langle \Delta \rangle \quad \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

CP Transformation

- Canonical CP transformation

$$\phi(x) \stackrel{\mathcal{C}\mathcal{P}}{\longmapsto} \eta_{\mathcal{C}\mathcal{P}}\ \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \stackrel{\widetilde{\mathcal{C}\mathcal{P}}}{\longmapsto} U_{\mathrm{CP}}\, \Phi^*(\ \mathcal{P}\ \ x)$$

unitary matrix

Generalized CP Transformation

☞ setting w/ discrete symmetry G

G and CP transformations do not commute

☞ generalized CP transformation

Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)

☞ invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1 y_1 + \omega^2 x_2 y_2 + \omega x_3 y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

☞ canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

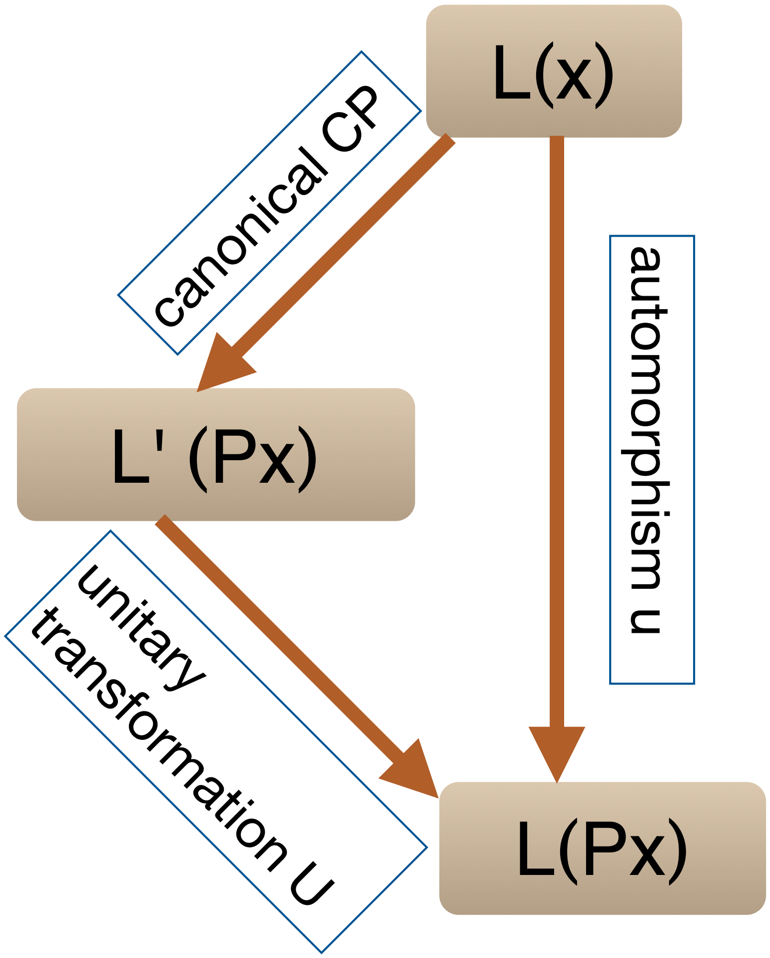
➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ➪ G and physical CP transformations do not commute



$$\Phi(x) \xrightarrow{\widetilde{CP}} U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\; x)$$

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a **class-inverting, involutory** automorphism of G

➪ non-existence of such automorphism in certain groups

➪ calculable physical CP violation in generic setting

examples: T_7, $\Delta(27)$, …..

Novel Origin of CP (Time Reversal) Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ⇨ CP symmetry cannot be defined for certain groups

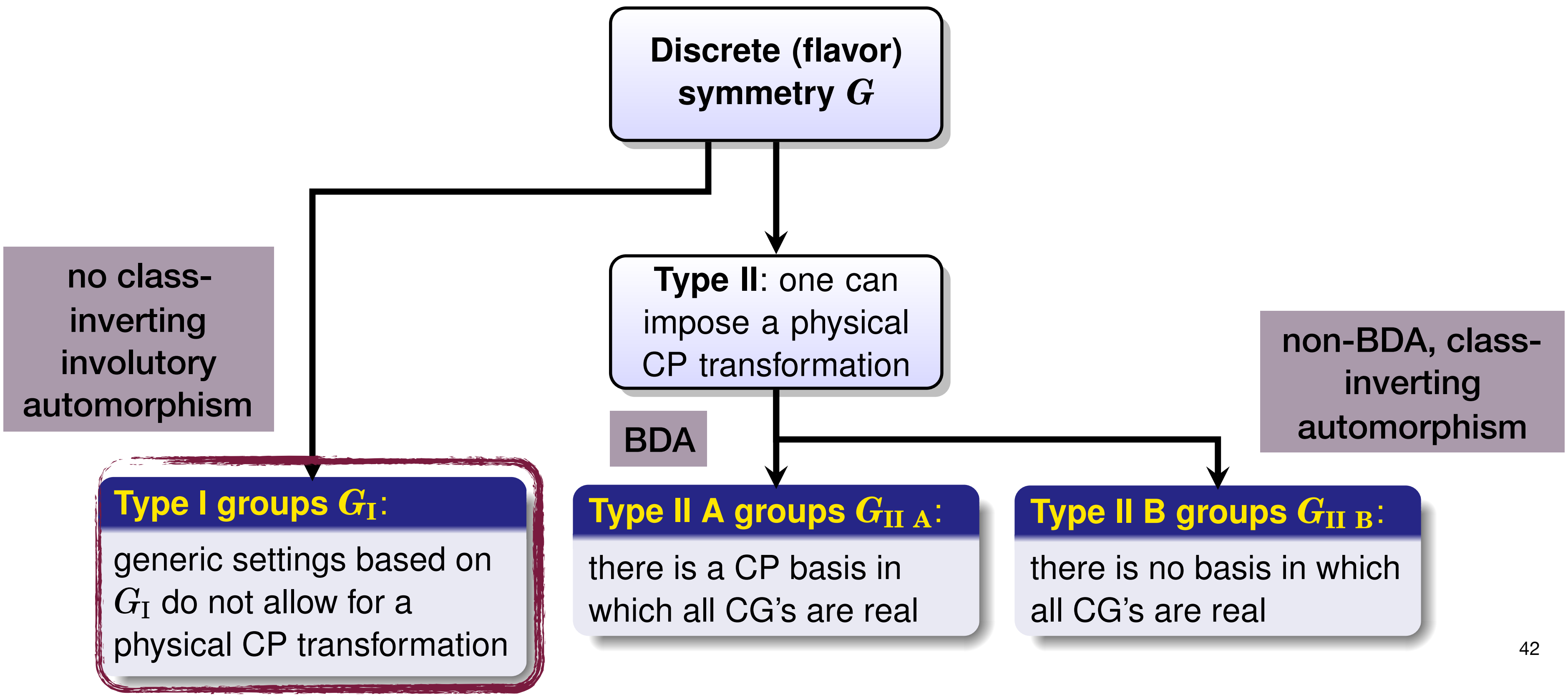
CP Violation from Group Theory!

A Novel Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

- **For discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ Physical CP violation**

CP Violation from Group Theory!



Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Outlook

Summary

- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM

- Symmetries:
  - can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry ⇒ predictive power
  - Symmetries ⇒ Correlations, Correlations, Correlations!!!

- **Dirac vs Majorana?** - should remain open minded!
  - naturally light Dirac neutrinos from discrete R-symmetry
  - suppressed nucleon decays and naturally small mu term

Summary

- **Discrete Groups (of Type I) affords a Novel origin of CP violation:**
  - **Complex CGs ⇒ Group Theoretical Origin of CP Violation**
- **NOT all outer automorphisms correspond to physical CP transformations**
- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\,\rho_{\boldsymbol{r}_i}(g)^*\,U^\dagger_{\boldsymbol{r}_i} \qquad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

class inverting, involutory automorphisms ⇔ physical CP transformations