Cosmic Censorship as Quantum Effect

Marcelo's 65th Birthday
December 5-7, 2017

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arXiv:160506078 [PLB(2016)]; arXiv:1608.05366 [PRL(2017)]

NON PERTURBATIVE ASPECTS OF QFT AND LOEWE'S 65 FEST

5 - 7 DECEMBER 2017

TOPICS

QCD PHASE TRANSITION
ANALYTIC QCD
CGC | BFKL-BK EQUATIONS
QCD IN EXTREME CONDITIONS
SCHWINGER-DYSON EQUATIONS
LOW DIMENSIONAL QFT
EFFECTIVE FIELD THEORIES
NEW TRENDS IN PARTICLES PHYSICS
LATTICE AND HIGHER ORDER CORRECTIONS
GRIBOV'S AMBIGUITY AND CONFINEMENT

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QCD PHASE TRANSITION
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LOW DIMENSIONAL QFTS (including **BLACK HOLES)**
EFFECTIVE FIELD THEORIES
NEW TRENDS IN PARTICLES PHYSICS
LATTICE AND HIGHER ORDER CORRECTIONS
GRIBOV'S AMBIGUITY AND CONFINEMENT

All spherically symmetric solutions in GR turn out to be singular.

DEMONSTRATION OF THE NON-EXISTENCE OF GRAVITATIONAL FIELDS WITH A NON-VANISHING TOTAL MASS FREE OF SINGULARITIES

BY A. EINSTEIN
(Institute for Advanced Study, Princeton, New Jersey)

Schwarzschild's solution for a gravitational field with central symmetry, as it is well known, becomes singular in the neighborhood of the origin. It is also generally regarded as unlikely that within the frame of the generalized theory of relativity of the pure gravitational field, any solutions may exist that represent particles of finite no vanishing total mass without singularities. In this paper I give a proof of the non-existence of such solutions.

We shall confine ourselves here to such solutions which are plonged in an euclidean space.

Revista de la Universidad Nacional de Tucumán, **A2** (1941) 11.

Nakedness and censorship

c.f., T. P. Singh, *J. Astrophys. Astr.* **20**, 221 (1999)

Under reasonable, generic, initial conditions in GR, singular solutions inevitably arise in GR (Penrose-Hawking theorem).

A singularity represents a failure in the spacetime continuum, where the notion of geometry breaks down, the "normal" physical laws do not apply and it is no longer possible to predict the outcome of experiments.

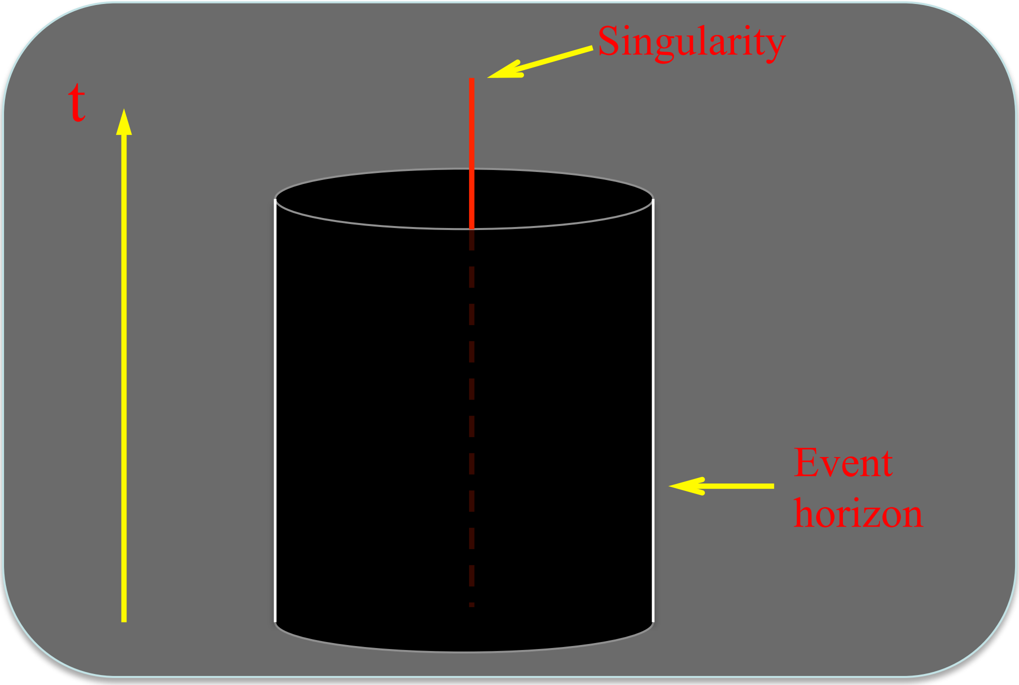
Singularities causally connected to us –*naked singularities*- give rise to serious conceptual problems: physics becomes unpredictable (useless).

"Green slime, lost socks and broken TV sets could emerge from naked singularities" (J. Earman)

Static Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

For $m>0$ the horizon at $r=2m$ hides the curvature singularity at $r=0$, implementing cosmic censorship.



If $m<0$ there is no horizon, the curvature singularity at $r=0$ is a Naked Singularity (NS).

Cosmic Sensorhip (CC): Roger Penrose (1968) conjetured that NSs cannot exist in nature.

- CC seems to be true, but there is no proof of it.
- Christodoulu: Collapsing matter can form NSs.
- This, however, requires finely tuned initial conditions.

➔ The need of "*fine tuning*" suggests that *NS*s could be perturbatively unstable: can quantum mechanics rule out *NS*s?

➔ Can quantum effects avoid NSs as they prevent the collapse of the electron to r=0 in the hydrogen atom?

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Intractable problem in 3+1 dimensions: Go down to 2+1

The 2+1 black hole

M. Bañados, C.Teitelboim, J.Z. (1992)

All solutions of Einstein's equations in 2+1dimensions are spacetimes of constant negative curvature:

$$R^{ab} + l^{-2} e^{a} e^{b} = 0$$

$$\left(R^{\alpha\beta}_{\mu\nu} = -[\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu} - \delta^{\alpha}_{\nu}\delta^{\beta}_{\mu}] l^{-2} \right)$$

- 2+1 black holes are spherically symmetric, stationary solutions labeled by two constants of integration: *mass* (*M*) and *angular momentum* (*J*).

- These spaces have the *same constant negative curvature* ($-l^{-2}$) for all values of *M* and *J*.

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(In what follows we use l=1)

Black hole in 2+1 dimensions

Static case

$J=0$

$$ds^2 = -(r^2 - M)dt^2 + \frac{dr^2}{(r^2 - M)} + r^2 d\phi^2$$

$$r_+^2 = M$$

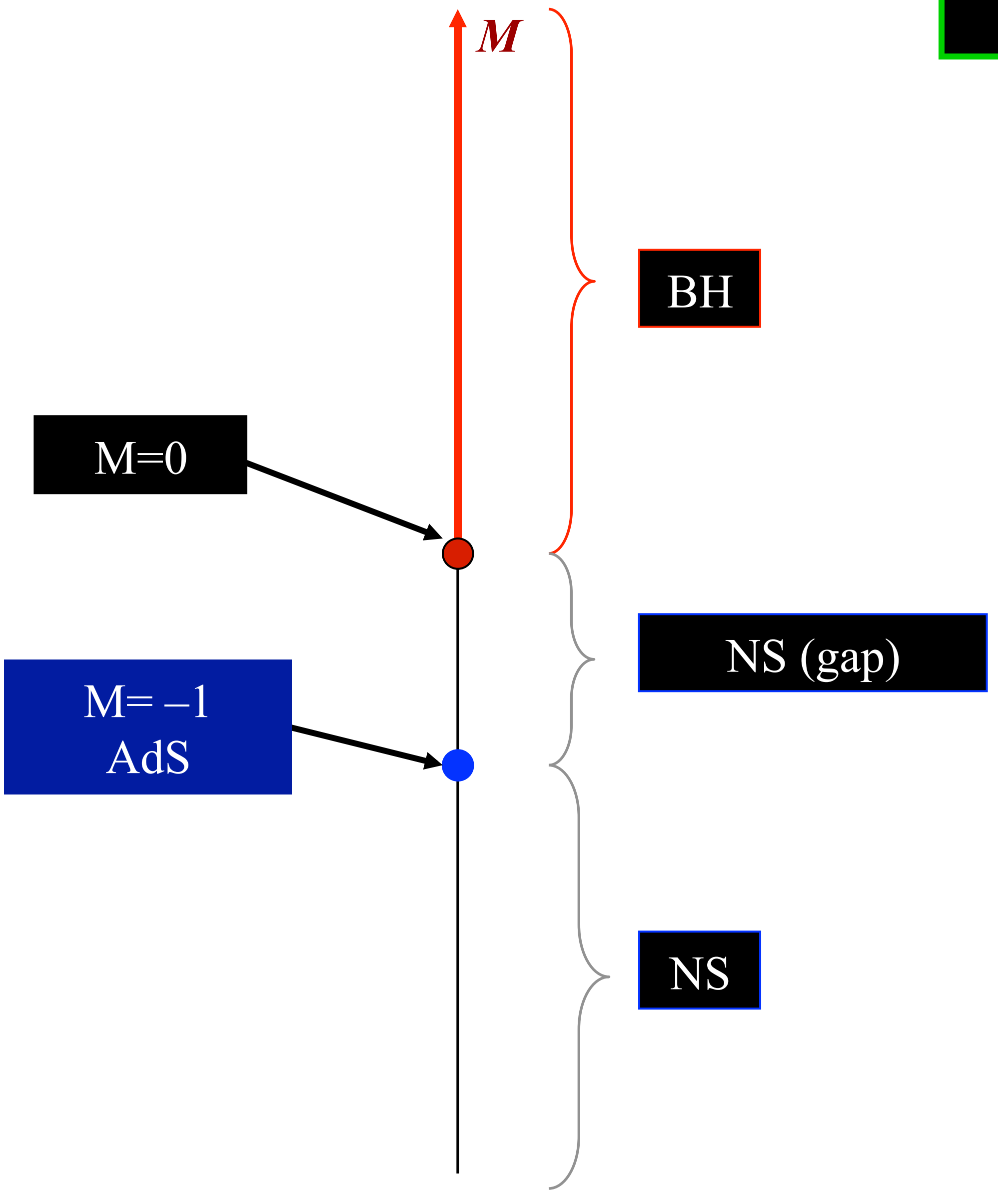
$M \geq 0$ → BH; horizon at $r_+ = M^{1/2}$

$M = -1$ → AdS spacetime ($\Lambda = -1$)

$M<0$ → No horizon: *Naked singularity*

2+1 BH spectrum

(J=0)



Spinning 2+1 black hole

$J \neq 0$

$$ds^2 = -f^2(r)dt^2 + \frac{dr^2}{f^2(r)} + r^2(Ndt + d\phi)^2$$

$$f^2 = -M + r^2 + \frac{J^2}{4r^2}$$

$$N = -\frac{J}{2r^2}$$

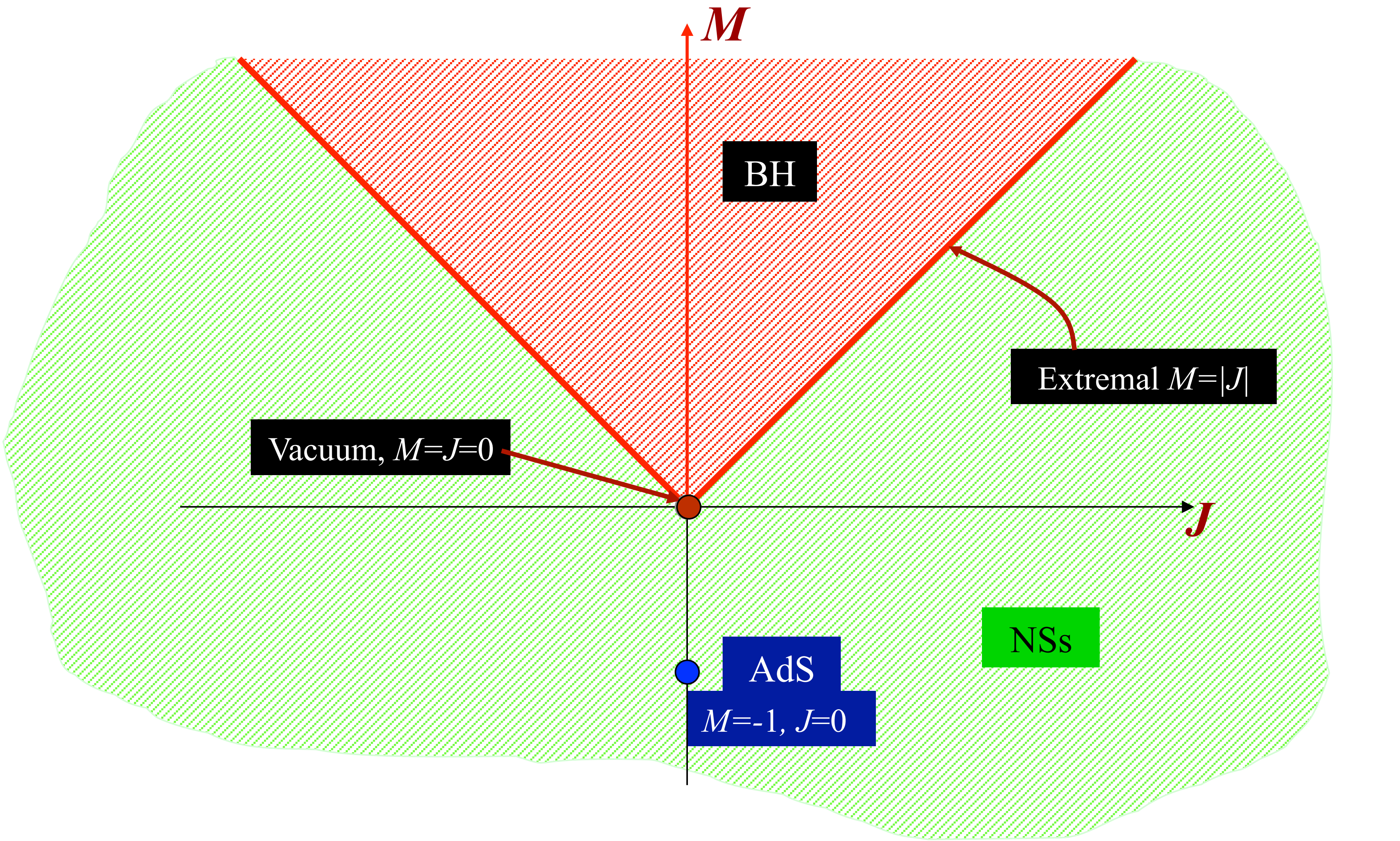
$$r_\pm^2 = \frac{M}{2}\left(1 \pm \sqrt{1 - \frac{J^2}{M^2}}\right) \in \boxed{\mathrm{R}^+} \Leftrightarrow M \geq |J|$$

$M \geq |J| > 0$ → BH; horizons $r_\pm \geq 0$

$M = -1$, $J = 0$ → AdS

$M < |J|, \neq -1$ → Naked singularities

Spectrum of rotating 2+1 *BHs*



How is a 2+1 BH made?

M. Bañados, C.Teitelboim, M.Henneaux, J.Z. (1993)

- The 2+1 BHs are obtained by identifications in AdS_{2+1}, defined by the pseudoesphere

$$-(x^0)^2-(x^1)^2+(x^2)^2+(x^3)^2=-1$$

which has 6 Killing vectors:

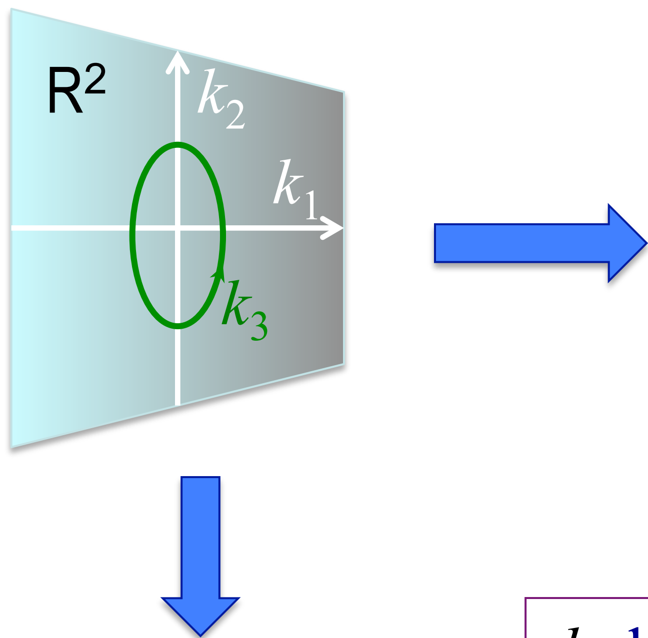
$$k=\frac{1}{2}k^{ab}(x_a\partial_b-x_b\partial_a)=\frac{1}{2}k^{ab}J_{ab}\in so(2,2)$$

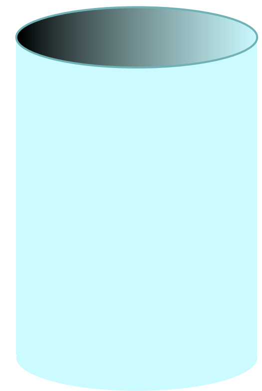
Thus, the BH geometry is *locally* AdS (not *globally*)

- Identifying (quotienting) by up to two commuting Killing vectors does not change the local geometry: *the* 2+1 *BHs are locally isometric to* AdS_{2+1}
- Not all Killing vectors yield BHs.

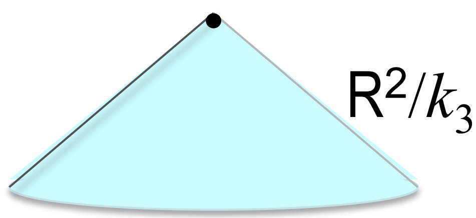
Identifications by Killing vectors respect the *local geometry*:

k_1, k_2, k_3: isometries





k_1 leaves no *fixed* points
➡ no singularities



r=0 fixed point of k_3
➡ conical singularity

Boosting black holes

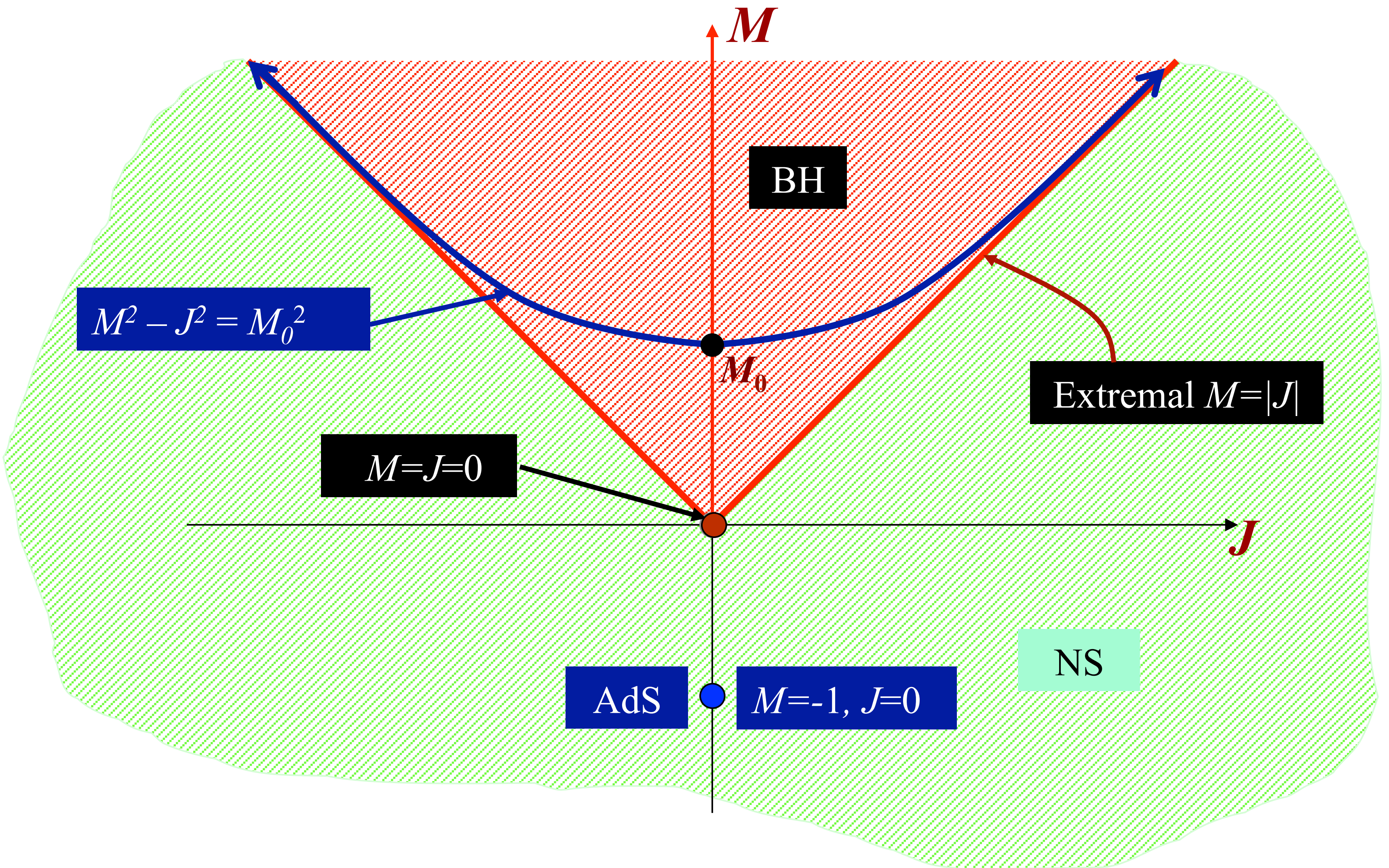
The freedom to make Lorentz transformations in AdS_{2+1} can be exploited to boost the identifying Killing vectors. The resulting black holes have different M and J .

$$\frac{AdS_{2+1}}{k} \longrightarrow \frac{AdS_{2+1}}{k'}, \qquad k' = \Lambda k$$

In particular, a static BH ($M_0 \neq 0$, $J_0 = 0$) can be turned into a spinning one ($M \neq 0$, $J \neq 0$) by a "Lorentz" boost:

$$M = \frac{1+\Omega^2}{1-\Omega^2} M_0, \quad J = \frac{2\Omega}{1-\Omega^2} M_0, \quad \boxed{M^2 - J^2 = M_0^2}$$

Spinning 2+1 black hole states

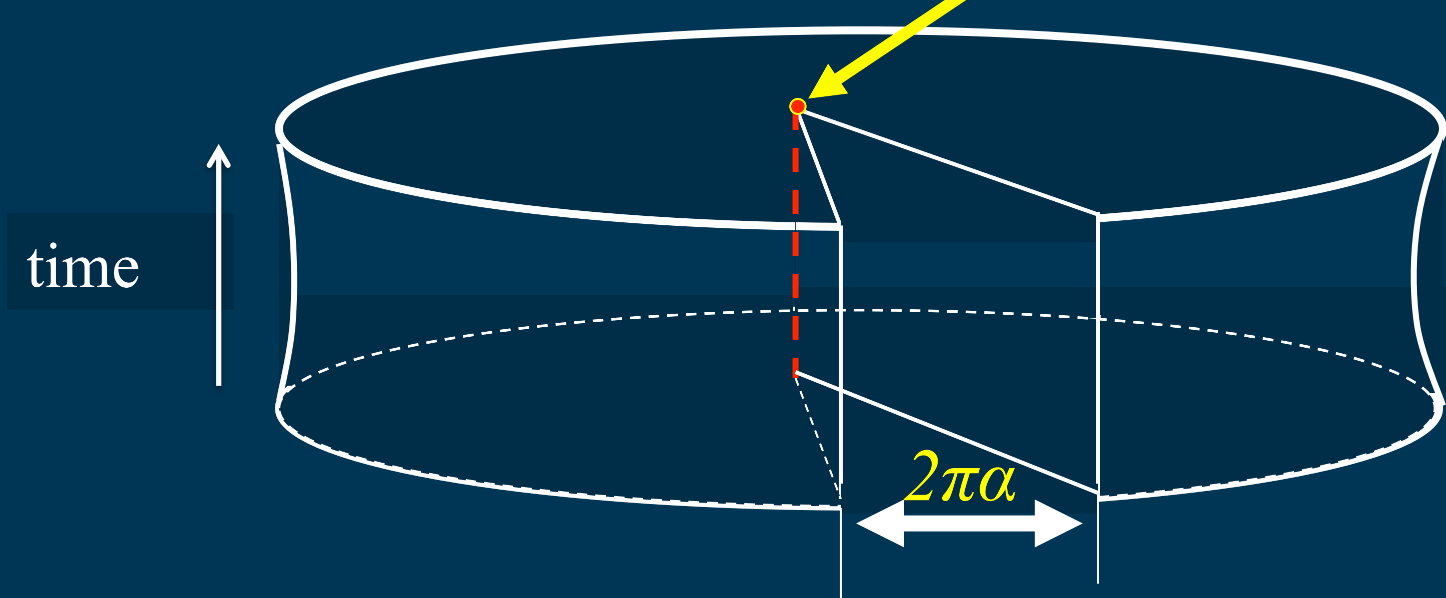


Conical singularities

O.Mišković, J.Z. (2009)

Angular defect in 2+1 D

1D Defect



Identification in the x^1-x^2 plane generates a conical singularity in the set of fixed points of the Killing vector:

$$k = -2\pi\alpha\,(x_2\partial_3 - x_3\,\partial_2)$$
$$= -2\pi\alpha\,\partial_\varphi$$

Angular defect ⇔ δ-like curvature singularity

A conical defect/excess in 2+1 dim. is a localized, static/stationary, spherically symmetric geometry.

A conical singularity is indistinguishable from a black hole at large distance (like planets and black holes).

Unlike black holes, a conical singularity is not surrounded by an event horizon → Naked Singularity orbifold AdS_3/k)

The conical geometry looks like a BH:

$$ds^2 = -\left(r^2 - M\right)d\boldsymbol{\tau}^2 + \left(r^2 - M\right)^{-1} dr^2 + r^2 d\boldsymbol{\phi}^2$$

where the "mass" is negative, $M = -(1-\boldsymbol{\alpha})^2$, and related to the deficit angle, $\Delta\varphi = 2\pi\alpha = 2\pi\left[1-\sqrt{-M}\right]$.

The exceptional cases are:

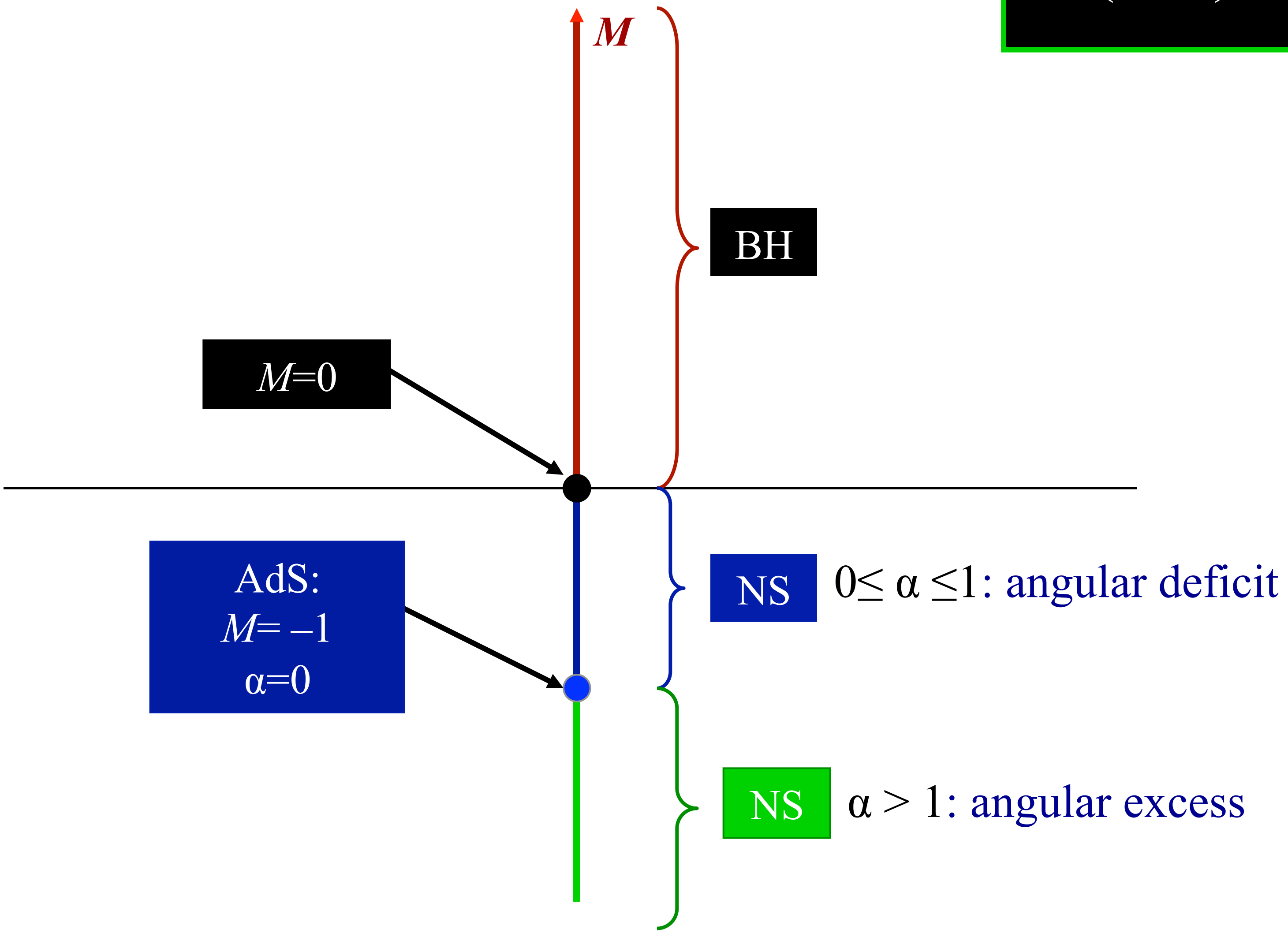
$\alpha = 0, M = -1$ → anti-de Sitter; no deficit

$\alpha = 1, \ M = 0$ → zero mass; maximum deficit (2π)

For $-1 < M < 0$ these are ***naked*** singularities that behave as point particles; quite harmelss otherwise.

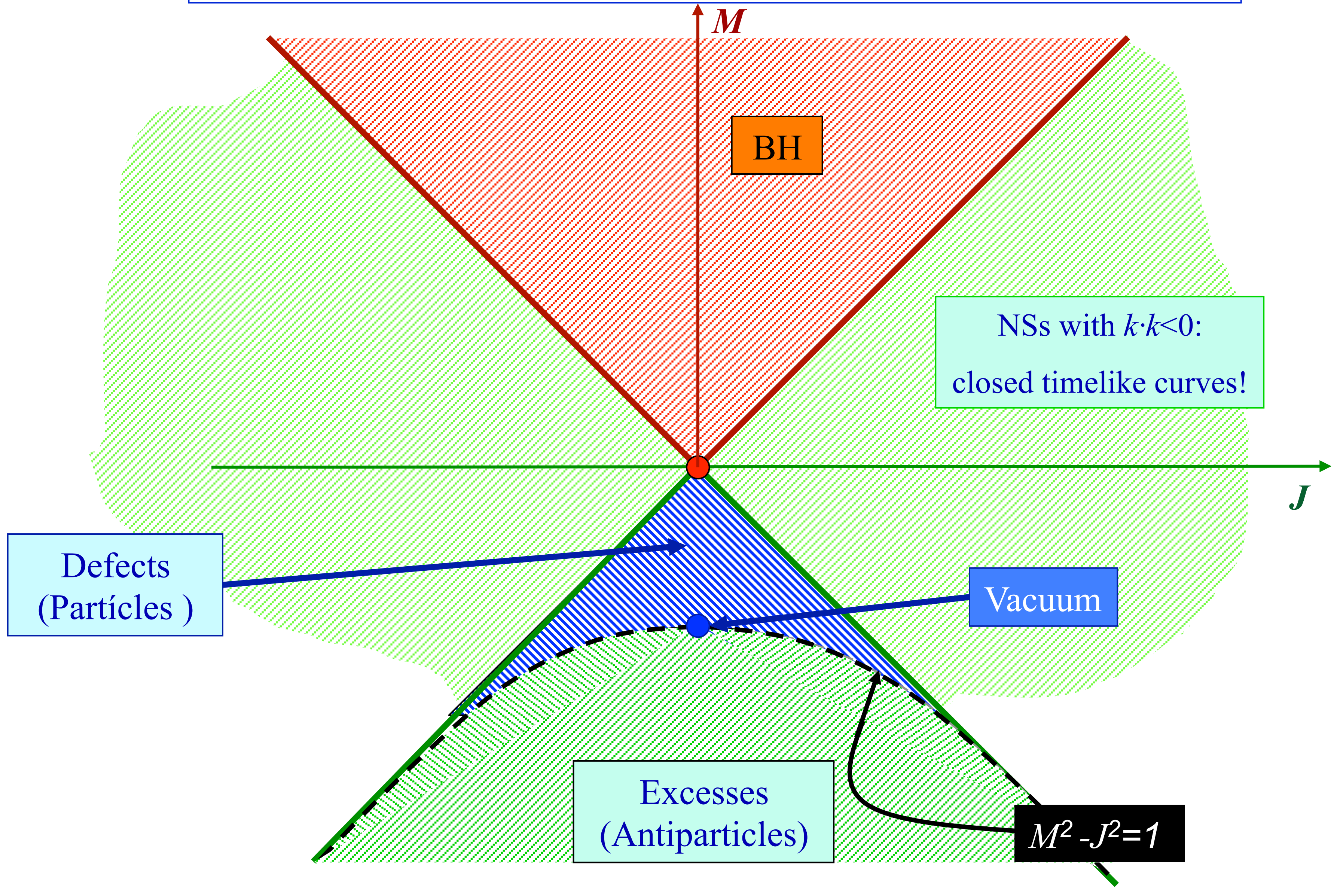
2+1 *BH-NS spectrum*

(J=0)



Like BHs, conical singularities can also acquire angular momentum…

2+1 *BHs and NSs: extended spectrum*



Black hole identifications

AdS_{2+1}: $-(x^0)^2-(x^1)^2+(x^2)^2+(x^3)^2=-1$

- Generic BH: $r_+ > r_- > 0$, $M > |J| \neq 0$

$$\xi_{+-}=\alpha_+ J_{12}-\alpha_- J_{03} \qquad \alpha_\pm = 2r_\pm$$

- Extremal BH: $r_+ = r_- > 0$, $M = J \neq 0$

$$\xi_{Ext}=\alpha_+(J_{01}-J_{23})+\tfrac{1}{2}(J_{12}+J_{03}+J_{02}-J_{13})$$

- Zero mass BH: $r_+ = r_- = 0$, $M = J = 0$

$$\xi_0=\tfrac{1}{2}(J_{12}+J_{03}+J_{02}-J_{13})$$

These are all non-compact elements of SO(2,2)

- $\xi \cdot \xi > 0$ → *No closed timelike curves*
- No fixed points → *No conical singularities*

Conical identifications

AdS_{2+1}: $-(x^0)^2 - (x^1)^2 + (x^2)^2 + (x^3)^2 = -1$

- Generic spinning cone, $M<-|J|\neq 0$

$$\xi_{+-} = \beta_+ J_{01} + \beta_- J_{23} \qquad \beta_\pm = \sqrt{-M+J} \pm \sqrt{-M-J}$$

- Extremal cone, $M=-|J|\neq 0$

$$\xi_{Ext} = \beta_+ (J_{01} - J_{23}) + \tfrac{1}{2}(J_{12} + J_{01} + J_{23} - J_{13})$$

- Zero mass cone, $M=J=0$.

$$\xi_0 = \tfrac{1}{2}(J_{12} + J_{20} + J_{03} + J_{31})$$

These are all compact elements of SO(2,2)

- $\xi\cdot\xi>0$ ⟹ *No closed timelike curves*
- $r = 0$ fixed point ⟹ *Conical singularity*

Quantum effects

M.Casals, A.Fabbri, C.Martínez, J.Z. (2016, 2017)

Quantization

In 2+1 dimensions GR has no local degrees of freedom

- No gravity waves
- No gravitons → no gravitational quantum corrections.

Hence, the only quantum effects may be due to matter.

Strategy:

- ➔ Consider a conformally(*) coupled scalar field $\hat{\phi}$, with transparent boundary conditions.
- ➔ Compute the renormalized stress-energy tensor $\left\langle T^{\mu}{}_{\nu} \right\rangle$ for the quantum fluctuations around $g_{\mu\nu} = \bar{g}_{\mu\nu},\ \hat{\phi} = 0$.
- ➔ Compute the modified geometry (back-reaction).

(*) Enormous simplification. Exact solutions; no tail; analytic results

Renormalized Stress-Energy Tensor (RSET):

Since the BH and the conical geometries are obtained by identifications in the AdS covering space, the stress-energy tensor can be obtained from the one in the embedding spacetime by the method of images.

In the covering space the RSET for a conformally coupled scalar is

$$\left\langle \hat{T}_{\mu\nu}(x) \right\rangle_{ren} = \lim_{x'\to x} \frac{\hbar}{4}\left[3\nabla_\mu^x \nabla_\nu^{x'} - g_{\mu\nu} g^{\alpha\beta} \nabla_\alpha^x \nabla_\beta^{x'} - \nabla_\mu^x \nabla_\nu^x - \frac{1}{4l^2} g_{\mu\nu}\right]\bar{G}(x',x)$$

where $G(x',x)$ is the two-point function,

$$\bar{G}(x',x) = (4\pi\,|x-x'|)^{-1}$$

and $|x-x'| = \sqrt{(x-x')^a (x-x')_a}$ is the geodesic distance measured in the embedding space.

Method of images:

The two-point function in the BH/cone can be obtained by applying the identification operator to x':

$$G(x',x) = \sum_{n\in Z} \bar{G}(x, H^n(\xi)x'|),$$

where $H(\xi)$ is the matrix corresponding to the identification vector ξ .

For the generic (spinning) BH,

$$H^{BH}(\xi) = \begin{bmatrix} \cosh(\pi\alpha_+) & \sinh(\pi\alpha_+) & 0 & 0 \\ \sinh(\pi\alpha_+) & \cosh(\pi\alpha_+) & 0 & 0 \\ 0 & 0 & \cosh(\pi\alpha_-) & \sinh(\pi\alpha_-) \\ 0 & 0 & \sinh(\pi\alpha_-) & \cosh(\pi\alpha_-) \end{bmatrix}$$

where $\alpha_\pm = \sqrt{M+J} \pm \sqrt{M-J}$.

Method of images:

Similarly, for the generic (spinning) cone,

$$H^{Cone}(\xi) = \begin{bmatrix} \cos(\pi\beta_-) & 0 & 0 & -\sin(\pi\beta_-) \\ 0 & \cos(\pi\beta_+) & -\sin(\pi\beta_+) & 0 \\ 0 & \sin(\pi\beta_+) & \cos(\pi\beta_+) & 0 \\ \sin(\pi\beta_-) & 0 & 0 & \cos(\pi\beta_-) \end{bmatrix}$$

where $\beta_{\pm} = \sqrt{-M+J} \pm \sqrt{-M-J}$.

With these matrices, can compute H^n and finally $\langle T^{\mu}{}_{\nu} \rangle$.

N.B.: *The conical geometry is obtained from the BH by analytic continuation* $M \rightarrow -M$

A (very long!) direct and calculation yields, for a massless scalar field on a static BH (J=0)

$$\kappa \left\langle \hat{T}^{\mu}{}_{\nu} \right\rangle^{BH} = \frac{l_P M^{3/2}}{2\sqrt{2} r^3} \sum_{n=1}^{\infty} \frac{\cosh(2n\pi\sqrt{M})+3}{[\cosh(2n\pi\sqrt{M})-1]^{3/2}} \, diag(1,1,-2) \ ,$$

and on the static conical singularity,

$$\kappa \left\langle \hat{T}^{\mu}{}_{\nu} \right\rangle^{NS} = \frac{l_P (-M)^{3/2}}{2\sqrt{2} r^3} \sum_{n=1}^{N_0} \frac{\cos(2n\pi\sqrt{-M})+3}{[\cos(2n\pi\sqrt{-M})-1]^{3/2}} \, diag(1,1,-2) \ ,$$

which corresponds to the analytic continuation $M \longrightarrow -M$.

In both cases, $\left\langle T^{\mu}{}_{\nu} \right\rangle = \frac{F(M)}{r^3}$.

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($l_P=\hbar G$)

and on the static conical singularity,

why?

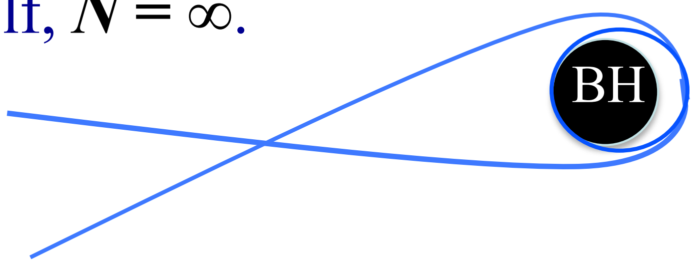
$$\kappa\left\langle \hat{T}^{\mu}_{\ \nu}\right\rangle^{NS} = \frac{l_P (-M)^{3/2}}{2\sqrt{2}r^3}\sum_{n=1}^{N_0}\frac{\cos(2n\pi\sqrt{-M})+3}{[\cos(2n\pi\sqrt{-M})-1]^{3/2}}\, diag(1,1,-2)\ ,$$

which corresponds to the analytic continuation $M \longrightarrow -M$.

In both cases, $\left\langle T^{\mu}_{\ \nu}\right\rangle = \frac{F(M)}{r^3}$.

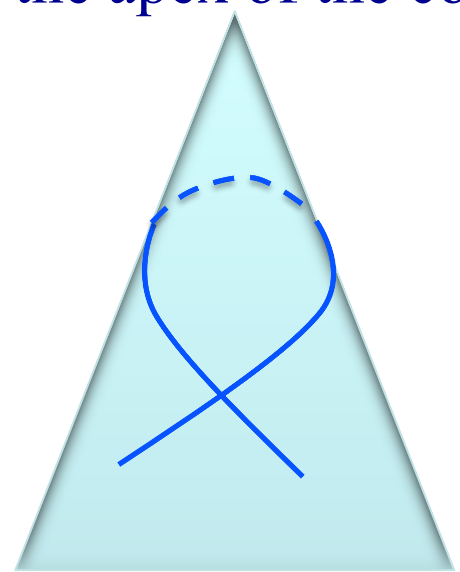
The summation $\sum_{n=1}$ results from the fact that these geometries are multiply connected.

- For the BH there are infinitely many null geodesics connecting a point to itself, $N = \infty$.



- In a conical geometry, the number or self-intersecting null paths is finite and depends on the angular deficit at the apex of the cone:

$$N_0 = [1 - \Delta/(2\pi)]^{-1} = [-M]^{-1/2}$$



These contributions to the stress-energy modify Einstein's equations, and change the geometry. A direct calculation in both cases gives

$$ds^2 = -\left(r^2 - M - \frac{F(M)}{r}\right)dt^2 + \left(r^2 - M - \frac{F(M)}{r}\right)^{-1} dr^2 + r^2 d\theta^2$$

where $F(M) \sim O(\hbar) > 0$.

Effect of quantum corrections

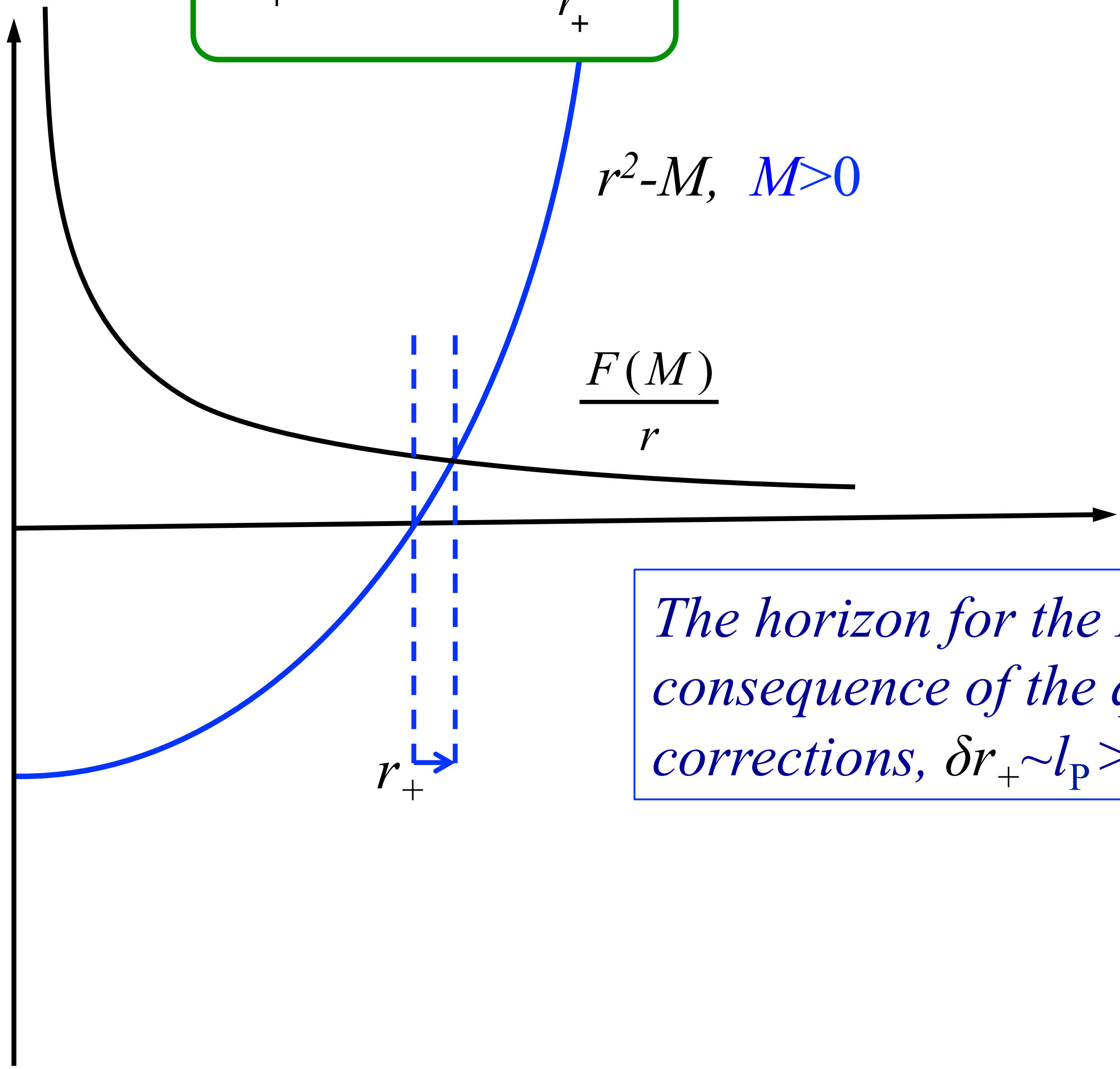
The modified geometries have a horizon both for $M > 0$ and $M < 0$, since

$$r^2 - M - \frac{F(M)}{r} = 0$$

always has real solutions for $F(M) > 0$.

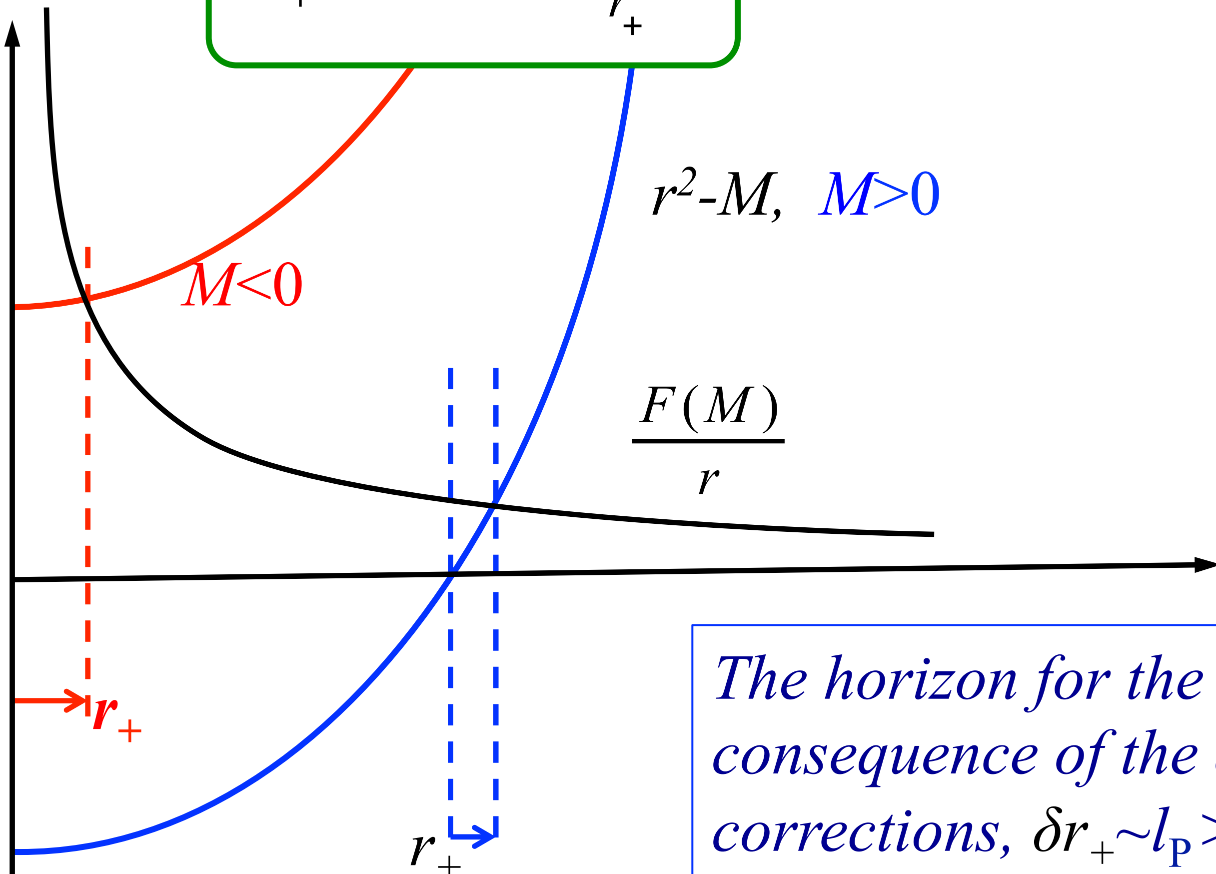
No matter how large the conical defect is, the quantum corrections of the vacuum end up dressing the naked singularity.

$$r_+^2 - M = \frac{F(M)}{r_+}$$



The horizon for the BH grows as a consequence of the quantum corrections, $\delta r_+ \sim l_P > 0$.

$$r_+^2 - M = \frac{F(M)}{r_+}$$



The horizon for the BH grows as a consequence of the quantum corrections, $\delta r_+ \sim l_P > 0$.

The quantum corrections generate a horizon for the conical singularity, dressing up its nakedness, $\boldsymbol{r}^{NS}_+ > 0$.
The naked singularity becomes a black hole.

Caveat:

A static BH is an extremely exeptional case: it would require infinitely fine-tuned initial conditions to produce one by collapsing matter.

Similarly, a NS corresponding to a real particle is likely to have nonvanishing spin.

Will our results survive if the BH or NS were not exactly static?

(*Our conclusions may be accidentally due to the exceptional fine-tuned static case. The horizon could go away if the BH / NS have nonzero angular momentum...*)

Are our conclusions still valid for $J \neq 0$?

This is a difficult question. There are several different cases to be considered, depending on the regions connected by the geodesics in a rotating BH:

$$0 < r < r_- \,, \qquad r_- < r < r_+ \,, \qquad r_+ < r < \infty$$

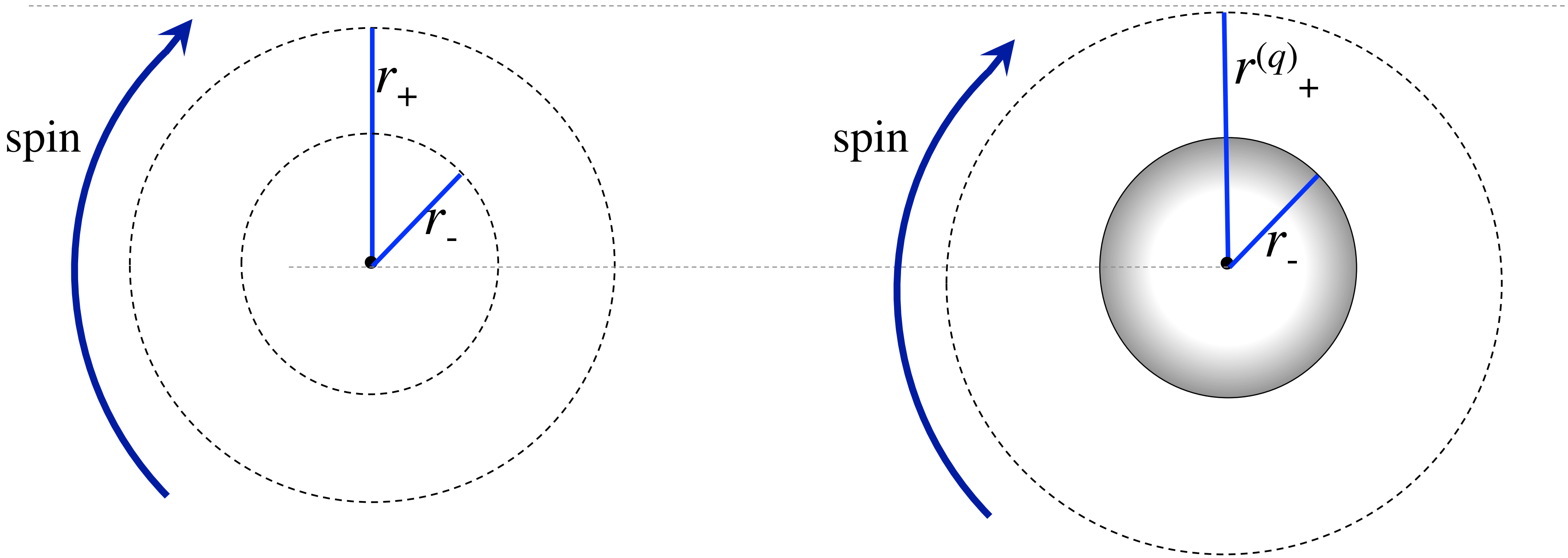
For a spinning conical singularity life is even harder.

Problem of resonance: If $\beta_+ = (rational) \times \beta_-$, then $(H^{Cone})^n$ becomes proportional to the identity and $\langle T^{\mu}{}_{\nu} \rangle$ blows up!

Luckily, this happens for angular momentum above a finite threshold, $J > J^*$

(See arXiv:1608.05366 [PRL (2017)] and forthcoming paper.)

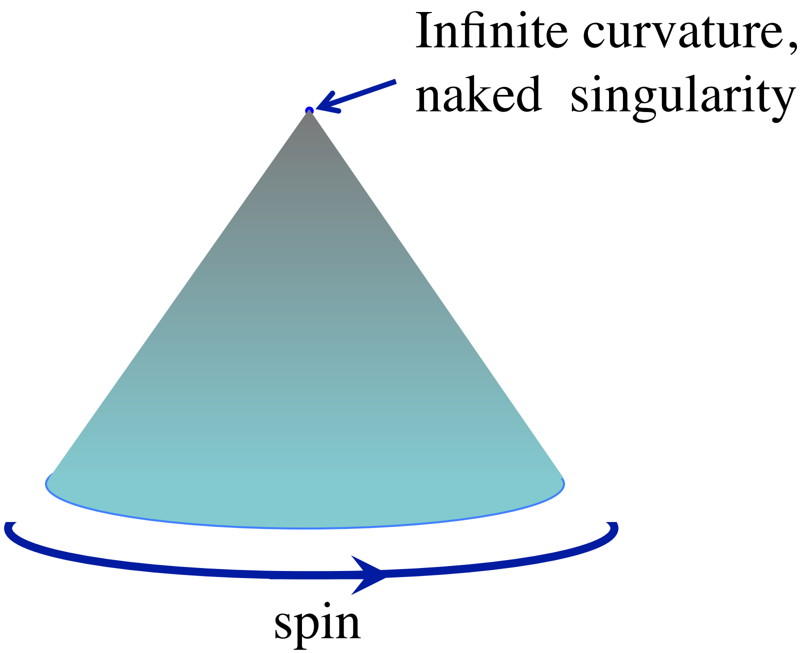
Backreacted BH geometry ($J \neq 0$)

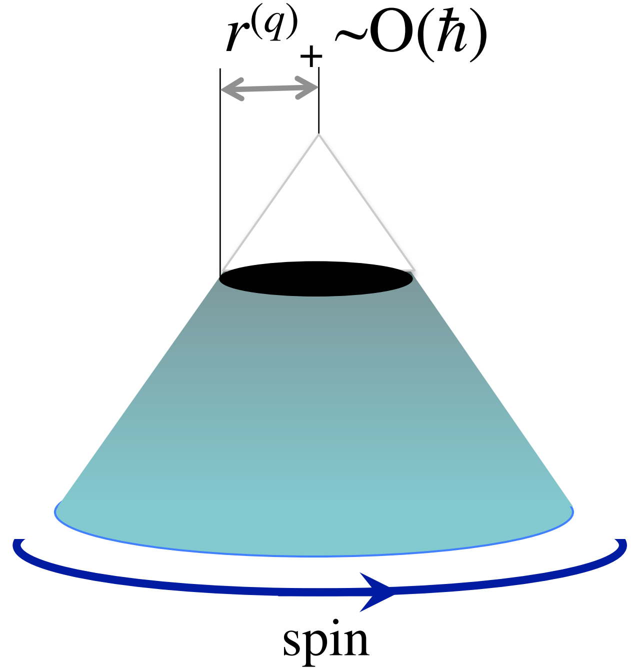


Classical BTZ black hole: The two horizons at radii r_+ and r_- respectively.

Quantum-corrected black hole: The outer horizon is slightly larger than its classical counterpart, $r^{(q)}_+ = r_+ + O(\hbar)$. A hard surface forms at the inner horizon r_- .

Backreacted NS geometry ($J \neq 0$)





Classical naked singularity: No horizon surrounds the conical singularity.

Quantum-corrected singularity: A horizon forms around the conical singularity so that for an external observer it looks like a black hole.

Summary

- Black holes ($M \geq |J|$) and conical geometries ($M \leq -|J|$) are complementary parts of the $3D$ BH spectrum

- Including a quantum scalar field makes the BH horizon grow, $r^q{}_+ > r^c{}_+$.

- The quantum corrections produce a horizon around the otherwise naked singularity.

- These effects hold for generic *spinning* BHs and NSs.

- Cosmic censorship is a result of quantum mechanics.

Quantum effects not only prevent the collapse of the electron into the atomic nucleus, they prevent the formation of naked singularities. They could also provide mechanisms to avoid other singularities, like the BB.

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It seems that Einstein was right after all: Nature does abhor singularities.

This is not, however, a feature of the classical theory, but the result of a quantum effect.

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Feliz cumpleaños, Marcelo!