Cosmology, Magnetic Seed and QHE

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How a physical system should be written?

a) A physical system requires having good variables

b) should be, perhaps, mathematically well defined

However, could happen that the equations are difficult to solve

Finding the midpoint is part of the art of theoretical physics!

Exactly in this context we would like to study

Primordial (Seed) Magnetic Field

What means that?

Basically,

what is the origin of magnetic fields in the universe?

No answer presently known

Standard Solution (cosmology, GUT, …)

1. **First a magnetic seed**
2. **Second a dynamo mechanism (Bierman Battery, e.g.)**

Battery mechanism is accepted (plasma +electrodynamics)

Why are important magnetic fields

1) Magnetic field are a major agent in the interstellar of spiral, dwarf galaxies …

2) Are essential for the onset of star formation

3) (Galactic) magnetic fields are observed in the optical range (e.g using Zeeman effect)

Magnetic field in M51 contours total emission 6cm

Zeemen effect observed ——— **The origin of B is a mistery**

Ideas: Magnetogenesis (Grasso and Rubinstein, Kandus et al …. good reviews)
Very active field (but not easy) by several reasons

a) **Cosmologically B is an extra parameter**
b) **particle physics (neutrinos) beyond standard model**
c) **New Physics in the context of GUTs (astroparticles…)**

Assumptions in cosmology

COSMOLOGICAL PRINCIPLE (NEWTON, LAPLACE, ...)

UNIVERSE IS HOMOGENEOUS AND ISOTROPIC (Einstein 1917)

Mathematically CP means

$$ds^2 = -dt^2 N^2(t) + a^2(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

and

$$S = \int d^4x \sqrt{g}(R + \Lambda)$$

spatial derivative disappears! **and the equations of motion (FLRW)**

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \Lambda$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\Lambda$$

ANOTHER EQUIVALENT VIEWPOINT

$$L = \frac{1}{2N} a \; \dot{a}^2 + \frac{1}{6} N \; \Lambda \; a^3$$

which gives for a(t)

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \Lambda$$

and for N

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\Lambda$$

Same idea + Trick

Let's assume

$$x = \frac{2}{3} a^{3/2}$$

Then the Lagrangean is

$$L = \frac{1}{2N}\dot{x}^2 - \frac{\omega^2}{2}x^2$$

and the eq. motion are

$$\ddot{x} + \omega^2 x = 0$$
$$\dot{x}^2 + \omega^2 x^2 = 0$$

with

$$\omega^2 = -\frac{3}{4}\Lambda$$

and solutions

$$x(t) \to e^{\pm i\omega t}$$

which are dS or AdS ones

- FRW equations can be written "harmonic oscillator" + constraint

- Each $x(t)$ denotes "the universe" → Null strings Strong coupling gravity

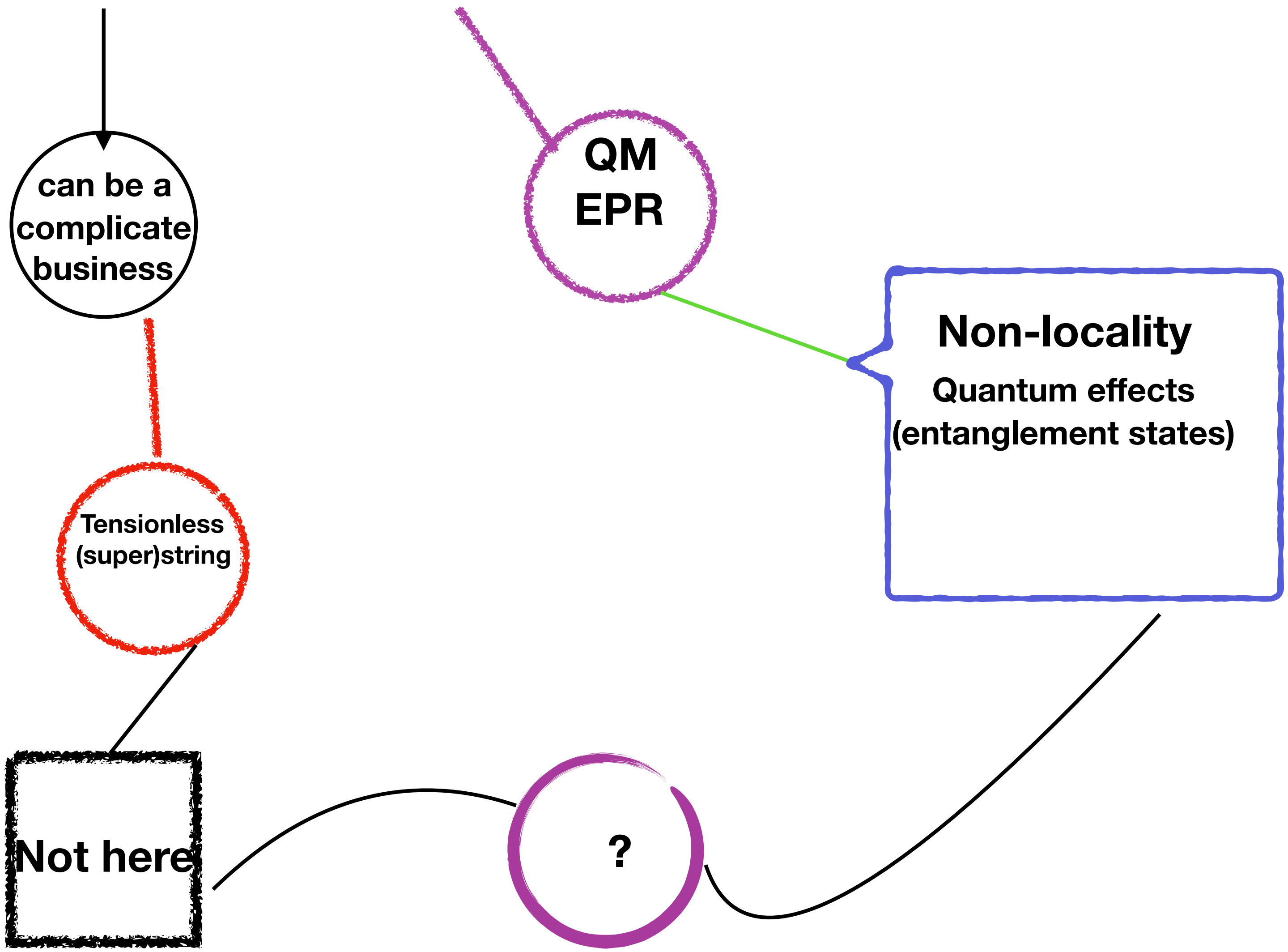
- The allowed solutions are dS or AdS.

BASIC QUESTIONS

- How do the points interact?
- Which symmetr(ies)y should be respected?

At the end of the day we are interested in QM

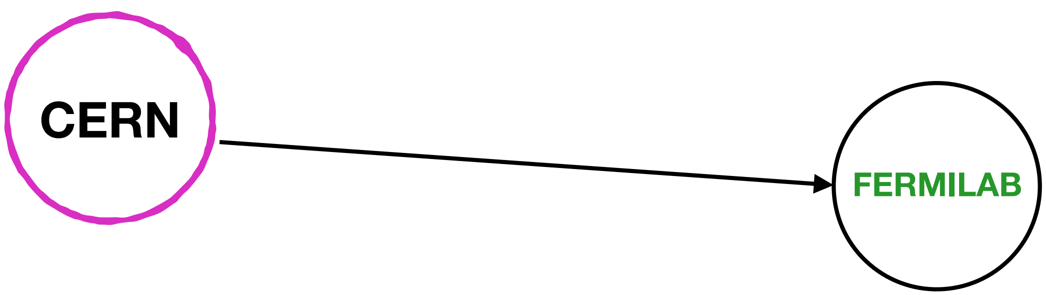
Diffemorphisms and interactions



QFT DELIKATESSEN; one of the axioms

Cluster decomposition axiom

$$[\mathcal{H}(\varphi(x,t)), \mathcal{H}(\varphi(y,t))] = 0 \qquad (x-y)^2 > 0$$



Physics should be independent!

- **Cluster decomposition is consistent with CP**

BUT CAN NOT BE CONSISTENT WITH STRUCTURE FORMATION

AT THE END THE CP IS A (VERY GOOD) APPROXIMATION,

- **But non-local effects are (very) consistent with QM (Bell theorem, 1964)**
- **and does not violate relativity principles (Einstein, Rosen and Podolsky, 1935)**

Cosmological Variations: Divertimento

Let me invent the following model

$$L = \frac{1}{2N}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{N}{2}\left(\omega_1^2 x_1^2 + \omega_2^2 x_2^2\right) - \frac{\theta}{2}\left(x_1 \dot{x}_2 - \dot{x}_1 x_2\right)$$

and the Hamiltonian is

$$H = \frac{N}{2}\,[p_1^2 + p_2^2 + (\omega_1^2 + \frac{\theta^2}{4})\,x_1^2 + (\omega_2^2 + \frac{\theta^2}{4})\,x_2^2 + \theta\,(x_1 p_2 - x_2 p_1)],$$

with the Poisson Brackets

$$[x_i, x_j] = 0, \qquad [x_i, p_j] = \delta_{ij}$$

and

$$[\mathbf{p_i}, \mathbf{p_j}] = \epsilon_{\mathbf{ij}}\theta$$

AT THIS LEVEL

$\theta \longrightarrow$ **Is not a magnetic field**

1. Cosmological constants are not essential (at this every)

2. Cluster decomposition principle is applicable to 1-“cluster”

3. θ plug the bubbles

The equations of motion are

$$\ddot{x}_1 + \omega_1^2\ x_1 + \theta\ \dot{x}_2 = 0,$$
$$\ddot{x}_2 + \omega_2^2 x_2 - \theta \dot{x}_1 = 0,$$

and the constraint

$$\dot{\mathbf{x}}_1^2 + \dot{\mathbf{x}}_2^2 + \omega_1^2 \mathbf{x}_1^2 + \omega_2^2 \mathbf{x}_2^2 = \mathbf{0}$$

which is identically satisfied

$$\frac{d}{dt}\left[\dot{x}_1^2 + \dot{x}_2^2 + \omega_1^2 x_1^2 + \omega_2^2 x_2^2\right] = 0$$

constant chosen =0!!!!!

THE CHANGE OF VARIABLE

$$x_1 = a^{3/2}(t), \qquad x_2 = b^{3/2}(t)$$

We get the modified FLRW equations

$$2\,\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -\frac{4}{3}\omega_1^2 - 2\,\theta\sqrt{a\,b}\;\frac{\dot{b}}{a^2},$$

$$2\,\frac{\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 = -\frac{4}{3}\omega_2^2 + 2\,\theta\sqrt{a\,b}\,\frac{\dot{a}}{b^2}$$

DE

plus the constraint

$$\left(\frac{\dot{a}}{a}\right)^2 = -\left(\frac{2}{3}\omega_1\right)^2 - \frac{1}{a^3}\left(\left(\frac{2}{3}\omega_2\right)^2 b^3 + \dot{b}^2 b\right)$$

Frequencies

$$-\omega_1^2 \to \frac{3}{4}\Lambda_1, \quad -\omega_2^2 \to \frac{3}{4}\Lambda_2.$$

From FLRW equations we find

$$-8\pi G\, p = 2\,\theta\sqrt{a\,b}\;\frac{\dot{b}}{a^2},$$

$$\frac{8\pi G}{3}\rho = -\frac{1}{a^3}\left(\frac{4}{9}\,\omega_2^2\,b^3 + \dot{b}^2 b\right)$$

Therefore

$$\rho + \frac{\mathbf{6\pi G}}{\theta^{\mathbf{2}}}\mathbf{p^2} = \frac{\mathbf{3\,\Lambda_2}}{\mathbf{8\pi G}}\left(\frac{\mathbf{b}}{\mathbf{a}}\right)^{\mathbf{3}}.$$

and as a consequence the Chapligyn state equation is FOUND (Dark energy)

FIRST CONCLUSIONS

A) We reproduce FLRW equations (two-metric)
B) There is inflation without inflaton (flatness and … Gondolo et al PRD 2017)
C) In terms of x-variables one see

$$[\mathbf{p_1}, \mathbf{p_2}] = \theta$$

Therefore θ is a constant magnetic field

in principle tiny!

PHYSICAL SCALES

$$H = \frac{N}{2} \left[p_1^2 + p_2^2 + \left(\omega_1^2 + \frac{\theta^2}{4} \right) x_1^2 + \left(\omega_2^2 + \frac{\theta^2}{4} \right) x_2^2 + \theta \left(x_1 p_2 - x_2 p_1 \right) \right],$$

IDENTIFY

a) $\sqrt{|\Lambda|} \gg \sqrt{|\theta|}$ **(era-** $\sqrt{|\Lambda|}$ **)**

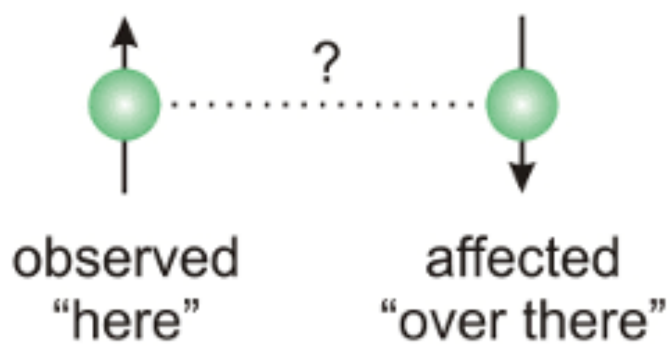
b) $\sqrt{|\Lambda|} \ll \sqrt{|\theta|}$ **(magnetic-era)**

c) $\sqrt{|\theta|} \gg H$ **(present-era)**

Going to Quantum Mechanics

Why?, because we are around inflation era

Between regions there is quantum entanglement



The entanglement is responsible for the magnetic seed

In the magnetic era we take

$$\omega_{1,2}^2 \ll |\theta|^2$$

and therefore the LANDAU Hamiltonian

$$H = \frac{\theta^2}{4}\left(\bar{p}_1^2 + \bar{p}_2^2\right) + \left(\bar{x}_1^2 + \bar{x}_2^2\right) + \theta\left(x_1 p_2 - x_2 p_1\right)$$

$$= \aleph$$

$\aleph$ "primal" as in J. L. Borges!. **(M. Loewe et al)**

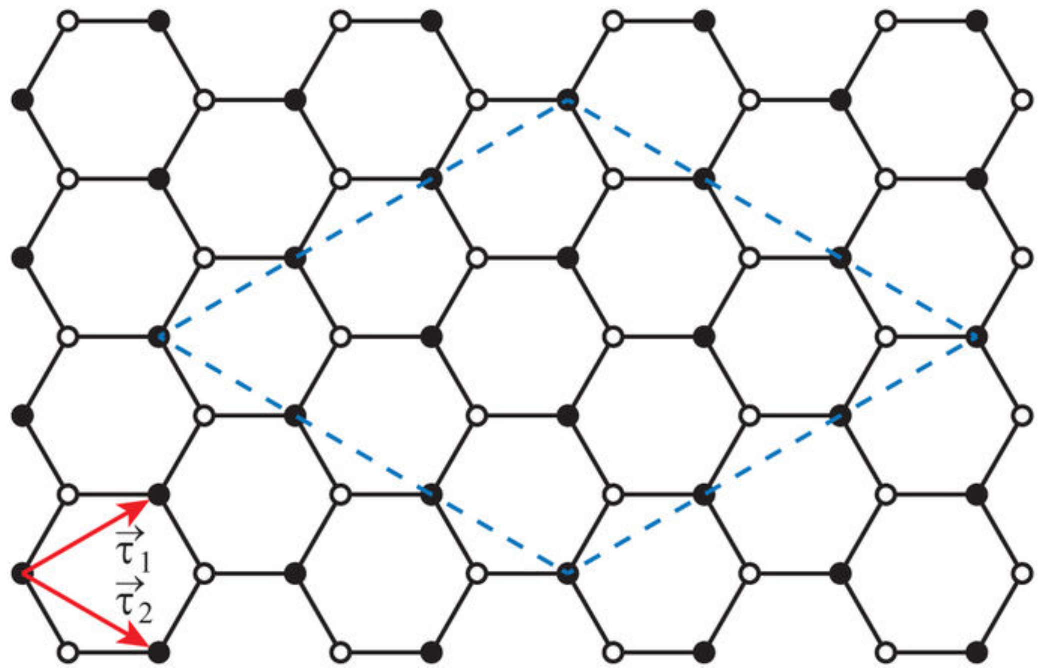
$\aleph$ has the spectrum

$$E_{n_-,n_+} = \theta\left(2n_- + 1\right)$$

and ground state is LLL

$$\psi_(x_1, x_2) = e^{-\frac{1}{2}\theta|x_1 - x_2|^2}$$

more sites (honeycomb)



Conclusions

1) The magnetic seed is a quantum effect

2) Effects of gravitational radiation, in principle, are calculable (Synchrotron radiation)

3) The gravitational radiation effects are dipolar

The model is exacly soluble!

in Daniel’s words

יהי אור ויהי אור

y dijo Yahvé, “Sea la luz: y fué la luz”