Effective field theory with higher-order Lorentz violation

Marat Reyes

Grupo de Física de Altas Energías-UBB, Chillán.

NON PERTURBATIVE ASPECT OF QFT AND
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Outline

1. Higher-order Lorentz-invariance violation
2. Perturbative unitarity
3. Renormalization
4. Concluding Remarks

Spacetime foam

There is consensus that at the Planck scale spacetime should depart from its continuum structure. This vision is shared by a number of quantum gravity versions.

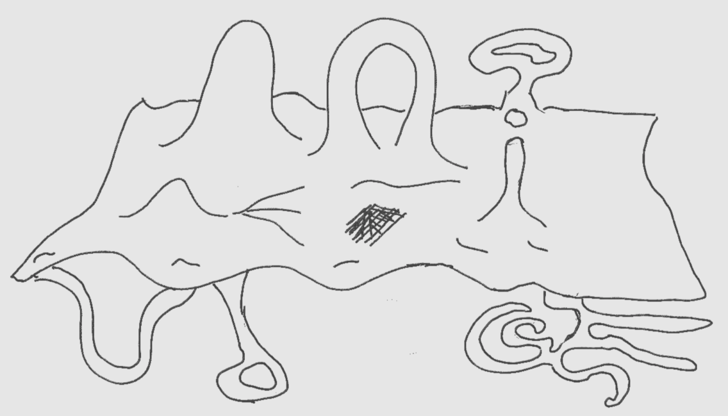

Figure 1: Spacetime at Planck scales.

Effective approach

The difficulty to reach the scale $m_P \approx 10^{19}$ GeV has stimulated the search of new physics in the form of small departures to the standard model. In particular, suppressed effects of **Lorentz invariance violation** described within the framework of **effective field theory**.

- Standard Model Extension (SME), mass dimension $d \leq 4$ [D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998); S. R. Coleman and S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).]
- Non Minimal framework and higher-order models, $d > 4$, [R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90**, 211601 (2003); V. A. Kostelecky and M. Mewes, Phys. Rev. D **80**, 015020 (2009); Phys. Rev. D **85**, 096005 (2012); M. Schreck, Phys. Rev. D **93**, no. 10, 105017 (2016).]

Alternatively, studies of CPT and Lorentz-invariance violation have been given in

- Modified dispersion relations [G. Amelino-Camelia, J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and S. Sarkar, Nature **393**, 763 (1998).]
- String/M theory [V. A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); V. A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991).]
- Loop quantum gravity [R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999); J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000); H. Sahlmann and T. Thiemann, Class. Quant. Grav. **23**, 867 (2006).]
- Horava gravity [P. Horava, Phys. Rev. D **79**, 084008 (2009).]

Beyond the Standard Model

In the effective framework one includes modified terms in the Lagrangian, which can be classified according to its mass dimension d

$$S = \int d^4x \left(\mathcal{L}^{SM} + \delta\mathcal{L}^{(d=3,4)} + \delta\mathcal{L}^{(d=5)} + \dots \right) . \tag{1}$$

In particular, the Myers and Pospelov model [R. C. Myers and M. Pospelov Phys. Rev. Lett. **90**, 211601 (2003)],

$$\delta\mathcal{L}^{(5)}_{fermion} = \frac{1}{m_P}\bar{\psi}(\eta_1 \not{n} + \eta_2 \not{n}\gamma_5)(n\cdot\partial)^2\psi\,, \tag{2}$$

and the standard model extension [V. A. Kostelecky and N. Russell, Rev. Mod. Phys. **83**, 11 (2011),].

In general the Lorentz symmetry breakdown is implemented with a preferred four vector n, which is believed to arise from expectation values in an underlying theory.

What to expect in the presence of Lorentz-invariance violation with higher-order operators?

1) We may have more degrees of freedom (some become complex for certain range of energies).

$$\mathcal{L}(q,\dot{q},\ddot{q}) \to \quad \pi_{\dot{q}} = \frac{\partial L}{\partial \ddot{q}} \qquad p_q = \frac{\partial L}{\partial \dot{q}} - \frac{\partial \pi_{\dot{q}}}{\partial t} \tag{3}$$

2) Improved convergence properties.

$$\Delta = \frac{1}{p^2 - m^2 - p^4/M^2} = \frac{1}{p^2 - m_1^2} - \frac{1}{p^2 - m_2^2}, \tag{4}$$

For $M > m$ we have two poles at $p^2 = m^2$ and $p^2 = M^2$.

3) An indefinite metric in Hilbert space η, which in general leads to a pseudo-unitary condition for the S matrix, i.e., $S^{\dagger}\eta S = \eta$. [T. D. Lee and G. C. Wick, Nucl. Phys. B9, 209 (1969); T. D. Lee, G. C. Wick, Phys. Rev. D2, 1033 (1970).]. Some years ago Lee-Wick showed that unitarity can be preserved by requiring that only positive metric particles be stable.

4) Non renormalizability, large Lorentz violations, modified asymptotic Hilbert space.

In this talk we will show some approaches to prove unitarity and to study renormalization in the presence of higher-order theories with Lorentz violation.

Perturbative Unitarity

Let us focus on the Myers-Pospelov Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\,\psi + g\bar{\psi}\not{n}(n\cdot\partial)^2\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{5}$$

where n is a privileged four-vector and g a small parameter. We fix $n = (1,0,0,0)$ and from the dispersion relation we found the four solutions

$$\omega_1 = \frac{1-\sqrt{1-4gE}}{2g}, \qquad \omega_2 = \frac{1-\sqrt{1+4gE}}{2g},$$
$$W_1 = \frac{1+\sqrt{1-4gE}}{2g}, \qquad W_2 = \frac{1+\sqrt{1+4gE}}{2g},$$

where $E = \sqrt{\vec{p}^2 + m^2}$.

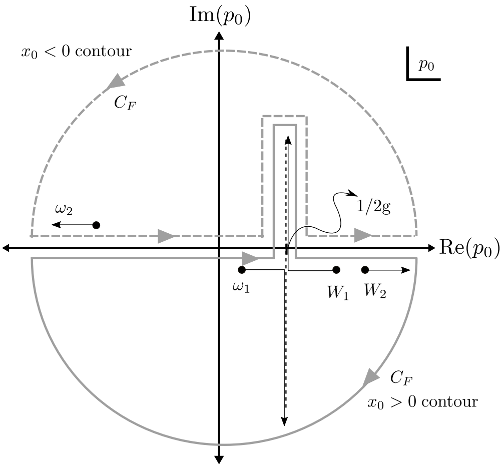

Figure 2: The poles in the complex plane

The optical theorem

The unitarity of the S-matrix and considering $S = 1 + iT$ implies

$$-i(T - T^\dagger) = T^\dagger T. \tag{6}$$

Between initial states $|i\rangle$ and final states $\langle f|$ and inserting a complete set of intermediate states $\langle m|$, we rewrite the above equation as

$$\langle f| T |i\rangle - \langle f| T^\dagger |i\rangle = i \sum_m \int d\Pi_m \langle f| T^\dagger |m\rangle \langle m| T |i\rangle . \tag{7}$$

We write $\mathcal{M}_{fi} - \mathcal{M}^*_{if} = i \sum_m \int d\Pi_m \mathcal{M}_{fm}\mathcal{M}^*_{im}$, and in the special case of forward scattering $f = i$, we arrive at the unitarity condition

$$2\,\mathrm{Im}(\mathcal{M}_{ii}) = \sum_m \int d\Pi_m |\mathcal{M}_{im}|^2 . \tag{8}$$

Implementation: Lee-Wick prescription

In our modified QED we will check unitarity for the graph

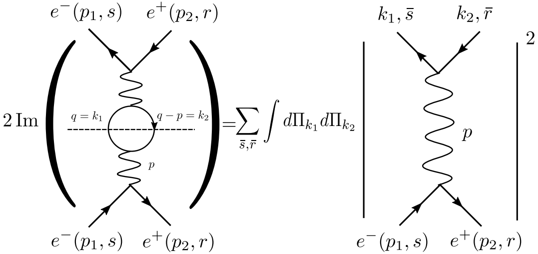

Figure 3: The scattering diagram

Some central points to satisfy the perturbative constraint are:

1) The sum over physical states in the amplitude diagram must be carried only over positive metric states

2) In the loop diagram a suitable prescription for the path C_{LW} is needed to avoid the poles and to compute the residues

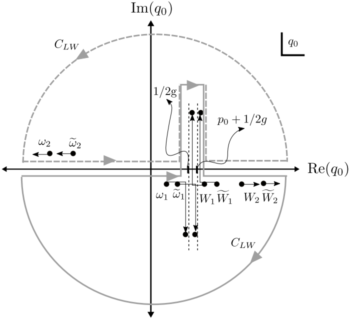

Figure 4: The path C_{LW}

The LHS of the unitarity condition is

$$M(p) = \frac{-e^4 J_1^{\mu}(p) J_2^{\nu}(p)}{p^4} \int \frac{d^3q}{(2\pi)^4} \; Tr\left(\gamma_\mu (\not{R} + m)\gamma_\nu(\not{Q} + m)\right) I(q_0.p_0)$$

where $R^\mu = ((p_0 + q_0)(1 - g(p_0 + q_0)), \vec{q})$, $J_1^\mu(p) = \bar{v}^r(p_2)\gamma^\mu u^s(p_1)$ and $J_2^\mu(p) = \bar{u}^s(p_1)\gamma^\mu v^r(p_2)$.
The important integral is

$$I(q_0, p_0) = -i \int_{C_{LW}} \frac{dq_0}{g^4(q_0 - \omega_1)(q_0 - \omega_2)(q_0 - W_1)(q_0 - W_2)}$$
$$\times \quad \frac{1}{(q_0 - p_0 - \omega_1)(q_0 - p_0 - \omega_2)(q_0 - p_0 - W_1)(q_0 - p_0 - W_2)}.$$

We can compute the imaginary part of $M(p)$ considering

$$Disc(M) = M(p_0 + i\epsilon) - M(p_0 - i\epsilon) = 2iImM(p_0 + i\epsilon).$$

The RHS of the unitarity condition is the amplitude $\mathcal{A}$

$$\mathcal{A} = \sum_{\text{phys}\to\omega_1,\omega_2} \sum_{r,r'} \int \frac{d^3 k_2}{(2\pi)^3} \frac{1}{E_2} \frac{d^3 k_1}{(2\pi)^3} \frac{1}{E_1} (2\pi)^4 \delta^4(k_2 + k_1 - p) |M|^2$$

where we have used the Lee-Wick prescription in order to sum only over physical states. In the center of mass system one has

$$\begin{aligned}\mathcal{A} \;&=\; \frac{e^4 J_1^{\mu}(p) J_2^{\nu}(p)}{p^4} \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^2 2EN_1 2EN_2}\; Tr\left(\gamma_\mu(\gamma^0 E + \gamma^i k_{i2} - m)\right.\\ &\qquad \left.\gamma_\nu(\gamma^0 E + \gamma^i k_{i1} + m)\right) \delta^4(-\omega_1 + \omega_2 - p_0)\delta^3(\vec{k}_1 + \vec{k}_2).\end{aligned}$$

Finally, by comparing both sides one is able to prove the unitarity constraint for the considered one-loop scattering process.

Renormalization and Lorentz violating asymptotic states

It has been shown that in the presence of Lorentz invariance violation new operators induced via radiative corrections in the effective Lagrangians may modify the pole masses of the two-point functions. [R. Potting, Phys. Rev. D **85**, 045033 (2012); M. Cambiaso, R. Lehnert and R. Potting, Phys. Rev. D **90**, no. 6, 065003 (2014)].

To explain the idea consider the standard Yukawa Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\phi^2 + \lambda\bar{\psi}\phi\psi + \psi(i\partial\!\!\!/ - M)\psi\,, \tag{9}$$

and let us compute the one-loop radiative correction to the scalar self energy Σ_2.

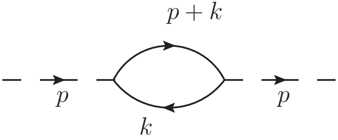

Figure 5: Scalar self energy

The standard computation gives divergencies proportional to p^2 and M^2 which can be cancelled by counterterms.
One can write the two-point function (or Green function) as

$$G(p^2) = \frac{1}{p^2 - m_R^2 + \Sigma(p^2)}\,, \tag{10}$$

with

$$\Sigma(p^2) = \Sigma_2(p^2) + p^2\delta_\phi - (\delta_\phi + \delta_m)m_R^2\,, \tag{11}$$

The on-shell conditions are

$$\Sigma(m_{Pole}^2) = m_R^2 - m_{Pole}^2\,, \qquad \left.\frac{d\Sigma(p^2)}{dp^2}\right|_{p^2=m_{Pole}^2} = 0 \tag{12}$$

1) In the presence of Lorentz violations one has a different structure that may lead to modifications in the renormalization conditions.
2) This can be generally proved at the non perturbative level with the spectral density in the Kállén-Lehmann representation.

Renormalization

Based on the work [J. R. Nascimento, A. Y. Petrov and C. M. Reyes, arXiv:1706.01466 [hep-th].]

We focus on the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 + \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi + g_2\bar{\psi}n\!\!\!/(n\cdot\partial)^2\psi + g\bar{\psi}\phi\psi\,. \quad (13)$$

We impose the simplification of considering $m = M$ and choose the preferred four-vector to be purely timelike $n = (1, 0, 0, 0)$.

We have the usual dispersion relation for the scalar with solutions $p_0 = \pm E$ with $E = \sqrt{\vec{p}^2 + m^2}$, and the modified ones for the fermion

$$(p_0 - g_2 p_0^2)^2 - \vec{p}^2 - m^2 = 0\,, \quad (14)$$

The Yukawa-like model

The standard, that is, non-singular at $g_2 \to 0$, solutions are

$$\omega_1 = \frac{1-\sqrt{1-4g_2E}}{2g_2}\,, \qquad \omega_2 = \frac{1-\sqrt{1+4g_2E}}{2g_2}\,, \tag{15}$$

and the Lee-Wick ones

$$W_1 = \frac{1+\sqrt{1-4g_2E}}{2g_2}\,, \qquad W_2 = \frac{1+\sqrt{1+4g_2E}}{2g_2}\,. \tag{16}$$

The fermion propagator is

$$S(p) \;=\; \frac{i\left((p_0-g_2p_0^2)\gamma^0+p_i\gamma^i+m\right)}{g_2^2(p_0-\omega_1+i\epsilon)(p_0-W_1+i\epsilon)(p_0-\omega_2-i\epsilon)(p_0-W_2+i\epsilon)}\,, \tag{17}$$

The Lee-Wick prescription

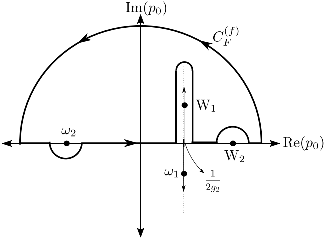

Figure 6: The Lee-Wick prescription

To define the contour $C_F^{(f)}$ we use an heuristic argument. At energies below the critical one we consider the Lee-Wick prescription which rounds the negative pole from below and the three positive ones from above. Now we define the new contour as the one obtained by continuously deforming the curve such to avoid any crossing and singularity with the poles, as shown in the above figure.

Radiative corrections

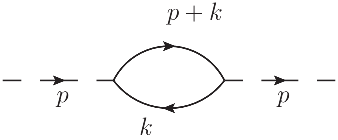

Figure 7: Scalar self energy

$$\Pi(p) = -\frac{g^2}{2}\phi(-p)\phi(p)\int \frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left((Q_\mu\gamma^\mu + m)(R_\nu\gamma^\nu + m)\right)}{(Q^2-m^2)(R^2-m^2)}\,, \qquad (18)$$

where we define

$$\begin{aligned} Q_\mu &= k_\mu - g_2 n_\mu (n\cdot k)^2\,, \\ R_\mu &= k_\mu + p_\mu - g_2 n_\mu (n\cdot(k+p))^2\,. \end{aligned} \qquad (19)$$

$$\Pi(p) = \Pi(0) + p_\mu\left(\frac{\partial\Pi}{\partial p_\mu}\right)_{p=0}\frac{1}{2}p_\mu p_\nu\left(\frac{\partial^2\Pi}{\partial p_\mu \partial p_\nu}\right)_{p=0} + \ldots\,. \qquad (20)$$

The contributions

$$\Pi(p) = -2g^2m^2q_0 - 2g^2p^2q_1 - 2g^2(n\cdot p)^2q_n\,, \tag{21}$$

where

$$\begin{aligned} q_0 &= -\frac{i}{48\pi^2 g_2^2 m^2} + \frac{i}{48\pi^2}\left(6\gamma_E - 0,46 + 12i\pi - 18\ln\left(\frac{g_2 m}{2}\right)\right)\,, \\ q_1 &= -\frac{i}{2\pi^2}\left(i\pi - \ln\left(\frac{g_2 m}{2}\right) - \frac{1}{3}\right)\,, \\ q_n &= \frac{i}{\pi^2}\,. \end{aligned} \tag{22}$$

The contributions

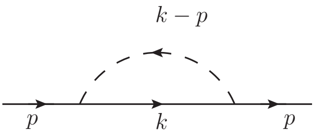

Figure 8: Fermion self energy

The fermion self-energy graph is represented by the integral

$$\Sigma(p) = g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\not{Q} + m}{((k-p)^2 - m^2)(Q^2 - m^2)} \, . \tag{23}$$

We find

$$\begin{aligned} \Sigma_2 &= g^2 \not{n} f_1^n + g^2 m f_0 + g^2 \not{p} f_1 + g^2 m (n \cdot p) f_2^n \\ &+ g^2 \not{n} (n \cdot p) f_3^n + g^2 p^2 \not{n} f_6^n + g^2 (n \cdot p) \not{p} f_4^n + g^2 (n \cdot p)^2 \not{n} f_5^n \, . \end{aligned} \tag{24}$$

The scalar pole mass

We start with

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_R \partial^\mu \phi_R - \frac{1}{2} m_R^2 \phi_R^2 \,, \tag{25}$$

Consider the renormalized two-point function

$$(\Gamma_R^{(2)})^{-1} = p^2 - m_R^2 + \Pi_R(p) \,, \tag{26}$$

where

$$\Pi_R(p) = p^2 A_\phi + m_R^2 B_\phi + (n \cdot p)^2 C_\phi \,, \tag{27}$$

and

$$\begin{aligned} A_\phi &= -2g^2 q_1 \,, \\ B_\phi &= -2g^2 q_0 \,, \\ C_\phi &= -2g^2 q_n \,. \end{aligned} \tag{28}$$

The scalar pole mass

In order to find the pole we consider the ansatz

$$\bar{P}_\phi^2 = p^2 - M_{\rm ph}^2 + \bar{y}(n\cdot p)^2\,, \tag{29}$$

where $M_{\rm ph}$ and $\bar{y}$ are the unknown constants we want to find. We demand the renormalized two-point function to satisfy the condition at $\bar{P}_\phi^2 = 0$

$$(\Gamma_R^{(2)})^{-1}(\bar{P}_\phi^2 = 0) = 0\,, \tag{30}$$

From (26) replacing the value of p^2 given in (29) and using the condition (30), we arrive at the equation

$$\begin{aligned} 0 \;\; = \;\; & M_{\rm ph}^2 - \bar{y}(n\cdot p)^2 - m_R^2 + A_\phi\left(M_{\rm ph}^2 - \bar{y}(n\cdot p)^2\right) + B_\phi m_R^2 \\ & + C_\phi (n\cdot p)^2\,. \end{aligned} \tag{31}$$

The scalar pole mass

Due to the independence of each term, and after some algebra, we find

$$M_{\mathrm{ph}}^2 = m_R^2 \frac{(1 - B_\phi)}{(1 + A_\phi)} , \tag{32}$$

and

$$\bar{y} = \frac{C_\phi}{1 + A_\phi} . \tag{33}$$

Conclusions

- QFT with higher-order Lorentz violation are intrinsically different from the SME minimal models even at the tree level.
- One can preserve unitarity at one loop order in higher-order Lorentz violating theories using the Lee-Wick prescription.
- UV divergences in some model as the Yukawa-like model are not present and one can show the finiteness of the S-matrix.
- Several works are planned for the near future: Kallen-Lehman representation for higher-order theories, study of more models, induced finite corrections (vertex).