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Quasi-relativistic systems in strong magnetic field

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NON PERTURBATIVE ASPECTS OF QFT AND LOEWE'S 65 FEST

5 - 7 DECEMBER 2017

Chiral fermions

- *Massless* Dirac fermions:

$$\left(\gamma^0 p_0 - \vec{\boldsymbol{\gamma}} \cdot \vec{\boldsymbol{p}}\right)\Psi = 0 \quad \Rightarrow \quad \frac{\vec{\Sigma}\cdot\vec{\boldsymbol{p}}}{|\vec{\boldsymbol{p}}|}\Psi = \mathrm{sign}(p_0)\gamma^5\Psi$$

For particles ($p_0 > 0$): chirality = helicity

For antiparticles ($p_0 < 0$): chirality = - helicity

- Massive Dirac fermions in *ultrarelativistic* regime
  - High temperature: $T >> m$
  - High density: $\mu >> m$

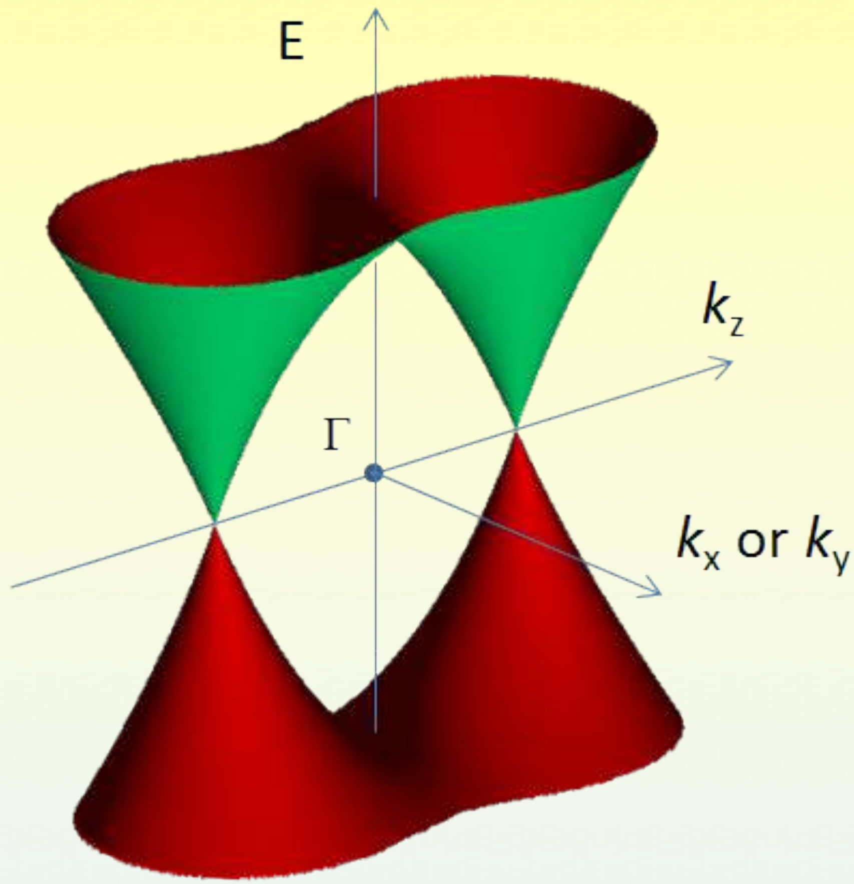
Chiral matter

- Matter made of chiral fermions with $n_{\mathrm{L}} \neq n_{\mathrm{R}}$
- Unlike the electric charge $(n_{\mathrm{R}} + n_{\mathrm{L}})$, the chiral charge $(n_{\mathrm{R}} - n_{\mathrm{L}})$ is **not** conserved

$$\frac{\partial(n_R + n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

$$\frac{\partial(n_R - n_L)}{\partial t} + \vec{\nabla} \cdot \vec{j}_5 = \frac{e^2 \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}{2\pi^2 c}$$

- The chiral symmetry is anomalous in quantum theory



Credits: Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

DIRAC & WEYL MATERIALS

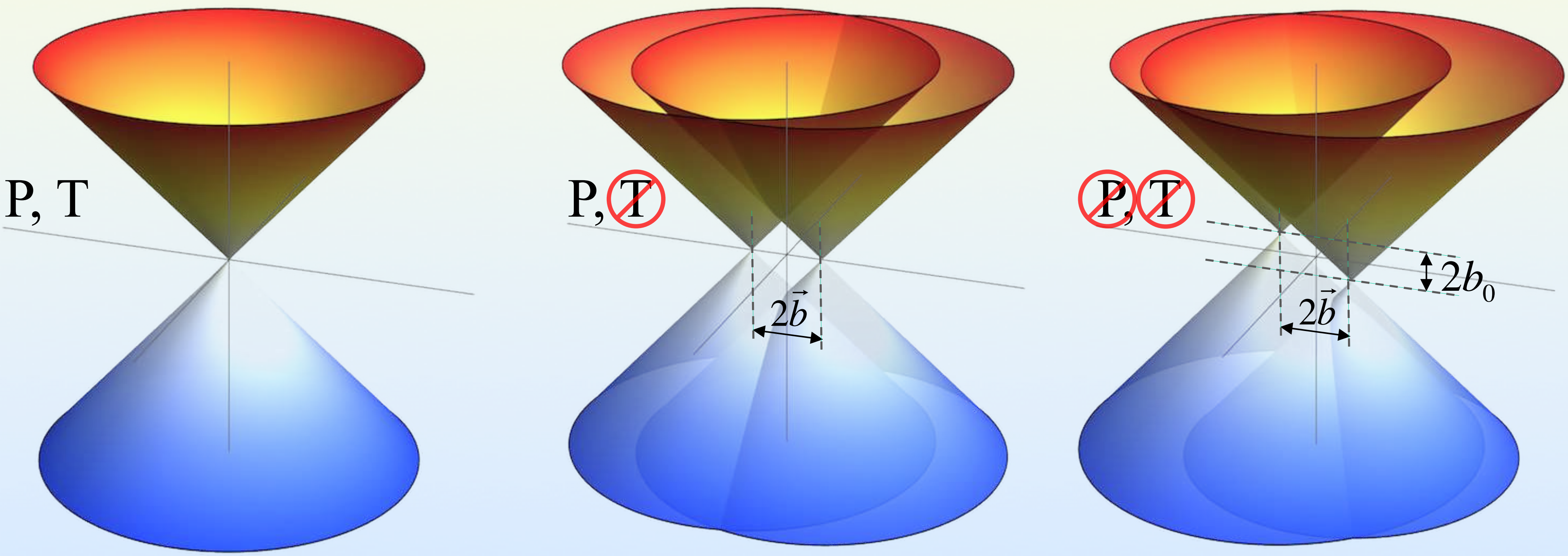
Dirac vs. Weyl materials

- Low-energy Hamiltonian of a Dirac/Weyl material

T P

$$H = \int d^3\mathbf{r}\, \bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right) - \left(\vec{b}\cdot\vec{\gamma}\right)\gamma^5 + b_0\gamma^0\gamma^5\right]\psi$$

Dirac (e.g., Na_3Bi, Cd_3As_2, $ZrTe_5$) **Weyl** (e.g., TaAs, NbAs, TaP, NbP, WTe_2)



Low-energy Hamiltonian

- The Hamiltonian for massless Dirac fermions is given by

$$H_D = \begin{pmatrix} v_F(\vec{k}\cdot\vec{\sigma}) & 0 \\ 0 & -v_F(\vec{k}\cdot\vec{\sigma}) \end{pmatrix}$$

- This can we viewed as a combination of two Weyl fermions

$$H_\lambda = \lambda v_F(\vec{k}\cdot\vec{\sigma})$$

where $\lambda = \pm 1$ is a chirality

The Weyl energy eigenstates are given by

$$\psi_k^\lambda = \frac{1}{\sqrt{2\epsilon_k}k_\perp}\begin{pmatrix} \sqrt{\epsilon_k + \lambda k_z}\, k_- \\ \lambda\sqrt{\epsilon_k - \lambda k_z}\, k_\perp \end{pmatrix}$$

They described particles of energy $\epsilon_k = v_F\sqrt{k_x^2 + k_y^2 + k_z^2}$

The mapping $k \to \psi_k^\lambda$ has a nontrivial topology

Berry connection & curvature

- Consider evolution from $\psi_{\boldsymbol{k}}$ to $\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}$:

$$\langle\psi_{\boldsymbol{k}}|\psi_{\boldsymbol{k}+\delta\boldsymbol{k}}\rangle \approx 1 + \delta\boldsymbol{k}\cdot\langle\psi_{\boldsymbol{k}}|\boldsymbol{\nabla}_{\boldsymbol{k}}|\psi_{\boldsymbol{k}}\rangle \approx e^{i\boldsymbol{a}_{\boldsymbol{k}}\cdot\delta\boldsymbol{k}}$$

where $\boldsymbol{a}_{\boldsymbol{k}} = -i\langle\psi_{\boldsymbol{k}}|\boldsymbol{\nabla}_{\boldsymbol{k}}|\psi_{\boldsymbol{k}}\rangle$ is the Berry connection

- The Berry curvature is defined as follows:

$$\boldsymbol{\Omega}_k = \boldsymbol{\nabla}_k \times \boldsymbol{a}_{\boldsymbol{k}}$$

- Note the similarity with gauge fields, but $\boldsymbol{a}_{\boldsymbol{k}}$ and $\boldsymbol{\Omega}_k$ are defined in the momentum space
- It is convenient to define the Chern number (flux of $\boldsymbol{\Omega}_k$)

$$C = \frac{1}{2\pi}\oint \boldsymbol{\Omega}_k \cdot d\boldsymbol{S}_{\boldsymbol{k}}$$

- A nonzero (integer) Chern number indicates a nonzero (topological) charge inside the k-volume surrounded by the closed surface (Gauss's law)

Berry curvature for Weyl fermions

- In the case of Weyl fermions,

$$\mathbf{\Omega}_k = \lambda \frac{\vec{k}}{2k^3}$$

(Note: this looks like a field of a monopole at $\vec{k} = 0$)

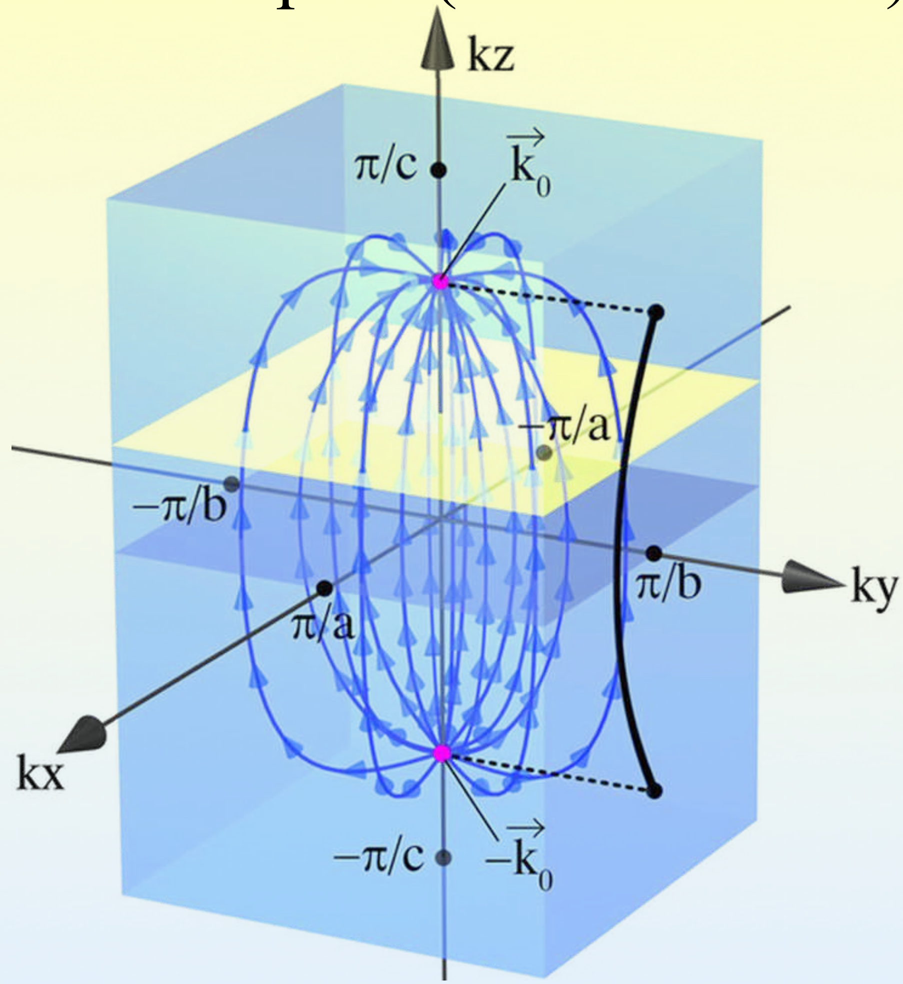
- Let us calculate the total flux of $\mathbf{\Omega}_k$-field through the spherical surface of radius K with the center at $\vec{k} = 0$

$$C = \frac{1}{2\pi} \oint \lambda \frac{\vec{k}}{2k^3} \cdot \frac{\vec{k}}{k} k^2 \sin\theta \, d\theta d\varphi = \lambda = \pm 1$$

- Thus, the electronic structure of massless Weyl fermions is characterized by a topological monopole at $\vec{k} = 0$
- Is the Berry monopole just a mathematical curiosity?
- Are there any observable consequences?

Weyl fermions on a lattice

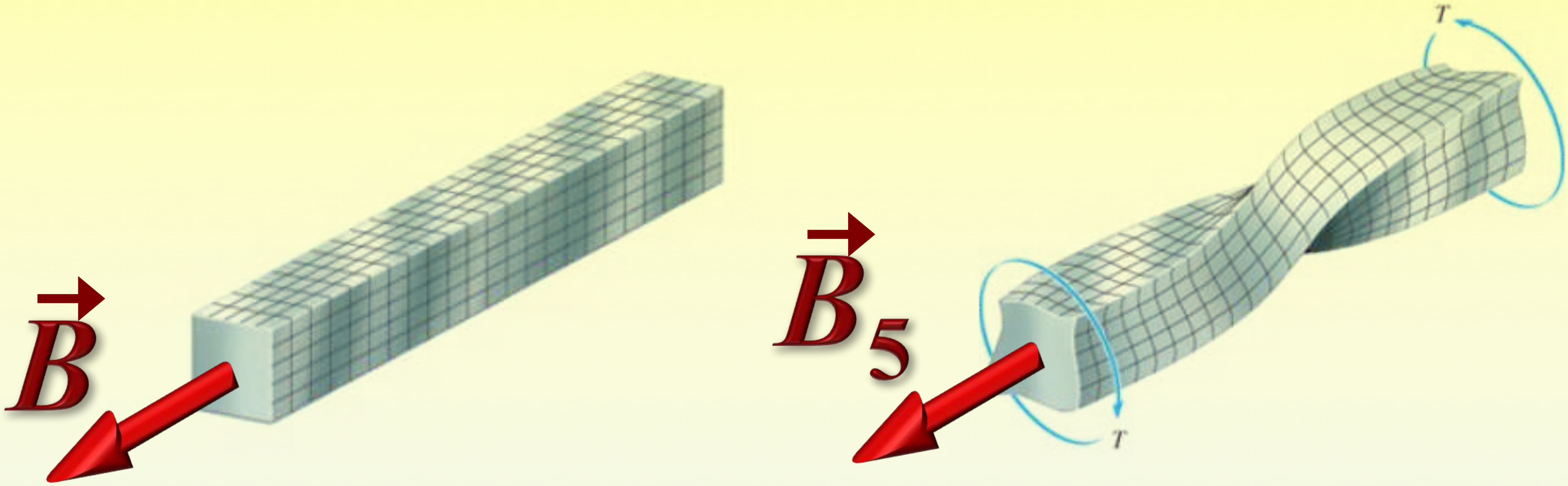
- In solid state physics, the momentum space (Brillouin zone) is compact
- A closed surface around a single Weyl node is also a closed surface (of opposite orientation) around a the rest of the Brillouin zone
- Flipping surface orientation changes the sign of the flux
- There must be an opposite charge somewhere in the rest of the zone



[Morimoto & Nagaosa, Scientific Reports **6**, 19853 (2016)]

- Thus, Weyl fermions come in pairs of opposite chirality

[Nielsen & Ninomiya, Nucl. Phys. B **193**, 173 (1981); B **185**, 20 (1981)]



PSEUDO-ELECTROMAGNETIC FIELDS

$$\mathbf{E}_\lambda = \mathbf{E} + \lambda \mathbf{E}_5 \text{ and } \mathbf{B}_\lambda = \mathbf{B} + \lambda \mathbf{B}_5$$

[Zubkov, Annals Phys. **360**, 655 (2015)]
[Cortijo, Ferreiros, Landsteiner, Vozmediano. Phys. Rev. Lett. **115**, 177202 (2015)]
[Grushin, Venderbos,Vishwanath, Ilan, Phys. Rev. X 6, 041046 (2016)]
[Cortijo, Kharzeev, Landsteiner, Vozmediano, Phys. Rev. B 94, 241405 (2016)]
[Pikulin, Chen, Franz, Phys. Rev. X **6**, 041021 (2016)]

Strain in Weyl materials

- Strains affect low-energy quasiparticles in Weyl materials

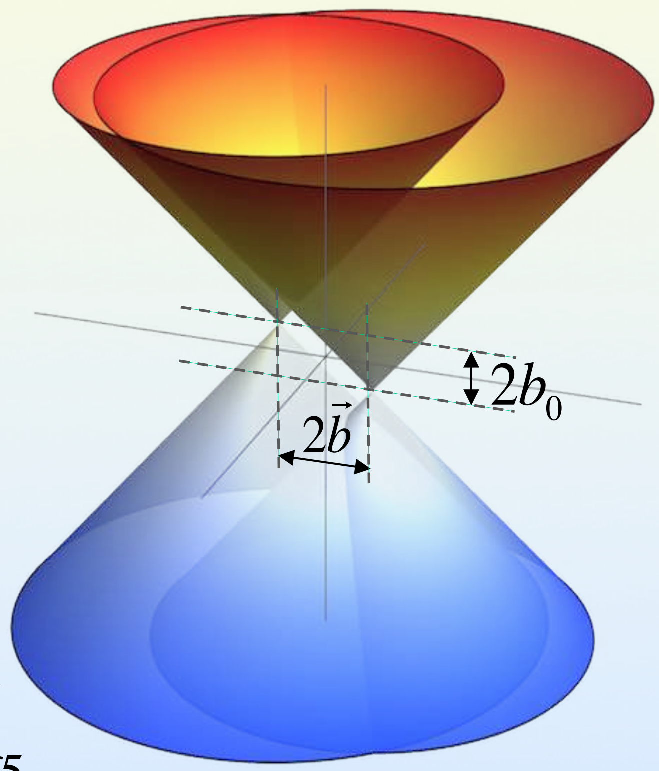
$$H = \int d^3\mathbf{r}\,\bar{\psi}\left[-iv_F\left(\vec{\gamma}\cdot\vec{\mathbf{p}}\right)-\left(\vec{b}+\vec{A}_5\right)\cdot\vec{\gamma}\,\gamma^5+\left(b_0+A_{5,0}\right)\gamma^0\gamma^5\right]\psi$$

where the components of the chiral gauge fields are

$$A_{5,0} \propto b_0\left|\vec{b}\right|\partial_{||}u_{||}$$

$$A_{5,\perp} \propto \left|\vec{b}\right|\partial_{||}u_{\perp}$$

$$A_{5,||} \propto \alpha\left|\vec{b}\right|^2\partial_{||}u_{||}+\beta\sum_i\partial_i u_i$$



The associated pseudo-EM fields are

$$\vec{B}_5 = \vec{\nabla}\times\vec{A}_5 \quad \text{and} \quad \vec{E}_5 = -\vec{\nabla}A_0 - \partial_t\vec{A}_5$$

Chiral effects in Weyl materials

- Any qualitative properties of Weyl materials directly sensitive to b_0 and $\vec{b}$?
- Some proposals:
  - Anomalous Hall effect
  - Anomalous Alfven waves
  - Strain/torsion induced CME
  - Strain/torsion induced quantum oscillations
  - Strain/torsion dependent resistance
  - etc.
- How about collective modes?

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

CONSISTENT CHIRAL KINETIC THEORY

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral kinetic theory

[Son and Yamamoto, Phys. Rev. D **87**, 085016 (2013)]
[Stephanov and Yin, Phys. Rev. Lett. **109**, 162001 (2012)]

- Kinetic equation:

$$\frac{\partial f_\lambda}{\partial t} + \frac{\left[e\tilde{\mathbf{E}}_\lambda + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_\lambda) + \frac{e^2}{c}(\tilde{\mathbf{E}}_\lambda\cdot\mathbf{B}_\lambda)\mathbf{\Omega}_\lambda\right]\cdot\nabla_{\mathbf{p}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)} + \frac{\left[\mathbf{v} + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda) + \frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda\right]\cdot\nabla_{\mathbf{r}} f_\lambda}{1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)} = 0$$

where $\tilde{\mathbf{E}}_\lambda = \mathbf{E}_\lambda - (1/e)\nabla_{\mathbf{r}}\epsilon_{\mathbf{p}}$, $\quad \mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$,

$$\epsilon_{\mathbf{p}} = v_F p\left[1 - \frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]$$

and $\mathbf{\Omega}_\lambda = \lambda\hbar\frac{\hat{\mathbf{p}}}{2p^2}$ is the Berry curvature

Current and chiral anomaly

- The definitions of density and current are

$$\rho_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[1+\frac{e}{c}(\mathbf{B}_\lambda\cdot\mathbf{\Omega}_\lambda)\right]f_\lambda,$$

$$\mathbf{j}_\lambda = e\int \frac{d^3p}{(2\pi\hbar)^3}\left[\mathbf{v}+\frac{e}{c}(\mathbf{v}\cdot\mathbf{\Omega}_\lambda)\mathbf{B}_\lambda + e(\tilde{\mathbf{E}}_\lambda\times\mathbf{\Omega}_\lambda)\right]f_\lambda$$
$$+e\boldsymbol{\nabla}\times\int\frac{d^3p}{(2\pi\hbar)^3}f_\lambda\epsilon_{\mathbf{p}}\mathbf{\Omega}_\lambda,$$

They satisfy the following anomalous relations:

$$\frac{\partial\rho_5}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j}_5 = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B})+(\mathbf{E}_5\cdot\mathbf{B}_5)\Big]$$ ✔

$$\frac{\partial\rho}{\partial t}+\boldsymbol{\nabla}\cdot\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}\Big[(\mathbf{E}\cdot\mathbf{B}_5)+(\mathbf{E}_5\cdot\mathbf{B})\Big]$$ ✘

Consistent definition of current

- Additional Bardeen-Zumino term is needed,

$$\delta j^{\mu} = \frac{e^3}{4\pi^2\hbar^2 c}\epsilon^{\mu\nu\rho\lambda}A^5_{\nu}F_{\rho\lambda}$$

- In components,

$$\delta\rho = \frac{e^3}{2\pi^2\hbar^2 c^2}(\mathbf{A}^5\cdot\mathbf{B})$$

$$\delta\mathbf{j} = \frac{e^3}{2\pi^2\hbar^2 c}A^5_0\mathbf{B} - \frac{e^3}{2\pi^2\hbar^2 c}(\mathbf{A}^5\times\mathbf{E})$$

- Its role and implications:
  - Electric charge is conserved locally ($\partial_\mu J^\mu = 0$)
  - Anomalous Hall effect is reproduced
  - CME vanishes in equilibrium ($\mu_5 = -eb_0$)

Collective modes

We search for plane-wave solutions with

$$\mathbf{E}' = \mathbf{E} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{B}' = \mathbf{B} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

and the distribution function $f_\lambda = f_\lambda^{(\mathrm{eq})} + \delta f_\lambda$,

where

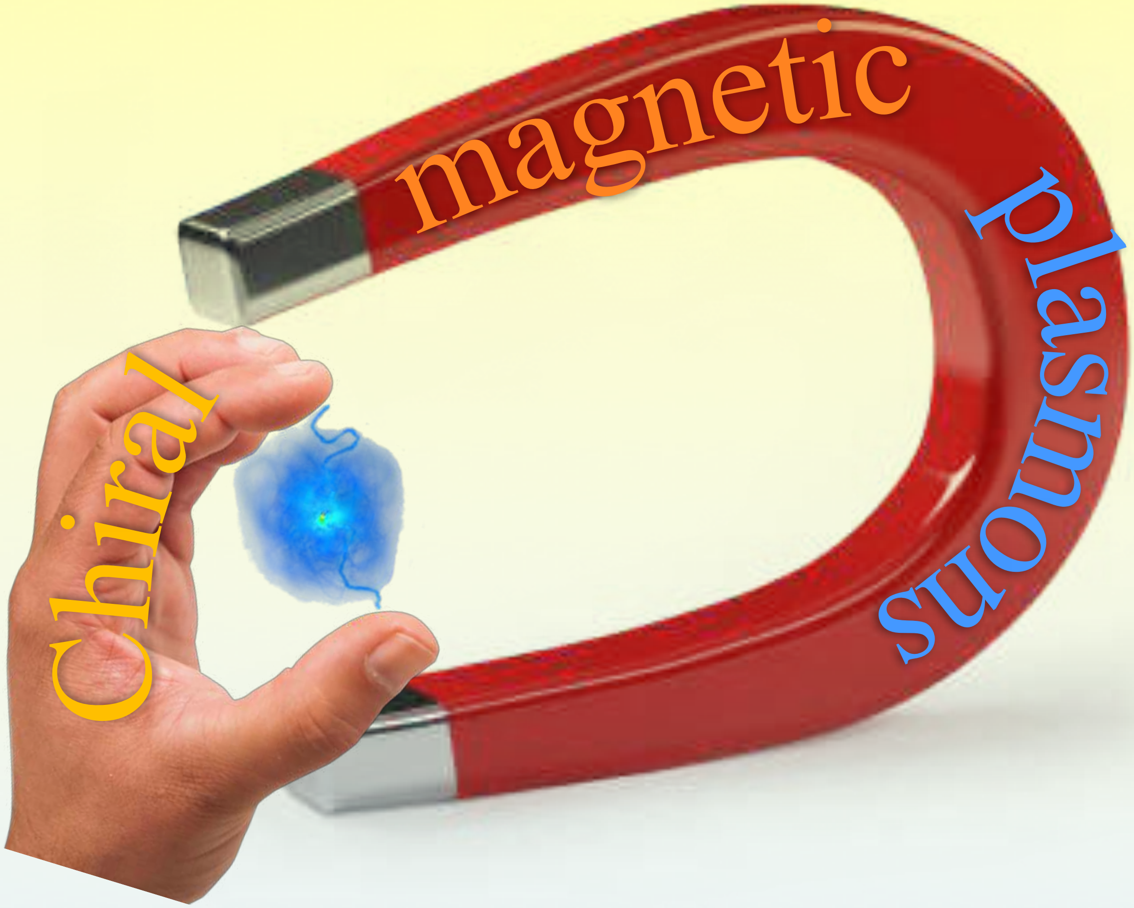
$$\delta f_\lambda = f_\lambda^{(1)} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}}$$

The polarization vector & susceptibility tensor:

$$P^m = i\frac{J'^m}{\omega} = \chi^{mn} E'^n$$

The plasmon dispersion relations follow from

$$\det\left[\left(\omega^2 - c^2 k^2\right)\delta^{mn} + c^2 k^m k^n + 4\pi\omega^2\chi^{mn}\right] = 0$$



RESULTS:

CHIRAL MAGNETIC PLASMONS

[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Chiral magnetic plasmons

Non-degenerate plasmon frequencies @ k=0:

$$\omega_l = \Omega_e, \qquad \omega_{\rm tr}^{\pm} = \Omega_e \sqrt{1 \pm \frac{\delta\Omega_e}{\Omega_e}}$$

where the Langmuir frequency is

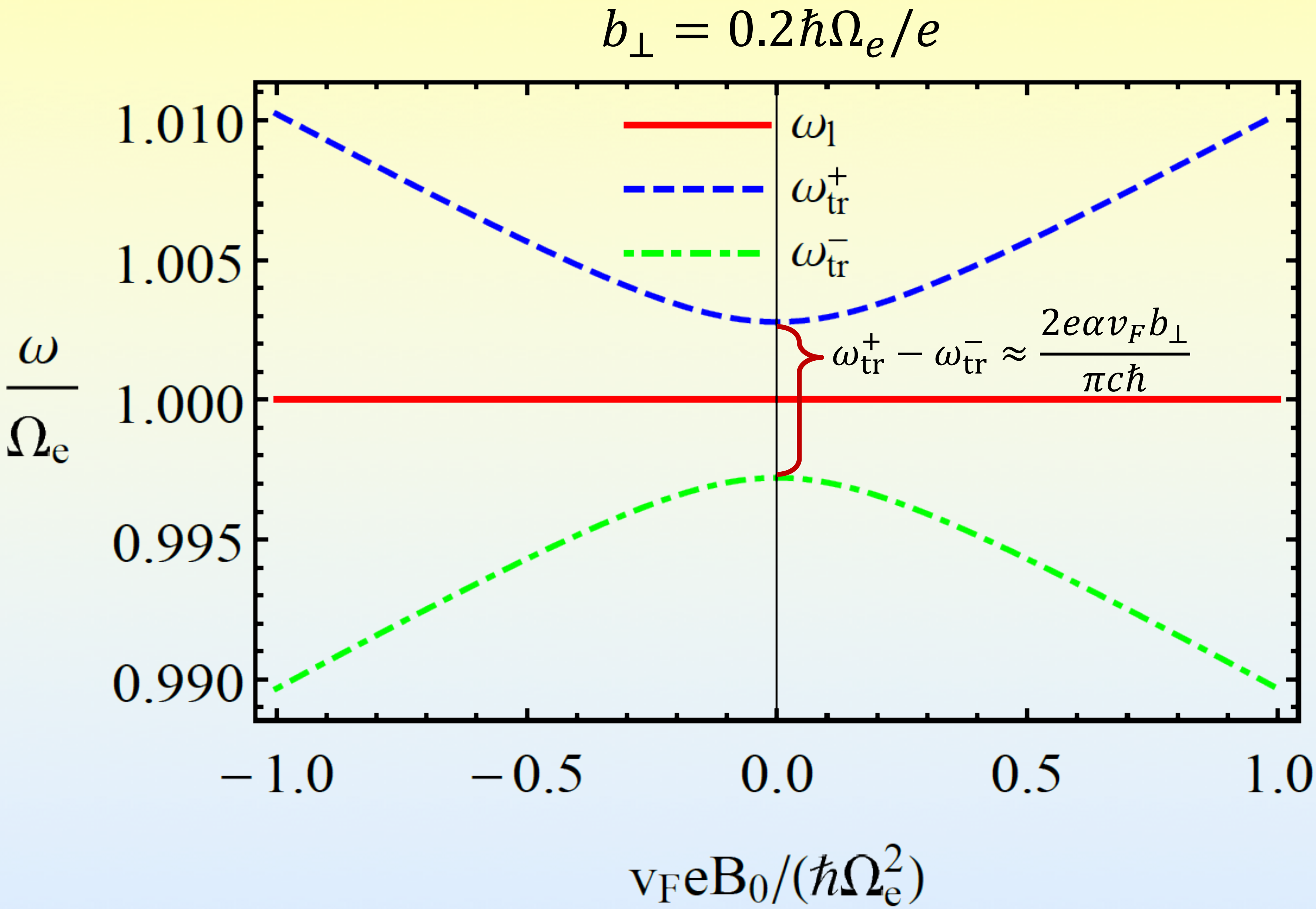
$$\Omega_e \equiv \sqrt{\frac{4\alpha}{3\pi\hbar^2}\left(\mu^2 + \mu_5^2 + \frac{\pi^2 T^2}{3}\right)}$$

and

$$\delta\Omega_e = \frac{2e\alpha v_F}{3\pi c\hbar^2}\Bigg\{9\hbar^2 b_\perp^2 + \Big[\frac{2v_F}{\Omega_e^2}(B_0\mu + B_{0,5}\mu_5) - 3\hbar b_\parallel - \frac{v_F\hbar^2}{4T}\sum_{\lambda=\pm} B_{0,\lambda} F\left(\frac{\mu_\lambda}{T}\right)\Big]^2\Bigg\}^{1/2}$$

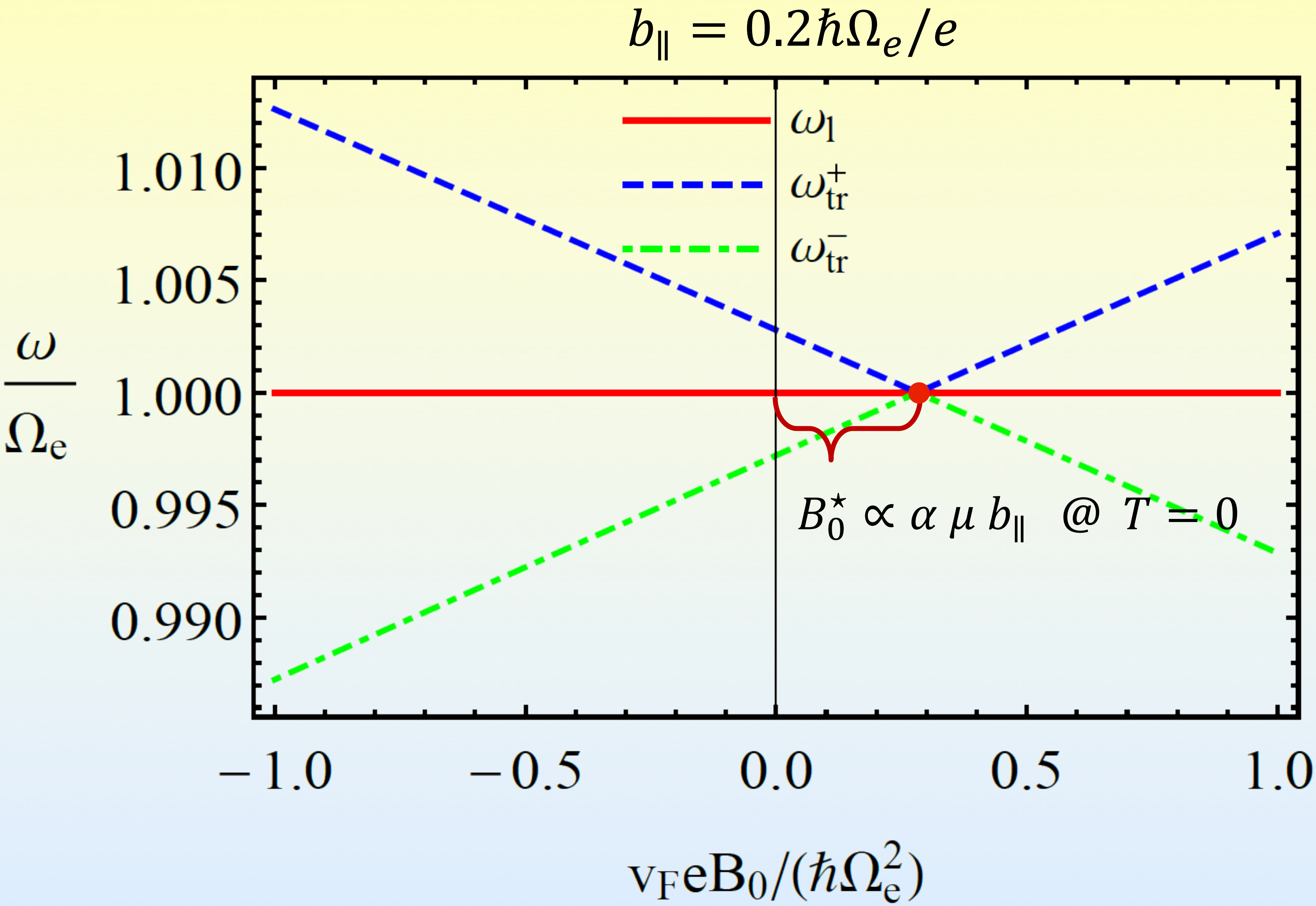
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]

Plasmon frequencies, $\vec{B} \perp \vec{b}$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

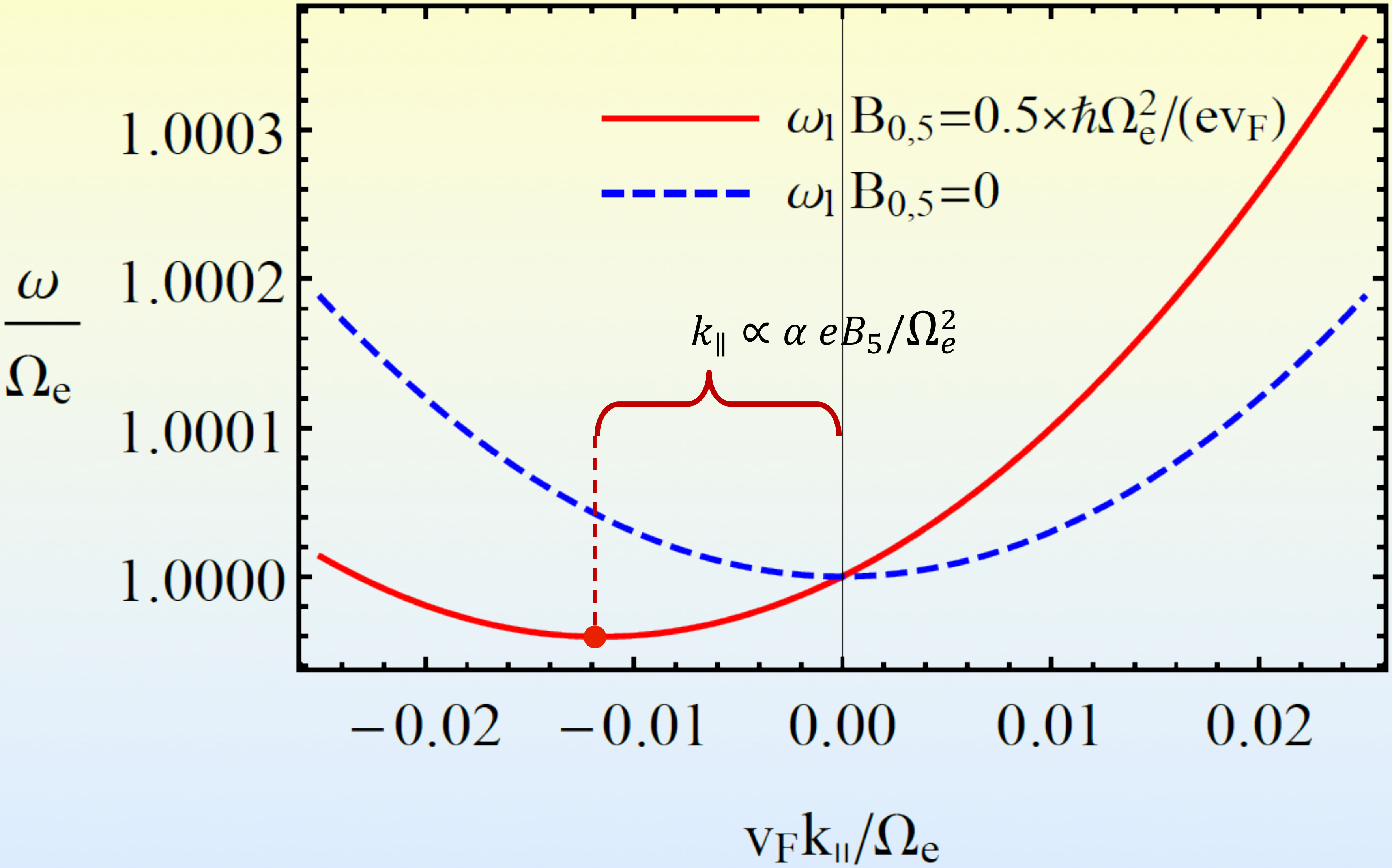
Plasmon frequencies, $\vec{B} \parallel \vec{b}$



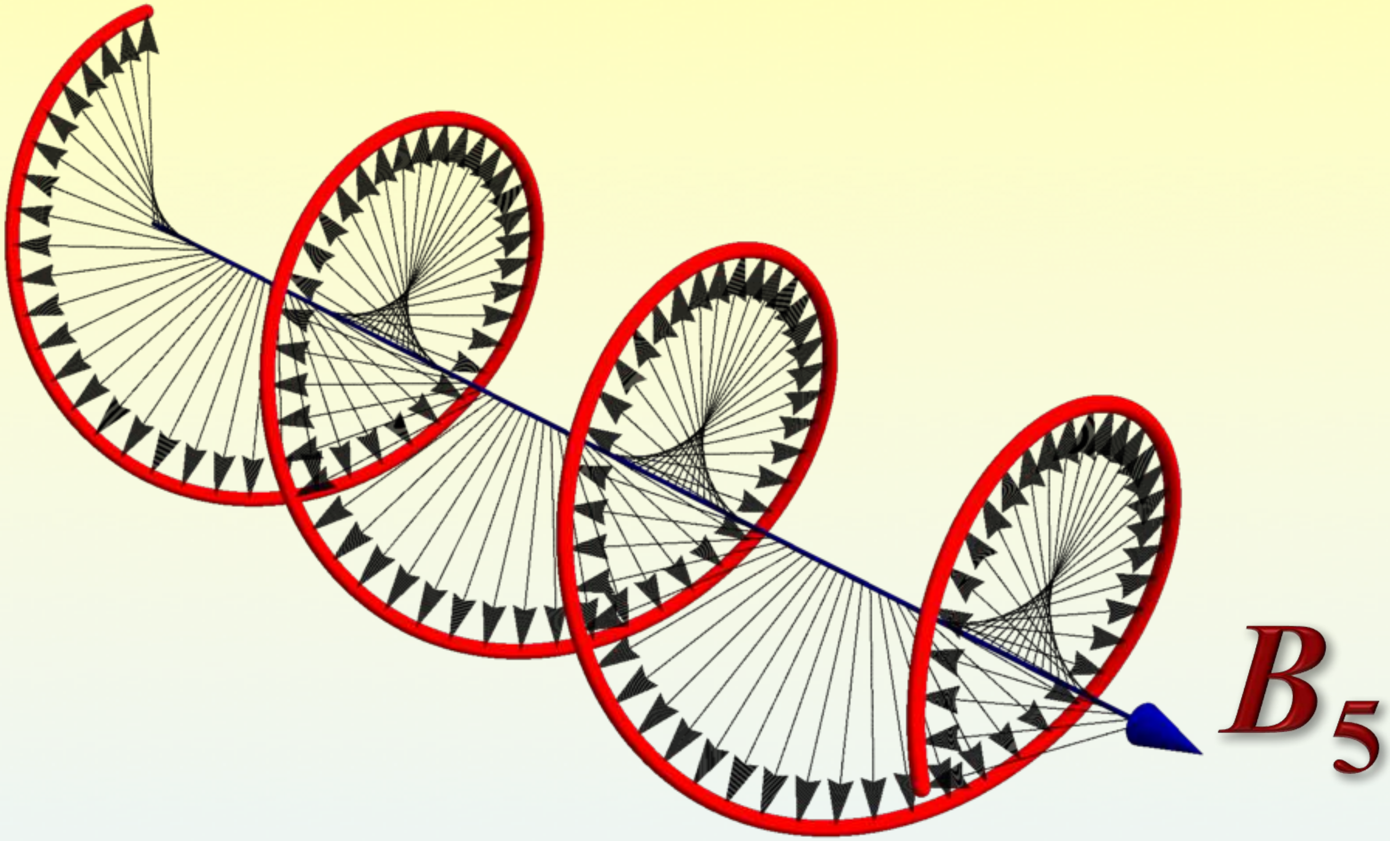
[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. Lett. **118**, 127601 (2017)]

Plasmons with $\vec{k} \neq 0$, $\vec{k} \parallel \vec{B}_5$

- The longitudinal mode is sensitive to $\vec{B}_5$



[Gorbar, Miransky, Shovkovy, Sukhachov, Phys. Rev. B **95**, 115202 (2017)]



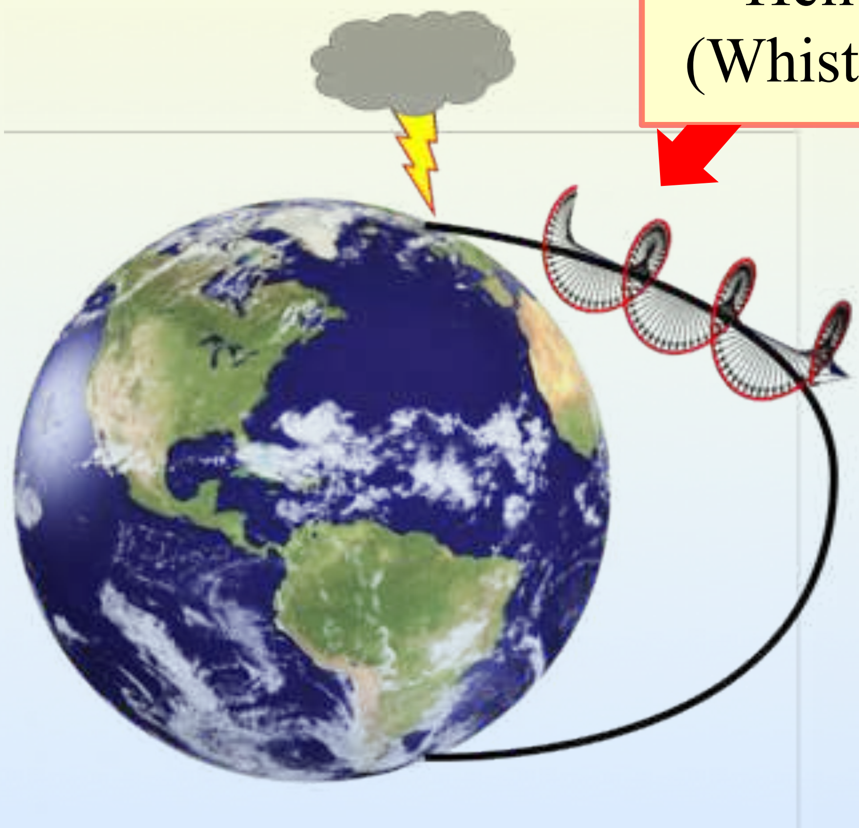
RESULTS:
PSEUDOMAGNETIC HELICONS

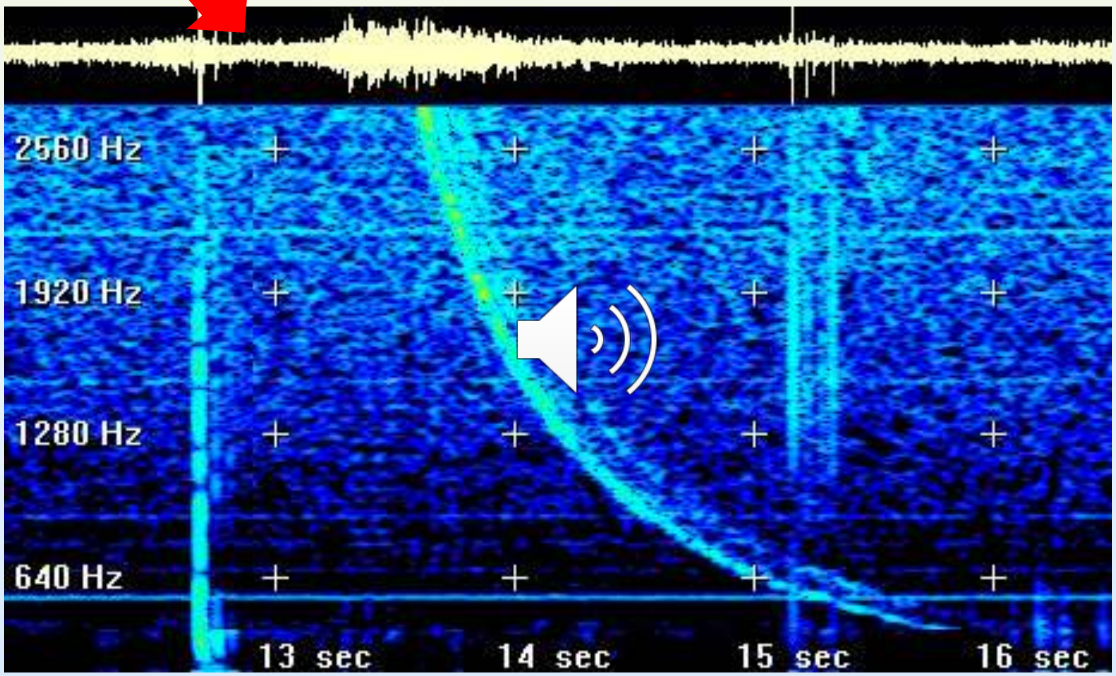
[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

Pseudo-magnetic helicons

- Usual helicons are transverse low-energy gapless excitations propagating along the background magnetic field $\vec{B}_0$ in uncompensated media (e.g., metals with different electron and hole densities)

Helicon in the ionosphere (Whistler) and its spectrogram





(Pseudo-)magnetic helicon

- Helicon dispersion law at $T \to 0$:

$$\omega_h|_{B_{0,5}\to 0,\mu_5\to 0} \overset{b_0\to 0}{=} \frac{eB_0c^3\hbar^2\pi v_F^2k^2}{\pi\hbar^2c^2\Omega_e^2\mu + 2B_0e^4v_F^2b_\parallel} + O(k^3)$$

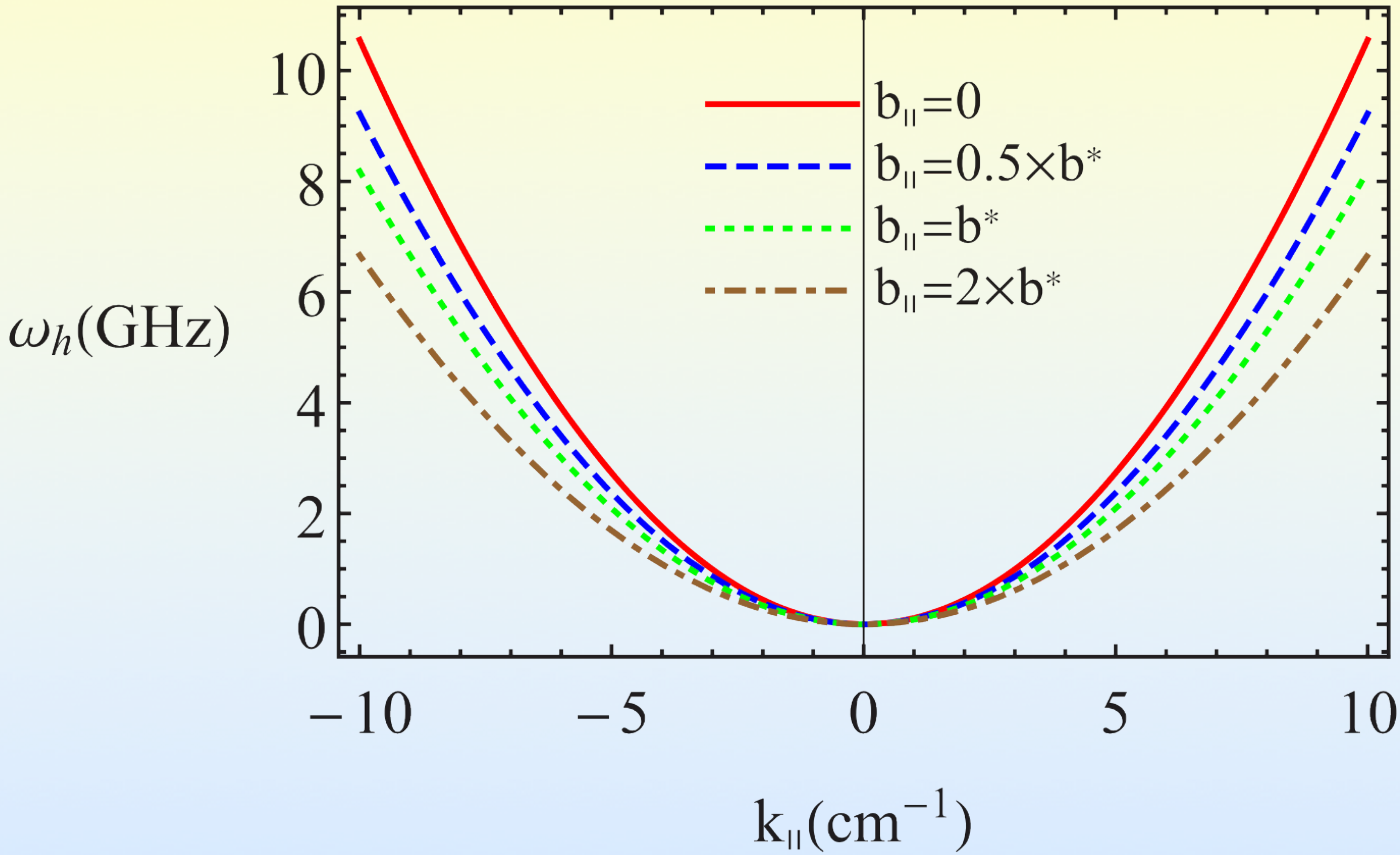
$$\omega_h|_{B_0\to 0,\mu\to 0} \overset{b_0\to -\mu_5/e}{=} \frac{eB_{0,5}c^3\hbar^2\pi v_F^2k^2}{\pi\hbar^2c^2\Omega_e^2\mu_5 + 2B_{0,5}e^4v_F^2b_\parallel} + O(k^3)$$

- Properties:
  - Gapless electromagnetic wave propagates in metals **without magnetic field**!
  - Chiral shift modifies effective helicon mass
  - In the equilibrium regime $eb_0 = -\mu_5$, the linear in the wave vector term is **absent**

[E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and P.O. Sukhachov, Phys. Rev. B **95**, 115422 (2017)]

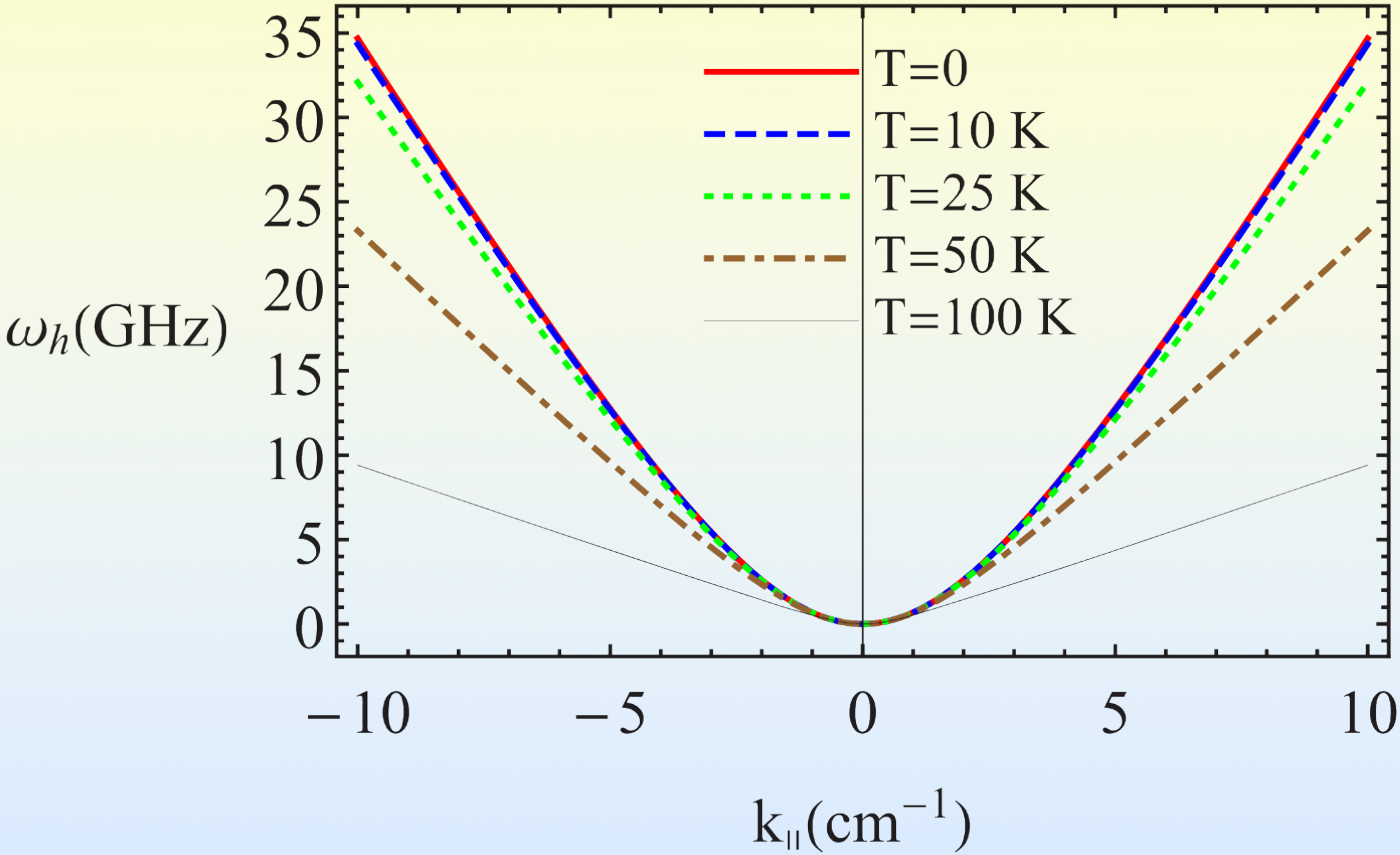
Helicons at different $b_{\parallel}$

$eb_0 = -\mu_5, B_{0,5} = 10^{-2}\mathrm{T}, \mu_5 = 5\ \mathrm{meV}, \mu = 0, eb^* = 0.3\pi\hbar v_F/c_3$



Helicons at different T

$$eb_0 = -\mu_5,\ B_{0,5} = 10^{-2}\text{T},\ b_\parallel = 0.5b^*,\ \mu_5 = 5\text{ meV},\ \mu = 0$$



Summary

- Consistent chiral kinetic theory is needed
- Chiral magnetic plasmons (χMPs) are sensitive to local charge (non-)conservation
- Properties of χMPs carry information about b_0 and $\vec{b}$
- χMPs are not only due to the oscillation of *electric* charge, but also *chiral* charge
- New types of collective modes, pseudomagnetic helicons, may exist in Weyl materials
- Other unusual collective modes are possible (e.g., see arXiv:1712.01289 for anomalous Hall waves)