On the compatibility of thermodynamic equilibrium conditions with lattice propagators

F. Canfora (CECS), A.Giacomini (UACh), Pablo Pais (CECS) L. Rosa (INFN, Napoli), A. Zerwekh (UTFSM)

Loewe´s Fest

Santiago de Chile 05.12.2017

Talk based on Eur. Phys. J. C76 (2016), 443 (arXiv:1606.02271)

Physical motivations

- Astrophysical evidences supporting the existence of quark stars
- In order to find self gravitating solutions an equation of state is needed.
- In many physical cases the exact equation of state is not known or too difficult to plug it in the gravitational equations.
- It is possible to find bounds on the mass of self gravitating objects assuming some reasonable properties of the eq. of state but without knowing its exact form: absence of superluminal signal propagation $\frac{\mathrm{d}P}{\mathrm{d}e} \leq 1$ and Le Chatelier principle $\frac{\mathrm{d}P}{\mathrm{d}e} \geq 0$. For neutron stars this has been done in Phys. Rev. Lett. 32, 324 (1972)
- Absolute upper bound for self gravitating objets < 3,2 M
- Question: the equation of state derived from non-perturbabative quark and gluon propagators will be compatible with the above principles?

- The lattice propagator for quarks is

$$S(p) = -\frac{\gamma_\mu p_\mu + 1_4\ M_0(p)}{p^2 + M_0^2(p)}\ , \quad M_0(p) = \frac{M_3}{p^2 + m^2} + m_0\ ,$$

$$M_3 = 0.196\ GeV^3, m^2 = 0.639\ GeV^2, m_0 = 0.014\ GeV$$

- This propagator can be expanded in three "standard" fermion propagators, two with complex conjugate and one with real poles. The poles are given by

$$p^2\left(p^2 + m^2\right)^2 + \left[M_3 + m_0\left(p^2 + m^2\right)\right]^2 = \left(p^2 + \alpha_1\right)\left(p^2 + \alpha_2\right)\left(p^2 + \alpha_3\right)$$

Partition function from propagator

- It is possible to compute the partition function from the propagator as sum of very fast convergent series of suitable Bessel functions

$$\frac{\log Z(T,\mu)}{2\beta V N_c N_f} = \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \Lambda^{-2} \left[p^2 + \left(\frac{M_3}{(\omega_n - i\mu)^2 + p^2 + m^2} + m_0 \right)^2 + (\omega_n - i\mu)^2 \right]$$

$$= \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \Lambda^{-2} \left[\frac{\left(p^2 + (\omega_n - i\mu)^2\right)\left(p^2 + (\omega_n - i\mu)^2 + m^2\right)^2 + \left(M_3 + m_0\left(p^2 + (\omega_n - i\mu)^2 + m^2\right)\right)^2}{\left(p^2 + (\omega_n - i\mu)^2 + m^2\right)^2} \right]$$

$$= \sum_{i=1}^{i=4} c_i \sum_{n=-\infty}^{+\infty} \int \frac{d^3p}{(2\pi)^3} \ln \Lambda^{-2} \left[p^2 + \alpha_i^2(\omega_n, \mu) \right] =: \sum_{i=1}^{i=4} I^{(\alpha_i)} c_i,$$

where $\{\alpha_i^2(\omega_n,\mu),\ (i = 1,2,3)\}$ are minus the three roots of the numerator, $\alpha_4^2 = (\omega_n - i\mu)^2 + m^2$, $\{c_i = 1,\ (i = 1,2,3)\}$ and $c_4 = -2$. So, everything reduces to find for a generic α_i, the quantity

$$I(T,\mu,\alpha^2) = \frac{(\alpha^2)^2}{32\pi^2} \left(\ln\left(\frac{\alpha^2}{\Lambda^2}\right) - \frac{3}{2} \right) + \frac{\alpha^2 T^2}{\pi^2} \sum_{n=1}^{+\infty} (-1)^{n+1} n^{-2} K_2\left(n\frac{\sqrt{\alpha^2}}{T}\right) \cosh(n\mu/T),$$

- It is now possible to visualize the equation of state as a parametric plot using

$$P(T,\mu) = \frac{T}{V}\log Z(T,\mu),$$
$$s(T,\mu) = \frac{\partial P}{\partial T}(T,\mu),$$
$$n(T,\mu) = \frac{\partial P}{\partial \mu}(T,\mu),$$
$$e(T,\mu) = Ts - P + \mu n.$$

- In order to have a single valued function P(e) the functions P(T) and e(T) must be strictly monotonic functions (or the shape of both non-monotonicities should exactly coincide). However this condition fails for some range of T

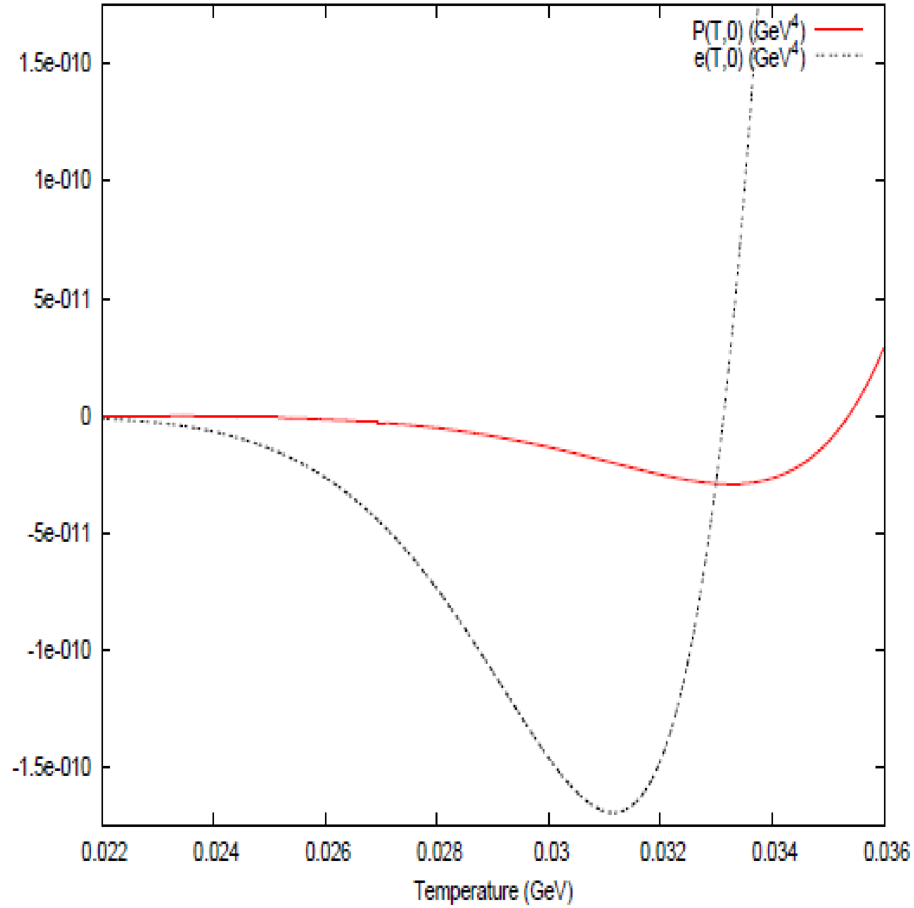


FIG. 1. The pressure P of the quark sector (red line) and its energy density e (black dots) as function of the temperature for $\mu = 0$. It is worth to note the detail of the plot due the critical zone is very narrow compared to the entire unit range and the values of negative P is less than 2×10^{-5} GeV.

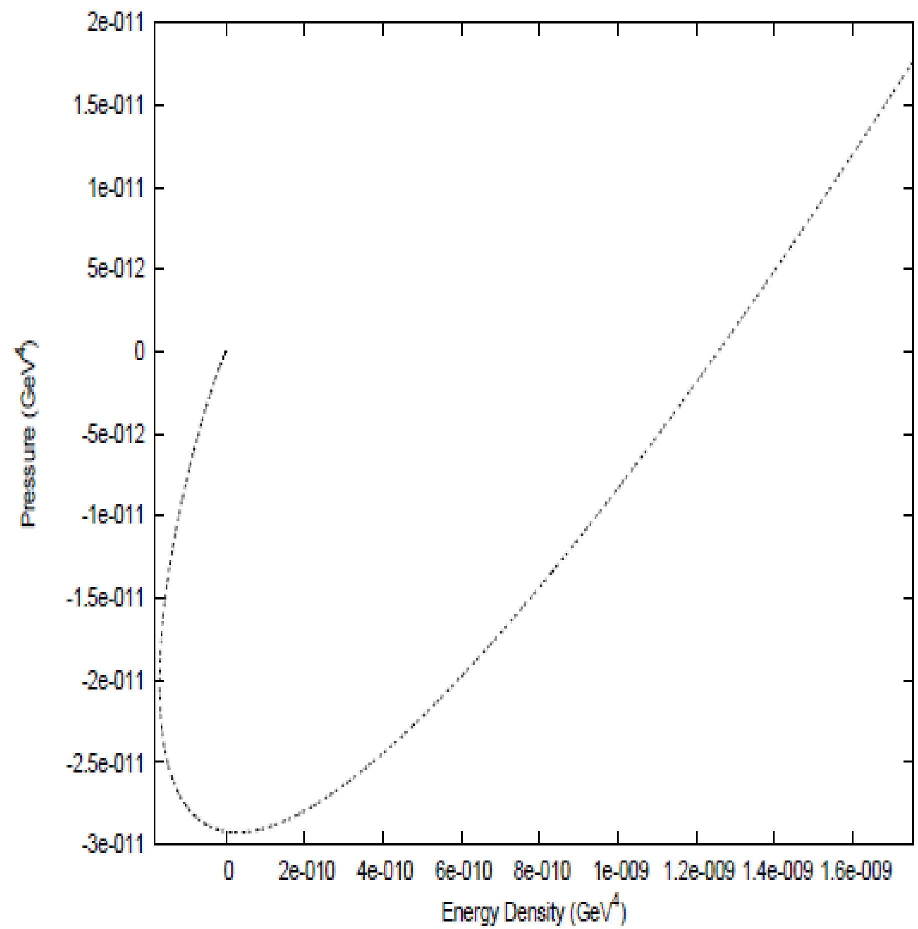


FIG. 2. The pressure P of the quark sector as function of the energy density e for $\mu = 0$. We can see if $e \leq 0$ then the EOS cannot be defined.

Changing the lattice parameters

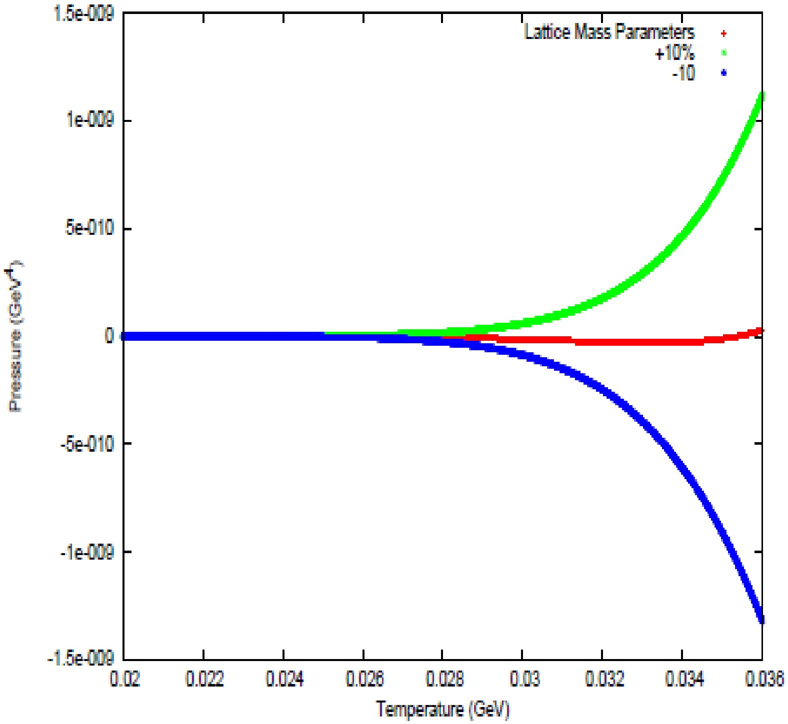


FIG. 3. Comparison of the pressure of the quark sector as function of temperature in the critical zone for $\mu = 0$, M_3, m^2 and m_0 given in 2, and a $\pm 10\%$ modification of these lattice mass parameters.

- Once the existence of the EOS has been insured the causality and Le Chatelier´s principle are satisfied

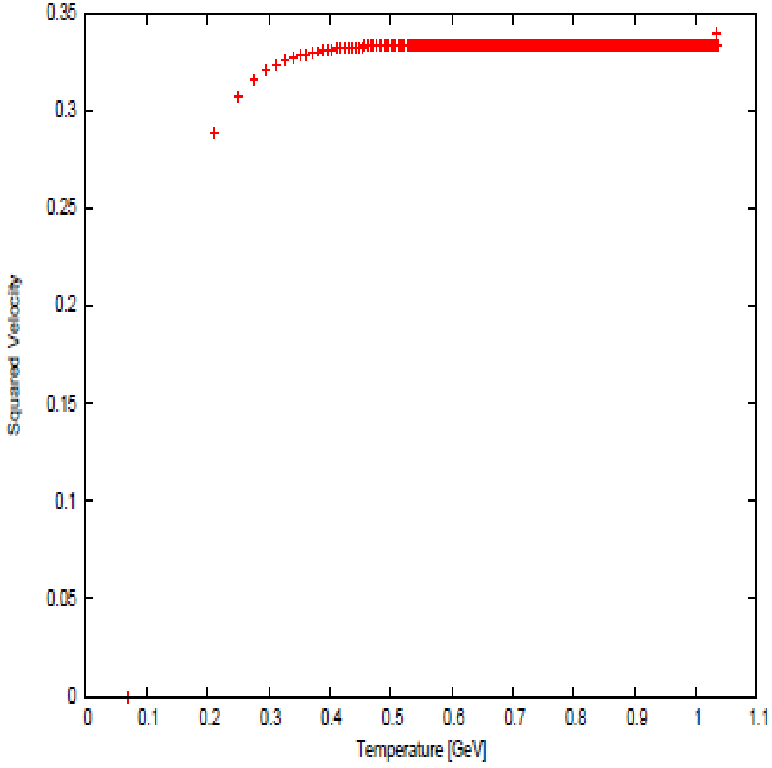


FIG. 5. The squared sound velocity of the quark sector as function of the temperature for $\mu = 0$.

Adding gluons

- In the case of gluons the propagator can be calculated from first principles using the Gribov Prescription i.e. restrict the path integral to a region of configuration space where the gauge fixing is not ambiguous.
- This prescription can be implemented with local action principle (Zwanziger)
- GZ action introduces extra fields which if “taken seriously” may have non zero VEV (condensates) giving rise to RGZ

Gauge theories and gauge fixing

- Yang Mills theories are described by a gauge potential.

$$L = -\frac{1}{4g^2} trF_{\mu\nu}F^{\mu\nu}, \quad (F_{\mu\nu})^a = (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^a$$

- This gauge potential is not unique as two gauge potentials related by a gauge transformations describe the same physical configuration.

$$A_\mu \to A'_\mu = U^\dagger A_\mu U + U^\dagger \partial_\mu U, \quad U^\dagger = U^{-1}$$

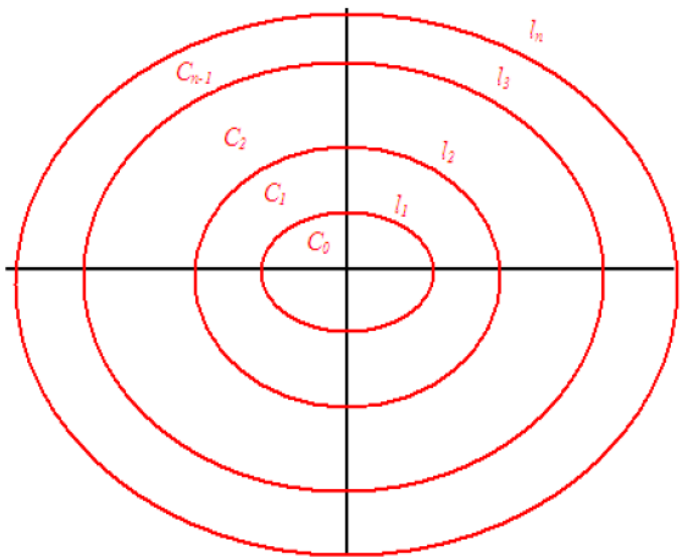
- A gauge fixing is needed. The most famous gauge fixing is the Coulomb or Lorenz gauge

$$\partial_\mu A^\mu = 0; \quad \partial_i A^i = 0$$

- In non-Abelian theories this works well locally but NOT globally

The Gribov Horizon and confinement

- The gauge potential space can be divided in the form



- Every region is a copy of the fundamental region. In order to avoid overcounting of states in path integral it must be restricted up to the first horizon. The implementation of this procedure modifies the gluon propagator which becomes suppressed in the infrared (CONFINEMENT!).

Refined GZ propagator

- Propagator form the (refined) Gribov-Zwanziger approach

$$\Delta^{ab}_{\mu\nu}(p) = \delta^{ab}\frac{p^2+N^2}{p^4+p^2(N^2+m^2)+m^2N^2+\lambda^4}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right) = \delta^{ab}\Delta(p^2)\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right) ,$$

$$N^2 = 2.51\ \mathrm{GeV}^2 ,$$
$$m^2 = -1.92\ \mathrm{GeV}^2 ,$$
$$\lambda^4 = 5.3\ \mathrm{GeV}^4 .$$

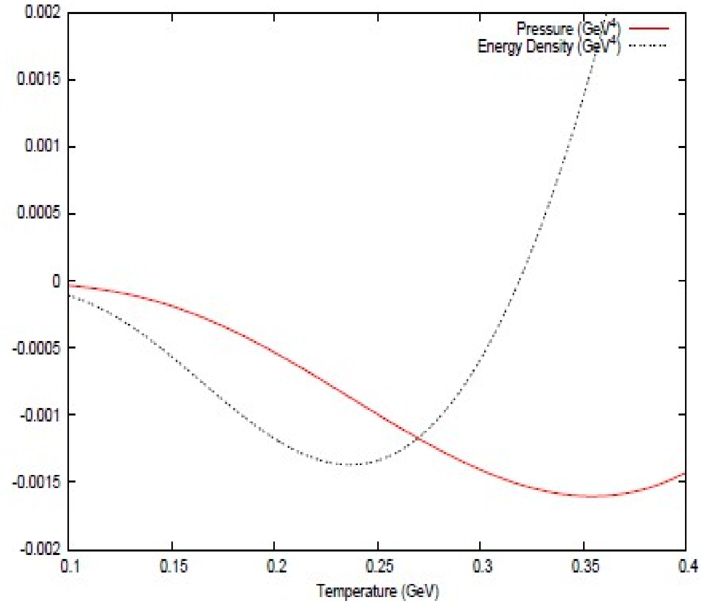
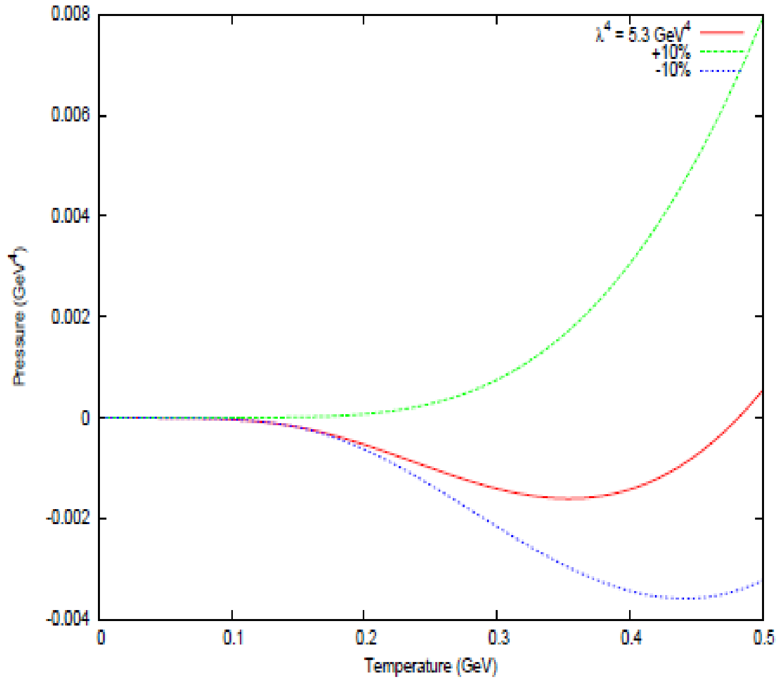
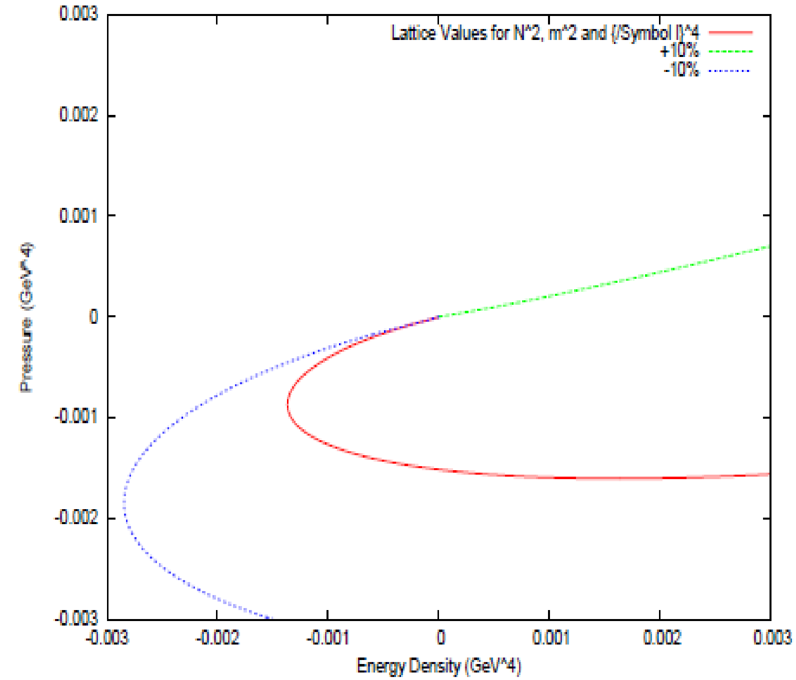


FIG. 10. The gluon pressure (red line) and gluon energy density (black dots) as function of temperature for $\mu = 0$ in RGZ approach.



(a)



(b)

FIG. 11. (a) The pressure as function on temperature for $\mu = 0$ and different values of the lattice mass parameters in RGZ approach. We see in this critical region, the EOS is not well-defined. (b) The pressure as a function of the energy density for different values of the RGZ parameters N^2, m^2 and λ^4. We observe an EOS seems to be almost well-defined when the RGZ parameters are modified +10%.

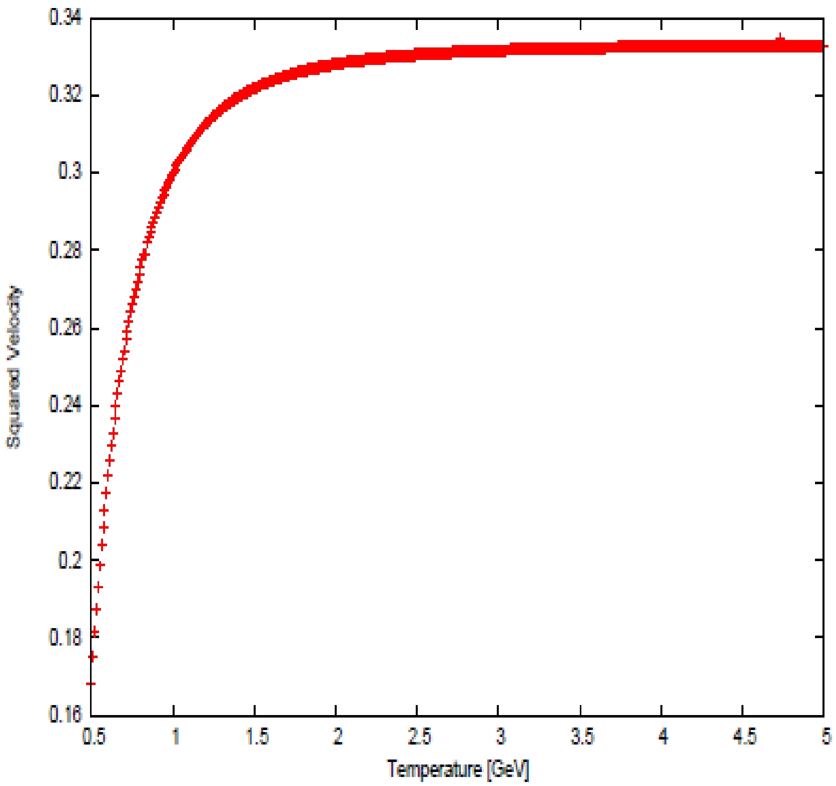


FIG. 8. The squared velocity as a function of the temperature in GZ approach.

Conclusions

- We analyzed the existence and properties of equations of state arising from non perturbative quark and gluon propagators.
- In order to reduce as much as possible the numerical error we used the ζ function regularization method
- In the case of quarks we have found that using the usual value of lattice parameters the pressure and energy density are not monotonic functions of the temperature so there is a range where the equation of state is not single valued.
- Changing the fit parameters of at least 10% the feature disappears.
- In the gluon case we used the RGZ propagator. Also in this case there is a range where the equation of state is not single valued. Also in this case changing the parameters of at least 10% can make the feature disappear.

- The modification of the parameters physically corresponds to change the complex conjugate poles in the non-perturbative propagator for real ones. This is only way to have monotonic functions. The real poles still violate positivity .
- The problem is that the lattice data today are already much more precise than 10% son it does not seem a viable option to modify them to solve the problem of non-existence of eq. of state. The same problem for the RGZ propagator as modifying the parameters would make the propagator off scale with the lattice data.
- An explanation for non-single valued eq. of state can be some missing information when considering strongly interacting matter i.e. some extra physical parameters are necessary to label the equilibrium states.