The Gribov confinement prescription in a Lorentz breaking scenario

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Outline

Yang-Mill quantization / Gribov ambiguities

- Analytical computations require gauge fixing;
- The Landau gauge: Lorentz covariant; the Faddeev-Popov (FP) operator is Hermitian

$$
\begin{aligned}
Z_{\mathrm{FP}} &= V \int \mathscr{D}A^U\, \delta(f^a) |\det -\partial D|\, \mathrm{e}^{-S_{\mathrm{YM}}} \\
&\neq \int \mathscr{D}A \mathscr{D}c \mathscr{D}\overline{c}\, \mathrm{e}^{-S_{\mathrm{FP}}} ,
\end{aligned}
$$

with

$$
S_{\mathrm{FP}} = S_{\mathrm{YM}} + \int d^4x \left\{ \overline{c}^a(x) \partial_\mu D^{ab}_\mu c^b(x) + \frac{(\partial_\mu A^a_\mu)^2}{2\xi} \right\}
$$

Though, the gauge is not completely fixed

The Gribov-Zwanziger action

Gribov's proposal: restrict the path integral of the gauge field to the region

$$\Omega \;=\; \left\{A_\mu,\, \partial_\mu A_\mu \;=\; 0,\; -\partial_\mu D_\mu \;\geq\; 0\right\}$$

The FP operator is closely related to the **inverse of the Ghost two point function**

- **Gribov:** $\left\langle \bar{c}^a(k) c^b(-k) \right\rangle \approx \frac{1}{k^2}\left(\frac{1}{1-\sigma(k,A)}\right)\delta^{ab}\,,\;\; 1-\sigma(k,A) \leq 0$
- **Zwanziger:** It amounts to enforce the positivity over **the trace of the lowest lying eigenvalue of the FP operator** $(\lambda_{\min})$
- Both formulations are equivalents[1]

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[1] Capri et. al Phys.Lett. B719 (2013) 448-453

The Gribov-Zwanziger action

Zwanziger:

- Compute perturbativelly all lowest lying eigenvalue of the FP operator
- Take the trace over all of them and impose positivity over the trace:

$$\mathrm{Tr}\Lambda \;=\; 2\left(\frac{2\pi}{L}\right)^2\left(d(N^2-1)-\frac{1}{V}H(A)\right)>0$$

with **the horizon function**

$$H(A) \;=\; g^2\int d^4x d^4y\; f^{abc}f^{adl}\,A^b_\mu(x)\left[\partial_\nu D_\nu{}^{-1}\right]^{cl}A^d_\mu(y)\delta(x-y)$$

The restricted partition function

$$Z_{\mathrm{GZ}} \;=\; V\int \mathscr{D}A^U\;\theta(dV(N^2-1)-H(A))\,\mathrm{e}^{-S_{\mathrm{FP}}}$$

The Gribov-Zwanziger action: The gap equation

In the thermodynamic limit $\theta \to \delta$

$$Z_{\rm GZ} = V \int \mathscr{D}A^U \exp\left\{ -S_{\rm FP} + \gamma^{*4} H(A) + 4\gamma^{*4} V(N^2-1) \right\} = \mathrm{e}^{-V\mathscr{E}_v}$$

satisfying the gap equation
(a consistency condition of the saddle-point approx.)

$$\frac{\partial \mathscr{E}_v}{\partial \gamma^2} = 0$$

That amounts to

$$\langle H(A) \rangle = 4(N^2-1)$$

The Localized Gribov-Zwanziger action

In order to localize the GZ action BRST doublets of fields should be added

$$
\begin{aligned}
S_{GZ} &= S_{\mathrm{YM}} + \int d^4x \Big(\bar{\varphi}^{ac}_{\mu} (\partial_\nu D^{ab}_{\nu}) \varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu} (\partial_\nu D^{ab}_{\nu}) \omega^{bc}_{\mu} \\
&- g f^{amb} (\partial_\nu \bar{\omega}^{ac}_{\mu}) (D^{mp}_{\nu} c^p) \varphi^{bc}_{\mu} \Big) + \gamma^{*2} \int d^4x \left(g f^{abc} A^a_\mu (\varphi^{bc}_{\mu} + \bar{\varphi}^{bc}_{\mu}) \right) \\
&- 4\gamma^4 V (N^2 - 1)
\end{aligned}
$$

with

$$
\boxed{\begin{aligned}
s\overline{\omega} &= \overline{\varphi} \qquad & s\overline{\varphi} &= 0 \\
s\varphi &= \omega \qquad & s\omega &= 0
\end{aligned}}
$$

The gap equation in a general YM theory

Considering only quadratic terms in the fields, within perturbation theory

$$
\begin{aligned}
Z_{GZ}^{\text{quad}} &= \int [\mathrm{d}A] \left[\det -\partial^2\right] \exp\left\{ -\frac{1}{2}\int \frac{\mathrm{d}^d q}{(2\pi)^d} A_\mu^a(q) K_{\mu\nu}^{ab} A_\nu^b(-q) - 4V\gamma^{*4}(N^2-1)\right\} \\
&= \mathrm{e}^{-V\mathscr{E}_v}
\end{aligned}
$$

with

$$
K_{\mu\nu}^{ab} = \delta^{ab}\left[\left(q^2+\frac{2Ng^2\gamma^{*4}}{q^2}\right)\delta_{\mu\nu}+\left(\frac{1}{\alpha}-1\right)q_\mu q_\nu\right]
$$

So that

$$
\mathscr{E}_v = -4\gamma^4(N^2-1)+\frac{1}{V}\ln\gamma^4+\frac{3(N^2-1)}{4}\int\frac{d^4q}{(2\pi)^4}\ln\left(q^2+\frac{2\gamma^4 Ng^2}{q^2}\right)
$$

The gap equation and gauge field propagator

- The gap equation finally reads

$$\left.\frac{d\mathscr{E}_v}{d\gamma^2}\right|_{\gamma=\gamma^*} = 0$$

$$1 = \frac{3Ng^2}{8}\int\frac{d^d p}{(2\pi)^d}\frac{1}{p^4+2\gamma^{*4}Ng^2}$$

- Taking the inverse of $K^{ab}_{\mu\nu}$ and then recovering the Landau gauge $\alpha \to 0$

$$\begin{aligned}\langle A^a_\mu(k)A^b_\nu(-k)\rangle &= \delta^{ab}\frac{k^2}{k^4+\gamma^4}P_{\mu\nu}(k)\\ &= \delta^{ab}\frac{1}{2}\left(\frac{1}{k^2+i\gamma^{*2}}+\frac{1}{k^2-i\gamma^{*2}}\right)P_{\mu\nu}(k)\end{aligned}$$

Why should we care about Lorentz symmetry breaking?

- It started with V. A. Kostelecky and S. Samuel, Phys. Rev. D **39** (1989) 683.

  "Spontaneous Breaking of Lorentz Symmetry in String Theory,"

  Though it should occur at the order of the Planck scale, 10^{19} GeV.

- It is possible that some IR effects could be due to Lorentz symmetry breaking started at the Planck level;
- Some very high precision experiments on IR effects searching for the breaking of the Lorentz Invariance are on development
  Y. Michimura, *et al.*, Phys. Rev. Lett. **110** (2013) no.20, 200401
  F. Allmendinger *et al.*, Phys. Rev. Lett. **112** (2014) no.11, 110801
- After Kostelecky's paper more than 500 works have cited Lorentz symmetry breaking, thus it is worth have a look ;)
  V. Vasileiou, J. Granot, T. Piran and G. Amelino-Camelia, Nature Phys. **11** (2015) no.4, 344.

Lorentz symmetry breaking

- In the 90s Carroll, Field and Jackiw S. Carroll, G. Field, R. Jackiw, Phys. Rev. D **41**, 1231 (1990) proposed the first consistent effective model of Lorentz symmetry breaking by introducing a constant axial vector field a_μ defining the preferred direction.

$$S_{LB} = \int d^4x \left[\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 - \frac{i\xi}{2} a_\rho \varepsilon_{\rho\mu\nu\lambda} \left(A^a_\nu \partial_\lambda A^a_\mu - \frac{2}{3} g A^a_\mu A^b_\nu A^c_\lambda f^{abc} \right) \right]$$

Still has gauge invariance (by total derivative)

- The gauge fixed action reads

$$S = S_{LB} + \int d^4x \left(\frac{(\partial_\mu A_\mu)^2}{2\alpha} + \bar{c}^a \partial_\mu D^{ab}_\mu c^{ab} \right)$$

it is plagued by *FP's zero modes*

The Gribov restriction

The restricted partition function reads

$$Z = \mathcal{N} \int \mathscr{D}A^U \, \theta(dV(N^2-1) - H(A)) \, e^{-S}$$

or even

$$Z = \mathcal{N} \int \mathscr{D}A^U \exp\left\{ -S - \gamma^4 H(A) + 4\gamma^4 V(N^2-1) \right\}$$

I First, we should localize the action (including the auxiliary fields $\bar{\phi}, \phi$ and $\bar{\omega}, \omega$

II Consider only quadratic terms of the action, and the constant term (working within perturbative regime)

The Gribov restriction

The partition function then reads

$$Z = \mathscr{N} \int \mathscr{D}\phi \exp\left\{ \int \frac{d^4k}{(2\pi)^4} \left(-\frac{1}{2} A^a_\mu(k) K^{ab}_{\mu\nu} A^b_\nu(-k) - \bar{c}^a P^{ab} c^b \right) \right\}$$

with

$$K^{ab}_{\mu\nu} = \delta^{ab} \left(\frac{k^4 + 2g^2 N\gamma^4}{k^2} \delta_{\mu\nu} + \left(\frac{1}{\alpha} - 1 \right) k_\mu k_\nu - \xi a_\rho \varepsilon_{\rho\nu\mu\lambda} k_\lambda \right)$$

and

$$P^{ab} = \delta^{ab} p^2$$

- In order to compute the gauge field propagator the inverse of $K^{ab}_{\mu\nu}$ should be computed

The gauge propagator

The Landau gauge is recovered in the limit $\alpha \to 0$

$$
\begin{aligned}
\langle A^a_\mu(k) A^b_\nu(-k)\rangle &= \delta^{ab} F(k) \Bigg\{ \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{k^4}{(k^4+\gamma^4)} a_\beta \varepsilon_{\beta\nu\mu\alpha} \frac{\xi k_\alpha}{k^2} \frac{k^6}{(k^4+\gamma^4)^2} \\
&\quad + \left[\frac{(\xi a\cdot k) k_\mu}{k^2} - \xi a_\mu\right]\left[\frac{(\xi a\cdot k) k_\nu}{k^2} - \xi a_\nu\right] \Bigg\}
\end{aligned}
$$

- If $a_\mu \to 0$, the GZ results is recovered
- The *red* term is new compared with 3D Yang-Mills_Chern-Symons model [2]

  e*topological* CFJ term

[2] F. Canfora, A. J. Gmez, S. P. Sorella and D. Vercauteren, Annals Phys. **345** (2014) 166

The gauge propagator

Regimes of the theory

- By looking at the parameter space of (ξ, M) we could find regions where the gauge propagator has physical and non-physical one-particle interpretation;
- Analysis of the poles through the overall factor

$$F(k) \;=\; \frac{(k^4+\gamma^4)k^2}{(k^4+\gamma^4)^2+k^6M^2}$$

This is qualitatively equivalent to the results of F. Canfora, *et al.* Annals Phys. **345** (2014) 166:

The gauge propagator

- The poles can be read of from

$$\begin{aligned} P(k^2) &= k^8 + 2Gk^4 + G^2 + M^2 k^6 \\ &= (k^2+m_1^2)(k^2+m_2^2)(k^2+m_3^2)(k^2+m_4^2) \end{aligned}$$

- The regions can be identified through the discriminant

$$\Delta = 256 G^5 M^4 - 27 G^4 M^8$$

Analysing the roots we have the following possible regions:

1. $G < 27\left(\frac{M}{4}\right)^4$ $(\Delta < 0)$ we have two real roots and two complex conjugate roots.
2. $G > 27\left(\frac{M}{4}\right)^4$ $(\Delta > 0)$ we have all four roots to be real or complex.

G and M are redefinitions of the Gribov parameter and of the topological mass ξa_μ

Final words

Outcomes

- We have studied the rule of Gribov copies in an effective Lorentz broken model;
- We could see that the behavior of the gauge propagator strongly depends on the preferred direction (given by a_μ);
- Regions in the parameter space (ξ and M) where the gauge propagator displays (non)physical one-particle interpretation is equivalent to the results of F. Canfora, *et al.* Annals Phys. **345** (2014) 166

Works in progress

- Take into account the *aether-like term*, such as $\kappa^{\nu\lambda}_{\mu\rho} F^{\mu\rho} F_{\nu\lambda}$ in Yang-Mills;
- Study three-point functions of the gauge field up to one-loop correction

Thanks for all of you :)