Meson-Hybrid Mixing in 1^{--} Heavy Quarkonium from QCD Sum-Rules

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Motivation

- Mesons:
  - Constituent quark model, $q\overline{q}$

Gell-Mann & Zweig (1964)

- Hybrids:
  - Hadrons with explicit quark and gluon constituents, $q\overline{q}g$
  - Allowed by QCD
  - Outside of the constituent quark model
  - Not yet definitively observed

Horn & Mandula (1978)

Meyer & Swanson (2015)

➡ Maybe hybrid states exist as quantum mechanical superpositions of $q\overline{q}$ and $q\overline{q}g$.

➡ Perhaps the resonances we've assigned to conventional $q\overline{q}$ states are actually mixed.

Currents & Cross-Correlator

- Currents: $j_\mu^{(\mathrm{m})} = \overline{Q}\gamma_\mu Q$

$$j_\nu^{(\mathrm{h})} = g_s \overline{Q}\gamma^\rho\gamma^5 t^a \widetilde{G}^a_{\nu\rho} Q \quad \text{with} \quad \widetilde{G}^a_{\nu\rho} = \frac{1}{2}\epsilon_{\nu\rho}{}^{\omega\zeta} G^a_{\omega\zeta}$$

where here Q is a heavy quark field (i.e. charm or bottom)

- Cross-correlator: $\Pi_{\mu\nu}(q) = i \int d^dx\, e^{iq\cdot x} \langle \Omega | T[\, j_\mu^{(\mathrm{m})}(x)\, j_\nu^{(\mathrm{h})}(0)] | \Omega \rangle$

$$= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi(q^2)$$

where Π probes 1^{--} states which couple to the $j_\mu^{(\mathrm{m})}$ and $j_\nu^{(\mathrm{h})}$ currents, i.e. mixing

Cross-Correlator Calculation

- Calculate the cross-correlator within the operator product expansion (OPE) to leading order (LO).

- Include OPE terms proportional to the 4d and 6d gluon condensates and the 6d quark condensate given by:

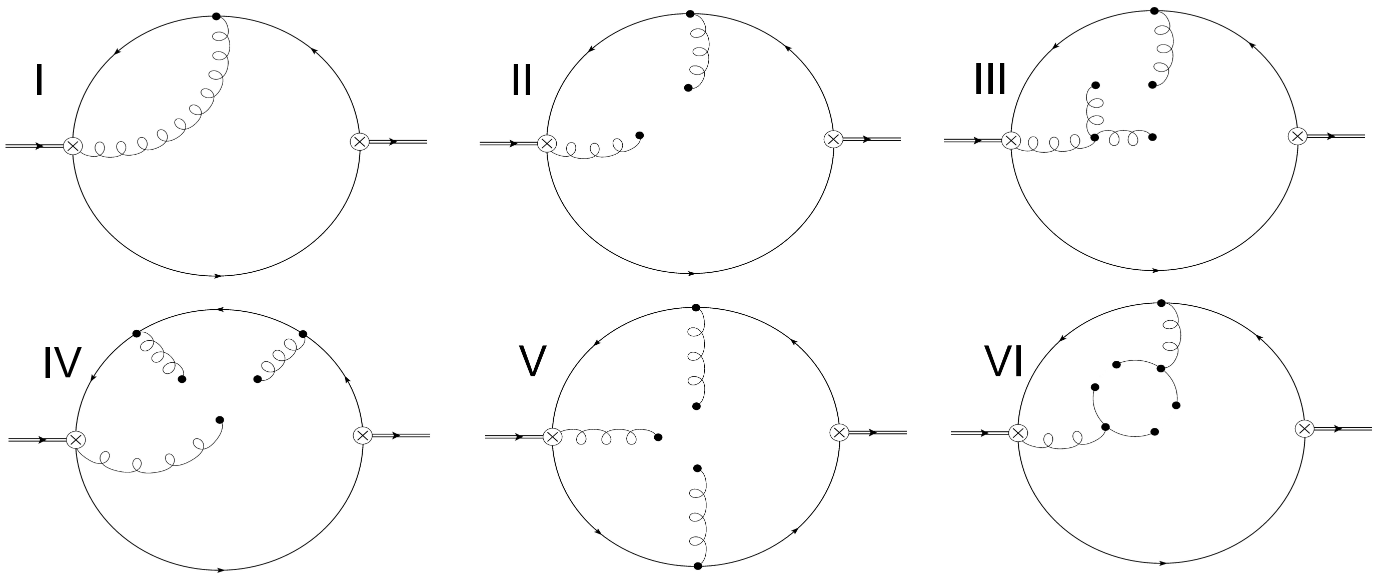
$$\left\langle \alpha G^2 \right\rangle = \alpha_s \left\langle : G^a_{\omega\phi} G^a_{\omega\phi} : \right\rangle$$

$$\left\langle g^3 G^3 \right\rangle = g_s^3 f^{abc} \left\langle : G^a_{\omega\zeta} \, G^b_{\zeta\rho} G^c_{\rho\omega} : \right\rangle$$

$$\left\langle J^2 \right\rangle = \frac{2}{3} \kappa \, g_s^4 \left\langle : \overline{q}_i^\alpha q_i^\alpha : \right\rangle^2$$

- Each of these non-perturbative corrections is the product of a perturbatively computed Wilson coefficient and one of these QCD condensates.

Diagrams



- The cross-correlator will now take the form:

$$\Pi = \Pi^{(\mathrm{I})} + \Pi^{(\mathrm{II})} + \Pi^{(\mathrm{III})} + \Pi^{(\mathrm{IV})} + \Pi^{(\mathrm{V})} + \Pi^{(\mathrm{VI})}$$

- Diagrams arranged in order of magnitude of contribution to LSR.

- Resulting divergent integrals handled using dimensional regularization in $D = 4 + 2\epsilon$ dimensions.

Renormalization

- $\Pi^{(\mathrm{I})}(q^2) = \dfrac{2\alpha_s m^4 z(1+4z)\, {}_2F_1\left(1,1;\frac{5}{2};z\right)}{9\pi^3\epsilon} + \cdots$ where $z = \dfrac{q^2}{4m^2}$

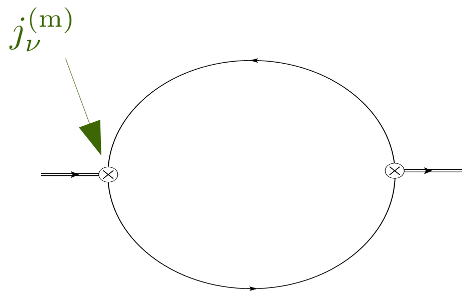
  ➡ Non-polynomial divergence requires renormalization

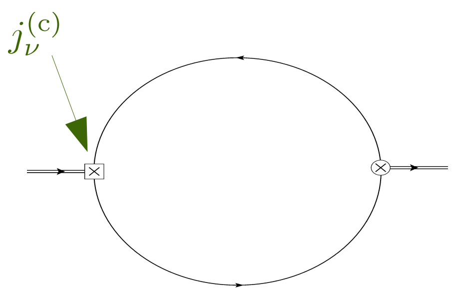
- $j_\nu^{(\mathrm{m})}$ is renormalization group invariant.

- $j_\nu^{(\mathrm{h})} \to j_\nu^{(\mathrm{h})} + \dfrac{C_1}{\epsilon} j_\nu^{(\mathrm{m})} + \dfrac{C_2}{\epsilon} j_\nu^{(\mathrm{c})}$ with $j_\nu^{(\mathrm{c})} = \overline{Q} i D_\nu Q$

  C_1 and C_2 chosen to remove non-polynomial divergences.

- Renormalization induced diagrams:





➡ $C_1 = -\dfrac{10m^2\alpha_s}{9\pi}$ and $C_2 = \dfrac{4m\alpha_s}{9\pi}$

Renormalized Cross-Correlator

$$\Pi^{(\mathrm{I})}(z) = \frac{2\alpha_s m^4 z}{81\pi^3}\Bigg(18(z-1)\,{}_3F_2\left(1,1,1;\tfrac{3}{2},3;z\right) - 2z(4z+1)\,{}_3F_2\left(1,1,2;\tfrac{5}{2},4;z\right) + 3\left(3(4z+1)\log\left(\frac{m^2}{\mu^2}\right) + 26z + 6\right)\,{}_2F_1\left(1,1;\tfrac{5}{2};z\right)\Bigg)$$

$$\Pi^{(\mathrm{II})}(z) = \frac{z\left(3 - {}_2F_1\left(1,1;\tfrac{5}{2};z\right)\right)}{18\pi(1-z)}\langle\alpha G^2\rangle$$

$$\Pi^{(\mathrm{III})}(z) = \frac{\left(-2-5z+4z^2+(2-7z+10z^2-4z^3)\,{}_2F_1\left(1,1;\tfrac{5}{2};z\right)\right)}{2304\pi^2 m^2(1-z^3)}\langle g^3G^3\rangle$$

$$\Pi^{(\mathrm{IV})}(z) = \frac{\left(-22+41z-16z^2+(10-25z+22z^2-8z^3)\,{}_2F_1\left(1,1;\tfrac{5}{2};z\right)\right)}{4608\pi^2 m^2(z-1)^3}\langle g^3G^3\rangle$$

$$\Pi^{(\mathrm{V})}(z) = \frac{\left(-15+12z+(3-2z)\,{}_2F_1\left(1,1;\tfrac{5}{2};z\right)\right)}{4608\pi^2 m^2(z-1)^2}\langle g^3G^3\rangle$$

$$\Pi^{(\mathrm{VI})}(z) = \frac{2\left(2+5z-4z^2+(-2+7z-10z^2+4z^3)\,{}_2F_1\left(1,1;\tfrac{5}{2};z\right)\right)}{81m^2(z-1)^3}\alpha_s^2\langle\overline{q}q\rangle^2$$

Laplace Sum-Rule (LSR)

- The correlator satisfies the dispersion relation

$$\Pi(Q^2) = \frac{Q^6}{\pi}\int_{4m^2}^{\infty}\frac{\mathrm{Im}\Pi(t)}{t^3(t+Q^2)}dt + \cdots\,, \quad Q^2 > 0$$

- Unsubtracted LSR:

Borel transform

$$R_0(\tau) = \frac{1}{\tau}\boxed{\lim_{\substack{N,Q^2\to\infty\\ \tau=N/Q^2}}\frac{(-Q^2)^N}{\Gamma(N)}\left(\frac{d}{dQ^2}\right)^N}\left[-Q^2\Pi(Q^2)\right] = \int_{t_0}^{\infty} e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi(t)dt$$

- Apply resonance plus continuum model:

$$\frac{1}{\pi}\mathrm{Im}\Pi(t) = \rho^{\mathrm{had}}(t) + \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)\theta(t-s_0)$$

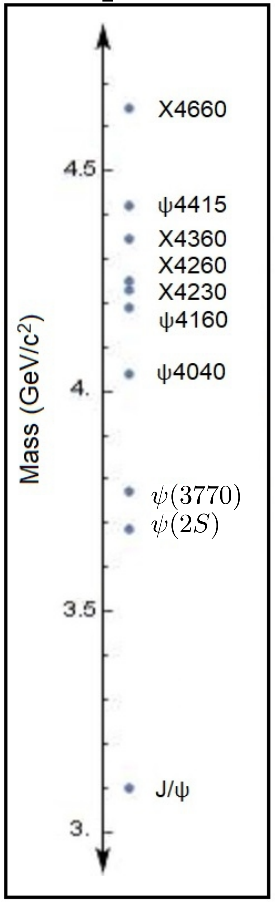
- Continuum subtracted LSR:

$$R_0(\tau, s_0) = R_0(\tau) - \int_{s_0}^{\infty} e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)dt = \int_{t_0}^{s_0} e^{-t\tau}\rho^{\mathrm{had}}(t)dt$$

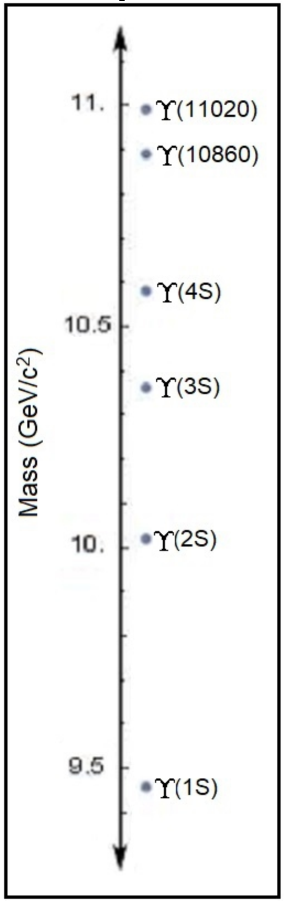
Modelling 1^{--} Heavy Quarkonium

- Replace $\int_{t_0}^{s_0} e^{-t\tau} \rho^{\text{had}}(t)dt$ with several different resonance models
- Chosen models depend on experimental data
- Models consist of between 1 and 4 wide and/or narrow "resonances"
- Wide "resonances" cluster experimental results that are close together

Charmonium 1^{--} spectrum



Bottomonium 1^{--} spectrum



(All values from PDG)

Analysis

- The hadronic side of the LSR now takes the form:

$$R_0(\tau, s_0) = \sum_i \xi_i \; F_i(\tau) \;, \qquad i \in \{1,2,3,4\}$$

➡ s_0 and ξ_i (mixing parameters) are extracted as best fit parameters

- ξ_i are a measure of mixing in a given "resonance" or "cluster"

➡ F_i represent either narrow or wide "resonances" in our model given by:

$$F_i(\tau) \to e^{-M_i^2 \tau} \quad \text{or} \quad F_i(\tau) \to e^{-M_i^2 \tau} \frac{\text{Sinh}(\Gamma_i M_i \tau)}{\Gamma_i M_i \tau}$$

Example: (2 narrow, 1 wide model, used in charmonium sector)

$$R_0(\tau, s_0) = \xi_1 \; e^{-M_1^2 \tau} + \xi_2 \; e^{-M_2^2 \tau} + \xi_3 \; e^{-M_3^2 \tau} \frac{\text{Sinh}(\Gamma_3 M_3 \tau)}{\Gamma_3 M_3 \tau}$$

Borel Window

- Lower bound: Require resonance contribution be no less than 10% that of the total (resonance + continuum).

- Upper bound: Require that the perturbative contribution be no less than 3 times that of the 4d contribution and 9 times that of the 6d contribution.

(GeV^{-2})	$\tau_{\min}$	$\tau_{\max}$
Charmonium	0.1 ± 0.05	0.6 ± 0.05
Bottomonium	0.01 ± 0.005	0.2 ± 0.005

- Borel window then discretized and χ^2 minimization used to find the optimal s_0 and the ξ_i (mixing parameters) are generated

QCD Parameters

Charmonium Parameters

μ	(1.275 ± 0.100) GeV
$\alpha(M_\tau)$	(0.330 ± 0.014)
$\overline{m}_c$	(1.275 ± 0.025) GeV

Bottomonium Parameters

μ	(4.18 ± 0.10) GeV
$\alpha(M_Z)$	(0.1185 ± 0.0006)
$\overline{m}_b$	(4.18 ± 0.03) GeV

Condensates

$\langle \alpha G^2 \rangle$	(0.075 ± 0.02) GeV^4
$\langle g^3 G^3 \rangle$	$((8.2 \pm 1.0)$ GeV$^2)\langle \alpha G^2 \rangle$
$\langle : \overline{q}_i^\alpha q_i^\alpha : \rangle$	$-(0.23 \pm 0.03)^3$ GeV^3

Charmonium: Models & Results

Model			Fit		Results			
m_1 , Γ_1 (GeV)	m_2 , Γ_2 (GeV)	m_3 , Γ_3 (GeV)	s_0 (GeV^2)	$\chi^2 \times 10^6$ (GeV^{12})	ζ (GeV^6)	$\frac{\xi_1}{\zeta}$	$\frac{\xi_2}{\zeta}$	$\frac{\xi_3}{\zeta}$
3.10 , 0	-	-	12.5	4.33	0.514 ± 0.021	1	–	–
3.10 , 0	3.73 , 0	-	13.9	3.17	0.734 ± 0.040	0.726 ± 0.034	0.274 ± 0.034	–
3.10 , 0	3.73 , 0	4.30 , 0	24.1	0.164	2.88 ± 0.25	0.215 ± 0.012	-0.022 ± 0.049	0.762 ± 0.030
3.10 , 0	3.73 , 0	4.30 , 0.30	24.2	0.162	2.97 ± 0.26	0.210 ± 0.012	-0.032 ± 0.048	0.758 ± 0.025
3.10 , 0	3.73 , 0.05	4.30 , 0.30	24.2	0.162	2.97 ± 0.26	0.210 ± 0.012	-0.032 ± 0.048	0.758 ± 0.025
3.10 , 0	-	4.30 , 0	23.7	0.184	2.68 ± 0.25	0.228 ± 0.019	–	0.772 ± 0.019
3.10 , 0	-	4.30 , 0.30	23.6	0.204	2.66 ± 0.25	0.228 ± 0.020	–	0.772 ± 0.019

J/ψ — $\psi(2S)$ $\psi(3770)$ — 7 state cluster $\psi(4040) \to X(4660)$ — $\zeta = \sum_{i=1}^{n} |\xi_i|$ — Ratios of mixing parameters ξ_i

- Inclusion of the 3rd resonance cluster significantly improves the fit
- Inclusion of resonance widths has almost no impact on the fit
- Inclusion of a 4th resonance causes χ^2 to minimize at $s_0 \approx m_4^2$
- The 1st 2 resonance model also merges with the continuum

True for bottomonium sector too

- Non-zero mixing in the J/ψ
- No evidence for mixing in the $\psi(2S), \psi(3770)$ cluster
- Large mixing parameter for the 4.3 GeV cluster
- Excluding the $\psi(2S), \psi(3770)$ cluster still captures the physics

Note:
The X(4260) has often been interpreted as having significant hybrid content, our results are consistent with this idea.

Bottomonium: Models & Results

Model			Fit		Results			
m_1 , Γ_1 (GeV)	m_2 , Γ_2 (GeV)	m_3 , Γ_3 (GeV)	s_0 (GeV^2)	$\chi^2 \times 10^4$ (GeV^{12})	ζ (GeV^6)	$\frac{\xi_1}{\zeta}$	$\frac{\xi_2}{\zeta}$	$\frac{\xi_3}{\zeta}$
9.46 , 0	-	-	107	42.0	140 ± 1	1	$-$	$-$
9.46 , 0	10.02 , 0	-	100	36.5	189 ± 9	0.774 ± 0.014	-0.226 ± 0.014	$-$
9.46 , 0	10.02 , 0	10.47 , 0	132	0.086	1377 ± 33	0.203 ± 0.002	-0.380 ± 0.003	0.418 ± 0.005
9.46 , 0	10.02 , 0	10.47 , 0.22	132	0.088	1375 ± 32	0.203 ± 0.002	-0.379 ± 0.003	0.418 ± 0.005

$\Upsilon(S1)$ $\Upsilon(S2)$ 2 state cluster $\Upsilon(S3)$ & $\Upsilon(S4)$ $\zeta = \sum_{i=1}^{n} |\xi_i|$ Ratios of mixing parameters ξ_i

- Inclusion of the 3rd resonance cluster significantly improves the fit
- Inclusion of resonance widths has almost no impact on the fit
- Inclusion of a 4th resonance causes χ^2 to minimize at $s_0 \approx m_4^2$
- The 2 resonance model also merges with the continuum

As in the charmonium sector

- No clearly unmixed states here
- We find non-zero mixing for all 3 resonances, the $\Upsilon(S1)$, the $\Upsilon(S2)$ and the 2 state cluster of the $\Upsilon(S3)$ & $\Upsilon(S4)$

Summary

- The best agreement between QCD and experiment is achieved when using 3 resonance fits in both the charmonium and bottomonium sectors.

- **Charmonium:**

  Mixing in the charmonium sector is well described by a 2 resonance model including the J/ψ and a 4.3 GeV resonance. This result is consistent with interpretations of the X(4260) as having significant hybrid content.

- **Bottomonium:**

  The $\Upsilon(S1)$, $\Upsilon(S2)$ and the $\Upsilon(S3)$, $\Upsilon(S4)$ cluster all have non-zero mixing parameters.

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