

# QCD SUM RULES AT FINITE TEMPERATURE: A REVIEW OF RECENT APPLICATIONS

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# QCD SUM RULES

(Shifman, Vainshtein, Zakharov)

1979 – to date

(a few kP)

- **ANALYTICAL METHOD TO *SOLVE* QCD AT FERMI SCALES**
- **OPERATOR PRODUCT EXPANSION OF CURRENT CORRELATORS AT SHORT DISTANCES**
- **CAUCHY THEOREM IN THE COMPLEX ENERGY PLANE**
- **(QUARK-HADRON DUALITY) R. Shankar, Phys. Rev. D 15, 755 (1977).**
- **COMPLEMENTARY TOOL TO LATTICE QCD**

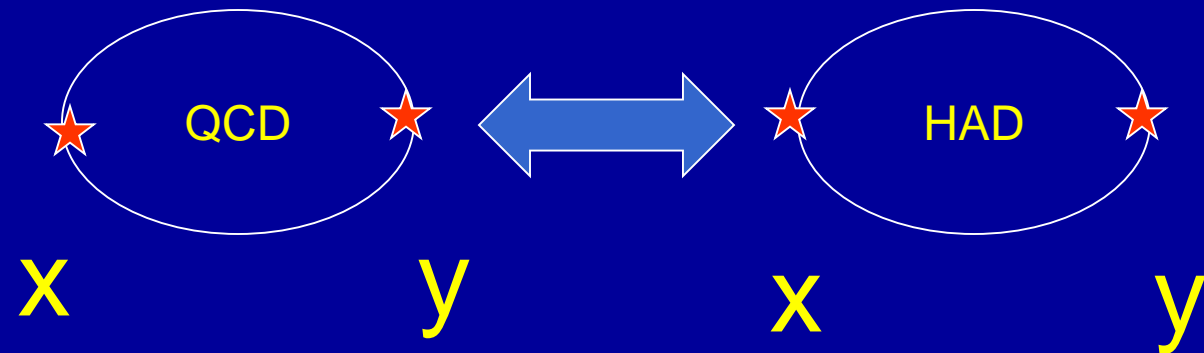
# QCD SUM RULES

## CURRENT CORRELATION FUNCTIONS

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T(J(x) J^\dagger(0)) | 0 \rangle$$

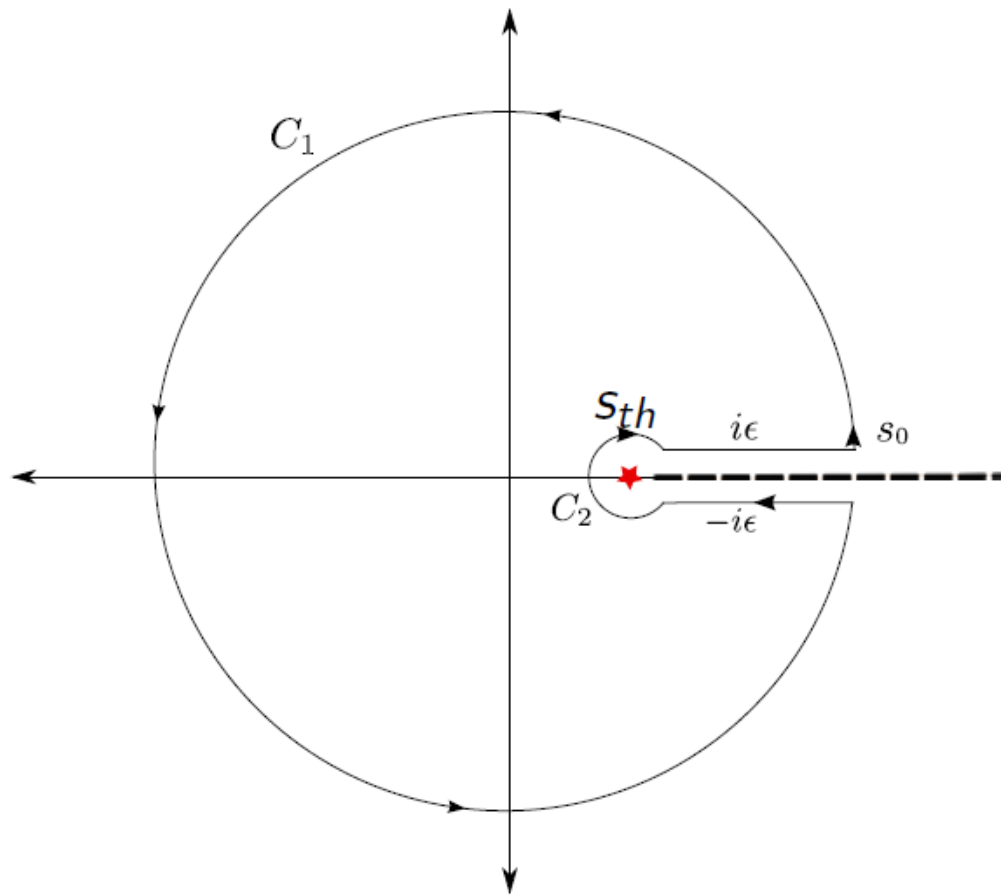
$$J(x) : \bar{\psi}(x) \gamma_\mu \psi(x); \quad \bar{\psi}(x) \gamma_\mu \gamma_5 \psi(x); \quad G_{\mu\nu}^a(x) G_{\mu\nu}^a(x); \text{ etc.}$$

$$\Pi(q^2)_{QCD} \Leftrightarrow \Pi(q^2)_{HAD}$$



# CAUCHY'S THEOREM IN THE COMPLEX ENERGY PLANE

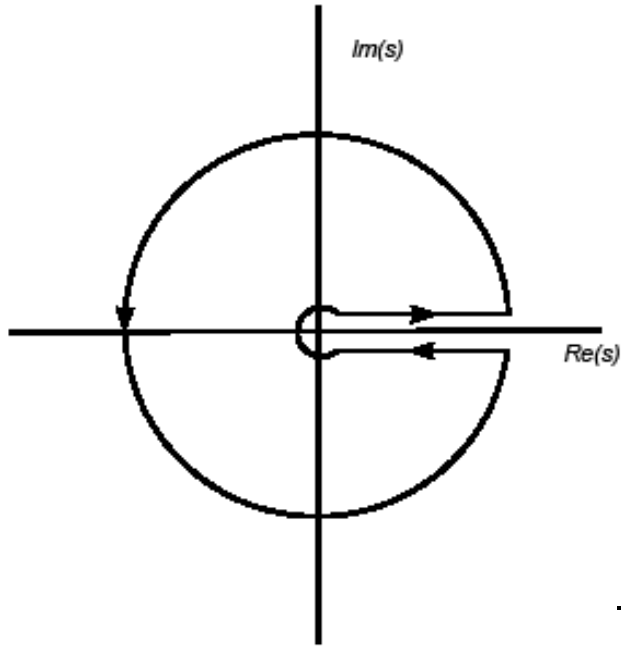
R. SHANKAR 1977



## The Residue Theorem

$$\oint \Pi(s)p(s) ds = 2\pi i \cdot \text{Res}[\Pi(s)p(s), s = 0] \quad (4)$$

# QUARK-HADRON DUALITY



$$\oint_C \Pi(s) ds = 0$$

$$-\frac{1}{2\pi i} \oint_{C(|s_0|)} ds \Pi(s) = \int_{s_{th}}^{s_0} ds \frac{1}{\pi} \text{Im} \Pi(s)$$

## Q C D: OPERATOR PRODUCT EXPANSION

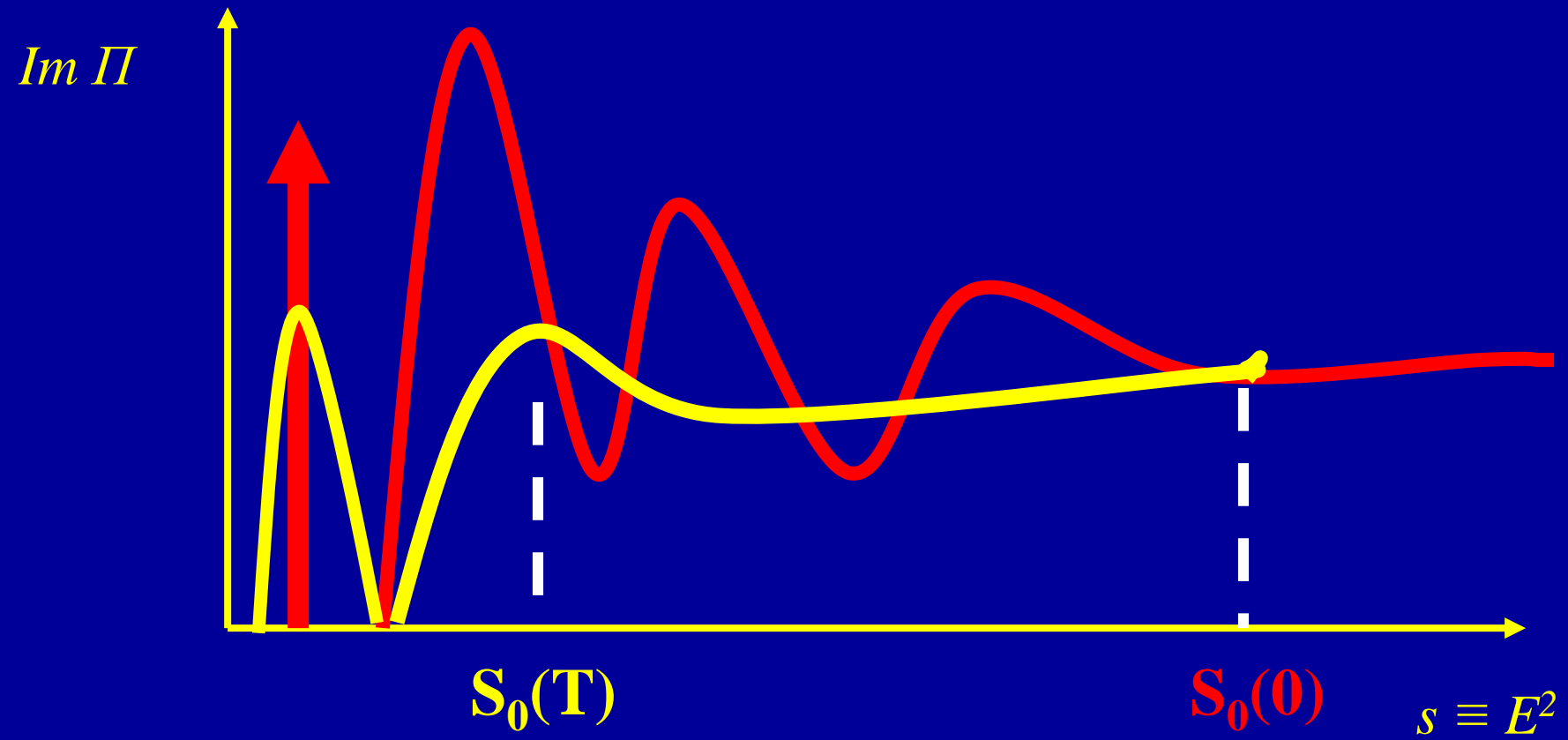
$$\Pi(q^2) = \int d^4x e^{iqx} \langle 0 | T (J(x) J^+(0)) | 0 \rangle$$

$$\Pi(q^2)|_{QCD} = I + \sum_{N=0} C_{2N+2}(q^2, \mu^2) \langle 0 | \hat{O}_{2N+2}(\mu^2) | 0 \rangle$$

$$I \Rightarrow O(\alpha_s^4) \quad C_{2N+2} \Rightarrow \frac{1}{(-q)^{2N+2}}$$

$$m_q \langle 0 | \bar{q} q | 0 \rangle, \quad \langle 0 | \alpha_s G_{\mu\nu} G^{\mu\nu} | 0 \rangle, \quad \text{etc.}$$

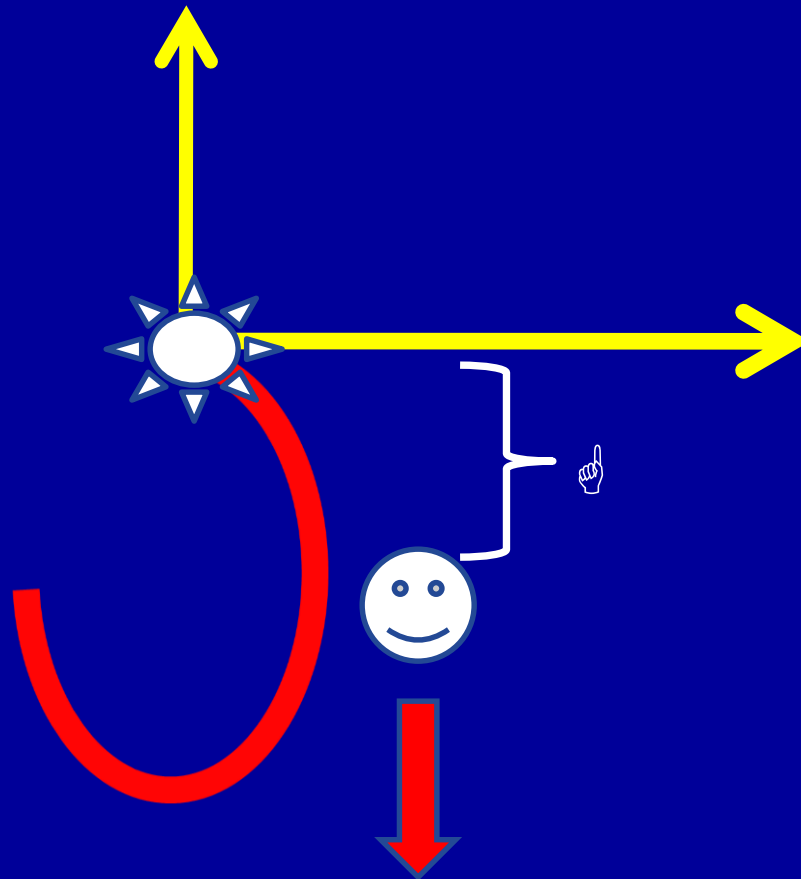
# Realistic Spectral Function (T)





# DECONFINEMENT ORDER PARAMETERS

$S_0(T)$  & WIDTH(T) [M(T)] & f(T)



$$S_0(T)$$

**Threshold for PQCD: Deconfinement Order Parameter**

[Bochkarev & Shaposnikov (1986); Dominguez & Loewe (1989- to date)]

**Related to LQCD (Polyakov Loop)**

[ **Carlomagno & Loewe (2016-2017)**]

# RESONANCE BROADENING

(Dominguez & Loewe 1988)

- **WIDTH:  $\Gamma(T)$       LIFETIME:  $\tau = \Gamma^{-1}$**
- **HADRONIC INTERPRETATION:**

## ABSORPTION OF PARTICLES IN MEDIUM

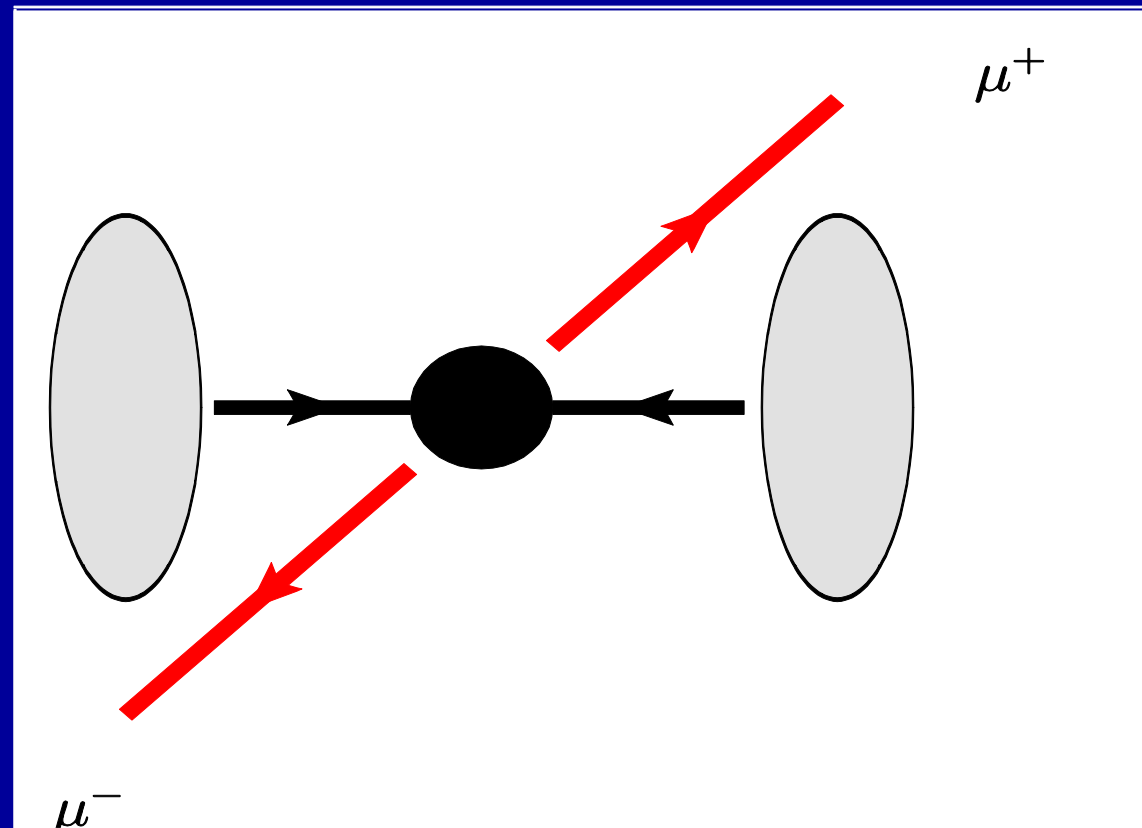
- **QCD INTERPRETATION:**
  - QUARK-GLUON DECONFINEMENT

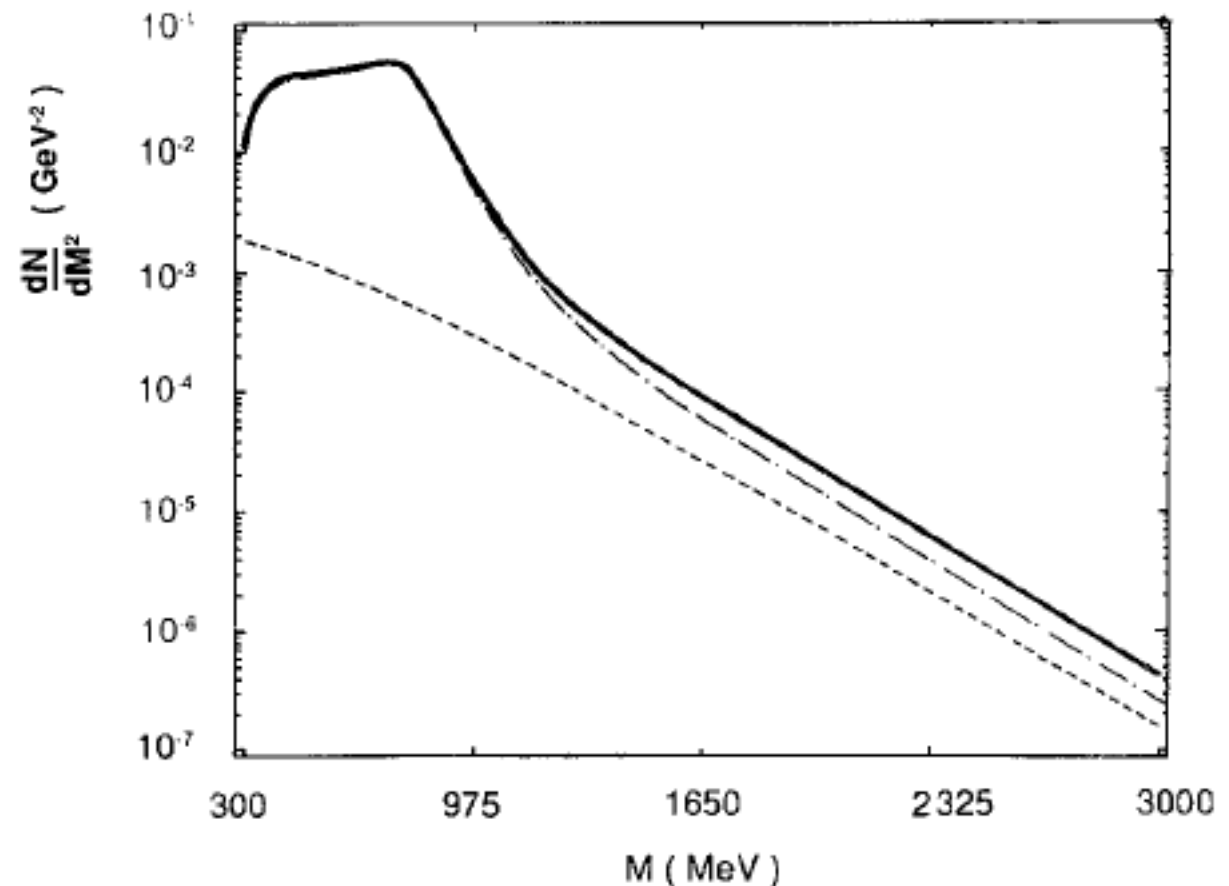
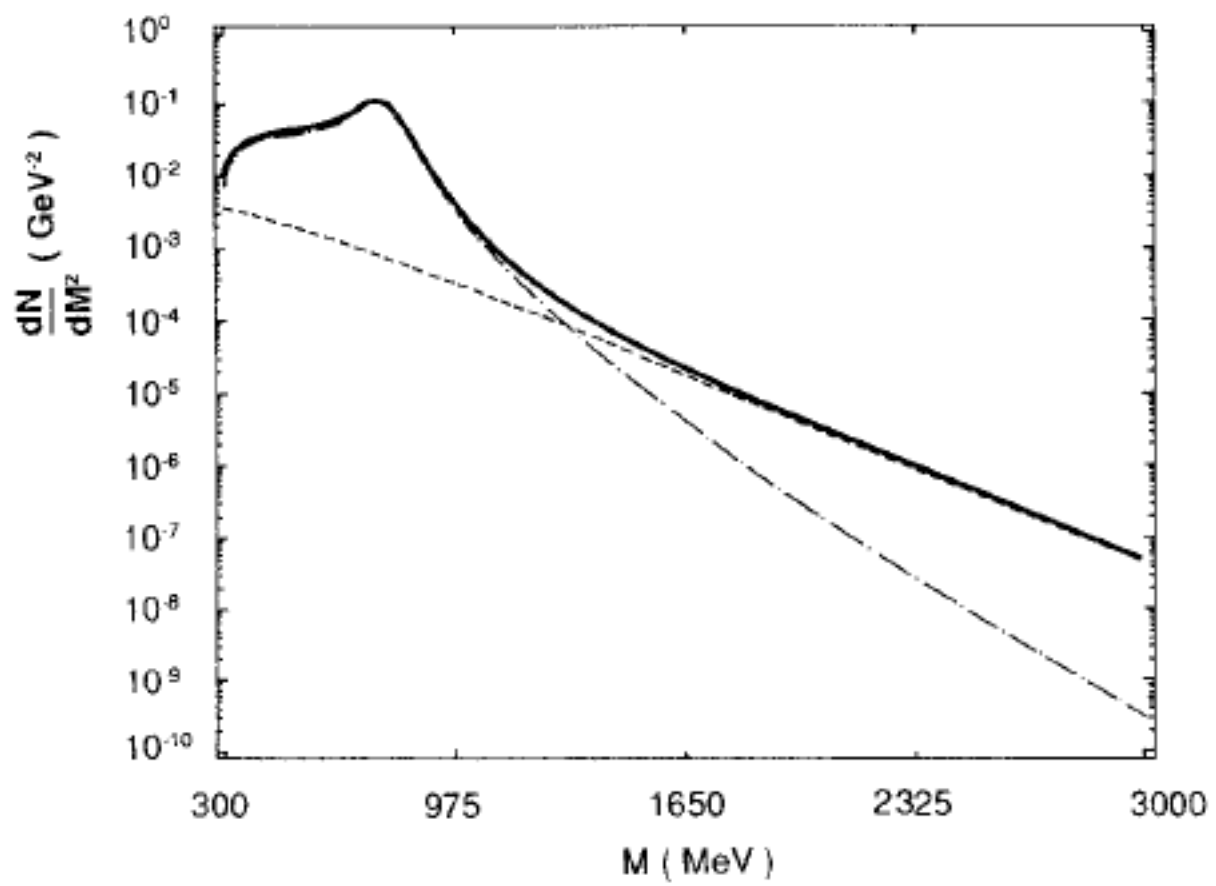
# DIMUON PRODUCTION

J. Cleymans, J. Fingberg, K. Redlich (1987)  
(without resonance broadening)

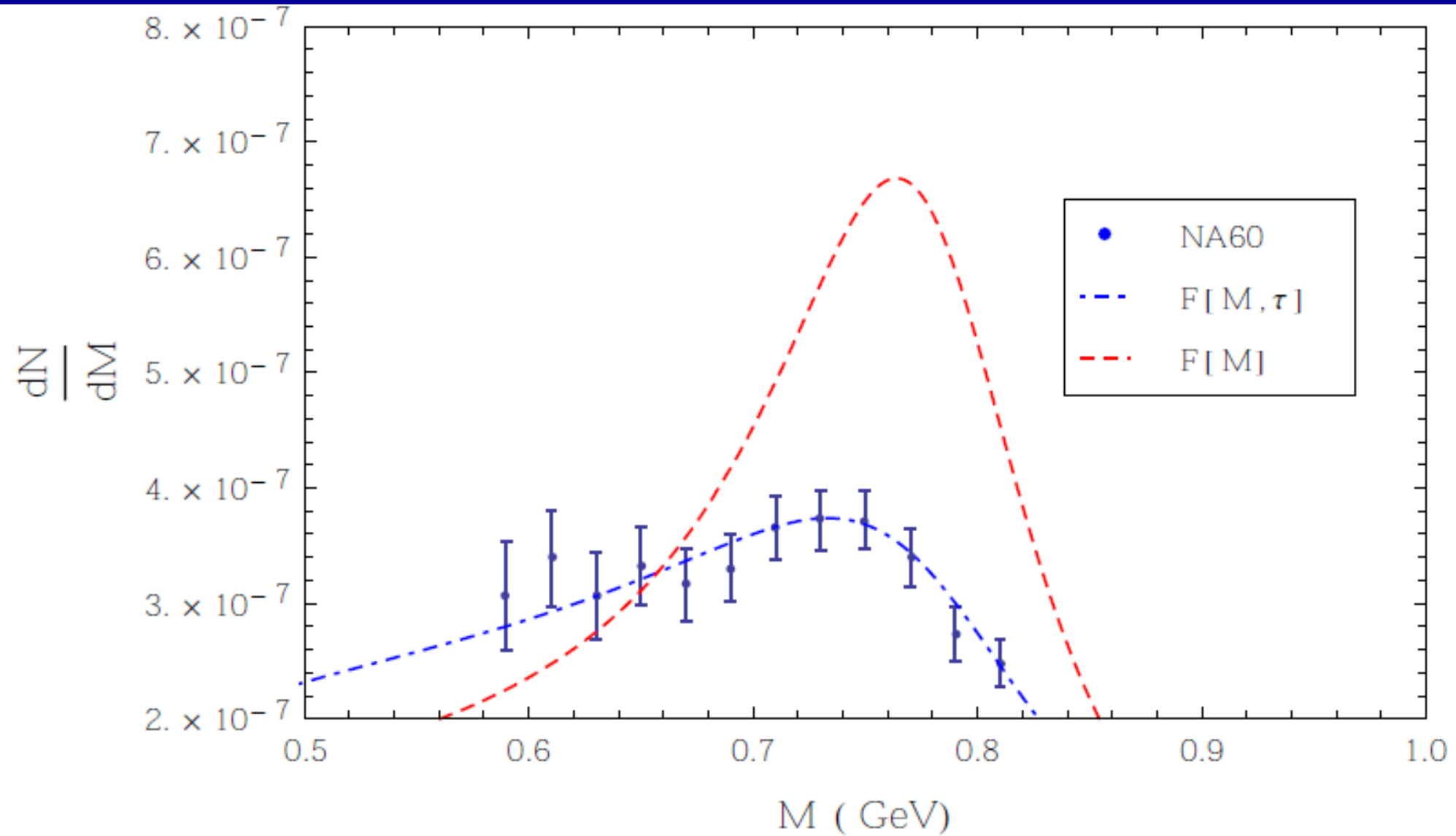
C.A. Dominguez & M. Loewe (1990)  
(with resonance broadening)

Na60 (CERN) 2006 - 2009

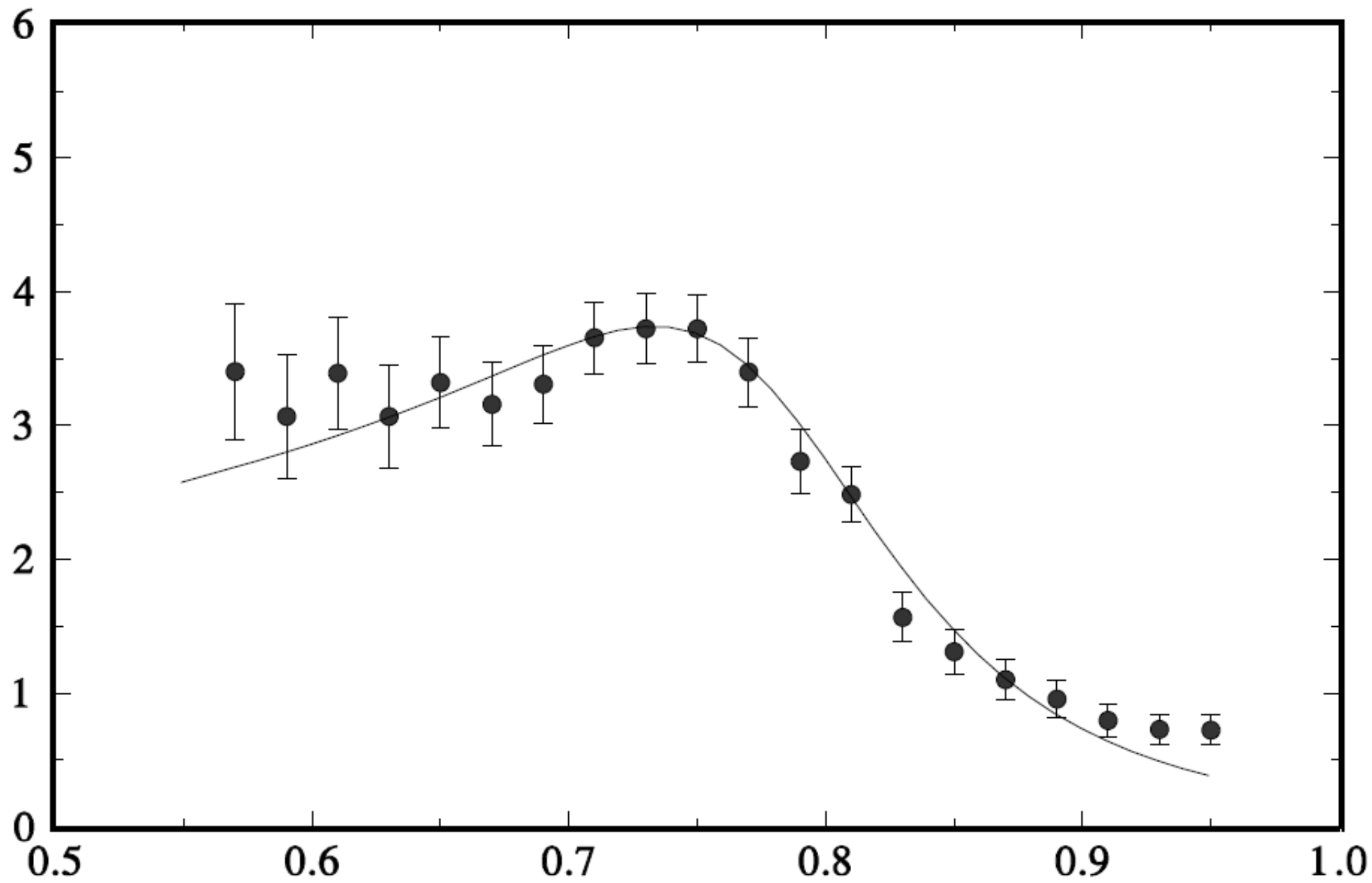




[Ayala, Dominguez, Hernandez, Loewe, Mizher PRD (2013)]



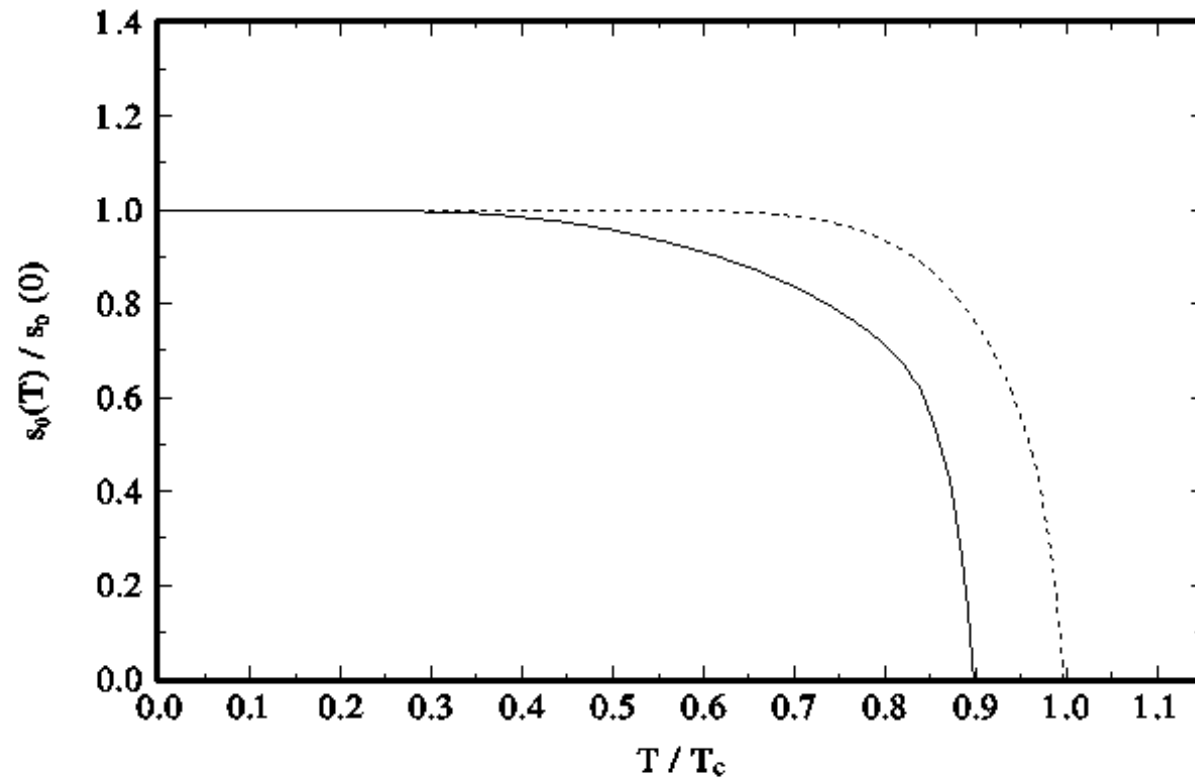
$dN/dM$  (in  $10^{-7} \text{ GeV}^{-1}$ )



**f(T)**

**e.g. CURRENT-HADRON COUPLING:  $f_\pi(T) \propto \langle \bar{q} q \rangle(T)$**

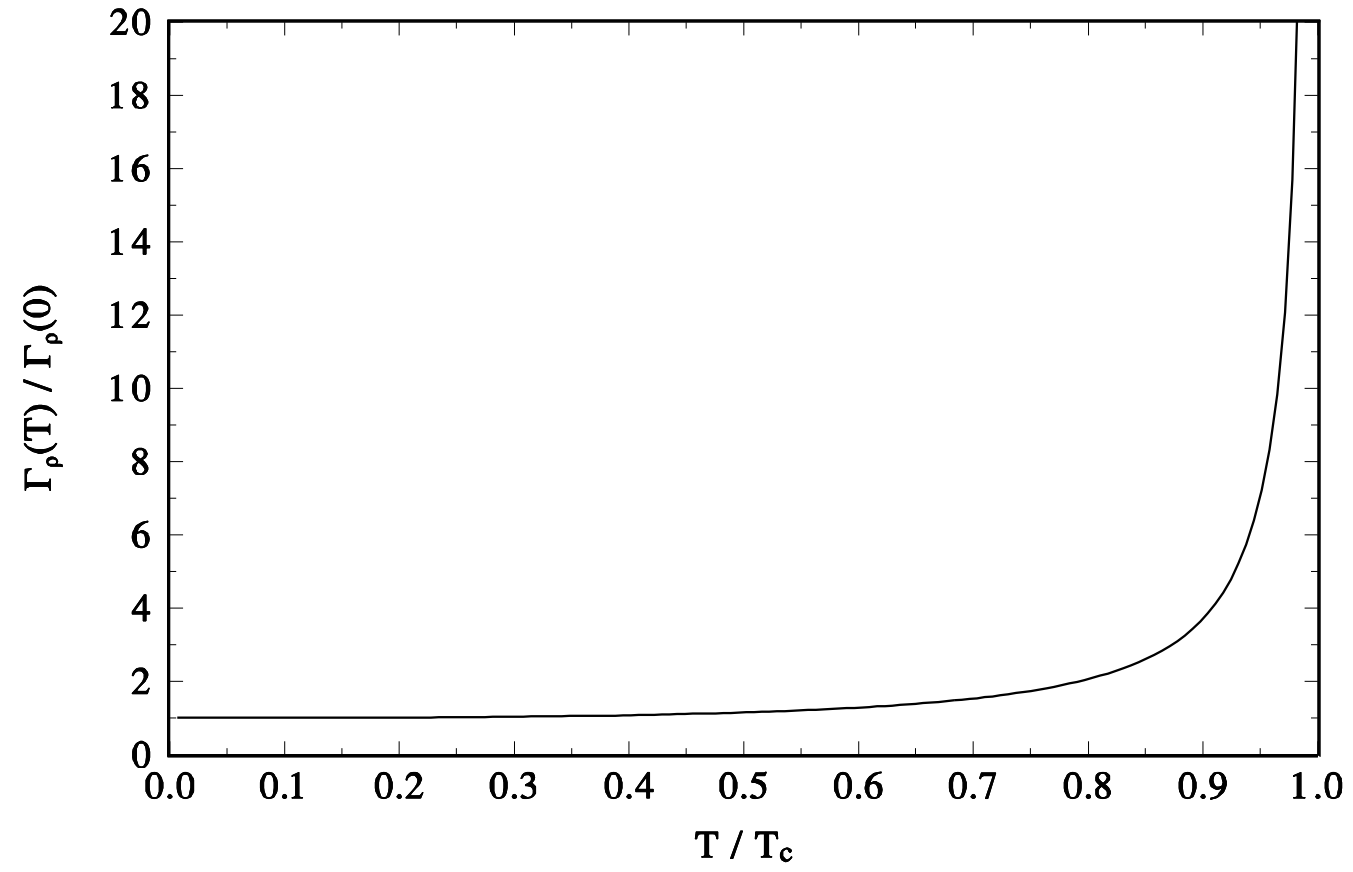
**$f_\pi(T)$  (---)  $\propto s_0(T)$  (---)** [Dominguez & Loewe (1989)]



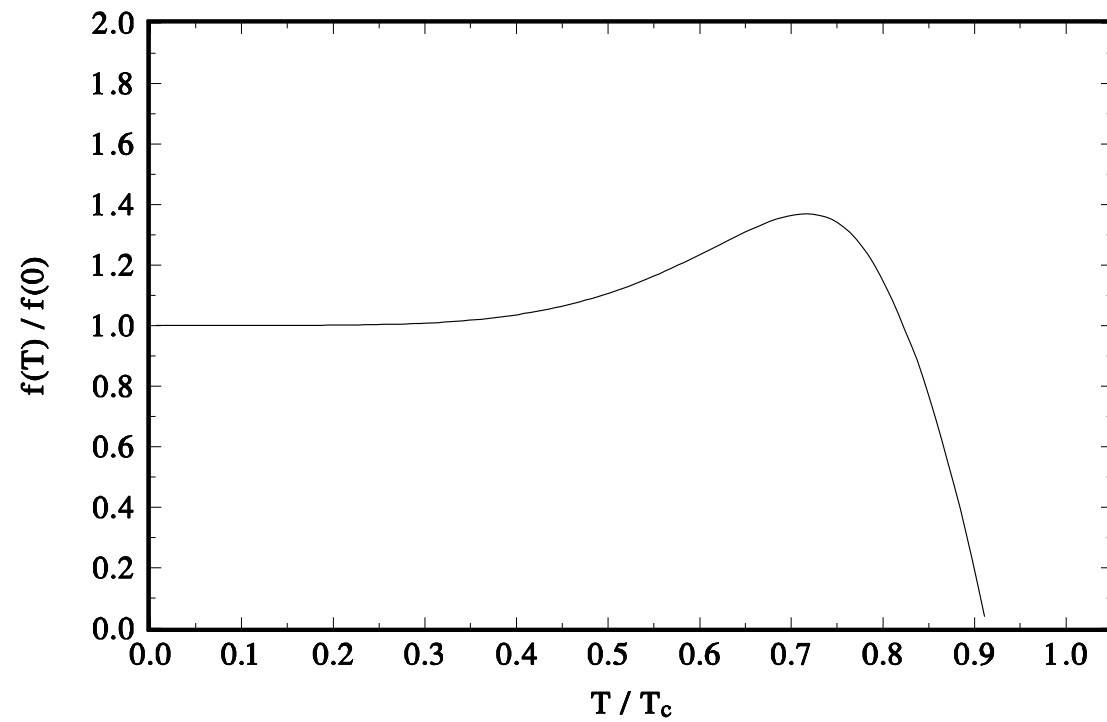


# **Light & Heavy-Light Quark Hadrons (T)**

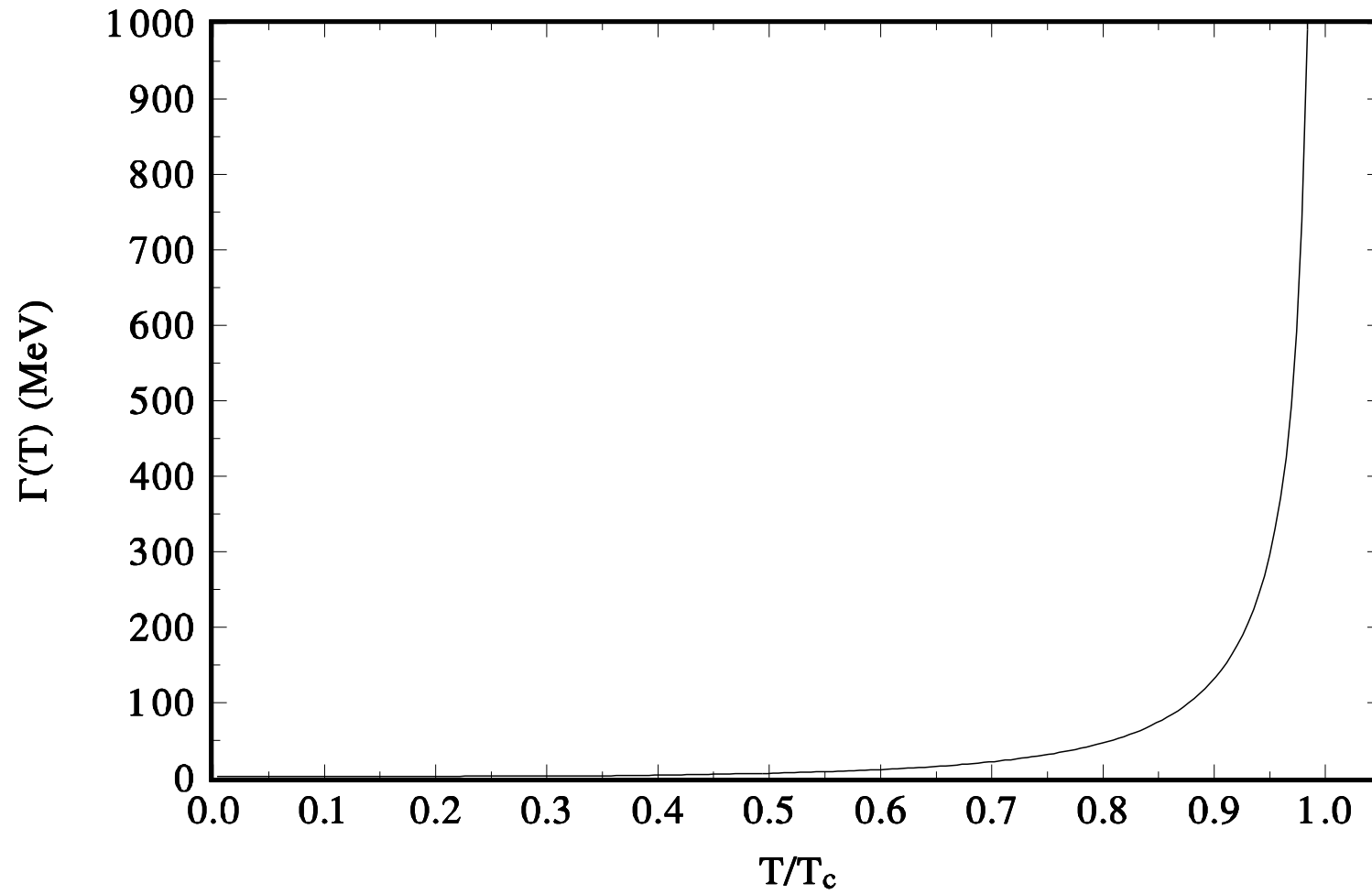
$\Gamma_\rho(T)$



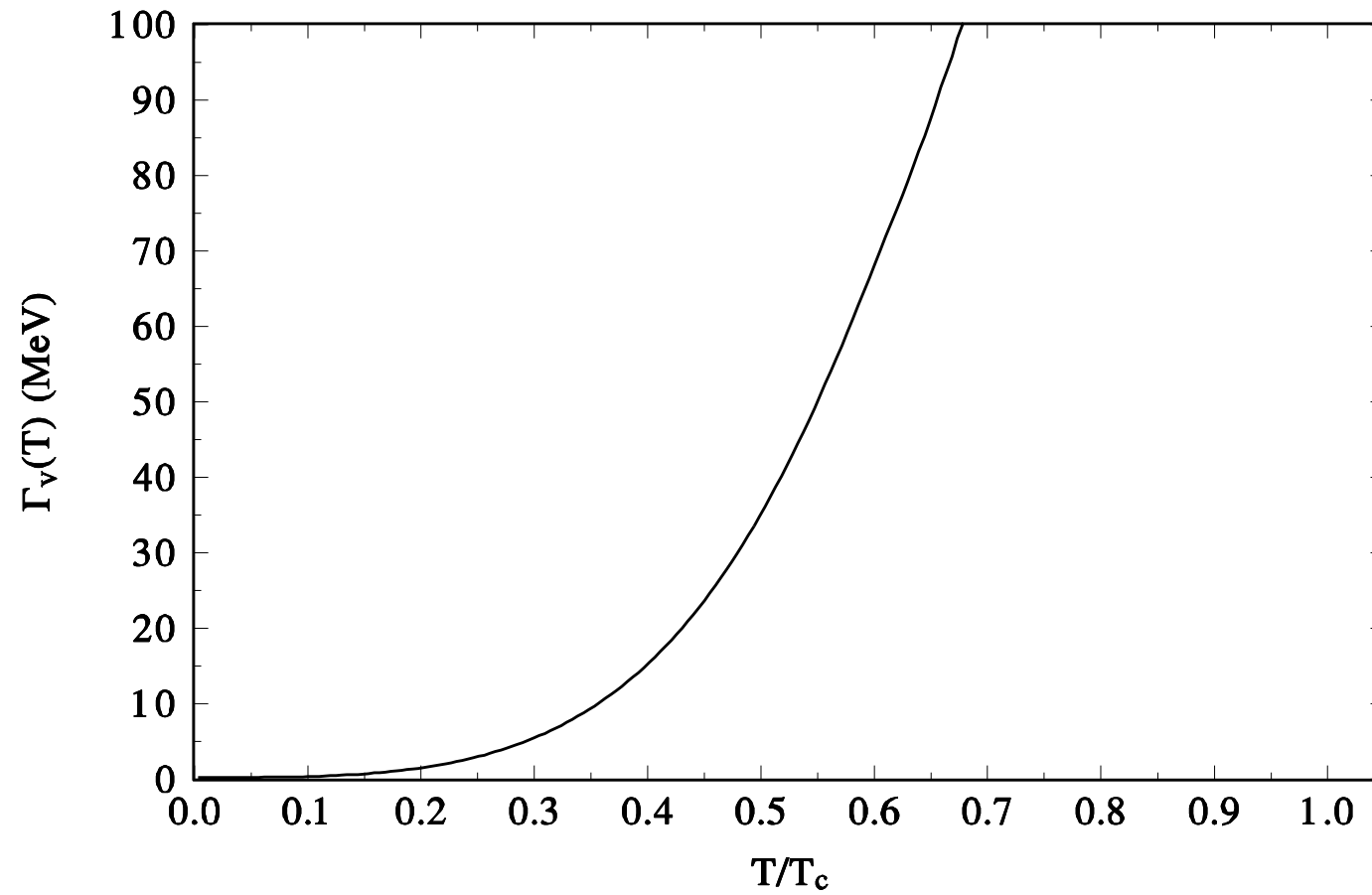
$f_{\rho}(T)$



# D-meson width



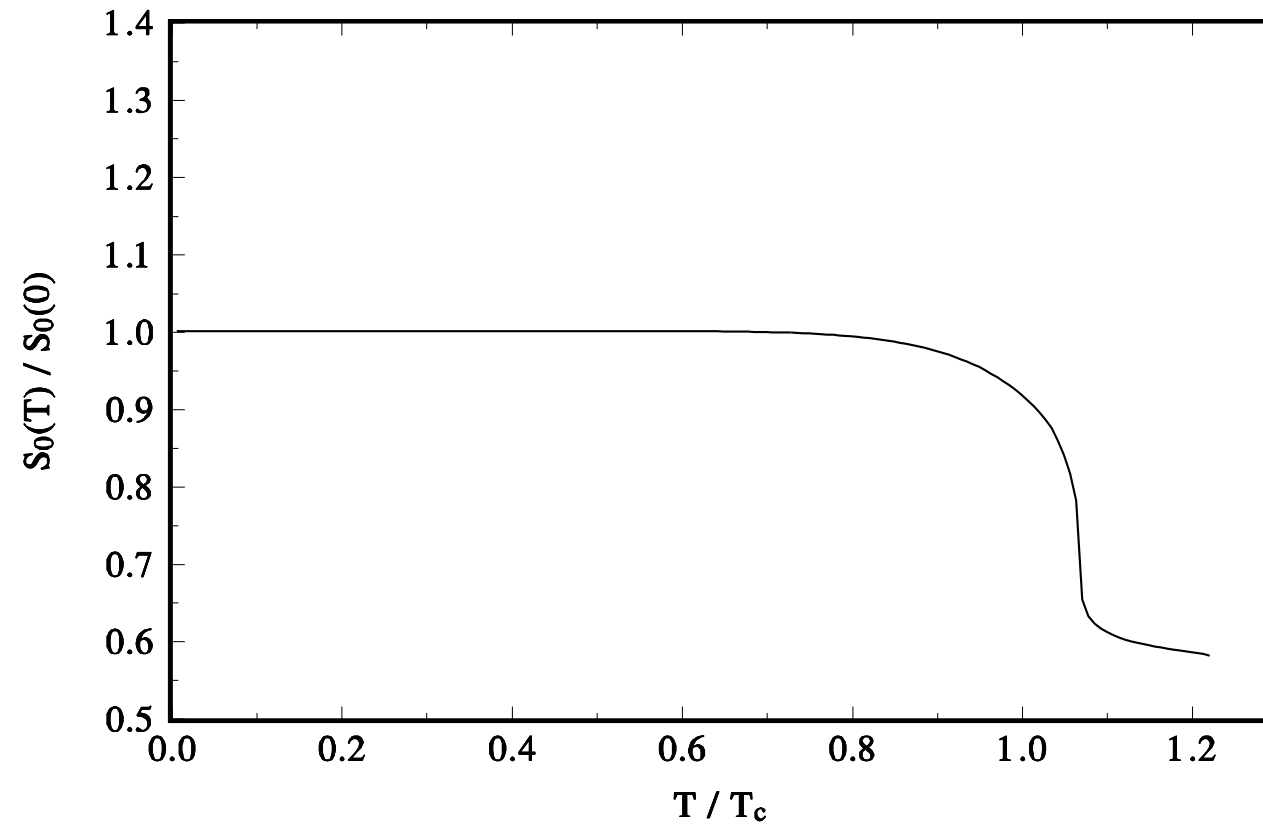
# D\* -meson width

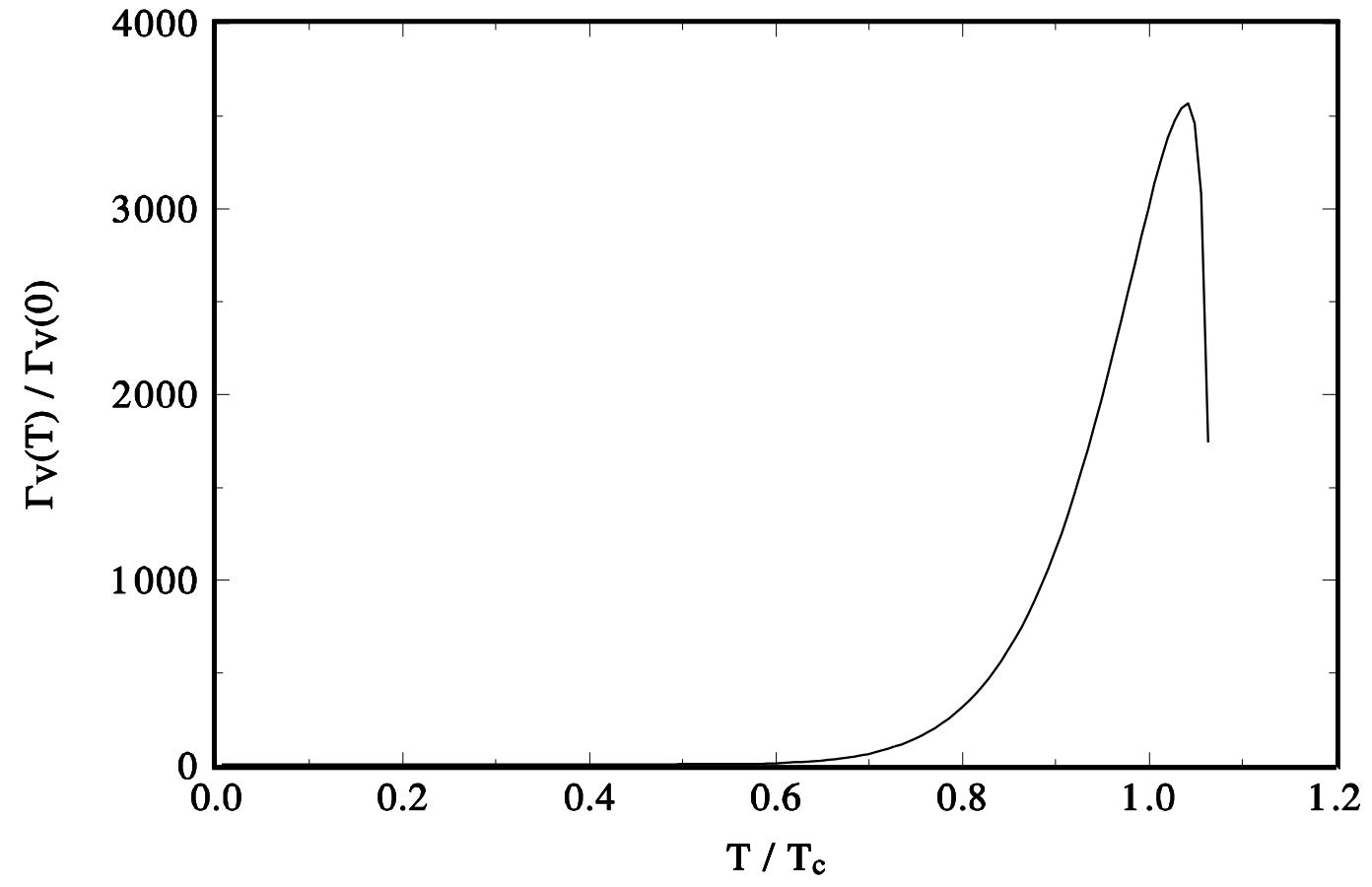


# Heavy-Quark Hadrons (T) (c & b)

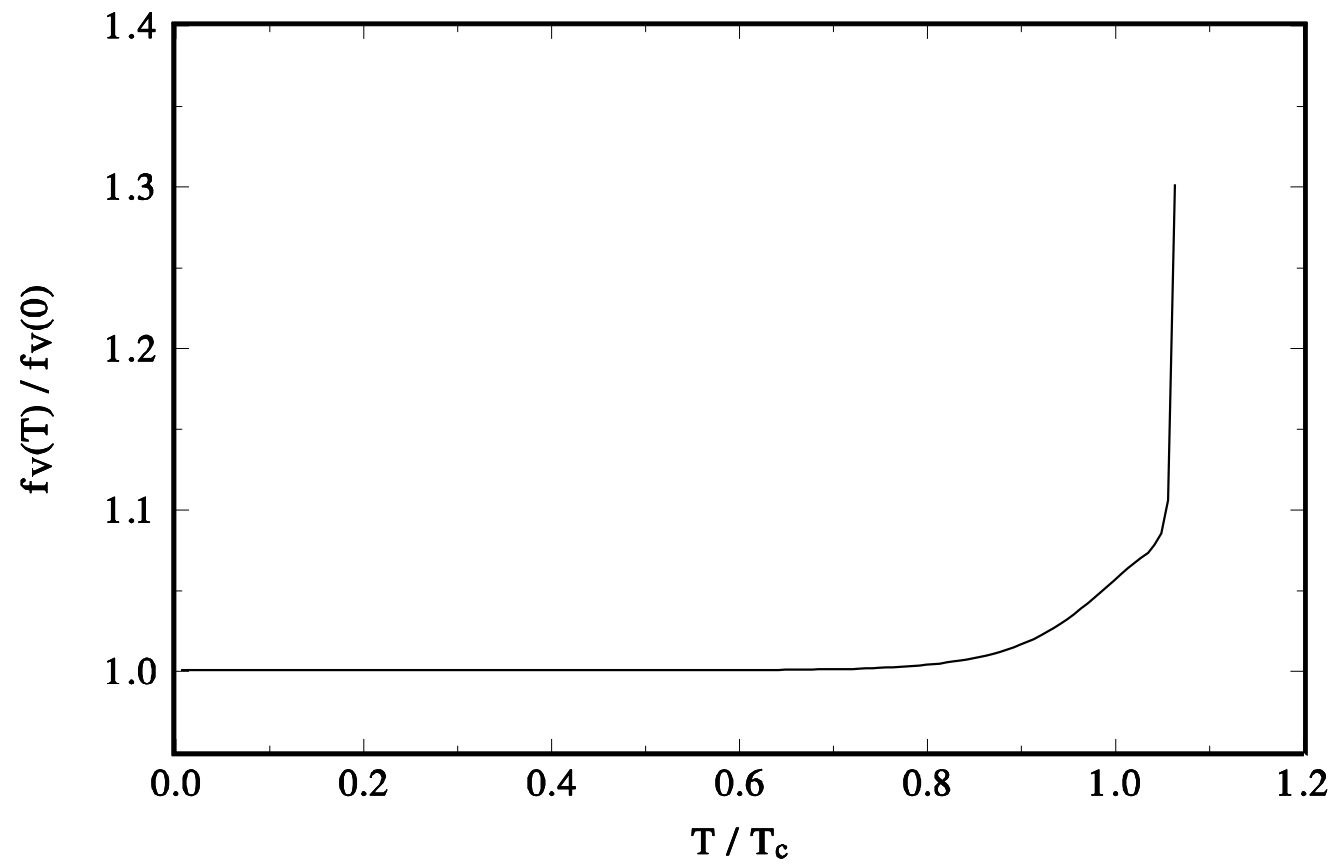
$J/\psi$  ( $c - \bar{c}$ ),  $\Upsilon$  ( $b - \bar{b}$ ),  $\eta_c$ ,  $\chi_c$ ,  $\eta_b$

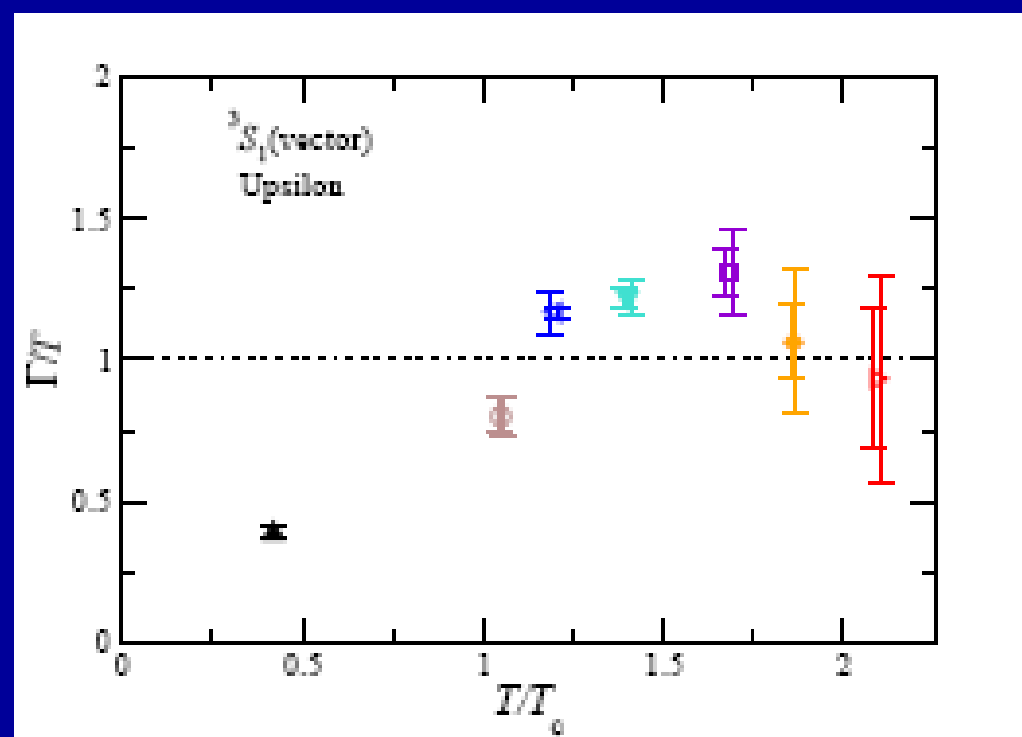
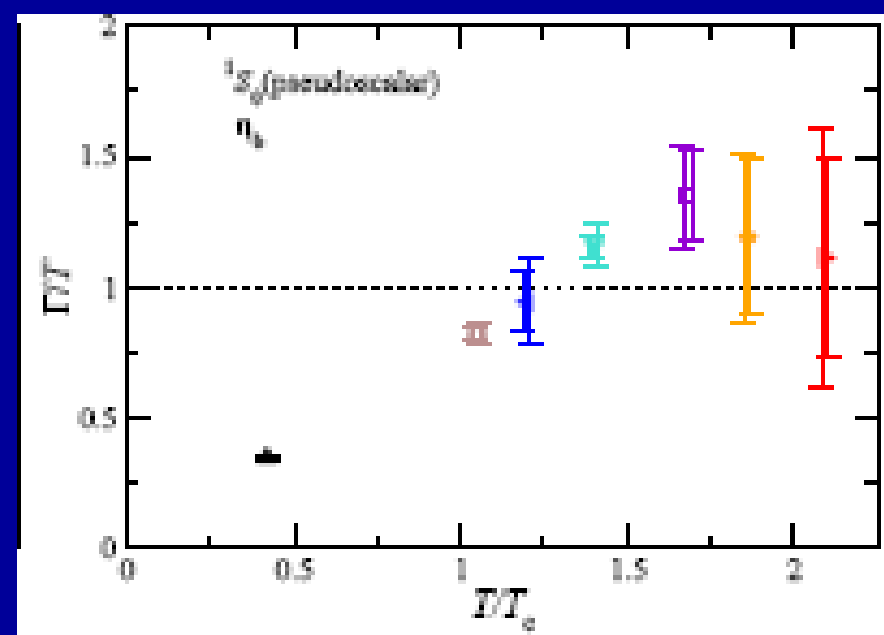
Dominguez, Loewe, Rojas & Zhang (2010-2011)











**THANK YOU**

# CABO DE LA BUENA ESPERANZA (CPT)

## Cabo da Boa Esperança

**Bartolomeu Dias (1488)**



