QCD Sum Rule Analysis of Heavy-light Hybrids for $J^P = \{0^\pm, 1^\pm\}$

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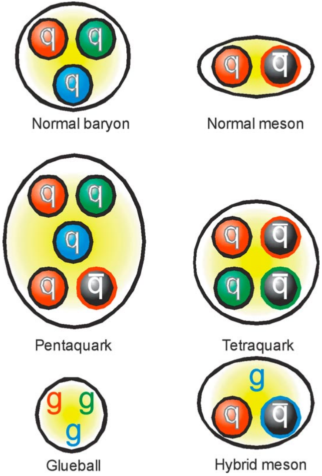
QCD sum-rules analysis of open-charm hybrid mesons

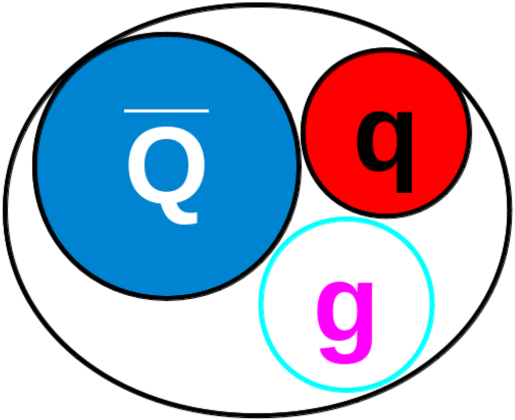
Hybrid Mesons



Experimental Searches

- XYZ Resonances
- GlueX (JLab)
- $Y(4260)$ $\bar{c}c$ hybrid candidate observed by BaBar (BABAR Collaboration, Phys. Rev. Lett. 95, 142001).
- Planned experiments: PANDA (FAIR).
- $Z_c(4430)$ four-quark state (Belle Collaboration, Phys. Rev. D 90, 112009).
- $P_c(4450)^+$ and $P_c(4380)^+$ five-quark states (LHCb Collaboration, Phys. Rev. Lett. 115, 072001).

Heavy-light Hybrids



- $J^P = \{0^\pm, 1^\pm\}$.
- JH, D. Harnett, T.G. Steele JHEP05(2017)149.
- J. Govaerts, L.J. Reinders, J. Weyers, Nucl. Phys. B **262** (1985)

Calculation of Correlation Function

- The mass prediction is obtained through calculation of the correlation function using a hybrid meson current.

$$\Pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_v\left(q^2\right) + \frac{q_\mu q_\nu}{q^2}\Pi_s\left(q^2\right)$$
$$= i\int d^d x\, e^{iq\cdot x}\langle\Omega|T\, j_\mu(x)\, j_\nu^\dagger(0)\,|\Omega\rangle.$$
$$\text{where } j_\mu(x) = g_s \bar{Q}^a(x)\,\Gamma^\nu \mathcal{G}^n_{\mu\nu}(x)\, t^n_{ab} q^b(x)$$

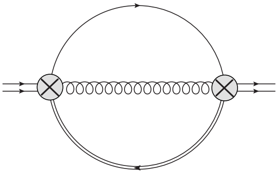
Γ^ρ	$\mathcal{G}^a_{\mu\rho}$	$J^{P(C)}$
γ^ρ	$G^a_{\mu\rho}$	$0^{+(+)}$, $1^{-(+)}$
γ^ρ	$\tilde{G}^a_{\mu\rho}$	$0^{-(+)}$, $1^{+(+)}$
$\gamma^\rho\gamma_5$	$G^a_{\mu\rho}$	$0^{-(-)}$, $1^{+(-)}$
$\gamma^\rho\gamma_5$	$\tilde{G}^a_{\mu\rho}$	$0^{+(-)}$, $1^{-(-)}$

where $\tilde{G}^a_{\mu\rho} = \frac{1}{2}\epsilon_{\mu\rho\nu\sigma} G^a_{\nu\sigma}$

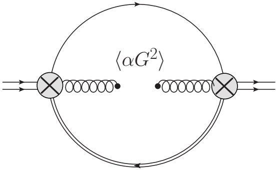
- Correlators calculated within the Operator Product Expansion (OPE):

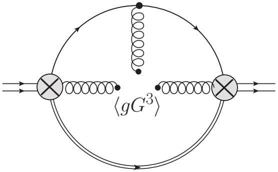
$$\lim_{x\to y}\mathcal{O}_1(x)\mathcal{O}_2(y) = \sum_n C_n(x-y)\mathcal{O}_n(x)$$

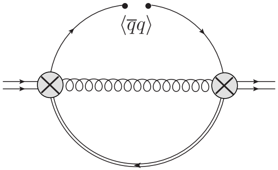
Perturbative and Non-Perturbative Contributions

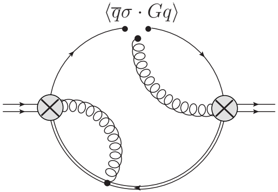


Feynman diagrams created with Jaxodraw (D. Binosi, L. Theussl, Comput. Phys. Commun. **161** (2004)).









Fourteen distinct condensate diagrams contribute (up to mass dimension six).

Correlation Function Calculation

- For each contributing Feynman diagram in the OPE,
  1. Expand any non-local vacuum expectation values (VEVs) in terms of local VEVs.
  2. Perform NLO quark mass expansion (in perturbation theory)
  3. Use Tarasov recursion relations (implemented by TARCER) to replace complicated numerator structure with additional denominator factors.
  4. Implement remaining master integrals and dimensional regularization.
- Perform QCD Laplace sum-rule analysis.
  1. Determine valid τ window which ensures OPE convergence.
  2. Extract continuum parameter s_0 and mass prediction m_H.
  3. Perform uncertainty analysis.
  4. Investigate mixing effects.

Correlation Function Results - Perturbation Theory + $\mathcal{O}(m_q)$ Correction

$$
\begin{aligned}
\Pi^{PT+\mathcal{O}(m_q)}(z) = & \frac{M_Q^6 \alpha_s}{960\pi^3 z^2}\left[f_1^{(\text{pert})}(z)\log(1-z) + f_2^{(\text{pert})}(z)\mathrm{Li}_2(z) + c^{(\text{pert})} z\right] \\
& + \frac{M_Q^5 m_q \alpha_s}{\pi^3 z^2}\left[f_1^{(m)}(z)\log(1-z) + f_2^{(m)}(z)\mathrm{Li}_2(z) + c^{(m)} z\right],
\end{aligned}
$$

$$
z \equiv \frac{q^2}{M_Q^2}
$$

Correlation Function Results - 3D and 4D Condensates

$$\Pi^{3D+4D}(z) = \frac{M_Q^3 \alpha_s \langle \overline{q} q \rangle}{6\pi z^2} \left[f^{(qq)}(z) \log(1-z) + c^{(qq)} z \right] + \frac{M_Q^2 \langle \alpha G^2 \rangle}{144\pi z^2} \left[f^{(GG)}(z) \log(1-z) + c^{(GG)} z \right]$$

Correlation Function Results - 5D Condensate

$$\Pi^{5D}(z) = \frac{M_Q \alpha_s \langle g\overline{q}\sigma Gq\rangle}{3456\pi z^2}\left[f_1^{(qGq)}(z)\log(1-z) + f_2^{(qGq)}(z)\frac{z^2}{1-z}\log\left(\frac{M_Q^2}{\mu^2}\right) + f_3^{(qGq)}(z)\frac{z}{(1-z)}\right]$$

Correlation Function Results - 6D Gluon Condensate

$$\Pi^{6D}(z) = \frac{\langle g^3 G^3 \rangle}{192\pi^2 z^2}\left[f^{(GGG)}(z)\log(1-z) + c^{(GGG)} z\right]$$

QCD Laplace Sum Rules

- M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **159** (1979)
- Dispersion relation

$$\Pi\left(Q^2\right) = \frac{1}{\pi}\int ds\, \frac{\mathrm{Im}\,\Pi\left(s\right)}{s+Q^2} + \left(\mathrm{polynomials\,in\,Q}^2\right)$$

relates information on the quarks on the left (our expansion of the correlation function) to hadronic features on the right.

- To accentuate the ground state resonance and eliminate constant and polynomial terms, we apply the Borel transform $\hat{\mathcal{B}}$, given by

$$\hat{\mathcal{B}} = \lim \frac{1}{\Gamma\left(n\right)}\left(-Q^2\right)^n\left(\frac{d}{dQ^2}\right)^n,\ \left\{Q^2, n\right\} \to \infty,\ \frac{n}{Q^2} \equiv \tau.$$

QCD Sum-Rules

- Borel transform may be expressed as an inverse Laplace transform
$$\frac{1}{\tau}\hat{\mathcal{B}}[f\left(Q^2\right)] = \mathcal{L}^{-1}[f\left(Q^2\right)]$$
- Forms the Laplace sum rule,
$$\mathcal{R}_k(\tau) \equiv \int_{M^2}^{\infty} dt\, t^k e^{-t\tau} \frac{1}{\pi}\mathrm{Im}\Pi(t).$$
Using a resonance plus continuum model
$$\frac{1}{\pi}\mathrm{Im}\Pi(t) = M_H^8 f_H^2 \delta(t - M_H^2) + \theta(t - s_0)\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)$$
we can extract the hadronic mass
$$M_H^2 = \frac{\mathcal{R}_{k+1}(\tau, s_0)}{\mathcal{R}_k(\tau, s_0)}.$$
where subtracted sum rule is
$$\mathcal{R}_k(\tau, s_0) = \mathcal{R}_k(\tau) - \int_{M^2}^{\infty} dt\, t^k e^{-t\tau}\frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{OPE}}(t)$$

Borel Window

- Sum rule is analyzed in a range of τ values where OPE convergence is ensured, and ground state is selected
- τ upper bound:

$$\left|\frac{\mathcal{R}_k^{4D}(\tau,\infty)}{\mathcal{R}_k^{PT}(\tau,\infty)}\right| \leq \frac{1}{3} \qquad \left|\frac{\mathcal{R}_k^{6D}(\tau,\infty)}{\mathcal{R}_k^{4D}(\tau,\infty)}\right| \leq \frac{1}{3}$$

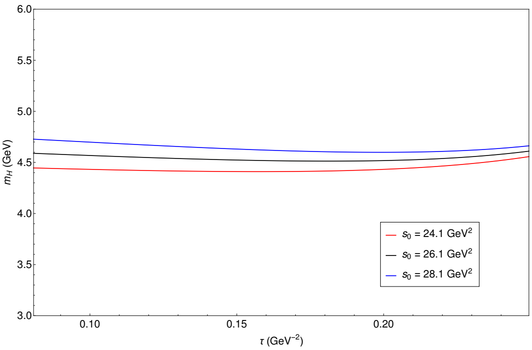
- τ lower bound:

$$\mathrm{PC}\left(s_0,\tau\right) = \frac{\int_{M_Q^2}^{s_0} e^{-t\tau}\mathrm{Im}\Pi(t)\mathrm{d}t}{\int_{M_Q^2}^{\infty} e^{-t\tau}\mathrm{Im}\Pi(t)\mathrm{d}t} \geq \frac{1}{10}$$

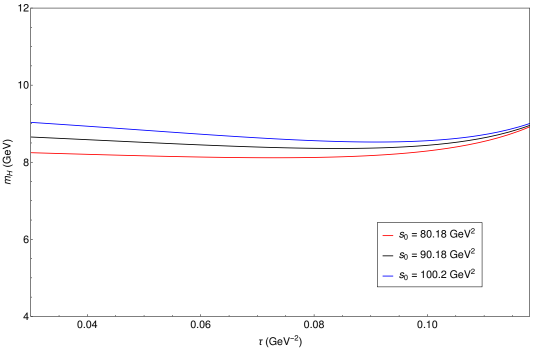
- Minimize

$$\sum\left(\frac{1}{m_H}\sqrt{\frac{\mathcal{R}_{k+1}(\tau_i,\, s_0)}{\mathcal{R}_k(\tau_i,\, s_0)}} - 1\right)^2$$

Borel Window



$0^{+(+)}$ Charm-light mass



$0^{+(+)}$ Bottom-light mass

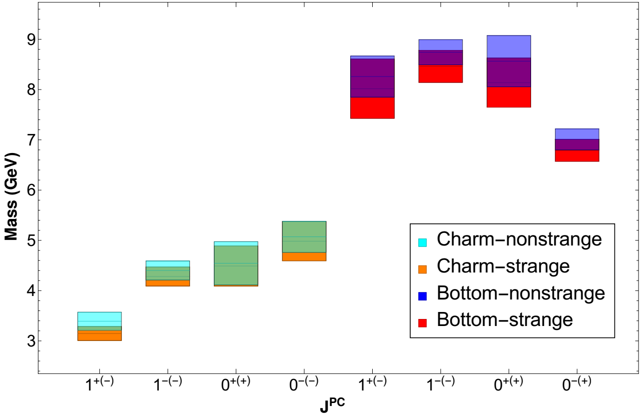
Source: Ho, Harnett, and Steele, JHEP05(2017)149.

QCD Parameters

Parameter	Value
$\alpha_s(M_\tau)$	0.330 ± 0.014
$\alpha_s(M_Z)$	0.1185 ± 0.0006
M_c	(1.275 ± 0.025) GeV
M_b	(4.18 ± 0.03) GeV
$m_n(2\text{ GeV})$	(3.40 ± 0.25) MeV
$m_s(2\text{ GeV})$	(93.5 ± 2.5) MeV
f_π	(92.2 ± 3.5) MeV
f_K	(110.0 ± 4.2) MeV

Parameter	Value
$\frac{M_c}{m_n}$	322.6 ± 13.6
$\frac{M_b}{m_n}$	1460.7 ± 64.0
$\frac{M_c}{m_s}$	11.73 ± 0.25
$\frac{M_b}{m_s}$	52.55 ± 1.30
$\langle \alpha G^2 \rangle$	$(0.075 \pm 0.020)\ \text{GeV}^4$
$\langle g^3 G^3 \rangle$	$((8.2 \pm 1.0)\ \text{GeV}^2)\ \langle \alpha G^2 \rangle$
$\frac{\langle g\overline{q}\sigma G q\rangle}{\langle \overline{q}q\rangle} \equiv M_0^2$	$(0.8 \pm 0.1)\ \text{GeV}^2$

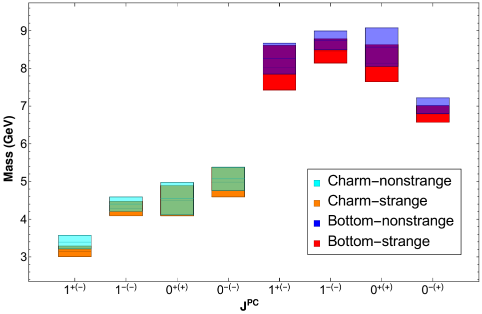
Results



Source: Ho, Harnett, and Steele, JHEP05(2017)149.

Results

- Predictions are heavier than previous GRW analysis, except in 1^+ charm-nonstrange and 0^- bottom-nonstrange channels.
- Similar spectrum hierarchy seen in charm and bottom channels.
- Discrepancies in 0^- consistent with predictions by Hilger, Krassnigg (Eur. Phys. J. A (2017) 53: 142).



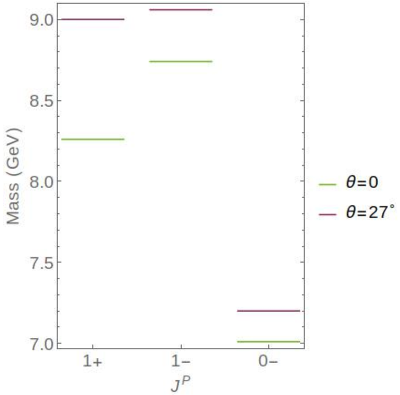
Mixing

- Possibility of mixing with conventional currents complicates sum rule analysis (Chen, W., JH et al. J. High Energ. Phys. (2013)).

$$\rho^{\mathrm{had}}(t) = \pi M_{conv}^8 f_{conv}^2 \delta(t - M_{conv}^2) + \pi M_H^8 f_H^2 \delta(t - M_H^2)$$

- Define mixing angle:

$$\sin^2(\theta) = \frac{f_{conv}^2}{f_{conv}^2 + f_H^2},\ |f_{conv}| \le \frac{1}{2}|f_H|$$



Mass predictions of B-hybrids without and with meson-hybrid mixings.

Concluding Remarks

- Mass predictions for heavy-light open flavour hybrid systems for $J^P = \{0^{\pm},\, 1^{\pm}\}$ have been obtained.
- Mixing with conventional mesons tends to increase predicted hybrid masses.
- Nonstrange/strange mass predictions consistent with degeneracy.
- A complete investigation of the operator mixing effects and coupling to conventional states is reserved for future work.

Acknowledgements

University of Saskatchewan

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