

Dynamical Systems and Applications



Report of Contributions

Contribution ID: 5

Type: **not specified**

On the Fibonacci Mandelbrot set

For $\beta \in \mathbb{C}$ with $|\beta| < 1$ define the contractions

$h_0(z) = \beta z$ and $h_1(z) = \beta z + 1$

and consider the attractor A_β of the iterated function system $\{h_0, h_1\}$. In 1985 Barnsley and Harrington introduced the “Mandelbrot set for pairs of linear maps” which is the set of all β with connected attractor A_β . This set has been thoroughly studied by many authors.

In the present talk we consider a variant of this Mandelbrot

set. In particular, we consider the attractors of the

iterated function system $\{f_0, f_1\}$ given by

$\backslash[$

$f_0(z) = \beta z, f_1(z) = 1 + \beta^2 z$

$\backslash]$

and study the associated Mandelbrot set \mathcal{M} . Among other things we show that \mathcal{M} is connected.

The structure of the iterated function system $\{f_0, f_1\}$ is related to the Fibonacci Language which is the subshift of finite type over $\{0, 1\}$ defined by forbidding the occurrence of two consecutive ones. This language and its difference language play an important role in the construction.

Presenter: SIRVENT, Víctor (UCN)

Contribution ID: 6

Type: **not specified**

Ergodic optimization in negative curvature

Thursday 5 December 2019 12:10 (30 minutes)

In this talk we discuss the existence of maximizing measures for uniformly continuous potentials on negatively curved non-compact manifolds. This is a joint work with Aníbal Velozo.

Presenter: RIQUELME, Felipe (PUCV)

Contribution ID: 7

Type: **not specified**

Generic Birkhoff Spectra

Friday 6 December 2019 11:30 (30 minutes)

This is a joint work with Zolt'an Buczolic and Bal'azs Maga. Let (Ω, σ) be the full-shift of two alphabets, and f be a continuous, real-valued function on it. Let L_f be the set of all of the possible limiting values of the Birkhoff averages of f , i.e.

$L_f := \left\{ \alpha \in \mathbb{R} : \exists \omega \in \Omega \text{ such that } \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} f(\sigma^n \omega) = \alpha \right\}$. For each $\alpha \in L_f$, we define the level set $E_f(\alpha) := \left\{ \omega \in \Omega : \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} f(\sigma^n \omega) = \alpha \right\}$, and we define a function $S_f : \mathbb{R} \rightarrow \mathbb{R}$, which we refer to as the Birkhoff spectra, as follows: $S_f(\alpha) := \begin{cases} \dim_H(E_f(\alpha)) & \alpha \in L_f, \\ 0 & \alpha \notin L_f, \end{cases}$ where \dim_H is the Hausdorff dimension.

$\mathbb{R} \rightarrow \mathbb{R}$, which we refer to as the Birkhoff spectra, as follows: $S_f(\alpha) := \begin{cases} \dim_H(E_f(\alpha)) & \alpha \in L_f, \\ 0 & \alpha \notin L_f, \end{cases}$ where \dim_H is the Hausdorff dimension.

In this talk, we will discuss shapes and properties of the Birkhoff spectrum S_f for generic/typical continuous functions f in the sense of Baire category. In particular, we will be interested in the behavior of the spectrum near the boundary of L_f , such as the continuity and the values of one-sided derivatives.

For more information, please refer to: <https://arxiv.org/abs/1905.06001> {arXiv:1905.06001}.

Presenter: RYO, Moore (UC)

Contribution ID: 8

Type: **not specified**

Lyapunov exponents of area preserving endomorphisms

Friday 6 December 2019 09:30 (30 minutes)

We consider a family of area preserving non-invertible maps on the two-torus, which is the composition of the well-known Chirikov standard family (s_r) with a linear expansion E . If E is an homothety then our family can be seen as a “randomized” version of the standard family. We show on one hand that the Lyapunov exponents are different for all small values of r . On the other hand, for large enough expansion and values of the shear parameter r , we also obtain lower bounds for the difference between the two Lyapunov exponents.

We will discuss ome possible generalization.

This is joint work with Martin Andersson, Pablo Carrasco and Jiagang Yang.

Presenter: SAGHIN, Radu (PUCV)

Contribution ID: 9

Type: **not specified**

Approximating integrals with respect to stationary probability measures

Friday 6 December 2019 11:00 (30 minutes)

From a dynamical approach, the problem of approximation of integrals with respect to stationary probability measures is analogue to the problem of approximation of integrals with respect to the Lebesgue measure studied by Jenkinson and Pollicott in [”A dynamical approach to accelerating numerical integration with equidistributed points.”Proceedings of the Steklov Institute of Mathematics 256.1 (2007): 275-289]. In this talk, I will enunciate a theorem and give an algorithm that yields accelerated approximations of integrals with respect to stationary probability measures. Finally, I will explain how this result can be applied to estimate the Wasserstein distance between certain stationary probability measures. This is joint work with Natalia Jurga.

Presenter: CIPRIANO, Italo (UC)

Contribution ID: **10**Type: **not specified**

The projective derivative as a dynamical tool

Wednesday 4 December 2019 12:25 (30 minutes)

When a group action of positive circle diffeomorphisms is considered, the projective derivative gives rise to a cocycle of Möbius transformations. By deducing precise expressions of this cocycle, we obtain several results about reducibility and almost reducibility to the group of rotations $SL(2, \mathbb{R})$. We also discuss novel and classical results about the projective derivative for real maps. This is a joint work with Andrés Navas (USACH).

Presenter: PONCE, Mario (UC)

Contribution ID: 11

Type: **not specified**

Escape of entropy for countable Markov shifts

Wednesday 4 December 2019 11:10 (30 minutes)

In the context of countable Markov shifts I will present a result that relates the escape of mass, the measure theoretic entropy and the entropy at infinity of the system. This relation has several consequences. For example, that the entropy map is upper semi-continuous for finite entropy Markov shifts. This is joint work with Mike Todd and Anibal Velozo.

Presenter: IOMMI, Godofredo (UC)

Contribution ID: 14

Type: **not specified**

Singular-expansive properties of flows

Thursday 5 December 2019 09:30 (55 minutes)

(joint with X. Wen and Y. Yang) A new kind of expansiveness for flows namely the singular-expansivity is proposed. We prove that it coincides with the rescaling expansivity for C^1 generic vector fields. We give sufficient conditions for a k^* -expansive flow to be singular-expansive. We prove that a singular-expansive flow has countably many periodic orbits and, if the set of singularities is isolated, then it has finitely many periodic orbits of bounded period. We give an example of a singular-expansive flow with the shadowing property which is not expansive. Further examples are given.

Presenter: MORALES , Carlos (Universidade Federal do Rio de Janeiro)

Contribution ID: 15

Type: **not specified**

Measures maximizing entropy for Kan's example

Friday 6 December 2019 12:15 (30 minutes)

In 1994, Ittai Kan provided the first examples of maps with intermingled basin. The Kan's example corresponds to a partially hyperbolic endomorphism defined on a surface with boundary exhibiting two intermingled hyperbolic physical measures. Both measures are supported on the boundary and are also measures maximizing the topological entropy. In this talk we will discuss the existence of a third measure maximizing the entropy, supported in the interior of the surface.

This is a work in progress joint with Bárbara Núñez and Sebastian Ramirez from PUCV.

Presenter: VÁSQUEZ, Carlos (PUCV)

Contribution ID: 16

Type: **not specified**

Birkhoff stability and Nekhoroshev stability of elliptic equilibrium positions in Hamiltonian systems

Wednesday 4 December 2019 09:45 (55 minutes)

We present a survey of stability results for elliptic equilibrium positions in Hamiltonian systems under generic assumptions.

References:

Giorgilli, A., Delshams, A., Fontich, E., Galgani, L., and Simo, C., Effective Stability for a Hamiltonian System near an Elliptic Equilibrium Point, with an Application to the Restricted Three-Body Problem, *J. Differential Equations*, 1989, vol. 77, no. 1, pp. 167–198.

Niederman, L., Generic super-exponential stability of elliptic equilibrium positions for symplectic vector fields, *Regul. Chaotic Dyn.* 18 (2013), no. 6, 719–731.

Bounemoura, A., Fayad, B., Niederman, L., Nekhoroshev estimates for steep real-analytic elliptic equilibrium points, to appear in *Nonlinearity*, preprint HAL <https://hal.archives-ouvertes.fr/hal-02074689>

Bounemoura, A., Fayad, B., Niederman, L., Super-exponential stability for generic real-analytic elliptic equilibrium points, preprint HAL <https://hal.inria.fr/hal-01188980v1>

Presenter: NIEDERMAN, Laurent (Laboratoire Mathématiques d’Orsay and IMCCE/ASD Orsay, France)

Contribution ID: 17

Type: **not specified**

On the dynamics of minimal topological finite rank systems

Thursday 5 December 2019 10:55 (30 minutes)

In this talk we will discuss about the dynamics of topological finite rank systems. This class of Cantor systems arise naturally in symbolic dynamics and also in the study of interval exchange maps. It is an extension of the so called substitution systems and linearly recurrent. Surprisingly, many dynamical properties of this last systems can be extended to topological finite rank systems, many times with complicated combinatorial arguments. That dynamical properties we will discuss are: expansiveness, combinatorial complexity, spectra, symbolic factors, automorphism group.

Presenter: MAASS , Alejandro (U. de Chile)

Contribution ID: 21

Type: **not specified**

Characteristic factors and joint ergodicity for commuting transformations and polynomial iterates

Wednesday 4 December 2019 11:40 (30 minutes)

In this talk I will review the notion of joint ergodicity in the context of multiple ergodic averages. Essentially, this property says that a multiple average converges to the “correct” limit, namely a product of integrals. This property was discovered by Furstenberg for linear iterates in weakly mixing systems and extended for polynomials (also in weakly mixing systems) by Bergelson. When considering several commuting transformations results are more scarce, mainly because there is no a well understood notion of “characteristic factor”. By exploiting a recent work by Tao and Ziegler, I will present some developments on the understanding of characteristic factors for multiple averages for commuting transformations, and show some applications of this to the joint ergodicity property in the context of several commuting transformations. This is joint work with Andreas Koutsogiannis and Wenbo Sun.

Presenter: DONOSO, Sebastián (U. de Chile)

Contribution ID: 22

Type: **not specified**

Measure separating systems

Wednesday 4 December 2019 15:00 (30 minutes)

The notion of expansive homeomorphism is important in topological dynamics,[2]. Different authors have extended some of these notions in more general contexts, in particular the notion of separating theory [1]. In this direction, we show some basic properties of some levels of separating homeomorphisms since the measurable point of view.

Acknowledgement

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References

- [1] A. Artigue, Separating homeomorphisms, *Advances in Dynamical Systems and Applications*, 12 (2017), 17–24.
- [2] M.R. Utz, Unstable homeomorphisms, *Proc. Amer. Math. Soc.*, 1 (1950), 769–774.

Presenter: CARRASCO, Dante (UBB)

Contribution ID: 23

Type: **not specified**

Self-induced system

Thursday 5 December 2019 11:25 (30 minutes)

A minimal Cantor system is said to be self-induced whenever it is conjugate to one of its induced systems. Substitution subshifts and some odometers are classical examples, in a common work with F. Durand and N. Ormes, we show that these are the only examples in the equicontinuous or expansive case. Nevertheless, we exhibit a zero entropy self-induced system that is neither equicontinuous nor expansive. We also provide non-uniquely ergodic self-induced systems with infinite entropy. Moreover, we give a characterization of self-induced minimal Cantor systems in terms of substitutions on finite or infinite alphabets.

Presenter: PETITE, Samuel (Université de Picardie Jules Verne)

Contribution ID: 25

Type: **not specified**

Thermodynamic formalism for a certain class of subadditive sequences on countable Markov shifts

Thursday 5 December 2019 15:50 (30 minutes)

In this talk, we consider a class of asymptotically subadditive sequences on countable Markov shifts. This type of sequences appears naturally in the theory of factors of Gibbs measures and also in some dimension problems of non conformal maps.

We show that the type of sequences we consider

generalizes almost additive sequences under certain finiteness conditions on the space. We define the topological pressure, show the variational principle and study the existence of Gibbs equilibrium states for this type of sequences. Finally

we give an application of the results to the theory of

factors of Gibbs measures. The results here are special cases of the results obtained in the paper by Iommi, Lacalle and Yayama.

Presenter: LACALLE, Camilo (UBB)

Contribution ID: 26

Type: **not specified**

Topological stability for fuzzy expansive maps

Thursday 5 December 2019 15:20 (30 minutes)

Authors: L. BADILLA, D. CARRASCO-OLIVERA, V.F. SIRVENT AND H. VILLAVICENCIO

Abstract: We introduce the definitions of expansivity and topological stability for homeomorphism on fuzzy metric space. We show some basic properties of fuzzy expansive homeomorphisms. Moreover we prove Walters' theorem in the context of fuzzy metric spaces, i.e., a fuzzy expansive system with the fuzzy shadowing property is fuzzy topologically stable.

Presenter: BADILLA, Leonel (UBB)

Contribution ID: 28

Type: **not specified**

Complete characterization of multiplication CA with respect to pre-expansivity

Wednesday 4 December 2019 14:30 (30 minutes)

In the context of Cellular Automata (CA), “pre-expansivity” is the property of being positively expansive on asymptotic pairs of configurations (i. e. configurations that differ in only finitely many positions). Pre-expansivity therefore lies between positive expansivity and pre-injectivity, two important notions of CA theory.

In this talk we consider the family of reversible one-way cellular automata that corresponds to multiplication by k in base $m=kk'$, where “directional” pre-expansivity can be completely characterized in terms of k and k' . This family was already considered by Blanchard-Maass, and positive left-expansivity was characterized in terms of k and k' , our result shows that these two notions are completely independent (orthogonal). On the other hand, the “directional” pre-expansive CA in this family coincide with the universal pattern generators of the family, i.e., CAs where every word is present somewhere in the positive orbit of every finite initial configuration.

Presenter: GAJARDO, Anahí (U. de Concepción)

Contribution ID: 29

Type: **not specified**

Limit cycles and Abelian integrals

Thursday 5 December 2019 14:30 (30 minutes)

We will show how limit cycles, which form part of a dynamic object: an ordinary differential equation, are related with zeros of an analytic object: an Abelian integral. We will show when such a relation is an equivalence and we will exhibit some recent results about these two objects.

Presenter: REBOLLO, Salomón (UBB)

Contribution ID: 30

Type: **not specified**

SMART is Topologically Mixing

Friday 6 December 2019 10:00 (30 minutes)

Turing machines have traditionally been studied as computational models, but we center our line of research on the dynamical properties of Turing machines, thus focusing on their behavior rather than the final results. This approach, in the context of Turing machines, has been fruitful since its inception by Kůrka in 1997 [2], with studies on immortality [4, 5], entropy, equicontinuity, periodicity and, recently, transitivity and minimality [1, 3]. The existence of a Topological Transitive and Topological Minimal Turing machine was presented recently [1], concepts ligated to reaching finite configurations in the orbits of the Turing machine. Nevertheless, Mixing notions have not been studied in this field. Total Transitivity, Weak Mixing and Topological Mixing endorse time restriction to reaching finite configurations inside the orbits of the machine, therefore requiring a deeper understanding on the dynamics of the studied machines. Although, known Turing machines with all the previous properties have very simple dynamics. In this presentation, we prove that the complex machine SMART [1] presents the property Topological Mixing.

[1] J. Cassaigne, N. Ollinger and R. Torres-Aviles. A Small Minimal Aperiodic Reversible Turing Machine. *Journal of Computer and System Science*, 84C:288-301, 2017.

[2] P. Kůrka. On topological dynamics of Turing machines. *Theoretical Computer Science*, 174(1-2):203-216, 1997.

[3] A. Gajardo, N. Ollinger and R. Torres-Aviles. The Transitivity Problem of Turing Machines. *Mathematical Foundation of Computer Science 2015 (Milano, Italy) Lecture Notes of Computer Science*, 9234:231-242, 2015.

[4] J. Kari and N. Ollinger. Periodicity and Immortality in Reversible Computing. *Mathematical Foundation of Computer Science 2008 (Roun, Poland) Lecture Notes of Computer Science*, 5162:419-430, 2008.

[5] E. Jeandel. On immortal configurations in Turing machines. *Conference on Computability in Europe 2012 (Cambridge, UK) Lecture Notes of Computer Science*, 7318:334-343, 2012.

Presenter: TORRES, Rodrigo (UBB)