Aspects of Chiral Symmetry Breaking in RQED

Alfredo Raya
IFM-UMSNH, CEE-UBB

In collaboration with
Luis Albino Fernández, Adnan Bashir, Ana J. Mizher
Juan Angel Casimiro, Cristián Villavicencio and others

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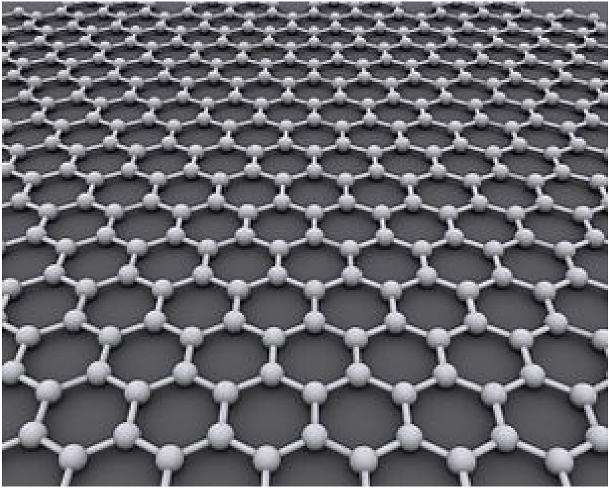
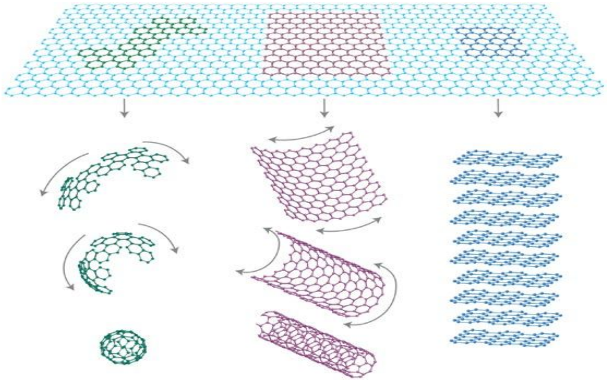
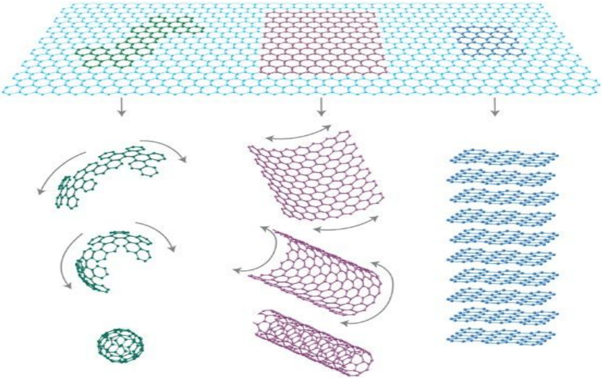
Motivation

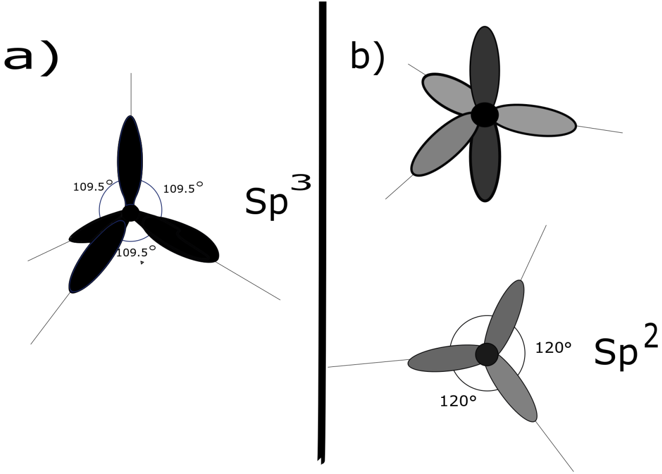


Mother of all graphitic structures

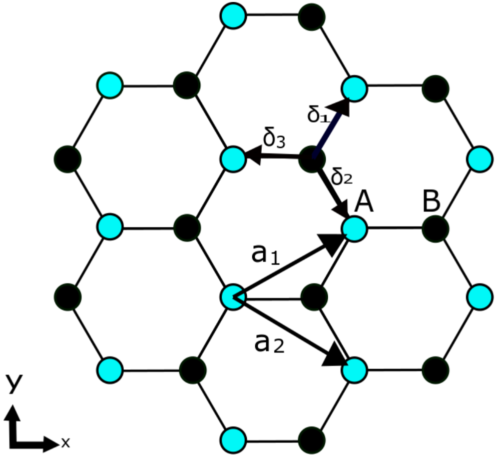


Mother of all graphitic structures *a la mexicana*



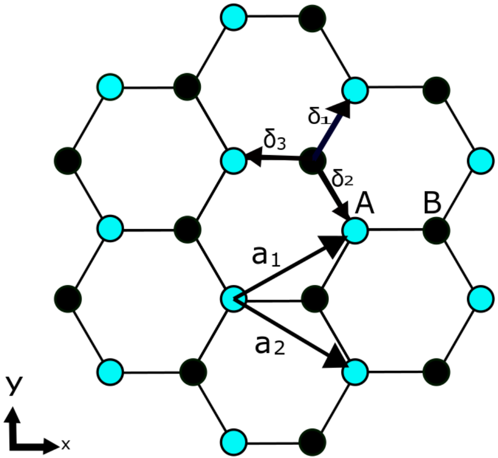


a)
109.5°
109.5°
109.5°
Sp3
b)
120°
120°
Sp2

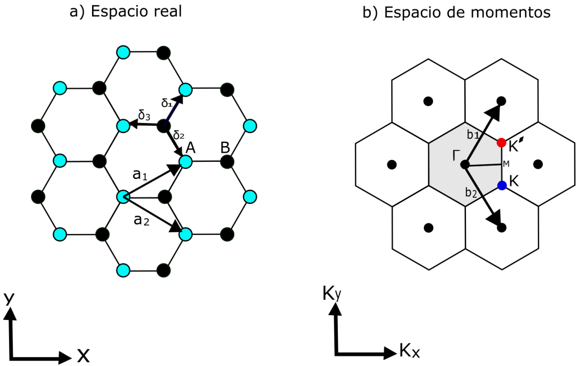


$$a_1 = \frac{a}{2}\left(3, \sqrt{3}\right), \qquad a_2 = \frac{a}{2}\left(3 - \sqrt{3}\right),$$

Crystalline structure
Graphene



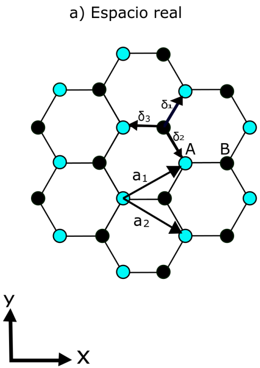
$$\delta_1 = \frac{a}{2}\left(1, \sqrt{3}\right), \qquad \delta_2 = \frac{a}{2}\left(1, -\sqrt{3}\right), \qquad \delta_3 = (-a, 0).$$

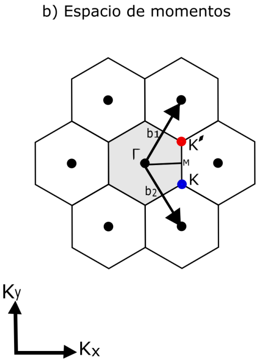


$$a_i \cdot b_j = 2\pi\delta_{ij}.$$

$$b_1 = \left(\frac{4\pi}{a\sqrt{3}}, 0\right), \qquad b_2 = \left(\frac{2\pi}{a\sqrt{3}}, \frac{2\pi}{a}\right).$$

Dirac points
Reciprocal lattice





$$K' = \left(\frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a}\right), \qquad K = \left(\frac{2\pi}{3a}, -\frac{2\pi}{3\sqrt{3}a}\right),$$

Lineal combination

$$E\psi_A(r_A) = t\psi_B(r_B) + t\psi_B(r_B - a_2) + t\psi_B(r_B - a_1),$$
$$E\psi_B(r_B) = t\psi_A(r_A) + t\psi_A(r_A + a_1) + t\psi_A(r_A + a_2),$$

Lineal combination

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$$E\psi_B(r_B) = t\psi_A(r_A) + t\psi_A(r_A + a_1) + t\psi_A(r_A + a_2),$$

Bloch's Theorem

$$\psi_k(r + T) = e^{ik\cdot T}\psi_k(r),$$

T is a crystal translation

Lineal combination

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Bloch's Theorem

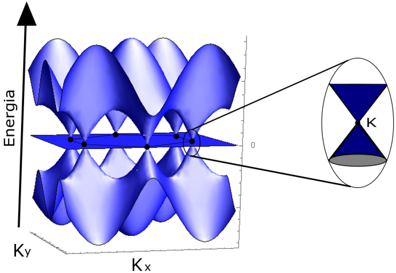
$$\psi_k(r + T) = e^{ik\cdot T}\psi_k(r),$$

T is a crystal translation

$$\hat{H}(k)\begin{bmatrix}\Psi_A\\ \Psi_B\end{bmatrix} = \begin{pmatrix} 0 & t\sum_\delta e^{-ik\cdot\delta} \\ t\sum_\delta e^{+ik\cdot\delta} & 0\end{pmatrix}\begin{bmatrix}\Psi_A\\ \Psi_B\end{bmatrix}.$$

Graphene

$$E_\pm = \pm t \left| \sum_\delta e^{ik\cdot\delta} \right|$$
$$= \pm t\sqrt{3 + \cos\left(\sqrt{3}k_y a\right) + 4\cos(\sqrt{3}k_y a/2)\cos(3k_x a/2)}.$$



Taylor expanding the Hamiltonian around the Dirac points $K = k + q$ with $|q| \ll |k|$,

$$\hat{H}_{K,K'} = \hbar v_F \begin{pmatrix} 0 & q_x \mp iq_y \\ q_x \pm iq_y & 0 \end{pmatrix},$$

where

$$v_F = \frac{3at}{2\hbar} \simeq \frac{1}{300} c.$$

Taylor expanding the Hamiltonian around the Dirac points $K = k + q$ with $|q| \ll |k|$,

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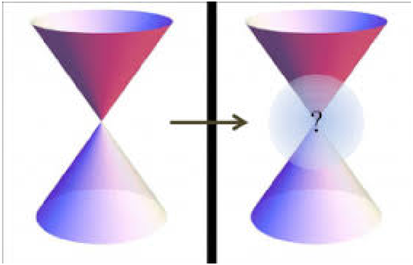
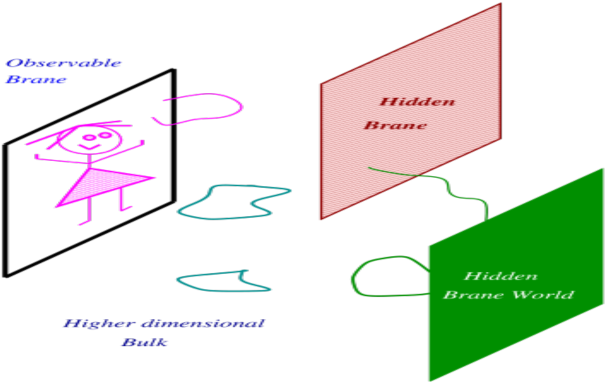
$$\hat{H}_{K'} = \sigma_1 H_K \sigma_1$$

Big question

Mind the gap

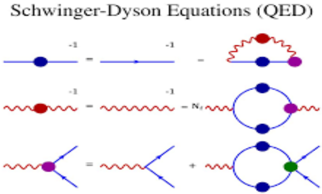




Observable Brane
Hidden Brane
Hidden Brane World
Higher dimensional Bulk

SDEs
Infinite tower

SDE form an infinite, tower of nonlinear relations among the Green functions of a given QFT



We express the electron propagator $S(k)$ as

$$S(k) = \frac{F(k^2,\Lambda^2)}{i\gamma\cdot k + \mathcal{M}(k^2,\Lambda^2)}\,.$$

(Tree level $\rightarrow F(k^2,\Lambda^2) = 1$ and $\mathcal{M}(k^2,\Lambda^2) = m_0$).

The photon propagator takes the form

$$\begin{aligned}
\Delta_{\mu\nu}(q) &= \Delta^T_{\mu\nu}(q) + \Delta^L_{\mu\nu}(q) \\
\Delta^T_{\mu\nu}(q) &= \frac{\varrho}{q}\left[\delta_{\mu\nu} - \chi\frac{q_\mu q_\nu}{q^2}\right]\,, \\
\Delta^L_{\mu\nu}(q) &= \varrho\,\kappa\frac{q_\mu q_\nu}{q^3}\,,
\end{aligned}$$

where $\chi \equiv 1 - \epsilon_e$, $\kappa \equiv \chi\xi$, $\varrho = \Gamma\left[\chi\right]/\left(4\pi\right)^{\epsilon_e}$. It is important to notice that RQED ($\epsilon_e = 1/2$) yields $\chi = \varrho = 1/2$ and $\kappa = \xi/2$.

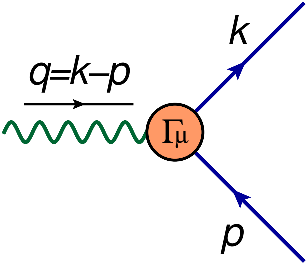


Figure: Diagrammatic representation of the full electron-photon vertex $\Gamma_\mu(k,p)$, with momentum flow indicated.

The electron-photon vertex can be written in terms of twelve independent spin structures.

The WFGTI associated with this vertex takes the form

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)\,, \qquad q = k - p.$$

This identity allows us to split

$$\Gamma_\mu(k,p) = \Gamma_\mu^L(k,p) + \Gamma_\mu^T(k,p)\,.$$

The longitudinal part $\Gamma_\mu^L(k,p)$

$$\Gamma_\mu^L(k,p) = \lambda_1(k,p)\gamma_\mu + \lambda_2(k,p) t_\mu \gamma\cdot t - i\lambda_3(k,p) t_\mu\,,$$

with $t = k + p$.

Thus,

$$\begin{aligned}
\lambda_1(k,p) &= \frac{1}{2}\left[\frac{1}{F(k^2,\Lambda^2)}+\frac{1}{F(p^2,\Lambda^2)}\right],\\
\lambda_2(k,p) &= \frac{1}{2}\left[\frac{1}{F(k^2,\Lambda^2)}-\frac{1}{F(p^2,\Lambda^2)}\right]\frac{1}{k^2-p^2},\\
\lambda_3(k,p) &= \left[\frac{\mathcal{M}(k^2,\Lambda^2)}{F(k^2,\Lambda^2)}-\frac{\mathcal{M}(p^2,\Lambda^2)}{F(p^2,\Lambda^2)}\right]\frac{1}{k^2-p^2},
\end{aligned}$$

The transverse part $\Gamma_\mu^T(k,p)$ of the vertex satisfies

$$q_\mu \Gamma_\mu^T(k,p) = 0\,.$$

It can be expanded out as

$$\Gamma_\mu^T(k,p) = \sum_{i=1}^{8} \tau_i(k,p) T_\mu^i(k,p)\,.$$

Using

$$\sigma_{\nu\rho} = \frac{i}{2}\left[\gamma_\nu, \gamma_\rho\right]\,,$$

we use

$$
\begin{aligned}
T^1_\mu(k,p) &= i\left[p_\mu(k\cdot q) - k_\mu(p\cdot q)\right] \,, \\
T^2_\mu(k,p) &= \left[p_\mu(k\cdot q) - k_\mu(p\cdot q)\right](\gamma\cdot t) \,, \\
T^3_\mu(k,p) &= q^2\gamma_\mu - q_\mu\,(\gamma\cdot q) \,, \\
T^4_\mu(k,p) &= iq^2\left[\gamma_\mu\,(\gamma\cdot t) - t_\mu\right] + 2q_\mu p_\nu k_\rho \sigma_{\nu\rho} \,, \\
T^5_\mu(k,p) &= \sigma_{\mu\nu} q_\nu \,, \\
T^6_\mu(k,p) &= -\gamma_\mu\left(k^2 - p^2\right) + t_\mu\,(\gamma\cdot q) \,, \\
T^7_\mu(k,p) &= \frac{i}{2}(k^2 - p^2)\left[\gamma_\mu\,(\gamma\cdot t) - t_\mu\right] + t_\mu p_\nu k_\rho \sigma_{\nu\rho} \,, \\
T^8_\mu(k,p) &= -i\gamma_\mu p_\nu k_\rho \sigma_{\nu\rho} - p_\mu\,(\gamma\cdot k) + k_\mu\,(\gamma\cdot p) \,,
\end{aligned}
$$

The full vertex must have the same transformation properties as the bare vertex under C.

This requires all the τ_i should be symmetric under $k \leftrightarrow p$, except τ_4 and τ_6, which are

$$\begin{aligned} \tau_i(k,p) &= \tau_i(p,k)\,, \qquad && i = 1,2,3,5,7,8, \\ \tau_i(k,p) &= -\tau_i(p,k)\,, && i = 4,6. \end{aligned}$$

For the longitudinal components, $\lambda_1(k,p)$, $\lambda_2(k,p)$ and $\lambda_3(k,p)$ are symmetric.

Scalar factors
Basis

An educated proposal is

$$
\begin{aligned}
\tau_1(k,p) &= \frac{a_1}{(k^2+p^2)}\,\lambda_3(k,p)\,,\\
\tau_2(k,p) &= \frac{2a_2}{(k^2+p^2)}\,\lambda_2(k,p)\,,\\
\tau_3(k,p) &= 2a_3\,\lambda_2(k,p)\,,\\
\tau_4(k,p) &= \frac{a_4(k^2-p^2)}{4k^2p^2}\,\lambda_3(k,p)\,,\\
\tau_5(k,p) &= -a_5\,\lambda_3(k,p)\,,\\
\tau_6(k,p) &= -\frac{2a_6(k^2+p^2)}{(k^2-p^2)}\,\lambda_2(k,p)\,,\\
\tau_7(k,p) &= -\left[\frac{a_4q^2}{2k^2p^2}+\frac{a_7}{k^2+p^2}\right]\,\lambda_3(k,p)\,,\\
\tau_8(k,p) &= 2a_8\,\lambda_2(k,p)\,.
\end{aligned}
$$

Effective modification:

$$\tau_6(k,p) \;=\; a_6 \frac{(k^2-p^2)}{(k^2+p^2)}\,\lambda_2(k,p)\,.$$

The one-loop corrections to the electron-photon vertex in the asymptotic limit$p^2 \gg k^2 \gg m_0^2$ is

$$\Gamma^T_\mu(k,p) \;\overset{p^2\gg k^2}{=}\; -\frac{\alpha}{8\pi p^2}\left(\xi-\frac{1}{3}\right)\log\left(\frac{p^2}{k^2}\right)T^{asy}_\mu\,,$$

with

$$T^{asy}_\mu = p^2\gamma_\mu - p_\mu\gamma\cdot p\,.$$

Hence, in this limit,

$$\Gamma^T_\mu(k,p) \;\overset{p^2\gg k^2}{=}\; \frac{1}{2}\left[\frac{1}{F(k^2)}-\frac{1}{F(p^2)}\right]\frac{T^{asy}_\mu}{k^2-p^2}\,.$$

On the other hand, the leading structure of the transverse vertex is

$$\Gamma_\mu^T(k,p) \stackrel{p^2 \gg k^2}{=} (\tau_3 + \tau_6)\, T_\mu^{asy}\,.$$

Moreover,

$$T_\mu^{3\,asy} = T_\mu^{6\,asy} \equiv T_\mu^{asy}\,.$$

As the simplest construction of the transverse vertex, we can single out $\tau_3(k,p)$ and $\tau_6(k,p)$

$$\begin{aligned}
\tau_3(k,p) &= a_3 \left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right] \frac{1}{k^2 - p^2}\,,\\
\tau_6(k,p) &= a_6 \left[\frac{1}{F(k^2)} - \frac{1}{F(p^2)}\right] \frac{1}{k^2 + p^2}\,.
\end{aligned}$$

The one-loop behavior of the vertex in the asymptotic limit requires

$$a_3 - a_6 = 1/2\,.$$

The SDE for the electron propagator is

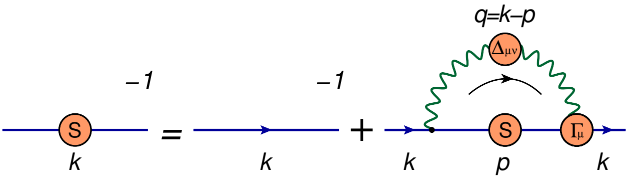


Figure: The gap equation for the electron propagator.

Mathematically,

$$S^{-1}(k) \quad = \quad S_0^{-1}(k) + \frac{\alpha}{2\pi^2}\int_E d^3p\,\gamma_\nu S(p)\Gamma_\mu(k,p)\Delta_{\mu\nu}(q)\,,$$

Gap equation

SDE for the fermion propagator

It is equivalent to the coupled integral equations

$$\frac{\mathcal{M}(k^2)}{F(k^2)} = m_0 + \frac{\alpha\varrho\kappa}{2\pi^2}\int_E \frac{d^3p}{q^3}\frac{F(p^2)}{p^2+\mathcal{M}^2(p^2)}\frac{1}{F(k^2)} \times\left\{\mathcal{M}(p^2)\,q\cdot k - \mathcal{M}(k^2)\,q\cdot p\right\} + \frac{\alpha\varrho}{2\pi^2}\int_E d^3p\,\frac{F(p^2)}{p^2+\mathcal{M}^2(p^2)}\,G_{\mathcal{M}}(k,p)\,,$$

$$\frac{1}{F(k^2)} = 1 - \frac{\alpha\varrho\kappa}{2\pi^2}\int_E \frac{d^3p}{q^3}\frac{F(p^2)}{p^2+\mathcal{M}^2(p^2)}\frac{1}{F(k^2)} \times\left\{q\cdot p + \mathcal{M}(k^2)\mathcal{M}(p^2)\,\frac{q\cdot k}{k^2}\right\} + \frac{\alpha\varrho}{2\pi^2}\int_E \frac{d^3p}{k^2}\frac{F(p^2)}{p^2+\mathcal{M}^2(p^2)}\,G_F(k,p)\,,$$

with

$$
\begin{aligned}
\frac{G_{\mathcal{M}}(k,p)}{\Delta_0(q^2)} &= (3-\chi)\mathcal{M}(p^2)\lambda_1 + \left[t^2 - \frac{(k^2-p^2)^2}{q^2}\chi\right]\mathcal{M}(p^2)\lambda_2 \\
&- \left[t\cdot p - \frac{(q\cdot p)(k^2-p^2)}{q^2}\chi\right]\lambda_3 + \nabla(k,p)\tau_1 + 2\nabla(k,p)\mathcal{M}(p^2)\tau_2 \\
&+2q^2\mathcal{M}(p^2)\tau_3 - 2\left[(k^2-p^2)(q\cdot p) + \nabla(k,p)\right]\tau_4 \\
&-2(q\cdot p)\,\tau_5 - 2(k^2-p^2)\mathcal{M}(p^2)\tau_6 \\
&- \left[(k^2-p^2)(t\cdot p) - \nabla(k,p)\right]\tau_7\,,
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\nabla(k,p) &= k^2p^2 - (k\cdot p)^2\,, \\
u(k,p) &= 2\,k\cdot p - \frac{\nabla(k,p)}{q^2}\,.
\end{aligned}
$$

Gap equation
SDE for the fermion propagator

$$
\begin{aligned}
\frac{G_F(k,p)}{\Delta_0(q^2)} \quad = \quad & [(1-3\chi)k\cdot p + 2\chi\, u(k,p)]\,\lambda_1 \\
& -\left[(k\cdot p)t^2 + 2\nabla(k,p) - \frac{(k\cdot p)(k^2-p^2)^2}{q^2}\chi\right]\lambda_2 \\
& -\left[t\cdot k - \frac{(q\cdot k)(k^2-p^2)}{q^2}\chi\right]\mathcal{M}(p^2)\lambda_3 \\
& +\nabla(k,p)\mathcal{M}(p^2)\tau_1 - (k^2+p^2)\nabla(k,p)\tau_2 \\
& +2(q\cdot k)(q\cdot p)\tau_3 + 2\left[q^2(t\cdot k) - \nabla(k,p)\right]\mathcal{M}(p^2)\tau_4 \\
& +2(q\cdot k)\mathcal{M}(p^2)\tau_5 - 2(k^2-p^2)(k\cdot p)\,\tau_6 \\
& +\left[(k^2-p^2)(t\cdot k) + \nabla(k,p)\right]\mathcal{M}(p^2)\tau_7 + \nabla(k,p)\tau_8\,,
\end{aligned}
$$

Solution to the gap equation

Bare vertex

Bare vertex, with $\lambda_1 = 1$ and $\lambda_{2,3} = \tau_{1,2,\ldots,8} = 0$.

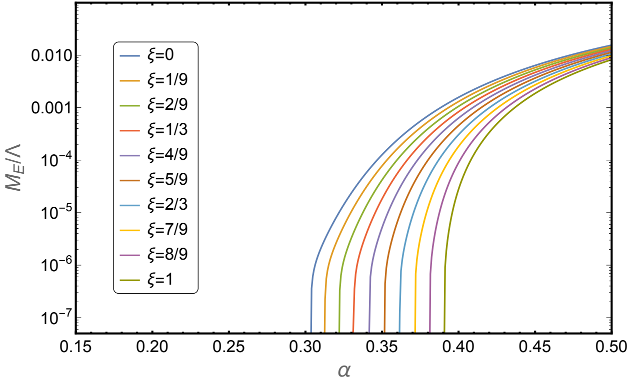


Figure: Dynamically generated mass M_E/Λ as a function of the electromagnetic coupling α for the bare vertex *ansatz*

Our vertex

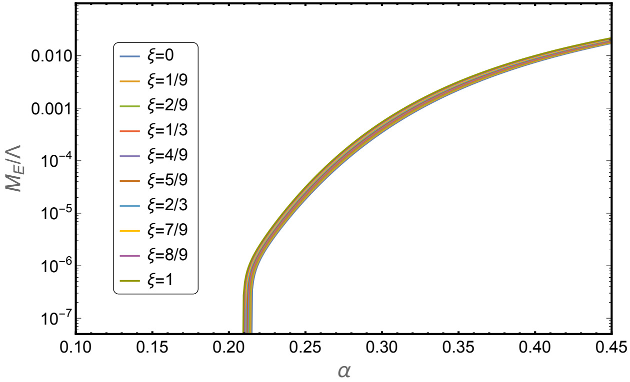


Figure: Dynamically generated mass M_E/Λ as a function of the electromagnetic coupling α for our proposed vertex.

Solution to the gap equation
Miransky scaling

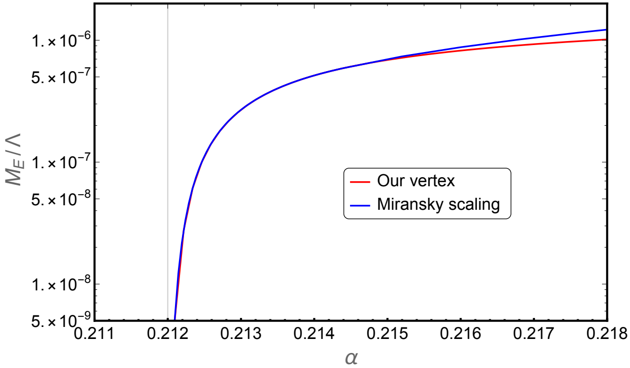


Figure: Miransky Scaling using our vertex for $\xi = 1/3$.

Final remarks
Concluding

- Symmetry preserving truncations of SDE are important to achieve physically reliable results
- Constraints arising from gauge symmetry guarantee correct physical interpretation
- Chiral symmetry breaking ocurrs in RQED provided the coupling exceeds a critical value
- Educated interactions bring closer to experimentally achievable results

¡GRACIAS!