The Standard Model and Beyond

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Abstract

This set of lectures is aimed at graduate students in particle physics, and is heavily based on content written by Yosef Nir for previous iterations of this school (who thus deserves most credit for this material). Following an overview of the Lagrangian formalism and the significance of symmetries in fundamental physics, we outline the Standard Model of Particle Physics as a gauge theory where the local $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken into $SU(3)_C \times U(1)_{EM}$ through the Brout-Englert-Higgs mechanism. The elementary particles and interactions are discussed along with the predictions and experimental tests of the model. Finally, the free parameters are enumerated, and outstanding questions and possible extensions to the Standard Model are discussed.

Outline

1 Introduction to the Lagrangian formalism

In modern physics, we encode the fundamental laws of nature through the quantum interpretation of the principle of least action. In Quantum Field Theory (QFT) the action is the integral over spacetime of the "Lagrangian density", $\mathcal{L}$:

$$S = \int d^4x \mathcal{L}[\phi_i(x), \partial_\mu \phi_i(x)], \tag{1}$$

where $d^4x = dx^0dx^1dx^2dx^3$ is the volume element in 4D Minkowski spacetime and the index i runs from 1 to the number of fields, and $\phi(x)$ refers to a generic field. In general the Lagrangian density (hereafter referred to just as the Lagrangian) must satisfy the following requirements:

1. It is only a function of the fields and their derivatives (this ensures translational invariance)

and must be invariant under the Poincaré group (spacetime translations and Lorentz transformations).

2. It only depends on the fields taken at a single point x^μ (to give a local theory).

3. It is real (so total probability is conserved).

4. It must be an analytic function of the fields (this is not a general requirement but is common to all field theories solved using perturbation theory, where we expand about a minimum leading to Lagrangians that are polynomials in the fields.

5. It must be invariant under certain internal symmetry groups that correspond to conserved quantities and reflect the fundamental symmetries of nature.

Two additional requirements are important when model-building:

7. Naturalness- every term not forbidden by a symmetry should appear.

8. Renormalisability[1] - a renormalisable Lagrangian should only contain terms with a dimension less than or equal to four in the fields and their derivatives.

The renormalisability requirement ensures that the Lagrangian contains no more than two ∂_μ operators, giving classical equations of motion with no higher than second order partial derivatives. We expect that the Lagrangian for the complete "theory of everything" in nature should indeed be renormalisable, however if we assume that the theory in question, here the Standard Model, is only an effective field theory valid up to some energy scale Λ then the non-renormalisable terms must also be considered. Such terms have coefficients with inverse mass dimensions $1/\Lambda^n$ for $n = 1, 2, 3....$ In practise, the renormalisable terms are the leading terms in an expansion in E/Λ where E is the energy scale of the process being studied, which means that the renormalisable part of the Lagrangian is a good starting point to study.

[1]The topic of renormalisaton goes far beyond this handout. This effectively refers to techniques that can be applied to handle infinities arising in calculated quantities by absorbing the divergances into the values of the physical quantities. A "nice" definition I found on the internet of a renormalisable theory is one where the adjustment of a finite number of parameters (such as the bare electron charge and mass) allows us to calculate the results of all observable in finite terms. A theory is not renormalizable if you need infinitly many quantities to absorb the infinities, and such theories are usually only considered as effective theories below some higher energy cut-off Λ.

Properties (i)-(iv) are covered extensively in many QFT courses and will not be discussed further here. However the final three, and particularly (v) will be covered extensively in this course. In particular, we will see how the particle content and interactions of the Standard Model can be explained by a spontaneously broken $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, as well as seeing how the predictions of this model have been tested experimentally to an impressive precision in the last half-century.

2 Symmetries in fundamental physics

Table 1 summarises the consequences in nature of imposing different types of symmetries on QFTs. It should be noted that *accidental symmetries* that can arise in QFTs but are not imposed as external constraints. These often arise when Lagrangian's are truncated, i.e. renormalisable terms in the Lagrangian often have accidental symmetries that are broken by non-renormalisable terms or anomalies [2]. These will be discussed for the renormalisable SM Lagrangian in these lectures.

Type	Consequence
Spacetime	Conservation of energy, momentum, angular momentum
Discrete	Selection rules
Global (exact)	Conserved charges
Global (spontaneously broken)	Massless scalars
Local (exact)	Interactions, massless spin-1 mediators
Local (spontaneously broken)	Interactions and massive spin-1 mediators

Table 1: Symmetries typically encountered in QFTs and their consequences

In the SM we only impose *local* symmetries, whilst in extensions of the SM we typically only impose *local* and *global discrete* symmetries. It is possible in principle to impose *global continuous symmetries* but this is rarely done in model building for two reasons: firstly there are arguments suggesting global continuous symmetries are always broken by gravitational effects so can only arise as accidental symmetries, and secondly, there are not obvious phenomenological motivations for such

[2] In Quantum Field Theory anomalies refer to symmetries that exist at a classical level but are broken in the theory. To be more specific, is the failure of a symmetry of a theory's classical action to be a symmetry of any regularization of the full quantum theory. Gauge anomalies refer to effects, often one-loop diagrams, that break the gauge symmetry of the theory. A well-known example is the chiral or " Adler–Bell–Jackiw" anomaly in electroweak interactions, whereby a symmetry of classical electrodynamics is violated by quantum corrections. It originally referred to the observed anomalous decay rate of the neutral pion into two photons.

symmetries.

When considering the implications of imposing symmetries on the particle content of the theory, the following consequences must be considered in addition to those in Table 1.

- The lightest particle charged under a symmetry is stable.
- Charged fermions cannot have Majorana masses.
- Chiral fermions cannot have Dirac masses. [3]

When considering the Majorana vs Dirac nature of fermions, the following differences should be noted:

- Dirac fermions have 4 degrees of freedom compared to 2 for Majorana fermions.
- Dirac fermions have a $m \times n$ general mass matrix, compared to Majorana fermions which have an $(m+n) \times (m+n)$ symmetric matrix.

Within the SM, the quarks and leptons are Dirac Fermions, whereas neutrinos *could* be Majorana fermions (more to come on this later).

Combining these observations it should thus be noted that *charged fermions in a chiral representation are massless* which means that there's a way that the existence of massless fermions in nature can be explained through symmetry principles.

3 Constructing the Standard Model

To construct a model describing elementary particles/interactions the following must be specified:

- The symmetry
- The transformation properties of the fermions and scalars under that symmetry.
- The pattern of spontaneous symmetry breaking

[3] A chiral fermion is just a field which transforms in one of the Weyl representations of the Lorentz group. A Dirac fermion can be composed into two chiral fermions. A key feature of the SM is that the fermions are chiral, in that the left- and right- Weyl spinor components of the would-be Dirac spinor representations couple differently to the Gauge fields.

The most general Lagrangian depending on the scalars and fermions that is invariant under that symmetry can then be written out. If the symmetry is local (see Table 1) then the required vector fields must also be added. The Lagrangian is written up to some order in the fields. If imposing renormalisability the Lagrangian is truncated at dimension four in the field, and can be decomposed into:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\psi} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi} \tag{2}$$

where $\mathcal{L}_{\text{kin}}$ describes the free spacetime propagation of all the dynamical fields and gauge interactions, $\mathcal{L}_{\psi}$ gives the fermion mass terms, $\mathcal{L}_{\text{Yuk}}$ describes the Yukawa interactions between the scalar sector and fermions, and $\mathcal{L}_{\phi}$ gives the scalar potential.

The resulting Lagrangian will have a number of free parameters that must be determined experimentally, with the number of free parameters (N) determining the number of independent experiments that must be performed to extract the parameters. Additional measurements then provide tests of the theory.

For the Standard Model is defined as follows:

- The gauge symmetry is a local

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \tag{3}$$

  which is spontaneously broken into

$$G_{SM} \to SU(3)_C \times U(1)_{EM}, \quad (Q_{EM} = T_3 + Y) \tag{4}$$

- There are three generations of fermions, each of which consists of five representations of G_{SM}

$$Q_{Li}(3,2)_{+\frac{1}{6}}, \quad U_{Ri}(3,1)_{+\frac{2}{3}}, \quad D_{Ri}(3,1)_{-\frac{1}{3}}, \quad L_{Li}(1,2)_{-\frac{1}{2}}, \quad E_{Ri}(1,1)_{-1} \quad (i = 1,2,3), \tag{5}$$

  and a single scalar field

$$\phi(1,2)_{+\frac{1}{2}} \tag{6}$$

In the notation $(A, B)_Y$, A is the representation under $SU(3)_C$, B is the representation under $SU(2)_L$ and Y is the hypercharge. The fermions that transform as triplets under $SU(3)_C$ are the quarks, whilst those that transform as singlets are the leptons.

Referring back to Equation (2), we will now discuss the explicit form of the Lagrangian made of the fermion fields Q_{Li}, U_{Ri}, D_{Ri}, L_{Li} and E_{Ri} in Equation (5) and the scalar field ϕ in Equation (6), subject to the gauge symmetry in Equation (3) and generating the SSB in Equation (4).

3.1 $\mathcal{L}_{\text{kin}}$

The local symmetries in the SM gauge group require the introduction of three types of gauge boson degrees of freedom

$$G_a^\mu(8,1)_0, \quad W_a^\mu(1,3)_0, \quad B^\mu(1,1)_0 \tag{7}$$

which have corresponding field strengths

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu \tag{8}$$

$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu \tag{9}$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \tag{10}$$

where $f_{abc}(\epsilon_{abc})$ are the $SU(3)$ ($SU(2)$) structure constants. The covariant derivative is

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y \tag{11}$$

where the L_a's are $SU(3)_C$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_L$ generators (the 2×2 Pauli matices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets) and the Y's are the $U(1)_Y$ charges. The explicit forms of the covariant derivative acting on the fermion and scalar fields are

$$\begin{aligned}
D^\mu \phi &= (\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu)\phi, \\
D^\mu Q_{Li} &= (\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu) Q_{Li}, \\
D^\mu U_{Ri} &= (\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{2i}{3} g' B^\mu) U_{Ri}, \\
D^\mu D_{Ri} &= (\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{3} g' B^\mu) D_{Ri}, \\
D^\mu L_{Li} &= (\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu) L_{Li}, \\
D^\mu E_{Ri} &= (\partial^\mu - ig' B^\mu) E_{Ri},
\end{aligned} \tag{12}$$

giving a resulting Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{kin}} = &-\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
&- i\bar{Q_{Li}} \not{D} Q_{Li} - i\bar{U_{Ri}} \not{D} U_{Ri} - i\bar{D_{Ri}} \not{D} D_{Ri} - i\bar{L_{Li}} \not{D} L_{Li} - i\bar{E_{Ri}} \not{D} E_{Ri} - (D^\mu \phi)^\dagger (D_\mu \phi)
\end{aligned} \tag{13}$$

This part of the Lagrangian is flavour-universal and conserves CP. (Note the "slash" notation $\not{D}$ corresponds to $\gamma^\mu D_\mu$

3.2 $\mathcal{L}_\psi$

There are no explicit mass terms for fermions in the SM Lagrangian. Dirac mass terms cannot be written as the fermions are assigned to chiral representations of the gauge symmetry. Majorana mass terms also cannot be used as they have $Y \neq 0$ so $\mathcal{L}_\psi^{SM} = 0$

3.3 $\mathcal{L}_{\text{Yuk}}$

In the SM the Yukawa part of the Lagrangian is

$$\mathcal{L}_{\text{Yuk}}^{SM} = Y_{ij}^d \bar{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \bar{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \bar{L_{Li}} \phi E_{Rj} + h.c \tag{14}$$

where $\tilde{\phi} = i\tau_2 \phi^\dagger$ and the Y^f are general 3×3 matrices of dimensionless couplings. In general this part of the Lagrangian is flavour-dependent (i.e. $Y^f \not\propto \mathbf{1}$) ansd CP violating. WLOG, we can perform a bi-unitary transformation[4] to diagonalise the lepton Yukawa couplings ($\hat{Y}^e$ is diagonal and real):

$$Y^e \to \hat{Y}^e = U_{eL} Y^e U_{eR}^\dagger = \text{diag}(y_e, y_\mu, y_\tau). \tag{15}$$

In the basis defined by the transformation in Equation (15) the components of the lepton $SU(2)$-doublets and the three singlets are:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \; e_R, \; \mu_R, \; \tau_R. \tag{16}$$

where e, μ, τ are ordered by the size of their (increasing) Yukawa couplings.

A similar approach can be taken in the quark sector:

$$Y^u \to \hat{Y}^u = V_{uL} Y^u V_{uR}^\dagger = \text{diag}(y_u, y_c, y_t). \tag{17}$$

In this basis, the components of the quark $SU(2)$- doublets and the three "up-type" quark singlets are:

$$\begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \begin{pmatrix} c_L \\ s_{cL} \end{pmatrix}, \begin{pmatrix} t_L \\ b_{tL} \end{pmatrix}, \; u_R, \; c_R, \; t_R. \tag{18}$$

where as before u, c, t are ordered by their increasing Yukawa couplings. A different bi-unitary transformation can be performed to diagonalise the "down-type" quark masses:

[4] For a general mass matrix m_{ij} there exist unitary matrices S and T such that $S^\dagger m T = m_d$ is diagonal. S is the unitary matrix that diagonalises the Hermitian combination $mm^\dagger$ i.e. $S^\dagger(mm^\dagger)S = m_d^2$

$$Y^d \to \hat{Y}^d = V_{dL} Y^d V_{dR}^\dagger = \text{diag}(y_d, y_s, y_b). \tag{19}$$

In this basis the components of the quark $SU(2)$- doublets and the three "down-type" quark singlets are:

$$\begin{pmatrix} u_{dL} \\ d_L \end{pmatrix}, \begin{pmatrix} c_{sL} \\ s_L \end{pmatrix}, \begin{pmatrix} t_{bL} \\ b_L \end{pmatrix}, \ d_R, \ s_R, \ b_R. \tag{20}$$

where as before d, s, b are ordered by their increasing Yukawa couplings. In general, $V_{uL} \neq V_{dL}$,meaning that the interaction bases defined by Equation (17) and Equation (19) are different. Choosing the "up-type" quark basis, we can Y^d as a product of a unitary matrix and a diagonal one

$$Y^u = \hat{Y}^u, \quad Y^d = V\hat{Y}^d \tag{21}$$

and similarly in the "down-type" quark basis

$$Y^d = \hat{Y}^d, \quad Y^u = V^\dagger \hat{Y}^u \tag{22}$$

where in both cases the matrix V is given by

$$V = V_{uL} V_{dL}^\dagger \tag{23}$$

It should be noted that whilst V_{uL}, V_{uR}, V_{dL} and V_{dR} depend on the basis from which we start the diagonalisation, the combination $V = V_{uL} V_{dL}^\dagger$ does not which suggests that V is indeed related to physical quantities. We will soon see that it plays a crucial role in determining charged current interactions.

3.4 $\mathcal{L}_\phi$

The scalar potential in the SM Lagrangian is:

$$\mathcal{L}_\psi^{SM} = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \tag{24}$$

Note that this part also conserves CP. $\mu^2 < 0$ and $\lambda > 0$ generates the required spontaneous symmetry breaking (SSB). Defining $v^2 = -\frac{\mu^2}{\lambda}$ the Lagrangian can be re-written (up to a constant term) as

$$\mathcal{L}_\psi^{SM} = -\lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 \tag{25}$$

This form of the scalar potential implies that the scalar field acquires a vacuum expectation value (VEV) of $|\langle \phi \rangle| = \frac{v}{\sqrt{2}}$. Chosing the direction of $\langle \phi \rangle$ in the real direction of the "down" component

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{26}$$

This VEV breaks the $SU(2) \times U(1)$ electroweak symmetry down to a $U(1)$ subgroup. This corresponds to there being one (and only one) linear combination of generators that annihilates the vaccuum state. For the choice in Equation (26) this is $T_3 + Y$ which we identify as the generator Q of the unbroken subgroup $U(1)_{EM}$.

3.5 Summary

The renormalisable part of the SM Lagrangian is

$$\begin{aligned}\mathcal{L}_{SM} = &-\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - (D^\mu\phi)^\dagger(D_\mu\phi)\\ &- i\bar{Q_{Li}}\not{D}Q_{Li} - i\bar{U_{Ri}}\not{D}U_{Ri} - i\bar{D_{Ri}}\not{D}D_{Ri} - i\bar{L_{Li}}\not{D}L_{Li} - i\bar{E_{Ri}}\not{D}E_{Ri}\\ &+ (Y_{ij}^d\bar{Q_{Li}}\phi D_{Rj} + Y_{ij}^u\bar{Q_{Li}}\tilde{\phi}U_{Rj} + Y_{ij}^e\bar{L_{Li}}\phi E_{Rj} + h.c)\\ &- \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2\end{aligned} \tag{27}$$

where $i, j = 1, 2, 3$

4 The SM particle spectrum

4.1 Scalars

If we go back to $\mathcal{L}_\phi$, and denote the four real components of the scalar doublet as three phases $\theta_a(x)$ (with $a = 1, 2, 3$) and one magnitude $h(x)$. The three phases are the "would-be" Goldstone bosons. In the SM the broken generators are T_1, T_2 and $T_3 - Y$, so we can write:

$$\phi(x) = \exp\left[\frac{i}{2}(\sigma_a\theta_a(x) - I\theta_3(x)\right]\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix} \tag{28}$$

The local $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian allows one to rotate away the explicit dependence on the three $\theta_a(x)$. These would-be Goldstone bosons are "eaten" by the three gauge bosons that acquire masses as a result of SSB. In this gauge the one degree of freedom in $\phi(x)$ can then be seen through:

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix} \tag{29}$$

The scalar $h(x)$ is the Higgs boson, which is an $SU(3)_C$ singlet and is $U(1)_{EM}$. By plugging Equation (29) into Equation (25) we can identify the mass as

$$m_h^2 = 2\lambda, v^2 \tag{30}$$

Experimentally, the best combined value quoted by the Particle Data Group (PDG) based on measurements by the ATLAS and CMS collaborations at CERN is 125.25 ± 0.17 GeV [1]

4.2 Vector bosons

As the $SU(3)_C$ gauge symmetry is unbroken after SSB, the gluon, which exists as an $SU(3)_C$ colour-octet, and is $U(1)_{EM}$-neutral, is massless with $m_g = 0$.

For the broken $SU(2)_L \times U(1)_Y$, three of the four vector bosons (corresponding to the spontaneously broken generators) acquire masses whilst one remains massless. Considering $(D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle)$ and using the expression for $D^\mu(\phi)$ from Equation (12)

$$D^\mu\langle\phi\rangle = \frac{i}{\sqrt{8}}(gW_a^\mu\sigma_a + g'B^\mu)\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{i}{\sqrt{8}}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1^\mu - iW_2^\mu) \\ g(W_1^\mu + iW_2^\mu) & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{31}$$

meaning that the mass terms of the vector bosons are given by

$$\mathcal{L}_{M_V} = \frac{1}{8}\begin{pmatrix} 0 & v \end{pmatrix}\begin{pmatrix} gW_{3\mu} + g'B_\mu & g(W_1 - iW_2)_\mu \\ g(W_1 + iW_2)_\mu & -gW_{3\mu} + g'B_\mu \end{pmatrix}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1 - iW_2)^\mu \\ g(W_1 + iW_2)^\mu & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} \tag{32}$$

Defining the weak mixing angle θ_W through

$$\tan\theta_W = \frac{g'}{g}, \tag{33}$$

we can then define four gauge boson states as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)_\mu,\; Z_\mu^0 = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu,\; A_\mu^0 = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu \tag{34}$$

The $W_\mu^\pm$ are charged under electromagnetism (EM) whilst A_μ^0 and Z_μ^0 are neutral. In terms of these gauge boson fields the Lagrangian can be written

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2v^2W^{+\mu}W_\mu^- + \frac{1}{8}(g^2 + g'^2)v^2Z^{0\mu}Z_\mu^0 \tag{35}$$

which allow us to write

$$m_W^2 = \frac{1}{4}g^2v^2,\;\; m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2,\;\; m_A^2 = 0 \tag{36}$$

(Note that for a complex field ϕ with mass m the mass term is $m^2|\phi|^2$ whilst for a real field it is $\frac{m^2\phi^2}{2}$).

At this point we should note:

- As expected, three vector bosons acquire a mass with one remaining massless.
- $m_A^2 = 0$ should be seen as a consistency check on our calculation rather than a prediction.
- θ_W represents a rotation angle of the two neutral vector bosons from the interaction basis, where the fields have well-defined transformation properties under the full gauge symmetry (W_3, B) into the mass bases for the vector bosons (Z, A)

The SSB in the theory leads to relationships between observables that would otherwise have been independent in the absence of the symmetry. For example:

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2} \tag{37}$$

This relationship can be tested experimentally, with the left hand side being derived from the measured mass spectrum and the right from the interaction rates. This relationship is conventionally expressed in terms of θ_W as

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \tag{38}$$

The $\rho = 1$ results from SSB by $SU(2)$-doublets and thus provides a test of this specific ingredients of the SM. The experimental values of the weak gauge boson masses are [1]:

$$m_W = 80.377 \pm 0.012\text{GeV}, \quad m_Z = 91.1876 \pm 0.0021\text{GeV} \tag{39}$$

By requiring $\rho = 1$ the ratio of these can be used to determine $\sin^2\theta_W$ which can also be determined experimentally through various interaction rates. Taking the values above

$$\frac{m_W}{m_Z} = 0.8814 \pm 0.0001 \to \sin^2\theta_W = 1 - \left(\frac{m_W}{m_Z}\right) 0.2231 \tag{40}$$

which can be then compared to values determined experimentally.

4.3 Fermions

As the SM does not allow bare mass terms for the charged fermions, their masses can only arise from the Yukawa part of the Lagrangian in Equation (14). With $\langle\phi_0\rangle = \frac{v}{\sqrt{2}}$ this has a piece corresponding to charged lepton masses

$$m_e = \frac{y_e v}{\sqrt{2}}, \quad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{y_\tau v}{\sqrt{2}}, \tag{41}$$

a piece corresponding to up-type quark masses

$$m_u = \frac{y_u v}{\sqrt{2}}, \quad m_c = \frac{y_c v}{\sqrt{2}}, \quad m_t = \frac{y_t v}{\sqrt{2}}, \tag{42}$$

and a piece corresponding to down-type quark masses.

$$m_d = \frac{y_d v}{\sqrt{2}}, \quad m_s = \frac{y_s v}{\sqrt{2}}, \quad m_b = \frac{y_b v}{\sqrt{2}}. \tag{43}$$

In other words, all charged fermions acquire Dirac masses as a result of the SSB in the SM. This means that whilst they are in chiral representations of the full SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ they are in vector-like representations of the $SU(3)_C \times U(1)_{EM}$ group:

- The LH and RH charged lepton fields e, μ, τ are in the $(0)_{-1}$ representation.
- The LH and RH up-type quarks fields u, c, t are in the $(3)_{+\frac{2}{3}}$ representation.
- The LH and RH down-type quark fields d, s, b are in the $(3)_{-\frac{1}{3}}$ representation.

On the other hand the neutrinos remain massless

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \tag{44}$$

In the construction we have followed, this is the case in spite of the fact that neutrinos transform as $(1)_0$ under the unbroken gauge group, which in principle then allows Majorana masses. As we will soon see, this masslessness is an *accidental symmetry* of the SM.

The experimental values of the charged fermion masses are [1]:

$$\begin{aligned}
m_e &= 0.51099895000 \pm 0.0000000001 \text{ MeV} \\
m_\mu &= 105.6583755 \pm 0.0000023 \text{ MeV}, \\
m_\tau &= 1776.86 \pm 0.12 \text{ MeV},
\end{aligned} \tag{45}$$

$$m_u = 2.16^{+0.49}_{-0.26} \text{ MeV}, \quad m_c = 1.27 \pm 0.02 \text{ GeV}, \quad m_t = 172.69 \pm 0.30 \text{ GeV}$$

$$m_d = 4.67^{+0.48}_{-0.17} \text{ MeV}, \quad m_s = 93.4^{+8.6}_{-3.4} \text{ MeV}, \qquad m_b = 4.18^{+0.03}_{-0.02} \text{ GeV}$$

4.4 Summary

A summary of the mass eigenstates of the SM, their $SU(3)_C \times U(1)_{EM}$ quantum numbers and masses in units of the VEV v is provided in Table 2.

Particle	Spin	Colour	Q	Mass [v]
$W^{\pm}$	1	(1)	± 1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e, μ, τ	$\frac{1}{2}$	(1)	-1	$\frac{y_{e,\mu,\tau}}{\sqrt{2}}$
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	(1)	0	0
u, c, t	$\frac{1}{2}$	(3)	$+\frac{2}{3}$	$\frac{y_{u,c,t}}{\sqrt{2}}$
d, s, b	$\frac{1}{2}$	(3)	$-\frac{1}{3}$	$\frac{y_{d,s,b}}{\sqrt{2}}$

Table 2: Summary of SM particle spectrum

All masses are proportional to the VEV of the scalar field, v, which for the massive gauge bosons and fermions is expected. In the absence of SSB gauge boson mass terms would be forbidden by the gauge symmetry and for fermions they would be protected by their chiral nature. The situation is difficult for the Higgs boson as a mass-squared term doesn't violate any symmetry. $m_h \propto v$ is a manifestation of the fact the SM has a single dimensionful parameter, which can be taken to be v, and thus all masses must be proportional to this parameter.

5 SM interactions

5.1 EM and strong interactions

As discussed previously, a local $SU(3)_C \times U(1)_{EM}$ survives SSB in the SM, which means that massless photon and gluon gauge fields exist in the SM. All charged fermions interact with the photon giving the QED Lagrangian

$$\mathcal{L}_{QED,\psi} = -\frac{2e}{3}\bar{u}_i \not{A} u_i + \frac{e}{2}\bar{d}_i \not{A} d_i + e\bar{\ell}_i \not{A} \ell_i \tag{46}$$

where $u_{1,2,3} = u, c, t$, $d_{1,2,3} = d, s, b$ and $\ell_{1,2,3} = e, \mu, \tau$.

The following points should be emphasised about QED:

- The photon couplings are *vector-like* and *parity conserving*.
- *Diagonality*: the photon couples to e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ but not between generations i.e. to $e^\pm\mu^\mp$, $e^\pm\tau^\mp$ or $\mu^\pm\tau^\mp$ pairs, and similarly in the up-type and down-type quark sectors.
- *Universality*: the photon couplings to different generations are universal.

All coloured fermions (i.e. the quarks) interact with the gluon

$$\mathcal{L}_{QCD,\psi} = -\frac{g_s}{2}\bar{q}\lambda_a \not{G}_a q, \tag{47}$$

where $q = u, c, t, d, s, b$. For QCD the following points should be emphasised:

- The gluon couplings are *vector-like* and *parity conserving*.
- *Diagonality*: the gluon couples to $q\bar{q}$ but doesn't allow flavour changing pairs $q'\bar{q}$ for example $\bar{t}c$.
- *Universality*: the gluon couplings to different quark generations are universal.

The universality of photon and gluon couplings arises from the $SU(3)_C \times U(1)_{EM}$ gauge invariance and thus holds in any model, not just the SM.

5.2 Z-boson mediated weak interactions

All SM fermions couple to the Z-boson as described by

$$\begin{aligned}\mathcal{L}_{Z,\psi} = \frac{e}{s_W c_W}\Bigg[&-\left(\frac{1}{2}-s_W^2\right)\bar{e_{Li}}\not{Z}e_{Li} + s_W^2\bar{e_{Ri}}\not{Z}e_{Ri} + \frac{1}{2}\bar{\nu_{L\alpha}}\not{Z}\nu_{L\alpha} + \left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\bar{u_{Li}}\not{Z}u_{Li} \\ &-\frac{2}{3}s_W^2\bar{u_{Ri}}\not{Z}u_{Ri} - \left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\bar{d_{Li}}\not{Z}d_{Li} + \frac{1}{3}s_W^2\bar{d_{Ri}}\not{Z}d_{Ri}\Bigg]\end{aligned} \tag{48}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. The following points should be noted:

- The Z-boson couplings are *chiral* and *parity violating*.
- *Diagonality*: the Z-boson couples diagonally, which means there are no Z-mediated flavour-changing neutral current (FCNC) processes.
- *Universality*: The couplings of the Z-boson to different fermion generations are universal.

The universality of the Z-boson weak interactions results from a special feature of the SM: all fermions of a given chirality and given charge come from the same $SU(2)_L \times U(1)_Y$ representation. In terms of experimental tests of this diagonality and universality, consider the leptonic sector. If we consider the experimental values for the Z- boson branching fractions according to the particle data group (PDG) [1]

$$\begin{aligned} BR(Z \to e^+e^-) &= 3.3632 \pm 0.0042\% \\ BR(Z \to \mu^+\mu^-) &= 3.3662 \pm 0.0066\% \\ BR(Z \to \tau^+\tau^-) &= 3.3696 \pm 0.0083\% \end{aligned} \tag{49}$$

The ratios of these values (also quoted from PDG [1]) are consistent with universality.

$$\begin{aligned} \frac{\Gamma(\mu^+\mu^-)}{\Gamma(e^+e^-)} &= 1.0001 \pm 0.0024 \\ \frac{\Gamma(\tau^+\tau^-)}{\Gamma(e^+e^-)} &= 1.0020 \pm 0.0032 \end{aligned} \tag{50}$$

Similarly, the diagonality can be tested by experimental searches for processes that would violate this

$$\begin{aligned} BR(Z \to e^\pm\mu^\mp &< 7.5 \times 10^{-7} \\ BR(Z \to e^\pm\tau^\mp &< 5.0 \times 10^{-6} \\ BR(Z \to \mu^\pm\tau^\mp &< 6.5 \times 10^{-6} \end{aligned} \tag{51}$$

If we omit common factors, particularly a factor of $\frac{e^2}{4s_W^2c_W^2}$, and phase space factors, the following predictions for Z-boson decays can be obtained for Z-decays into a single-generation fermion-antifermion pair of each type

$$\begin{aligned} \Gamma(Z \to \nu\bar{nu}) &\propto 1, \\ \Gamma(Z \to \ell\bar{\ell}) &\propto 1 - 4s_W^2 + 8s_W^4, \\ \Gamma(Z \to u\bar{u}) &\propto 3\left(1 - \frac{8}{3}s_W^2 + \frac{32}{9}s_W^4\right), \\ \Gamma(Z \to d\bar{d}) &\propto 3\left(1 - \frac{4}{3}s_W^2 + \frac{8}{9}s_W^4\right), \end{aligned} \tag{52}$$

Subbing in $s_W^2 = 0.225$ this gives

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.51 : 1.74 : 2.24 \tag{53}$$

Experimentally, the current experimental values (from PDG [1], with the $u\bar{u}$ and $d\bar{d}$ numbers taking from the older 2010 update) are

$$\begin{aligned} BR(Z \to \nu\bar{\nu}) &= 6.6667 \pm 0.01833\% \\ BR(Z \to \ell\bar{\ell}) &= 3.3658 \pm 0.0023\% \\ BR(Z \to u\bar{u}) &= 11.6 \pm \pm 0.6\% \\ BR(Z \to d\bar{d}) &= 15.6 \pm 0.4\% \end{aligned} \tag{54}$$

which, taking the central values only, gives

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.505 : 1.740 : 2.340 \tag{55}$$

which are fairly consistent.

5.3 W-boson

When considering charged vector boson $W^\pm$ to fermion pairs. For the lepton mass eigenstates, things are relatively simple as there exists an interaction basis that is also a mass basis, which means the W-boson interactions are universal in the mass basis:

$$\mathcal{L}_{W,\ell} = -\frac{g}{\sqrt{2}}(\bar{\nu_e}_L \not{W}^+ e_L^- + \bar{\nu_\mu}_L \not{W}^+ \mu_L^- + \bar{\nu_\tau}_L \not{W}^+ \mu_L^- + h.c.) \tag{56}$$

This equation reveals some important features of the SM:

- Only left-handed leptons take part in charged-current interactions, which means parity is violated.
- *Diagonality*: the charged current (CC) interactions couple each charged lepton to a single neutrino, and vice versa. Note that a global $SU(2)$ symmetry would allow off-diagonal couplings, but it is the local symmetry that generates this diagonality.
- *Universality*: the couplings of the W-boson to $\tau\nu_\tau$, $\mu\nu_\mu$ and $e\nu_e$ are equal. As above, a global symmetry would have accommodated independent couplings to each lepton pair.

All of these predictions have been experimentally tested. For universality, lets consider the branching ratios of the W-boson to the three lepton flavour pairs [1]

$$\begin{aligned} BR(W^+ \to e^+\nu_e) &= (10.71 \pm 0.16)\times 10^{-2} \\ BR(W^+ \to \mu^+\nu_\mu) &= (10.63 \pm 0.15)\times 10^{-2} \\ BR(W^+ \to e^+\nu_e) &= (11.38 \pm 0.21)\times 10^{-2} \end{aligned} \tag{57}$$

which provide a nice confirmation of universality

$$\begin{aligned}\frac{\Gamma(\mu^+\nu_\mu}{\Gamma(e^+\nu_e)} &= 0.996 \pm 0.008\\ \frac{\Gamma(\tau^+\nu_\mu}{\Gamma(e^+\nu_e)} &= 1.043 \pm 0.024\end{aligned} \tag{58}$$

For the quark sector things are more complicated as there is no interaction basis that is also a mass basis. In the interaction basis where the down-type quarks are the mass eigenstates, the CC interactions have the following form:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}}(\bar{u_{dL}}\not{W}^+ d_L + \bar{u_{sL}}\not{W}^+ s_L + \bar{u_{bL}}\not{W}^+ b_L + h.c.) \tag{59}$$

The Yukawa matrices in this basis have the form given in Equation (22) and in particular in the up-quark sector we have

$$\mathcal{L}^u_{\text{Yuk}} = \begin{pmatrix} \bar{u_{dL}} & \bar{u_{sL}} & \bar{u_{bL}} \end{pmatrix} V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \tag{60}$$

which tells us that the transformation to the mass basis is given by

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}. \tag{61}$$

This equation allows us to write the form of the CC interactions in the mass basis as:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \begin{pmatrix} \bar{u_L} & \bar{c_L} & \bar{t_L} \end{pmatrix} V \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c. \tag{62}$$

We could easily convince ourselves that we would have obtained the same form as Equation (62) starting from any arbitrary interaction basis. It should be noted again at this point that $V = V_{uL} V^\dagger_{dL}$ is basis independent. The equation above reveals the following important features:

- Only left-handed quarks take place in CC interactions, which means that again, parity is violated.
- The W-boson couplings to the quarks are *neither universal nor diagonal*. The universality of gauge interactions is hidden in the unitarity of V.

The hidden universality in the quark sector can be tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \text{hadrons}) \tag{63}$$

Experimentally (from the PDG [1]),

$$\frac{\Gamma(W \to cX)}{\Gamma(W \to \text{hadrons})} = 0.49 \pm 0.04 \tag{64}$$

The matrix V is called the CKM matrix after Nicola Cabibbo, Makoto Kobayashi and Toshihide Maskawa. The form is not unique. Firstly, there is freedom in that we can permute between generations. This can be fixed by ordering the up-type and down-type quarks by their masses, i.e. $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$, which gives

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{65}$$

Ommitting common factors (particularly a factor $\frac{g^2}{4}$) and phase-space factors, we get the following predictions for the W-boson decays:

$$\begin{aligned} &\Gamma(W^+ \to \ell^+ \nu_\ell) \propto 1 \\ &\Gamma(W^+ \to u_i \bar{d}_j) \propto 3|V_{ij}|^2, \quad (i = 1, 2;\ j = 1, 2, 3) \end{aligned} \tag{66}$$

where the top quark isn't included as it is heavier than the W-boson so the decay is kinematically forbidden. Taking this, and the CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \tag{67}$$

into account, we obtain

$$\Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons}) \tag{68}$$

Experimentally (from the PDG [1]),

$$BR(W \to \text{leptons}) = (32.58 \pm 0.27), \quad BR(W \to \text{hadrons })(67.41 \pm 0.27)\% \tag{69}$$

which has a ratio of 2.07 (no uncertaities calculated) which is consistent with the SM prediction.

5.4 Interactions of the Higgs

The Higgs boson has self-interactions, weak interactions and Yukawa interactions through

$$
\begin{aligned}
\mathcal{L}_h =& \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4 \\
&+ m_W^2 W_\mu^- W^{\mu +}\left(\frac{2h}{v} + \frac{h^2}{v^2}\right) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \left(\frac{2h}{v} + \frac{h^2}{v^2}\right) \\
&- \frac{h}{v}(m_e \bar{e_L} e_R + m_\mu \bar{\mu_L}\mu_R + m_\tau \bar{\tau_L}\tau_R \\
&+ m_u \bar{u_L} u_R + m_d \bar{d_L} d_R + m_c \bar{c_L} c_R + m_s \bar{s_L} s_R + m_t \bar{t_L} t_R + m_b \bar{b_L} b_R + h.c)
\end{aligned}
\tag{70}
$$

The Higgs couples diagonally to the quark mass eigenstates. This is because the Yukawa couplings determine both the masses and the Higgs couplings to the fermions, which means that in the mass basis the Yukawa interactions are also diagonal. A formal derivation which starts from an arbitrary interaction basis goes as follows:

$$
\begin{aligned}
h\bar{D_L} Y^d D_R &= h\bar{D_L}(V_{dL}^\dagger V_{dL}) Y^d (V_{dR}^\dagger V_{dR}) D_R \\
&= h(\bar{D_L} V_{dL}^\dagger)(V_{dL} Y^d V_{dR}^\dagger)(V_{dR} D_R) \\
&= h(\bar{d_L}, \bar{s_L}, \bar{b_L}) \hat{Y}^d (d_R, s_R, b_R)^T
\end{aligned}
\tag{71}
$$

The Higgs couplings to the fermion mass eigenstates have the following properties:

- *Diagonality*.
- *Non-universality* .
- *Proportionality* to the fermion masses: the heavier the fermion, the stronger the coupling. The factor of proportionality is $\frac{m_\psi}{v}$.

This means the Higgs boson decay width is dominated by the heaviest particle that can be pair-produced in the decay. For m_h 125 GeV this is the bottom quark. The SM predicts the following branching ratios for the leading decay modes:

$$
BR_{b\bar{b}} : BR_{WW^*} : BR_{gg} : BR_{\tau^+\tau^-} : BR_{ZZ^*} : BR_{c\bar{c}} = 0.58 : 0.21 : 0.09 : 0.06 : 0.03 : 0.03. \tag{72}
$$

The following comments should be made about the branching ratios (BRs) in Equation (72):

- Of the six BRs, three (b, τ, c) correspond to two-body tree-level decays, meaning that at tree-level they obey $BR_{b\bar{b}} : BR_{\tau^+\tau^-} : BR_{c\bar{c}} = 3m_b^2 : m_\tau^2 : 3m_c^2$. In reality, QCD radiative corrections somewhat suppress the two modes with quark final states (b, c) compared to the one with the lepton final state (τ).

- The WW^* and ZZ^* modes stand for the three-body tree-level decays where one vector boson is on-shell and one is off-shell.
- The Higgs doesn't have tree-level couplings to gluons since it has no colour and the gluons have no mass. The decay to gluons proceeds via loop diagrams, with the dominant contribution coming from the top-quark loop.
- Similarly the decay to photons proceeds via loop diagrams with a small $(BR_{\gamma\gamma}\ 0.002)$ but observable rate, with dominant contributions coming from the W and top-quark loops which interfere destructively.

One of the very nice summary plots from ATLAS experiment in 2022 shows that the Higgs couplings measured during the second data-taking run appear to satisfy the expected relationships with the particle masses (see Figure Figure 1) [2].

5.5 Summary

Within the SM the fermions have 5 types of interaction which are summarised in Table 3.

Interaction	Fermion	Force carrier	Coupling	Flavour
Electromagnetic	u, d, ℓ	A^0	eQ	Universal
Strong	u, d	g	g_s	Universal
NC weak	all	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	Universal
CC weak	$\bar{u}d/\bar{\ell}\nu$	$W^\pm$	gV/g	Non-universal/ universal
Yukawa	u, d, ℓ	h	$y_{u,d,\ell}$	Diagonal

Table 3: Summary of SM fermion interactions

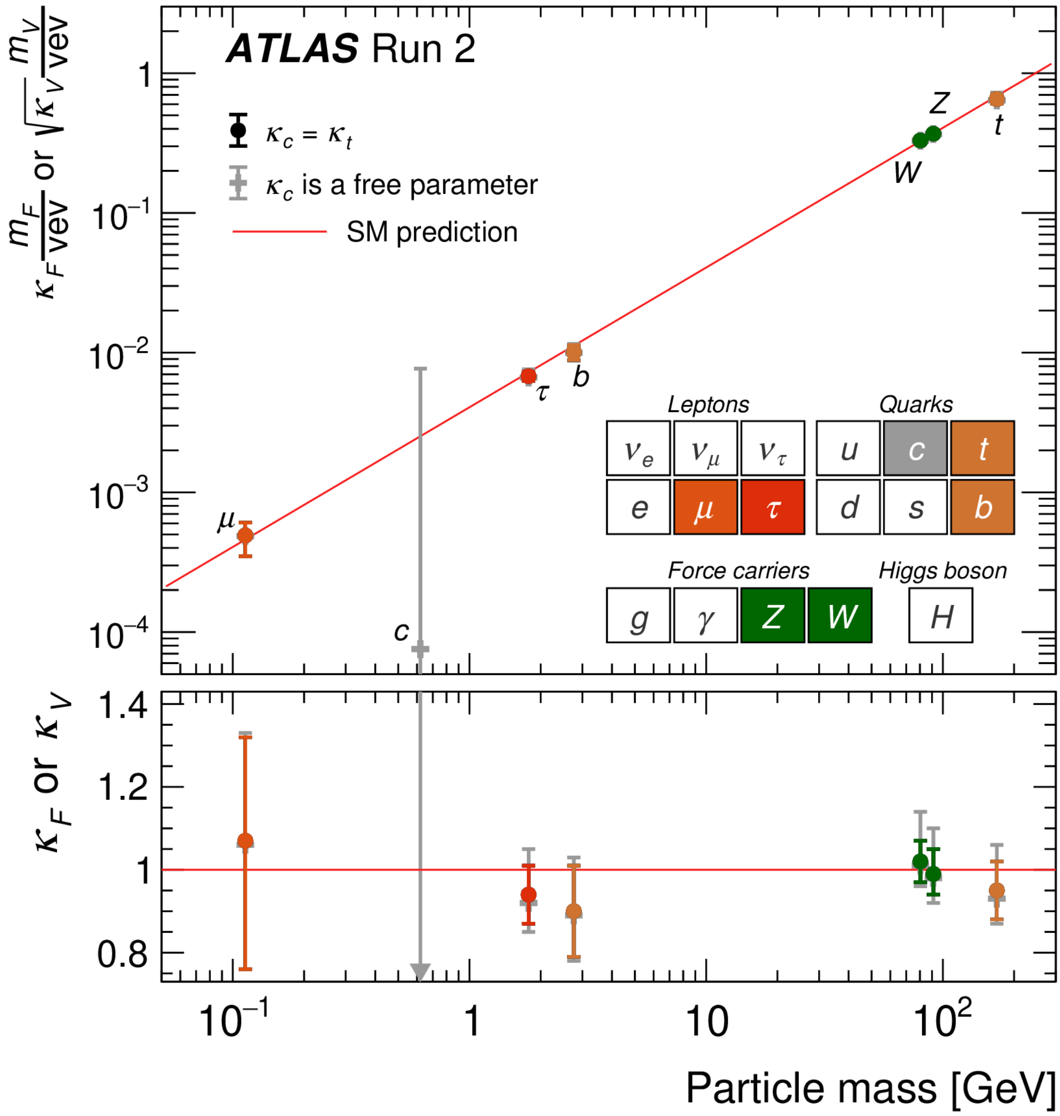

Figure 1: Reduced Higgs boson coupling strength modifiers and their uncertainties based on the ATLAS run 2 coupling measurements. The fact that the points lie on a straight line shows that these are following the expected scaling as a functiuon of particle mass.

6 Accidental symmetries of the SM

In the absence of the Yukawa matrices, $\mathcal{L}_{\text{Yuk}}$, the SM has a large $U(3)^5$ global symmetry:

$$G^{SM}_{global}(Y^{u,d,e} = 0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{73}$$

where

$$\begin{aligned} SU(3)^3_q &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)^2_\ell &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{PQ} \times U(1)_E. \end{aligned} \tag{74}$$

Here, (Q_{L1}, Q_{L2}, Q_{L3}) transform as a triplet under $SU(3)_Q$, and so on. Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y) which are still respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

It is important to note that $\mathcal{L}_{\text{Kin}}$ respects the non-Abelian flavour symmetry $SU(3)^3_q \times SU(3)^2_\ell$ under which:

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{75}$$

where V_i are unitary matrices. The Yukawa interactions in Equation (14) break the global symmetry,

$$G^{SM}_{global}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{76}$$

Under $U(1)_B$, all quarks (anti-quarks) carry charge $+\frac{1}{3}$ $\left(-\frac{1}{3}\right)$, while all other fields are neutral. This explains why proton decay has not been observed. Possible proton decay modes such as $p \to \pi^0 e^+$ or $p \to K^+ \nu$ are not forbidden by the $SU(3)_C \times U(1)_{EM}$ symmetry. However they violate $U(1)_B$ and so do not occur within the SM. This is an example of a quite general lesson: *the lightest particle that carries a conserved charge is stable.* This accidental $U(1)_B$ symmetry also explains why neutron-anti-neutron oscillations have not been observed. The remaining $U(1)$ factors correspond to electron number, muon number and tau number respectively. The charges of ν_e and e are (1,0,0), those of ν_μ and μ are (0,1,0) and finally those of ν_τ and τ are (0,0,1). This situation, for example, allows the muon decay mode $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ but forbids $\mu^- \to e^- \gamma$ and $\mu^- \to e^- e^+ e^-$. It is useful to define the total lepton number, which is the sum of the of the three lepton flavours and corresponds to a $U(1)_L$ symmetry. This is another accidental symmetry of the SM.

It should be noted that $U(1)_B$ as well as each of the lepton numbers are violated by chiral anomalies. The combination of $B-L$, however, is anomaly free as the anomalies cancel. Due to the anomalies, baryon and lepton number violating processes occur non-perturbatively in the SM. However the non-perturbative effects obey $\Delta B = \Delta L = 3n$ where n is an integer, and so do not lead to proton decay. Furthermore, they are very small and can be neglected in almost all cases we study, so they will not be discussed further here.

The accidental symmetries of the renormalisable part of the SM also explain the vanishing of neutrino masses. Since all neutrinos are charged under $U(1)_L$, a Majorana mass term violates the accidental $B-L$ symmetry by two units which prevents mass terms not only at tree level but to all orders in perturbation theory. Since the $B-L$ symmetry is non-anomalous , Majorana mass terms do not even arise at the non-perturbative level. We thus conclude that the renormalisable SM gives the *exact* prediction that

$$m_\nu = 0. \tag{77}$$

The transformations of Equation (75) are not a symmetry of $\mathcal{L}_{SM}$. Instead they correspond to a change of the interaction basis. These observations also provide a definition of flavour physics: it refers to interactions that break the $SU(3)^5$ symmetry (Equation (75)), meaning that "flavour violation" often describes processes or parameters that break this symmetry. The quark Yukawa couplings can be thought of as spurions[5] that break the global $SU(3)^3_q$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \begin{pmatrix} 3 & \bar{3} & 1 \end{pmatrix}_{SU(3)^3_q}, \quad Y^d \begin{pmatrix} 3 & 1 & \bar{3} \end{pmatrix}_{SU(3)^3_q}, \tag{78}$$

and the lepton Yukawa couplings can be thought of as spurions that break the global $SU(3)^2_\ell$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \; (3, \bar{3})_{SU(3)^2_\ell} \tag{79}$$

This spurion formalism is convenient for parameter counting, identification of flavour suppression factors and for the idea of minimal flavour violation.

6.1 Counting parameters in the SM

To work out the number of independent parameters in $\mathcal{L}^q_{\text{Yuk}}$, we start by considering that the two Yukawa matrices Y^u and Y^d are 3×3 and complex. This means there are 18 real and 18 imaginary

[5] Spurions are fictitious auxiliary fields in QFT that can be used to parametrise symmetry breaking and determine all parameters invariant under that symmetry

parameters in these matrices, however not all of them are physical. The pattern of G_{global} breaking means there is freedom to remove 9 real and 17 imaginary parameters (this is the number of parameters in three 3×3 matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$ to lead to the following interaction basis:

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u \tag{80}$$

where $\lambda_{d,u}$ are diagonal

$$\lambda_d = \text{diag}(y_d, y_s, y_b), \quad \lambda_u = \text{diag}(y_u, y_c, y_t) \tag{81}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We thus conclude that there are 10 quark flavour parameters: 9 real ones and a single phase. In the mass basis we identify the nine real parameters as six quark masses and three mixing angles, while the single phase is δ_{KM}.

To consider $\mathcal{L}_{\text{Yuk}^\ell}$, the Yukawa matrix Y^e is 3×3 and complex, and consequently there are 9 real and 9 imaginary parameters. There is freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 matrices minus the minus the phases related to $U(1)^3$. Its a bit more involved to justify why the remaining parameters are real.). For example, we can use the unitary transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$ to give the following interaction basis

$$Y^e = \lambda_e = \text{diag}(y_e, y_\mu, y_\tau). \tag{82}$$

We then conclude that there are 3 real lepton flavour parameters. In the mass basis, we identify these parameters as the three charged lepton masses. However we must modify the model in order to take into account neutrino masses.

To summarise, combining these parameters with the other parameters of the Standard Model associated with the interactions and Higgs sector, there are 19 free parameters in total, that are summarised in Table 4. These have to be determined experimentally, after which they can be used in the theory to make predictions to provide further tests of the SM. The relatively large number of free parameters in the SM, along with, for example, the existance of three generations of fermions, and the particular patterns of fermion masses, are often used as motivations for BSM physics.

Parameter	Desrcription
m_e	Electron mass
m_μ	Muon mass
m_τ	Tau mass
m_u	Up quark mass
m_d	Down quark mass
m_c	Charm quark mass
m_s	Strange quark mass
m_t	Top quark mass
m_b	Bottom quark mass
θ_{12}	CKM 12-mixing angle
θ_{23}	CKM 23-mixing angle
θ_{13}	CKM 13-mixing angle
δ	CKM CP-violating phase
g'	$U(1)_Y$ Gauge coupling
g	$SU(2)_L$ Gauge coupling
g_s	$SU(3)_C$ Gauge coupling
θ_{QCD}	QCD vacuum angle
v	Higgs vacuum expectation value
m$_h$	Higgs mass

Table 4: The 19 free parameters of the SM (assuming massless neutrinos)

7 Beyond the SM

We know the SM cannot be a full theory of nature, and instead can be seen as a low energy effective field theory that is valid below some scale $\Lambda >> m_Z$. In this case the SM Lagrangian can be extended to include all non-renormalisable terms, suppressed by powers of Λ:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{O}_{d=5} + \frac{1}{\Lambda^2}\mathcal{O}_{d=6} + ... \tag{83}$$

where $\mathcal{O}_{d=n}$ represents operators that are products of SM fields, transforming as singlets under the SM gauge group, of overall dimension n in the fields. For physics at an energy scale E well below Λ, the effects of operators of dimension $n > 4$ are suppressed by $\left(\frac{E}{\Lambda}\right)^{n-4}$. So in general, the higher the dimension of an operator, the smaller its effect at low energies.

In the previous sections we have been mainly studying the SM at tree level and with only renormalisable terms. The effects of including loop corrections and non-renormalisable terms can be classified as follows:

- *Forbidden processes:* various processes are forbidden by the accidental symmetries of the SM. Non-renormalisable terms (but not loop corrections) can break these accidental symmetries and allow forbidden processes to occur. Examples include neutrino masses and proton decay.

- *Rare processes:* these are not allowed at tree-level but can be generated by loop corrections and non-renormalisable terms, and are often related to accidental symmetries that hold in a particular sector of the SM but not the full theory. Examples include flavour changing neutral current (FCNC) processes.

- *Tree-level processes:* tree-level processes in a particular sector often depend on a small subset of SM parameters, which can lead to relationships among different processes in that sector. These relationships are violated by both loop effects and non-renormalisable. Both experimental precision measurements and precise theory calculations are needed to observe these small effects. Examples include electroweak precision measurements (EWPM).

For the last two types, where both loop corrections and non-renormalisable terms can contribute, their use in phenomenology can be divided into two eras. Before all of the SM particles had been directly discovered and all SM parameters measured, one could assume the validity of the renormalisable SM and indirectly measure the properties of the yet to be discovered SM particles. The charm quark, top quark and Higgs boson masses were all predicted this way. Once all particles have been discovered, and the parameters measured directly, the loop corrections can quantitatively determined and the effects of non-renormalisable terms can be unambiguously probed. At present- all three processes serve as searches for new physics.

8 Neutrinos

In the SM neutrinos are exactly massless, however experimental observations of neutrino oscillations have established that neutrinos have masses. Whilst the individual neutrino mass eigenvalues are not known, the mass-squared differences are inferred to be [1].

$$\begin{aligned} \Delta m_{21}^2 &\equiv m_2^2 - m_1^2 = (7.53 \pm 0.18) \times 10^{-5} (\text{eV})^2 \\ \Delta m_{32}^2 &\equiv m_3^2 - m_2^2 = (2.453 \pm 0.033) \times 10^{-3} (\text{eV})^2 \text{ (normal ordering)} \\ \Delta m_{32}^2 &\equiv m_3^2 - m_2^2 = (-2.536 \pm 0.034) \times 10^{-3} (\text{eV})^2 \text{ (inverted ordering)} \end{aligned} \tag{84}$$

This is a clear indication of BSM physics. The SM prediction of massless neutrinos is related to the lepton number symmetry. The SM prediction that neutrinos do not mix is related to the lepton flavour symmetry. As with other predictions that depend on accidental symmetries of the SM, this can be violated in generic extensions of the SM. We will now show that $d = 5$ terms violate the accidental lepton number and flavour symmetries of the SM, so can be probed by measurements of neutrino masses and mixings. The model under consideration is the νSM.

There is a single class of dimension-five terms that depend on SM fields and obey the SM symmetries. These involve two $SU(2)$-doublet lepton fields and two $SU(2)$-doublet scalar fields:

$$\mathcal{L}_{\nu\text{SM}} = \mathcal{L}_{\text{SM}} + \frac{Z_{ij}^{\nu}}{\Lambda} \phi\phi L_i L_j \tag{85}$$

where Z^{nu} is a symmetric and complex 3×3 matrix of dimensionless couplings, and Λ is a high mass scale $\Lambda >> v$

8.1 The neutrino spectrum

With ϕ^0 acquiring a VEV, $\langle \phi_0 \rangle = \frac{v}{\sqrt{2}}$, the Lagrangian in Equation (85) has a piece corresponding to a Majorana mass matrix for the neutrinos:

$$\mathcal{L}_{\nu\text{SM,mass}} = \frac{1}{2} (m_\nu)_{ij} \nu_i \nu_j, \quad (m_\nu)_{ij} = \frac{v^2}{\Lambda} Z_{ij}^{\nu} \tag{86}$$

The matrix m_ν can be diagonalised by a unitary transformation

$$V_{\nu L} m_\nu V_{\nu L}^T = \hat{m}_\nu = \text{diag}(m_1, m_2, m_3). \tag{87}$$

Majorana mass matrices are always symmetric, which is why they can be diagonlised by a unitary transformation, as opposed to bi-unitary transformations used to diagonalise the general mass matrices earlier in the course $M_{\text{diag}} = V_L M V_R^T$. If we denote the neutrino mass eigenstates as ν_1, ν_2, ν_3. The convention here is that ν_1 and ν_2 are separated by the smallest mass-squared difference with $m_2 > m_1$. ν_3 is the state with the mass-squared difference from the other two that is largest. Experimentally, we do not yet know whether it is heavier or lighter than the other two. These two scenarios are referred to as "normal" and "inverted" ordering respectively. This convention maps on to Equation (84) as $|\Delta m_{32}^2| > \Delta m_{21}^2 > 0$

8.2 The energy scale of neutrino mass generation

The measured mass (differences) of neutrinos has implications in the model for the scale Λ where they are generated. For experiments probing the low-energy regime, it is the combination $\frac{Z^\nu}{\Lambda}$ that is measured, meaning there is an ambiguity in the definition of Λ and Z^ν. The separating of the coefficient of a $d = 5$ term into a dimensionless coupling and a scale becomes meaningful when we discuss possible high-energy theories that generate the effective term. The "scale" of a non-renormalisable term is $\frac{\Lambda}{Z^\nu}$ (or when Z^ν is a matrix it is $\frac{\Lambda}{Z^\nu_{\text{max}}}$ where Z^ν_{max} is the largest eigenvalue of Z^ν). If we combine a measurement of $\frac{\Lambda}{Z^\nu}$ with the assumption that Z^ν is generated by perturbative physics thus $Z^\nu_{\text{max}} \leq 1$ translates to an upper bound on Λ. The measured neutrino mass-squared differences in Equation (84) provide a lower bound on two mass eigenvalues. There must be at least on neutrino mass that is heavier than $\sqrt{|\Delta m_{32}^2|}$,

$$m_{\text{heaviest}} \geq \sqrt{|\Delta m_{32}^2|} \simeq 0.05\text{eV} \tag{88}$$

and there is at least one additional mass heavier than $\sqrt{\Delta m_{21}^2}$ 0.009eV. That said, additional experimental and cosmological constraints provide an upper bound on absolute mass scale of the neutrinos of the order of 1 eV.

The effective Lagrangian in Equation (85), where, by definition $\Lambda >> v$ predicts that the neutrino masses are much lighter than the weak scale

$$m_{1,2,3} \frac{v^2}{\Lambda} << v \tag{89}$$

and the fact that we have measured neutrino masses to be much lighter than the W-boson mass makes it plausable that they could be generated by $d = 5$ terms. Going further, all SM fermions except the top-quark are light compared to m_W. For the charged fermions this lightness relates to the smallness of their Yukawa couplings, but the neutrinos are at least six orders of magnitude lighter than all of the charge fermions, which could be explained by their being generated by $d = 5$ terms.

We know that the SM cannot be a valid theory above the Planck scale $\Lambda < M_{Pl}$ and thus expect that $m_i \gtrsim \frac{v^2}{m_{Pl}}\ 10^{-5}$eV. We can also consider the scale of Grand Unified Theories (GUTs), where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, the SM gauge group, is assumed to be a subgroup of a unifying group, such as $SU(5)$ which is spontaneously broken to G_{SM} at some scale $\Lambda_{GUT} = \mathcal{O}(10^{16}$GeV). If the $d = 5$ terms are generated at Λ_{GUT} then we get $m_\nu\ 10^{-2}$eV

On the other hand, an experimental lower bound on neutrino masses provides an upper bound on the scale of relevant new physics. Using the bound in Equation (88) and the Majorana relations in Equation (86), we could conclude that the SM cannot be a valid theory above the scale $\frac{v^2}{m_\nu}\ 10^{15}$GeV which proves the SM cannot be valid up to the Planck scale. Its also interesting to note that this upper bound is interestingly close to the GUT scale.

8.3 Neutrino interactions

Adding dimension 5 operators into $\mathcal{L}_{\nu\mathrm{SM}}$ leads to significant phenomenological changes in the lepton sector. Re-writing the neutrino-related terms in the renormalisable part of the SM Lagrangian in the mass basis we get

$$\mathcal{L}_{\mathrm{SM},\nu} = i\bar{\nu_\alpha}\partial\!\!\!/\nu_\alpha - \frac{g}{2c_W}\bar{\nu_\alpha}Z\!\!\!/\nu_\alpha - \frac{g}{\sqrt{2}}(\bar{\ell_{L\alpha}}W\!\!\!/^{-}\nu_\alpha + h.c) \tag{90}$$

where $\alpha = e, \mu, \tau$. Note that this Lagrangian describes massless neutrinos and so the basis $(\nu_e, \nu_\mu, \nu_\tau)$ serves as both an interaction basis and a mass basis.

The Lagrangian in Equation (85) gives

$$\mathcal{L}_{\nu\mathrm{SM},\nu} = i\bar{\nu_i}\partial\!\!\!/\nu_i - \frac{g}{2c_W}\bar{\nu_i}Z\!\!\!/\nu_i - \frac{g}{\sqrt{2}}(\bar{\ell_{L\alpha}}W\!\!\!/^{-}U_{\alpha i}\nu_i + h.c) + m_i\nu_i\nu_i + \frac{2m_i}{v}h\nu_i\nu_i + \frac{m_i}{v^2}hh\nu_i\nu_i \tag{91}$$

where now $\alpha = e, \mu, \tau$ are the charged lepton mass eigenstates, whilst $i = 1, 2, 3$ are the neutrino mass eigenstates. The neutrino mass parameters $m_{1,2,3}$ are real and the mixing matrix U is unitary. As with the quark sector previously, starting from an arbitrary interaction basis the matrix U is given by

$$U = V_{eL}V_{\nu L}^{\dagger}. \tag{92}$$

whilst each of the V_{eL} and $V_{\nu L}$ are basis dependent this combination isn't. Explicitly we write this as:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{93}$$

The most significant differences between $\mathcal{L}_{\text{SM},\nu}$ and $\mathcal{L}_{\nu\text{SM},\nu}$ are

- The leptonic charged current interactions are neither diagonal nor universal and instead involve the mixing matrix U.
- In $\mathcal{L}_{\nu\text{SM},\nu}$ the Higgs boson has Yukawa couplings to neutrinos. These break lepton number. However size of these Yukawa couplings of the order of $\frac{m_i}{v}\ 10^{-13}$ so tiny and would generate a negligible branching ratio for $h \to \nu\nu$.

The neutrinos in the νSM thus have three types of interaction mediated by massive bosons:

- Neutral current weak interactions mediated by the Z^0 with coupling $\frac{e}{2s_W c_W}$.
- Charged current weak interactions mediated by the $W^{\pm}$ bosons and couplings given by $\frac{gU}{\sqrt{2}}$.
- Yukawa couplings to the Higgs with coupling $\frac{2m}{v}$

8.4 Accidental symmetries and the lepton mixing parameters

The dimension five terms added into $\mathcal{L}_{\nu\text{SM}}$ break the $U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry of the SM, meaning that the only remaining term in $G^{\text{global}}_{\text{SM}}$ is the baryon number symmetry:

$$G^{\text{global}}_{\nu\text{SM}} = U(1)_B \tag{94}$$

However this symmetry is anomalous and broken by non-perturbative effects, as well as by any dimension 6 operators.

The counting of the flavour parameters in the quark sector is the same as before, with 6 quark masses and four mixing parameters defining the CKM matrix, one of which is imaginary. For the lepton sector we must now go back to Equation (85). This now includes the 3×3 matrix Y^e (9 real and 9 imaginary components) and the symmetric 3×3 matrix Z^ν (6 real and 6 imaginary parameters). The kinetic and gauge terms have a $U(3)_L \times U(3)_E$ accidental global symmetry that is completely broken by the Y^e and Z^ν terms. This reduces the number of physical lepton flavour parameters to

$(15_R + 15_I) - 2 \times (3_R + 6_I) = 9_R + 3_I$. 6 of the real parameters are the three charged lepton masses $m_{e,\mu,\tau}$ and the three neutrino masses $m_{1,2,3}$. This means that the 3×3 unitary matrix U depends on 3 real mixing angles and 3 phases.

The difference between the parameters in U and those in the CKM matrix (which only has a single phase) is that $\mathcal{L}_{\nu\text{SM}}$ generates Majorana masses for neutrinos. This means there's no freedom in changing the mass basis by redefining the neutrino phases as these would introduce phases into the neutrino mass terms. Re-definitions of the six quark fields allowed us to remove 5 non-physical phases from V, but in this case re-definition of the three charged lepton fields only remove three non-physical phases from U. The two additional phases in U are referred to as *Majorana phases* as they appear as a result of assuming Majorana mass terms for the neutrinos. They also impact lepton number violating processes.

U can be conveniently parametrised as follows

$$U = \begin{pmatrix} c_{12}c_{23} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}) \qquad (95)$$

where $\alpha_{1,2}$ are the two Majorana phases and $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$.

As of October 2021 the best-fit parameters from the "nu-Fit" collaboration () are as follows (these correspond to the 3σ confidence intervals:

$$|U| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.801 - 0.845 & 0.513 - 0.579 & 0.143 - 0.156 \\ 0.232 - 0.507 & 0.459 - 0.694 & 0.629 - 0.779 \\ 0.260 - 0.526 & 0.470 - 0.702 & 0.609 - 0.763 \end{pmatrix} \qquad (96)$$

When working with the mass basis the formalisms for quark and lepton flavour mixing are very similar. There are differences however in the way that neutrino experiments are done. Quarks and charged leptons are identified through their mass eigenstates, however neutrinos can only be identified as interaction eigenstates (i.e. as ν_e, ν_μ, ν_τ according to whether the produce an e, μ or τ in the detector.

8.5 Outstanding questions

Whilst we can argue that the νSM could provide a consistent model of the neutrino sector, many parameters are still to be determined:

- The absolute mass scale of neutrinos is still undetermined.

- It is not known whether the spectrum has normal or inverted ordering.
- The three phases are yet to be measured.

There are also other ways to explain the experimental data, and the following questions could be relevent to explaining whether the νSM or some other theory provides the best low-energy description of the neutrino sector:

- Are neutrinos Majorana or Dirac particles?
- Do sterile neutrinos exist? These would be neutrino states uncharged under the SM gauge group but that mix with the active neutrinos.
- Are there dimension 6 operators that could significantly impact the neutrino sector?

9 Conclusion and acknowledgements

This course has provided a whistle-stop tour of the construction of the Standard Model of particle physics as a Quantum Field theory with a local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry that spontaneously broken into $SU(3)_C \times U(1)_{EM}$ through the Brout-Englert-Higgs mechanism. The elementary particles and interactions are discussed along with the predictions and experimental tests of the model. Finally, the free parameters are enumerated, and outstanding questions and possible extensions to the Standard Model are discussed. In particular extensions to the SM Lagrangian involving dimension 5 operators that could accommodate Majorana neutrino masses are discussed. Sincere thanks are owed to Professor Yosef Nir for his kind help in providing a copy of the pedagogical textbook on the Standard Model that he has written with Professor Yuval Grossman.

References

[1] Workman R L, Others. Review of Particle Physics[J]. PTEP, 2022, 2022: 083C01. http://dx.doi.org/10.1093/ptep/ptac097.

[2] A detailed map of Higgs boson interactions by the ATLAS experiment ten years after the discovery[J]. Nature, 2022, 607 (7917): 52–59. http://dx.doi.org/10.1038/s41586-022-04893-w. [Erratum: Nature 612, E24 (2022)].

A Discrete symmetries and selection rules

A discrete symmetry is one that describes a non-continuous change of the system, and they sometimes involve “swapping”. *Parity* is an example of a discrete symmetry and corresponds to spatial inversion $\mathbf{x} \to -\mathbf{x}$. A key result in non-relativistic Quantum Physics is that eigenstates of a “symmetric” potential are also eigenstates of parity. Parity conservation leads to “selection rules” that determine which lines are possible in atomic emission spectra. In general the term selection rule applies to constraints on allowed transitions between quantum states, and they occur in both chemistry but also in nuclear physics.

In QFT, three important discrete spacetime symmetries are C, P and T where C stands for charge conjugation (which changes particles into antiparticles by conjugating all their quantum numbers), P stands for parity (as above) and T symmetry corresponds to invariance under time reversal. The *CPT theorem* states any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry, however each of the individual symmetries have been shown to be violated in nature. The second law of thermodynamics suggest that time inversion symmetry is violated in the universe on the macroscopic level. For C and P, As the impact of parity reverses the chirality of fermion fields, meaning that the combined CP transformation turns a LH particle into a RH antiparticle (and so on). Direct CP violation can be accommodated in the quark sector of the SM through the CKM matrix and in the neutrino sector through the PMNS matrix, and is required astrophysically to explain the excess of matter over antimatter in the universe. Extensions of the SM could provide additional sources of CP violation. If we assume the CPT theorem holds then CP and T can be considered equivalent.

To think about selection rules in the SM, an example is that parity is always conserved in strong and electromagnetic interactions (whilst it can be violated in the weak interaction). Parity conservation gives additional constraintsd and can forbid processes that might be otherwise allowed kinematically.