The Standard Model*

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Abstract

This is a written version of a series of lectures aimed at graduate students in particle physics. Starting from symmetry principles, we construct the Standard Model, and derive the elementary particles and fundamental interactions of the model. We describe the phenomenological predictions and some of their experimental tests.

* Lectures given at the 5th Chilean School of High Energy Physics, Universidad Técnica Federico Santa Maria, Chile, 15–18 January 2018

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I. MODEL BUILDING FROM SYMMETRY PRINCIPLES

A. Lagrangians

Modern physics encodes the basic laws of Nature in the action S, and postulates the principle of minimal action in its quantum interpretation. In Quantum Field Theory (QFT), the action is an integral over spacetime of the "Lagrangian density" or Lagrangian, $\mathcal{L}$, for short. For our purposes, it is enough to consider the Lagrangian, rather than the action. In these lectures we explain how particle physicists "construct" Lagrangians. We do so by explicitly constructing the Standard Model Lagrangian. Then we discuss how experimentalists determine the numerical values of the parameters that appear in the Lagrangian, and how they test whether a Lagrangian provides a viable description of Nature.

The action is given by

$$S = \int d^4x\, \mathcal{L}[\phi_i(x), \partial_\mu \phi_i(x)]\,, \tag{1}$$

where $d^4x = dx^0 dx^1 dx^2 dx^3$ is the integration measure in four-dimensional Minkowski space. The index i runs from 1 to the number of fields. Here we denote a generic field by $\phi(x)$.

In general, we require the following properties for the Lagrangian:

(i) It is a function of the fields and their derivatives only, so as to ensure translational invariance.

(ii) It depends on the fields taken at one spacetime point x^μ only, leading to a local field theory.

(iii) It is real, so that the total probability is conserved.

(iv) It is invariant under the Poincaré group, that is under spacetime translations and Lorentz transformations.

(v) It is an analytic function in the fields. This is not a general requirement, but it is common to all field theories that are solved via perturbation theory. In all of these, we expand around a minimum, and this expansion means that we consider a Lagrangian that is a polynomial in the fields.

(vi) It is invariant under certain internal symmetry groups. The invariance of S (or of $\mathcal{L}$) is in correspondence with conserved quantities and reflects basic symmetries of the physical system.

We impose two additional requirements:

(vii) Naturalness: Every term in the Lagrangian that is not forbidden by a symmetry should appear.

(viii) Renormalizability. A renormalizable Lagrangian contains only terms that are of dimension less than or equal to four in the fields and their derivatives.

The requirement of renormalizability ensures that the Lagrangian contains at most two ∂_μ operations, and leads to classical equations of motion that are no higher than second order derivatives. If the full theory of Nature is described by QFT, its Lagrangian should indeed be renormalizable. The theories that we consider and, in particular, the Standard Model are, however, only low energy effective theories, valid up to some energy scale Λ. Therefore, we must include also non-renormalizable terms. These terms have coefficients with inverse mass dimensions, $1/\Lambda^n$, $n = 1, 2, \ldots$. For most purposes, however, the renormalizable terms constitute the leading terms in an expansion in E/Λ, where E is the energy scale of the physical processes under study. Therefore, the renormalizable part of the Lagrangian is a good starting point for our study.

Properties (i)-(v) are not the subject of these lectures. You must be familiar with them from your QFT course(s). We do, however, deal intensively with the other requirements. In particular, we focus on property (vi). Actually, the most important message that we would like to convey is the following: *(Almost) all experimental data for elementary particles*

TABLE I: Symmetries

Type	Consequences
Spacetime	Conservation of energy, momentum, angular momentum
Discrete	Selection rules
Global (exact)	Conserved charges
Global (spon. broken)	Massless scalars
Local (exact)	Interactions, massless spin-1 mediators
Local (spon. broken)	Interactions, massive spin-1 mediators

and their interactions can be explained by the standard model of a spontaneously broken $SU(3) \times SU(2) \times U(1)$ *gauge symmetry.*[1]

B. Symmetries

Symmetries in QFT have a strong predictive, or explanatory, power. The main consequences of the various types of symmetries are summarized in Table I.

We emphasize here that there are symmetries that are not imposed and are called *accidental* symmetries. They are outputs of the theory rather than external constraints. Accidental symmetries arise due to the fact that we truncate our Lagrangian. In particular, the renormalizable terms in the Lagrangian often have accidental symmetries that are broken by non-renormalizable terms or by anomalies. Since we study mostly the renormalizable SM Lagrangian, we will indeed encounter accidental symmetries.

In the SM, only local symmetries are imposed. Similarly, in most of the extensions of the SM, only local and global discrete symmetries are imposed. While it is possible, in principle, to impose also global continuous symmetries, this is rarely done in current model building. The reason for that is twofold. First, there are arguments that suggest that continuous global symmetries are always broken by gravitational effects and thus can only arise as accidental, rather than imposed symmetries. Second, there is no obvious phenomenological motivation

[1] Actually, the great hope of the high-energy physics community is to prove this statement wrong, and to find an even more fundamental theory.

TABLE II: Dirac and Majorana masses

	Dirac	Majorana
# of degrees of freedom	4	2
Representation	vector	neutral
Mass matrix	$m \times n$, general	$(m+n) \times (m+n)$, symmetric
SM fermions	quarks, charged leptons	neutrinos (?)

to impose such symmetries.

Additional important consequences of symmetries, beyond those of Table I, include

- The lightest particle that is charged under a symmetry is stable.
- Charged fermions cannot have Majorana masses.
- Chiral fermions cannot have Dirac masses.

A short summary of the differences between Dirac and Majorana fermions is given in Table II.

The main lesson that we can draw from these observations is the following: *Charged fermions in a chiral representation are massless.* In other words, if we encounter massless fermions in Nature, there is a way to explain their masslessness from symmetry principles.

C. Model building

To construct a model, we provide as input the following ingredients:

(*i*) The symmetry;

(*ii*) The transformation properties of the fermions and the scalars.

(*iii*) The pattern of spontaneous symmetry breaking (SSB).

Then we write the most general Lagrangian that depends on the scalar and fermion fields and is invariant under the symmetry. If the imposed symmetry is local, corresponding vector fields must be added.

We write the Lagrangian up to some order in the fields. Unless explicitly stated otherwise, we truncate the Lagrangian at the renormalizable level, that is, at dimension four in the fields. The most general renormalizable Lagrangian with scalar, fermion and gauge fields can be decomposed into

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi. \tag{2}$$

Here $\mathcal{L}_{\rm kin}$ describes the free propagation in spacetime of all dynamical fields, as well as the gauge interactions, $\mathcal{L}_\psi$ gives the fermion mass terms, $\mathcal{L}_{\rm Yuk}$ describes the Yukawa interactions, and $\mathcal{L}_\phi$ gives the scalar potential.

The resulting Lagrangian has a finite number of parameters that we need to determine by experiment. In principle, for a theory with N independent parameters, we need to perform N appropriate measurements to extract the values of the parameters. Additional measurements test the theory.

II. THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking (SSB); (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:

- The symmetry is a local

$$G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{3}$$

which is spontaneously broken into

$$G_{\rm SM} \to SU(3)_C \times U(1)_{\rm EM} \quad (Q_{\rm EM} = T_3 + Y). \tag{4}$$

- There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1} \quad (i=1,2,3). \tag{5}$$

There is a single scalar field,

$$\phi(1,2)_{+1/2}. \tag{6}$$

III. THE SM LAGRANGIAN

The most general renormalizable Lagrangian with scalar and fermion fields can be decomposed into

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_\psi + \mathcal{L}_{\rm Yuk} + \mathcal{L}_\phi. \tag{7}$$

Here $\mathcal{L}_{\rm kin}$ describes free propagation in spacetime, as well as gauge interactions, $\mathcal{L}_\psi$ gives fermion mass terms, $\mathcal{L}_{\rm Yuk}$ describes the Yukawa interactions, and $\mathcal{L}_\phi$ gives the scalar potential. We now find the specific form of the Lagrangian made of the fermion fields Q_{Li}, U_{Ri}, D_{Ri}, L_{Li} and E_{Ri} (5), and the scalar field ϕ (6), subject to the gauge symmetry (3) and leading to the SSB of Eq. (4).

A. $\mathcal{L}_{\rm kin}$

The local symmetry requires that we introduce the following gauge boson degrees of freedom:

$$G_a^\mu(8,1)_0, \quad W_a^\mu(1,3)_0, \quad B^\mu(1,1)_0. \tag{8}$$

The corresponding field strengths are given by

$$\begin{aligned} G_a^{\mu\nu} &= \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu, \\ W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu, \\ B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu, \end{aligned} \tag{9}$$

where f_{abc} (ϵ_{abc}) are the structure constants of $SU(3)$ ($SU(2)$). The covariant derivative is

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y, \tag{10}$$

where the L_a's are $SU(3)_C$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_L$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_Y$ charges. Explicitly, the covariant derivatives acting on the various scalar and fermion fields are given by

$$\begin{aligned} D^\mu \phi &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{2} g' B^\mu\right)\phi, \\ D^\mu Q_{Li} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} g W_b^\mu \tau_b + \frac{i}{6} g' B^\mu\right) Q_{Li}, \end{aligned}$$

$$\begin{aligned}
D^\mu U_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{2i}{3} g' B^\mu\right) U_{Ri}, \\
D^\mu D_{Ri} &= \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a - \frac{i}{3} g' B^\mu\right) D_{Ri}, \\
D^\mu L_{Li} &= \left(\partial^\mu + \frac{i}{2} g W_b^\mu \tau_b - \frac{i}{2} g' B^\mu\right) L_{Li}, \\
D^\mu E_{Ri} &= \left(\partial^\mu - i g' B^\mu\right) E_{Ri}.
\end{aligned} \tag{11}$$

$\mathcal{L}_{\text{kin}}$ is given by

$$\begin{aligned}
\mathcal{L}_{\text{kin}}^{\text{SM}} &= -\frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} - \frac{1}{4} W_b^{\mu\nu} W_{b\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \\
&\quad - i\overline{Q_{Li}} \not{D} Q_{Li} - i\overline{U_{Ri}} \not{D} U_{Ri} - i\overline{D_{Ri}} \not{D} D_{Ri} - i\overline{L_{Li}} \not{D} L_{Li} - i\overline{E_{Ri}} \not{D} E_{Ri} \\
&\quad - (D^\mu \phi)^\dagger (D_\mu \phi).
\end{aligned} \tag{12}$$

This part of the interaction Lagrangian is flavor-universal. In addition, it conserves CP.

B. $\mathcal{L}_\psi$

There are no mass terms for the fermions in the SM. We cannot write Dirac mass terms for the fermions because they are assigned to chiral representations of the gauge symmetry. We cannot write Majorana mass terms for the fermions because they all have $Y \neq 0$. Thus,

$$\mathcal{L}_\psi^{\text{SM}} = 0. \tag{13}$$

C. $\mathcal{L}_{\text{Yuk}}$

The Yukawa part of the Lagrangian is given by

$$\mathcal{L}_{\text{Y}}^{\text{SM}} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \overline{L_{Li}} \phi E_{Rj} + \text{h.c.}, \tag{14}$$

where $\tilde{\phi} = i\tau_2 \phi^\dagger$, and the Y^f are general 3×3 matrices of dimensionless couplings. This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

Without loss of generality, we can use a bi-unitary transformation,

$$Y^e \to \hat{Y}_e = U_{eL} Y^e U_{eR}^\dagger, \tag{15}$$

to change the basis to one where Y^e is diagonal and real:

$$\hat{Y}^e = \text{diag}(y_e, y_\mu, y_\tau). \tag{16}$$

In the basis defined in Eq. (16), we denote the components of the lepton $SU(2)$-doublets, and the three lepton $SU(2)$-singlets, as follows:

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}; \qquad e_R, \qquad \mu_R, \qquad \tau_R, \tag{17}$$

where e, μ, τ are ordered by the size of $y_{e,\mu,\tau}$ (from smallest to largest).

Similarly, without loss of generality, we can use a bi-unitary transformation,

$$Y^u \to \hat{Y}_u = V_{uL} Y^u V_{uR}^\dagger, \tag{18}$$

to change the basis to one where $\hat{Y}^u$ is diagonal and real:

$$\hat{Y}^u = \mathrm{diag}(y_u, y_c, y_t). \tag{19}$$

In the basis defined in Eq. (19), we denote the components of the quark $SU(2)$-doublets, and the quark up $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_L \\ d_{uL} \end{pmatrix}, \quad \begin{pmatrix} c_L \\ d_{cL} \end{pmatrix}, \quad \begin{pmatrix} t_L \\ d_{tL} \end{pmatrix}; \qquad u_R, \qquad c_R, \qquad t_R, \tag{20}$$

where u, c, t are ordered by the size of $y_{u,c,t}$ (from smallest to largest).

We can use yet another bi-unitary transformation,

$$Y^d \to \hat{Y}_d = V_{dL} Y^d V_{dR}^\dagger, \tag{21}$$

to change the basis to one where $\hat{Y}^d$ is diagonal and real:

$$\hat{Y}^d = \mathrm{diag}(y_d, y_s, y_b). \tag{22}$$

In the basis defined in Eq. (22), we denote the components of the quark $SU(2)$-doublets, and the quark down $SU(2)$-singlets, as follows:

$$\begin{pmatrix} u_{dL} \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_{sL} \\ s_L \end{pmatrix}, \quad \begin{pmatrix} u_{bL} \\ b_L \end{pmatrix}; \qquad d_R, \qquad s_R, \qquad b_R, \tag{23}$$

where d, s, b are ordered by the size of $y_{d,s,b}$ (from smallest to largest).

Note that if $V_{uL} \neq V_{dL}$, as is the general case, then the interaction basis defined by (19) is different from the interaction basis defined by (22). In the former, Y^d can be written as a unitary matrix times a diagonal one,

$$Y^u = \hat{Y}^u, \quad Y^d = V\hat{Y}^d. \tag{24}$$

In the latter, Y^u can be written as a unitary matrix times a diagonal one,

$$Y^d = \hat{Y}^d, \quad Y^u = V^\dagger \hat{Y}^u. \tag{25}$$

In either case, the matrix V is given by

$$V = V_{uL} V_{dL}^\dagger, \tag{26}$$

where V_{uL} and V_{dL} are defined in Eqs. (18) and (21), respectively. Note that V_{uL}, V_{uR}, V_{dL} and V_{dR} depend on the basis from which we start the diagonalization. The combination $V = V_{uL} V_{dL}^\dagger$, however, does not. This is a hint that V is physical. Indeed, below we see that it plays a crucial role in the charged current interactions.

D. $\mathcal{L}_\phi$

The scalar potential is given by

$$\mathcal{L}_\phi^{\rm SM} = -\mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \tag{27}$$

This part of the Lagrangian is also CP conserving.

Choosing $\mu^2 < 0$ and $\lambda > 0$ leads to the required spontaneous symmetry breaking. Defining

$$v^2 = -\frac{\mu^2}{\lambda}, \tag{28}$$

we can rewrite Eq. (27) as follows (up to a constant term):

$$\mathcal{L}_\phi = -\lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2. \tag{29}$$

The scalar potential (29) implies that the scalar field acquires a VEV, $|\langle \phi \rangle| = v/\sqrt{2}$. We have to make a choice of the direction of $\langle \phi \rangle$, and we choose it in the real direction of the down component,

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \tag{30}$$

This VEV breaks the $SU(2) \times U(1)$ symmetry down to a $U(1)$ subgroup. This statement corresponds to the fact that there is one (and only one) linear combination of generators that annihilates the vacuum state. With our specific choice, Eq. (30), it is $T_3 + Y$. The unbroken subgroup is identified with $U(1)_{\rm EM}$, and hence its generator, Q, is identified as

$$Q = T_3 + Y. \tag{31}$$

E. Summary

The renormalizable part of the Standard Model Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{\rm SM} = &- \frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_b^{\mu\nu}W_{b\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - (D^\mu\phi)^\dagger(D_\mu\phi) \\
&- i\overline{Q_{Li}}\not{D}Q_{Li} - i\overline{U_{Ri}}\not{D}U_{Ri} - i\overline{D_{Ri}}\not{D}D_{Ri} - i\overline{L_{Li}}\not{D}L_{Li} - i\overline{E_{Ri}}\not{D}E_{Ri} \\
&+ \left(Y^u_{ij}\overline{Q_{Li}}U_{Rj}\,\tilde{\phi} + Y^d_{ij}\overline{Q_{Li}}D_{Rj}\,\phi + Y^e_{ij}\overline{L_{Li}}E_{Rj}\,\phi + \text{h.c.}\right) \\
&- \lambda\left(\phi^\dagger\phi - v^2/2\right)^2 ,
\end{aligned} \tag{32}$$

where $i, j = 1, 2, 3$.

IV. THE SM SPECTRUM

A. Scalars: back to $\mathcal{L}_\phi$

Let us denote the four real components of the scalar doublet as three phases, $\theta_a(x)$ ($a = 1, 2, 3$), and one magnitude, $h(x)$. We choose the three phases to be the three "would be" Goldstone bosons. In the SM, the broken generators are T_1, T_2, and $T_3 - Y$, and thus we write

$$\phi(x) = \exp\left[(i/2)\left(\sigma_a\theta_a(x) - I\theta_3(x)\right)\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{33}$$

The local $SU(2)_L \times U(1)_Y$ symmetry of the Lagrangian allows one to rotate away the explicit dependence on the three $\theta_a(x)$. They represent the three would-be Goldstone bosons that are eaten by the three gauge bosons that acquire masses as a result of the SSB. In this gauge $\phi(x)$ has one degree of freedom (DoF):

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{34}$$

The scalar h is the Higgs boson. It is an $S(3)_C$-singlet and $U(1)_{\rm EM}$-neutral. Its mass can be obtained by plugging (34) into (29), and is given by

$$m_h^2 = 2\lambda v^2. \tag{35}$$

Experiment gives [1]

$$m_h = 125.09 \pm 0.24 \text{ GeV}. \tag{36}$$

B. Vector bosons: back to $\mathcal{L}_{\rm kin}(\phi)$

The $SU(3)_C$ gauge symmetry remains unbroken. Thus, the gluon, an $SU(3)_C$-octet and $U(1)_{\rm EM}$-neutral, is massless:

$$m_g = 0. \tag{37}$$

As concerns the vector bosons related to the $SU(2)_L \times U(1)_Y$ symmetry, since the symmetry that is related to three out of the four generators is spontaneously broken, three of the four vector bosons acquire masses, while one remains massless. To see how this happens, we examine $(D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle)$. Using Eq. (11) for $D^\mu\phi$, we obtain:

$$D^\mu\langle\phi\rangle = \frac{i}{\sqrt{8}}\left(gW_a^\mu\sigma_a + g'B^\mu\right)\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{i}{\sqrt{8}}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1^\mu - iW_2^\mu) \\ g(W_1^\mu + iW_2^\mu) & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{38}$$

The mass terms for the vector bosons are thus given by

$$\mathcal{L}_{M_V} = \frac{1}{8}(0\ v)\begin{pmatrix} gW_{3\mu} + g'B_\mu & g(W_1 - iW_2)_\mu \\ g(W_1 + iW_2)_\mu & -gW_{3\mu} + g'B_\mu \end{pmatrix}\begin{pmatrix} gW_3^\mu + g'B^\mu & g(W_1 - iW_2)^\mu \\ g(W_1 + iW_2)^\mu & -gW_3^\mu + g'B^\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{39}$$

We define an angle θ_W via

$$\tan\theta_W \equiv \frac{g'}{g}. \tag{40}$$

We define four gauge boson states:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2)_\mu, \quad Z_\mu^0 = \cos\theta_W W_{3\mu} - \sin\theta_W B_\mu, \quad A_\mu^0 = \sin\theta_W W_{3\mu} + \cos\theta_W B_\mu. \tag{41}$$

The $W_\mu^\pm$ are charged under electromagnetism (hence the superscripts $\pm$), while A_μ^0 and Z_μ^0 are neutral. In terms of the vector boson fields of Eq. (41), Eq. (39) reads

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2v^2W^{+\mu}W_\mu^- + \frac{1}{8}(g^2+g'^2)v^2Z^{0\mu}Z_\mu^0. \tag{42}$$

We learn that the four states of Eq. (41) are the mass eigenstates, with masses-squared

$$m_W^2 = \frac{1}{4}g^2v^2, \qquad m_Z^2 = \frac{1}{4}(g^2+g'^2)v^2, \qquad m_A^2 = 0. \tag{43}$$

(Recall that for a complex field ϕ with mass m the mass term is $m^2|\phi|^2$ while for a real field it is $m^2\phi^2/2$.) Three points are worth emphasizing:

1. As anticipated, three vector boson acquire masses.

2. $m_A^2 = 0$ is not a prediction, but rather a consistency check on our calculation.

3. The angle θ_W represents a rotation angle of the two neutral vector bosons from the interaction basis, where fields have well-defined transformation properties under the full gauge symmetry, (W_3, B), into the mass basis for the vector bosons, (Z, A).

SSB leads to relation between observables that would have been independent in the absence of a symmetry. One such important relation involves the vector-boson masses and their couplings:

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2}. \tag{44}$$

This relation is testable. The left hand side can be derived from the measured spectrum, and the right hand side from interaction rates. It is conventional to express this relation in terms of θ_W, defined in Eq. (40):

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1. \tag{45}$$

The $\rho = 1$ relation is a consequence of the SSB by $SU(2)$-doublets. It thus tests this specific ingredient of the SM.

The experimental values of the weak gauge boson masses are given by [1]

$$m_W = 80.385 \pm 0.015 \text{ GeV}; \qquad m_Z = 91.1876 \pm 0.0021 \text{ GeV}. \tag{46}$$

We can then use the $\rho = 1$ relation to determine $\sin^2\theta_W$:

$$\frac{m_W}{m_Z} = 0.8815 \pm 0.0002 \quad \Longrightarrow \quad \sin^2\theta_W = 1 - (m_W/m_Z)^2 = 0.2229 \pm 0.0004. \tag{47}$$

Measurements determine $\sin^2\theta_W$ by various interaction rates. The $\rho = 1$ relation is indeed realized in Nature (within experimental errors, and up to calculable quantum corrections).

C. Fermions: back to $\mathcal{L}_{\rm Yuk}$

Since the SM allows no bare mass terms for the fermions, their masses can only arise from the Yukawa part of the Lagrangian, which is given in Eq. (14). Indeed, with $\langle \phi^0 \rangle = v/\sqrt{2}$, Eq. (14) has a piece that corresponds to charged lepton masses:

$$m_e = \frac{y_e v}{\sqrt{2}}, \qquad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \qquad m_\tau = \frac{y_\tau v}{\sqrt{2}}, \tag{48}$$

a piece that corresponds to up-type quark masses,

$$m_u = \frac{y_u v}{\sqrt{2}}, \qquad m_c = \frac{y_c v}{\sqrt{2}}, \qquad m_t = \frac{y_t v}{\sqrt{2}}, \tag{49}$$

and a piece that corresponds to down-type quark masses,

$$m_d = \frac{y_d v}{\sqrt{2}}, \qquad m_s = \frac{y_s v}{\sqrt{2}}, \qquad m_b = \frac{y_b v}{\sqrt{2}}. \tag{50}$$

We conclude that all charged fermions acquire Dirac masses as a result of the spontaneous symmetry breaking. The key to this feature is that, while the charged fermions are in chiral representations of the full gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, they are in vector-like representations of the $SU(3)_C \times U(1)_{\rm EM}$ group:

- The LH and RH charged lepton fields, e, μ and τ, are in the $(1)_{-1}$ representation.
- The LH and RH up-type quark fields, u, c and t, are in the $(3)_{+2/3}$ representation.
- The LH and RH down-type quark fields, d, s and b, are in the $(3)_{-1/3}$ representation.

On the other hand, the neutrinos remain massless:

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0. \tag{51}$$

This is the case in spite of the fact that the neutrinos transform as $(1)_0$ under the unbroken gauge group, allowing in principle for Majorana masses. As we discuss below, their masslessness is related to an accidental symmetry of the SM.

The experimental values of the charged fermion masses are [1]

$$\begin{aligned}
&m_e = 0.510998946(3)\ {\rm MeV}, \qquad m_\mu = 105.6583745(24)\ {\rm MeV}, \qquad m_\tau = 1776.86(12)\ {\rm MeV},\\
&m_u = 2.2^{+0.6}_{-0.4}\ {\rm MeV}, \qquad m_c = 1.27 \pm 0.03\ {\rm GeV}, \qquad m_t = 173.2 \pm 0.09\ {\rm GeV},\\
&m_d = 4.7^{+0.5}_{-0.4}\ {\rm MeV}, \qquad m_s = 96^{+8}_{-4}\ {\rm MeV}, \qquad m_b = 4.18^{+0.04}_{-0.03}\ {\rm GeV},
\end{aligned} \tag{52}$$

where the u-, d- and s-quark masses are given at a scale $\mu = 2$ GeV, the c- and b-quark masses are the running masses in the $\overline{\rm MS}$ scheme, and the t-quark mass is derived from direct measurement.

D. Summary

The mass eigenstates of the SM, their $SU(3)_C \times U(1)_{\rm EM}$ quantum numbers, and their masses in units of the VEV v, are presented in Table III. All masses are proportional to the VEV of the scalar field, v. For the three massive gauge bosons, and for the fermions, this is

TABLE III: The SM particles

particle	spin	color	Q	mass $[v]$
$W^\pm$	1	(1)	±1	$\frac{1}{2}g$
Z^0	1	(1)	0	$\frac{1}{2}\sqrt{g^2+g'^2}$
A^0	1	(1)	0	0
g	1	(8)	0	0
h	0	(1)	0	$\sqrt{2\lambda}$
e,μ,τ	1/2	(1)	-1	$y_{e,\mu,\tau}/\sqrt{2}$
ν_e,ν_μ,ν_τ	1/2	(1)	0	0
u,c,t	1/2	(3)	$+2/3$	$y_{u,c,t}/\sqrt{2}$
d,s,b	1/2	(3)	$-1/3$	$y_{d,s,b}/\sqrt{2}$

expected: In the absence of spontaneous symmetry breaking, the former would be protected from acquiring masses by the gauge symmetry and the latter by their chiral nature. For the Higgs boson, the situation is different, as a mass-squared term does not violate any symmetry: $m_h \propto v$ is just a manifestation of the fact that the SM has a single dimensionful parameter, which can be taken to be v, and therefore all masses must be proportional to this parameter.

V. THE SM INTERACTIONS

In this Section, we discuss the interactions of the fermion and scalar mass eigenstates of the SM.

A. EM and strong interactions

By construction, a local $SU(3)_C \times U(1)_{\rm EM}$ symmetry survives the SSB. The SM has thus the photon and gluon massless gauge fields. All charged fermions interact with the photon:

$$\mathcal{L}_{{\rm QED},\psi} = -\frac{2e}{3}\overline{u_i}\not{A}u_i + \frac{e}{3}\overline{d_i}\not{A}d_i + e\overline{\ell_i}\not{A}\ell_i, \tag{53}$$

where $u_{1,2,3} = u,c,t$, $d_{1,2,3} = d,s,b$ and $\ell_{1,2,3} = e,\mu,\tau$. We emphasize the following points:

1. The photon couplings are *vector-like* and *parity conserving.*

2. *Diagonality:* The photon couples to e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$, but not to $e^\pm\mu^\mp$, $e^\pm\tau^\mp$ or $\mu^\pm\tau^\mp$ pairs, and similarly in the up and down sectors.

3. *Universality:* The couplings of the photon to different generations are universal.

All colored fermions (namely, quarks) interact with the gluon:

$$\mathcal{L}_{\mathrm{QCD},\psi} = -\frac{g_s}{2}\overline{q}\lambda_a \not{G}_a q, \tag{54}$$

where $q = u, c, t, d, s, b$. We emphasize the following points:

1. The gluon couplings are *vector-like* and *parity conserving.*

2. *Diagonality:* The gluon couples to $\bar{t}t$, $\bar{c}c$, *etc.*, but not to $\bar{t}c$ or any other flavor changing pair.

3. *Universality:* The couplings of the gluon to different quark generations are universal.

The universality of the photon and gluon couplings is a result of the $SU(3)_C \times U(1)_{\mathrm{EM}}$ gauge invariance, and thus holds in any model, and not just within the SM.

B. Z-mediated weak interactions

All SM fermions couple to the Z-boson:

$$\begin{aligned}
\mathcal{L}_{Z,\psi} = \frac{e}{s_W c_W}\Big[&-\left(\frac{1}{2}-s_W^2\right)\,\overline{e_{Li}}\not{Z}e_{Li} + s_W^2\,\overline{e_{Ri}}\not{Z}e_{Ri} + \frac{1}{2}\,\overline{\nu_{L\alpha}}\not{Z}\nu_{L\alpha} \\
&+\left(\frac{1}{2}-\frac{2}{3}s_W^2\right)\,\overline{u_{Li}}\not{Z}u_{Li} - \frac{2}{3}s_W^2\,\overline{u_{Ri}}\not{Z}u_{Ri} - \left(\frac{1}{2}-\frac{1}{3}s_W^2\right)\,\overline{d_{Li}}\not{Z}d_{Li} + \frac{1}{3}s_W^2\,\overline{d_{Ri}}\not{Z}d_{Ri}\Big].
\end{aligned} \tag{55}$$

where $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$. We emphasize the following points:

1. The Z-boson couplings are *chiral* and *parity violating.*

2. *Diagonality:* The Z-boson couples diagonally and, as a result of this, there are no Z-mediated flavor changing neutral current (FCNC) processes.

3. *Universality:* The couplings of the Z-boson to different fermion generations are universal.

The universality is a result of a special feature of the SM: all fermions of given chirality and given charge come from the same $SU(2)_L \times U(1)_Y$ representation.

As an example to experimental tests of diagonality and universality, we can take the leptonic sector. The branching ratios of the Z-boson into charged lepton pairs [1],

$$\begin{aligned} \mathrm{BR}(Z \to e^+e^-) &= (3.363 \pm 0.004)\%\,, \\ \mathrm{BR}(Z \to \mu^+\mu^-) &= (3.366 \pm 0.007)\%\,, \\ \mathrm{BR}(Z \to \tau^+\tau^-) &= (3.370 \pm 0.008)\%\,. \end{aligned} \tag{56}$$

beautifully confirms universality:

$$\begin{aligned} \Gamma(\mu^+\mu^-)/\Gamma(e^+e^-) &= 1.001 \pm 0.003, \\ \Gamma(\tau^+\tau^-)/\Gamma(e^+e^-) &= 1.002 \pm 0.003. \end{aligned} \tag{57}$$

Diagonality is also tested by the following experimental searches:

$$\begin{aligned} \mathrm{BR}(Z \to e^+\mu^-) &< 7.5 \times 10^{-7}, \\ \mathrm{BR}(Z \to e^+\tau^-) &< 9.8 \times 10^{-6}, \\ \mathrm{BR}(Z \to \mu^+\tau^-) &< 1.2 \times 10^{-5}. \end{aligned} \tag{58}$$

Omitting common factors, particularly, a factor of $e^2/(4s_W^2c_W^2)$, and phase-space factors, we obtain the following predictions for the Z decays into a one-generation fermion-pair of each type:

$$\begin{aligned} \Gamma(Z \to \nu\bar{\nu}) &\propto 1, \\ \Gamma(Z \to \ell\bar{\ell}) &\propto 1 - 4s_W^2 + 8s_W^4, \\ \Gamma(Z \to u\bar{u}) &\propto 3\left(1 - \frac{8}{3}s_W^2 + \frac{32}{9}s_W^4\right), \\ \Gamma(Z \to d\bar{d}) &\propto 3\left(1 - \frac{4}{3}s_W^2 + \frac{8}{9}s_W^4\right). \end{aligned} \tag{59}$$

Putting $s_W^2 = 0.225$, we obtain

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.51 : 1.74 : 2.24. \tag{60}$$

Experiments measure the following average branching ratio into a single generation of each fermion species:

$$\mathrm{BR}(Z \to \nu\bar{\nu}) = (6.67 \pm 0.02)\%,$$

$$\begin{aligned}
\mathrm{BR}(Z \to \ell\bar{\ell}) &= (3.37 \pm 0.01)\%, \\
\mathrm{BR}(Z \to u\bar{u}) &= (11.6 \pm 0.6)\%, \\
\mathrm{BR}(Z \to d\bar{d}) &= (15.6 \pm 0.4)\%,
\end{aligned} \tag{61}$$

which, using central values, give

$$\Gamma_\nu : \Gamma_\ell : \Gamma_u : \Gamma_d = 1 : 0.505 : 1.74 : 2.34, \tag{62}$$

in very nice agreement with the predictions.

C. W-mediated weak interactions

We now study the couplings of the charged vector bosons, $W^\pm$, to fermion pairs. For the lepton mass eigenstates, things are simple, because there exists an interaction basis that is also a mass basis. Thus, the W interactions must be universal also in the mass basis:

$$\mathcal{L}_{W,\ell} = -\frac{g}{\sqrt{2}} \left(\overline{\nu_{eL}}\, \not{W}^+ e^-_L + \overline{\nu_{\mu L}}\, \not{W}^+ \mu^-_L + \overline{\nu_{\tau L}}\, \not{W}^+ \tau^-_L + \text{h.c.} \right). \tag{63}$$

Eq. (63) reveals some important features of the model:

1. Only left-handed leptons take part in charged-current interactions. Consequently, parity is violated.

2. *Diagonality*: the charged current interactions couple each charged lepton to a single neutrino, and each neutrino to a single charged lepton. Note that a global $SU(2)$ symmetry would allow off-diagonal couplings; It is the local symmetry that leads to diagonality.

3. *Universality*: the couplings of the W-boson to $\tau\bar{\nu}_\tau$, to $\mu\bar{\nu}_\mu$ and to $e\bar{\nu}_e$ are equal. Again, a global symmetry would have allowed an independent coupling to each lepton pair.

All of these predictions have been experimentally tested. As an example of how well universality works, consider the decay rates of the W-bosons to the three lepton pairs [1]:

$$\begin{aligned}
\mathrm{BR}(W^+ \to e^+\nu_e) &= (10.71 \pm 0.16) \times 10^{-2}, \\
\mathrm{BR}(W^+ \to \mu^+\nu_\mu) &= (10.63 \pm 0.15) \times 10^{-2}, \\
\mathrm{BR}(W^+ \to \tau^+\nu_\tau) &= (11.38 \pm 0.21) \times 10^{-2},
\end{aligned} \tag{64}$$

which beautifully confirms universality:

$$\begin{aligned}\Gamma(\mu^+\nu)/\Gamma(e^+\nu) &= 0.986 \pm 0.013,\\ \Gamma(\tau^+\nu)/\Gamma(e^+\nu) &= 1.043 \pm 0.024.\end{aligned} \tag{65}$$

As concerns quarks, things are more complicated, since there is no interaction basis that is also a mass basis. In the interaction basis where the down quarks are mass eigenstates (23), the W interactions have the following form:

$$\mathcal{L}_{W,q} = -\frac{g}{\sqrt{2}} \left(\overline{u_{dL}}\, \not{W}^+ d_L + \overline{u_{sL}}\, \not{W}^+ s_L + \overline{u_{bL}}\, \not{W}^+ b_L + \text{h.c.} \right). \tag{66}$$

The Yukawa matrices in this basis have the form (25), and in particular, for the up sector, we have

$$\mathcal{L}^u_{\text{Yuk}} = (\overline{u_{dL}}\ \overline{u_{sL}}\ \overline{u_{bL}}) V^\dagger \hat{Y}^u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}, \tag{67}$$

which tells us straightforwardly how to transform to the mass basis:

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} = V \begin{pmatrix} u_{dL} \\ u_{sL} \\ u_{bL} \end{pmatrix}. \tag{68}$$

Using Eq. (68), we obtain the form of the W interactions (66) in the mass basis:

$$-\frac{g}{\sqrt{2}}\ (\overline{u_L}\ \overline{c_L}\ \overline{t_L})\ V\ \not{W}^+ \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}. \tag{69}$$

You can easily convince yourself that we would have obtained the same form starting from any arbitrary interaction basis. We remind you that $V = V_{uL} V_{dL}^\dagger$ is basis independent.

Eq. (69) reveals some important features of the model:

1. Only left-handed quarks take part in charged-current interactions. Consequently, parity is violated by these interactions.

2. The W couplings to the quark mass eigenstates are neither universal nor diagonal. The universality of gauge interactions is hidden in the unitarity of the matrix V.

The (hidden) universality within the quark sector is tested by the prediction

$$\Gamma(W \to uX) = \Gamma(W \to cX) = \frac{1}{2}\Gamma(W \to \text{hadrons}). \tag{70}$$

Experimentally,

$$\Gamma(W \to cX)/\Gamma(W \to \text{hadrons}) = 0.49 \pm 0.04. \tag{71}$$

The matrix V is called the CKM matrix [2, 3]. The form of the CKM matrix is not unique. First, there is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are therefore written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{72}$$

Omitting common factors (particularly, a factor of $g^2/4$) and phase-space factors, we obtain the following predictions for the W decays:

$$\begin{aligned} \Gamma(W^+ \to \ell^+\nu_\ell) &\propto 1, \\ \Gamma(W^+ \to u_i\overline{d_j}) &\propto 3|V_{ij}|^2 \qquad (i = 1, 2;\ j = 1, 2, 3). \end{aligned} \tag{73}$$

The top quark is not included because it is heavier than the W boson. Taking this fact into account, and the CKM unitarity relations

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1, \tag{74}$$

we obtain

$$\Gamma(W \to \text{hadrons}) \approx 2\Gamma(W \to \text{leptons}). \tag{75}$$

Experimentally,

$$\text{BR}(W \to \text{leptons}) = (32.40 \pm 0.27)\%, \qquad \text{BR}(W \to \text{hadrons}) = (67.41 \pm 0.27)\%, \tag{76}$$

which leads to

$$\Gamma(W \to \text{hadrons})/\Gamma(W \to \text{leptons}) = 2.09 \pm 0.01, \tag{77}$$

in good agreement with the SM prediction.

D. Interactions of the Higgs boson

The Higgs boson has self-interactions, weak interactions, and Yukawa interactions:

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - \frac{m_h^2}{2v}h^3 - \frac{m_h^2}{8v^2}h^4 \tag{78}$$

$$
\begin{aligned}
&+\ m_W^2 W_\mu^- W^{\mu+}\left(\frac{2h}{v}+\frac{h^2}{v^2}\right)+\frac{1}{2}m_Z^2 Z_\mu Z^\mu\left(\frac{2h}{v}+\frac{h^2}{v^2}\right)\\
&-\ \frac{h}{v}\Big(m_e\,\overline{e_L}\,e_R+m_\mu\,\overline{\mu_L}\,\mu_R+m_\tau\,\overline{\tau_L}\,\tau_R\\
&\quad +m_u\,\overline{u_L}\,u_R+m_c\,\overline{c_L}\,c_R+m_t\,\overline{t_L}\,t_R+m_d\,\overline{d_L}\,d_R+m_s\,\overline{s_L}\,s_R+m_b\,\overline{b_L}\,b_R+\text{h.c.}\Big).
\end{aligned}
$$

Note that the Higgs boson couples diagonally to the quark mass eigenstates. The reason for this is that the Yukawa couplings determine both the masses and the Higgs couplings to the fermions. Thus, in the mass basis the Yukawa interactions are also diagonal. A formal derivation, starting from an arbitrary interaction basis, goes as follows:

$$
\begin{aligned}
h\overline{D_L}Y^d D_R &= h\overline{D_L}(V_{dL}^\dagger V_{dL})Y^d(V_{dR}^\dagger V_{dR})D_R\\
&= h(\overline{D_L}V_{dL}^\dagger)(V_{dL}Y^d V_{dR}^\dagger)(V_{dR}D_R)\\
&= h(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\hat{Y}^d(d_R\ s_R\ b_R)^T. \qquad (79)
\end{aligned}
$$

We conclude that the Higgs couplings to the fermion mass eigenstates have the following features:

1. *Diagonality.*

2. *Non-universality.*

3. *Proportionality* to the fermion masses: the heavier the fermion, the stronger the coupling. The factor of proportionality is m_ψ/v.

Thus, the Higgs boson decay is dominated by the heaviest particle which can be pair-produced in the decay. For $m_h \sim 125$ GeV, this is the bottom quark. Indeed, the SM predicts the following branching ratios for the leading decay modes:

$$
\text{BR}_{\bar{b}b} : \text{BR}_{WW^*} : \text{BR}_{gg} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{ZZ^*} : \text{BR}_{c\bar{c}} = 0.58 : 0.21 : 0.09 : 0.06 : 0.03 : 0.03. \qquad (80)
$$

The following comments are in order with regard to Eq. (80):

1. From the six branching ratios, three (b,τ,c) stand for two-body tree-level decays. Thus, at tree level, the respective branching ratios obey $\text{BR}_{\bar{b}b} : \text{BR}_{\tau^+\tau^-} : \text{BR}_{c\bar{c}} = 3m_b^2 : m_\tau^2 : 3m_c^2$. QCD radiative corrections somewhat suppress the two modes with the quark final states (b,c) compared to one with the lepton final state (τ).

TABLE IV: The SM fermion interactions

interaction	fermions	force carrier	coupling	flavor
Electromagnetic	u, d, ℓ	A^0	eQ	universal
Strong	u, d	g	g_s	universal
NC weak	all	Z^0	$\frac{e(T_3 - s_W^2 Q)}{s_W c_W}$	universal
CC weak	$\bar{u}d/\bar{\ell}\nu$	$W^\pm$	gV/g	non-universal/universal
Yukawa	u, d, ℓ	h	y_q	diagonal

2. The WW^* and ZZ^* modes stand for the three-body tree-level decays, where one of the vector bosons is on-shell and the other off-shell.

3. The Higgs boson does not have a tree-level coupling to gluons since it carries no color (and the gluons have no mass). The decay into final gluons proceeds via loop diagrams. The dominant contribution comes from the top-quark loop.

4. Similarly, the Higgs decays into final two photons via loop diagrams with small ($\mathrm{BR}_{\gamma\gamma} \sim 0.002$), but observable, rate. The dominant contributions come from the W and the top-quark loops which interfere destructively.

Experimentally, the decays into final ZZ^*, WW^*, $\gamma\gamma$ and $\tau^+\tau^-$ have been established [4] and there is recent evidence for the $b\bar{b}$ mode [5, 6]. Normalized to the SM rate, we have

$$\begin{aligned} \mu_{ZZ^*} &= 1.17 \pm 0.23, \\ \mu_{WW^*} &= 0.99 \pm 0.15, \\ \mu_{\gamma\gamma} &= 1.14 \pm 0.14, \\ \mu_{\tau\tau} &= 1.09 \pm 0.23, \\ \mu_{bb} &= 0.98 \pm 0.20. \end{aligned} \tag{81}$$

E. Summary

Within the SM, the fermions have five types of interactions. These interactions are summarized in Table IV.

VI. THE ACCIDENTAL SYMMETRIES OF THE SM

In the absence of the Yukawa matrices, $\mathcal{L}_{\rm Yuk} = 0$, the SM has a large $U(3)^5$ global symmetry:

$$G_{\rm global}^{\rm SM}(Y^{u,d,e}=0) = SU(3)_q^3 \times SU(3)_\ell^2 \times U(1)^5, \tag{82}$$

where

$$\begin{aligned} SU(3)_q^3 &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)_\ell^2 &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \end{aligned} \tag{83}$$

Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kin}$ respects the non-Abelian flavor symmetry $SU(3)_q^3 \times SU(3)_\ell^2$, under which

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{84}$$

where the V_i are unitary matrices. The Yukawa interactions (14) break the global symmetry,

$$G_{\rm global}^{\rm SM}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{85}$$

Under $U(1)_B$, all quarks (antiquarks) carry charge $+1/3$ ($-1/3$), while all other fields are neutral. It explains why proton decay has not been observed. Possible proton decay modes, such as $p \to \pi^0 e^+$ or $p \to K^+ \nu$, are not forbidden by the $SU(3)_C \times U(1)_{\rm EM}$ symmetry. However, they violate $U(1)_B$, and therefore do not occur within the SM. The lesson here is quite general: The lightest particle that carries a conserved charge is stable. The accidental $U(1)_B$ symmetry also explains why neutron-antineutron oscillations have not been observed.

Note that $U(1)_B$ as well as each of the lepton numbers are anomalous. The combination of $B-L$, however, is anomaly free. Due to the anomaly, baryon and lepton number violating processes occur non-perturbatively. However, the non-perturbative effects obey $\Delta B = \Delta L = 3n$, with $n =$integer, and thus do not lead to proton decay. Moreover, they are very small, and can be neglected in almost all cases we study, and thus we do not discuss them any further.

The accidental symmetries of the renormalizable part of the SM Lagrangian also explain the vanishing of neutrino masses. A Majorana mass term violates the accidental $B - L$ symmetry by two units. Thus, the symmetry prevents mass terms not only at tree level but also to all orders in perturbation theory. Moreover, since the $B - L$ symmetry is non-anomalous, Majorana mass terms do not arise even at the non-perturbative level. We conclude that the renormalizable SM gives the *exact* prediction:

$$m_\nu = 0. \tag{86}$$

We see that the transformations of Eq. (84) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also provide a definition of flavor physics: it refers to interactions that break the $SU(3)^5$ symmetry (84). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)^3_q$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)^3_q}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)^3_q}, \tag{87}$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)^2_\ell$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \sim (3, \bar{3})_{SU(3)^2_\ell}. \tag{88}$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors, and the idea of minimal flavor violation.

A. Counting parameters

How many independent parameters are there in $\mathcal{L}^q_{\rm Yuk}$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitary transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u, \tag{89}$$

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d = \mathrm{diag}(y_d, y_s, y_b), \quad \lambda_u = \mathrm{diag}(y_u, y_c, y_t), \tag{90}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we identify the nine real parameters as six quark masses and three mixing angles, while the single phase is δ_{KM}.

How many independent parameters are there in $\mathcal{L}^\ell_{\mathrm{Yuk}}$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitary transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \lambda_e = \mathrm{diag}(y_e, y_\mu, y_\tau). \tag{91}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

VII. BEYOND THE SM

The SM is not a full theory of Nature. It is only a low energy effective theory, valid below some scale $\Lambda \gg m_Z$. Then, the SM Lagrangian should be extended to include all non-renormalizable terms, suppressed by powers of Λ:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda} O_{d=5} + \frac{1}{\Lambda^2} O_{d=6} + \cdots, \tag{92}$$

where $O_{d=n}$ represents operators that are products of SM fields, transforming as singlets under the SM gauge group, of overall dimension n in the fields. For physics at an energy scale

E well below Λ, the effects of operators of dimension $n > 4$ are suppressed by $(E/\Lambda)^{n-4}$. Thus, in general, the higher the dimension of an operator, the smaller its effect at low energies.

In previous sections, we studied the SM mainly at tree level and with only renormalizable terms. We can classify the effects of including loop corrections and nonrenormalizable terms into three broad categories:

1. *Forbidden processes:* Various processes are forbidden by the accidental symmetries of the Standard Model. Nonrenormalizable terms (but not loop corrections!) can break these accidental symmetries and allow the forbidden processes to occur. Examples include neutrino masses and proton decay.

2. *Rare processes:* Various processes are not allowed at tree level. These effects can often be related to accidental symmetries that hold within a particular sector of, but not in the entire, SM. Here both loop corrections and nonrenormalizable terms can contribute. Examples include flavor changing neutral current (FCNC) processes.

3. *Tree level processes:* Often tree level processes in a particular sector depend on a small subset of the SM parameters. This situation leads to relations among different processes within this sector. These relations are violated by both loop effects and nonrenormalizable terms. Here, precision measurements and precision theory calculations are needed to observe these small effects. Examples include electroweak precision measurements (EWPM).

As concerns the last two types of effects, where loop corrections and nonrenormalizable terms may both contribute, their use in phenomenology can be divided to two eras. Before all the SM particles have been directly discovered and all the SM parameters measured, one could assume the validity of the renormalizable SM and indirectly measure the properties of the yet unobserved SM particles. Indeed, the charm quark, the top quark and the Higgs boson masses were predicted in this way. Once all the SM particles have been observed and the parameters measured directly, the loop corrections can be quantitatively determined, and effects of nonrenormalizable terms can be unambiguously probed. Thus, at present, all three classes of processes serve to search for new physics.

VIII. NEUTRINOS

In the SM, the neutrinos are exactly massless. Experiments, however, established that neutrinos have masses. While the individual neutrino mass eigenvalues are not known, two mass-squared differences are inferred from experiments:

$$\begin{aligned}\Delta m^2_{21} &\equiv m_2^2 - m_1^2 = (7.5 \pm 0.2) \times 10^{-5}\ \mathrm{eV}^2, \\ \Delta m^2_{32} &\equiv m_3^2 - m_2^2 = \pm(2.3 \pm 0.1) \times 10^{-3}\ \mathrm{eV}^2. \end{aligned} \tag{93}$$

This is a clear experimental indication of physics beyond the SM.

The SM prediction that the neutrinos are massless is related to the lepton number symmetry. The SM prediction that the neutrinos do not mix is related to the lepton flavor symmetry. Similar to other predictions that depend on accidental symmetries of the SM, these predictions are violated in generic extensions of the SM. In this section we show that $d = 5$ terms violate the accidental lepton number and lepton flavor symmetries of the SM, and consequently are probed by measurements of neutrino masses and mixing. Concretely, we study a model that we call the νSM. It is the SM extended to include the most general $d = 5$ terms.

There is a single class of dimension-five terms that depend on SM fields and obey the SM symmetries. These terms involve two $SU(2)$-doublet lepton fields and two $SU(2)$-doublet scalar fields:

$$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \frac{Z^\nu_{ij}}{\Lambda}\phi\phi L_i L_j, \tag{94}$$

where Z^ν is a symmetric and complex 3×3 matrix of dimensionless couplings, and Λ is a high mass scale, $\Lambda \gg v$.

A. The neutrino spectrum

With ϕ^0 acquiring a VEV, $\langle\phi^0\rangle = v/\sqrt{2}$, $\mathcal{L}_{\nu\mathrm{SM}}$ in Eq. (94) has a piece that corresponds to a Majorana mass matrix for the neutrinos:

$$\mathcal{L}_{\nu\mathrm{SM,mass}} = \frac{1}{2}(m_\nu)_{ij}\nu_i\nu_j, \qquad (m_\nu)_{ij} = \frac{v^2}{\Lambda}Z^\nu_{ij}. \tag{95}$$

The matrix m_ν can be diagonalized by a unitary transformation:

$$V_{\nu L} m_\nu V^T_{\nu L} = \hat{m}_\nu = \mathrm{diag}(m_1, m_2, m_3). \tag{96}$$

Majorana mass matrices are always symmetric. While the diagonalization of a general mass matrix M involves a general bi-unitary transformation, $M_{\rm diag} = V_L M V_R^\dagger$, for a symmetric mass matrix the diagonalization is by a unitary matrix and its transpose, as in Eq. (96).

We denote the corresponding neutrino mass eigenstates by ν_1, ν_2, ν_3. The convention here is that the states ν_1 and ν_2 are the ones separated by the smaller mass-squared difference, with $m_2 > m_1$. The state ν_3 is the one whose mass-squared difference from the other two is the largest. It is not yet known experimentally whether it is heavier ('normal ordering') or lighter ('inverted ordering') than the other two. This convention is in one-to-one correspondence with the way that the experimental results are presented in Eq. (93): $|\Delta m^2_{32}| > \Delta m^2_{21} > 0$.

B. The scale of generation of neutrino masses

In this section we explain the implications of the measured neutrino masses for the scale Λ where these masses are generated. As long as experiments probe only the low energy effective theory, what is measured is the combination Z^ν/Λ. Thus, there is an ambiguity in the definition of Λ and Z^ν. The separation of the coefficient of a $d = 5$ term to a dimensionless coupling and a scale is meaningful when we discuss a full high energy theory which generates the effective term. What we refer to as the scale of a non-renormalizable term is Λ/Z^ν (or, in case that Z^ν is a matrix, as in Eq. (94), $\Lambda/Z^\nu_{\rm max}$, where $Z^\nu_{\rm max}$ is the largest eigenvalue of Z^ν). Note, however, that the combination of a measurement of Λ/Z^ν and the assumption that Z^ν is generated by perturbative physics and therefore $Z^\nu_{\rm max} \lesssim 1$ translates into an upper bound on Λ.

The measurements of the neutrino mass-squared differences, Eq. (93), do not tell us the individual masses of the neutrinos, but they provide a lower bound on two mass eigenvalues: There is at least one neutrino mass heavier than $\sqrt{|\Delta m^2_{32}|}$,

$$m_{\rm heaviest} \geq \sqrt{|\Delta m^2_{32}|} \simeq 0.05 \text{ eV}, \tag{97}$$

and there is at least one additional mass heavier than $\sqrt{\Delta m^2_{21}} \sim 0.009$ eV. There is, however, additional information from experiments and cosmology which provides an upper bound on the absolute mass scale of the neutrinos of order 1 eV.

The effective low energy Lagrangian of Eq. (94) where, by definition, $\Lambda \gg v$, predicts that the neutrino masses are much lighter than the weak scale:

$$m_{1,2,3} \sim v^2/\Lambda \ll v. \tag{98}$$

The fact that experiments find that the neutrinos are indeed much lighter than the W mass makes the notion that neutrino masses are generated by $d = 5$ terms very plausible.

In fact, all fermions of the SM except for the top quark are light relative to m_W. The lightness of charged fermions is related to the smallness of the corresponding Yukawa couplings. The question of why Yukawa couplings are small may find an answer in a more fundamental theory, beyond the SM. The neutrinos, however, are not only much lighter than m_W, but also lighter by at least six orders of magnitude than all charged fermions. This extreme lightness of the neutrinos is explained if their masses are generated by $d = 5$ terms.

Clearly, the SM cannot be a valid theory above the Planck scale, $\Lambda \lesssim M_{\rm Pl}$. We thus expect that $m_i \gtrsim v^2/M_{\rm Pl} \sim 10^{-5}$ eV. A more relevant scale might be the scale of Grand Unified Theories (GUTs). In GUTs, the $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group of the SM is assumed to be a subgroup of a unifying group, such as $SU(5)$, which is spontaneously broken to $G_{\rm SM}$ at a scale $\Lambda_{\rm GUT} = \mathcal{O}(10^{16}$ GeV). If the $d = 5$ terms are generated at $\Lambda_{\rm GUT}$, then we expect $m_\nu \sim 10^{-2}$ eV.

Conversely, an experimental lower bound on neutrino masses provides an upper bound on the scale of relevant new physics. Using the lower bound of Eq. (97) and the relation of Eq. (95), we conclude that the SM cannot be a valid theory above the scale

$$\Lambda \lesssim \frac{v^2}{m_\nu} \sim 10^{15} \text{ GeV}. \tag{99}$$

This proves that the SM cannot be valid up to the Planck scale. Furthermore, this upper bound is intriguingly close to the GUT scale.

C. The neutrino interactions

The addition of the dimension-five terms leads to significant changes in the phenomenology of the lepton sector. The modifications can be understood by re-writing the neutrino-related terms in the mass basis. The renormalizable SM gives

$$\mathcal{L}_{\rm SM,\nu} = i\overline{\nu_\alpha}\partial\!\!\!/\nu_\alpha - \frac{g}{2c_W}\overline{\nu_\alpha}Z\!\!\!/\nu_\alpha - \frac{g}{\sqrt{2}}\left(\overline{\ell_{L\alpha}}W\!\!\!\!/^-\nu_\alpha + \text{h.c.}\right), \tag{100}$$

where $\alpha = e, \mu, \tau$. (The Lagrangian (100) describes massless neutrinos, and consequently the basis $(\nu_e, \nu_\mu, \nu_\tau)$ serves as both an interaction basis and a mass basis.) The Lagrangian of Eq. (94) gives

$$\mathcal{L}_{\nu\mathrm{SM},\nu} = i\overline{\nu_i}\partial\!\!\!/\nu_i - \frac{g}{2c_W}\overline{\nu_i}Z\!\!\!/\nu_i - \frac{g}{\sqrt{2}}\left(\overline{\ell_{L\alpha}}W\!\!\!\!/^{-}U_{\alpha i}\nu_i + \text{h.c.}\right) + m_i\nu_i\nu_i + \frac{2m_i}{v}h\nu_i\nu_i + \frac{m_i}{v^2}hh\nu_i\nu_i. \quad (101)$$

Here $\alpha = e, \mu, \tau$ denotes only the charged lepton mass eigenstates, while $i = 1, 2, 3$ denotes the neutrino mass eigenstates. The neutrino mass parameters $m_{1,2,3}$ are real, and the mixing matrix U is unitary. Starting from an arbitrary interaction basis, the matrix U is given by

$$U = V_{eL}V_{\nu L}^\dagger. \quad (102)$$

While each of V_{eL} and $V_{\nu L}$ is basis-dependent, the combination $V_{eL}V_{\nu L}^\dagger$ is not. Explicitly we write it as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \quad (103)$$

The most significant changes from (100) to (101) concerning neutrino interactions are the following:

- The leptonic charged current interactions are neither universal nor diagonal. Instead, they involve the mixing matrix U.
- The Higgs boson has Yukawa couplings to neutrinos. These couplings break lepton number. The size of the Yukawa couplings is, however, tiny, of order $m_i/v \sim 10^{-13}$, leading to unobservably small branching ratio for $h \to \nu\nu$.

The νSM-neutrinos thus have three types of interactions, mediated by massive bosons. These interactions are summarized in Table V.

D. Accidental symmetries and the lepton mixing parameters

The dimension-five terms in Eq. (94) break the $U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry of the SM. With the addition of only $d = 5$ terms, all that remains of the $G_{\mathrm{SM}}^{\mathrm{global}}$ symmetry of the SM [see Eq. (85)] is baryon number symmetry:

$$G_{\nu\mathrm{SM}}^{\mathrm{global}} = U(1)_B. \quad (104)$$

TABLE V: The neutrino interactions

interaction	force carrier	coupling
NC weak	Z^0	$e/(2s_W c_W)$
CC weak	$W^\pm$	$gU/\sqrt{2}$
Yukawa	h	$2m/v$

This symmetry is, however, anomalous and broken by non-perturbative effects. In addition, it is broken by dimension-six terms.

The counting of flavor parameters in the quark sector remains unchanged: six quark masses and four mixing parameters, of which one is imaginary. How many physical flavor parameters are involved in the lepton sector? The Lagrangian of Eq. (94) involves the 3×3 matrix Y^e (9 real and 9 imaginary parameters), and the symmetric 3×3 matrix Z^ν (6 real and 6 imaginary parameters). The kinetic and gauge terms have a $U(3)_L \times U(3)_E$ accidental global symmetry, that is completely broken by the Y^e and Z^ν terms. Thus, the number of physical lepton flavor parameters is $(15_R + 15_I) - 2 \times (3_R + 6_I) = 9_R + 3_I$. Six of the real parameters are the three charged lepton masses $m_{e,\mu,\tau}$ and the three neutrino masses $m_{1,2,3}$. We conclude that the 3×3 unitary matrix U depends on three real mixing angles and three phases.

Why does the lepton mixing matrix U depend on three phases, while the quark mixing matrix V depends on only a single phase? The reason for this difference lies in the fact that the Lagrangian of Eq. (94) leads to Majorana masses for neutrinos. Consequently, there is no freedom in changing the mass basis by redefining the neutrino phases, as such redefinition will introduce phases into the neutrino mass terms. While redefinitions of the six quark fields allowed us to remove five non-physical phases from V, redefinitions of the three charged lepton fields allows us to remove only three non-physical phases from U. The two additional physical phases in U are called "Majorana phases," since they appear as a result of the (assumed) Majorana nature of neutrinos. They affect lepton number violating processes.

A convenient parametrization of U is the following:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2}), \quad (105)$$

where $\alpha_{1,2}$ are the Majorana phases, $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$.

The present status of our knowledge of the absolute values of the various entries in the lepton mixing matrix can be summarized as follows (we quote here the 3σ ranges):

$$|U| = \begin{pmatrix} 0.80 - 0.85 & 0.51 - 0.58 & 0.14 - 0.16 \\ 0.22 - 0.52 & 0.44 - 0.70 & 0.61 - 0.79 \\ 0.25 - 0.53 & 0.46 - 0.71 & 0.59 - 0.78 \end{pmatrix}. \quad (106)$$

When working in the mass basis, the formalisms of quark and lepton flavor mixing are very similar. The difference between these two phenomena arises due to the way neutrino experiments are done. While quarks and charged leptons are identified as mass eigenstates, neutrinos are identified as interaction eigenstates. Explicitly, they are identified as ν_e or ν_μ or ν_τ according to whether they produce in the detector an e or μ or τ lepton, respectively.

E. Open questions

All the results in the neutrino sector so far are consistent with the νSM. The following parameters are still not experimentally determined:

- The absolute mass scale of the neutrinos is still unknown. On one extreme, they could be quasi-degenerate and as heavy as parts of eV. On the other extreme, they could be hierarchical, with the lightest possibly massless.
- It is not known whether the spectrum has normal or inverted ordering.
- None of the three phases has been measured.

While the results can be accommodated in the νSM, there are other ways to explain the data. The following questions are of interest as further tests of the idea that the νSM is the correct low energy description of the neutrino sector:

- Are the neutrinos Dirac or Majorana fermions?

- Are there sterile neutrinos, that is, other light states that mix with the active neutrinos?

- Are there dimension six operators that significantly affect the neutrino interactions?

Acknowledgments

I thank Yuval Grossman for his work with me on a pedagogical textbook on the Standard Model. I thank the postdocs and students of my research group – Claudia Frugiuele, Aielet Efrati, Avital Dery and Daniel Aloni – for many useful discussions. YN is supported by grants from the I-CORE program of the Planning and Budgeting Committee of the Israel Science Foundation (grant number 1937/12), from the Israel Science Foundation (grant number 394/16) and from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel (grant number 2014230).

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