neutrino physics: theory and phenomenology

JOSÉ W F VALLE

6th Chilean school in High Energy Physics

ASTROPARTICLES
Astroparticles and High Energy Physics Group
IFIC Institut de Física Corpuscular
Universitat de València
CSIC Consejo Superior de Investigaciones Científicas
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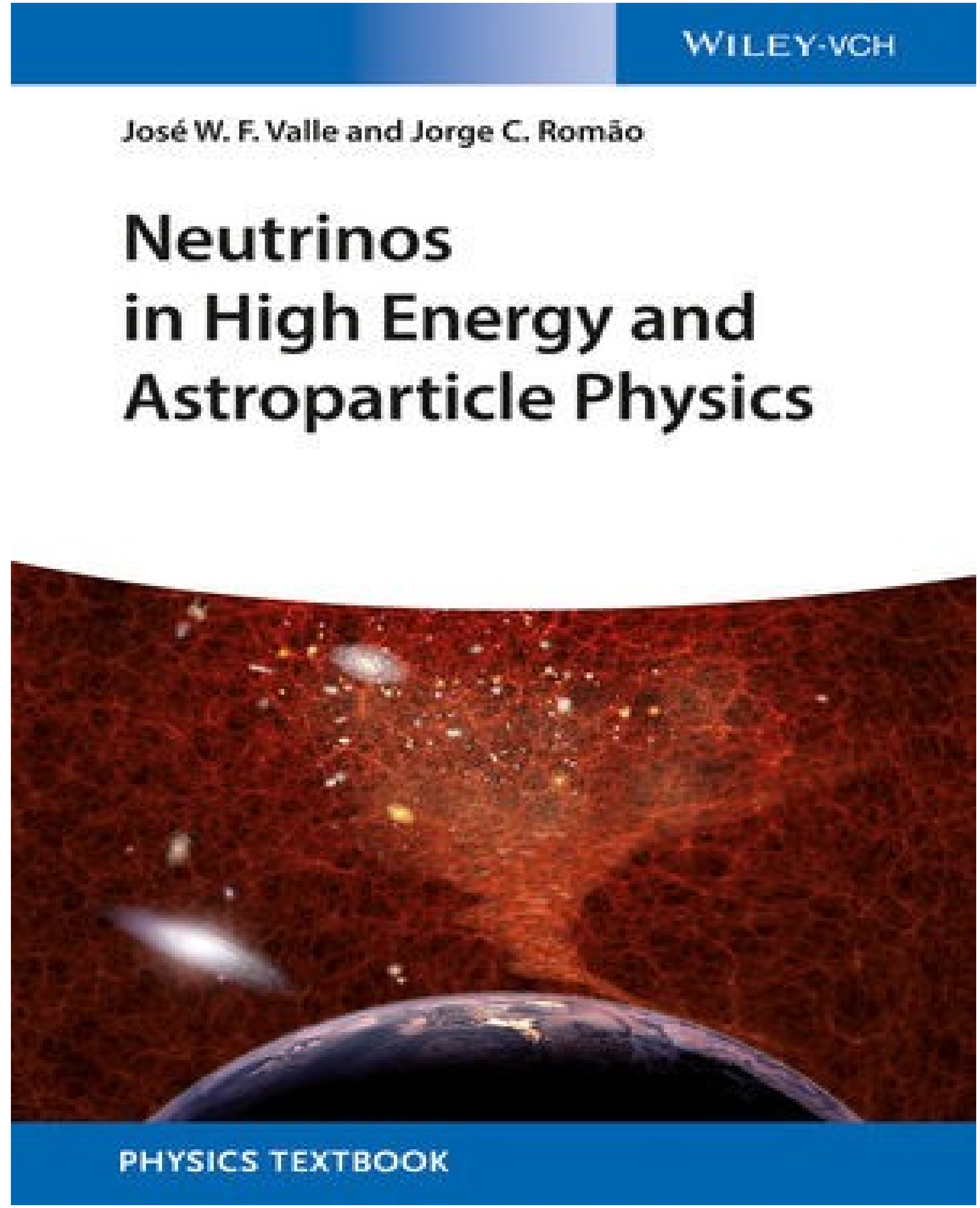

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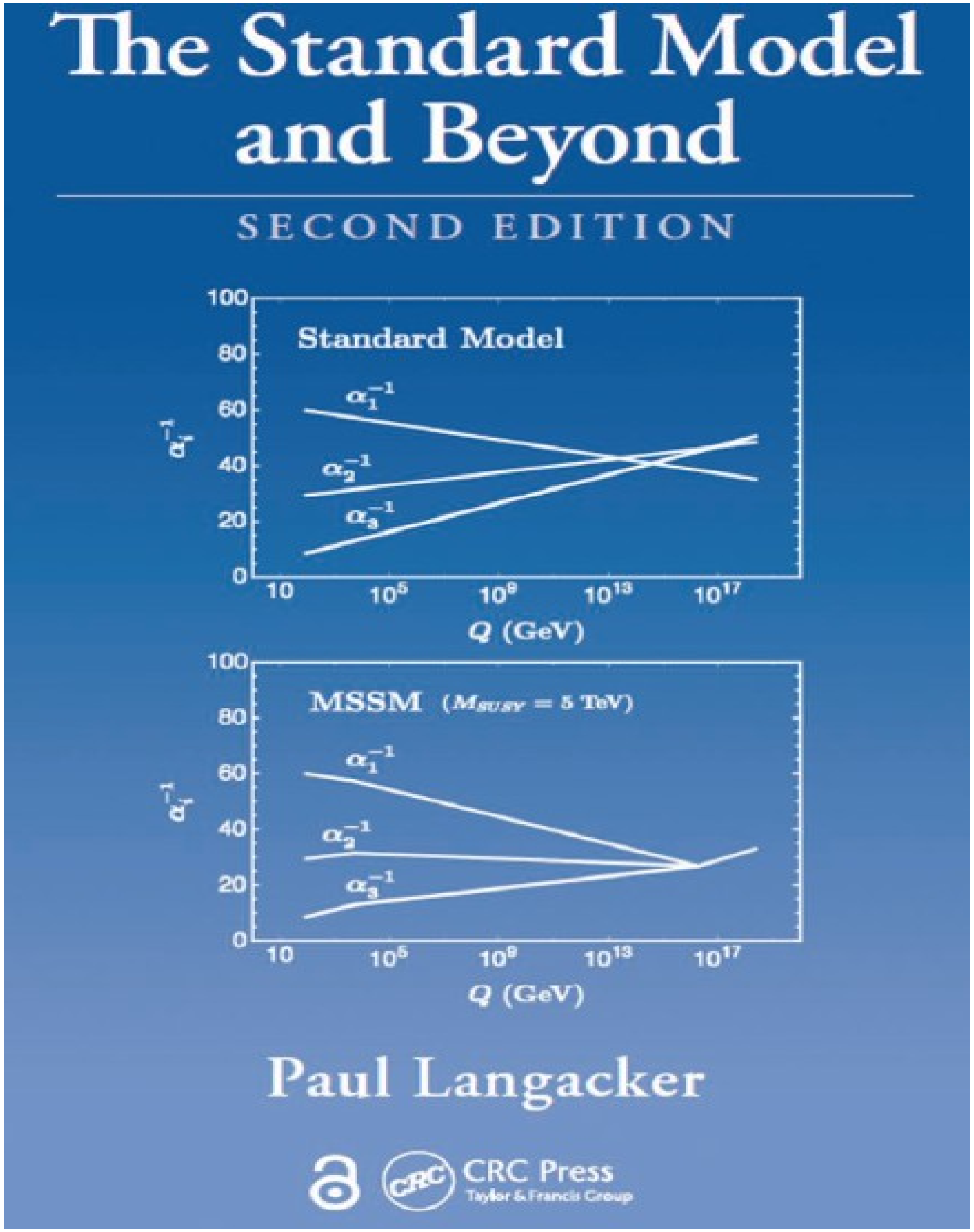
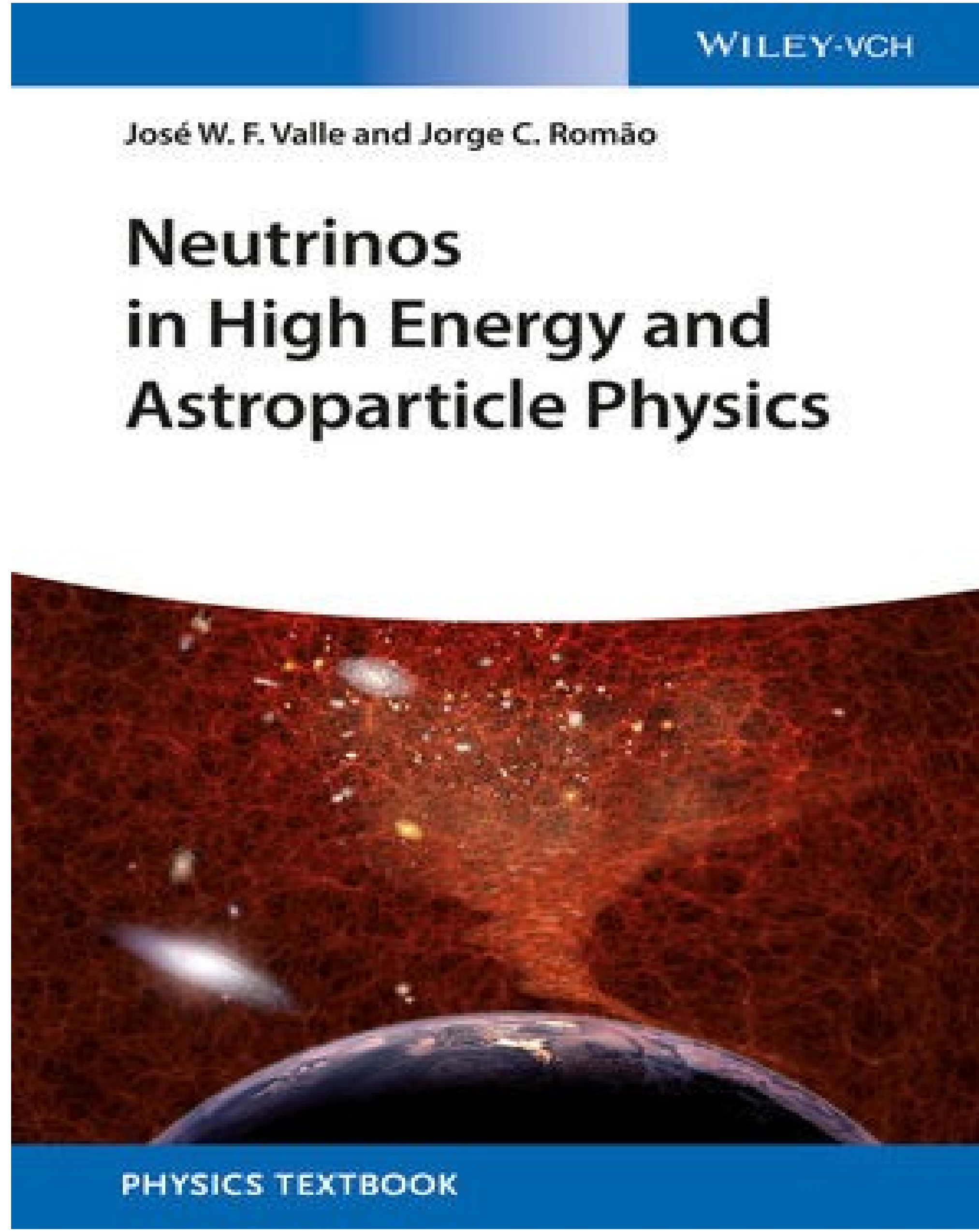




including results from various publications towards the end lecture 1

SM recap

The **standard model** (SM) of strong and electroweak interactions has provided the cornerstone of elementary particle physics for over 4 decades

The basic principle of the SM is **gauge invariance** which enables us to describe the interaction of the **matter** particles by **vector boson exchange**

The theory is based on the **321** gauge group with three sectors:

Quantum Chromodynamics (**QCD**) which deals with the strong interaction, **SU3**
weak interaction or **EW** sector **SU2**
Quantum Electrodynamics (QED) responsible for the electromagnetic force, U1

QED and **weak** forces get **combined** in the process of **symmetry breaking** required in order to reconcile the **short** range nature of the **weak** interaction with the **long** range of **EM**

Glashow, Salam and Weinberg
Higgs Englert

Electroweak gauge bosons

As we mentioned before there are four gauge bosons characterizing the electroweak sector of the standard model, three W^i_μ ($i = 1, 2, 3$, one for each generator T^i) transforming as the adjoint representation of $SU(2)_L$, and one B_μ for $U(1)_Y$. The corresponding field tensors are:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\,\epsilon_{abc} W^b_\mu W^c_\nu\,,$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,, \tag{2.1}$$

where we call g' and g the coupling constants of the $U(1)_Y$ and $SU(2)_L$ groups respectively, and ϵ_{abc} is the completely antisymmetric tensor in three dimensions. The kinetic Lagrangian for the bosons is given by

$$\mathcal{L}_G = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}\,, \tag{2.2}$$

and it is invariant under the (separate) local gauge transformations of the $SU(2)_L$ and $U(1)_Y$ groups. The general form of these gauge transformations for finite gauge parameters is,

$$\delta W^a_\mu = -\,\epsilon^{abc}\,\alpha^b\,W^c_\mu - \frac{1}{g}\,\partial_\mu \alpha^a$$

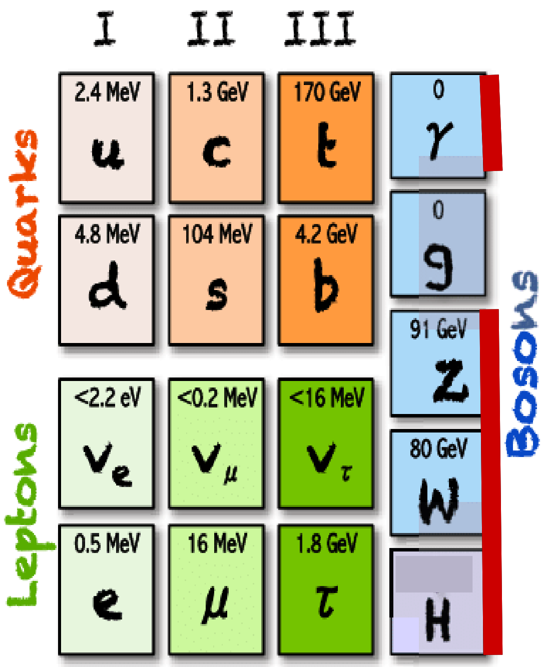
$$\delta B_\mu = -\,\frac{1}{g'}\,\partial_\mu \alpha_Y$$

$$W^a_\mu \frac{\sigma^a}{2} \to W'^a_\mu \frac{\sigma^a}{2} = \mathcal{U}_L\, W^a_\mu \frac{\sigma^a}{2}\, \mathcal{U}_L^{-1} + \frac{i}{g}\,\partial_\mu \mathcal{U}_L\, \mathcal{U}_L^{-1}\,,$$ SU(2)

$$B_\mu \to B'_\mu = B_\mu + \frac{i}{g'}\,\partial_\mu \mathcal{U}_Y\, \mathcal{U}_Y^{-1}\,, \tag{2.3}$$ U(1)

$$\mathcal{U}_L = e^{i\alpha^a \frac{\sigma^a}{2}}\,, \qquad \mathcal{U}_Y = e^{i\alpha_Y}\,, \tag{2.4}$$

matter fields



The matter fields of the standard model are all the known fermions which are classified in three generations. The two helicity states, left and right

$$\psi_L = \tfrac{1}{2}(1-\gamma_5)\psi\,, \quad \psi_R = \tfrac{1}{2}(1+\gamma_5)\psi\,, \qquad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.6}$$

of each charged fermion transform differently under the $\mathrm{SU(2)_L}$ group. Left handed components are assigned to doublet representation while right handed ones transform as singlets, that is,

$$L_L = \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}_L , \; Q_L = \begin{bmatrix} u \\ d \end{bmatrix}_L , \; e_R^-, u_R, d_R. \tag{2.7}$$

where we have only shown the particles in the first generation. The other two generations are just copies of the first. The quantum numbers with respect to the $\mathrm{SU(2)_L \otimes U(1)_Y}$ gauge group are given in Table 2.1, where the electric charge is given by

$$Q = T_3 + Y\,, \tag{2.8}$$

and $T_3 = \frac{1}{2}\sigma_3$.

Standard model matter fields

Particle	ν_{eL}	e_L	u_L	d_L	e_R	u_R	d_R
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0
Y	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	-1	$\frac{2}{3}$	$-\frac{1}{3}$
Q	0	-1	$\frac{2}{3}$	$-\frac{1}{3}$	-1	$\frac{2}{3}$	$-\frac{1}{3}$

Table 2.1 Quantum numbers of the particles of the first generation with respect to the $\mathrm{SU(3)_c \otimes SU(2)_L \otimes U(1)_Y}$ gauge group.

Under finite local gauge transformations the Ψ_L and ψ_R fields transform as follows

$$\begin{aligned} \Psi_L \to \Psi'_L &= e^{i\alpha^a \frac{\sigma^a}{2}} \, e^{i\alpha_Y Y} \, \Psi_L \,, \\ \psi_R \to \psi'_R &= e^{i\alpha_Y Y} \, \psi_R \,. \end{aligned} \tag{2.9}$$

The principle of gauge invariance establishes that the piece of the Lagrangian describing the gauge interactions of the fermions is obtained from the kinetic energy part of the Lagrangian, after substituting the derivative by the covariant derivative,

$$\begin{aligned} \partial_\mu \Psi_L \to \mathcal{D}_\mu \Psi_L &= \left(\partial_\mu + ig\frac{\sigma_a}{2} W^a_\mu + ig' Y B_\mu \right) \Psi_L \,, \\ \partial_\mu \psi_R \to \mathcal{D}_\mu \psi_R &= \left(\partial_\mu + ig' Y B_\mu \right) \psi_R \,. \end{aligned} \tag{2.10}$$

Using Eq. (2.3) and Eq. (2.9) one can easily verify that the covariant derivatives have the appropriate transformation properties (that is, they transform in the same way as the fields themselves),

$$\begin{aligned} \mathcal{D}_\mu \Psi_L \to \mathcal{D}_\mu \Psi'_L &= e^{i\alpha^a \frac{\sigma^a}{2}} \, e^{i\alpha_Y Y} \, \mathcal{D}_\mu \Psi_L \,, \\ \mathcal{D}_\mu \psi_R \to \mathcal{D}_\mu \psi'_R &= e^{i\alpha_Y Y} \, \mathcal{D}_\mu \psi_R \,. \end{aligned} \tag{2.11}$$

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After symmetry breaking (see below) the neutral gauge bosons W^3_μ and B_μ will mix to give one massless photon, A_μ and one massive Z_μ, through the relations,

weak basis W,B

$$W^3_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu ,$$
$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu , \quad (2.12)$$

mass basis A,Z

where θ_W is the weak mixing angle (also called Weinberg angle), satisfying the relations,

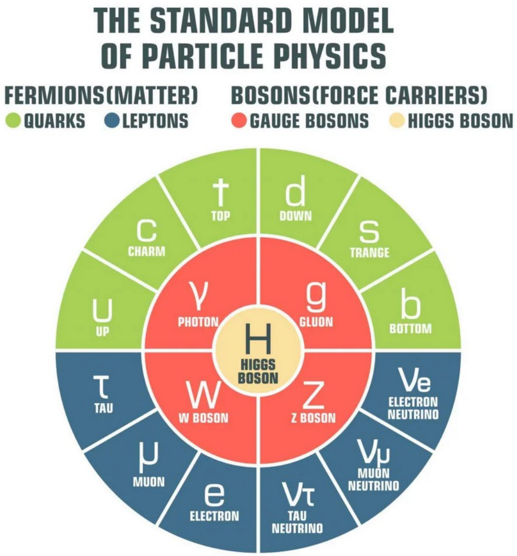
$$e = g \sin\theta_W = g' \cos\theta_W \quad ; \quad \frac{g'}{g} = \tan\theta_W , \quad (2.13)$$

and

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} . \quad (2.14)$$

Spontaneous symmetry breaking: mass generation

A spontaneously broken symmetry is preserved by the Lagrangian but it is not a symmetry of the ground state of the system, the vacuum state. In order to implement this idea in the standard model an $SU(2)_L$ scalar doublet Φ is introduced in the theory



$$\Phi = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$$

SU(2) doublet with Y=1/2

Particle	ϕ^+	ϕ^0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$
Y	$\frac{1}{2}$	$\frac{1}{2}$
Q	1	0

Lagrangian, invariant under the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge group,

$$\mathcal{L}_H = (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi) - \mu^2 \, \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi \right)^2 ,$$

$$D_\mu \Phi = \left[\partial_\mu + i \frac{g}{\sqrt{2}} \left(\tau^+ W_\mu^+ + \tau^- W_\mu^- \right) + i \frac{g}{2} \tau_3 W_\mu^3 + i \frac{g'}{2} B_\mu \right] \Phi$$

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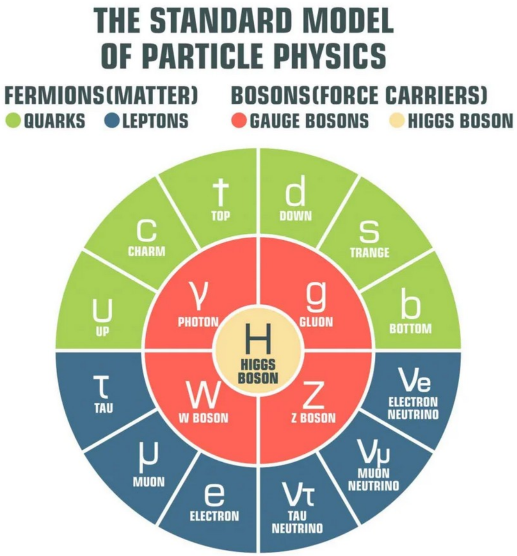

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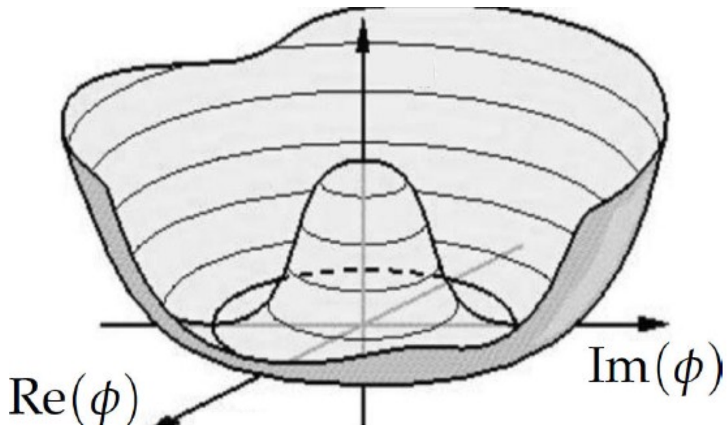
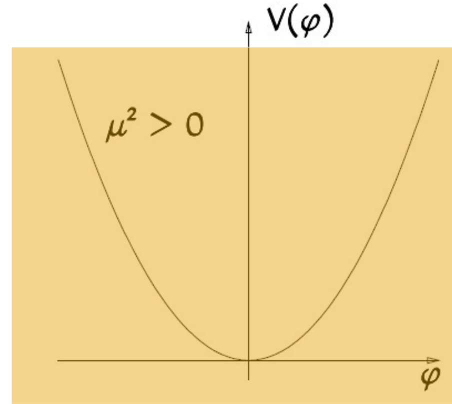
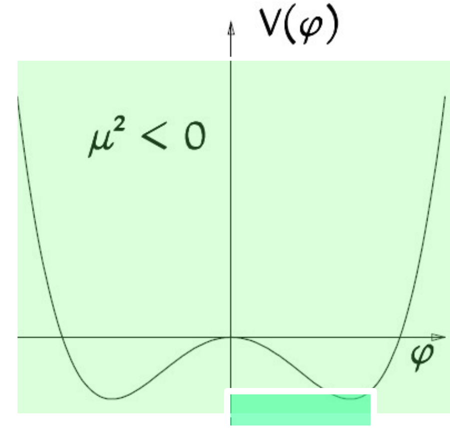
In Fig. 2.1 we sketch the potential part in $\mathcal{L}_H$, $V = \mu^2|\Phi|^2 + \lambda|\Phi|^4$ as a function of $|\Phi| = \sqrt{\Phi^\dagger\Phi}$. For $\mu^2 > 0$ V has a unique minimum at $|\Phi| = 0$. However when $\mu^2 < 0$ the classical ground state occurs at $|\Phi|^2 = -\frac{1}{2}\mu^2/\lambda$.

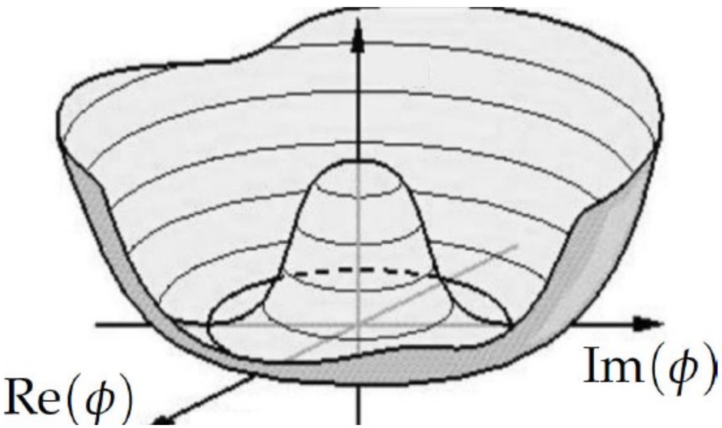


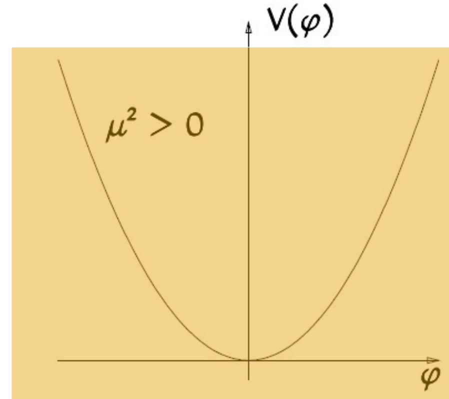


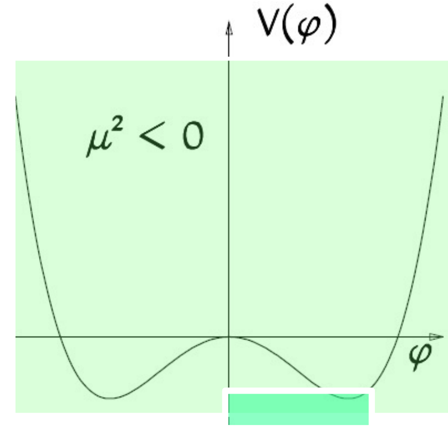


$SU(2)_L \times U(1)_Y \Rightarrow$ **U(1)**

gives $\langle|\Phi|\rangle = \frac{v}{\sqrt{2}} = \sqrt{-\frac{\mu^2}{2\lambda}}$







$SU(2)_L \times U(1)_Y \Rightarrow U(1)$

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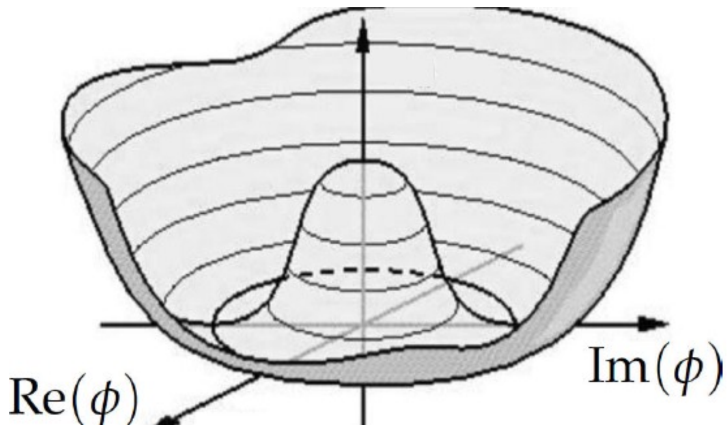
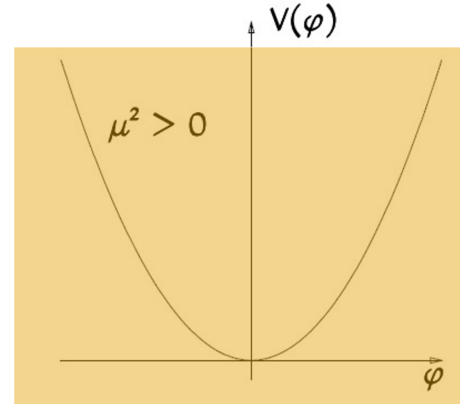
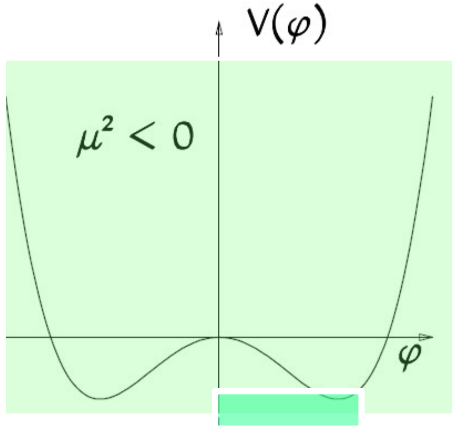
We now perform perturbation theory around one of the above continuous set of true vacua. To do this it is convenient to parametrize the scalar field as

$$\Phi = e^{i\frac{\theta^a(x)\sigma_a}{v}} \begin{bmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{bmatrix} , \tag{2.23}$$

where the fields θ^a and H are real and have zero vacuum expectation value. If the $SU(2)_L$ symmetry was a *global* symmetry of the Lagrangian the three θ^a fields would correspond to physical fields with zero mass since the potential is flat in those directions, as stated by the Goldstone Theorem







$\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ ➡ U(1)

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$\mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ / U(1)

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$$W^{\pm}_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} .$$

In the mass basis the SU(2)xU(1) covariant derivative reads

$$D_\mu \Phi = \left[\partial_\mu + i\frac{g}{\sqrt{2}} \left(\tau^+ W^+_\mu + \tau^- W^-_\mu \right) + ie\,Q\,A_\mu + i\frac{g}{\cos\theta_W} \left(\frac{\tau_3}{2} - Q \sin^2\theta_W \right) Z_\mu \right] \Phi$$

$$(\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi)$$

$$\mathcal{L}_H = \frac{1}{2}\,\partial_\mu H \partial^\mu H + \mu^2 H^2 + \frac{1}{2}\,\partial_\mu \theta^a \partial^\mu \theta^a + \frac{1}{4}\, g^2 v^2 W_\mu^+ W^{-\mu} + \frac{1}{8}\, g_Z^2 v^2 Z_\mu Z^\mu$$

$$+ \frac{gv}{2} W_\mu^- \partial^\mu \theta^+ + \frac{gv}{2} W_\mu^+ \partial^\mu \theta^- + \frac{gv}{2\cos\theta_W} Z_\mu \partial^\mu \theta^3 + \cdots ,$$

where $g_Z^2 = g^2 + g'^2$, and the dots stand for cubic and quartic terms and we have used Eq. (2.22). Looking at Eq. (2.24) one would think that the three θ^a fields are massless and the remaining H field has a mass squared $M_H^2 = -2\mu^2 > 0$. Notice however, that there is a mixing between the gauge fields and the θ^a fields, the *would be* Goldstone bosons. One has therefore to be more careful in the analysis of the spectrum. The best way to do it is to realize that the three θ^a fields can be gauged away by a finite transformation under the local $\mathrm{SU(2)_L}$ group.

Now we will rotate away the θ^a fields. We then have

$$\Phi(x) \to \Phi'(x) = e^{-i\frac{2\theta^a(x)}{v}\frac{\sigma^a}{2}}\,\Phi = \begin{bmatrix} 0 \\ \dfrac{v + H(x)}{\sqrt{2}} \end{bmatrix}$$

This particular choice of gauge is called the *unitary gauge*. In this gauge there is only one physical scalar field, the Higgs boson H, and the θ^a degrees of freedom become the longitudinal components of the 3 gauge bosons of $\mathrm{SU(2)_L}$ which are now massive.

physical Higgs boson as dynamical trace of SSB mechanism

Introducing Eq. (2.26) into the Higgs Lagrangian, Eq (2.20), and dropping the prime in W'^{a}_{μ}, we get after rotating the gauge bosons according to Eq. (2.12),

no photon quadratic term

$$\begin{aligned}\mathcal{L}_H =& \frac{1}{2}(\partial_\mu H)^2 - \frac{1}{2}M_H^2 H^2 - \frac{1}{4}\lambda H^4 - \lambda v H^3 + \frac{1}{2} v g^2 W_\mu^+ W^{-\mu} H \\ &+ \frac{1}{4} v \frac{g^2}{\cos\theta_W} Z_\mu Z^\mu H + \frac{1}{4} g^2 W_\mu^+ W^{-\mu} H^2 + \frac{1}{8}\frac{g^2}{\cos\theta_W} Z_\mu Z^\mu H^2 \\ &+ \frac{1}{2} M_Z^2 Z_\mu Z^\mu + M_W^2 W_\mu^+ W^{-\mu} + \lambda \frac{v^4}{4}, \end{aligned} \tag{2.27}$$

where the masses M_W, M_Z, and M_H are given by

$$M_W = \tfrac{1}{2} g v\,, \qquad M_Z = \tfrac{1}{2} g_Z v = \frac{M_W}{\cos\theta_W}\,, \qquad M_H = \sqrt{-2\mu^2} = 2\lambda v^2\,. \tag{2.28}$$

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where θ_W is the weak mixing angle (also called Weinberg angle), satisfying

$$e = g \sin\theta_W = g' \cos\theta_W \quad ; \quad \frac{g'}{g} = \tan\theta_W,$$

weak CC & NC

W/Z UA1, UA2

$$\Phi = \begin{bmatrix} 0 \\ \frac{v + H(x)}{\sqrt{2}} \end{bmatrix}$$

$$\tilde{\Phi} = \begin{bmatrix} \frac{v + H(x)}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\mathcal{L}_{\mathrm{Yuk}} = -\sum_{ij} \left[Y^l_{ij}\, \overline{l'}_{iL}\, \Phi\, l'_{jR} + Y^u_{ij}\, \overline{u'}_{iL}\, \tilde{\Phi}\, u'_{jR} + Y^d_{ij}\, \overline{d'_{iL}}\, \Phi\, d'_{jR} + h.c. \right],$$

$$\mathcal{L}_{\mathrm{Yuk}} = -\sum_{ij} \left[\overline{l'}_{iL}\, M^l_{ij} l'_{jR} + \overline{u'}_{iL}\, M^u_{ij} u'_{jR} + \overline{d'}_{iL}\, M^d_{ij} d'_{jR} + \frac{H}{\sqrt{2}} \overline{l'}_{iL}\, Y^l_{ij} l'_{jR} + \frac{H}{\sqrt{2}} \overline{u'}_{iL}\, Y^u_{ij} u'_{jR} + \frac{H}{\sqrt{2}} \overline{d'}_{iL}\, Y^d_{ij} d'_{jR} + h.c. \right]$$

where

$$M^l_{ij} = Y^l_{ij}\frac{v}{\sqrt{2}}, \quad M^u_{ij} = Y^u_{ij}\frac{v}{\sqrt{2}}, \quad M^d_{ij} = Y^d_{ij}\frac{v}{\sqrt{2}}.$$

Let us denote by l, u and d the mass eigenstates obtained via the rotations

$$l_{iL} = \boldsymbol{U^l_L}_{ij} l'_{jL} \qquad u_{iL} = \boldsymbol{U^u_L}_{ij} u'_{jL} \qquad d_{iL} = \boldsymbol{U^d_L}_{ij} d'_{jL},$$
$$l_{iR} = \boldsymbol{U^l_R}_{ij} l'_{jR} \qquad u_{iR} = \boldsymbol{U^u_R}_{ij} u'_{jR} \qquad d_{iR} = \boldsymbol{U^d_R}_{ij} d'_{jR},$$

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$$\mathcal{L}_{\text{Yuk}} = -\sum_{ij} \Big[\overline{l'}_{iL}\, M^l_{ij} l'_{jR} + \overline{u'}_{iL}\, M^u_{ij} u'_{jR} + \overline{d'}_{iL}\, M^d_{ij} d'_{jR}$$
$$+ \frac{H}{\sqrt{2}} \overline{l'}_{iL}\, Y^l_{ij} l'_{jR} + \frac{H}{\sqrt{2}} \overline{u'}_{iL}\, Y^u_{ij} u'_{jR} + \frac{H}{\sqrt{2}} \overline{d'}_{iL}\, Y^d_{ij} d'_{jR} + h.c. \Big]$$

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Let us denote by l, u and d the mass eigenstates obtained via the rotations

$$l_{iL} = \boldsymbol{U^l_L}_{ij} l'_{jL} \qquad u_{iL} = \boldsymbol{U^u_L}_{ij} u'_{jL} \qquad d_{iL} = \boldsymbol{U^d_L}_{ij} d'_{jL},$$
$$l_{iR} = \boldsymbol{U^l_R}_{ij} l'_{jR} \qquad u_{iR} = \boldsymbol{U^u_R}_{ij} u'_{jR} \qquad d_{iR} = \boldsymbol{U^d_R}_{ij} d'_{jR},$$

$$\mathcal{L}_F^{\text{kinetic}} = \sum_{\text{doub}} i\overline{\Psi}_L \gamma^\mu D_\mu \Psi_L + \sum_{\text{sing}} i\overline{\psi}_R \gamma^\mu D_\mu \psi_R,$$
$$= \sum_f i\overline{\psi}_f \gamma^\mu \partial_\mu \psi_f$$
$$- e\sum_f Q^f \overline{\psi}_f \gamma^\mu \psi_f A_\mu - \frac{g}{\cos\theta_W} \sum_f \overline{\psi}_f \gamma^\mu \left(g_V^f - g_A^f \gamma_5\right) \psi_f Z_\mu$$
$$- \frac{g}{\sqrt{2}} \sum_{\text{doub}} \overline{\psi_u} \gamma^\mu \frac{1-\gamma_5}{2} \psi_d W_\mu^+ - \frac{g}{\sqrt{2}} \sum_{\text{doub}} \overline{\psi_d} \gamma^\mu \frac{1-\gamma_5}{2} \psi_u W_\mu^-.$$

$$\mathcal{L}_{\text{Yuk}} = -\sum_i \left[m_i^l\, \overline{l_i}\, l_i + m_i^u\, \overline{u_i}\, u_i + m_i^d\, \overline{d_i}\, d_i \right] + \cdots$$

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \overline{u_i} \gamma^\mu (1 - \gamma^5) \boldsymbol{V}^{\text{CKM}}_{ij} d_j W_\mu^+ + \text{h.c.}$$

$$\boldsymbol{V}^{\text{CKM}} = \boldsymbol{U^u_L} \boldsymbol{U^d_L}^\dagger$$

CC quark
Weak interaction

The CKM matrix contains 4 free parameters,
3 angles and 1 phase which leads to CP violation

Like the value of the masses, the values of the angles in the **CKM** matrix have no explanation in the standard model and are fitted to experiment. **Flavor problem**

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$$U^{f\dagger} U^{f} = I.$$

Since the neutral current only connects fermions with the same electroweak charges

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$$\mathcal{L} = -e \sum_f Q^f \overline{\psi}_f \gamma^\mu \psi_f A_\mu - \frac{g}{\cos\theta_W} \sum_f \overline{\psi}_f \gamma^\mu \left(g_V^f - g_A^f \gamma_5 \right) \psi_f Z_\mu$$

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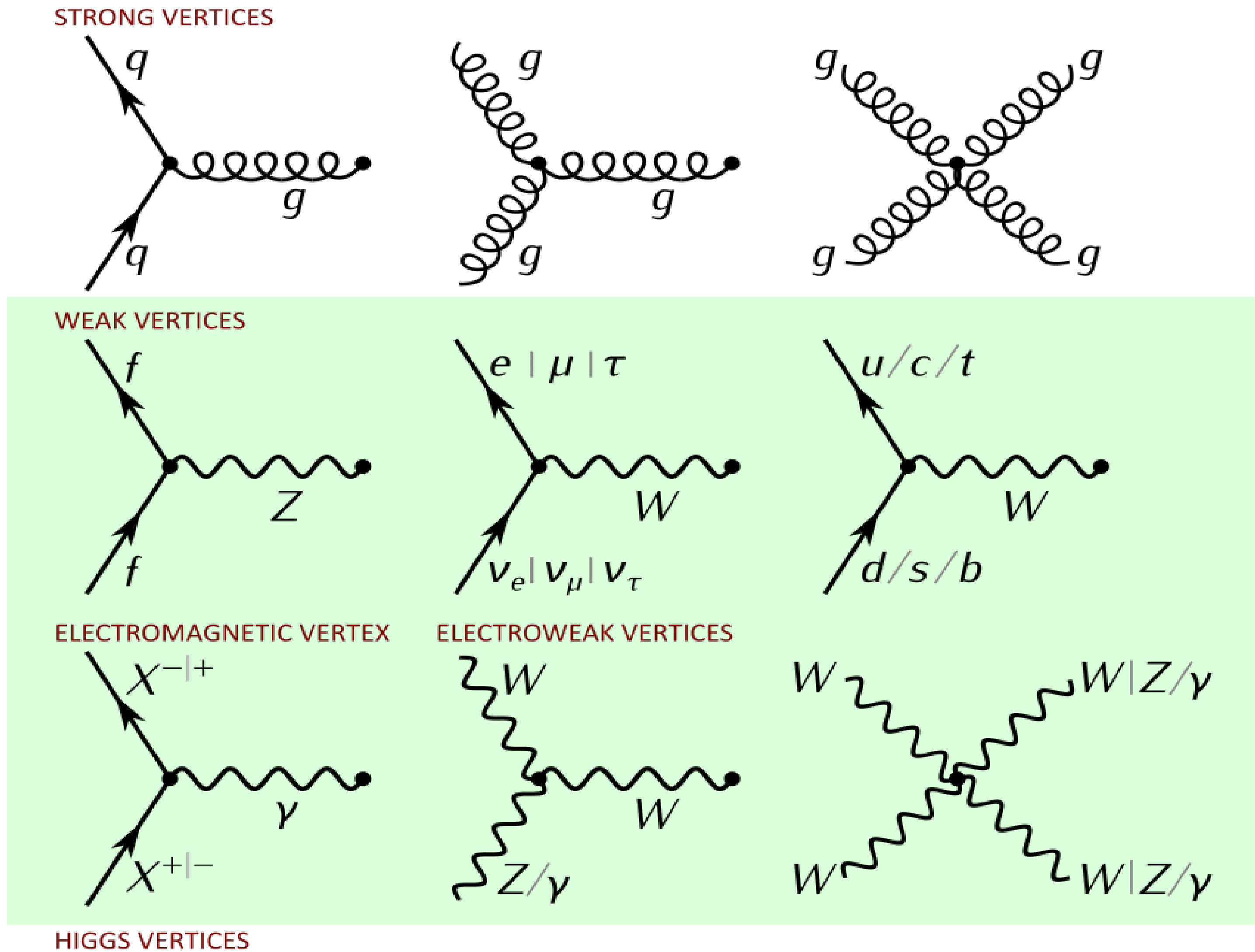
neutrinos remain massless since the standard model does not contain right-handed neutrinos nor any other ingredient capable of inducing neutrino masses.

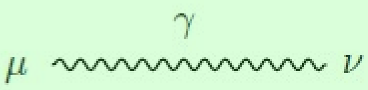
the charged current for **leptons** remains **trivial** because of the freedom to redefine the neutrino states by the same matrix as the charge leptons

some SM Feynman diagrams



In detail we have the propagator Feynman rules

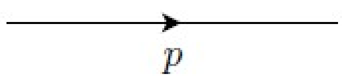
R_ξ gauge



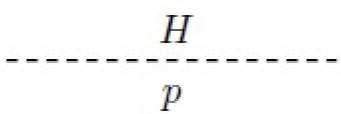
$$-i\left[\frac{g_{\mu\nu}}{k^2+i\epsilon}-(1-\xi_A)\frac{k_\mu k_\nu}{(k^2)^2}\right] \tag{C.24}$$

$$-i\frac{1}{k^2-M_W^2+i\epsilon}\left[g_{\mu\nu}-\frac{(1-\xi_W)k_\mu k_\nu}{k^2-\xi_W M_W^2}\right] \tag{C.25}$$

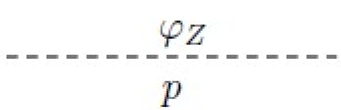
$$-i\frac{1}{k^2-M_Z^2+i\epsilon}\left[g_{\mu\nu}-\frac{(1-\xi_Z)k_\mu k_\nu}{k^2-\xi_Z M_Z^2}\right] \tag{C.26}$$



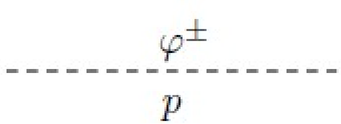
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2+i\epsilon} \tag{C.27}$$



$$\frac{i}{p^2-M_h^2+i\epsilon} \tag{C.28}$$



$$\frac{i}{p^2-\xi_Z M_Z^2+i\epsilon} \tag{C.29}$$

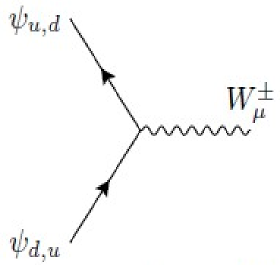


$$\frac{i}{p^2-\xi_W M_W^2+i\epsilon} \tag{C.30}$$

Charged current interaction

SM Vertices Feynman rules

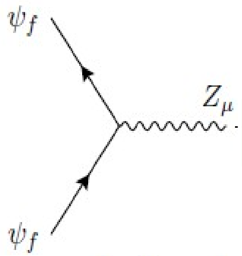
Complete SM Feynman rules
In appendix C of our book



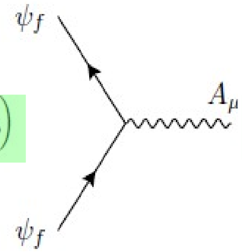
$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \qquad \text{(C.37)}$$

where we are neglecting $\boldsymbol{V}^{\mathrm{CKM}}$ that can be easily introduced using Eq. (2.34).

Neutral current interaction



$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$



$$ieQ_f\gamma_\mu \qquad \text{(C.38)}$$

where g_V^f, g_A^f are defined in Eq. (2.18).

SM Vertices Feynman rules

Complete SM Feynman rules
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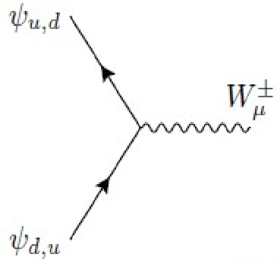

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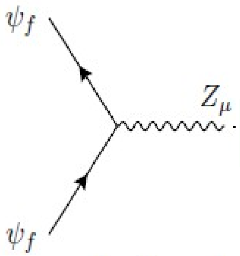
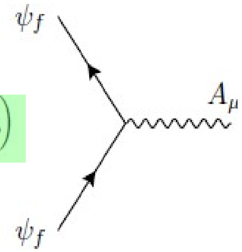


$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$



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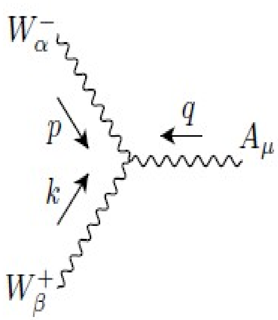
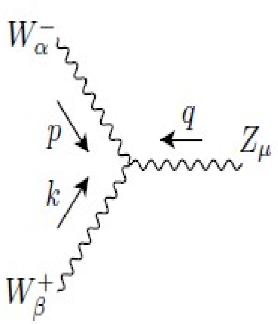
Triple gauge interactions



$$-ie\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right] \qquad \text{(C.31)}$$



$$-ig\cos\theta_W\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right] \qquad \text{(C.32)}$$

SM Vertices Feynman rules

Complete SM Feynman rules In appendix C of our book

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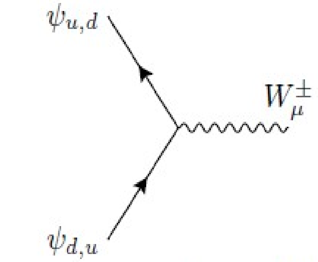

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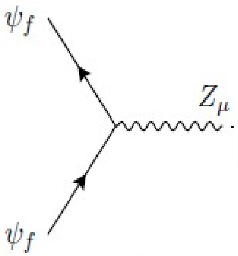

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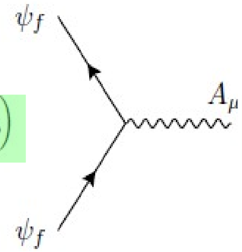

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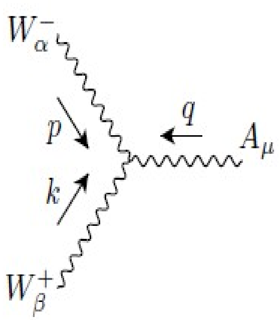
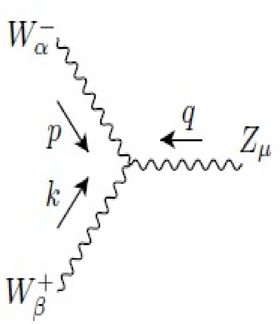
Triple gauge interactions

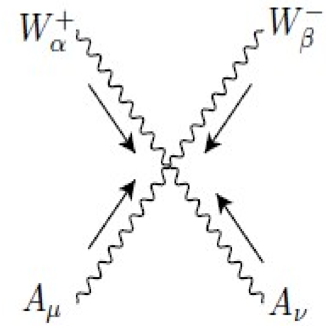


$$-ie\left[g_{\alpha\beta}(p-k)_\mu+g_{\beta\mu}(k-q)_\alpha +g_{\mu\alpha}(q-p)_\beta\right] \qquad \text{(C.31)}$$

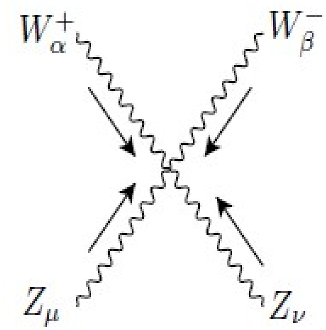


$$-ig\cos\theta_W\left[g_{\alpha\beta}(p-k)_\mu+g_{\beta\mu}(k-q)_\alpha +g_{\mu\alpha}(q-p)_\beta\right] \qquad \text{(C.32)}$$

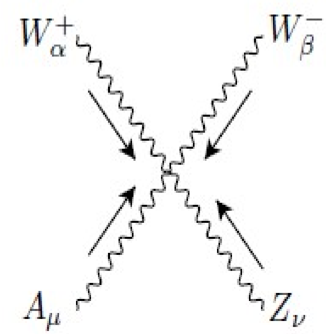
Quartic gauge interactions



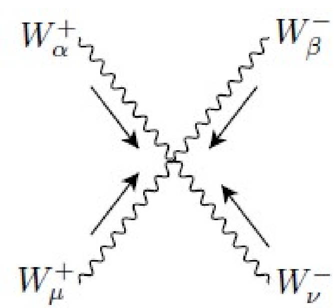
$$-ie^2\left[2g_{\alpha\beta}g_{\mu\mu}-g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\right] \qquad \text{(C.33)}$$



$$-ig^2\cos^2\theta_W\left[2g_{\alpha\beta}g_{\mu\nu}-g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\right] \qquad \text{(C.34)}$$



$$-ieg\cos\theta_W\left[2g_{\alpha\beta}g_{\mu\nu}-g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu}\right] \qquad \text{(C.35)}$$



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Higgs boson and unitarity in the standard model

The most important indication for physics beyond the standard model at the 1 TeV scale is the need to unitarize the weak interaction cross sections

For example the scattering of longitudinal gauge bosons only respects the unitarity limit if there is a Higgs boson, either elementary or composite but having the same effective coupling as in the standard model.

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One can show that cancellations occur for the Higgs boson of the **SM**. The conventions for the **SM** vertices are very important and are given in Appendix C of our book

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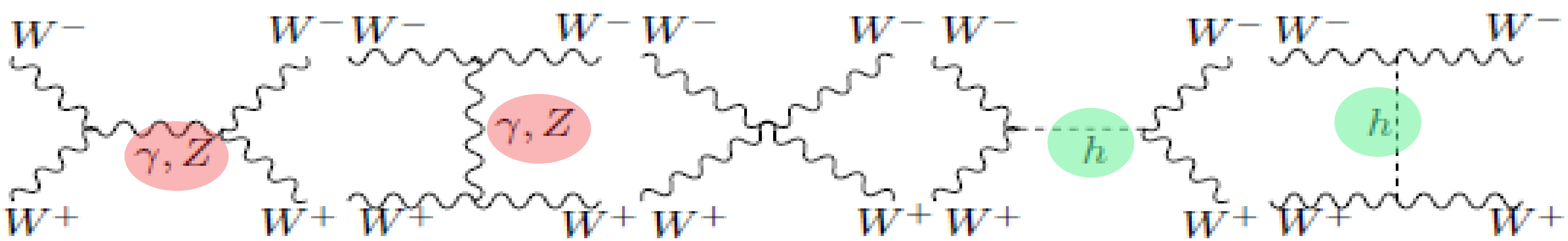
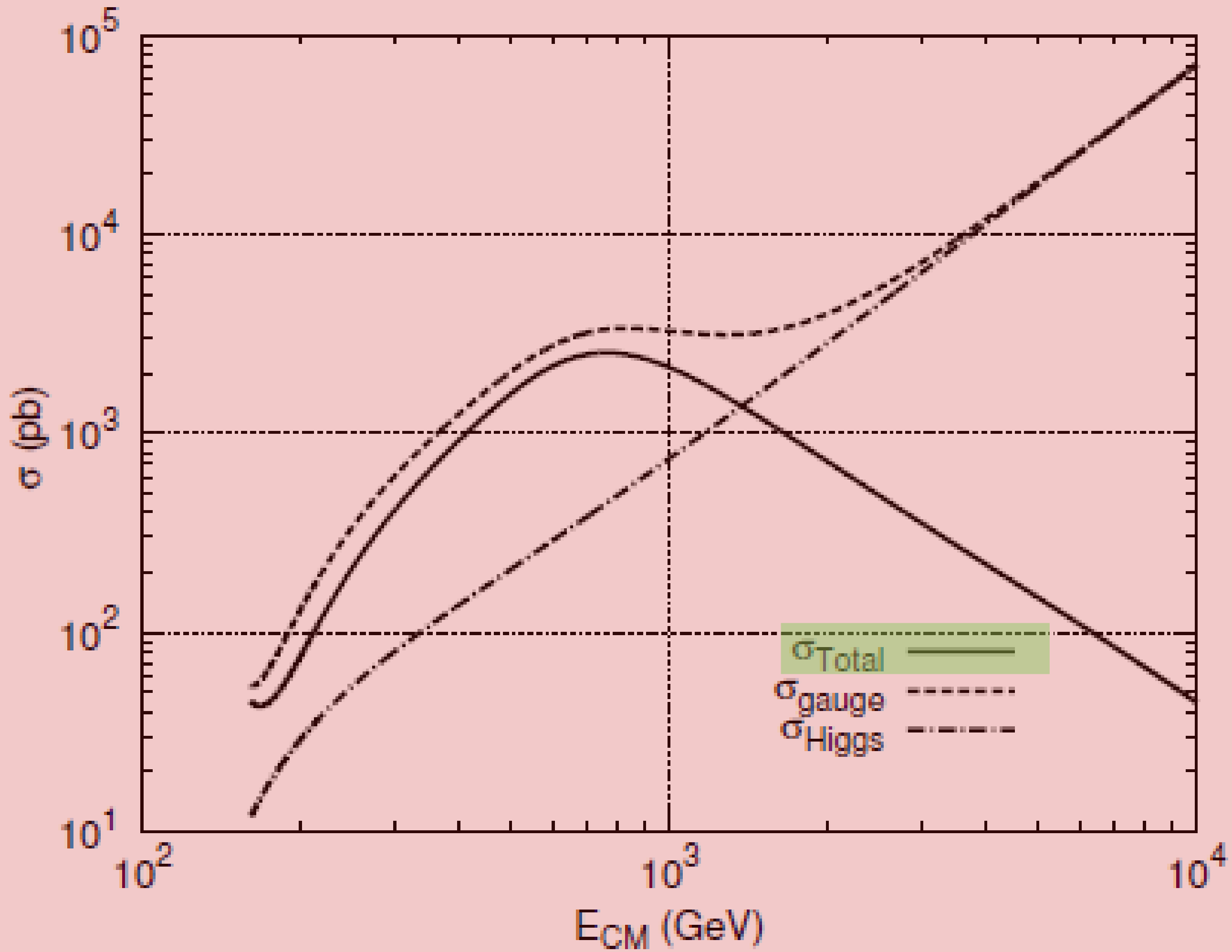
The process we consider is the scattering of longitudinal W

$$W_L^-(p_1) + W_L^+(p_2) \to W_L^-(q_1) + W_L^+(q_2)$$





Gauge terms quadratic in $s/(4M_W^2)$, cancel, but the linear term remains after we sum over the gauge part.

So, a theory of just intermediate vector bosons, is in trouble. This can be traced back to the fact that with mass the gauge invariance is lost, and the theory is not consistent without the Higgs boson diagrams

Hence the Higgs boson is crucial to make the **SM** model consistent.

Figure 2.4 Cross section for $W_L^- + W_L^+ \to W_L^- + W_L^+$. Shown are the contribution of the gauge diagrams (dashed), the contribution from the Higgs (dot-dashed) and the total cross section (solid line). The sum of the amplitudes from the gauge part have the opposite sign from those from the Higgs (not visible in the figure because we are plotting cross sections) forcing the cross section to decrease.

neutrino masses and mixing

Two types of massive fermions: **Dirac** or **Majorana**.
A Majorana fermion, is one that is its own antiparticle Majorana, 1937.

Except possibly for neutrinos **no** known elementary fermions are known to be their own antiparticle.

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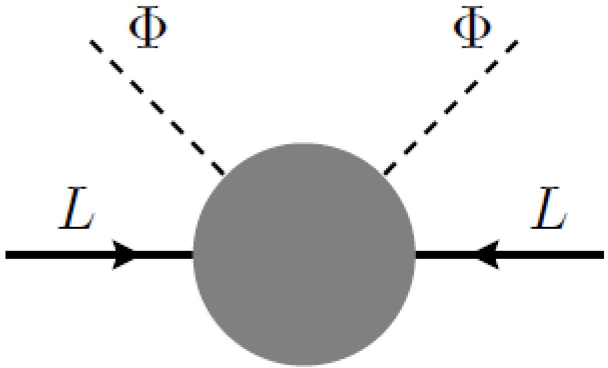
Massive **charged fermions** like the electron, the muon, the tau or the quarks must all be of Dirac-type.

However, **electrically neutral** fermions, like neutrinos, are generally **expected** to be Majorana-type, **irrespective** of how they acquire their mass. Schechter & JV PRD22 (1980) 2227

Note that the argument in favor of Majorana neutrinos goes beyond any particular neutrino mass generation mechanism, e.g. the **seesaw**, to be discussed later.

Nevertheless Majorana neutrinos fit well within the simplest effective source of neutrino mass, i.e. Weinberg's dimension five operator. Weinberg Phys.Rev.D 22 (1980) 1694.

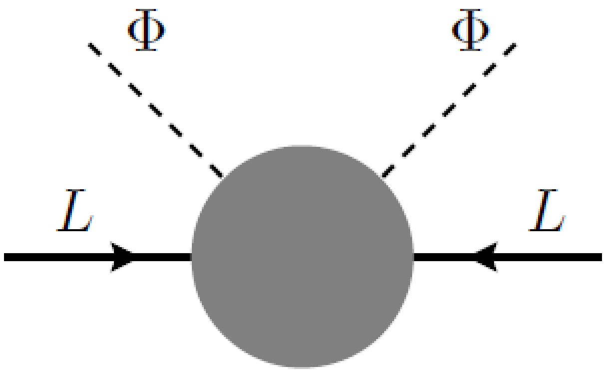


Lepton-number-violating dimension five operator responsible for generating neutrino mass after the electroweak symmetry breaking takes place.

Nothing is known regarding the mechanism that induces Weinberg's operator, its characteristic scale or flavor structure

we first describe Majorana masses at the **kinematical** level, before adding interactions

Schechter & JV PRD22 (1980) 2227



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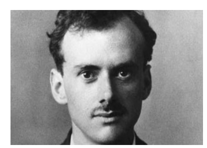
In order to derive in a simple way the 2-component description of Majorana fermion we start from usual theory of a massive spin-1/2 Dirac fermion, given by the Lagrangian

$$\mathcal{L}_{\mathrm{D}} = i\,\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - m\,\bar{\Psi}\Psi$$



$$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\,g^{\mu\nu}\,;$$

$$\gamma^{0} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \gamma^{i} = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix}, \quad \gamma_5 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} i\,\sigma_2 & 0 \\ 0 & i\,\sigma_2 \end{bmatrix}$$

$$C^{T} = -C\,, \quad C^{\dagger} = C^{-1}\,, \quad C^{-1}\,\gamma_{\mu}\,C = -\,\gamma_{\mu}^{T}$$

A Dirac spinor and its **conjugate** can then be written in terms of **two-component spinors**

$$\Psi_{\rm D} = \begin{bmatrix} \chi \\ i\,\sigma_2\,\phi^* \end{bmatrix}$$

$$\Psi_{\rm D}^c = \begin{bmatrix} \phi \\ i\,\sigma_2\,\chi^* \end{bmatrix}$$

$$\mathcal{L}_{\rm D} = i\,\phi\sigma^\mu\partial_\mu\overline{\phi} + i\,\overline{\chi}\,\overline{\sigma}^\mu\partial_\mu\chi - m\left(\phi\chi + \overline{\chi}\,\overline{\phi}\right)$$

S=S(Λ)

$$\sigma^\mu \equiv (1, \vec{\sigma}),\ \overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$$

$$S^\dagger\overline{\sigma}^\mu S = \Lambda^\mu{}_\nu\overline{\sigma}^\nu\ , \quad S\sigma^\mu S^\dagger = \Lambda^\mu{}_\nu\sigma^\nu$$

$$= i\sum_{a=1}^{2}\overline{\rho}_a\overline{\sigma}^\mu\partial_\mu\rho_a - \frac{1}{2}m\sum_{i=a}^{2}\left(\rho_a\rho_a + \overline{\rho}_a\overline{\rho}_a\right)$$

$$\chi = \frac{1}{\sqrt{2}}(\rho_1 + i\rho_2),$$

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$$\chi = \frac{1}{\sqrt{2}}(\rho_1 + i\rho_2),$$

$$\phi = \frac{1}{\sqrt{2}}(\rho_1 - i\rho_2)$$

this way the Dirac fermion is shown to be equivalent to two Majorana fermions of equal mass

$$\Psi_{\mathrm{D}} \to e^{i\alpha}\Psi_{\mathrm{D}}$$

The U(1) symmetry of the theory described by amounts to the continuous rotation symmetry

$$\rho_1 \to \quad \cos\theta\rho_1 + \sin\theta\rho_2$$

$$\rho_2 \to -\sin\theta\rho_1 + \cos\theta\rho_2$$

In any theory of neutrino masses the free field Lagrangian is given as

$$\mathcal{L}_{\rm M} = i \sum_{a=1}^{n} \overline{\rho}_a \overline{\sigma}_\mu \partial^\mu \rho_a - \frac{1}{2} \sum_{a,b=1}^{n} \left(\mathcal{M}_{\nu\, ab}\, \rho_a \rho_b + \text{h.c.} \right)$$

By Fermi statistics the mass coefficients we must have a **symmetric** matrix, in general **complex**.

One can show that this matrix can always be diagonalized by a complex unitary matrix as Schechter & JV PRD22 (1980) 2227

$$\mathcal{U}_\nu{}^T \mathcal{M}_\nu \, \mathcal{U}_\nu = \text{diag}(m_1, m_2, ..., m_n)$$

In general lepton number not fundamental

Quantization of Majorana and Dirac fermions

The solutions of the Majorana field equation can easily be obtained in terms of those of the Dirac equation, which are well known

Schechter & JV PRD22 (1980) 2227

$$\Psi_{\rm M} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2E} \sum_{r=1}^{2} \left[e^{-ik\cdot x} A_r(k) u_L(k,r) + e^{ik\cdot x} A_r^\dagger(k) v_L(k,r) \right]$$

where $u = C\bar{v}^T$ and $E(k) = (\vec{k}^2 + m^2)^{1/2}$ is the mass-shell condition

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Schechter & JV PRD22 (1980) 2227

$$\Psi_{\rm M} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2E} \sum_{r=1}^{2} \left[e^{-ik\cdot x} A_r(k) u_L(k,r) + e^{ik\cdot x} A_r^\dagger(k) v_L(k,r) \right]$$

where $u = C\bar{v}^T$ and $E(k) = (\vec{k}^2 + m^2)^{1/2}$ is the mass-shell condition

i. the creation and annihilation operators obey canonical **anti-commutation** rules

ii. the spinor **uL, vL** Dirac wave-functions are 2-component,
as there is a **chiral** projection in front

only one Fock space, instead of two characterizing the Dirac theory, corresponding to particle and anti-particle

u's and v's are the **same** wave functions in the Fourier decomposition the Dirac field

$$\Psi_{\rm D} = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{2E} \sum_{r=1}^{2} \left[e^{-ik\cdot x} a_r(k) u_r(k) + e^{ik\cdot x} b_r^\dagger(k) v_r(k) \right]$$

Two independent propagators

$$\langle 0|\, T(\rho_\alpha(x)\bar{\rho}_{\dot\beta}(y))\,|0\rangle = i(\sigma^\mu)_{\alpha\dot\beta}\,\partial_\mu \Delta_F(x-y;m),$$

characterizes fermion number **conserving** processes

$$\langle 0|\, T(\rho_\alpha(x)\rho_\beta(y))|0\rangle = -m\,\epsilon_{\alpha\beta}\Delta_F(x-y;m) = m(i\sigma_2)_{\alpha\beta}\,\Delta_F(x-y;m),$$

characterizes fermion number **violating** processes

Taking into account the free Lagrangian described above and the gauge interactions of the SM, one can derive all Feynman rules for processes involving Majorana (as well as Dirac) fermions from first principles

Using the helicity eigenstate wave-functions

$$\vec\sigma\cdot\vec k\, u_L^{\pm}(k) = \pm\,|\,\vec k\,|\,u_L^{\pm}(k)$$

$$\vec\sigma\cdot\vec k\, v_L^{\pm}(k) = \mp\,|\,\vec k\,|\,v_L^{\pm}(k)$$

one can show that, out of the 4 linearly independent wave functions $u_L^{\pm}(k)$ and $v_L^{\pm}(k)$, only two survive as the mass approaches zero, namely, $u_L^{-}(k)$ and $v_L^{+}(k)$ This way we recover the Lee-Yang two-component massless neutrino theory, namely as the massless limit of the Majorana theory.

Schechter, JV PRD24(1982)1883

lecture 2

Schechter & JV PRD22 (1980) 2227
W. Rodejohann, JV PRD 84 (2011) 073011

The lepton mixing matrix

We now turn to the structure of the charged and neutral current weak interactions **CC NC** associated to massive neutrinos

we diagonalize all mass matrices resulting from spontaneous gauge symmetry **SSB** breaking and then rewrite the gauge interactions in the mass eigenstate basis, where physical particles are clearly identified, as we did in the **CKM** matrix.

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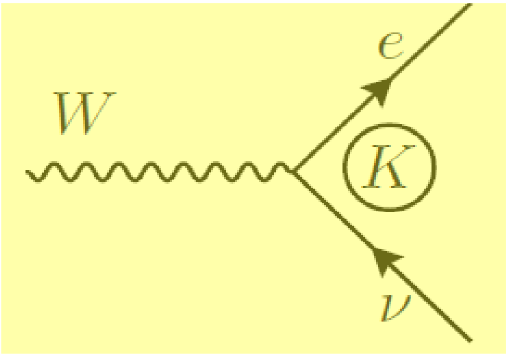
Dirac neutrinos

$$V^{\rm LEP} = {R_L^e}^\dagger R_L^\nu$$



like quarks, the CC weak interactions of massive neutrinos are described by a mixing matrix that also follows from the mismatch between the Yukawas of charged leptons and neutrinos

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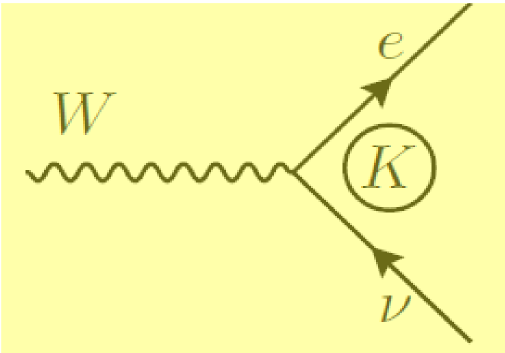

like quarks, the CC weak interactions of massive neutrinos are described by a mixing matrix that also follows from the mismatch between the Yukawas of charged leptons and neutrinos

$$V^{\mathrm{LEP}} = R_L^{e\,\dagger} R_L^{\nu}$$

$$V^{\mathrm{LEP}} = \omega_0(\gamma) \prod_{i<j}^{n} \omega_{ij}(\eta_{ij})$$

$$\omega_0(\gamma) = \exp i\left(\sum_{a=1}^{n} \gamma_a \,, A_a^a\right)$$

$$\omega_{ab}(\eta_{ab}) = \exp \sum_{a=1}^{n} (\eta_{ab} A_a^b - \eta_{ab}^* A_b^a)$$

$$\omega_{12}(\eta_{12}) = \begin{bmatrix} c_{12} & e^{i\theta_{12}} s_{12} & 0\ldots \\ -e^{-i\theta_{12}} s_{12} & c_{12} & 0\ldots \\ 0 & 0 & 1\ldots \\ \ldots & \ldots. & \ldots\ldots \end{bmatrix}$$

$$\eta_{ab} = |\eta_{ab}| \exp i\theta_{ab}.$$

once the charged leptons and Dirac neutrino mass matrices are diagonal, one can still rephase the corresponding fields, keeping the free Lagrangian invariant by $\omega_0(\alpha)$ and $\omega_0(\gamma - \alpha)$

$$V^{\rm LEP} = \omega_0(\alpha) \prod_{i<j}^{n} \omega_{ij}(\eta_{ij})\, \omega_0^\dagger(\alpha).$$

$$\omega_0(\alpha)\omega_{ab}(|\eta_{ab}|\exp i\theta_{ab})\,\omega_0^\dagger(\alpha) = \omega_{ab}[|\eta_{ab}|\exp i(\alpha_a + \theta_{ab} - \alpha_b)]$$

3 $n(n-1)/2$ mixing angles θ_{ij} and

1 $n(n-1)/2 - (n-1)$ independent CP phases.

Massive Dirac neutrinos mixing matrix has the same form as CKM matrix

$$\delta \equiv \phi_{12} + \phi_{23} - \phi_{13}$$

Affects neutrino oscillations

Majorana neutrinos: unitary approximation

The imposition of lepton number conservation in a gauge theory would be *ad hoc*
So neutrinos are generally expected to be Majorana

$$K = \omega_{23}(\theta_{23}, \phi_{23})\omega_{13}(\theta_{13}, \phi_{13})\omega_{12}(\theta_{12}, \phi_{12}),$$

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Our parametrization of the lepton mixing matrix is fully "**symmetrical**"
But there is a basic difference between Dirac and Majorana phases.

The rephasing invariant combination is the **Dirac** phase

$$\delta \equiv \phi_{12} + \phi_{23} - \phi_{13}$$

lepton-number conserving (**LNC**) processes

The other two phases are physical
J. Schechter, JV PRD 23 (1981) 1666
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these **Majorana phases** show up only in lepton-number violating (**LNV**) processes, e.g.

- **neutrinoless double beta decay**
- neutrino electromagnetic properties
- neutrino to anti-neutrino oscillation (thought-experiment)

but do **not** enter **LNC** processes like standard neutrino oscillations

Most general symmetrical form of the lepton mixing matrix

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the most general seesaw is described by **(n,m),**

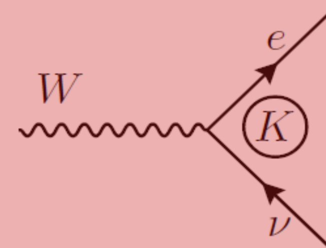
n=3 being the number of **321** isodoublets and **m** the number of extra leptons

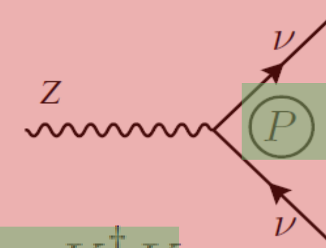
$$K^{(1)} = \prod_{a=2}^{n+m} \omega(\eta_{1a}) e^{(1)}$$

$$K^{(2)} = \prod_{a=2}^{n+m} \omega(\eta_{1a}) \prod_{b=3}^{n+m} \omega(\eta_{2b}) e^{(2)}$$

$$K^{(3)} = \prod_{a=2}^{n+m} \omega(\eta_{1a}) \prod_{b=3}^{n+m} \omega(\eta_{2b}) \prod_{c=4}^{n+m} \omega(\eta_{3c}) e^{(3)}$$

$$e_b^{(a)} = \delta_{ab}$$





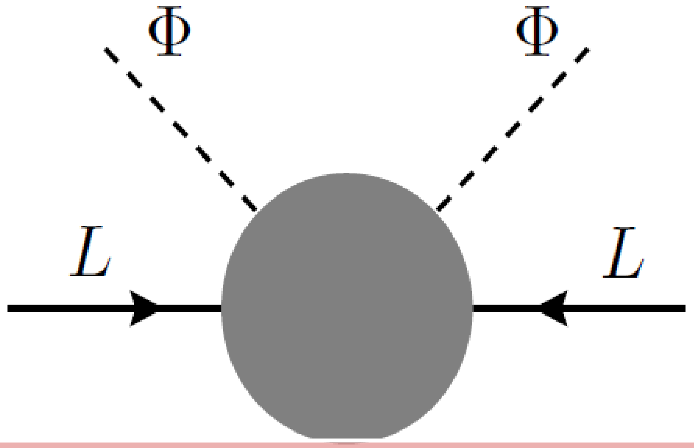
$$P = K^\dagger K.$$

same number of angles and phases

$$n(n+2m-1)/2$$

Indeed **lepton mixing is in general more complex in structure than quark mixing**

Origin of neutrino mass



new physics

$$\mathcal{O}_5 \propto LL\Phi\Phi \qquad m_\nu = \lambda_0 \frac{\langle\Phi\rangle^2}{M_X}$$

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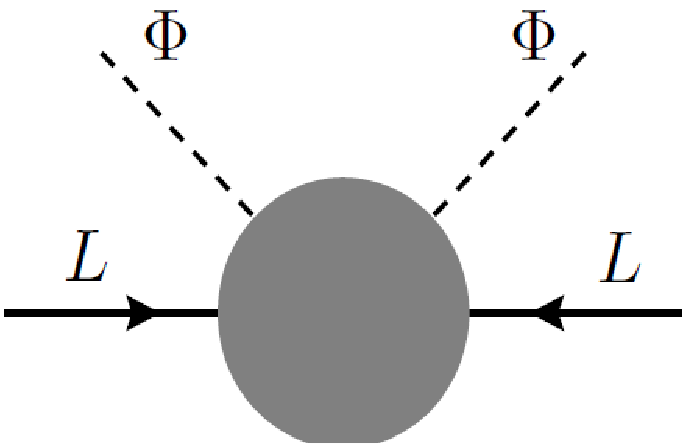


	$SU(3) \otimes SU(2) \otimes U(1)$
$L_a = (\nu_a, l_a)^T$	$(1, 2, -1/2)$
e_a^c	$(1, 1, 1)$
$Q_a = (u_a, d_a)^T$	$(3, 2, 1/6)$
u_a^c	$(\bar{3}, 1, -2/3)$
d_a^c	$(\bar{3}, 1, 1/3)$
Φ	$(1, 2, 1/2)$

+ new neutrals

new physics

$$\mathcal{O}_5 \propto LL\Phi\Phi \qquad m_\nu = \lambda_0 \frac{\langle\Phi\rangle^2}{M_X}$$

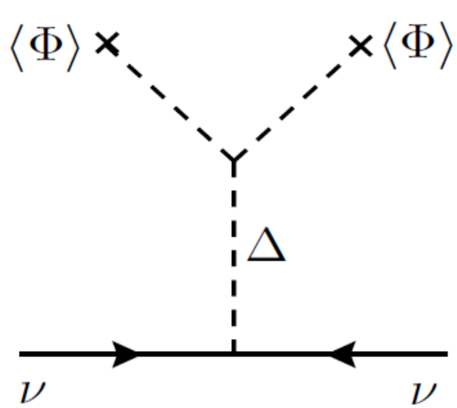
The seesaw mechanism postulate that additional **neutral heavy states** act as "**messenger**" particles to induce neutrino masses

explicit lepton number violation

spontaneous lepton number violation

Global symmetry
Gauge symmetry

Simplest or triplet seesaw
(now called type II)



$$\Delta = \begin{bmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^-}{\sqrt{2}} \end{bmatrix}$$

$$v_3 \equiv \langle \Delta^0 \rangle \qquad v_3 \ll v_2$$

$$v_2 \equiv \langle \Phi \rangle$$

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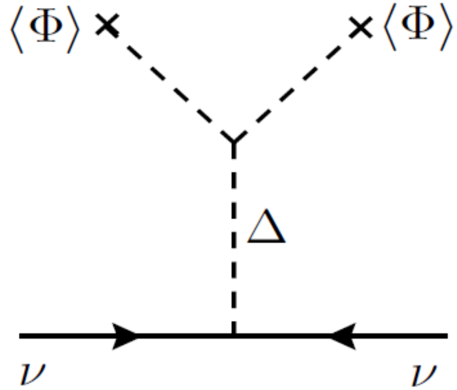

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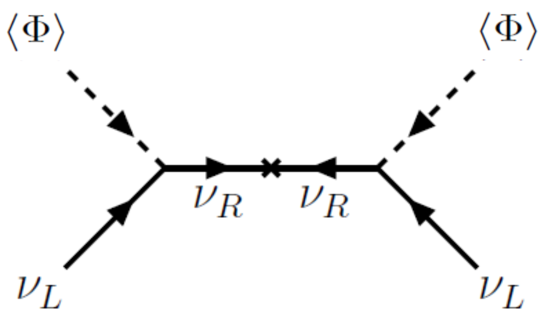

Type I seesaw

$$\mathcal{M}_\nu = \begin{bmatrix} M_1 & M_D \\ M_D^T & M_2 \end{bmatrix}$$

$$M_1 \ll M_D \ll M_2.$$

$$\mathcal{U}_\nu{}^T \mathcal{M}_\nu \; \mathcal{U}_\nu = \text{real, diagonal}$$

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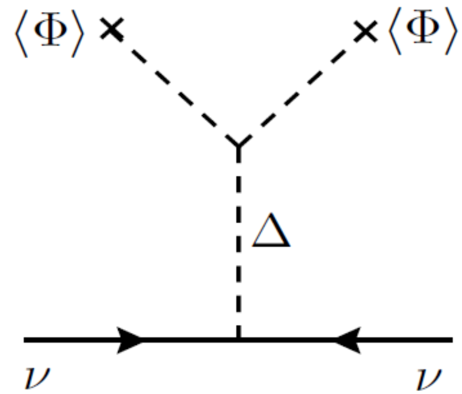

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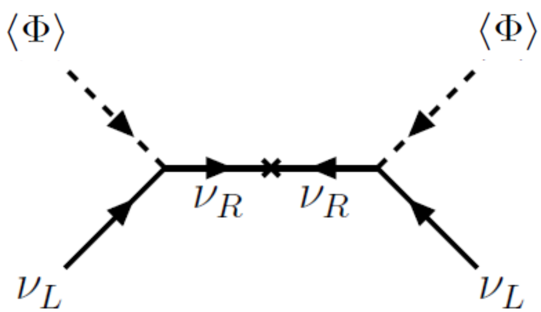

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$$\mathcal{U}_\nu = \exp(iH) \cdot V \,, \qquad H = \begin{bmatrix} 0 & S \\ S^\dagger & 0 \end{bmatrix}, \qquad V = \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix}$$

Seesaw expansion

$$\mathcal{U}_\nu = \begin{bmatrix} \left(I - \frac{1}{2} M_D^* (M_2^*)^{-1} M_2^{-1} M_D^T\right) V_1 & M_D^* (M_2^*)^{-1} V_2 \\ -M_2^{-1} M_D^T V_1 & \left(I - \frac{1}{2} M_2^{-1} M_D^T M_D^* (M_2^*)^{-1}\right) V_2 \end{bmatrix}$$

spontaneous L violation in SM seesaw

$$\mathcal{M}_\nu = \begin{bmatrix} Y_3 v_3 & Y_\nu v_2 \\ {Y_\nu}^T v_2 & Y_1 v_1 \end{bmatrix}$$

J. Schechter, JV, PRD25 (1982) 774

$$Y_1 \, \sigma \, {\nu_L^c}^T (i\sigma_2) \nu_L^c + \text{h.c.}$$

$$v_3 v_1 \sim {v_2}^2$$

$$v_1 \gg v_2 \gg v_3$$

$$m_\nu^{(I+II)} = M_1 - M_D M_2^{-1} M_D^T \simeq Y_3 v_3 - Y_\nu {Y_1}^{-1} {Y_\nu}^T \frac{\langle \Phi \rangle^2}{v_1}$$

$$V = m_\Phi^2 \Phi^\dagger \Phi + m_\sigma^2 \sigma^\dagger \sigma + \lambda_\Phi (\Phi^\dagger \Phi)^2 + \lambda_\sigma (\sigma^\dagger \sigma)^2 + \lambda_{\Phi\sigma} (\Phi^\dagger \Phi)(\sigma^\dagger \sigma)$$

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invariance of V under a global lepton-number transformation leads to

physical Nambu-Goldstone boson associated with its spontaneous breakdown

$$M_i^2 = \begin{bmatrix} 1 & \frac{v_2}{2v_1} & -\frac{v_2}{2v_3} \\ \frac{v_2}{2v_1} & \frac{v_2^2}{4v_1^2} & -\frac{v_2^2}{4v_1 v_3} \\ -\frac{v_2}{2v_3} & -\frac{v_2^2}{4v_1 v_3} & \frac{v_2^2}{4v_3^2} \end{bmatrix} \times \left\langle \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_i} \right\rangle$$

$$A = \Im\mathrm{m}\, \Phi^0 + \frac{v_2}{2v_1} \Im\mathrm{m}\, \sigma - \frac{v_2}{2v_3} \Im\mathrm{m}\, \Delta^0$$

$$G_Z \propto v_2 \, \Im\mathrm{m}\, \Phi^0 + 2v_3 \, \Im\mathrm{m}\, \Delta^0$$

Noether's theorem

$$J \propto v_3 {v_2}^2 \, \Im\mathrm{m}(\Delta^0) - 2 v_2 {v_3}^2 \, \Im\mathrm{m}(\Phi^0) + v_1 ({v_2}^2 + 4{v_3}^2) \, \Im\mathrm{m}(\sigma)$$

(Check the normalization factor)

perturbative expansion for the majoron (J) couplings to light neutrinos

$$\mathcal{L}_{\rm Yuk} = \frac{J}{2}\sum_{ij}\nu_i^T g_{ij}(i\sigma_2)\nu_j + \text{h.c.}$$

$$g_{ij} = -\frac{m_i}{v_1}\delta_{ij} + \left[\frac{m_i}{v_1}\left(V_1^\dagger D^* {M_2}^{*-1}{M_2}^{-1}D^T V_1\right)_{ij} + \text{transpose}\right] + \cdots$$

relevant for astrophysics

relevant for cosmology

relevant for $0\nu\beta\beta$ decay

Low-scale seesaw mechanism

Inverse seesaw

R.N. Mohapatra, JV: PRD34 (1986) 1642
M.C. Gonzalez-Garcia, JV: PLB 216 (1989) 360

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(3; 6), i.e. three **pairs** of SU(2) singlets are added to **SM** The **S** are charged under U(1)L global lepton number with the same charge as the doublet neutrinos, i.e. L = +1, but opposite to the others.

$$\mathcal{M}_\nu = \begin{bmatrix} 0 & Y_\nu^T \langle\Phi\rangle & 0 \\ Y_\nu \langle\Phi\rangle & 0 & M^T \\ 0 & M & \mu \end{bmatrix}$$

in the basis ν_L, ν_L^c, S_L
Y_ν is an arbitrary 3×3 complex Yukawa

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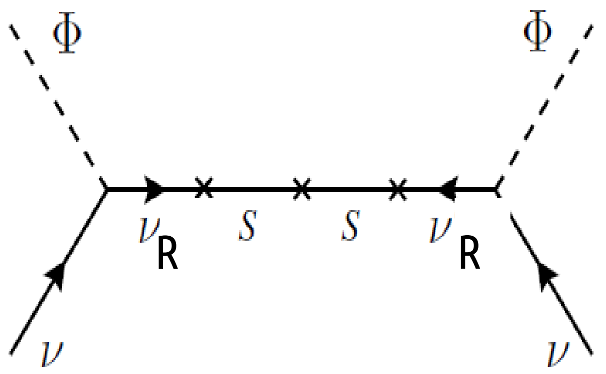
linear seesaw Evgeny K. Akhmedov et al: PRD53 (1996) 2752
PLB368 (1996) 270
Michal Malinsky, Romao, JV: PRL95 (2005) 161801

After electroweak symmetry breaking one gets the mass matrix

$$\mathcal{U}_\nu^T \cdot \mathcal{M}_\nu \cdot \mathcal{U}_\nu = \text{block diag}$$

$$\mathcal{U}_\nu \approx \begin{bmatrix} I & iS_2 & iS_1 \\ -i\frac{1}{\sqrt{2}}\left[S_1^\dagger - S_2^\dagger\right] & \frac{1}{\sqrt{2}}I & -\frac{1}{\sqrt{2}}I \\ i\frac{1}{\sqrt{2}}\left[S_1^\dagger + S_2^\dagger\right] & \frac{1}{\sqrt{2}}I & \frac{1}{\sqrt{2}}I \end{bmatrix} + \mathcal{O}\left(\epsilon^2\right)$$

$$iS^* = -\frac{1}{\sqrt{2}} m_D \left(M^T\right)^{-1} \sim \epsilon$$



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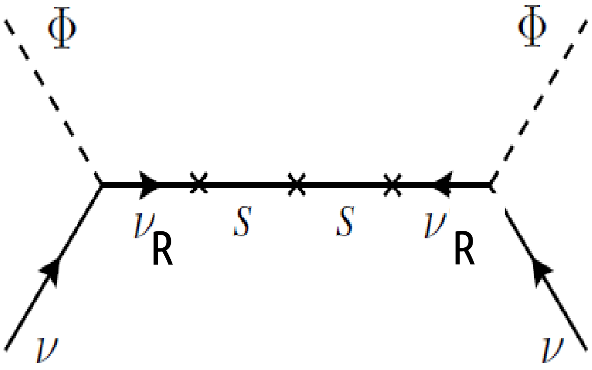


$$m_\nu{}^{\text{inverse}} = M_D^T {M^T}^{-1} \mu M^{-1} M_D^*$$

$$m_\nu \to 0 \quad \text{as} \quad \mu \to 0$$

t'Hooft

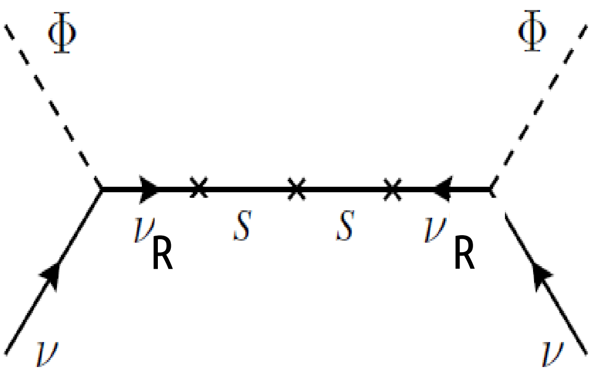
After electroweak symmetry breaking one gets the mass matrix

$$\mathcal{U}_\nu^T \cdot \mathcal{M}_\nu \cdot \mathcal{U}_\nu = \text{block diag}$$

$$\mathcal{U}_\nu \approx \begin{bmatrix} I & iS_2 & iS_1 \\ -i\frac{1}{\sqrt{2}}\left[S_1^\dagger - S_2^\dagger\right] & \frac{1}{\sqrt{2}}I & -\frac{1}{\sqrt{2}}I \\ i\frac{1}{\sqrt{2}}\left[S_1^\dagger + S_2^\dagger\right] & \frac{1}{\sqrt{2}}I & \frac{1}{\sqrt{2}}I \end{bmatrix} + \mathcal{O}\left(\epsilon^2\right)$$

$$iS^* = -\frac{1}{\sqrt{2}} m_D \left(M^T\right)^{-1} \sim \epsilon$$

$$m_\nu{}^{\text{inverse}} = M_D^T M^{T^{-1}} \mu M^{-1} M_D^*$$

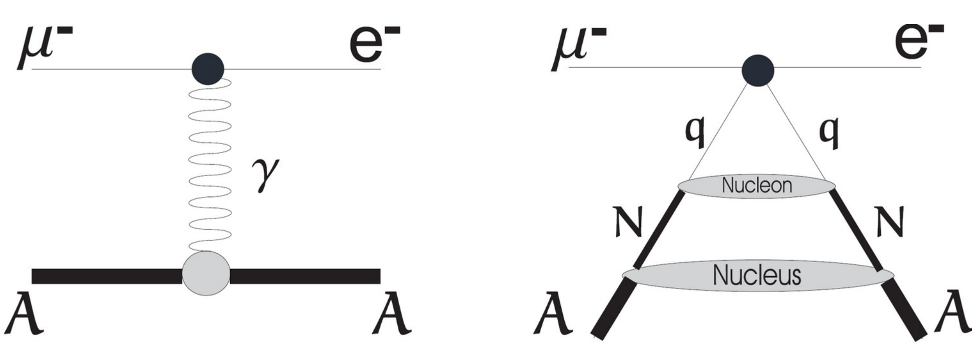


$$m_\nu \to 0 \quad \text{as} \quad \mu \to 0$$

t'Hooft

CLFV & Leptonic CPV that persist in the **massless limit**

J. Bernabeu et al, PLB187 (1987) 303
G. Branco, M. Rebelo, and JV, PLB 225 (1989) 385
N. Rius and JV, PLB 246 (1990) 249



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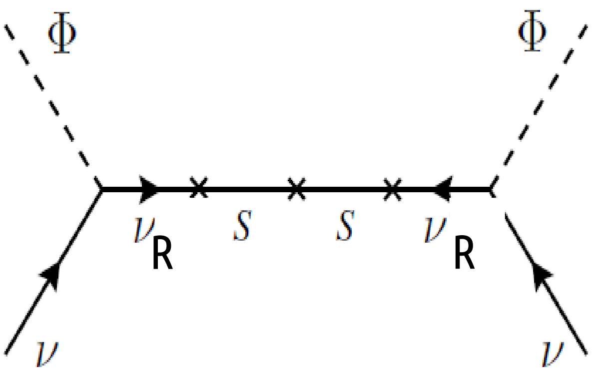

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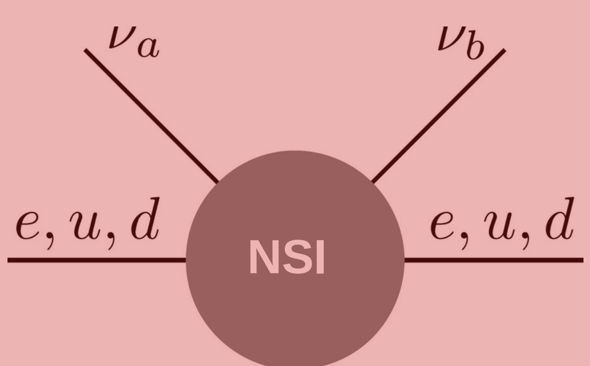
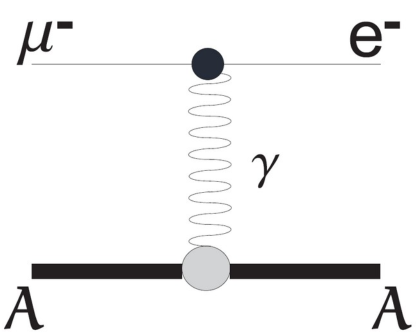
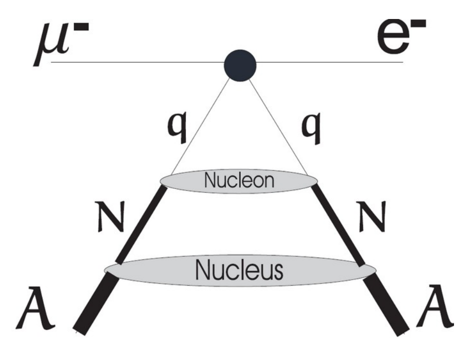
Low-scale seesaw phenomenology



Unitarity violation



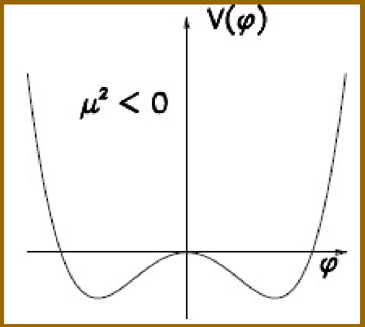


seesaw sum up

stability

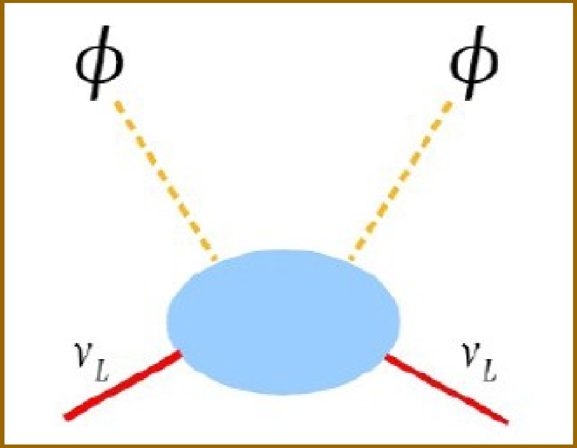
SEESAW dynamics

$$v_3 v_1 \sim {v_2}^2$$

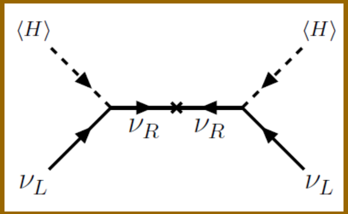


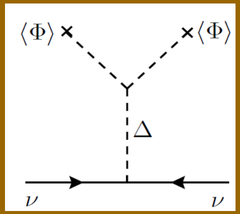
Mandal et al Phys.Rev.D 101 (2020) 115030

JHEP03(2021)212 & JHEP07(2021) 029



seesaw sum up





stability

TYPE I

Minkowski 77
Gellman Ramond Slansky 80
Glashow, Yanagida 79
Mohapatra Senjanovic 80
Lazarides Shafi Weterrich 81
Schechter-Valle 80 & 82

TYPE II

Schechter-Valle 80 & 82

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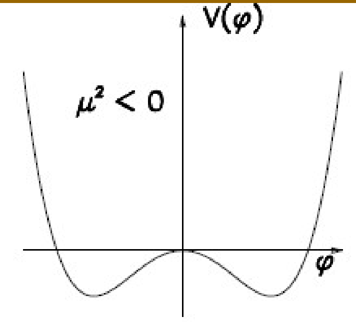
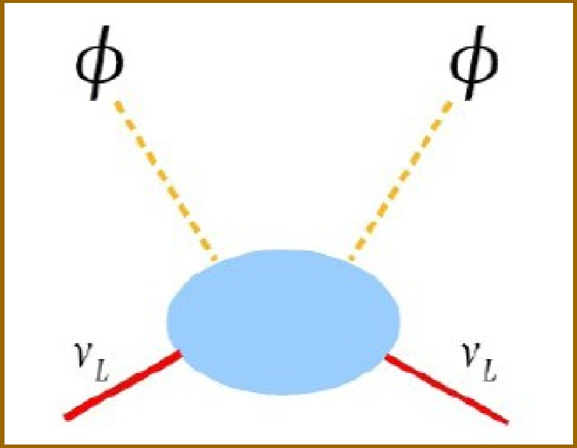
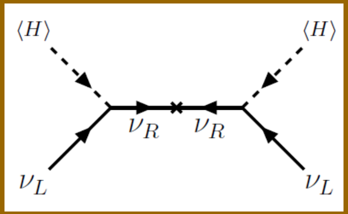


Mandal et al Phys.Rev.D 101 (2020) 115030

JHEP03(2021)212 & JHEP07(2021) 029



seesaw sum up



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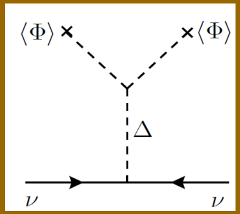


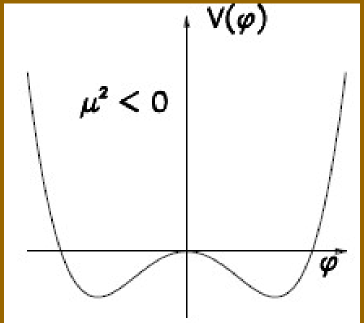
TYPE II

Schechter-Valle 80 & 82

stability

SEESAW dynamics

$$v_3 v_1 \sim {v_2}^2$$

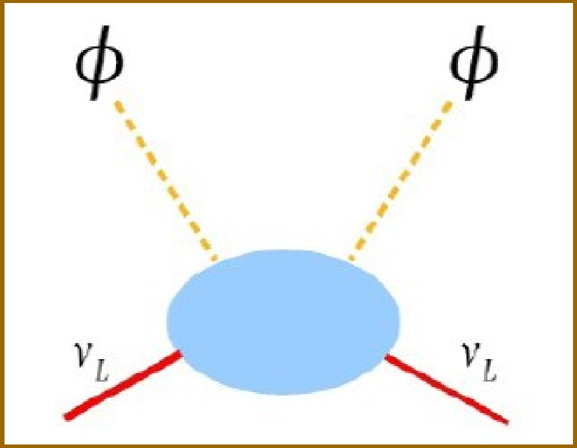


Mandal et al Phys.Rev.D 101 (2020) 115030

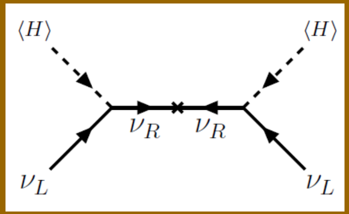
JHEP03(2021)212 & JHEP07(2021) 029

L-R seesaw # of Rs = # Ls

SM seesaw # of singlets arbitrary



seesaw sum up



TYPE I

Minkowski 77
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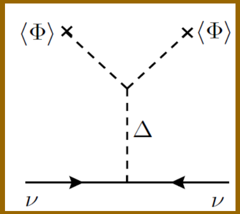
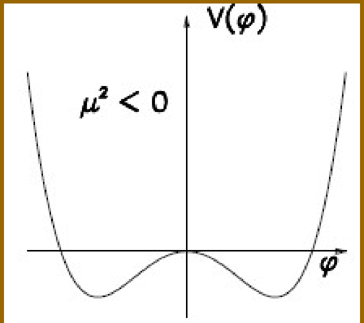


TYPE II

Schechter-Valle 80 & 82

stability



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Mandal et al Phys.Rev.D 101 (2020) 115030

JHEP03(2021)212 & JHEP07(2021) 029

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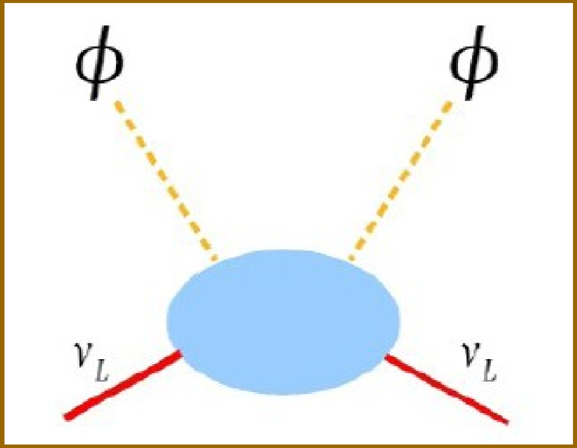


- **MISSING PARTNER**

(3,2) min viable type1 seesaw
(3,1) scoto-seesaw template

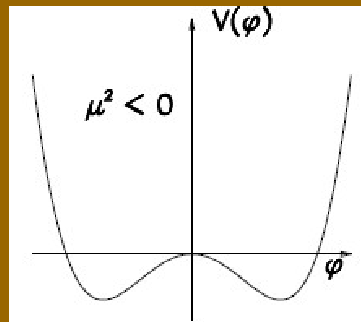
$m_{\beta\beta}$

seesaw sum up

SEESAW dynamics

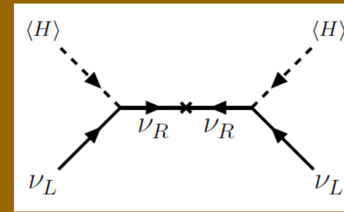
stability

$$v_3 v_1 \sim v_2{}^2$$



Mandal et al Phys.Rev.D 101 (2020) 115030

JHEP03(2021)212 & JHEP07(2021) 029



TYPE I

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Gellman Ramond Slansky 80
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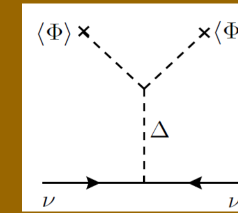
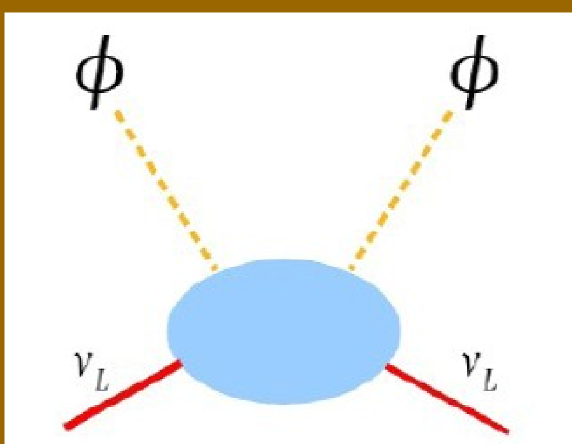


TYPE II

Schechter-Valle 80 & 82

L-R seesaw # of Rs = # Ls

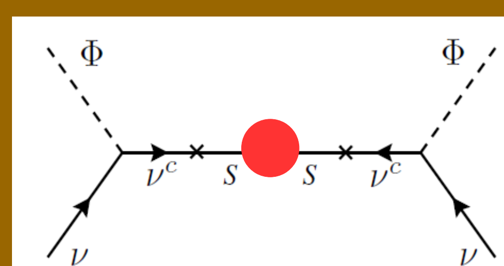
SM seesaw # of singlets arbitrary



- **MISSING PARTNER**
  (3,2) min viable type1 seesaw
  (3,1) scoto-seesaw template

$m_{\beta\beta}$

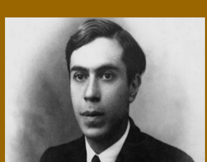
- **LOW-SCALE Type1 SEESAW (3,6) ISS & LSS**



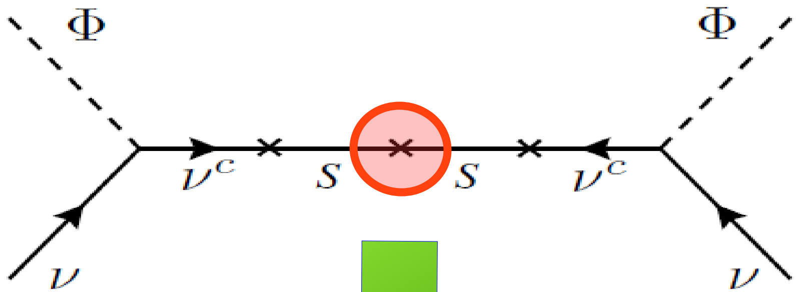
Mohapatra,Valle 86

Akhmedov et al Phys.Rev.D53 (1996) 2752
PhysLettB368 (1996) 270

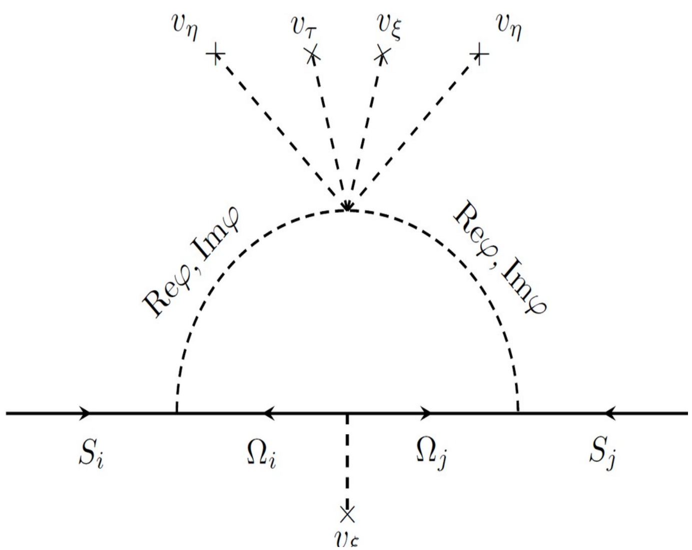
Malinsky et al PhysRevLett95(2005)161801



doubly protected inverse seesaw

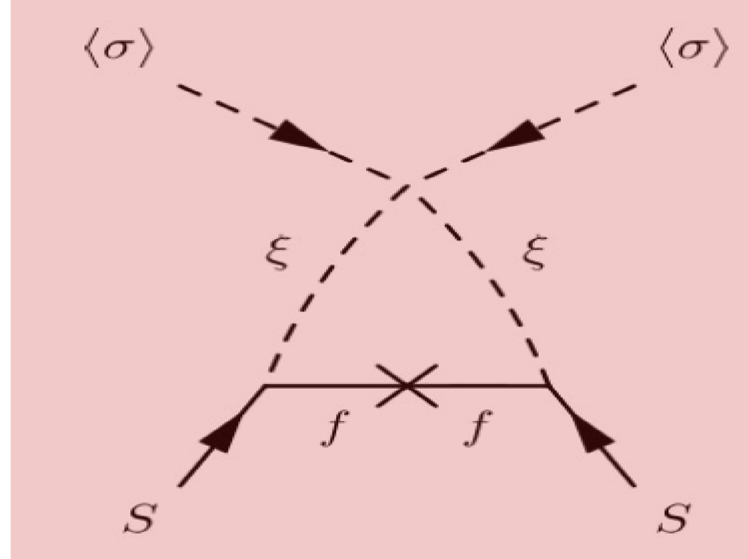


radiative inverse seesaw



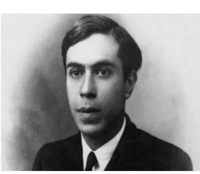
L-R scheme

:Cárcamo Hernández et al JHEP 1902 (2019) 065

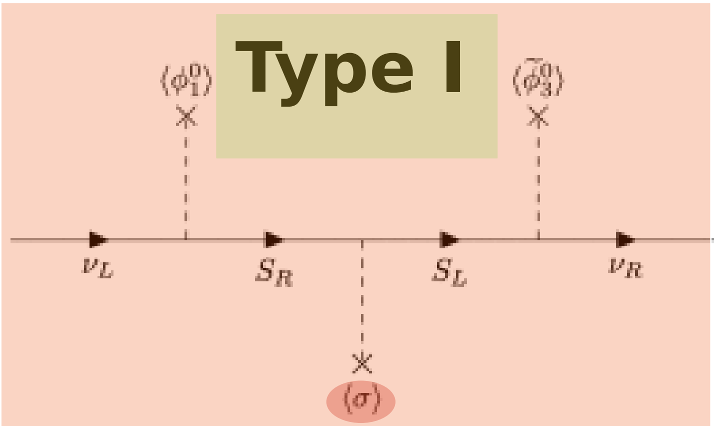


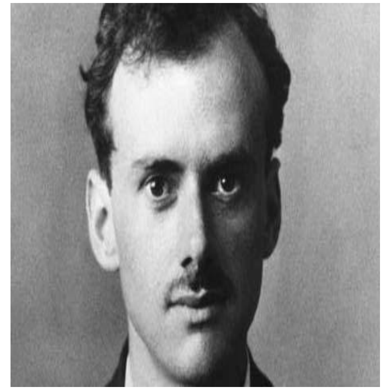
dark inverse seesaw

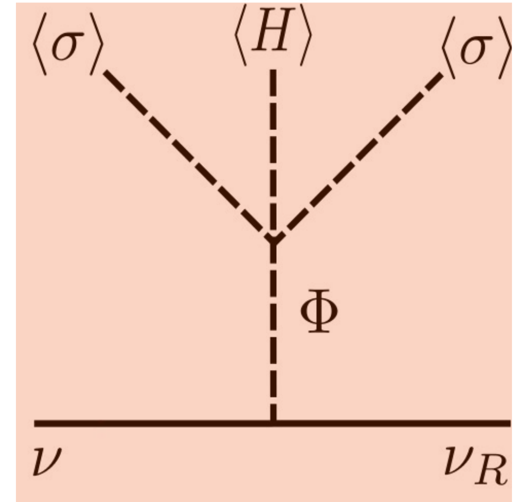
Mandal et al Phys.Lett.B821 (2021) 136609



Seesawing a la







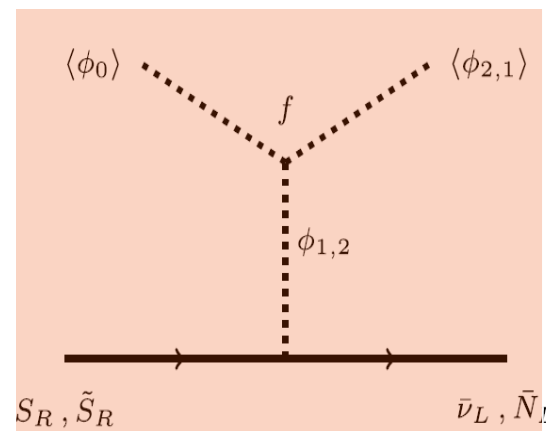
Type II

symmetry protecting small neutrino mass

+ Diracness

Phys.Lett. B762 (2016) 162-165

Phys.Rev. D94 (2016) 033012



Phys.Lett. B761 (2016) 431-436

Phys.Lett. B767 (2017) 209-213

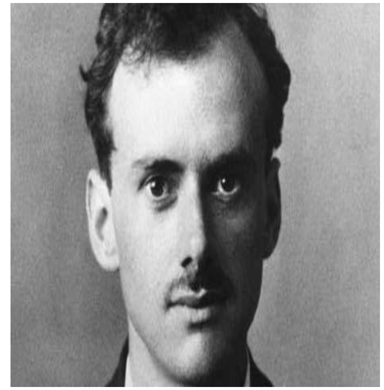
Phys.Rev. D98 (2018) 035009

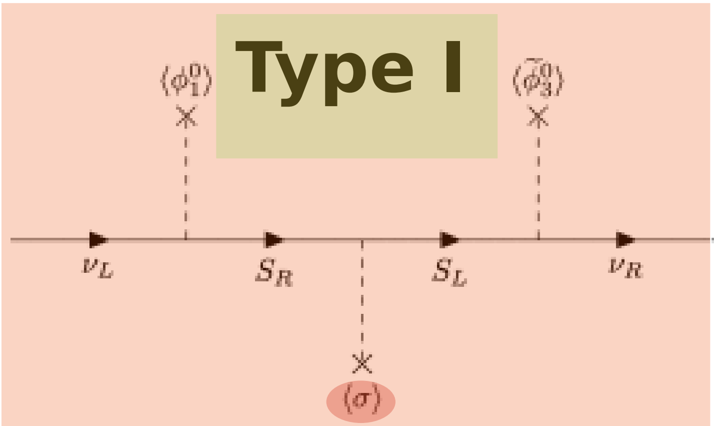
Phys.Lett. B781 (2018) 122-128

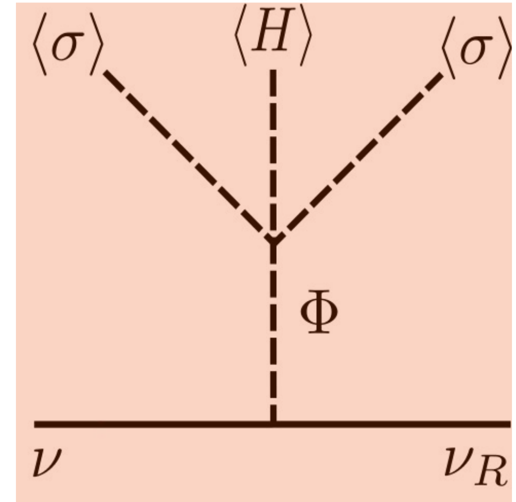
Addazi et al Phys.Lett. B759 (2016) 471-478

Phys.Lett. B755 (2016) 363-366

Seesawing a la







Type II

Phys.Lett. B762 (2016) 162-165

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symmetry protecting small neutrino mass

+ Diracness

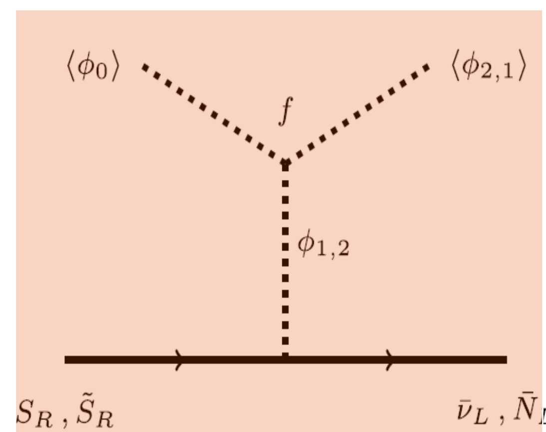
Peccei-Quinn symmetry

$$m_\nu^D \simeq \frac{y^{\nu_1}(y^S)^{-1}(y^{\nu_2})^T}{\sqrt{2}} \frac{v\, w}{v_\sigma}.$$

w ← SU3L

v_σ ← PQ

Revamped axion Phys.Lett.B 810 (2020) 135829



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Phys.Lett. B781 (2018) 122-128

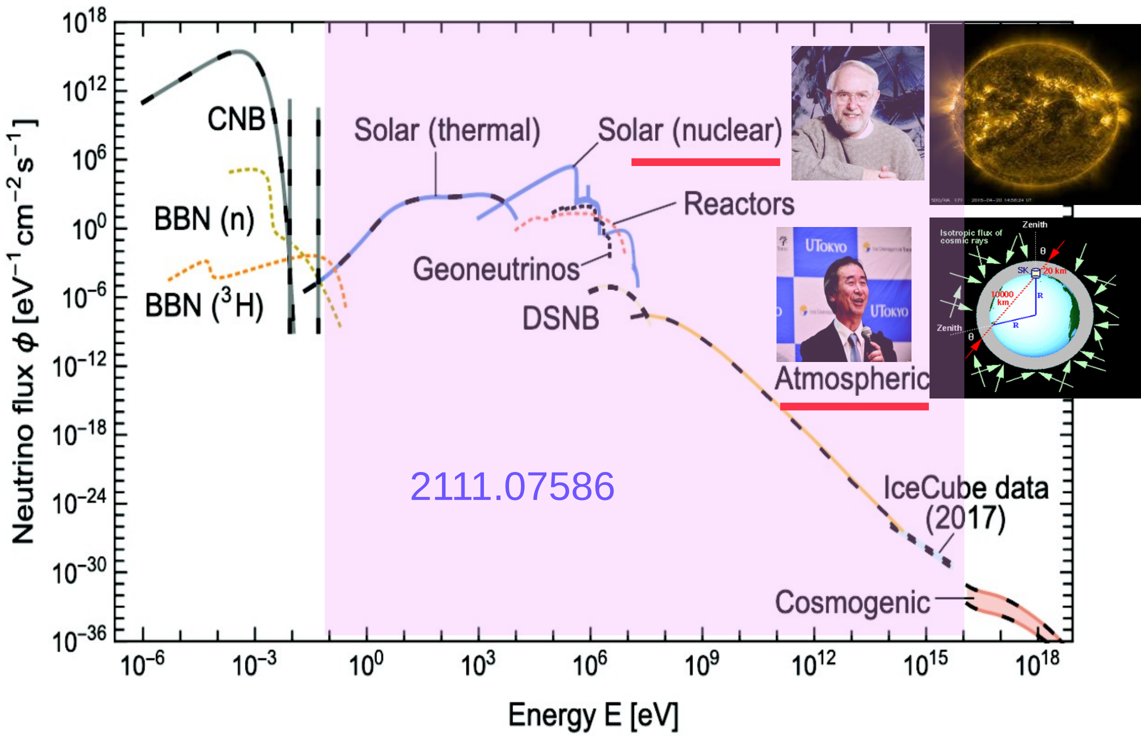
Addazi et al Phys.Lett. B759 (2016) 471-478

Phys.Lett. B755 (2016) 363-366

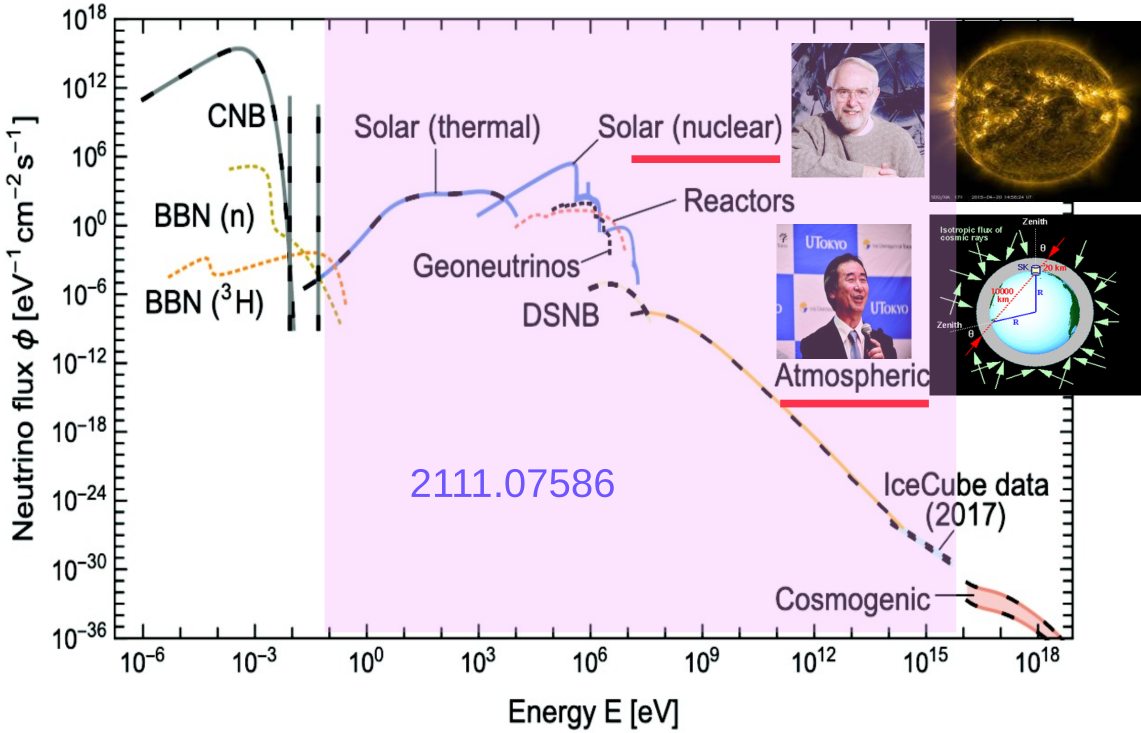
neutrino sources

Neutrinos arise in a variety of processes like **beta decays** in atomic nuclei. Besides Reactors, neutrinos are produced at accelerators.

Neutrinos are also produced by **natural** sources like solar neutrinos, and in cosmic ray interactions with atomic nuclei in the Earth: atmospheric



neutrino sources



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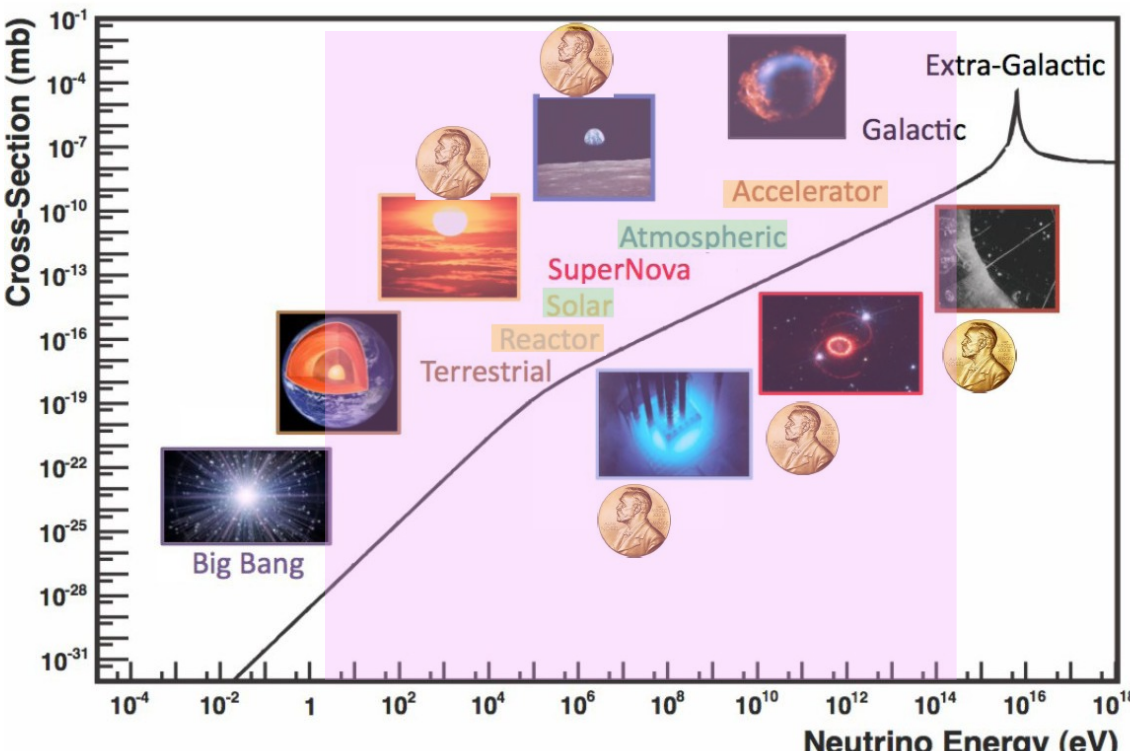
neutrino cross sections



Snowmass white paper:

beyond the standard model effects on neutrino flavor

Snowmass white paper:
beyond the standard model effects on neutrino flavor

Table 5 Neutrino detectors and neutrino telescopes sorted from smaller to larger energies. These are grouped into three categories separated by the double lines: large neutrino detectors (top), high-energy neutrino telescopes (HENT, middle), and extremely high-energy neutrino telescopes (EHENT, bottom). The detector technology and the neutrino interaction that they are sensitive to are shown in the right most columns with references. The label 'All Flavors' implies that they can also detect both charge- and neutral-current interactions. Table adapted from [696]

Energy range	Experiment	Technology	Detected flavor	References
$\lesssim 10^3$ GeV	JUNO	Liquid scintillator	All flavors	[236]
$\lesssim 10^3$ GeV	DUNE	LArTPC	All Flavors	[678]
$\lesssim 10^3$ GeV	THEIA	WbLS	All flavors	[488]
$\lesssim 10^3$ GeV	Super-Kamiokande	Gd-loaded water C	All flavors	[652]
$\lesssim 10^4$ GeV	Hyper-Kamiokande	Water Cherenkov	All flavors	[485]
$\lesssim 10^5$ GeV	ANTARES	Sea-Water Cherenkov	ν_μ, $\bar{\nu}_\mu$ (CC)	[679]
$\lesssim 10^6$ GeV	IceCube/IceCube-Gen2	Ice Cherenkov	All flavors	[435,680]
$\lesssim 10^6$ GeV	KM3NeT	Sea-water Cherenkov	All flavors	[681]
$\lesssim 10^6$ GeV	Baikal-GVD	Lake-Water Cherenkov	All flavors	[682]
$\lesssim 10^6$ GeV	P-ONE	Sea-Water Cherenkov	All flavors	[683]
1 − 100 PeV	TAMBO	Earth-skimming WC	ν_τ, $\bar{\nu}_\tau$ (CC)	[684]
$\gtrsim$ 1 PeV	Trinity	Earth-skimming Image	ν_τ, $\bar{\nu}_\tau$ (CC)	[685]
$\gtrsim$ 10 PeV	RET-N	Radar echo	All flavors	[686]
$\gtrsim$ 10 PeV	IceCube-Gen2	In-ice Radio	All flavors	[435]
$\gtrsim$ 10 PeV	ARIANNA-200	On-ice Radio	All Flavors	[687]
$\gtrsim$ 20 PeV	POEMMA	Space Air-shower image	ν_τ, $\bar{\nu}_\tau$ (CC)	[688]
$\gtrsim$ 100 PeV	RNO-G	In-ice radio	All flavors	[689]
$\gtrsim$ 100 PeV	ANITA/PUEO	Balloon radio	All Flavors	[690,691]
$\gtrsim$ 100 PeV	Auger/GCOS	Earth-skimming WC	ν_τ, $\bar{\nu}_\tau$ (CC)	[692,693]
$\gtrsim$ 100 PeV	Beacon	Earth-skimming radio	ν_τ, $\bar{\nu}_\tau$ (CC)	[694]
$\gtrsim$ 100 PeV	GRAND	Earth-skimming radio	ν_τ, $\bar{\nu}_\tau$ (CC)	[695]

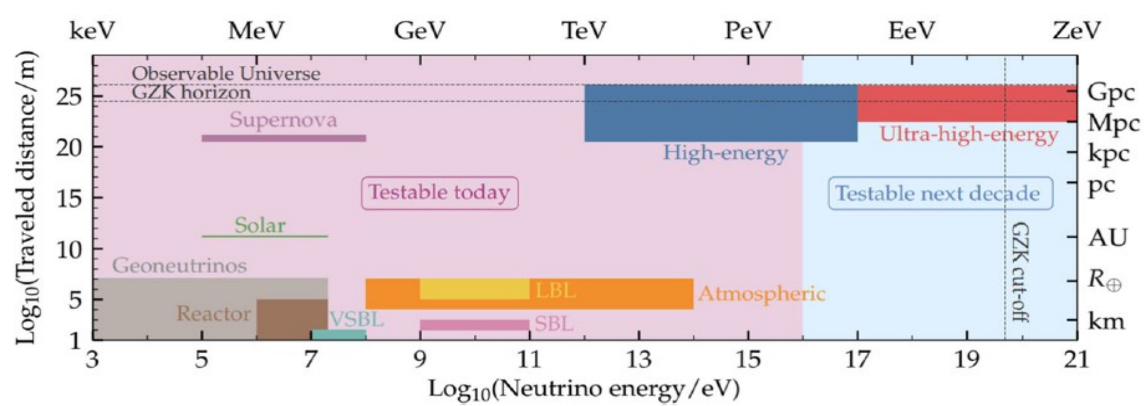
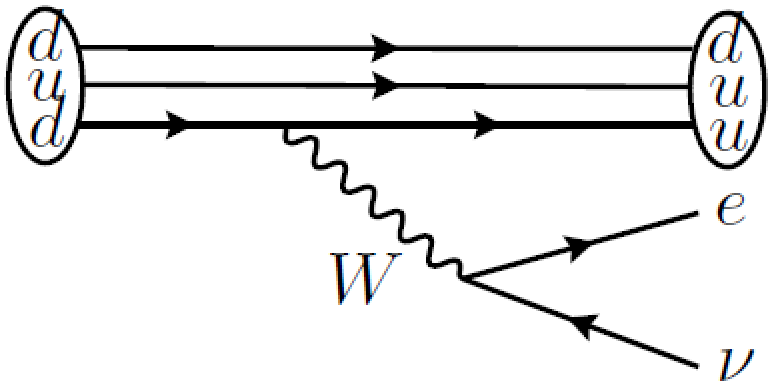


Fig. 14 Energy and distance scales relevant for neutrino telescopes. Three high-energy neutrino fluxes are labeled as atmospheric (orange), high-energy astrophysical (blue), and ultra-high-energy (red). The region explored by the current experiment is shown in pink, while next-generation is in light blue. Adapted from [697]

Beta decay

$$n \to p + e^- + \overline{\nu}_e$$

$${}^3H(\mathbf{0}, M) \to {}^3He^+(\boldsymbol{p'}, E') + e^-(\boldsymbol{p_e}, E_e) + \bar{\nu}_e(\boldsymbol{p_\nu}, E_\nu)$$

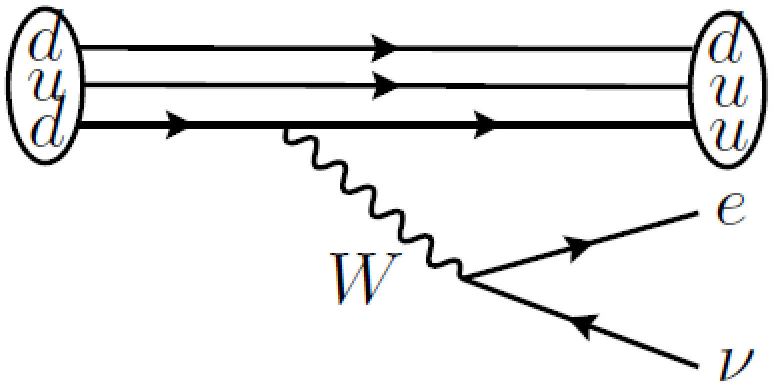


endpoint factor

Beta decay

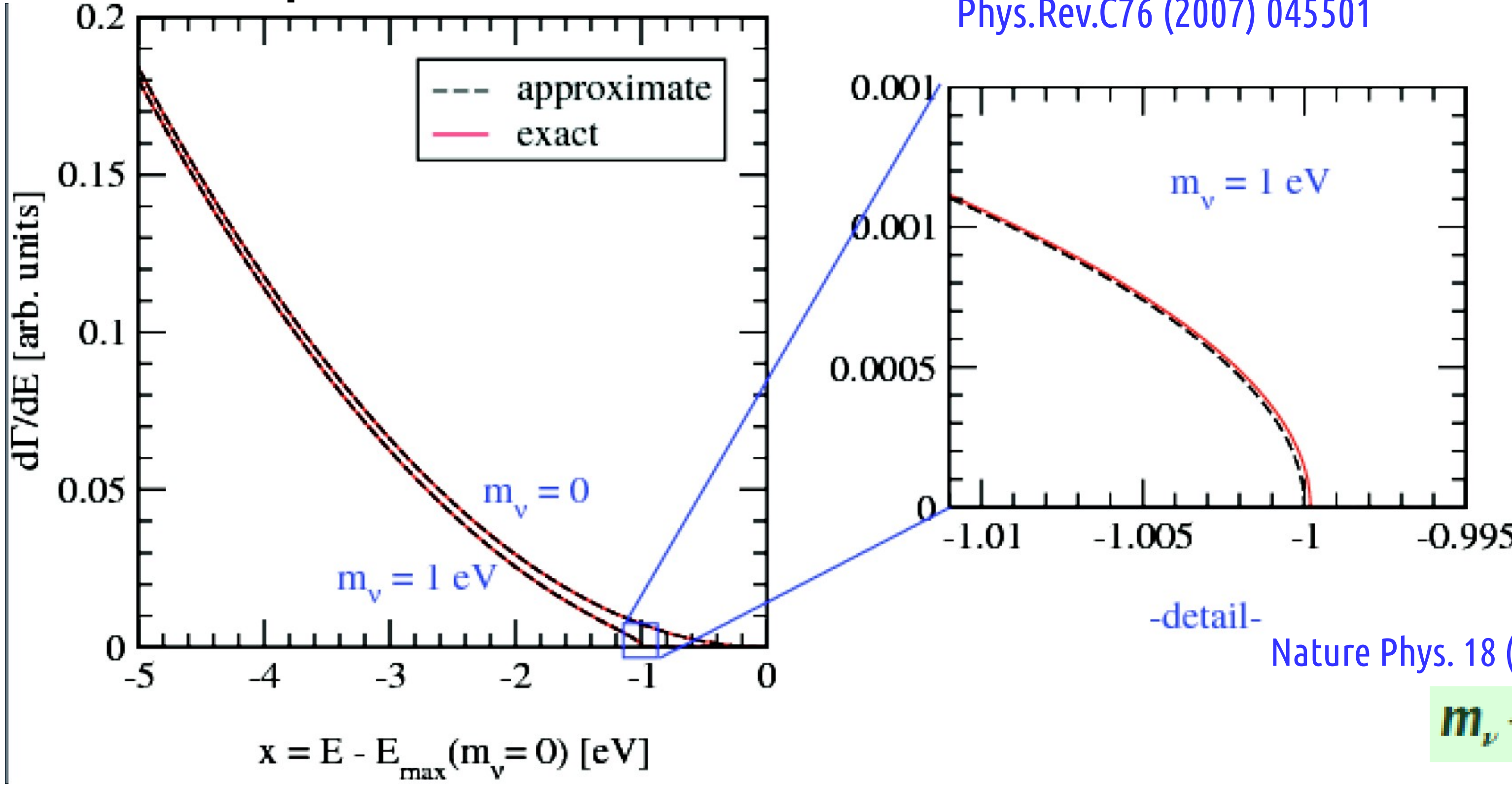
$n \to p + e^- + \overline{\nu}_e$

$^3H(\mathbf{0}, M) \to ^3He^+(\mathbf{p}', E') + e^-(\mathbf{p_e}, E_e) + \bar{\nu}_e(\mathbf{p_\nu}, E_\nu)$.



endpoint factor

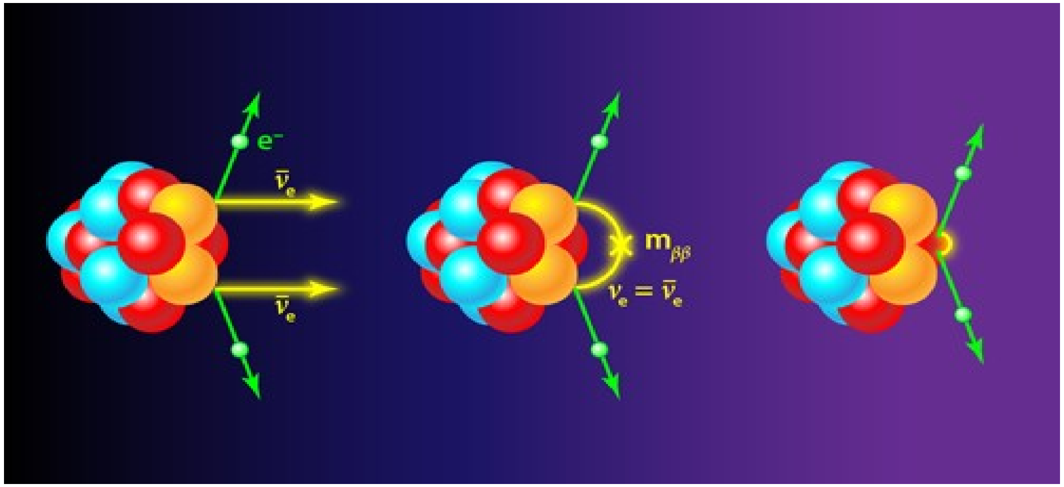
Phys.Rev.C76 (2007) 045501



Nature Phys. 18 (2022) 2, 160-166 • e-Print: 2105.08533

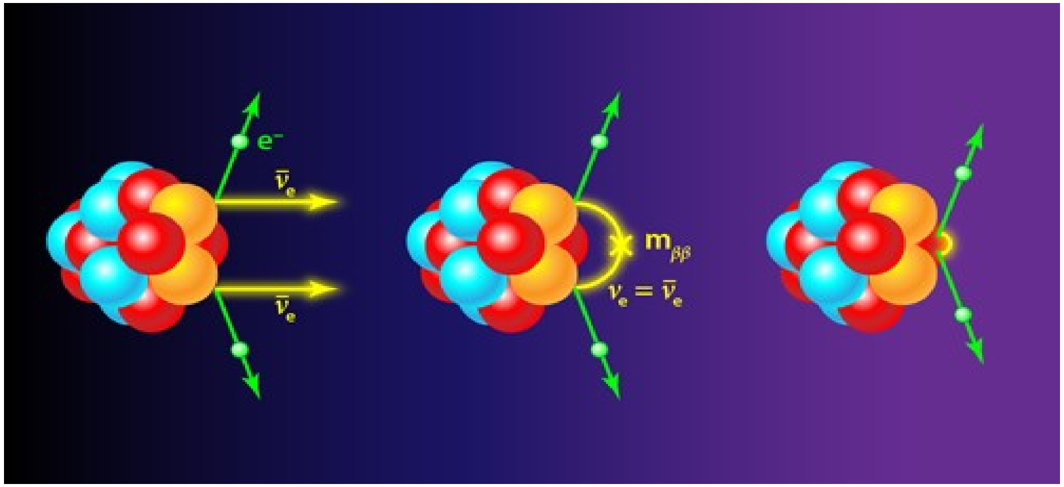
$m_\nu < 0.8$ eV c^{-2} at 90% CL.

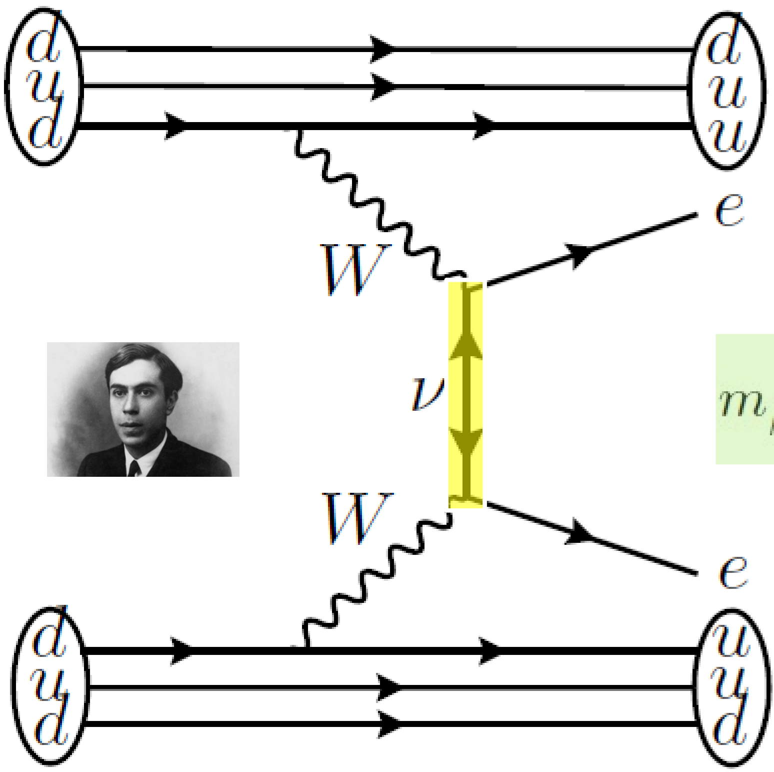
neutrinoless double beta decay



B.J.P. Jones 2108.09364 (TASI 2020)
C Adams et al 2212.11099

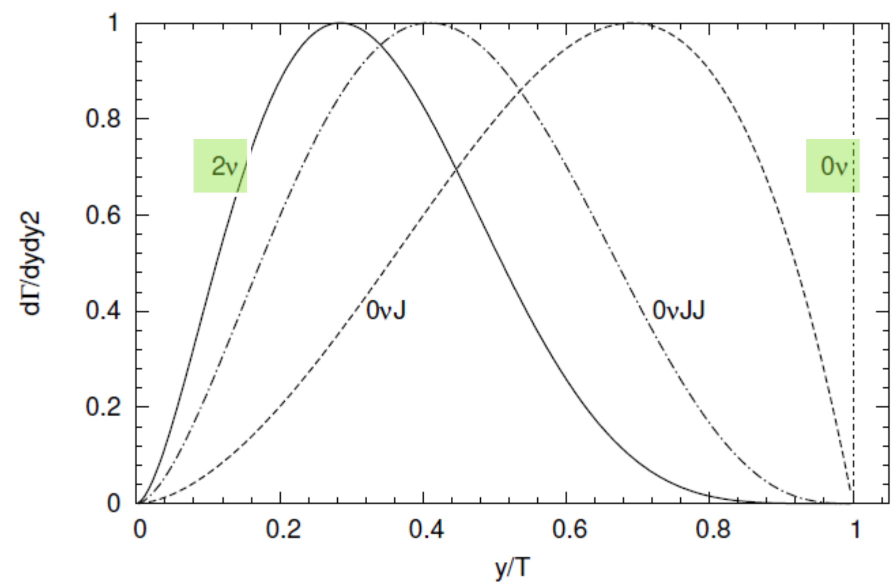
neutrinoless double beta decay



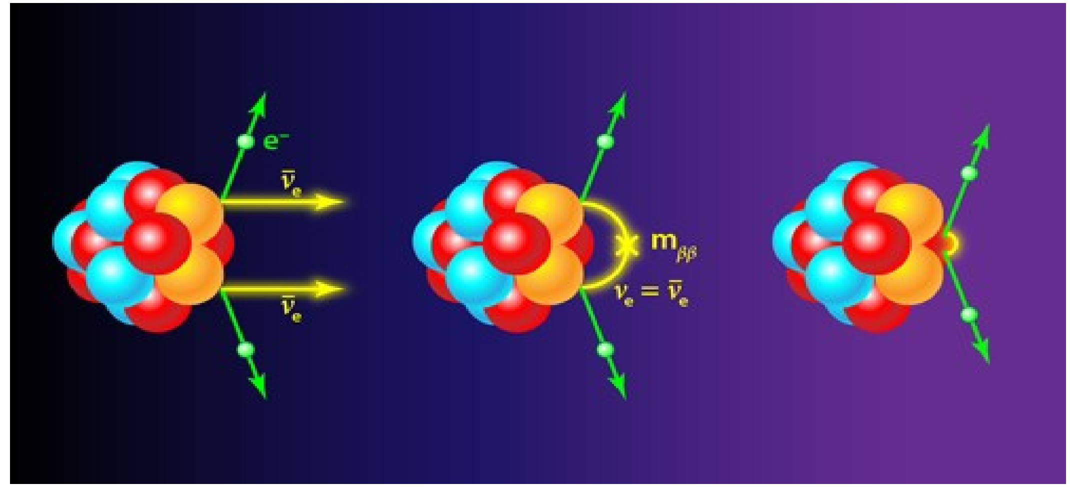


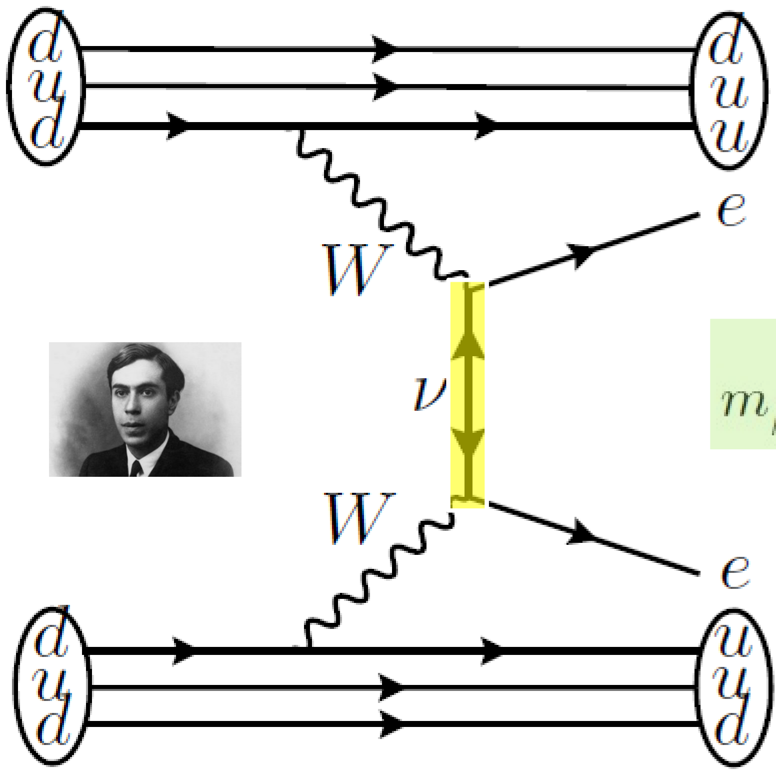
B.J.P. Jones 2108.09364 (TASI 2020)
C Adams et al 2212.11099

$$m_{\beta\beta} = \sum_i K_{ei}^2 m_i = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{2i\phi_{12}} + s_{13}^2 m_3 e^{2i\phi_{13}}|$$



neutrinoless double beta decay





B.J.P. Jones 2108.09364 (TASI 2020)
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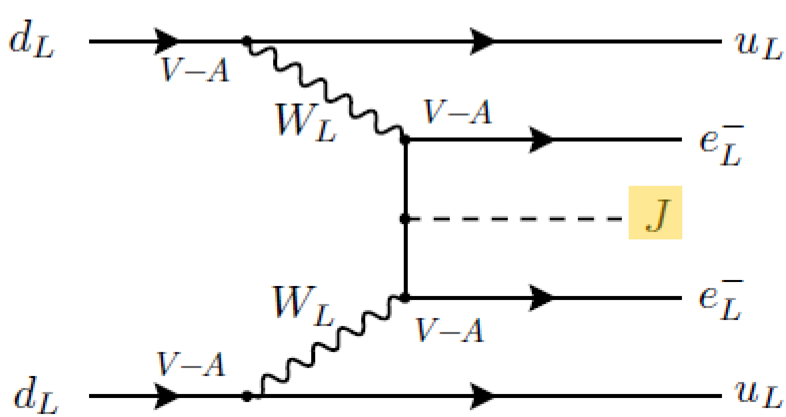
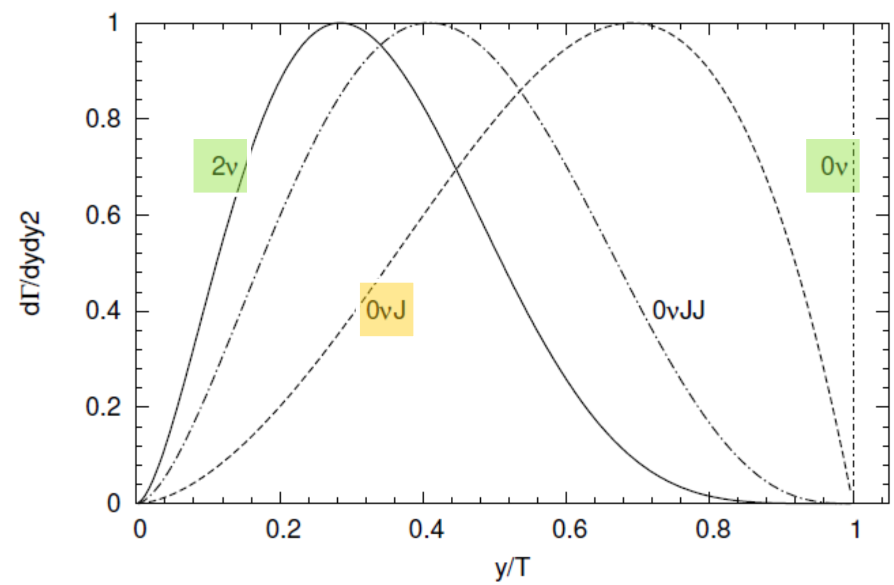


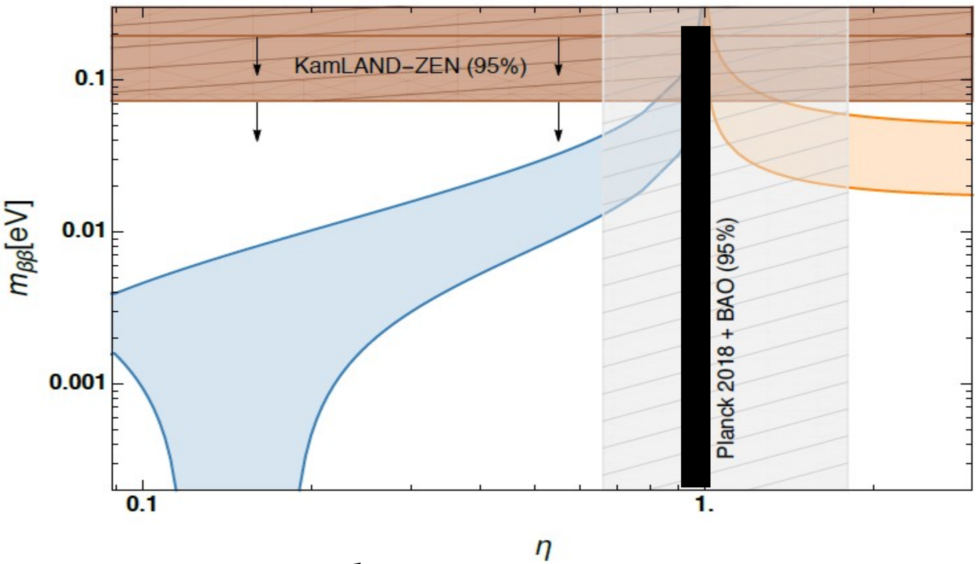


symmetrical parametrization is better than PDG

$$m_{\beta\beta} = \begin{cases} \left| c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{\frac{1}{2} i\alpha_{21}} + s_{13}^2\, m_3\, e^{\frac{1}{2} i(\alpha_{31} - 2\delta)} \right| & \text{(PDG)}, \\ \left| c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{2i\phi_{12}} + s_{13}^2\, m_3\, e^{2i\phi_{13}} \right| & \text{(symm.)} \end{cases}$$

Lattanzi et al JHEP 10 (2020) 213

Near degeneracy



degeneracy parameter

Agostini et al. Science 365 (2019) 1445
Final Gerda II … 2009.06079

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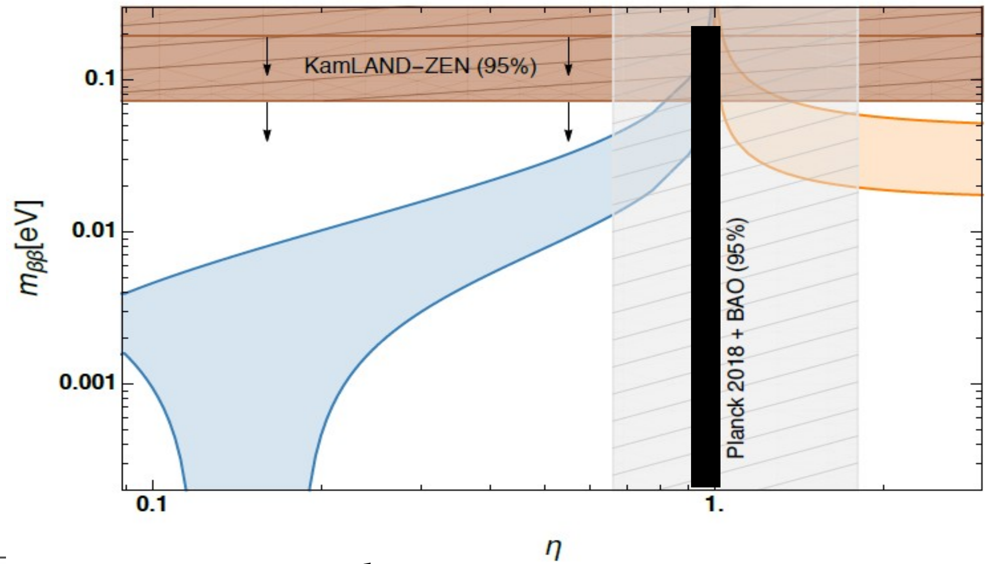
$$m_{\beta\beta} = \begin{cases} \left| c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{\frac{1}{2} i \alpha_{21}} + s_{13}^2\, m_3\, e^{\frac{1}{2} i (\alpha_{31} - 2\delta)} \right| & \text{(PDG)}\,, \\ \left| c_{12}^2 c_{13}^2\, m_1 + s_{12}^2 c_{13}^2\, m_2\, e^{2i\phi_{12}} + s_{13}^2\, m_3\, e^{2i\phi_{13}} \right| & \text{(symm.)} \end{cases}$$

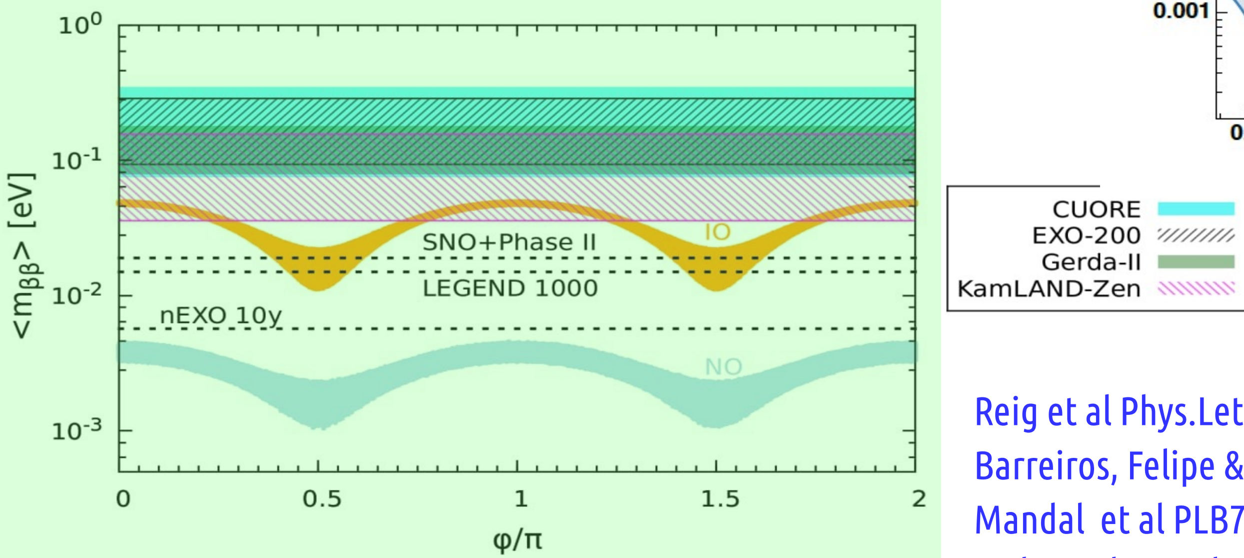
Lattanzi et al JHEP 10 (2020) 213

Near degeneracy



degeneracy parameter

One-massless neutrino



Agostini et al. Science 365 (2019) 1445
Final Gerda II … 2009.06079

Reig et al Phys.Lett. B790 (2019)303
Barreiros, Felipe & Joaquim JHEP (2019) 223
Mandal et al PLB789 (2019) 132
Avila et al Eur.Phys.J.C 80 (2020) 10, 908

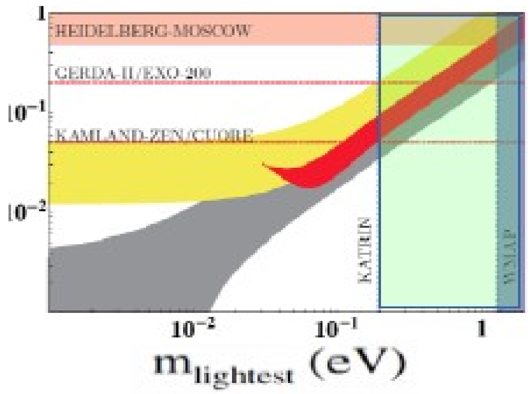
➢ general case

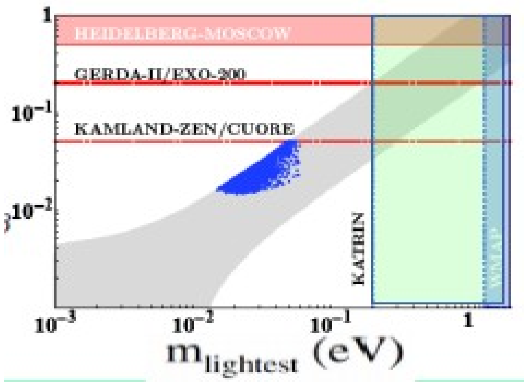
Lower bounds from family symmetries

Dorame et al PhysRevD86(2012)056001

Dorame et al Nucl.Phys.B 861 (2012) 259-270

King et al Phys.Lett. B 724 (2013) 68-72





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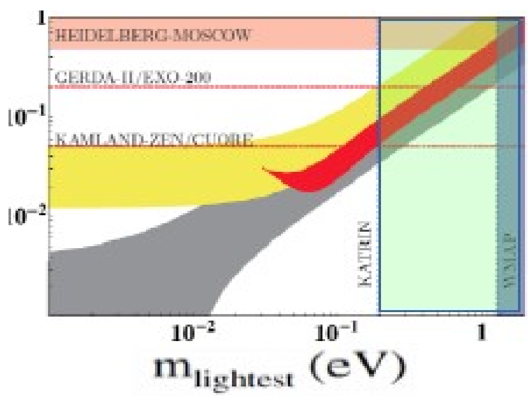

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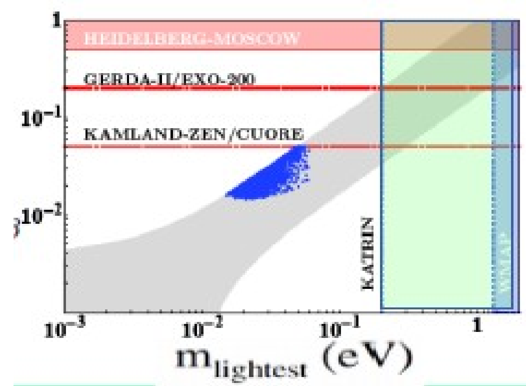

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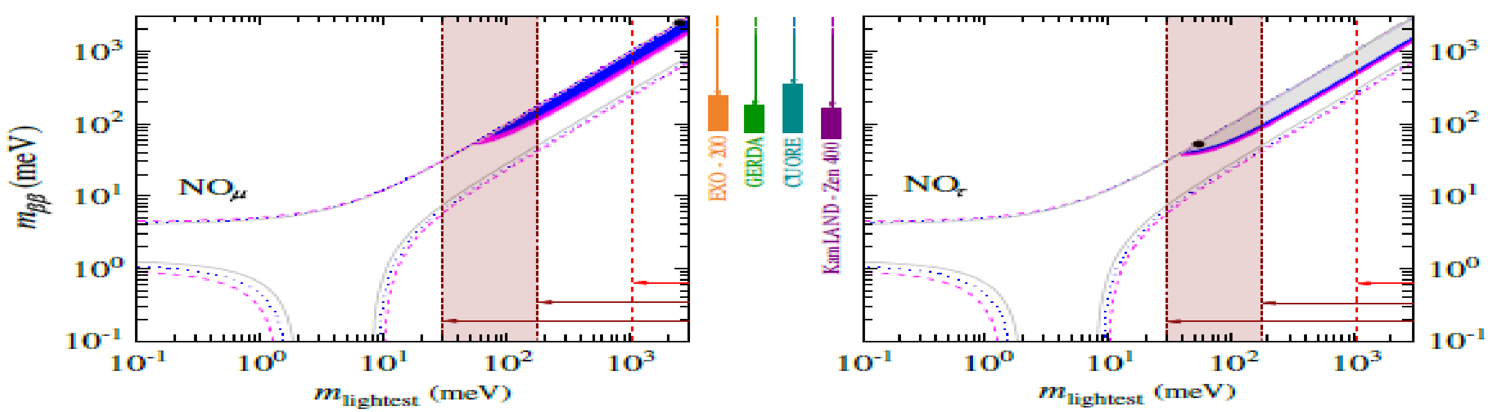
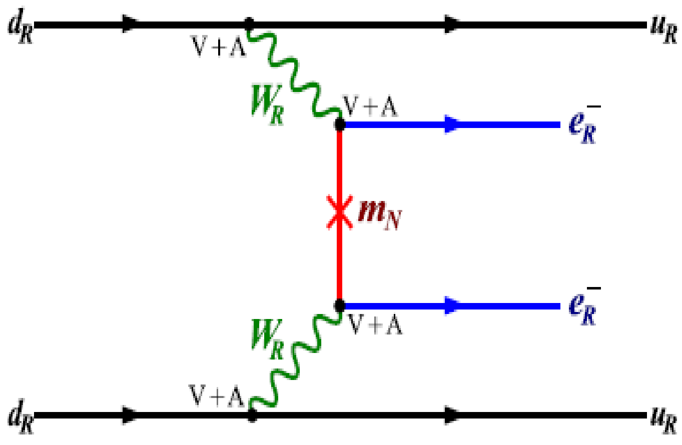
From Barreiros et al JHEP04(2021)249



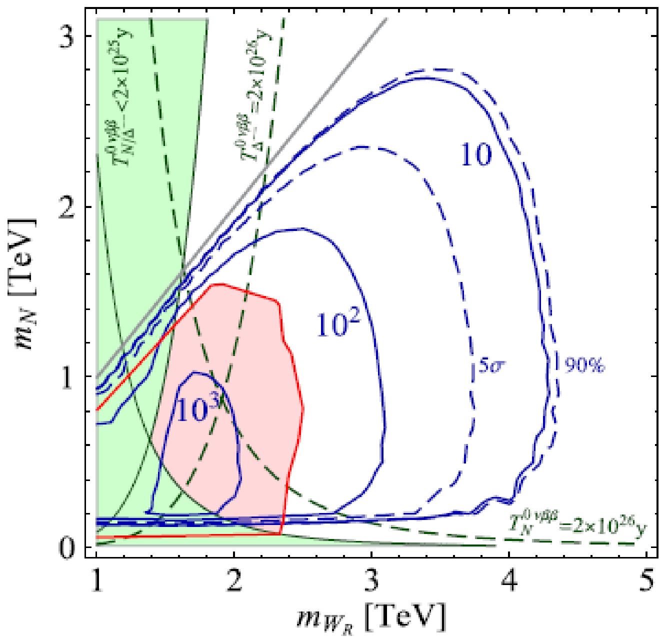
2 yrs ago

neutrinoless DBD induced by heavy mediators

PHYSICAL REVIEW D **86**, 055006 (2012)



PHYSICAL REVIEW D **86**, 055006 (2012)



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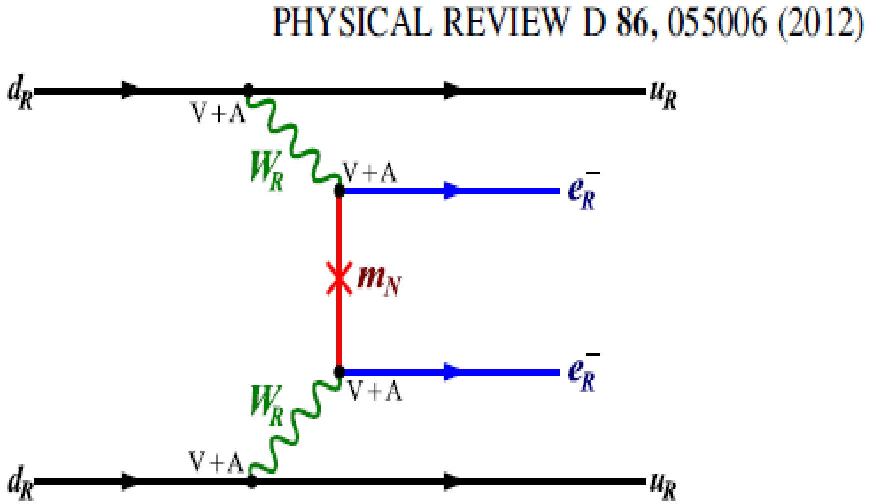
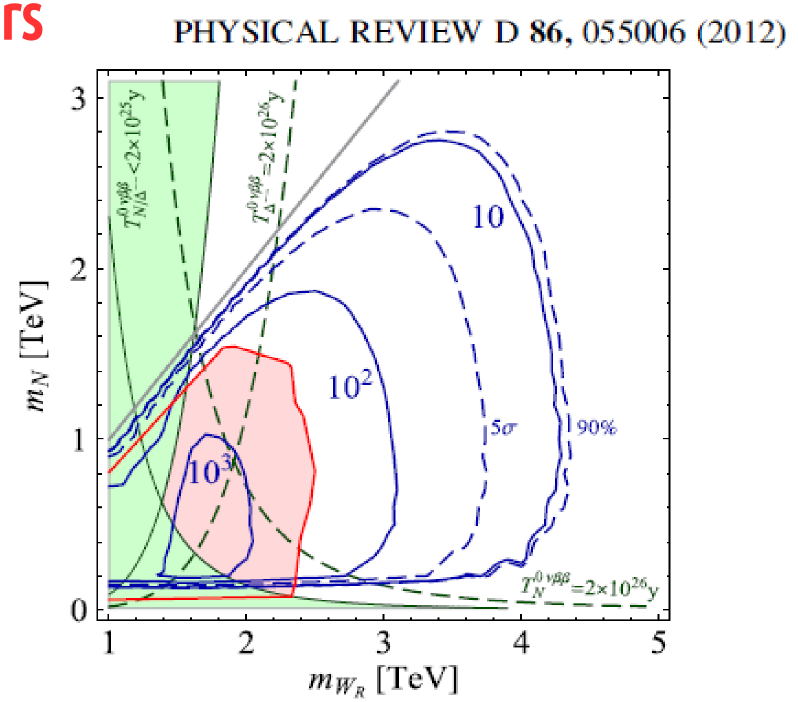




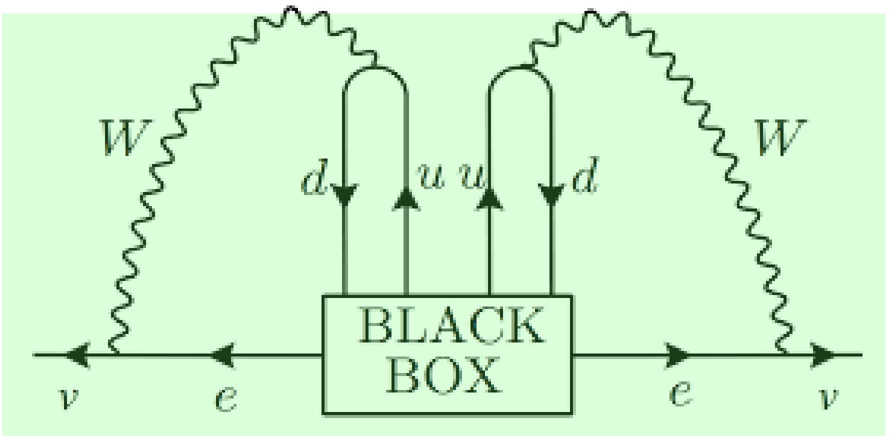
Theoretical significance

BLACK-BOX THEOREM

Schechter, Valle PhysRev D25 (1982) 2951

Duerr, Lindner, Merle JHEP06(2011)091

$0\nu\beta\beta$ **implies Majorana**



lecture 3

Neutrino oscillations

Current data indicates that there are three neutrinos that participate in the weak interactions

the precise measurement of the invisible width of the Z-boson at LEP we also know that there are three "active" isodoublet neutrinos (electron, muon or tau)

$$K \equiv \boldsymbol{V}^{\mathrm{LEP}} \equiv U,$$

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij},\ s_{ij} \equiv \sin\theta_{ij}.$$

$$\delta \equiv \phi_{13} - \phi_{12} - \phi_{23}$$

"Dirac phase"

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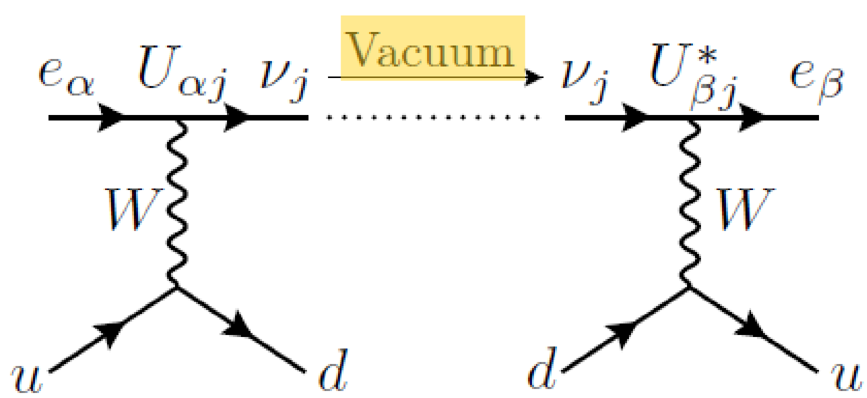
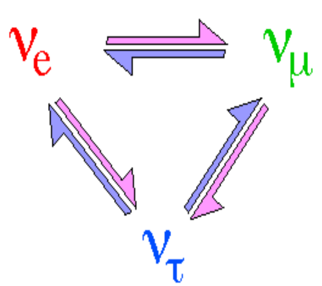
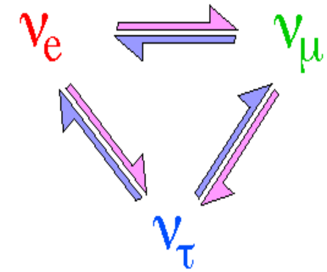
"Dirac phase"

The lepton mixing matrix implies a new phenomenon whereby a neutrino produced with a specific flavor can later be measured as a different flavor





$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\bar{l}_\alpha \gamma^\mu P_L \nu_k U_{\alpha k} W_\mu^- - \frac{g}{\sqrt{2}}\bar{\nu}_k \gamma^\mu P_L l_\alpha \, U^*_{\alpha k} W_\mu^+$$



Using the state vectors $$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i}|\nu_i\rangle, \quad (\alpha = e, \mu, \tau)$$

Derive

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= \left| \sum_j U^*_{\alpha j}\, U_{\beta j} \mathrm{e}^{-i\frac{m_j^2}{2E}L} \right|^2 \\ &= \delta_{\alpha\beta} - 4\sum_{i>j} \mathfrak{Re}(U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}) \sin^2\left(\frac{\Delta m^2_{ij}}{4E}L\right) \\ &\quad +2\sum_{i>j} \mathfrak{Im}(U^*_{\alpha i}U_{\alpha j}U_{\beta i}U^*_{\beta j}) \sin\left(\frac{\Delta m^2_{ij}}{2E}L\right) \end{aligned}$$

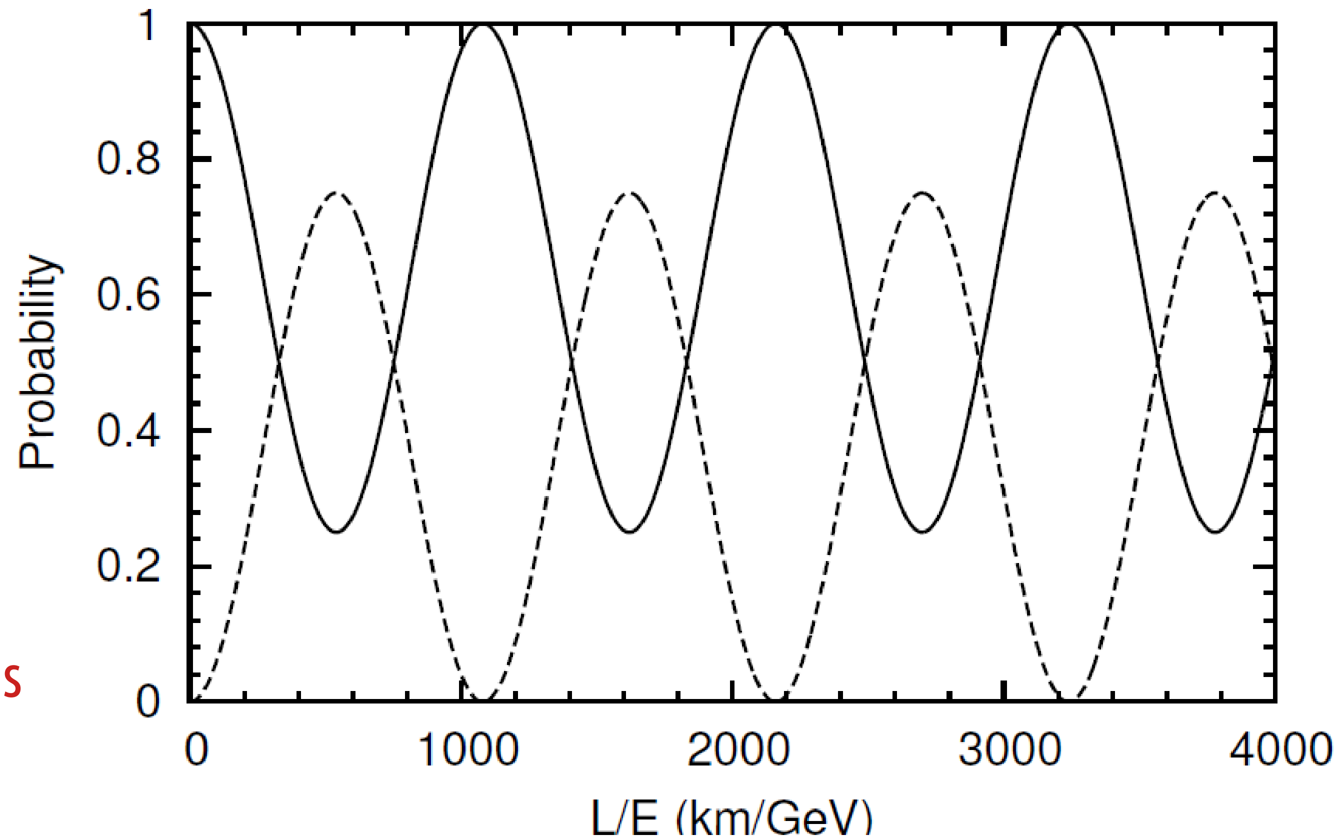
where E is the neutrino energy, L is the distance traveled by neutrino, and $\Delta m_{ij} \equiv m_i^2 - m_j^2$ (m_i being mass eigenvalues) are the mass squared differences. Here $\mathfrak{Re}$ and $\mathfrak{Im}$ denote real and imaginary parts.

E.g. two-neutrino approximation conversion probability takes a very simple form

$$P_{\mathrm{vacuum}}(\nu_e \to \nu_\mu) = \sin^2(2\theta)\ \sin^2(\frac{\Delta m^2 L}{4E})$$

a neutrino born as an electron neutrino will become a muon neutrino after traveling a distance L

solid curve is the probability for the original neutrino retaining its identity,
the dashed one is the probability of conversion to the other neutrino species



neutrino oscillations exist only if the mixing matrix U has non-vanishing non-diagonal matrix elements and the neutrino masses are non-degenerate

$$L_{ij}{}^{\mathrm{osc}} \equiv 2\pi \frac{2E}{\Delta m_{ij}^2}$$

neutrino oscillation lengths

the quantum mechanical phase evolves periodically, after some distance the state will return

The flavor content of the neutrino will then continue to oscillate as long as the quantum mechanical state maintains coherence. Oscillations can only be observed if the neutrino production, propagation and detection coherence conditions are satisfied

$$L_{ij}^{\text{coh}} \lesssim E[l_{ij}]^2 = \frac{16\pi^2 E^3}{[\Delta m_{ij}^2]^2}$$

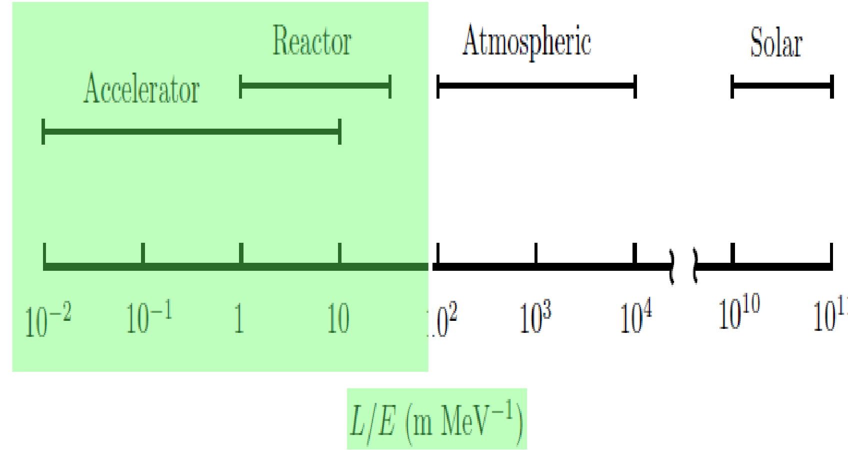
Since the mass differences between the neutrinos are so small the coherence length for neutrino oscillations will be very long, making this microscopic quantum effect observable over macroscopic distances.

For neutrinos in the electron-volt range, coherence is lost in the case of solar neutrinos, where what arrives at the underground detector on Earth is an incoherent neutrino admixture.

Oscillations arise from an interference between the different mass eigenstates in the neutrino wave function. Oscillations probe the squared mass splittings which appear in the oscillation length.

Information on the mixing coefficients is obtained from the oscillation amplitudes. The oscillation pattern depends on L/E, the distance/neutrino energy ratio

Experiment		L (m)	E (MeV)	Δm^2 (eV^2)
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	10^2–10^5	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^2	10^3–10^4	> 0.1
	LBL	$10^5 - 10^6$	10^4	$10^{-2} - 10^{-3}$

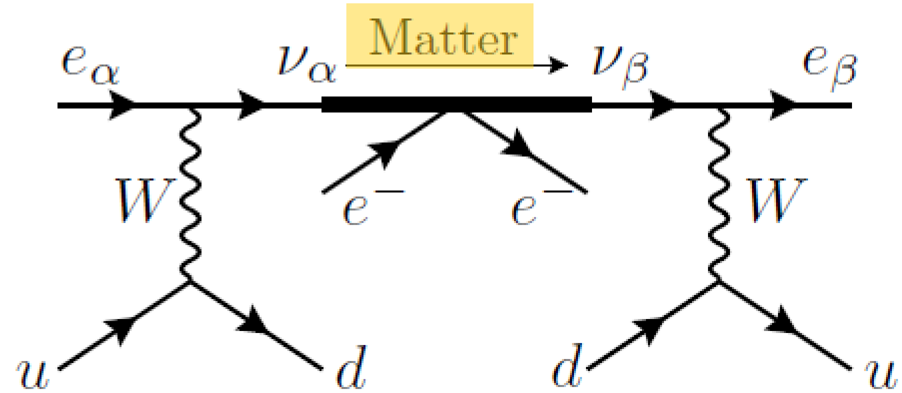


In **appearance** experiments one starts with a beam of neutrinos of a given flavor and observes neutrinos of a different flavor after traveling a distance L from the source.

If oscillations are present, the oscillation probability to the different flavor is nonzero.
one needs a neutrino beam of energy larger than the rest mass of the charged lepton to be created in the detection reaction.

In **disappearance** experiments, the detector probes the same flavor of the neutrinos originally present in the beam. $|\nu_\alpha\rangle \, (\alpha = e, \mu, \tau)$

If oscillations are present, the “survival” probability is smaller than unity. These experiments are “inclusive” i.e. one probes oscillations from the original flavor to all others to discuss actual experiments one must take **matter effects** into account. e.g. **solar** neutrinos are produced in the interior of the Sun and must cross the solar interior and also **Earth** matter before being detected in underground experiments such as Super-K



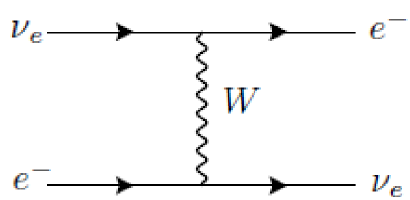
Earth matter effects relevant for atmospheric and, to some extent, also long baseline oscillation experiments. Kim, C. and Pevsner, A. (1993) *Neutrinos in Physics and Astrophysic*s, Harwood .

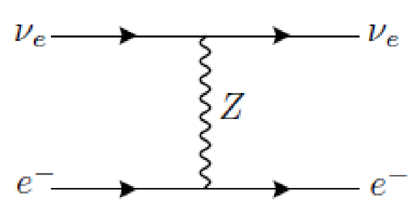
The presence of electrons in the medium changes the energy levels of the mass eigenstate neutrinos due to charged current coherent forward scattering of the electron neutrinos.

$$V(x) = \sqrt{2} G_F N_e(x)$$

8×10^{-12}eV

to 8×10^{-13}eV





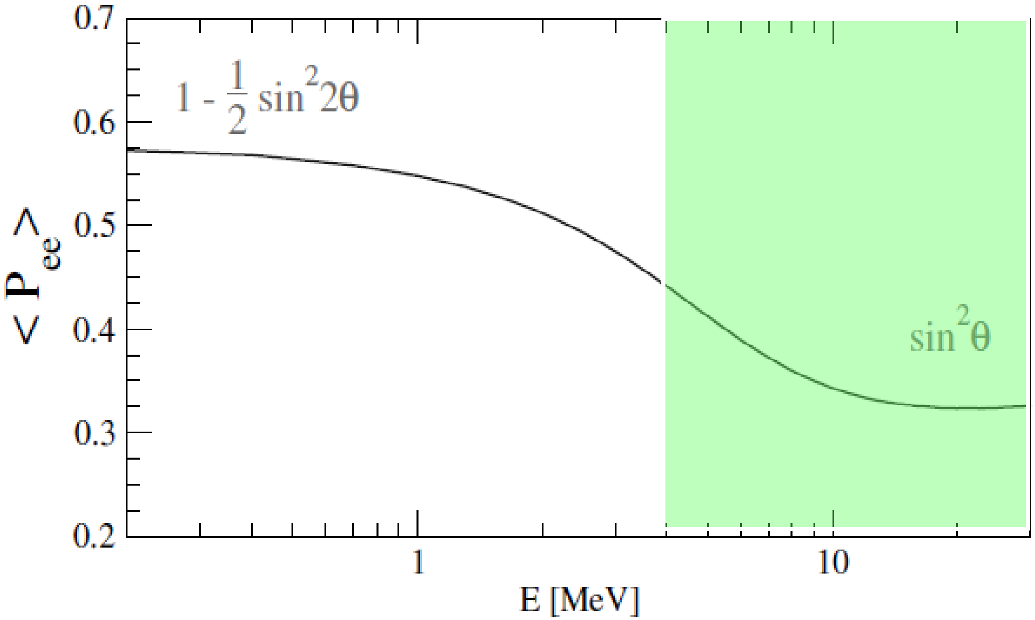
where G_F is the Fermi constant, and $N_e(x)$ is the electron number density at x.

MSW effect must be taken into account when considering the oscillations of neutrinos traveling through matter

this matter potential may substantially modify effective “masses” and mixing angles in the solar medium. E.g, in the adiabatic regime of slowly–varying matter densities, the mixing angle in matter is given by the MSW expression

$$\cos 2\theta_m = \frac{\Delta m^2 \cos 2\theta - 2\sqrt{2}\, E G_F N_e}{\sqrt{(\Delta m^2 \cos 2\theta - 2\sqrt{2}\, E G_F N_e)^2 + (\Delta m^2 \sin 2\theta)^2}}$$

to describe the experimental data we need the **neutrino survival probability**



the effect of matter is important for the suppression of high energy solar neutrinos

For which one has a stronger suppression in the flux reaching the detectors, as indicated by the data

Solar Neutrino Puzzle Davis

neutrino evolution equation in matter

$$i\frac{d}{dx}\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = H(x)\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix}$$

where ν_α $(\alpha = e, \mu, \tau)$ is the amplitude for the α-flavour. The Hamiltonian matrix H is

$$H(x) = U\begin{bmatrix}\frac{m_1^2}{2E} & 0 & 0\\ 0 & \frac{m_2^2}{2E} & 0\\ 0 & 0 & \frac{m_3^2}{2E}\end{bmatrix}U^\dagger + \begin{bmatrix}V(x) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

by re-phasing all the neutrino flavors

$$H(x) = U\mathrm{diag}\left[0, \frac{\Delta m_{21}^2}{2E}, \frac{\Delta m_{31}^2}{2E}\right]U^\dagger + \mathrm{diag}[V(x), 0, 0],$$

For anti-neutrinos $\quad V(x) \to -V(x)$ and $U \to U^*$

$$H(x) = U(N)\mathrm{diag}\left[0, \frac{\Delta m_{21}^2(N)}{2E}, \frac{\Delta m_{31}^2(N)}{2E}\right]U^\dagger(N)$$

A new generation of experiments including Super-K, the Sudbury Neutrino Observatory (SNO), KamLAND and accelerators have showed that neutrino flavors get inter-converted during their propagation mainly by oscillations 1

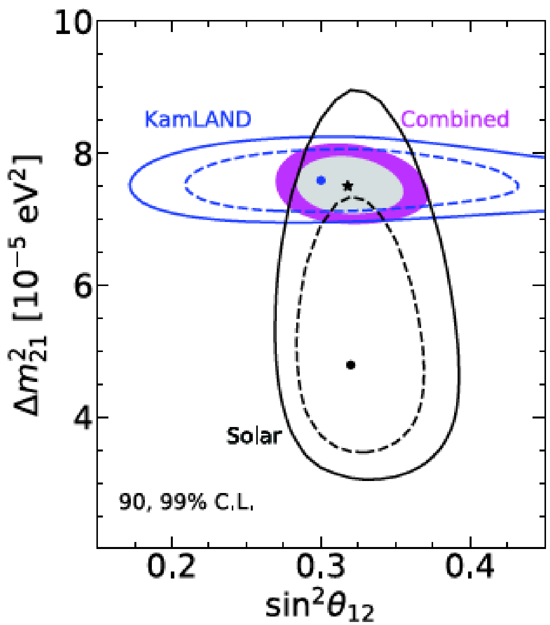
PF de Salas et al JHEP02(2021)071

solar sector

Cl, Ga, SK

SNO, Borexino

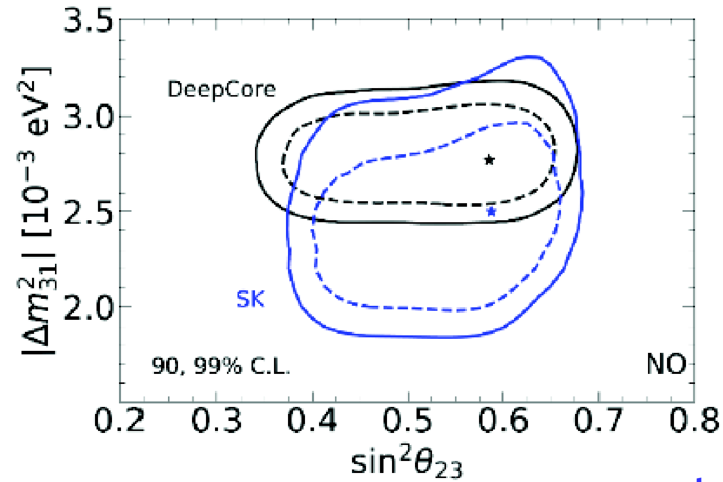
KamLAND



atmospheric results

Super-K

IC-DeepCore

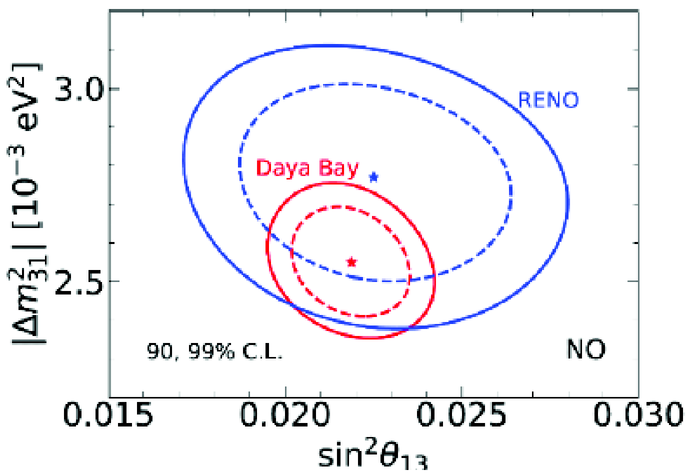


Tortola@DBD Lisbon2022

SBL reactors

Daya Bay

RENO

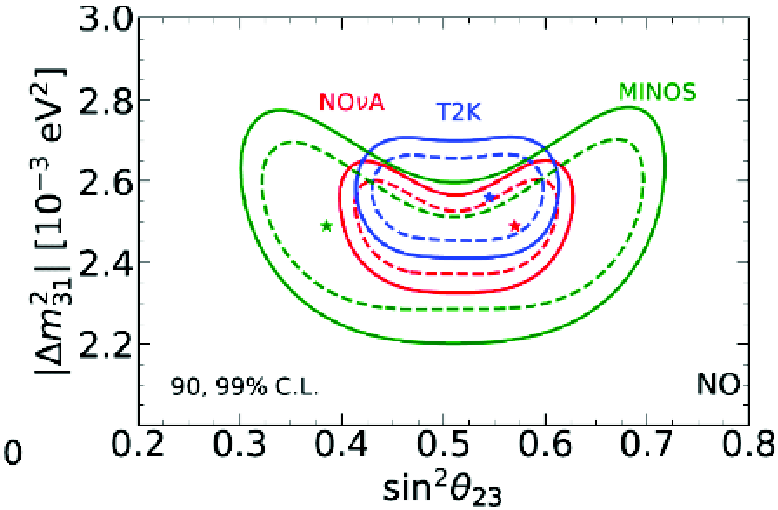


LBL experiments

MINOS

T2K

NOvA



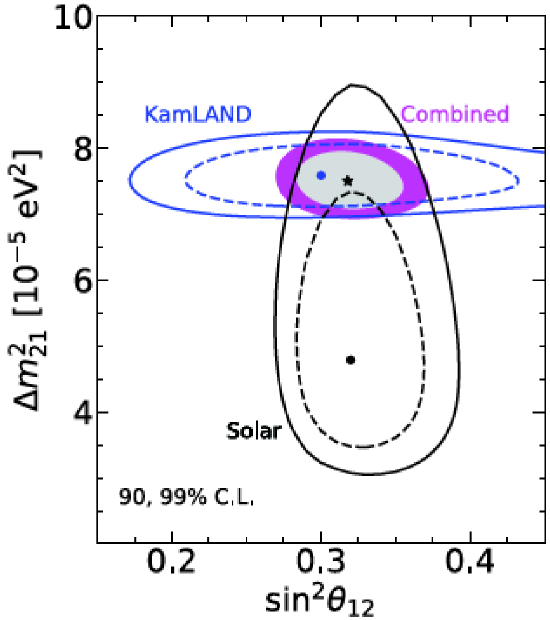
PF de Salas et al JHEP02(2021)071

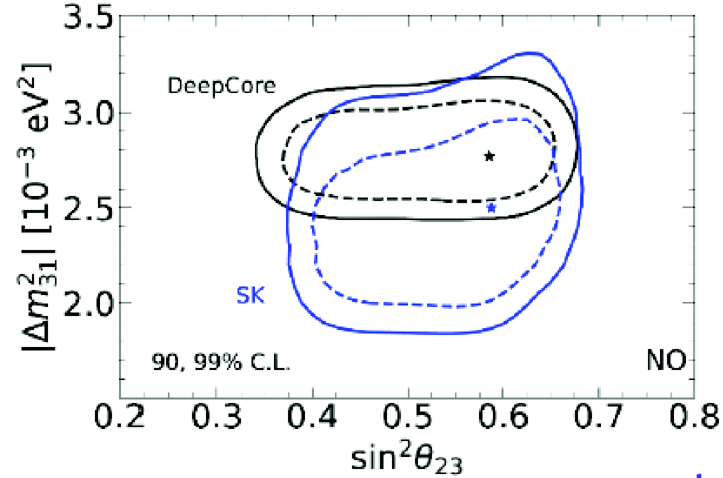
solar sector

Cl, Ga, SK

SNO, Borexino

KamLAND





atmospheric results

Super-K

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Tortola@DBD Lisbon2022

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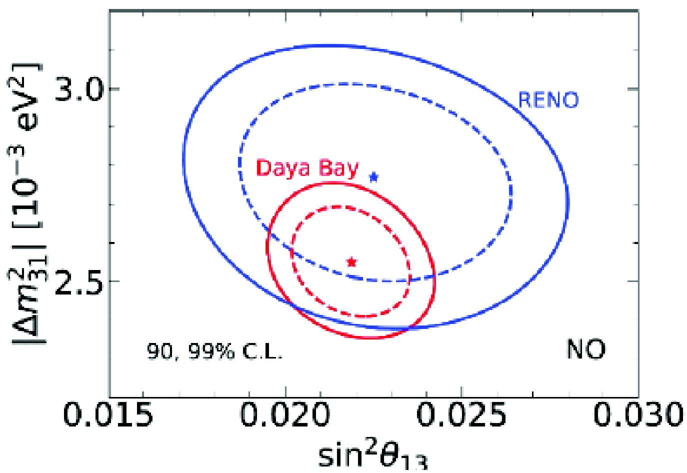
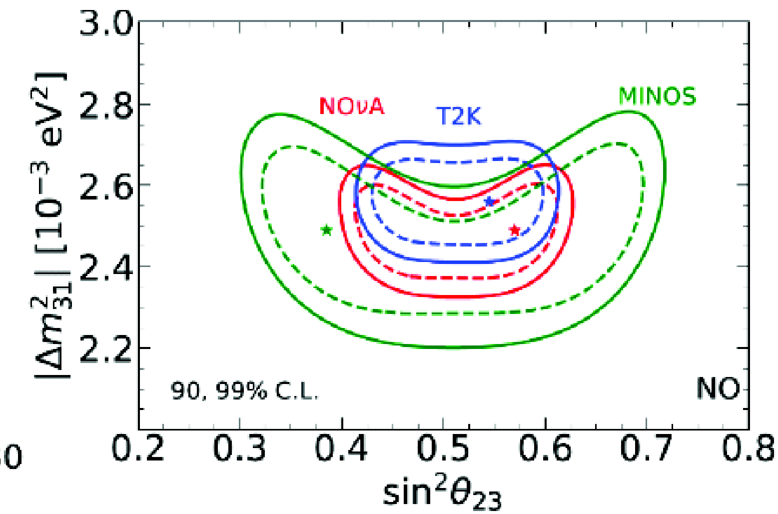




LBL experiments

MINOS

T2K

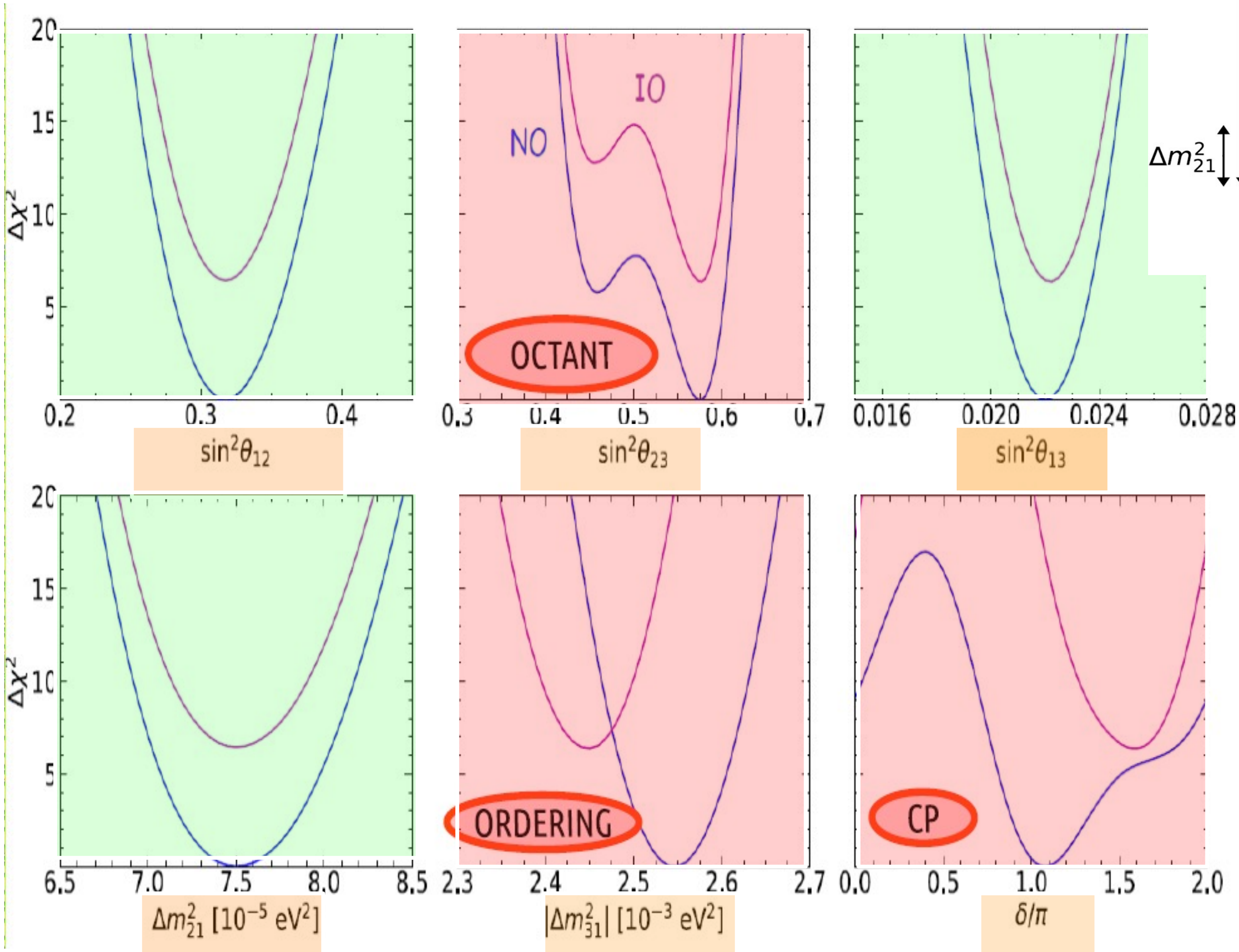
NOvA

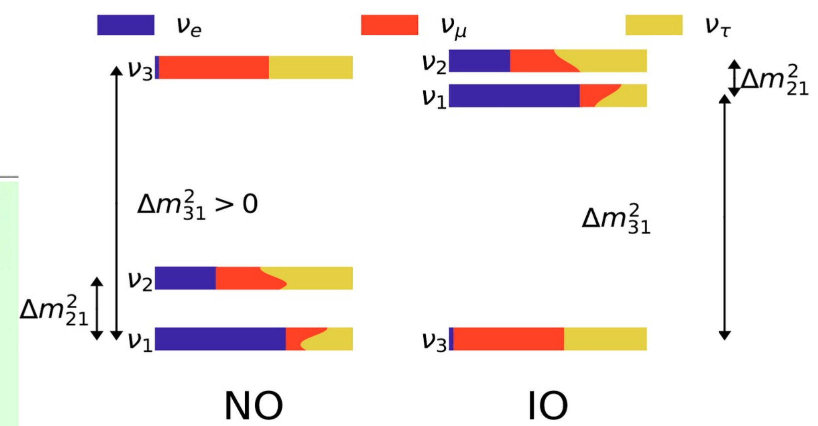
Note NC scattering is diagonal and does not distinguish flavors, it just gives an overall unphysical phase shift. This follows the **unitarity of the lepton mixing matrix** and consequent triviality of the neutral current matrix P. These features can be broken appreciably in low-scale seesaw. In this case the NC would also contribute relevant potentials to be taken into account in the neutrino evolution equation, and be relevant to describe neutrino propagation in very dense media like supernovae

neutrino oscillations

PF de Salas et al JHEP02(2021)071

https://zenodo.org/record 4593330#.YFoBVWNKjIo





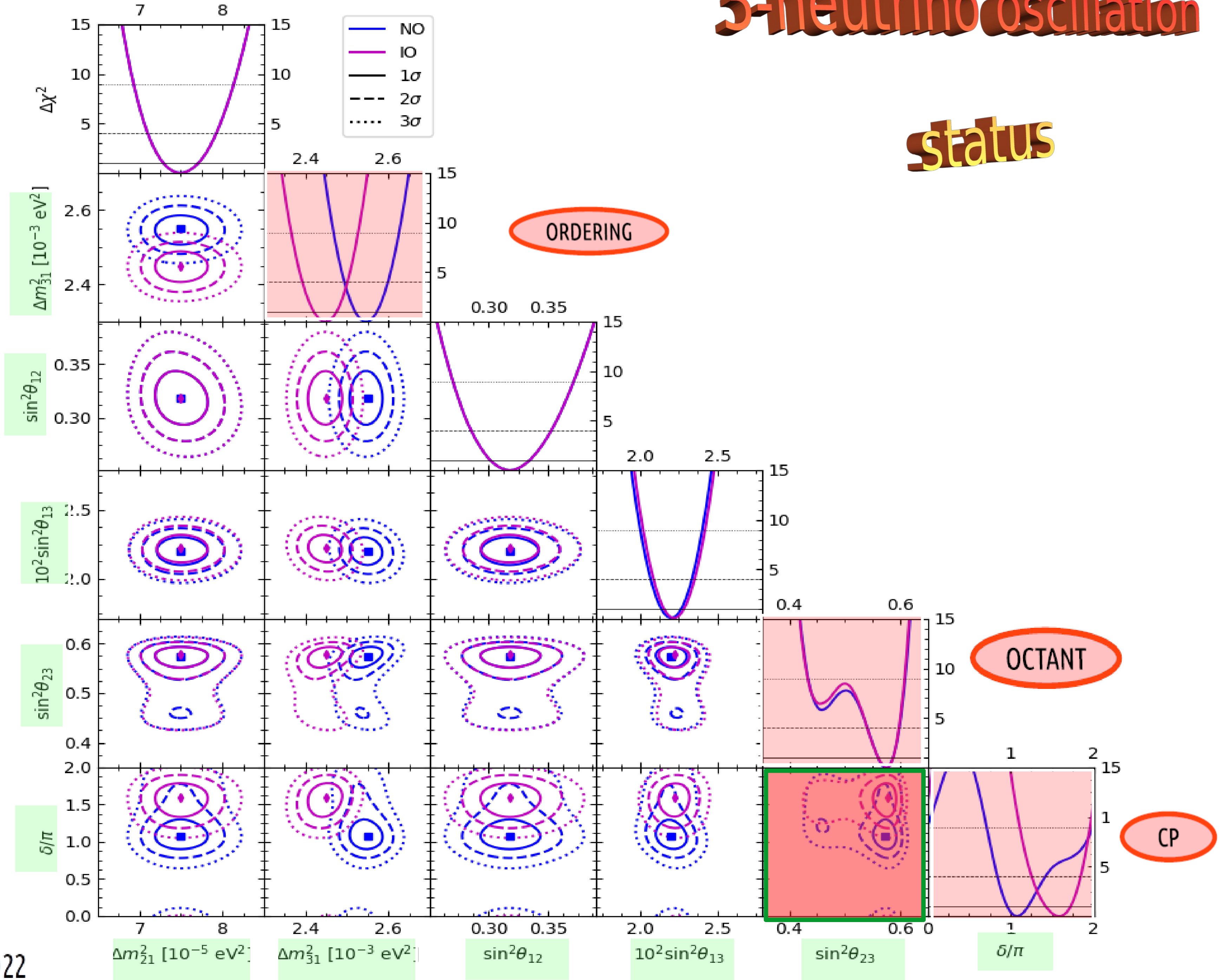
Similar results from Bari and NuFit groups

3-neutrino oscillation

status

PF de Salas et al JHEP02(2021)071

https://zenodo.org/record 4593330#.YFoBVWNKjIo

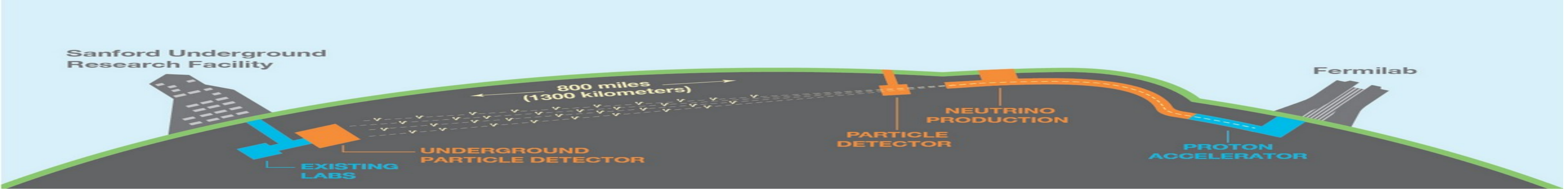


Updates from nu2022

$\sin^2 2\theta_{13} = 0.0853^{+0.0024}_{-0.0024}$ (2.8% precision)

CPV

DUNE
T2HK



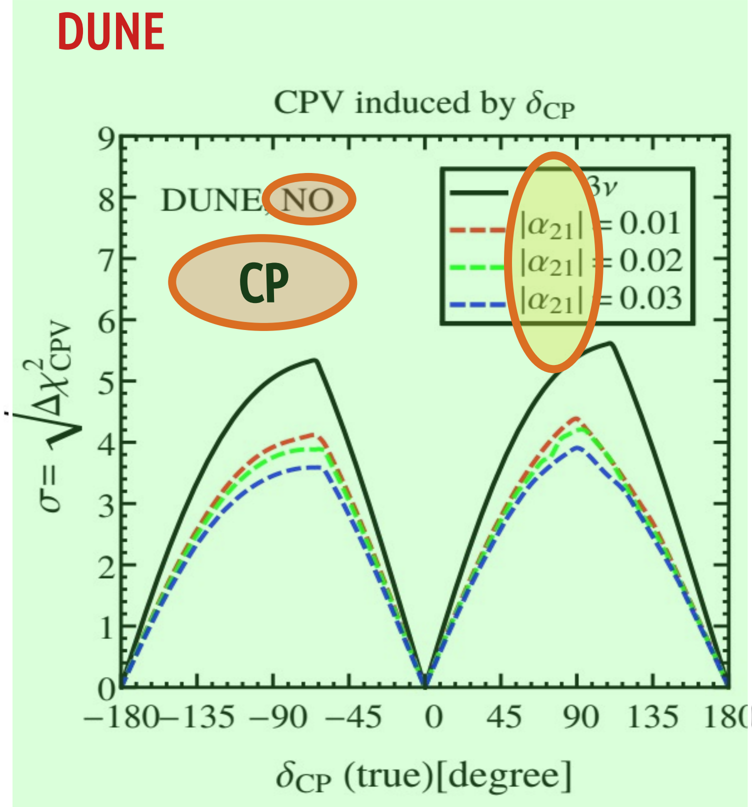
Leptonic CPV reviews

Nunokawa, Parke, JV

Prog.Part.Nucl.Phys. 60 (2008) 338

Branco, Felipe, Joaquim,
Rev.Mod.Phys. 84 (2012) 515

CPV



PhysRevLett117(2016)061804

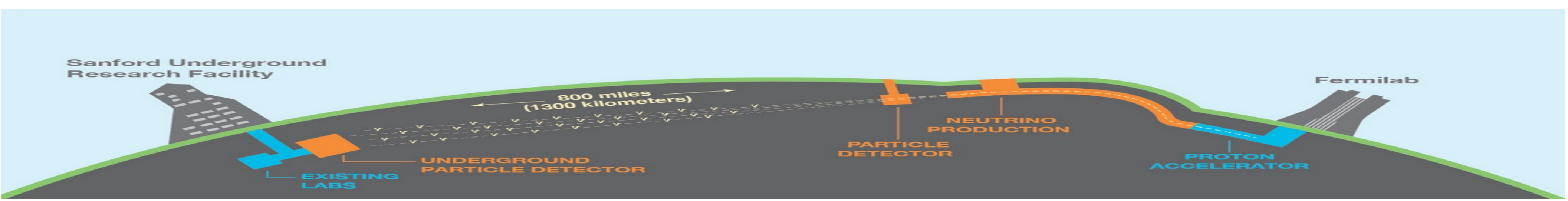
New J.Phys. 19 (2017) 9, 093005

PhysRevD97 (2018) 095026

2008.12769

DUNE
T2HK

Expected CP discovery Sensitivity: standard 3-nu vs Unitarity violation



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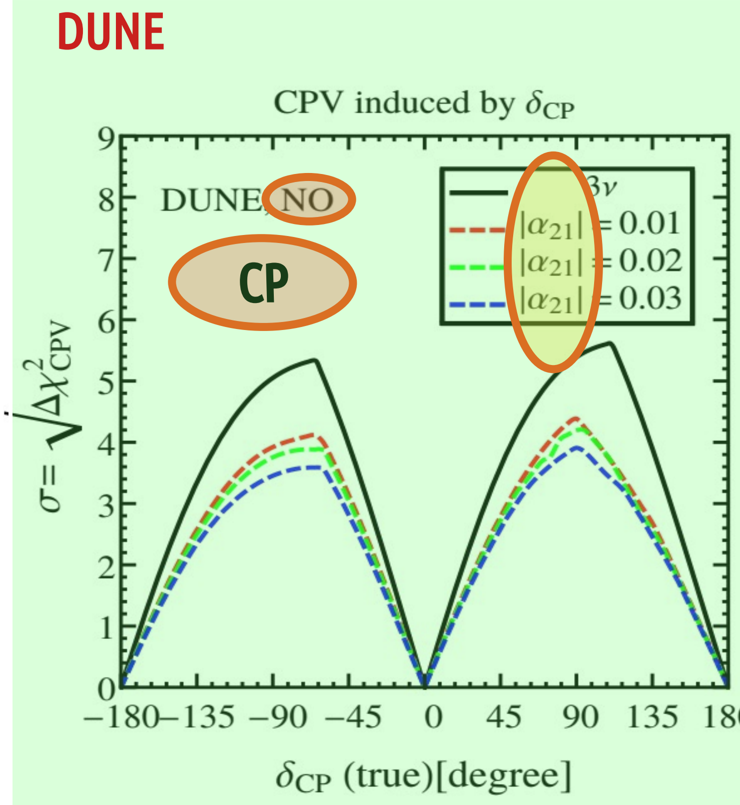

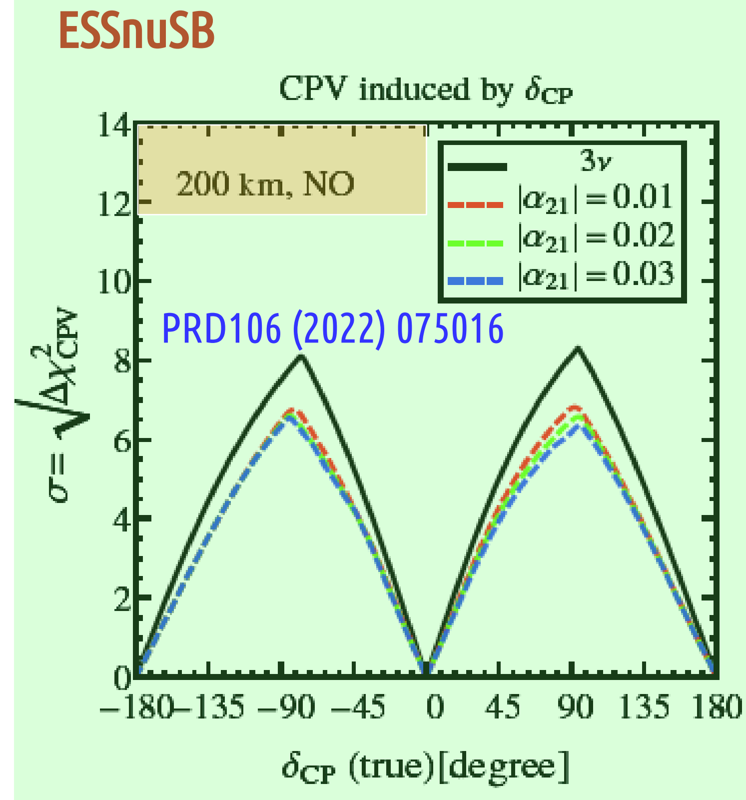

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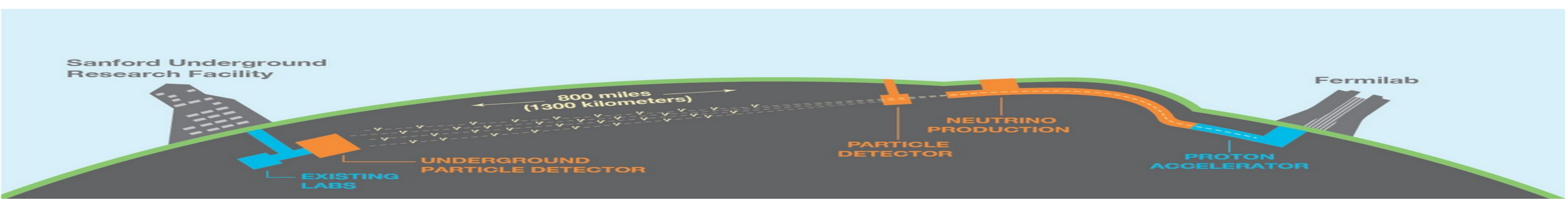

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TBM interpretation

Harrison, Scott
& Perkins 2002

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

θ_{13} → CP

systematic revamping

Chen et al
Phys.Lett. B753 (2016) 644
Phys.Rev. D94 (2016) 033002
JHEP 1807 (2018) 077
Phys.Lett. B792 (2019) 461
Phys.Rev. D99 (2019) 075005

TBM interpretation

Phys.Rev.D98(2018)055019

Harrison, Scott
& Perkins 2002

$$\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

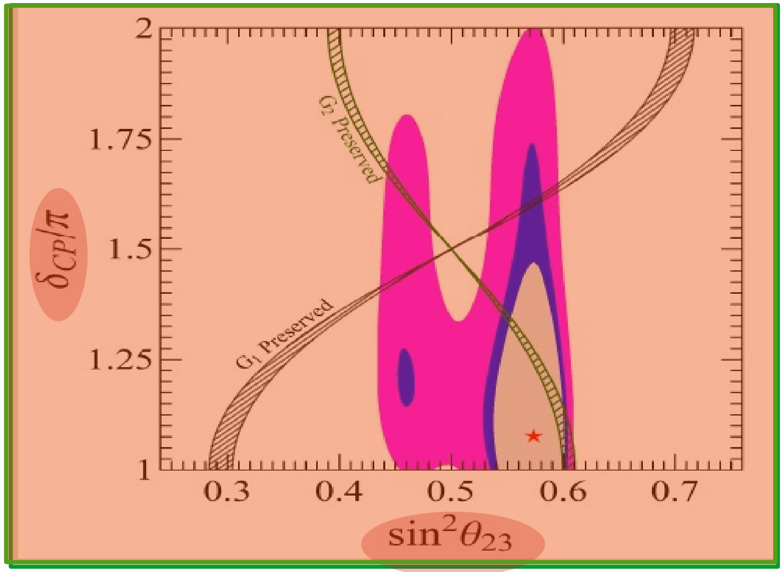
θ_{13} → CP

$$\sin^2\theta_{12}\cos^2\theta_{13} = \frac{1}{3}$$

$$\tan 2\theta_{23}\cos\delta_{CP} = \frac{\cos 2\theta_{13}}{\sin\theta_{13}\sqrt{2-3\sin^2\theta_{13}}}$$

an example

systematic revamping



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Bi-Large lepton mixing pattern

$$\begin{bmatrix} 1-\frac{1}{2}\lambda^2 & -\lambda e^{i\phi} & A\lambda^3 e^{i\phi} \\ \lambda e^{-i\phi} & 1-\frac{1}{2}\lambda^2 & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1-\frac{5\lambda^2}{2} & 2\lambda & -\lambda \\ -2\lambda+3\lambda^2 & 1-\frac{13\lambda^2}{2} & 3\lambda \\ \lambda+6\lambda^2 & -3\lambda+2\lambda^2 & 1-5\lambda^2 \end{bmatrix}$$

Largest Q-mixing similar to smallest L-mixing
Cabibbo angle as universal seed for flavor mixing

Phys.Rev. D86 (2012) 051301
Phys.Rev.D87 (2013) 053013
Phys.Lett. B748 (2015) 1-4

$\sin\theta_{12}^{\rm CKM} = \lambda$ and $\sin\theta_{23}^{\rm CKM} = A\lambda^2$, where $\lambda = 0.22453 \pm 0.00044$, $A = 0.836 \pm 0.015$

predicting solar & atm

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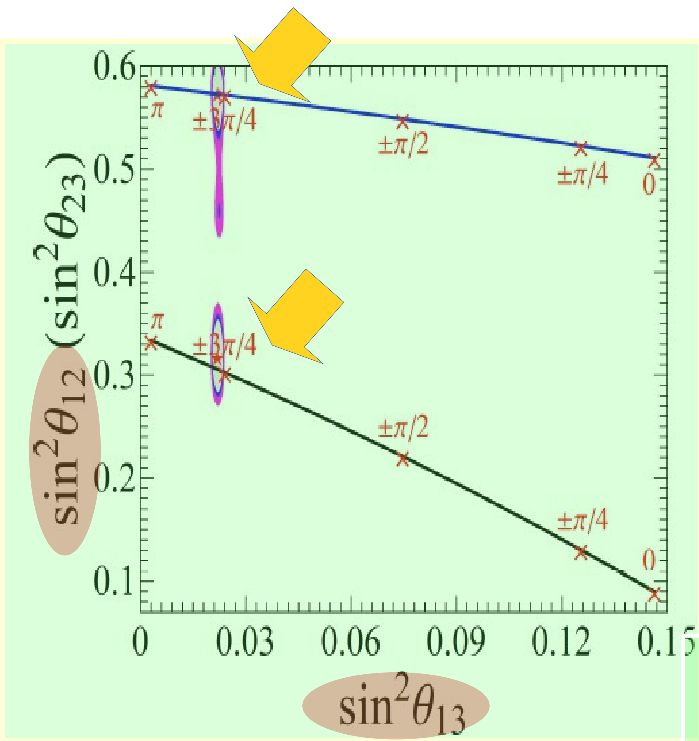


From Phys.Lett. B792 (2019) 461

looser realization
Phys.Lett.B 796 (2019) 162

Many other patterns, e.g. trimaximal, most can be probed at DUNE or T2HK

e.g. Phys.Rev.D97(2018)095025

robustness unitarity seesaw probe

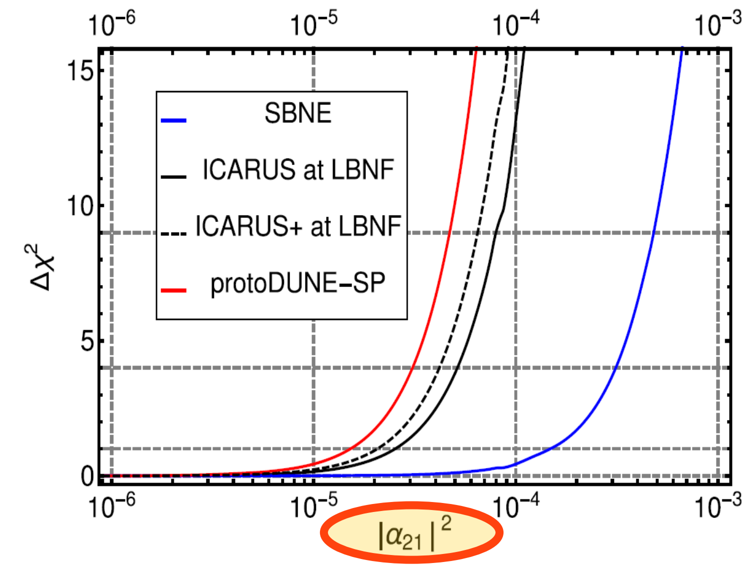
J.V. Phys.Lett. B199 (1987) 432
Miranda & J.V. Nucl.Phys. B908 (2016) 436
Escrihuela et al, Phys.Rev. D92 (2015) 053009
New J. Phys. 19 (2017) 093005

$$\begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U$$

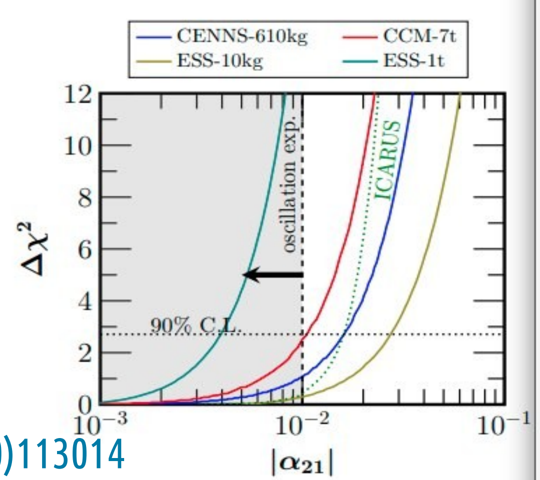
near measurements needed

Shao-Feng Ge et al
Phys.Rev. D95 (2017) 033005

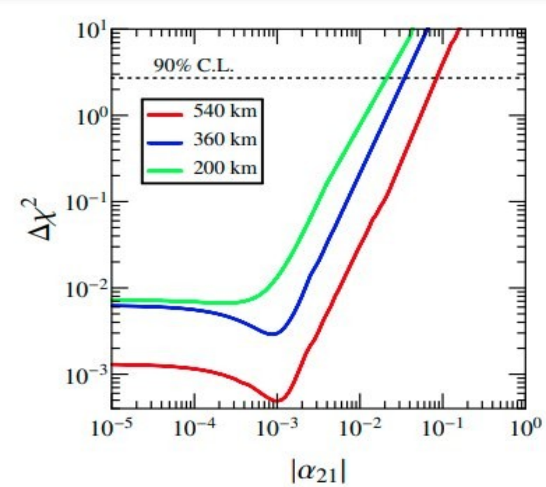
	One parameter (1 d.o.f.)		All parameters (6 d.o.f.)	
	90% C.L.	3σ	90% C.L.	3σ
	Neutrinos only			
$\alpha_{11}>$	0.98	0.95	0.96	0.93
$\alpha_{22}>$	0.99	0.96	0.97	0.95
$\alpha_{33}>$	0.93	0.76	0.79	0.61
$\lvert\alpha_{21}\rvert<$	1.0×10^{-2}	2.6×10^{-2}	2.4×10^{-2}	3.6×10^{-2}
$\lvert\alpha_{31}\rvert<$	4.2×10^{-2}	9.8×10^{-2}	9.0×10^{-2}	1.3×10^{-1}
$\lvert\alpha_{32}\rvert<$	9.8×10^{-3}	1.7×10^{-2}	1.6×10^{-2}	2.1×10^{-2}



PRD97 (2018) 095026



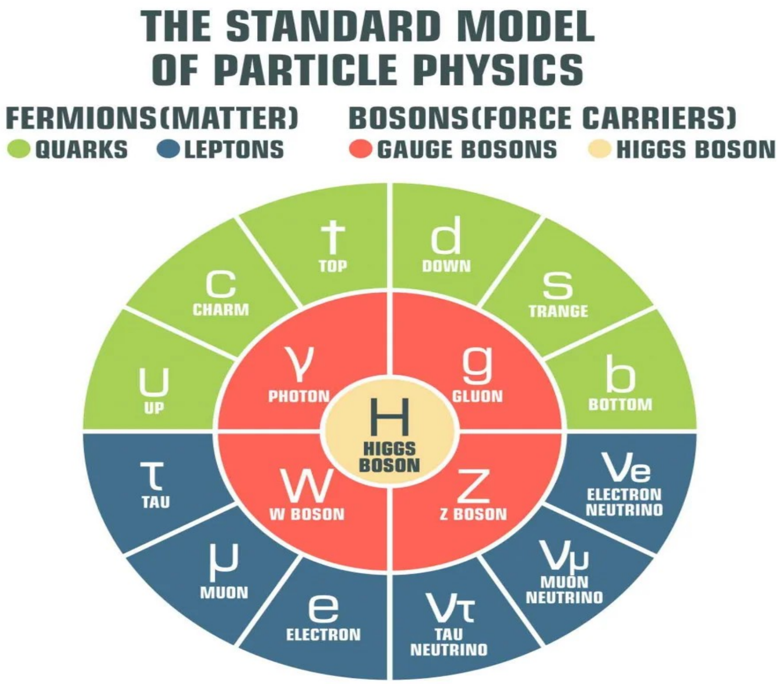
PhysRevD102(2020)113014



ESSnuB

PRD106 (2022) 075016

Higgs discovery is **not the last brick**



- neutrinos are massless because the SM does not contain right-handed neutrinos nor any other ingredient capable of inducing neutrino mass
- SM **fails** in the neutrino sector

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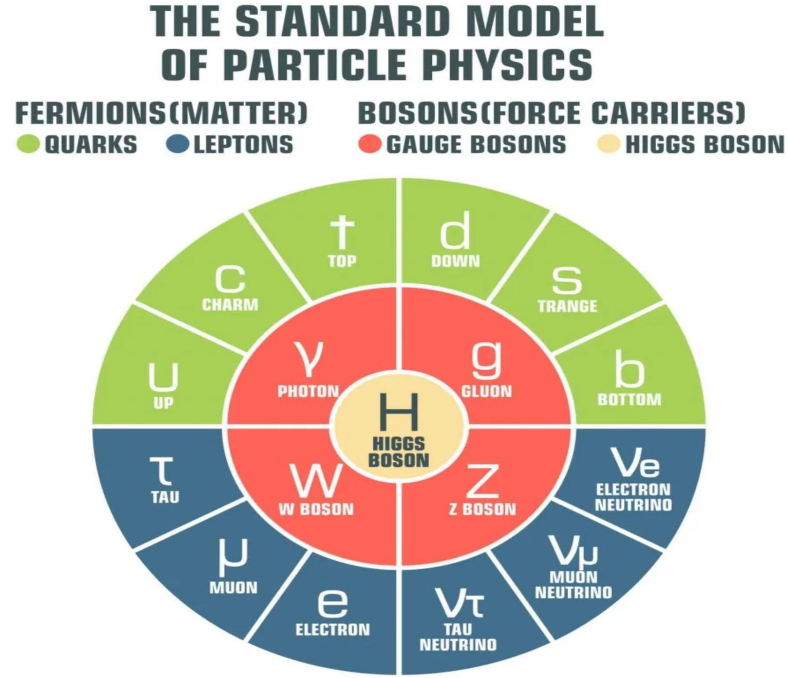

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SM drawbacks

- **neutrino mass**
- **EWSB,**
- **Flavor**
- **unification,**
- **Gravity**
-

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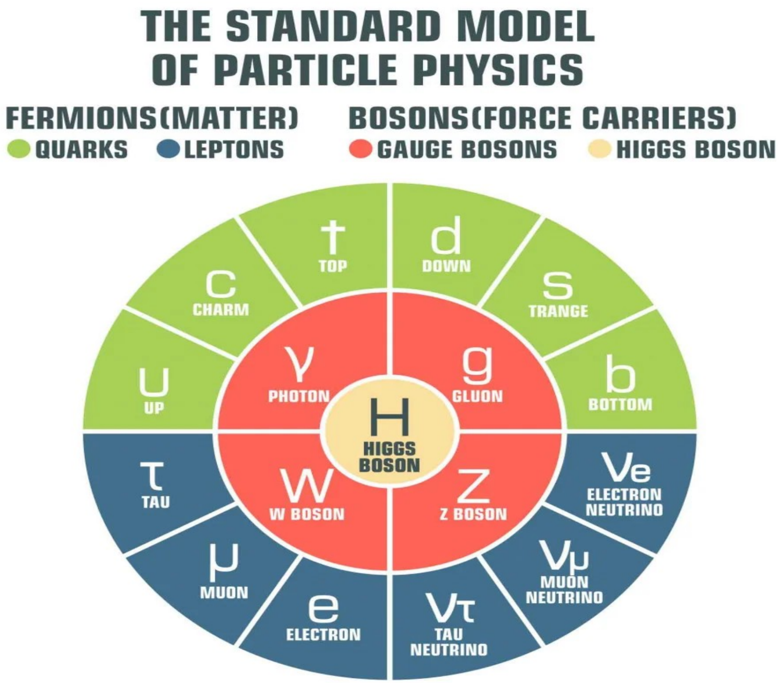

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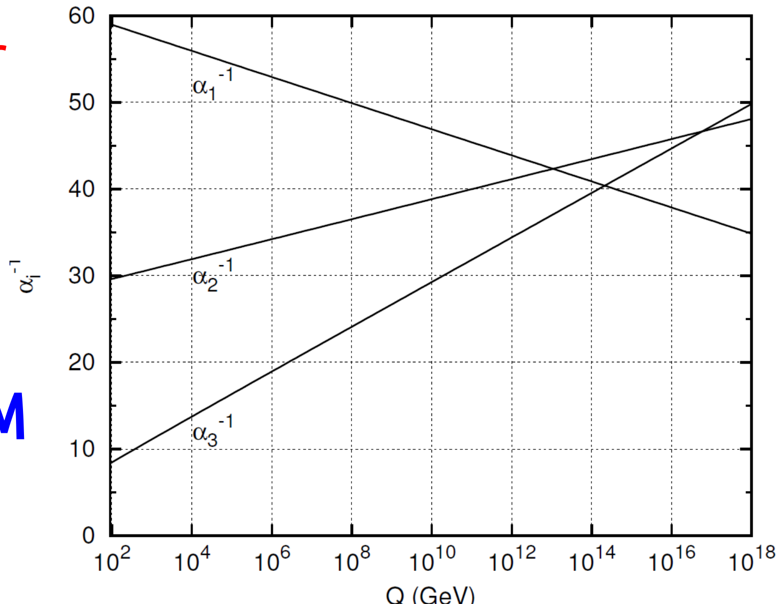

- **neutrino mass**
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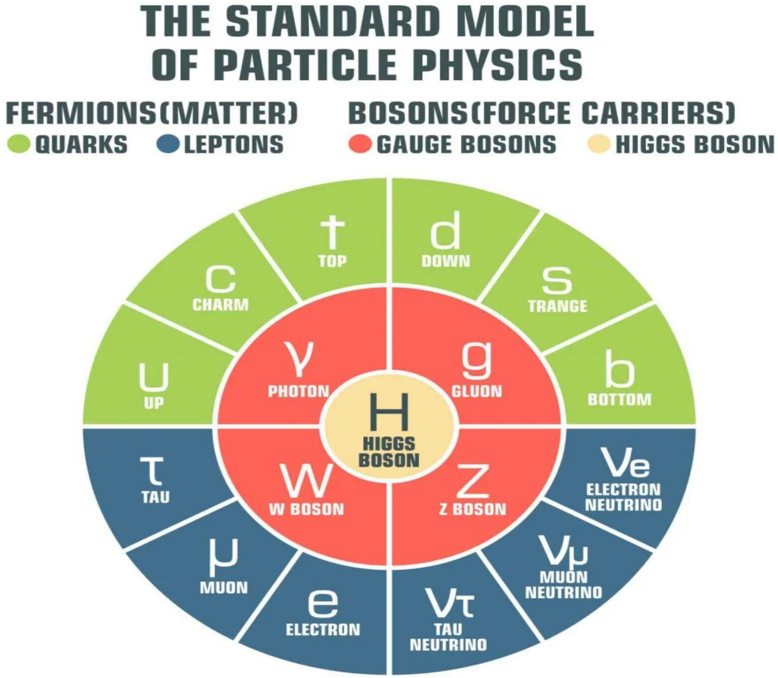

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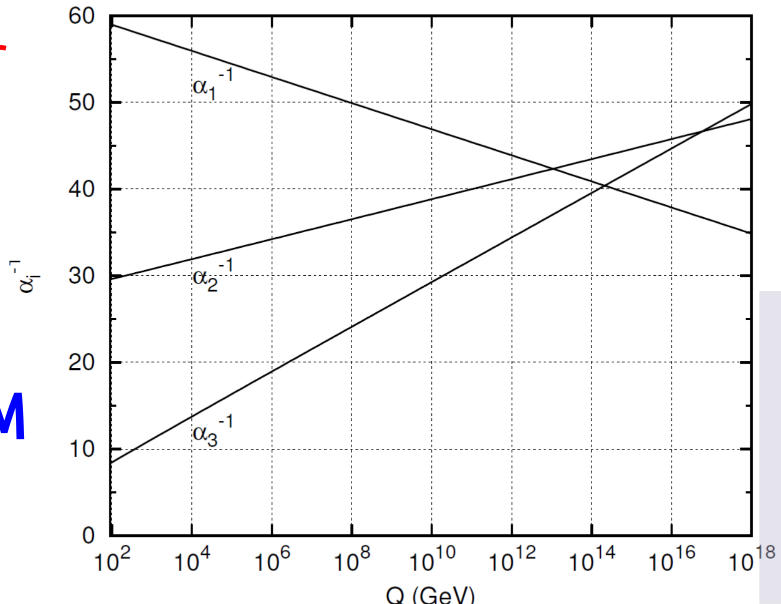


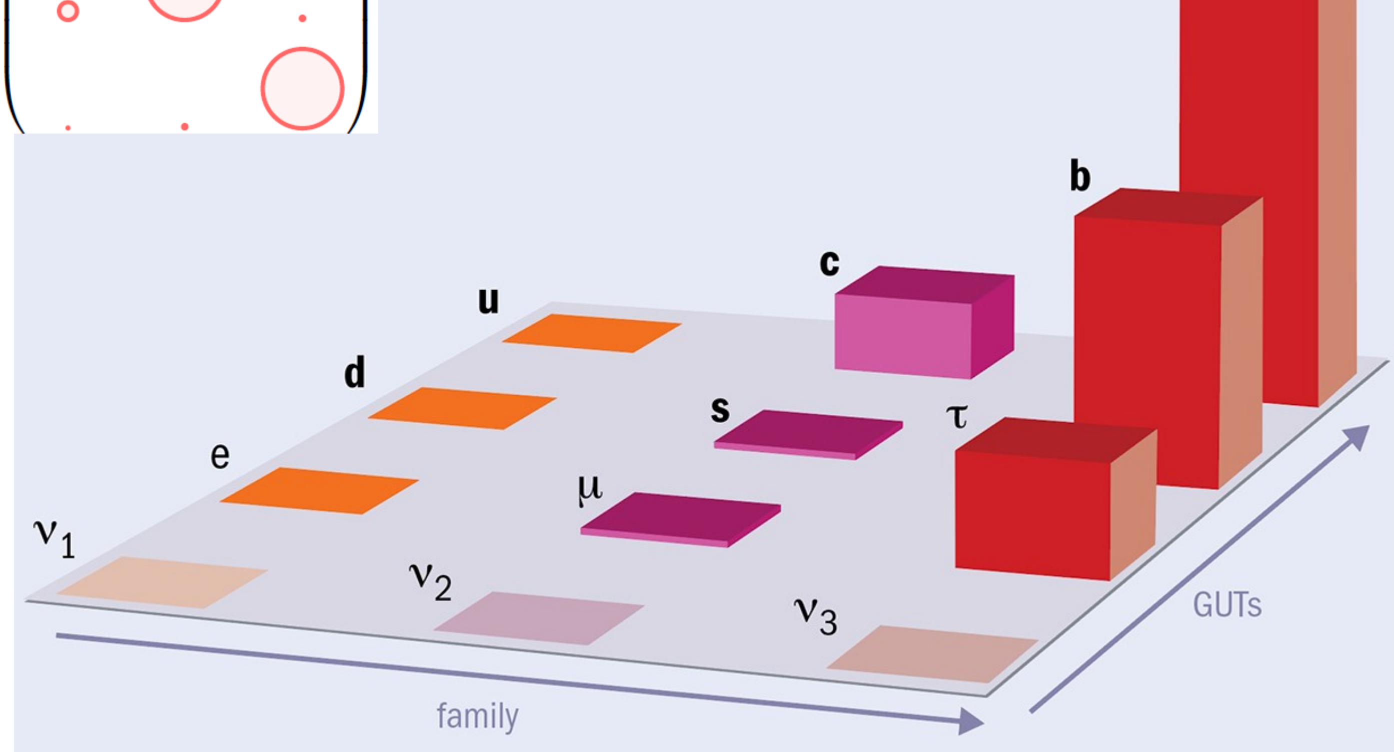
- **neutrino mass**
- **EWSB**
- **Flavor** …
- **unification**
- **Gravity**
-

Cosmology, e.g.

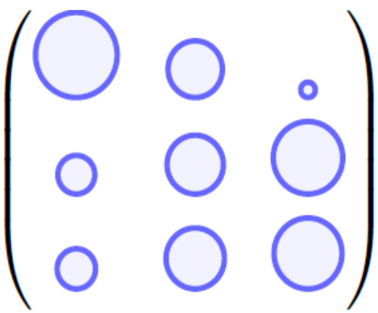
dark matter, baryogenesis, inflation, dark energy


Hierarchies of quark
and lepton masses
t/10
b
c
u
d
s
τ
e
μ
ν1
ν2
ν3
GUTs
family

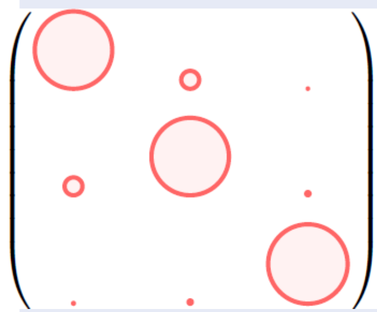
Hierarchies of quark and lepton masses



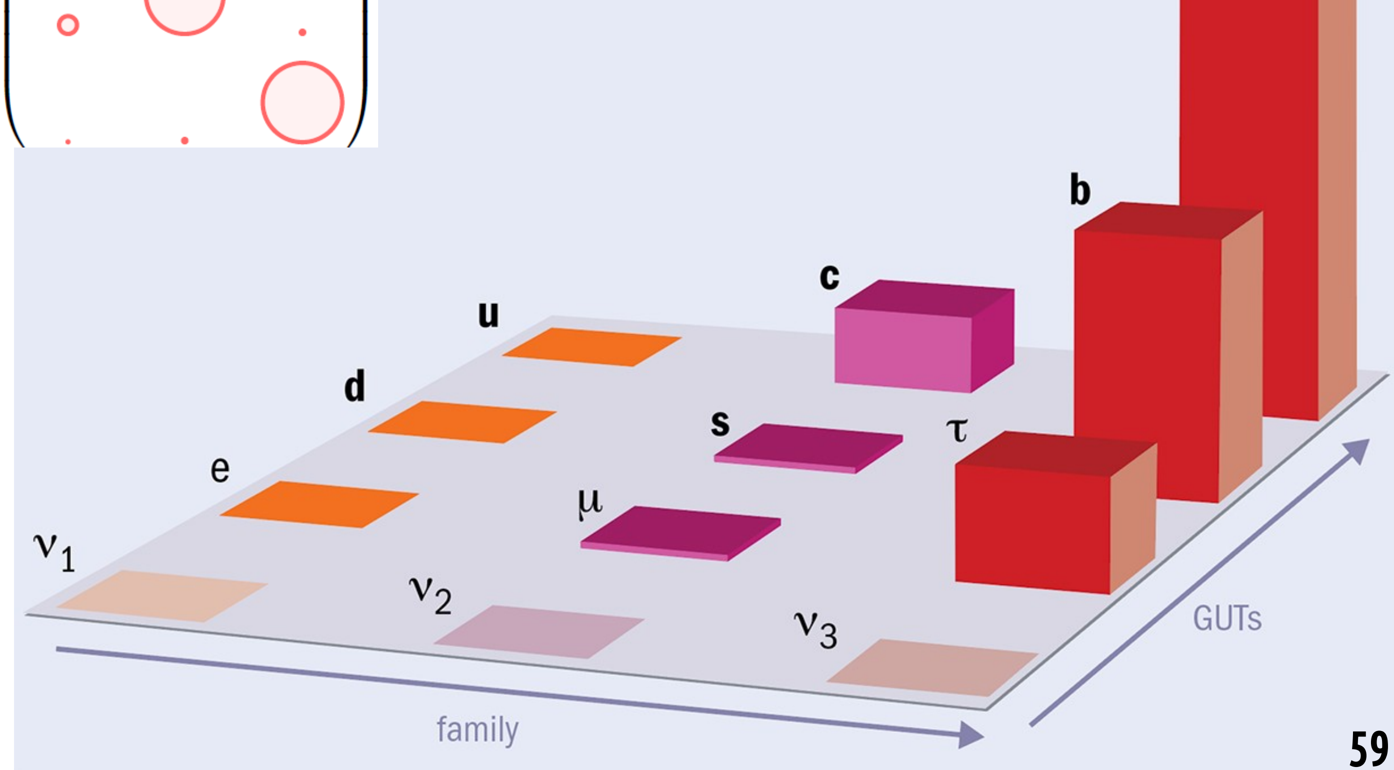
Pattern of quark & lepton mixings



versus



Hierarchies of quark and lepton masses



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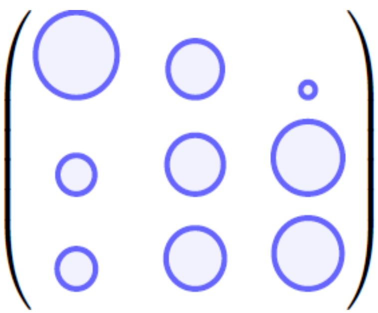

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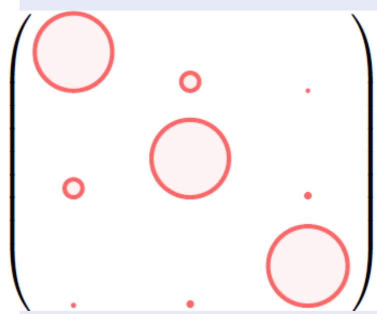
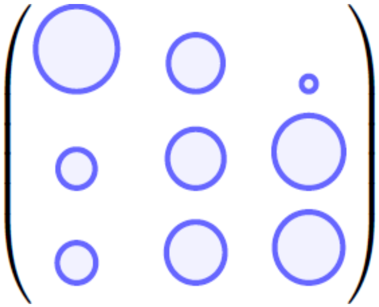


understanding flavor may require a more radical departure involving extra space-time dimensions

Q/L mixing pattern



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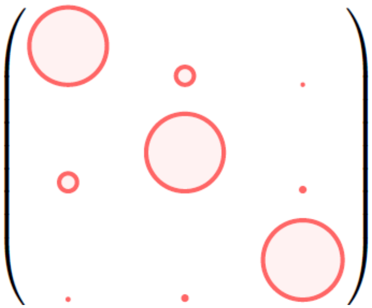

flavour legacy of oscillations

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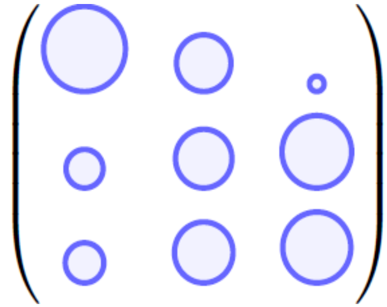

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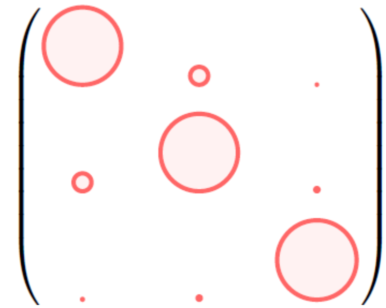
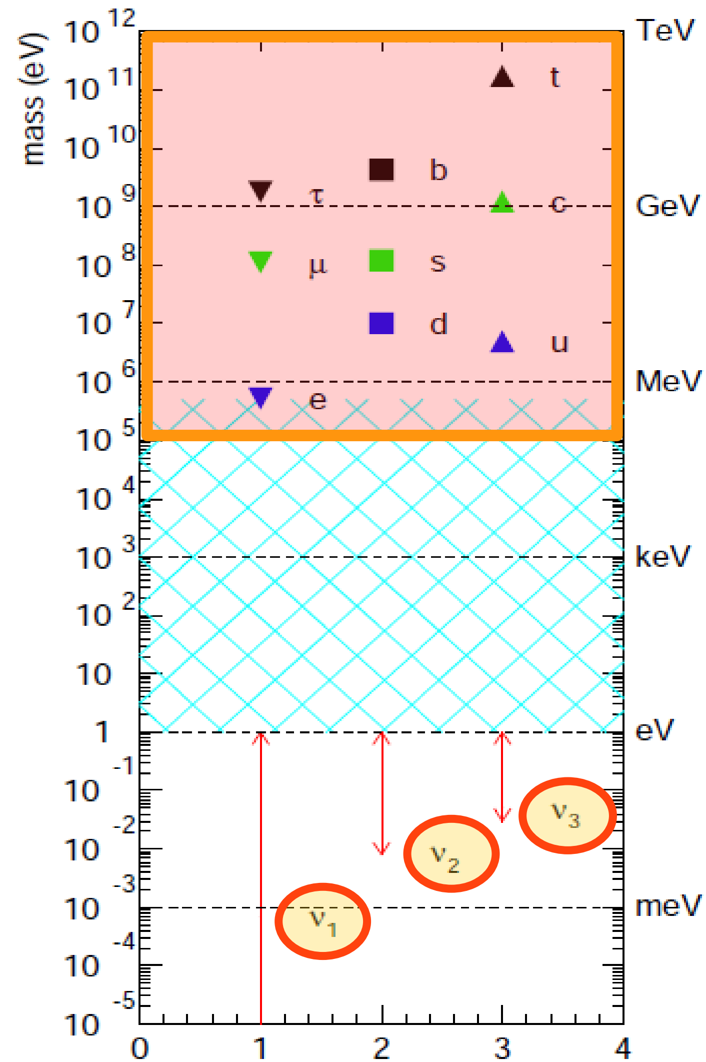


Q/L mass hierarchies



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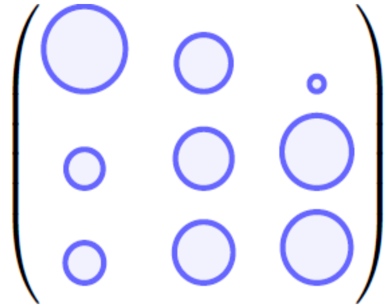

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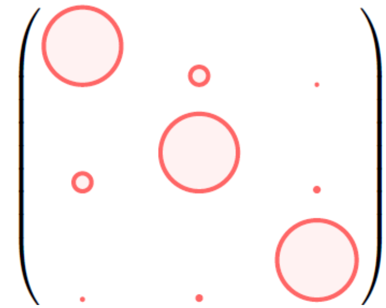

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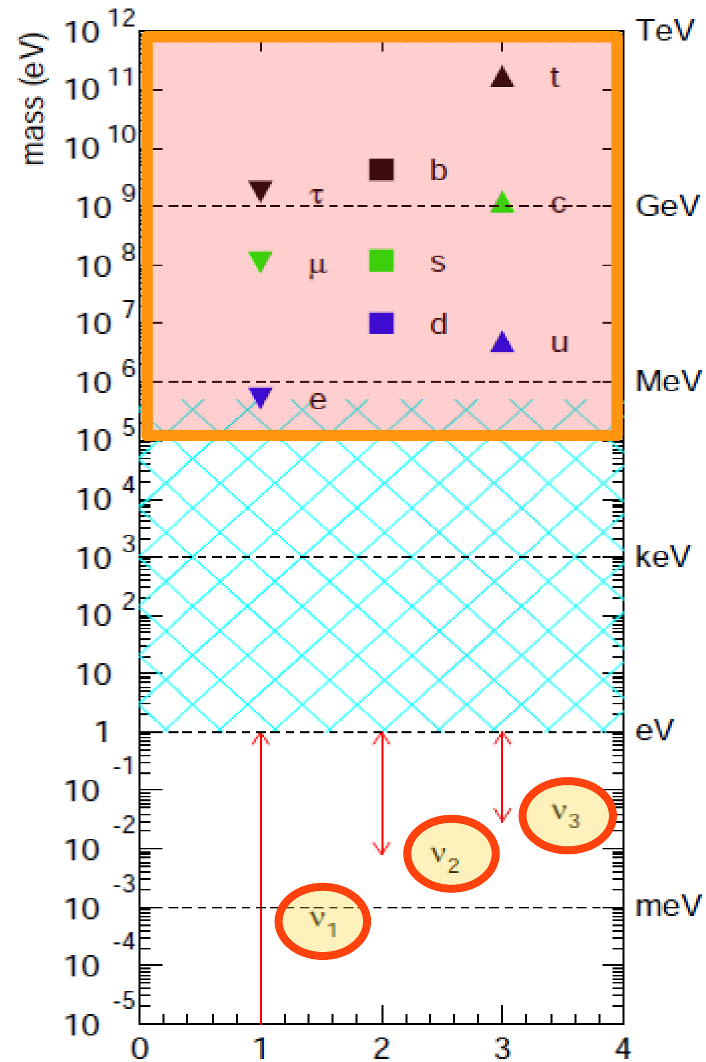

Morisi et al Phys.Rev. D84 (2011) 036003

King et al Phys. Lett. B 724 (2013) 68

Morisi et al Phys.Rev. D88 (2013) 036001

Bonilla et al Phys.Lett. B742 (2015) 99

Reig, JV, Wilczek Phys.Rev. D98 (2018) 095008

5D warped flavour dynamics

Randall-Sundrum Phys.Rev.Lett. 83 (1999) 3370

- mass hierarchies from geometry

  Arkani-Hamed & Schmaltz hep-ph/9903417

- mixing angles from family symmetry

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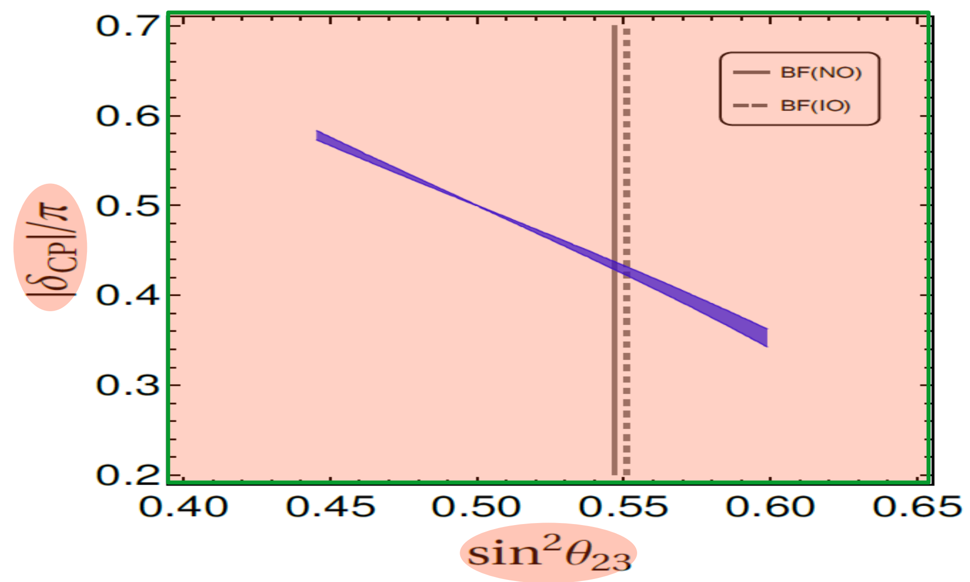
TM mixing pattern predicted from T’



TM1

$$\cos^2\theta_{12}\cos^2\theta_{13} = \frac{2}{3}$$

TM1 pattern

$$\cos\delta_{CP} = \frac{(3\cos 2\theta_{12} - 2)\cos 2\theta_{23}}{3\sin 2\theta_{23}\sin 2\theta_{12}\sin\theta_{13}}$$

Chen et al Phys. Rev. D 102, 095014 (2020)

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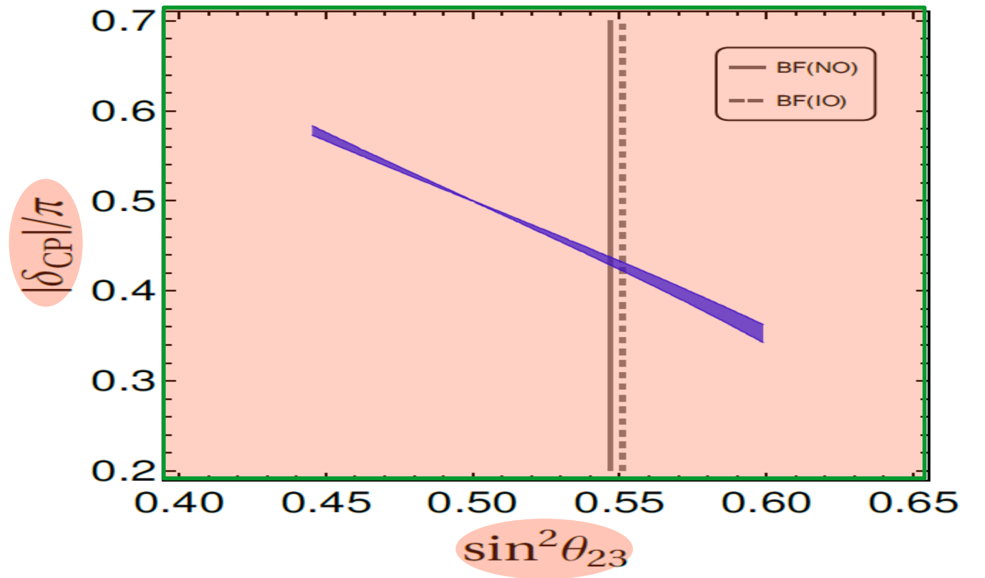

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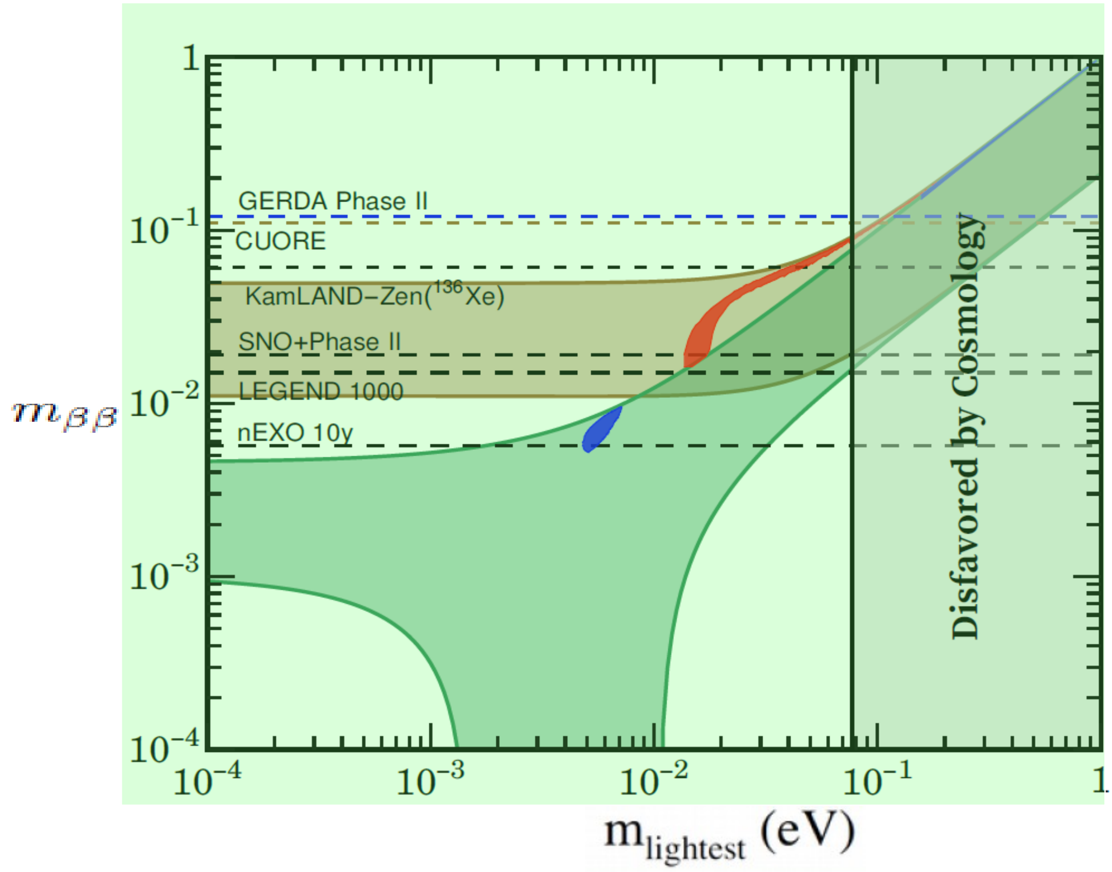

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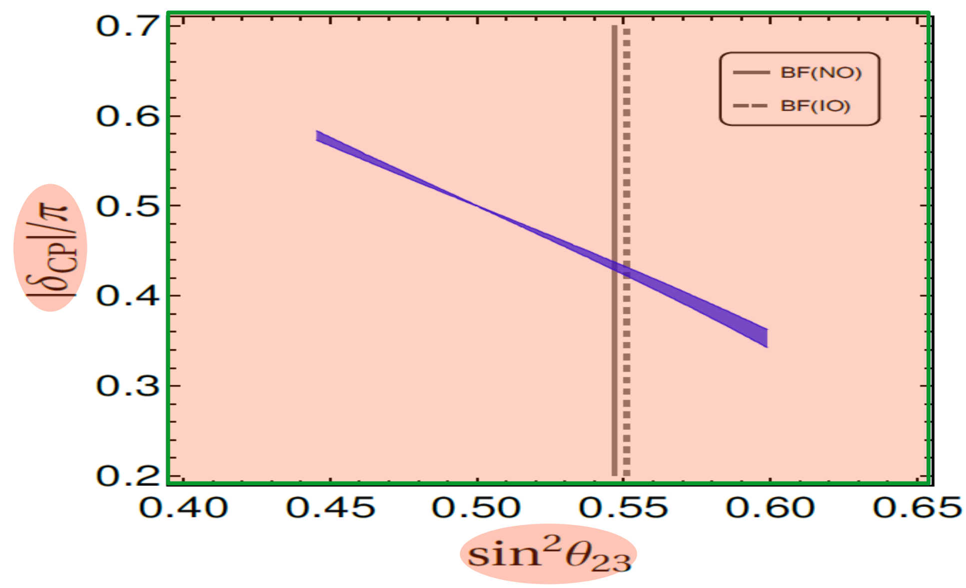

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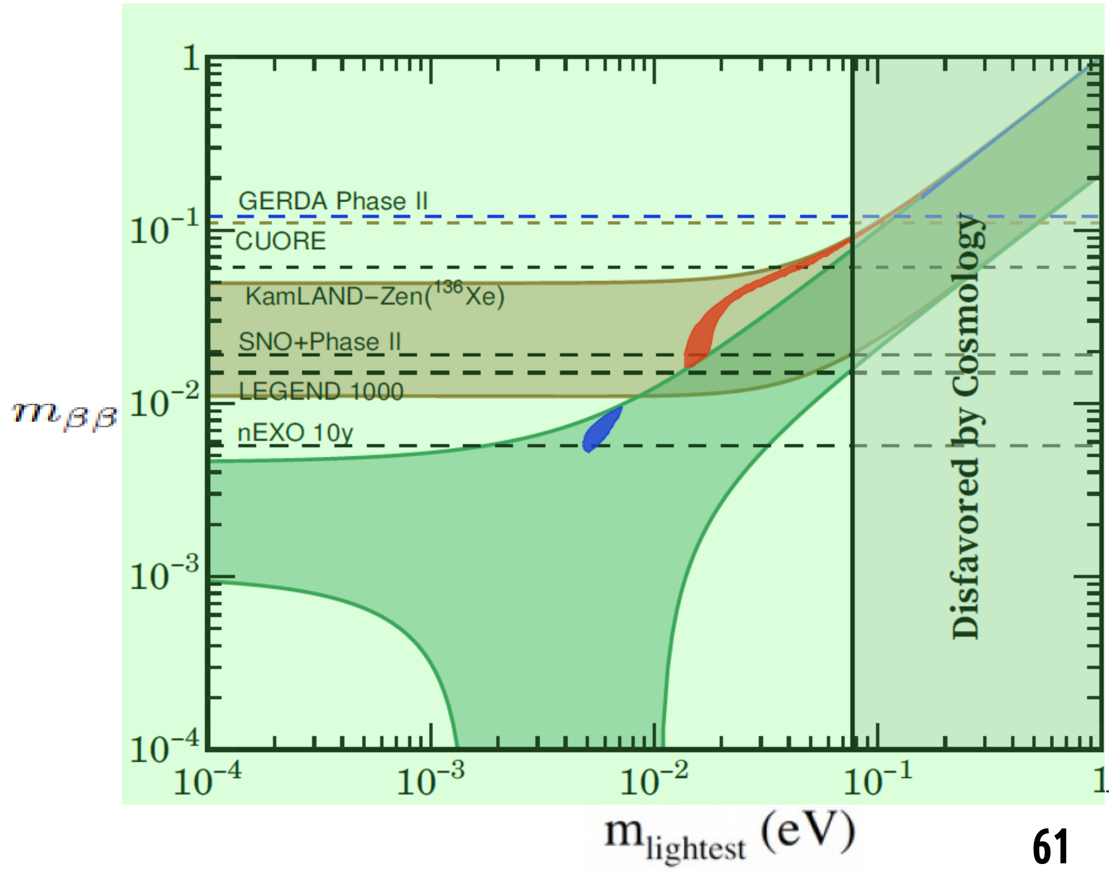


TM2 pattern

Dirac neutrino alternative
Chen et al JHEP01(2016)007
Phys. Rev. D95 (2017) 095030
Phys.Lett. B771 (2017) 524

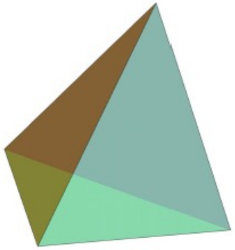
family symmetry from 6D Orbifold

$$\mathcal{M} = \mathbb{M}^4 \times (\mathbb{T}^2/\mathbb{Z}_2)$$

Phys.Lett.B 801 (2020) 135195

Phys.Rev.D 101 (2020) 11, 116012

A4 family symmetry “derived”



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Phys.Lett.B 801 (2020) 135195

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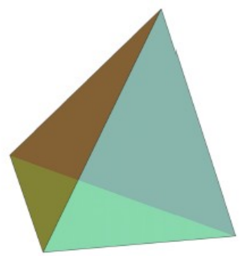
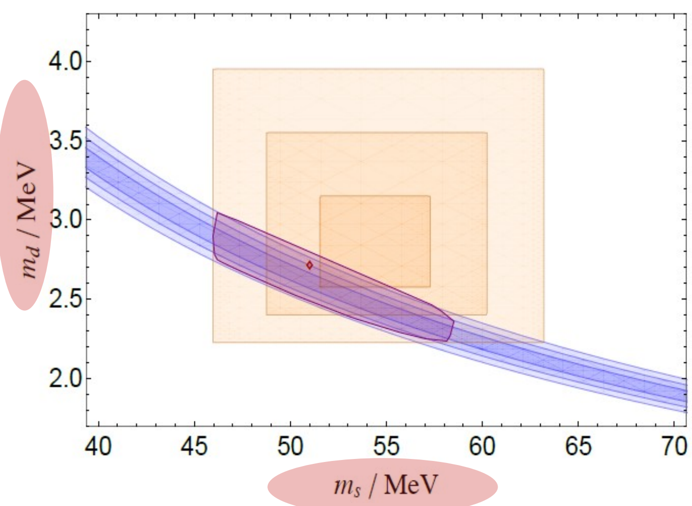
Golden Q-L relation

$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}}$$





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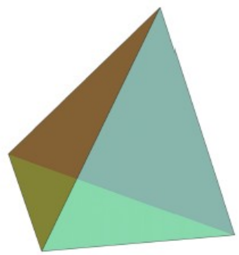
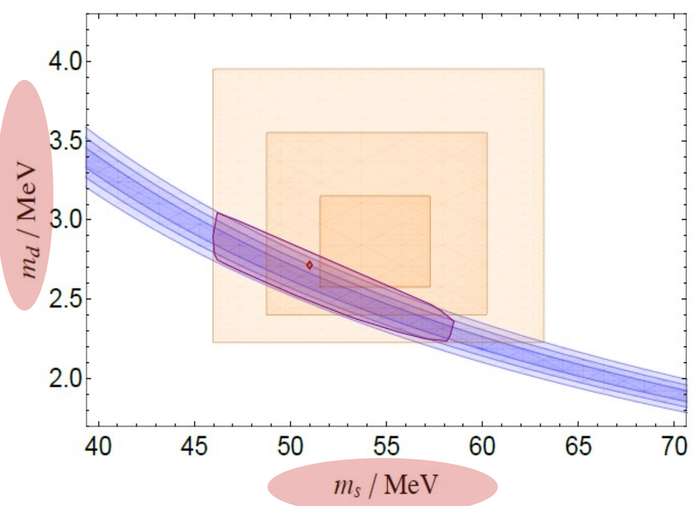
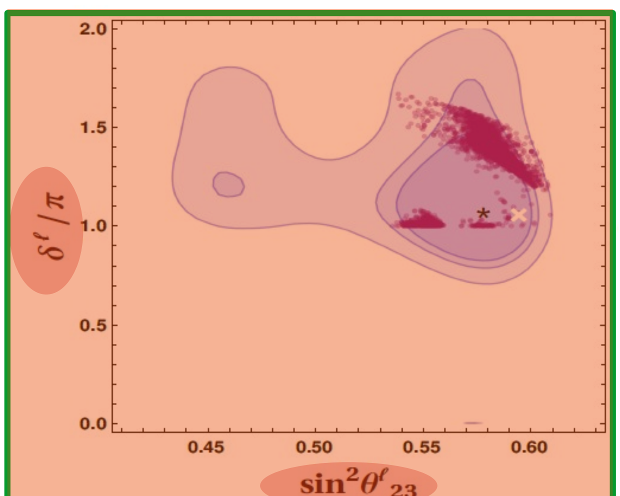
Golden Q-L relation

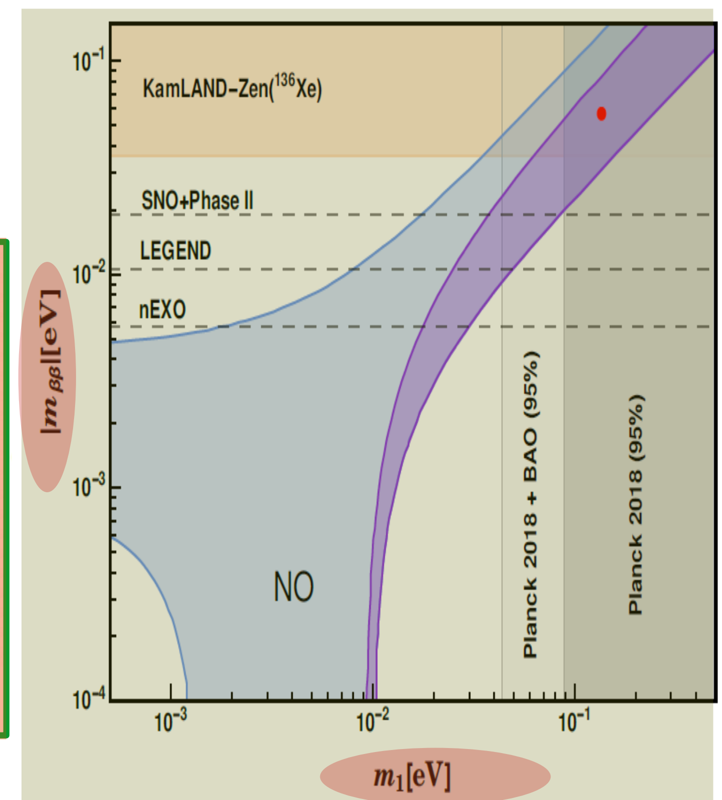
$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}}$$



NO

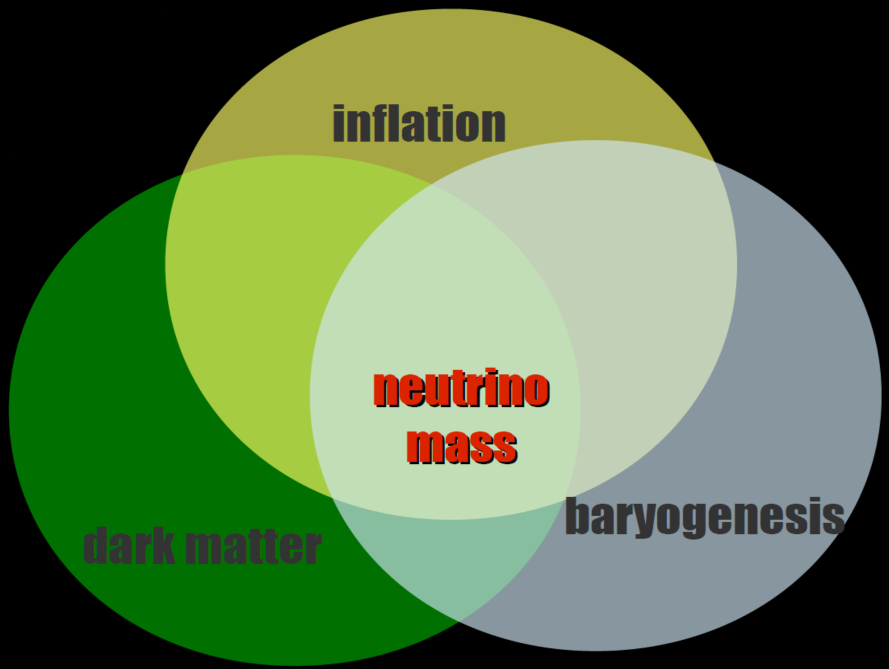






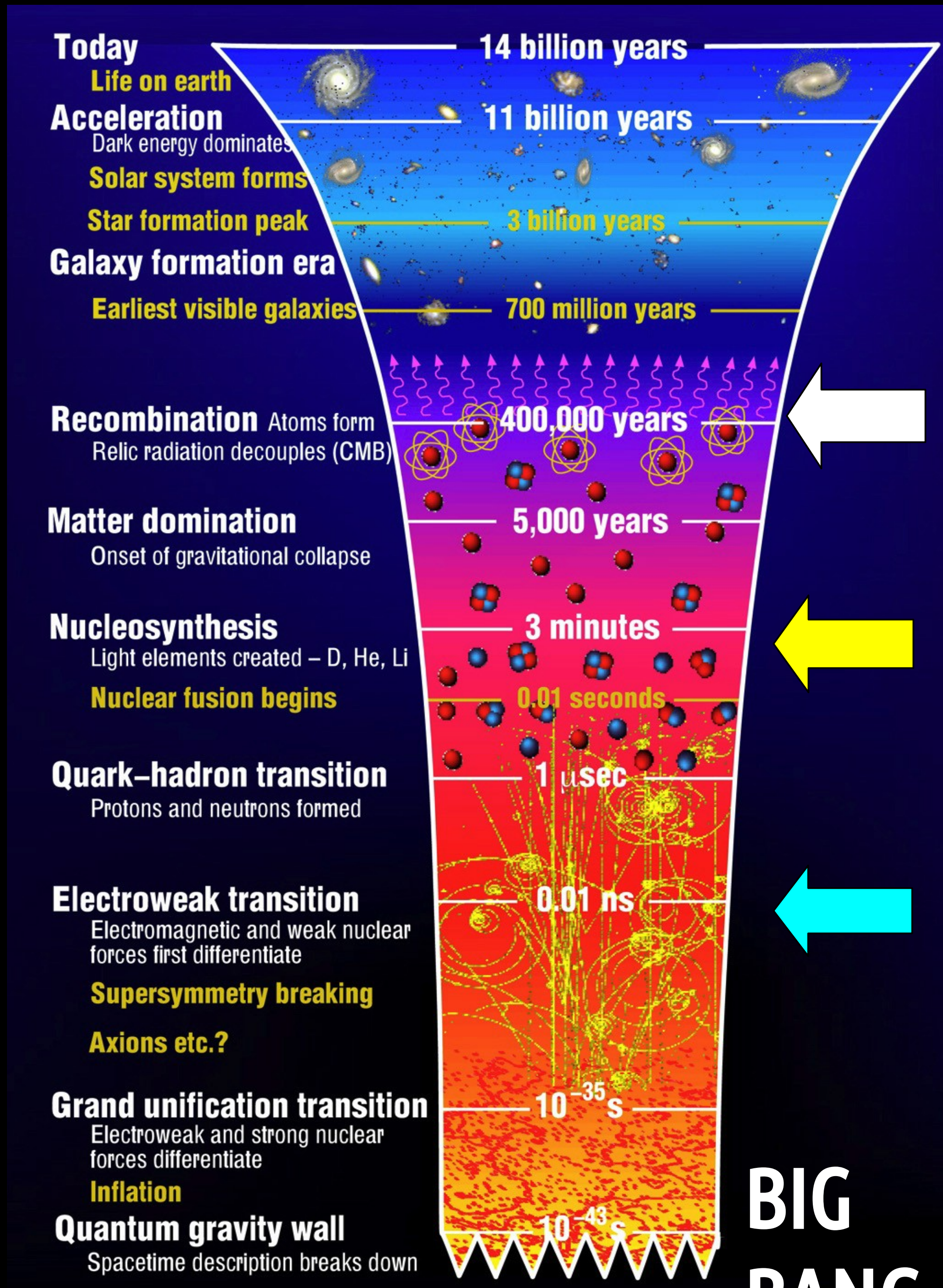
Good global fit of flavor observables

Phys.Rev.D 105 (2022) 055030



neutrinos can probe early stages of the Universe

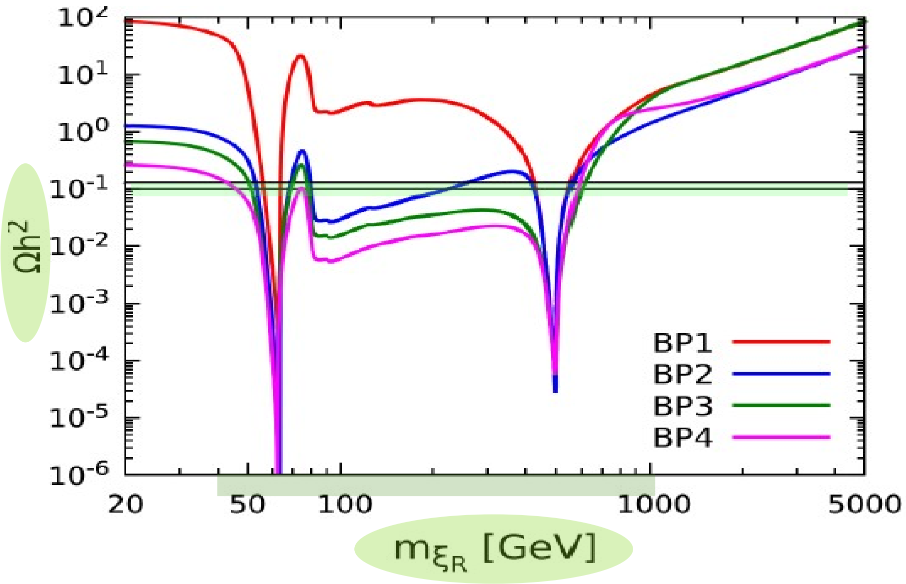
neutrinos may hold the key to “explaining” DM



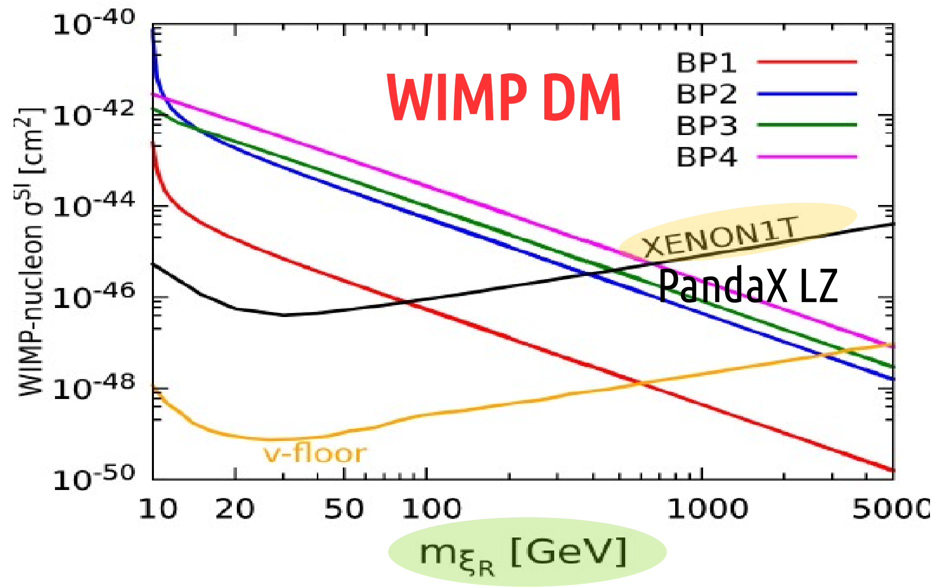
BIG BANG

dark inverse typeI seesaw mechanism

LambdaCDM Phys.Lett.B 821 (2021) 136609

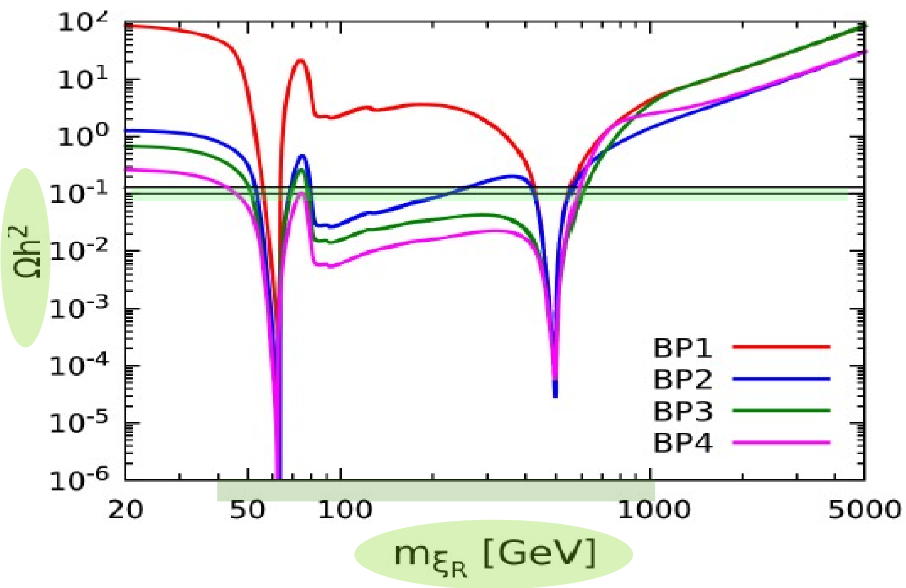


Xenon1T PhysRevLett.121.111302
PandaX Lux-Zepellin

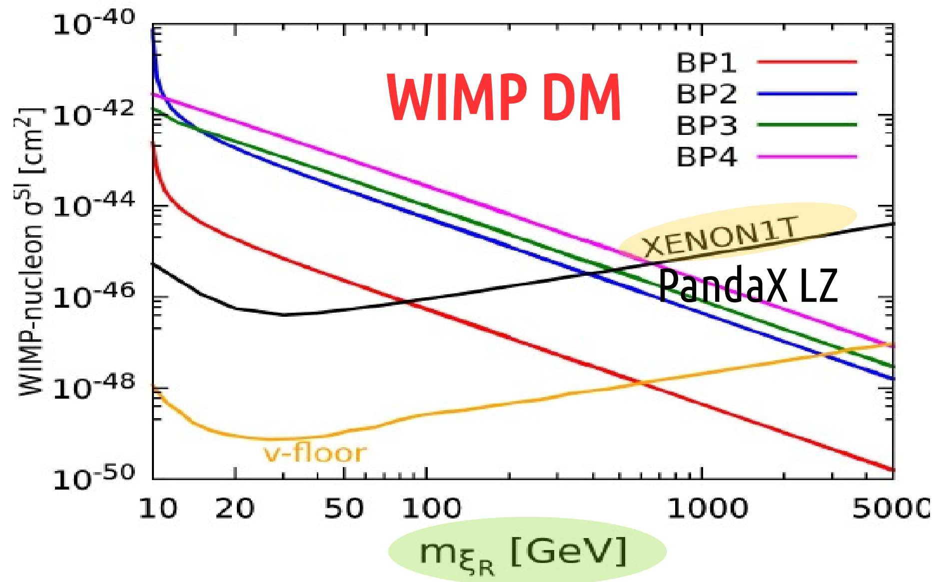


dark inverse typeI seesaw mechanism

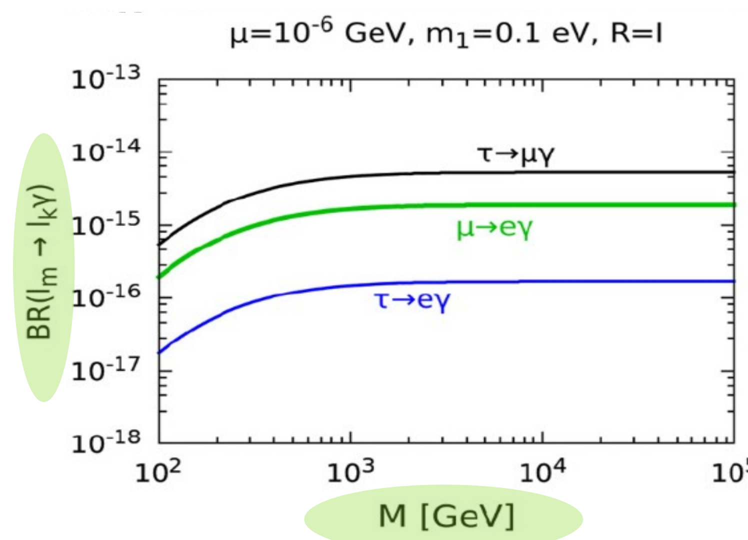
LambdaCDM Phys.Lett.B 821 (2021) 136609

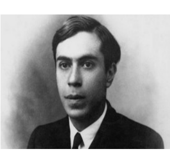


Xenon1T PhysRevLett.121.111302
PandaX Lux-Zepellin



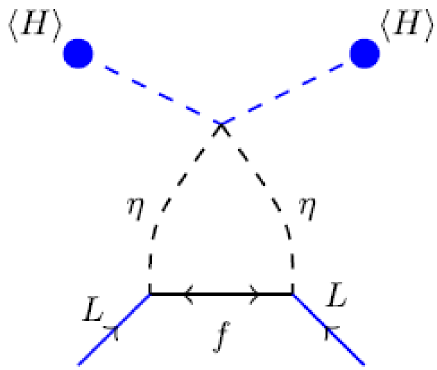
e.g. large cLFV from inverse type I seesaw
Mandal et al
Phys.Lett.B 821 (2021) 136609
(larger values possible)





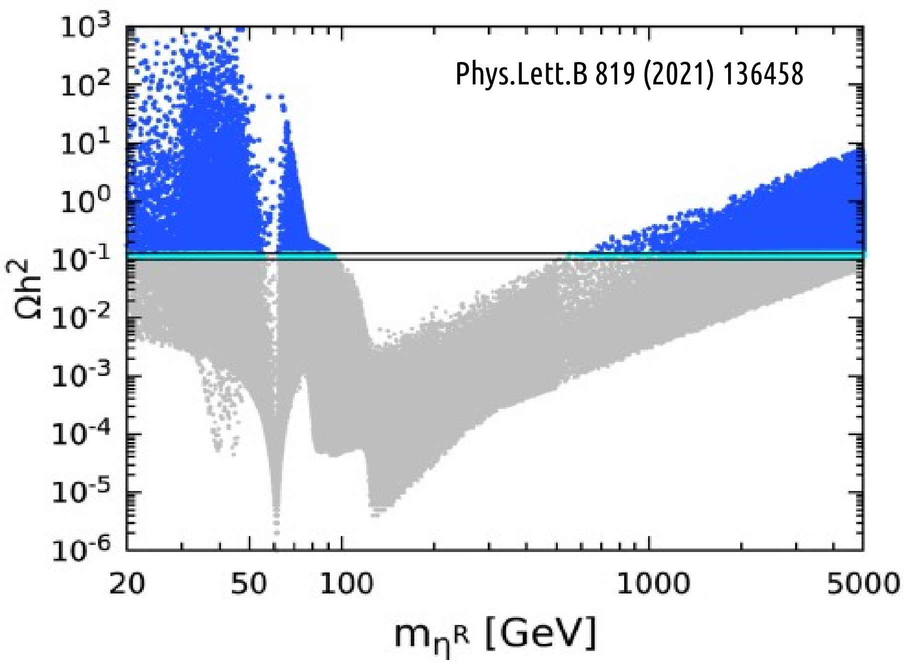
Scotoseesaw : combining WIMP & seesaw paradigms

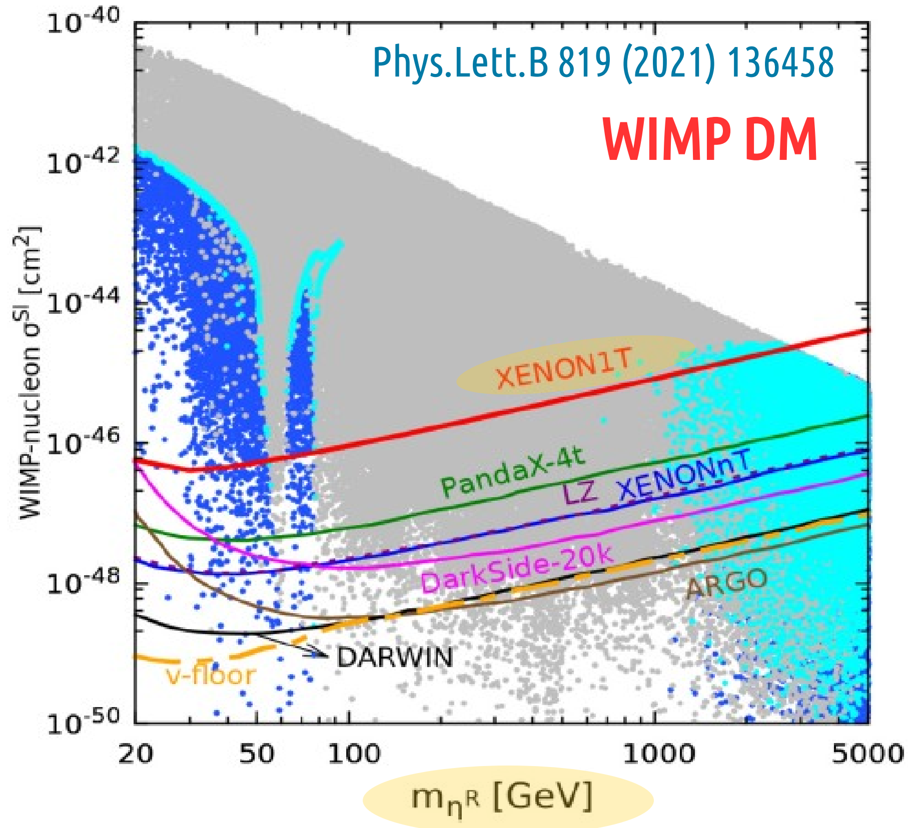
Loop solar scale



$$\Delta m^2_{\rm ATM} = \left(\frac{v^2}{2M_N}\mathbb{Y}_N^2\right)^2, \quad \Delta m^2_{\rm SOL} \approx \left(\frac{1}{32\pi^2}\right)^2 \left(\frac{\lambda_5 v^2}{M_f^2 - m_\eta^2} M_f \mathbb{Y}_f^2\right)^2$$

Tree atm scale from type-I seesaw

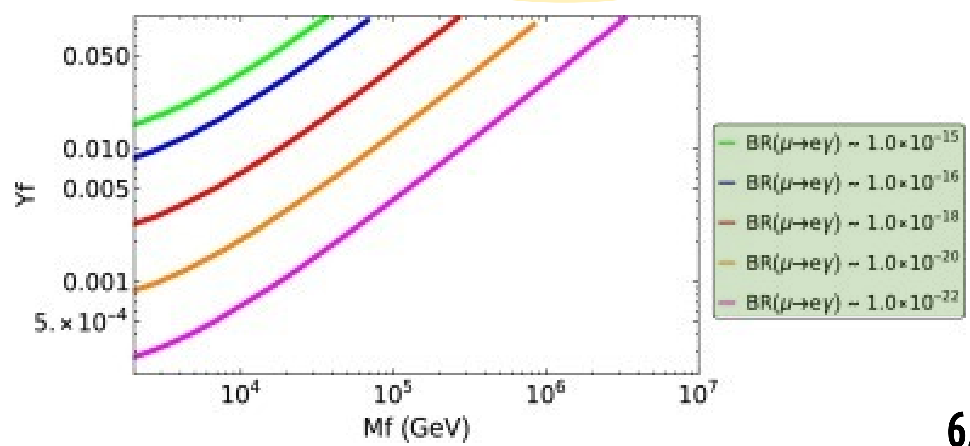




$0\nu\beta\beta$ **lower bound**

cLFV from "dark" loops

PLB789 (2019) 132

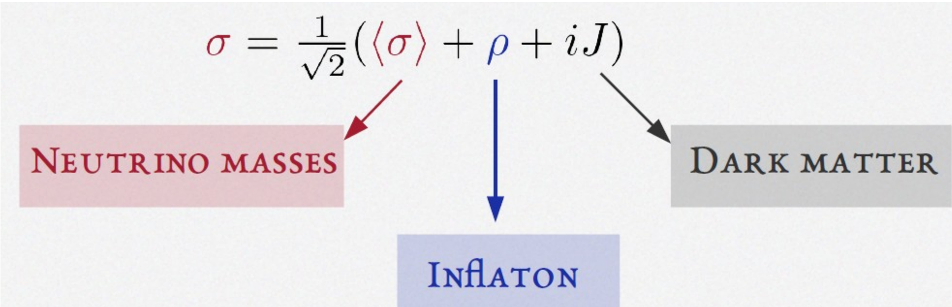


majoron dark matter

DM Berezinsky, Valle PLB318 (1993) 360

Inflation Boucenna, Morisi, Shafi, Valle Phys.Rev. D90 (2014) 055023

LG Aristizabal et al JCAP 1407 (2014) 052



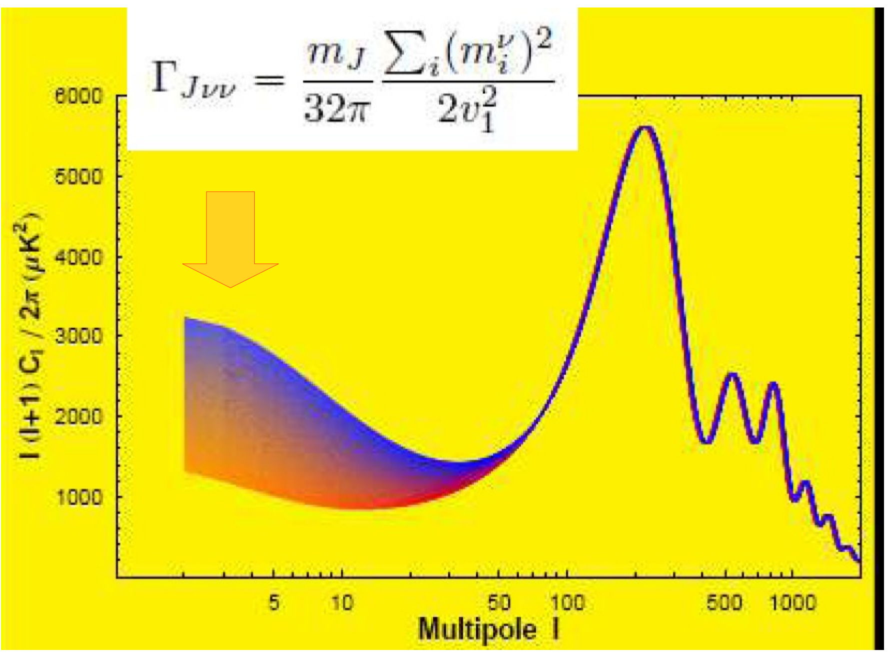
X-rays from DM decay

$\mathbf{J} \to \gamma\gamma$

Lattanzi et al PRD88 (2013) 063528

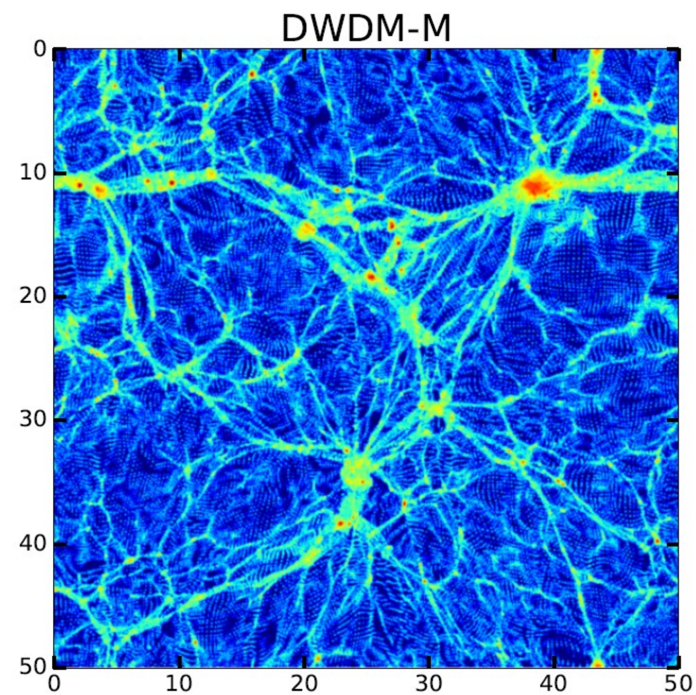
Consistency with CMB

Lattanzi & Valle, PRL99 (2007) 121301



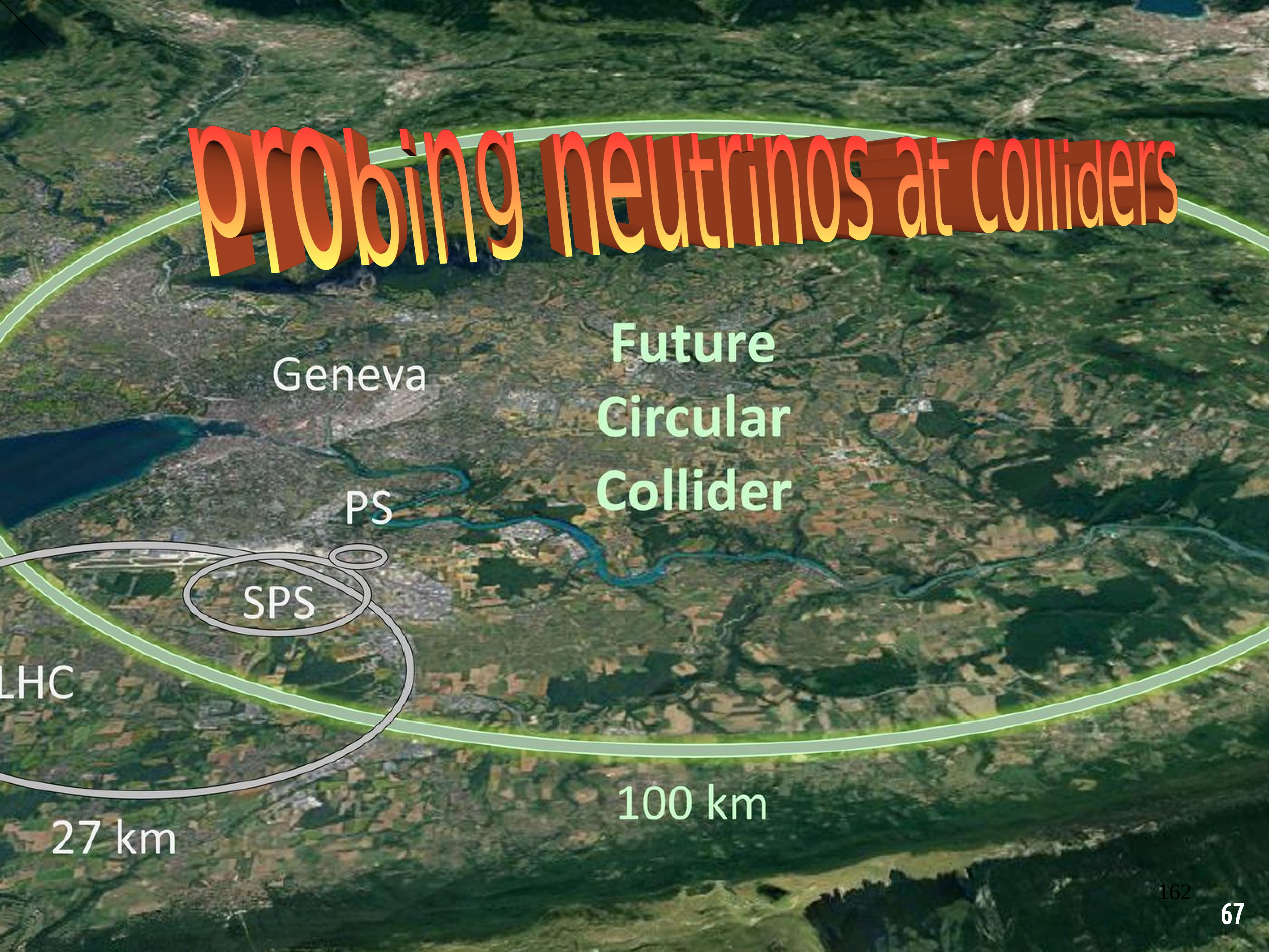
large scale structure

Kuo et al JCAP 1812 (2018) 026



Reig, Yamada, Valle JCAP 09 (2019) 029

probing neutrinos at colliders

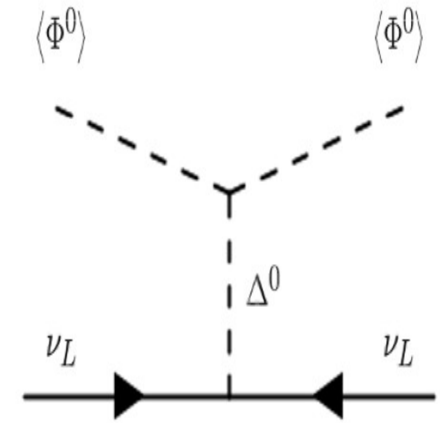


simplest seesaw

current oscillation data
can reconstruct triplet seesaw
so that it can be tested at
high-energies

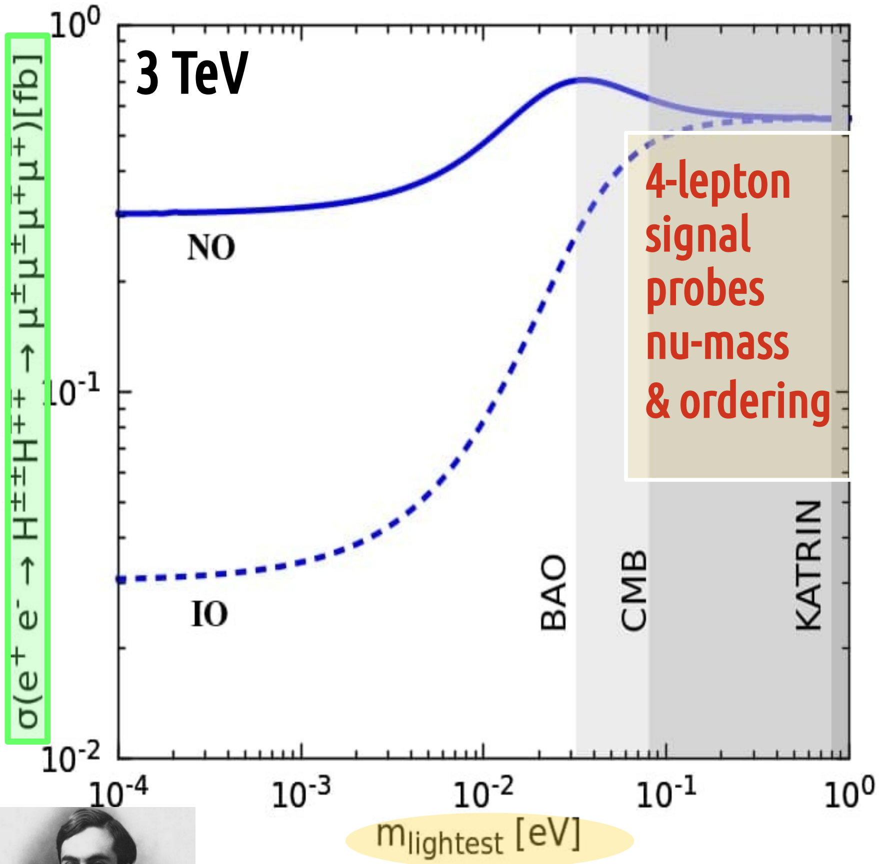
Schechter & JV PRD22 (1980) 2227

PRD25 (1982) 774

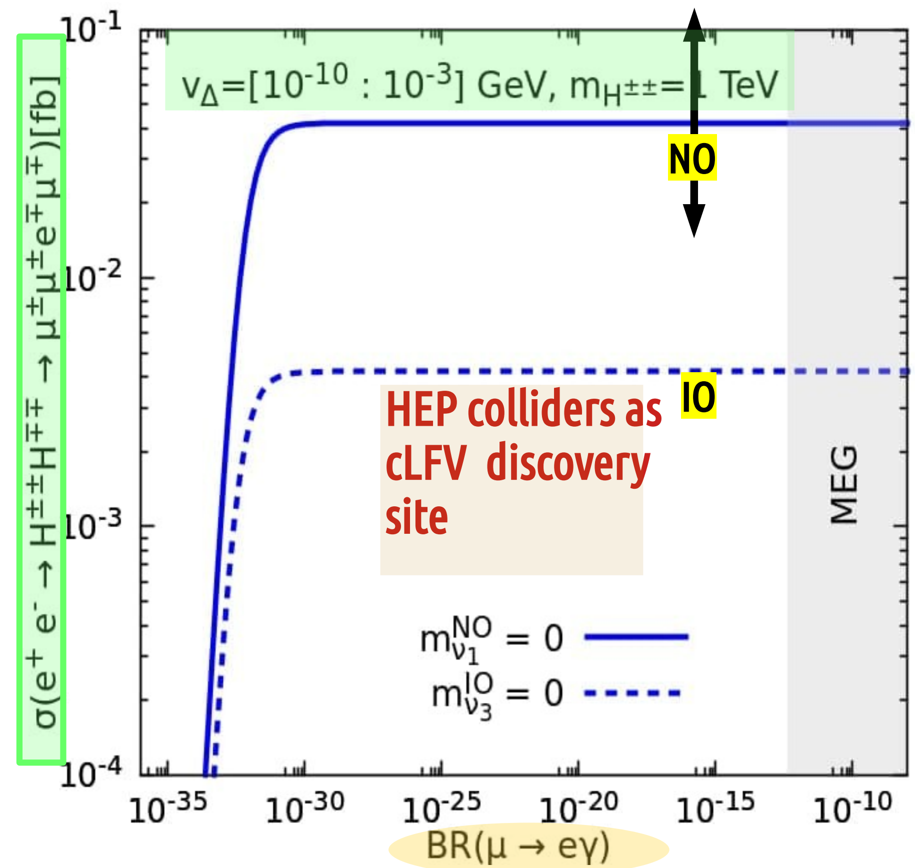


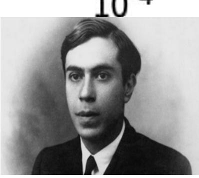
seesaw mediator produced in
@ e+e- / pp collisions

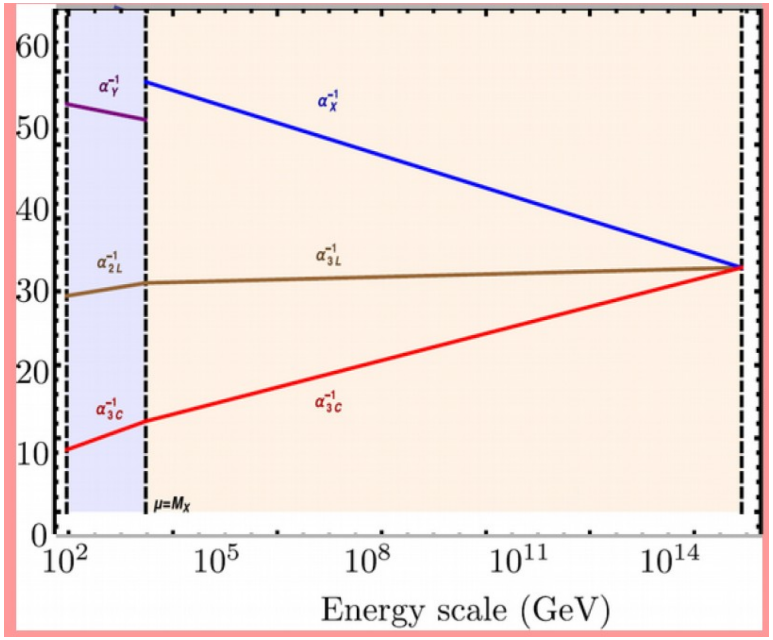
Miranda et al Phys.Rev.D105 (2022) 095020



Miranda et al PLB 829 (2022) 137110







the physics responsible for neutrino masses may also induce gauge coupling unification

neutrino path to unification

Boucenna et al Phys. Rev. D 91, 031702 (2015)

Deppisch et al Phys.Lett. B762 (2016) 432

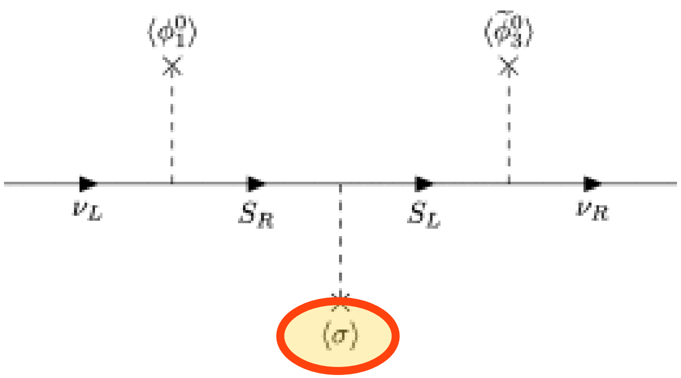
why 3 families

Old 331 model

PRD22(1980)738

From *Physics Letters B 810 (2020) 135829*

Diracness from PQ



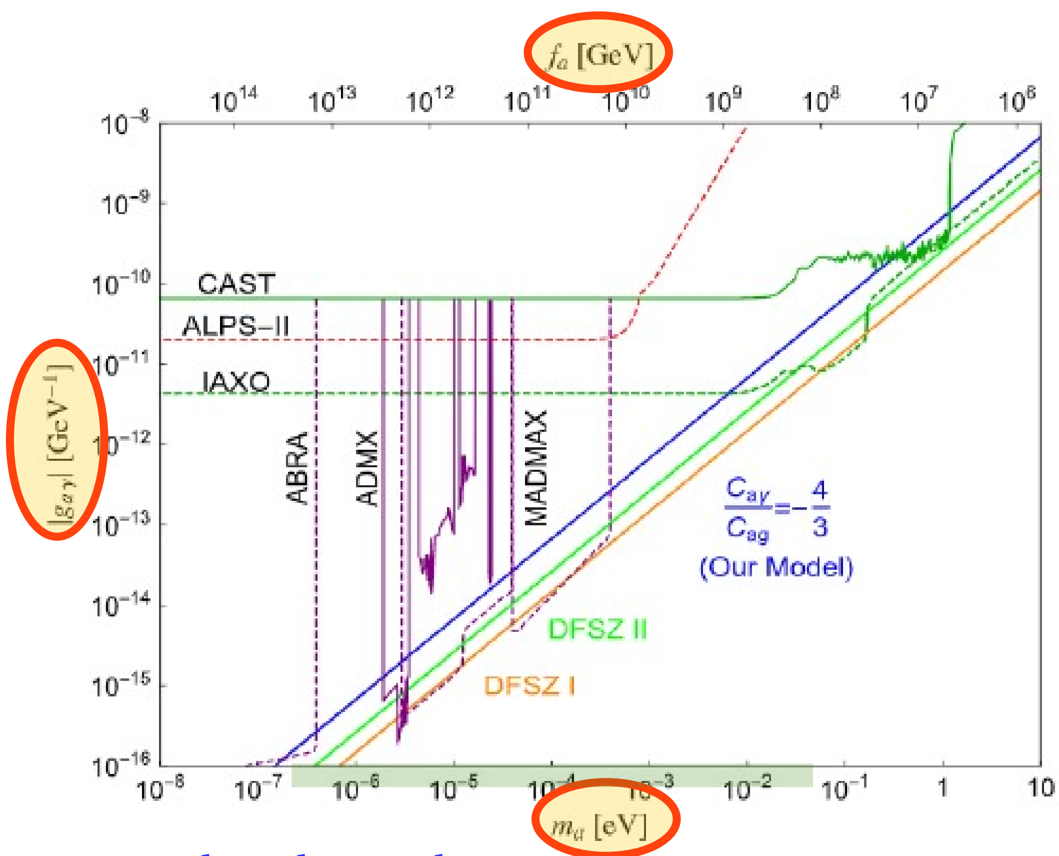
Peccei-Quinn symmetry

$$m_\nu^D \simeq \frac{y^{\nu_1}(y^S)^{-1}(y^{\nu_2})^T}{\sqrt{2}} \frac{v_W}{v_\sigma}.$$

SU3L

PQ

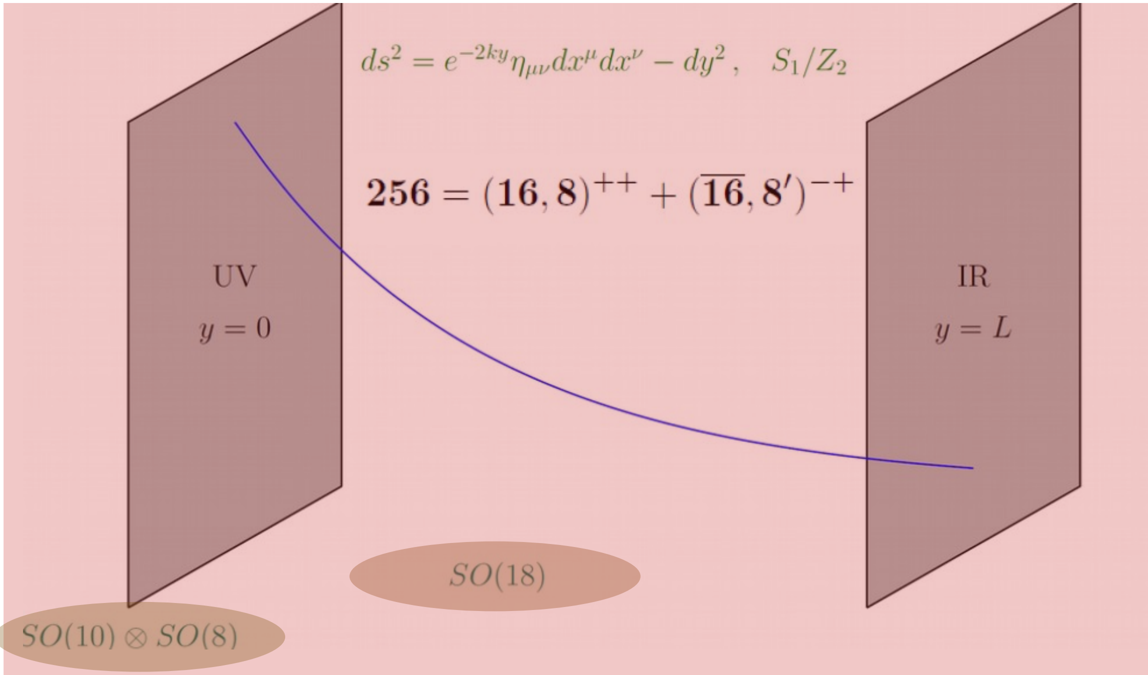
Dirac seesaw neutrino mass



tree-level quark FCNC

new path to family unification

inspired by beauty of neutrinos in SO10



promote M4 to AdS5

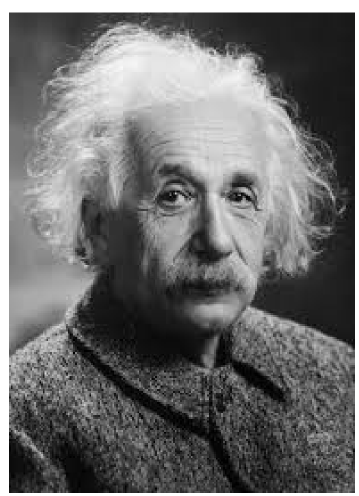
Reig, Valle, Vaquera-Araujo, Wilczek
Phys.Lett. B774 (2017) 667-670

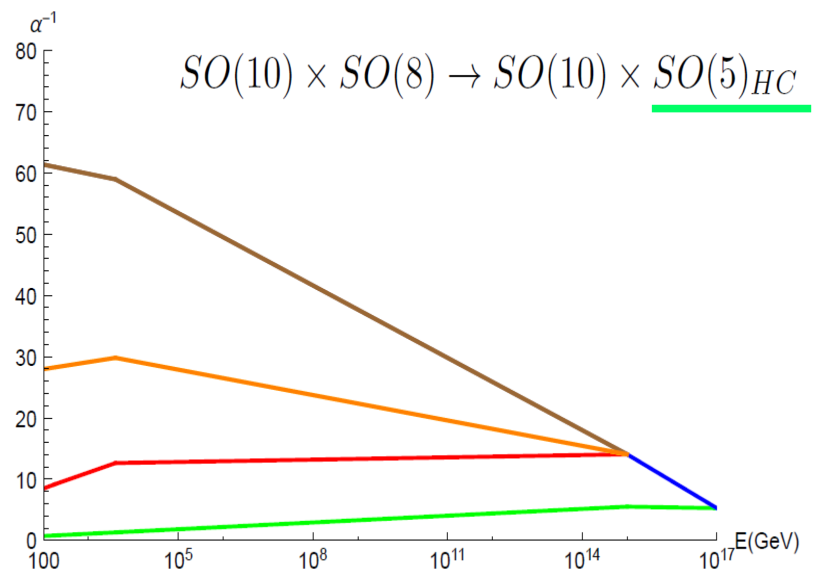
use orbifold BC to decouple mirrors

unwanted chiral families bound
by new hypercolor force above TeV

new spectroscopy

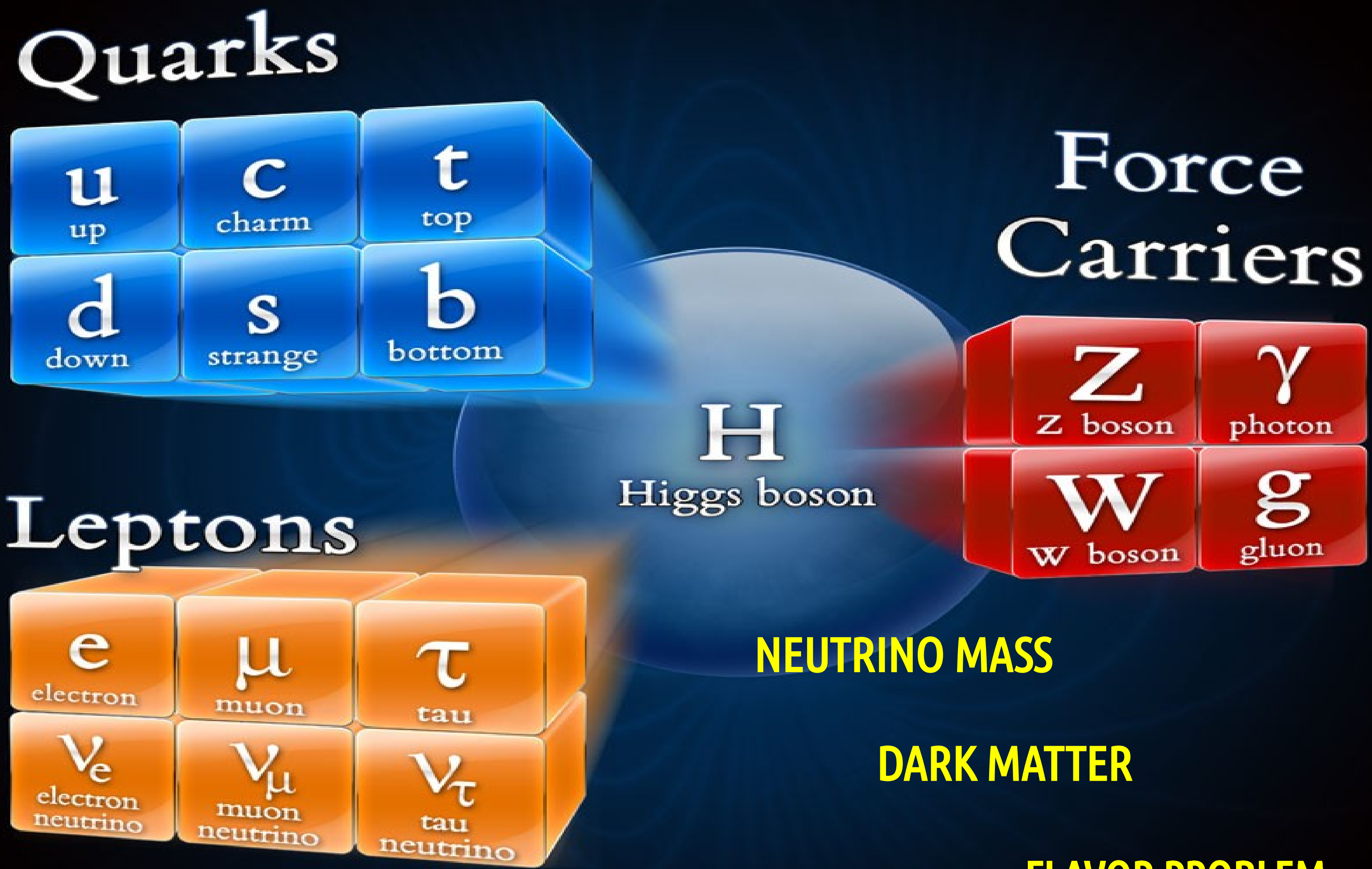
Reig, JV, Wilczek
Phys.Rev. D98 (2018) 095008





- viable SO3 family symmetry
- golden Q-L mass formula
- PQ symmetry & axion

NEUTRINO COMPLETION of SM NECESSARY



NEUTRINO MASS

DARK MATTER

FLAVOR PROBLEM

UNIFICATION