1. Introduction, Masses and Mixing
2. Oscillations
3. Neutrinoless Double Beta Decay Neutrino Mass Mechanisms

1. Introduction

- Observation of Beta Decay

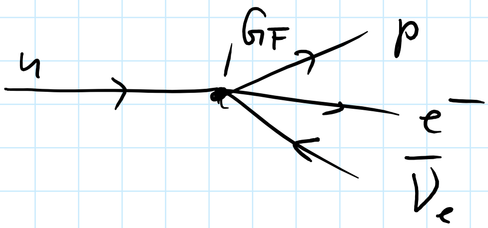
$$ {}^{A}_{Z}N \to {}^{A}_{Z+1}N' + e^- $$

$$ Q = m_N - m_{N'} - m_e $$

- Prediction of neutrinos by Pauli 1930 to conserve energy etc.

$\Rightarrow$ electrically neutral
weakly interacting

- E. Fermi:
Theory of beta decay



- Very difficult to detect

$$ \bar{\nu}_e + p \to e^+ + n $$

$$ \sigma = \left|\frac{G_F}{\sqrt{2}}\right|^2 \int \frac{dp_e}{(2\pi)^3} 2\pi\, \delta(m_p + E_\nu - m_N - E_e)\, |M|^2 $$

$$ \approx \frac{G_F^2}{2\pi} E_\nu^2 \qquad M \sim 1 $$

$2\pi \quad E_\nu$

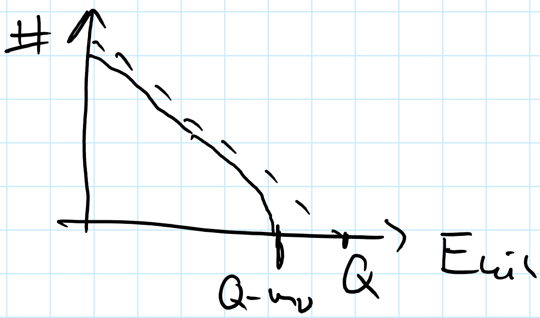
$\approx 10^{-44}\,\mathrm{cm}^2 \quad$ for $E_\nu \sim 1\,\mathrm{MeV}$

$\Delta x \approx \frac{1}{\rho\sigma} \approx 10^{21}\,\mathrm{cm} \approx 10^{11}$ earth diameter

$\rho \approx 10^{23}\,\frac{\mathrm{atoms}}{\mathrm{cm}^3}$

- Observation: Reines, Cowan 1956 (Nobel prize 1995)

- Mass



$m_{\nu_e} < 2\,\mathrm{eV}$

Mainz, Troitsk

- Limit on neutrino mass from SN 1987A

~ 170.000 ly

20 events, $\quad E_\nu \sim 20\,\mathrm{MeV}$

$\delta E_\nu \sim 20\,\mathrm{MeV}$

$\delta t \sim 10\,\mathrm{s}$

$\boxed{\text{Question: estimate the limit on the mass of neutrinos}}$

- $m_{\nu_\mu} \leq 190\,\mathrm{keV} \qquad \pi^+ \to \mu^+ + \nu_\mu$

$m_{\nu_\tau} \lesssim 18\,\mathrm{MeV} \qquad \tau \to 5\pi + \nu_\tau$

$\Rightarrow$ No direct evidence for neutrino masses

2. Neutrinos, SM, and their masses

$$SU(3)\times SU(2)\times U(1)_Y$$

$$L=\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\begin{matrix}+\frac{1}{2}\\ -\frac{1}{2}\end{matrix}\longleftrightarrow\left(\overset{SU(3)}{1},\overset{SU(2)}{2},\overset{Y}{-\tfrac{1}{2}}\right)\qquad Q=T_3+Y$$

$$e^c=e_R^*\longleftrightarrow(1,1,+1)$$

$$\nu^c=\nu_R^*\longleftrightarrow(1,1,0)$$

$$H=\begin{pmatrix}h^+\\ h^0\end{pmatrix}\longleftrightarrow\left(1,2,+\tfrac{1}{2}\right)$$

$$\mathcal{L}=-y^\ell_{\alpha\beta}\bar{L}_\alpha H e_{R\beta}-y^{\ell *}_{\alpha\beta}\bar{e}_{R\alpha}H L_\beta$$

$\Longrightarrow$ Gauge interactions = UV-completion of Fermi theory

$$\mathcal{L}\ni\frac{g}{\sqrt{2}}\sum_\alpha\bar{l}_\alpha\gamma^\mu P_L W^+_\mu\nu_\alpha\qquad P_L=\tfrac{1}{2}(1-\gamma_5)$$

$$-\frac{g}{2\cos\theta_W}\sum_\alpha\bar{\nu}_{L\alpha}\gamma^\mu P_L\nu_\alpha Z$$

Dirac

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/-m)\psi\qquad\bar{\psi}=\psi^\dagger\gamma_0$$

$$\psi=\psi_L+\psi_R=P_L\psi+P_R\psi$$

$$-m\bar{\psi}\psi=-m(\bar{\psi}_R\psi_L+\bar{\psi}_L\psi_R)$$

$$-m\bar{\psi}\psi = -m\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$$

$\Rightarrow$ Only possibility? $\quad \phi^2, \phi^\dagger\phi$ both Lorentz invariant

$$\psi^T A \psi$$

$$\rightarrow \psi^T \exp\left(-\frac{i}{4}\omega^{\mu\nu}\sigma_{\mu\nu}^T\right) A \exp\left(\frac{i}{4}\omega^{\mu\nu}\sigma_{\mu\nu}\right)\psi$$

$$\Rightarrow \quad \sigma_{\mu\nu}^T A + A\sigma_{\mu\nu} = 0$$

$$\Rightarrow \quad A = C$$

$$\Rightarrow m\,\psi^T C\psi \qquad \psi^c = C\bar{\psi}^T \quad C = i\gamma_0\gamma_2$$

$$E^2 = p^2 + m^2 \qquad = C_\gamma \psi^*$$

$\Rightarrow$ Two ways to introduce a mass

Dirac

$$-m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

$$= -\frac{1}{2}\begin{pmatrix}\psi_L^T & (\psi_R^c)^T\end{pmatrix}\begin{pmatrix}0 & m\\ m & 0\end{pmatrix} C \begin{pmatrix}\psi_L\\ \psi_R^c\end{pmatrix}$$

Majorana

$$= -\frac{m}{2}\left(\bar{\psi}_L C\psi_L + \bar{\psi}_L C\bar{\psi}_L^T\right)$$

$$= -\frac{m}{2}\begin{pmatrix}\psi_L^T & (\psi_R^c)^T\end{pmatrix}\begin{pmatrix}m & 0\\ 0 & 0\end{pmatrix} C \begin{pmatrix}\psi_L\\ \psi_R^c\end{pmatrix} + h.c.$$

$\Rightarrow$ Mass terms are not gauge invariant

$$\psi = \begin{pmatrix}\xi\\ \eta\end{pmatrix} \qquad \psi^c = \psi \quad \text{Majorana condition}$$

Particle = antiparticle

Particle = antiparticle

$$\phi^\dagger = \phi$$

Masses and mixing in the SM

Dirac : $\overline{\nu}_{L\alpha}\, m^\nu_{\alpha\beta}\, \nu_{R\alpha} + \overline{l}_{L\alpha}\, m^l_{\alpha\beta}\, l_{R\alpha} + h.c.$

Diagonalisation

$$m^\nu = U_\nu^\dagger \, \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\, V_\nu$$

$$m^l = U_l^\dagger \, \mathrm{diag}(m_e, m_\mu, m_\tau)\, V_l$$

$$\nu_R' = V_\nu \nu_R \ , \quad \nu_L' = U_\nu \nu_L$$

$$l_R' = V_l\, l_R \ , \quad l_L = U_l\, l_L \qquad m^\nu = (m^\nu)^T$$

Majorana $\frac{1}{2}\overline{\nu}_{L\alpha}\, m^\nu_{\alpha\beta}\, \nu^c_{L\beta} + \overline{l}_{L\alpha}\, m^l_{\alpha\beta}\, l_{R\alpha} + h.c.$

$$m^\nu = U_\nu^\dagger \, \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\, U_\nu^*$$

QUESTION : Show that m_ν is diagonalized like this with U_ν is unitary

CC no longer diagonal in generation space

$$-\frac{g}{\sqrt{2}}\, \overline{l}_\alpha \gamma_\mu P_L W_\mu \underbrace{(U_l^\dagger U_\nu)_{\alpha\beta}}_{U_{PMNS} \text{ matrix}} \nu_\beta' + h.c.$$

U_{PMNS}

N^2 parameters $= N(N-1)/2$ angles (3)

$+ N(N+1)/2$ phases (6)

Remove unphysical phases

Dirac : $l_{L\alpha} \to e^{i\theta_\alpha} l_L$, $\nu_\alpha \to e^{i\phi_\alpha} \nu_\alpha$

Dirac : $\ell_{L\alpha} \to e^{i\theta_\alpha} \ell_L$, $\nu_\alpha \to e^{i\phi_\alpha} \nu_\alpha$

Removes $2N-1$ phases $\Rightarrow (N-1)(N-2)/2$

(1)

Majorana case : $\cancel{\nu_\alpha \to e^{i\phi_\alpha} \nu_\alpha} \Rightarrow \nu_\alpha^T C \nu_\alpha$

$\to e^{2i\phi_\alpha} \nu_\alpha^T C \nu_\alpha$

Only remove N phases from ch. leptons

$\Rightarrow N(N-1)/2$ phases

(3)

$\Rightarrow$ Standard parametrization

$$U_{PMNS}^{Dirac} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

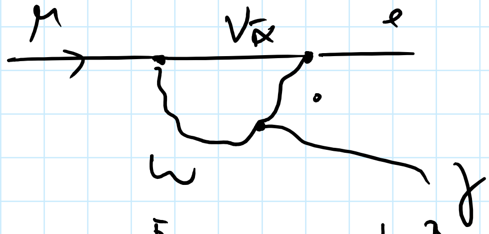
$$\times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad c_{12} = \cos\theta_{12}$$

$$U_{PMNS}^{Majorana} = U_{PMNS}^{Dirac} \cdot \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2})$$

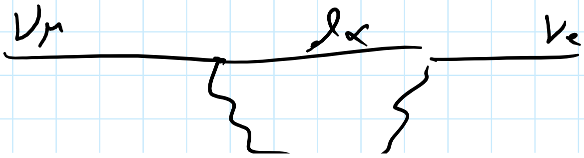
$0 \leq \theta_{12}, \theta_{13}, \theta_{23} < \frac{\pi}{2}$ $\qquad 0 \leq \delta, \phi_1, \phi_2 < 2\pi$

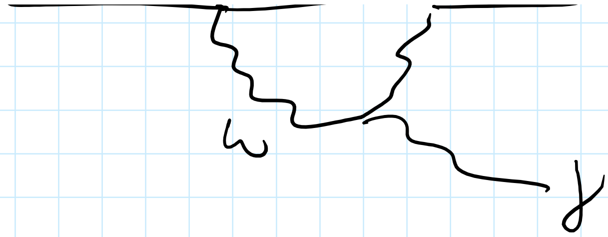
$0 \leq m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ positive, real

$\mu \to e\gamma$



$$\Gamma \approx \frac{m_\mu^5}{4\pi} e^2 G_F^2 \left| \sum_{\alpha=1}^{3} U_{\mu\alpha} U_{e\alpha}^* \left(\frac{m_{\nu_\alpha}}{m_W}\right)^2 \right|^2$$





3. Neutrino Oscillations

$$U_{(PMNS)} = \underset{\text{atmospheric}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \underset{\text{reactor}}{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}} \underset{\text{solar}}{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

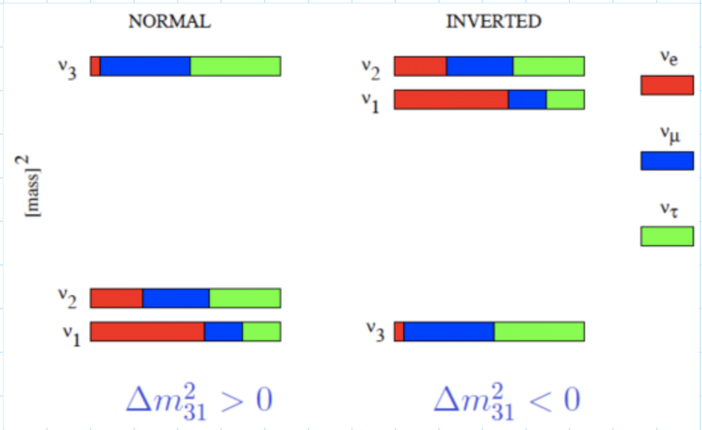
$$\left[\times \, \mathrm{diag}(1, e^{i\phi_1}, e^{i\phi_2}) \right]$$

$c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$

General: $0 \leq \theta_{ij} < \frac{\pi}{2}$, $0 \leq \delta < 2\pi$

Convention: $|U_{e1}| > |U_{e2}| > |U_{e3}|$ (ν_1 is most similar to ν_e)

$\Rightarrow 0 \leq \theta_{12}, \theta_{13} \leq \frac{\pi}{4}$



Mass splittings

$\Delta m^2_{31} = m^2_{\nu_3} - m^2_{\nu_1}$

(atmospheric)

$\Delta m^2_{21} = m^2_{\nu_2} - m^2_{\nu_1} > 0$

(solar)

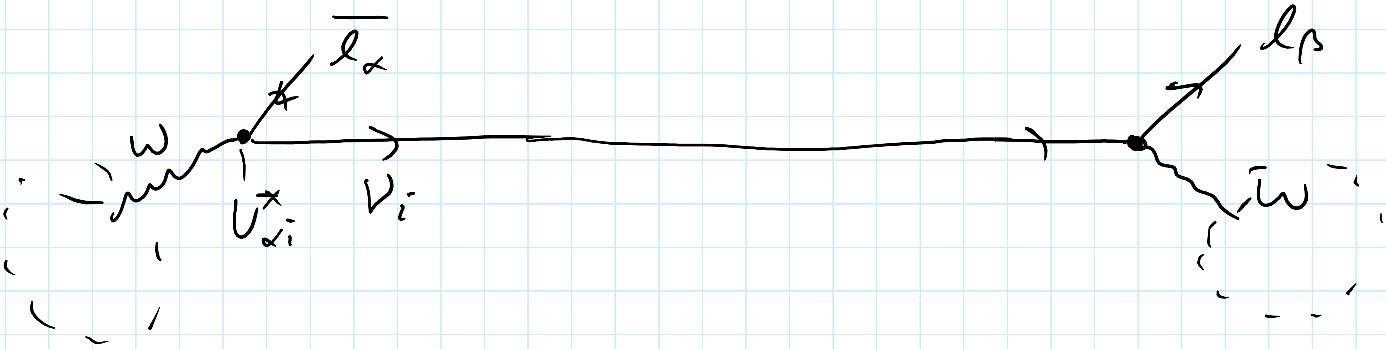
Experimental observation

parameter	best fit $\pm 1\sigma$	90% C.L. range
Δm^2_{21} [10^{-5}eV^2]	7.56 ± 0.19	7.26–7.87
Δm^2_{31} [10^{-3}eV^2] (NO)	2.55 ± 0.04	2.48–2.62
$\lvert\Delta m^2_{31}\rvert$ [10^{-3}eV^2] (IO)	$2.47^{+0.04}_{-0.05}$	2.40–2.53
$\sin^2\theta_{12}/10^{-1}$	$3.21^{+0.18}_{-0.16}$	0.294–0.352
$\sin^2\theta_{23}/10^{-1}$ (NO)	$4.30^{+0.20\,a}_{-0.18}$	0.403–0.466 & 0.577–0.608
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.98^{+0.17\,b}_{-0.15}$	0.569–0.623
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.155^{+0.090}_{-0.075}$	0.0201–0.0228
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.155^{+0.076}_{-0.092}$	0.0201–0.0228
δ/π (NO)	$1.40^{+0.31}_{-0.20}$	0.98–2.00
δ/π (IO)	$1.56^{+0.22}_{-0.26}$	1.15–1.90

[a] Local min. at $\sin^2\theta_{23}$=0.596 with $\Delta\chi^2 = 2.1$ w.r.t. the global min.
[b] Local min. at $\sin^2\theta_{23}$=0.426 with $\Delta\chi^2 = 3.0$ w.r.t. the global min. for IO.

$|\Delta m^2_{31}| \gg \Delta m^2_{21}$
Weak preference for NO
θ_{12} large
θ_{23} maximal? $\gtrless \frac{\pi}{4}$?
θ_{13} small
$\delta \approx \frac{3}{2}\pi$? (weak preference)

Farzan, Tortola, arxiv: 1710.09360



Plane wave derivation

$$\nu_\alpha = \sum U^*_{\alpha i}\,\nu_i \qquad E_i^2 = |\vec{p}|^2 + m^2_{\nu_i}$$

$$\hat{H}\,\nu_i = E_i\,\nu_i$$

$$\Rightarrow \nu_i(t) = e^{-iE_i t}\,\nu_i(0)$$

$$\Rightarrow \nu_\alpha(t) = \sum U^*_{\alpha i}\,e^{-iE_i t}\,\nu_i(0)$$

$$\Rightarrow P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t)\rangle|^2$$

$$= \Big|\sum_{i,j} U_{\beta j}\,U^*_{\alpha i}\,e^{-iE_i t}\,\langle \nu_j | \nu_i\rangle\Big|^2 \qquad \nu_\beta = \sum U_{\beta j}\,\nu_j$$

$$\langle \nu_i | \nu_j \rangle = \delta_{ij}$$

$$= \Big|\sum U_{\beta i}\,U^*_{\alpha i}\,e^{-i\frac{m^2_{\nu_i}}{2p}t}\Big|^2$$

$$\langle \nu_i | \nu_j \rangle = \delta_{ij}$$

$$= \left| \sum_i U_{\beta i} U^*_{\alpha i} e^{-i \frac{m^2_{\nu_i}}{2p} t} \right|^2$$

$$E_i = \sqrt{p^2 + m^2_{\nu_i}} \approx |\vec{p}| + \frac{1}{2} \frac{m^2_{\nu_i}}{|p|}$$

$$= \sum_{i,j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} e^{-i(m^2_{\nu_i} - m^2_{\nu_j}) \frac{L}{2}}$$

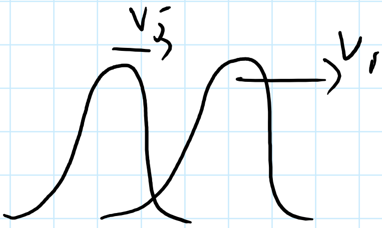
$\Rightarrow$ Oscillations require

- Quantum coherence $\leftrightarrow$ weak interaction
- Large enough momentum uncertainty

  e.g. $\pi^+ \to \mu^+ \nu_\mu$

Improved modelling using wave packet

$$\nu_\alpha = \sum_i U^*_{\alpha i} \int dp \exp\left[-i (p - p_0^i)^2 / (2\sigma^2) \right] \nu_i$$



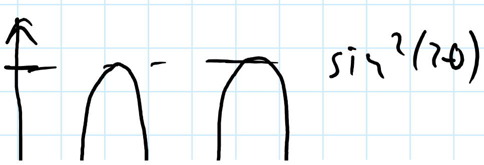
Exponential suppression:
- Distance $\gtrsim 1/\sigma$
- Energy difference $\gtrsim \sigma$

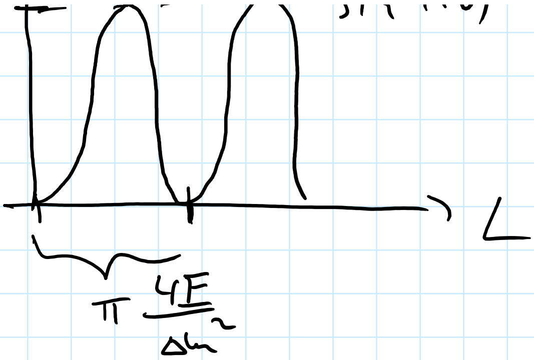
Two generation case: $\alpha, \beta = 1, 2$

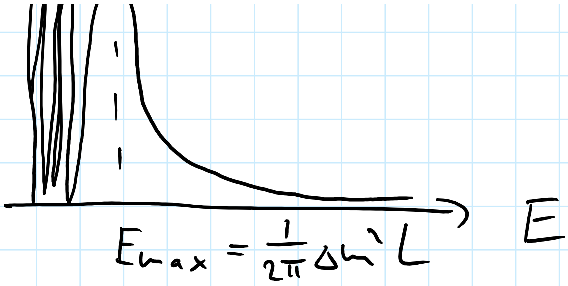
$$U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Appearance: $$P(\nu_\alpha \xrightarrow{\alpha \neq \beta} \nu_\beta) = \sin^2(2\theta) \sin^2\left(1.27 \frac{\Delta m^2[eV]\, L[km]}{E_\nu[GeV]}\right)$$

Disappearance: $$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

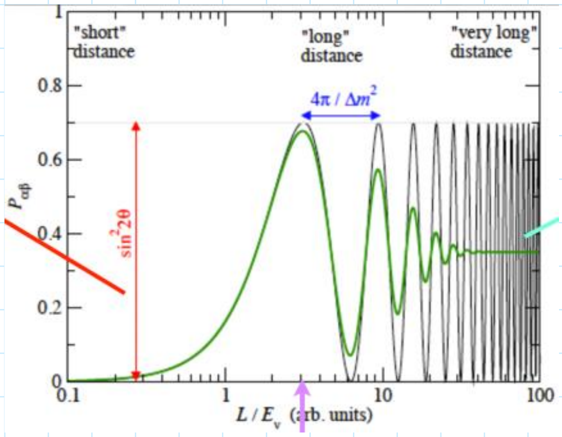






1) $E/L \gg \Delta m^2$

$$P(\nu_\alpha \to \nu_\beta) \approx \sin^2 2\theta \times \frac{\Delta m^2 L^2}{16 E^2}$$

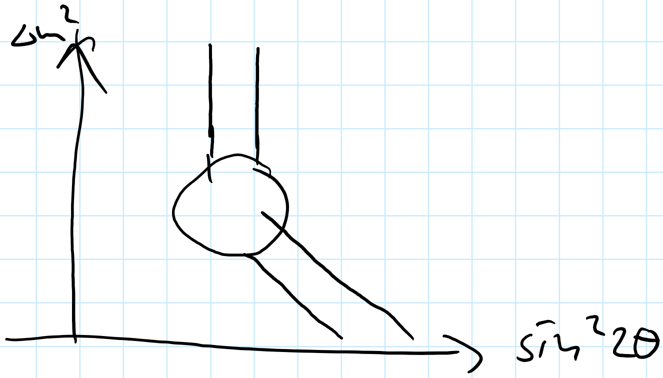


2) $E/L \ll \Delta m^2$

$$P(\nu_\alpha \to \nu_\beta) \to \frac{1}{2} \sin^2 2\theta$$

3) $E/L \approx \Delta m^2$

Optimal distance

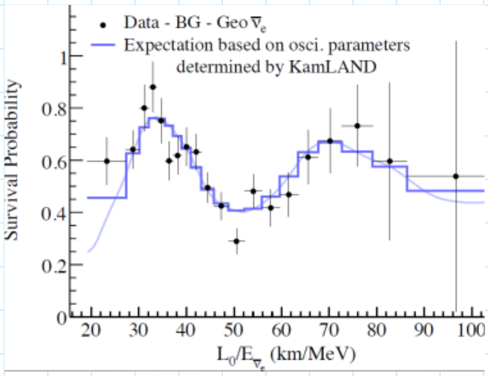


KamLAND

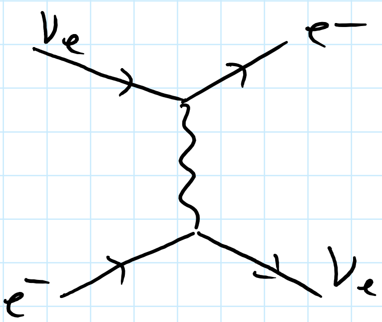
$E \approx 1\ \text{MeV}$

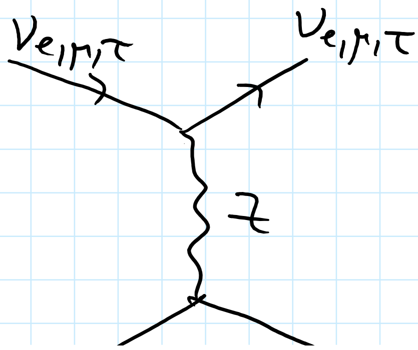
$L \approx 180\ \text{km}$

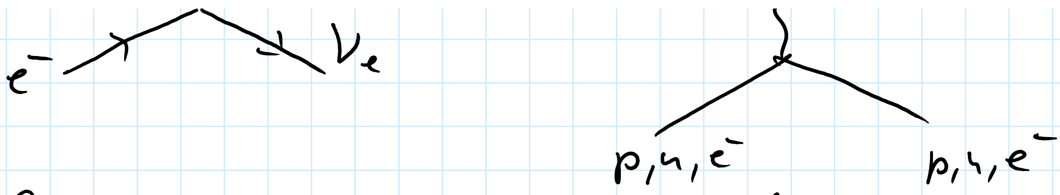
$\Rightarrow \frac{E}{L} \approx \Delta m^2 = 10^{-5}\ \text{eV}^2$



Neutrino propagation in matter







$$\Rightarrow \quad H = \frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha \gamma_\mu (1-\gamma_5) \nu_\alpha \sum_{f=p,n,e} \bar{f} \gamma^\mu \left(g_V^{\alpha f} - g_A^{\alpha, f} \gamma_5 \right) f$$

Non-relativistic: $\quad \langle \bar{f} \gamma^\mu f \rangle = \delta_{\mu 0} n_f$

Unpolarized: $\quad \langle \bar{f} \gamma^\mu \gamma_5 f \rangle = 0$

Neutral: $\quad n_e = n_p$

$$V^\alpha_{matter} = \frac{G_F}{\sqrt{2}} \mathrm{diag} \left(n_e - \frac{1}{2} n_n , - \frac{n_n}{2} , - \frac{n_n}{2} \right)$$

$$H^{vac} = U^T \cdot \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \cdot U = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$H^{matter} = H^{vac} + \begin{pmatrix} \frac{1}{\sqrt{2}} G_F n_e & 0 \\ 0 & 0 \end{pmatrix}$$

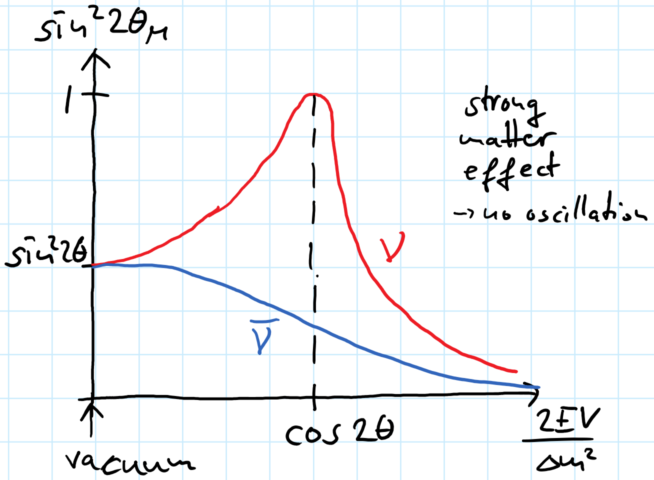
$$\equiv \frac{\Delta m_M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

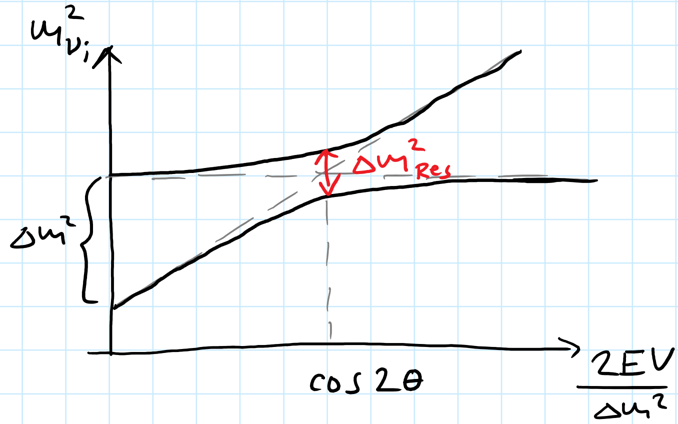
with Δm_M^2, θ_M effective values in matter replacing vacuum values Δm^2, θ:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(\cos 2\theta - \frac{2EV}{\Delta m^2} \right)^2}$$

$$\Delta m_M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{2EV}{\Delta m^2} \right)^2}$$

$$P(\nu_\alpha \to \nu_\beta)_{\text{matter}} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$





Resonance for $V = \frac{\Delta m^2}{2E} \cos 2\theta$

with $\sin^2 2\theta_M = 1$, $\theta_M = \frac{\pi}{4}$ maximal

$\Delta m_M^2 = \Delta m^2 \sin 2\theta < \Delta m^2$

MSW effect (Mikheyev, Smirnov, Wolfenstein)

Resonance depends on sign of Δm^2 and on ν vs $\bar{\nu}$
(for $\bar{\nu}$: $V \to -V$)