Lecture 4: Large momentum effective theory for parton physics

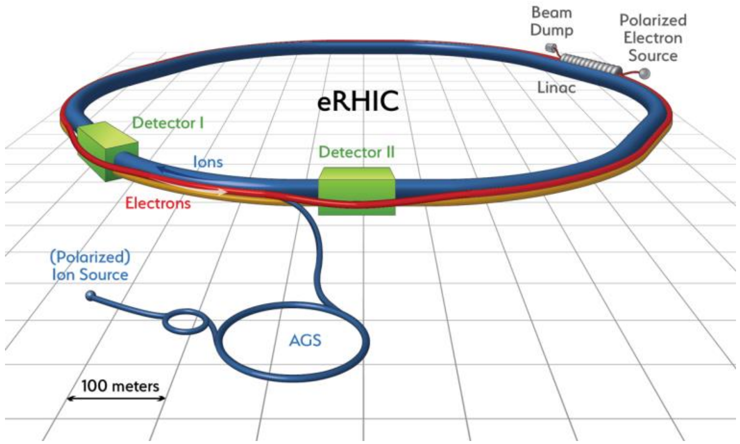
Xiangdong Ji (UMD)

Outline

- Proton structure and parton physics at Electron-Ion collider (EIC)
- Calculate partons on lattice:

Large momentum effective theory (LaMET)

- Recent progress

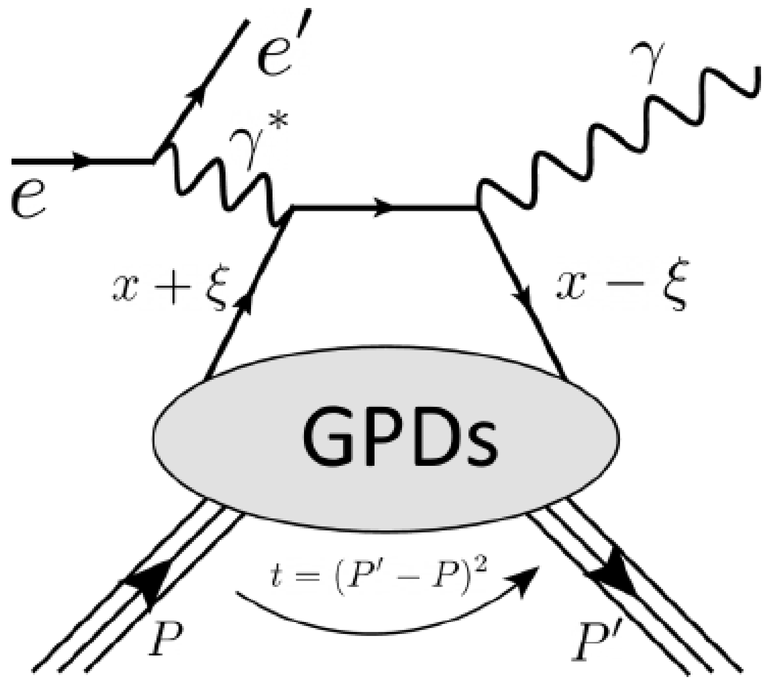


Properties of the proton

- One of the most important science drives for EIC is to understand the internal structure of the proton.
- Traditionally the proton was studied through either elastic scattering (form factors) or deep inelastic scattering (PDFs)
- A large class of new processes can be used to probe partonic structure of the proton
  - Semi-Inclusive DIS & TMD distributions (Collins et al)
  - DVCS and related process & GPD (Ji, Muller et al)
  - Jet production etc.

Deaply virtual comptons scattering

- Ji, 1996 PRL



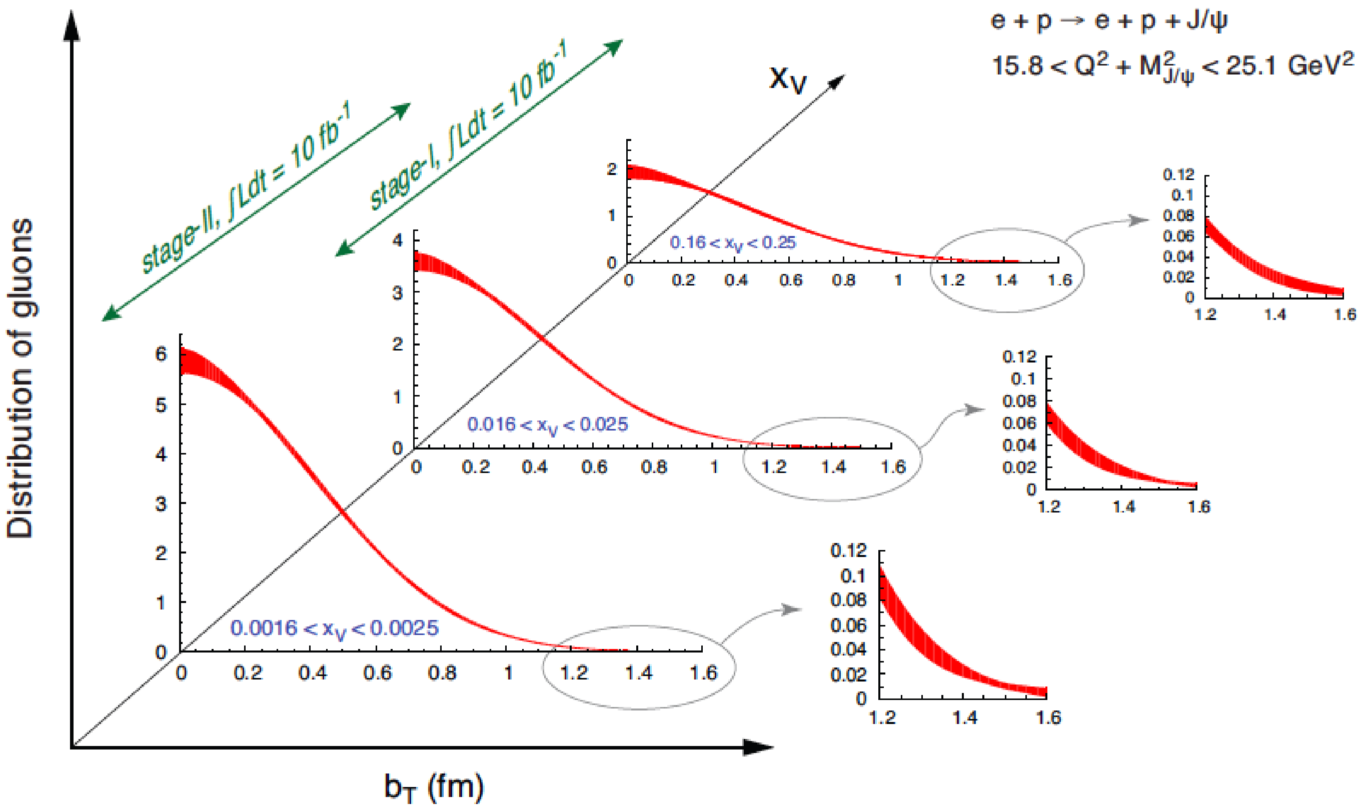
Mass, spin, etc..

- DVCS can be used to probe form factors of energy momentum tensor (1996)

$$\langle P'|T^{\mu\nu}_{q,g}|\rangle P\rangle = \bar{u}\,(P')\Big[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M + C_{q,g}(t)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)/M + \bar{C}_{q,g}(t)g^{\mu\nu}M\Big]u(P)\,.$$

where A, B, C, C-har are related the mass, spin, and stress distributions in the proton.

GPD & Parton distributions in impact parameter space (burkardt)

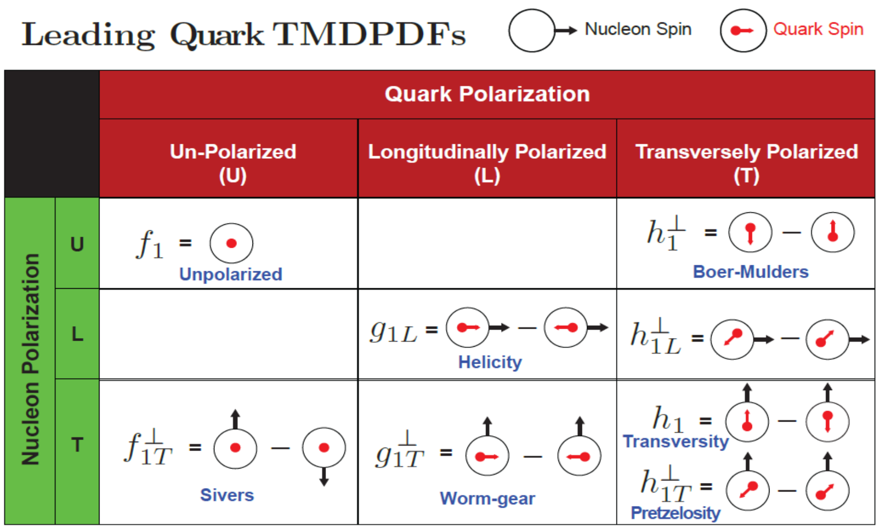


TMD distributions

- Partons have transverse momentum $\vec{k}_\perp$, from which one can define various TMD PDFs

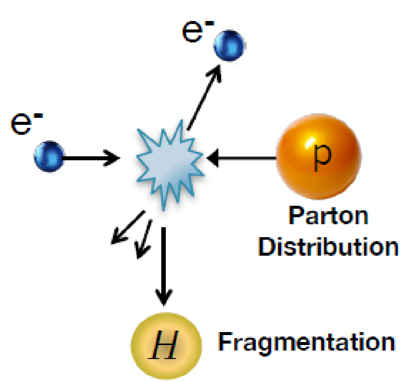
  (TMD Handbook, 2022)

- TMD distributions can be measured in various high-energy scattering processes



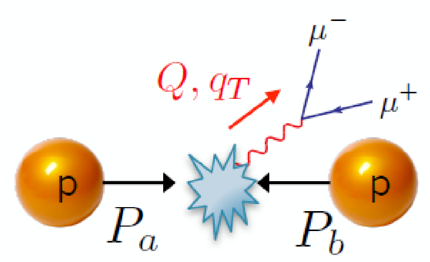
Semi-Inclusive DIS

$\sigma \sim D_{H/i}(z, k_T) \otimes f_{i/p}(x, p_T)$

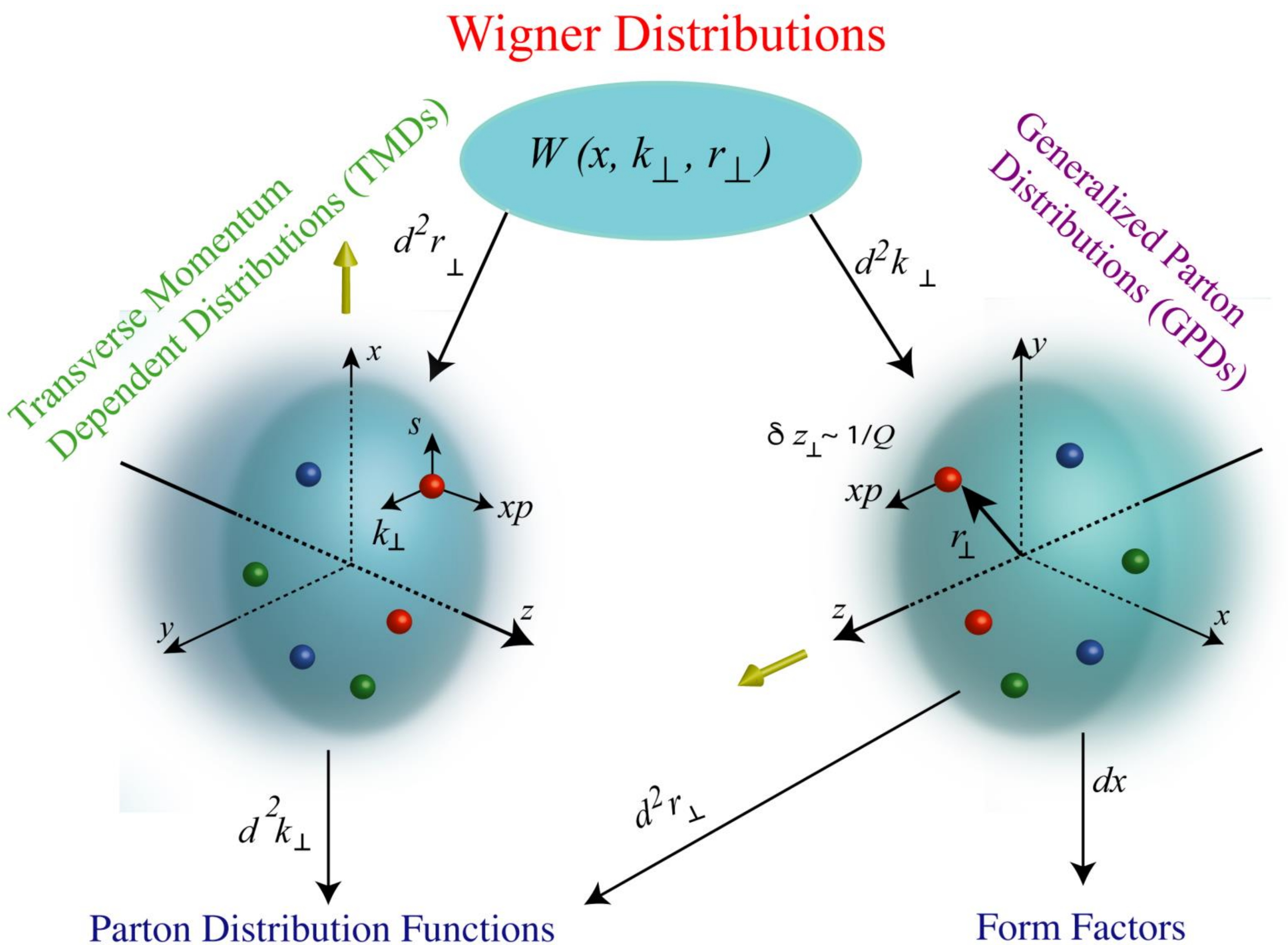


Drell-Yan

$\sigma \sim f_{i/p}(x_a, k_{aT}) \otimes f_{\bar{i}/p}(x_b, k_{bT})$



$q_T \ll Q$



Wigner Distributions
W (x, k⊥, r⊥)
Transverse Momentum Dependent Distributions (TMDs)
Generalized Parton Distributions (GPDs)
d²r⊥
d²k⊥
x
s
xp
k⊥
y
z
δ z⊥~ 1/Q
xp
r⊥
z
x
y
d²k⊥
d²r⊥
dx
Parton Distribution Functions
Form Factors

Large-momentum expansion and EFT

Refs: arXiv: 2007.06613 AAPPS & arXiv:2004.03543 RMP

Parton from Large momentum expansion a la Feynman

- Assuming large momentum limit exist, parton physics is obtained through physical observables at finite momentum by Taylor expansion at $P^z = \infty$,

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

where M is a bound-state scale,

P^z is a large-momentum scale.

Naïve dimensional analysis

- $\epsilon = \left(\frac{M}{P^z}\right)^2$ is an expansion parameter

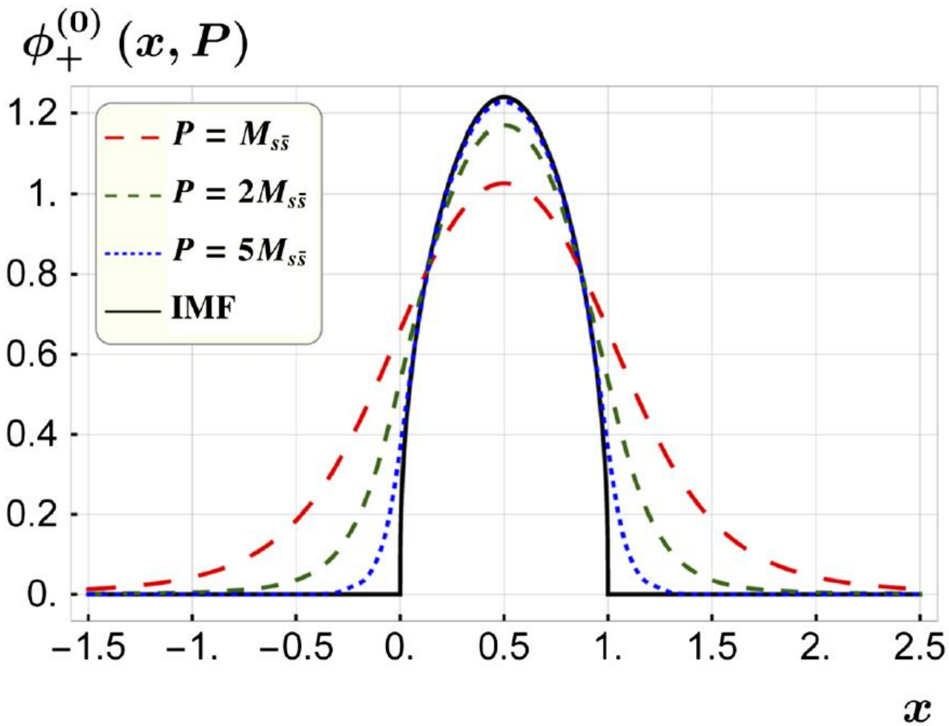
  M =1 GeV, $P^z = 2$ GeV

  $\epsilon = 1/4$

  the expansion may already work!

Thus a 2 GeV proton's structure is already similar to that at infinite momentum!

Wave functions as function of external momentum ('t Hooft model 1+1 D QCD with ∞ **color**)



Other examples in QFT

- This is what we frequently do in QCD

Lattice QCD: approximate a continuum

theory by a discrete one.

cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi/a$

0.1 fm ~ 2 GeV

HQET: $\epsilon = \Lambda_{QCD}/m_Q$

using $m_Q = \infty$ to approximate

m_c =1.5 GeV!

QFT subtleties (not considered by Feynman)

- There is a UV cut-off Λ_{UV}, $f(k^z, P^z)$ is not analytic at $P^z = \infty$!
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (HEP PDF)
- Calculating mom.dis. is done with the limit 1) and PDF is defined in limit 2).
- Solution: matching in EFT

  The difference is perturbative!

Large momentum effective theory & matching condition

Instead of the simple Taylor expansion,

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + ...$$

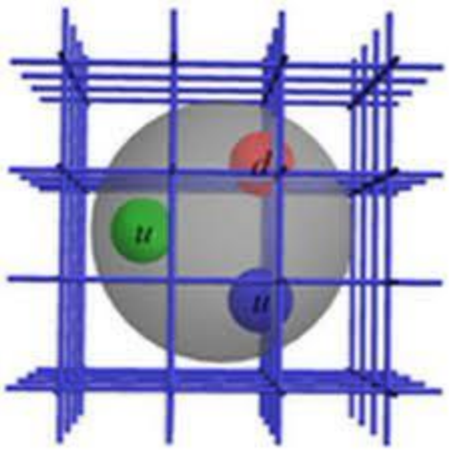
We have the relation between mom dis. in full QCD (a->0,P->∞) and PDFs in light-front theory (P->∞,a->0)

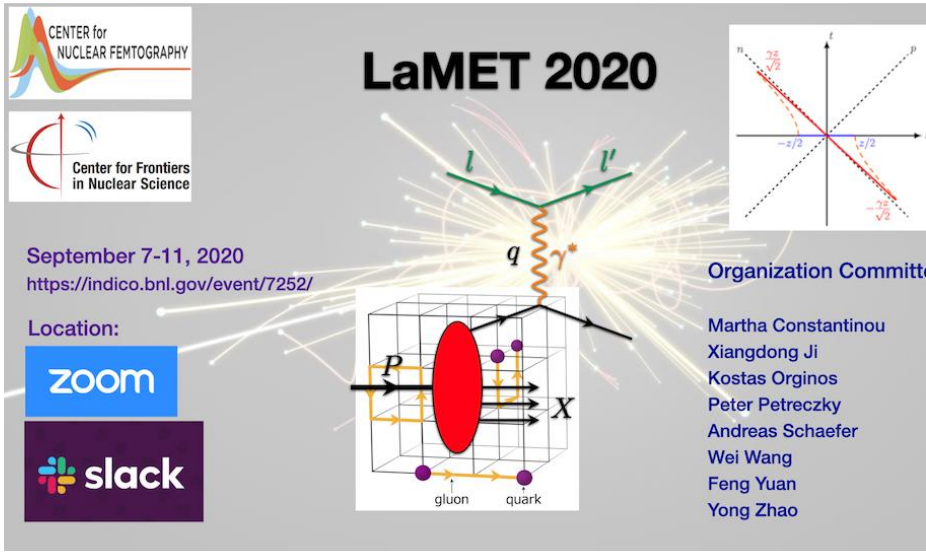
(Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x,\mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

Applying for lattice QCD calculations

- Challenges:
  - Renormalization of linear divergences
  - Computing correlations at large distance z
  - Continuum limit
  - Perturbative matching at high orders
- Very active direction in the last few years





First calculation (Yang et al, PRL (2017))

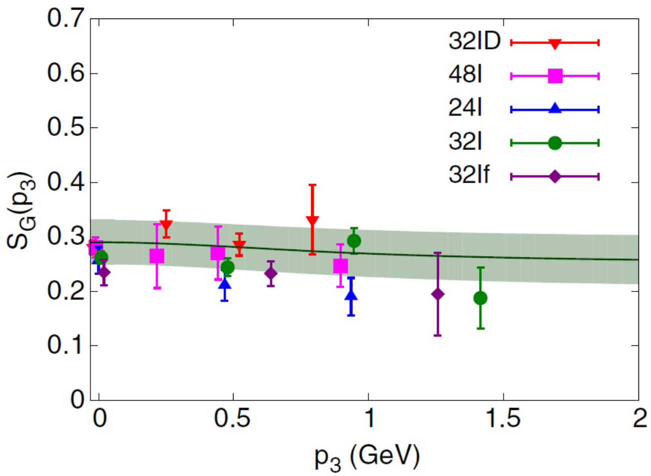
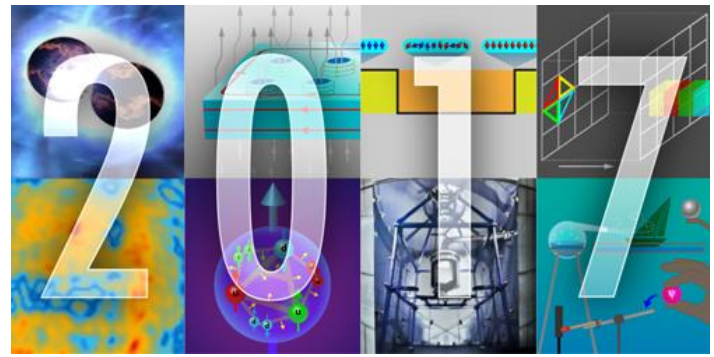
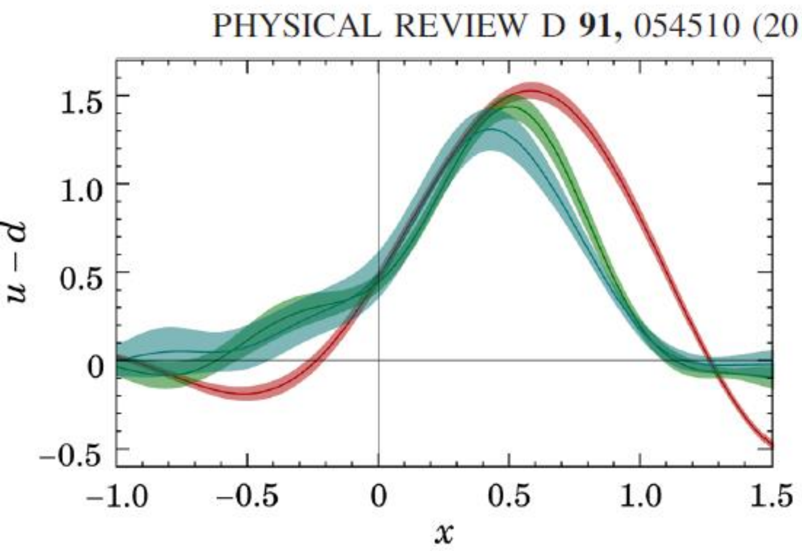


FIG. 4. The results extrapolated to the physical pion mass as a function of the absolute value of $\vec{p} = (0, 0, p_3)$, on all the five ensembles. All the results have been converted to $\overline{\mathrm{MS}}$ at $\mu^2 = 10\ \mathrm{GeV}^2$. The data on several ensembles are shifted horizontally to enhance the legibility. The green band shows the frame dependence of the global fit [with the empirical form in Eq. (11)] of the results.

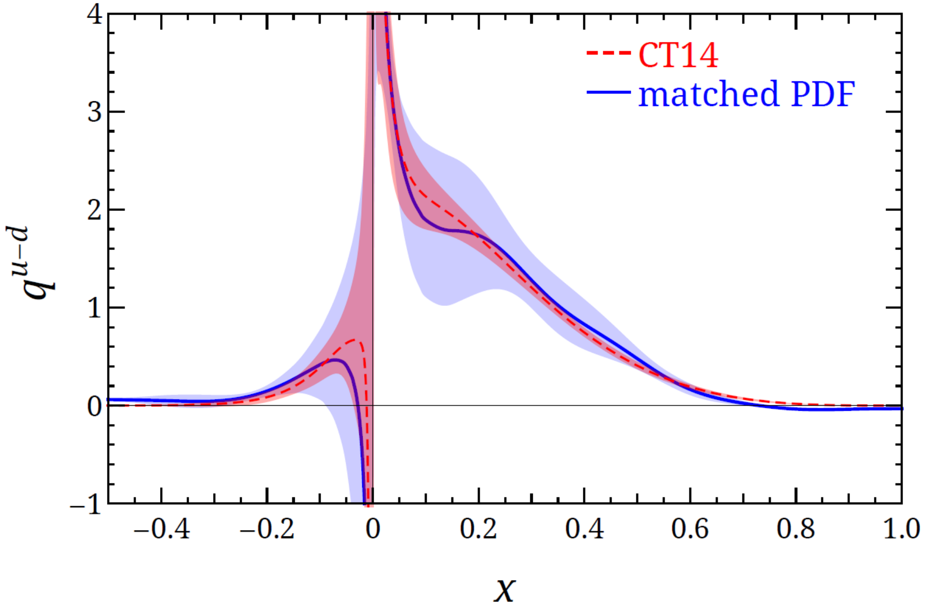


Gluons Provide Half of the Proton's Spin
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from

LP3 collaboration



H.W.Lin et al, PRD, 2015



2018

•*Rev.Mod.Phys.* 93 (2021) 3, 035005

•e-Print:
•2004.03543 [hep-ph]

Large-momentum effective theory

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(published 00 MONTH 0000)

Since the parton model was introduced by Feynman more than 50 years ago, much has been learned about the partonic structure of the proton through a large body of high-energy experimental data and dedicated global fits. However, limited progress has been made in calculating partonic observables such as the parton distribution function (PDFs) from the fundamental theory of strong interactions, quantum chromodynamics (QCD). Recently some advocated for a formalism, large-momentum effective theory (LaMET), through which one can extract parton physics from the properties of the proton traveling at a moderate boost factor such as $\gamma \sim 2$–5. The key observation behind this approach is that Lorentz symmetry allows the standard formalism of partons in terms of light-front operators to be replaced by an equivalent one with large-momentum states and time-independent operators of a universality class. With LaMET, the PDFs, generalized PDFs or generalized parton distributions, transverse-momentum-dependent PDFs, and light-front wave functions can all be extracted in principle from lattice simulations of QCD (or other nonperturbative methods) through standard effective field theory matching and running. Future lattice QCD calculations with exascale computational facilities could help one to understand the experimental data related to the hadronic structure, including those from the upcoming electron-ion colliders dedicated to exploring the partonic landscape of the proton. Here the progress made in the past few years in the development of the LaMET formalism and its applications is reviewed, with an emphasis on a demonstration of its effectiveness from initial lattice QCD simulations.

DOI:

CONTENTS

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Recent progress

How to get high-precision PDF

- Small lattice spacing limit, $a \rightarrow 0$
- Renormalization of linear divergences
- Matching up to two loops
- Physical point, $m_\pi \rightarrow m_\pi^{phys}$
- Larger momentum P
- …

Continuum limit

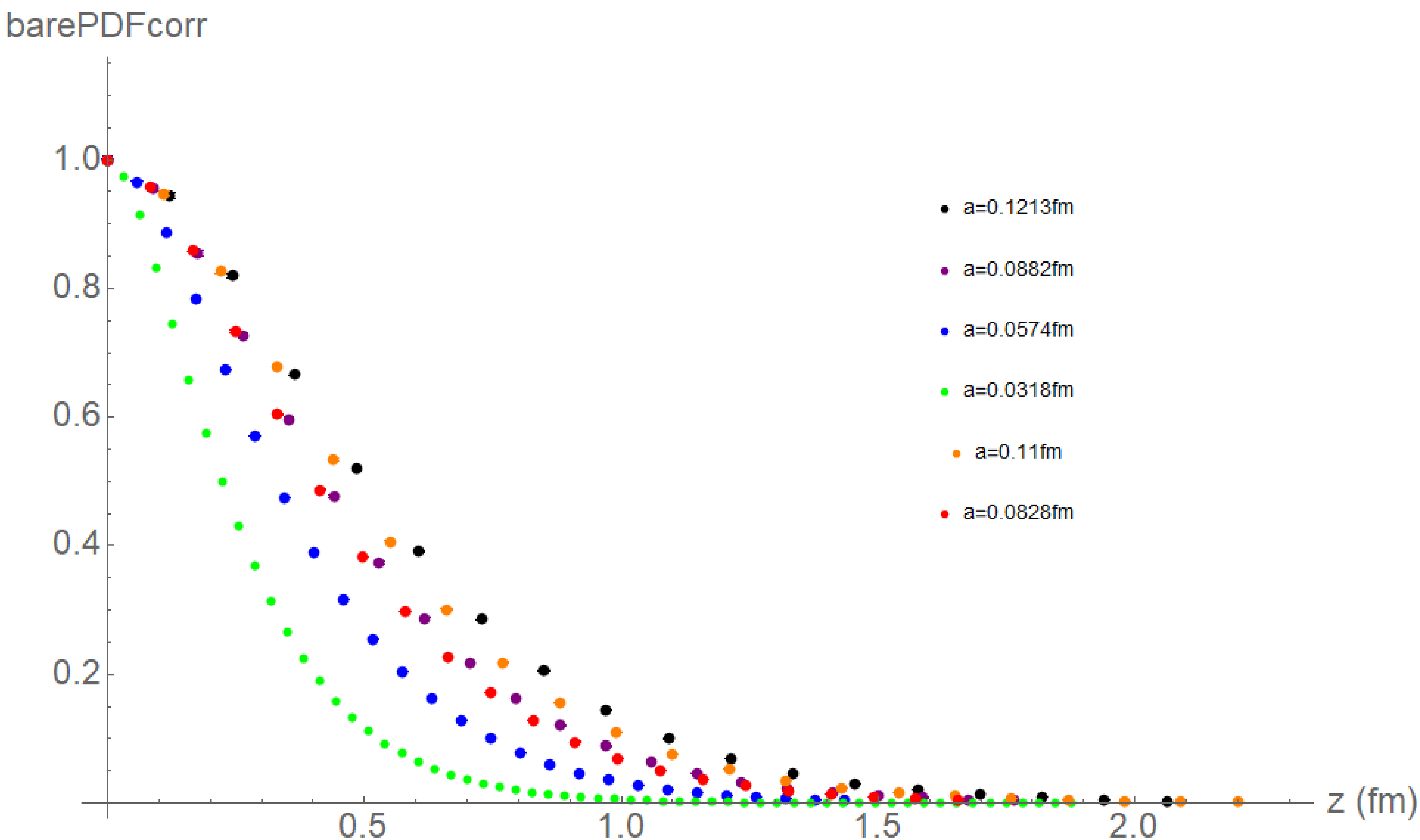
- For LaMET application, fine lattice spacing is extremely important
- Effective expansion parameter is

$$r = aP^z \sim a\gamma$$

which can be large.

- Controlled extrapolation to continuum…

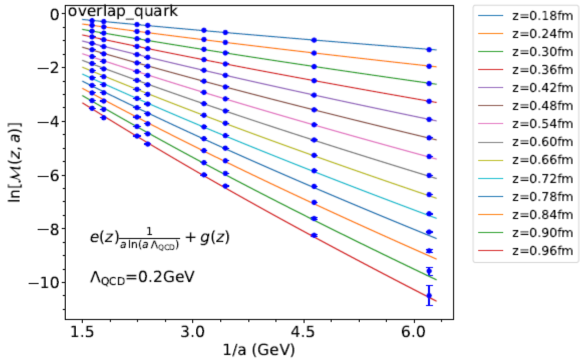
MSU, EMTC, BNL/ANL, LPC

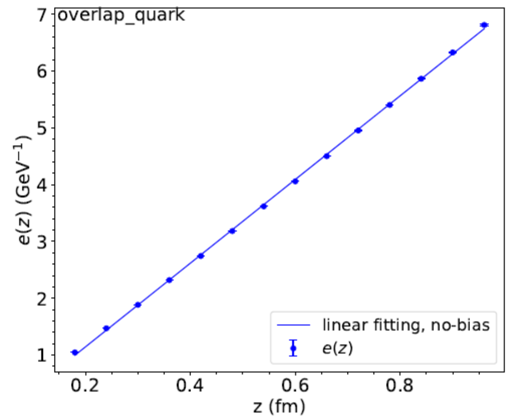


barePDFcorr
1.0
0.8
0.6
0.4
0.2
0.5
1.0
1.5
2.0
z (fm)
a=0.1213fm
a=0.0882fm
a=0.0574fm
a=0.0318fm
a=0.11fm
a=0.0828fm

Renormalization of linear divergences

- Renormalization of linear divergences causes large numerical uncertainties.
- The standard approaches (RI/MOM,ratio) could have large non-pert. effects
- New approaches
  - Auxiliary field approach (Green et al, Ji et al, 2018, C. Alexandrou et al, 2021)
  - Hybrid renormalization (Ji et al, 2021)

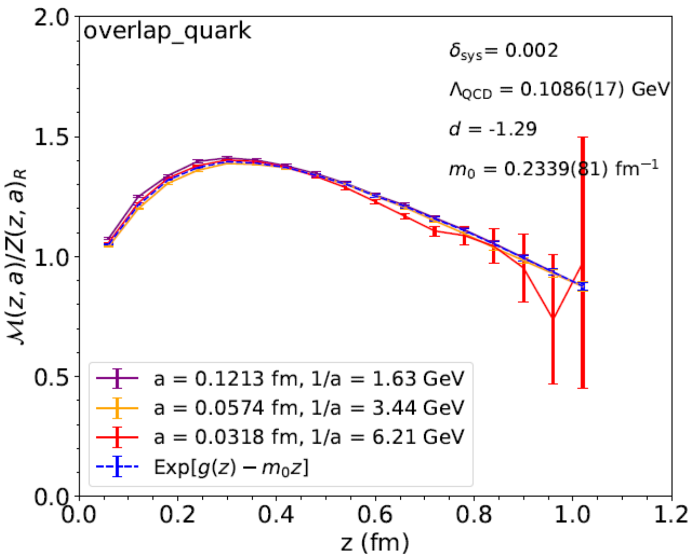




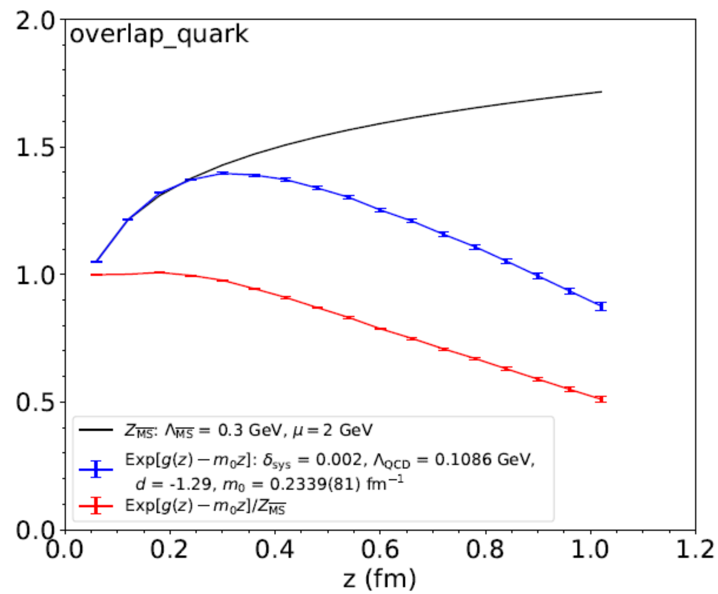
Hybrid Renormalization

Lattice Parton Collaboration (LPC)

Continuum limit



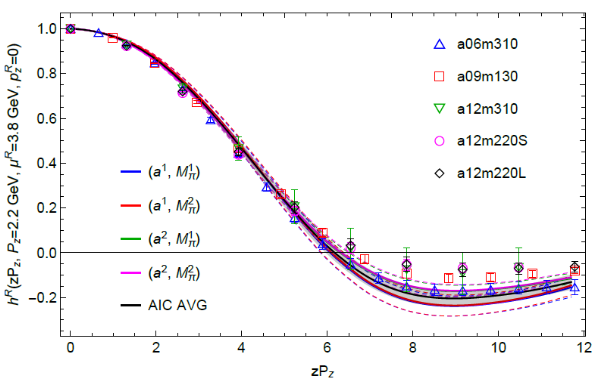
Pert.vs Non-pert Z



Continuum extrapolations

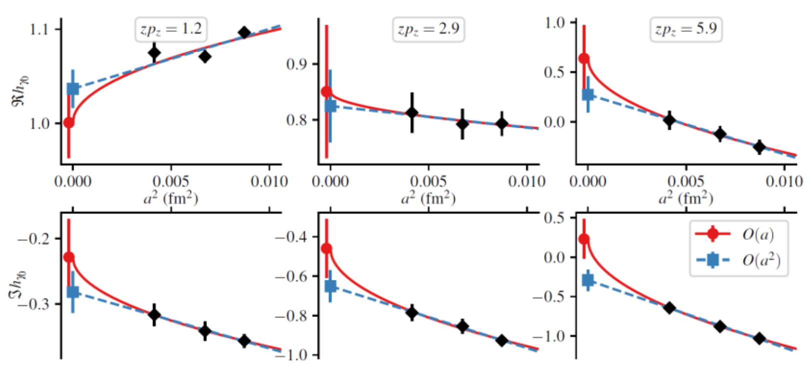
MSU

2011.14971
a=0.06,0.09,0.12



ETMC

2011.00964
a= 0.064,0.082,0.093



Valence pion PDF $(u(x) - \bar{u}(x))$

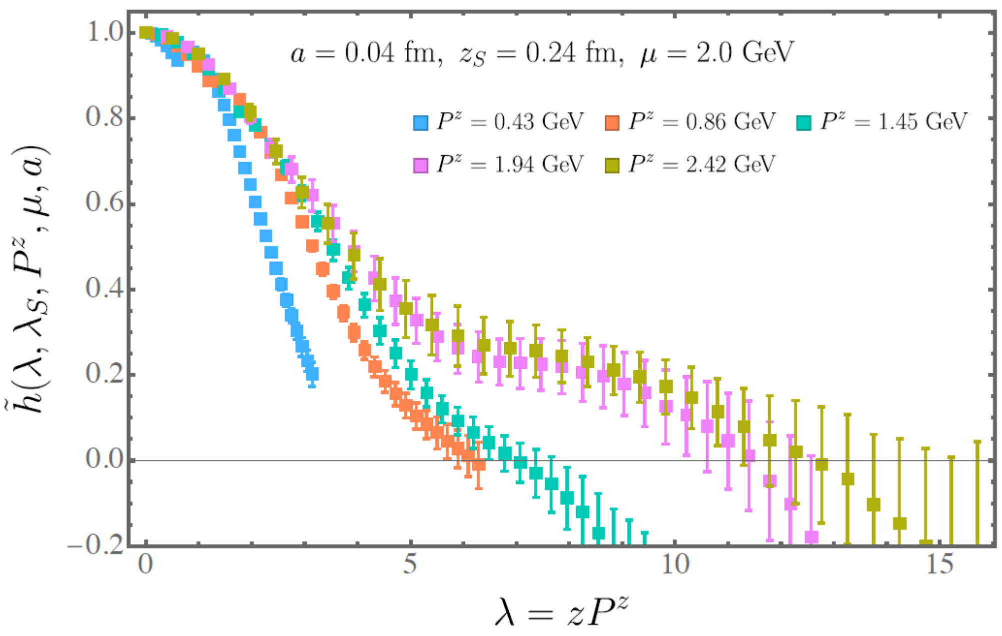
(ANL/BNL collaboration, arXiv: 2112.02208)

- Lattice data with 2+1 flavor HISQ configurations
- $m_\pi^{sea} = 160$ MeV, $m_\pi^{val} = 300$ MeV, a = 0.04, 0.06 fm
- Renormalized spatial correlation functions for fixed pion external mom.

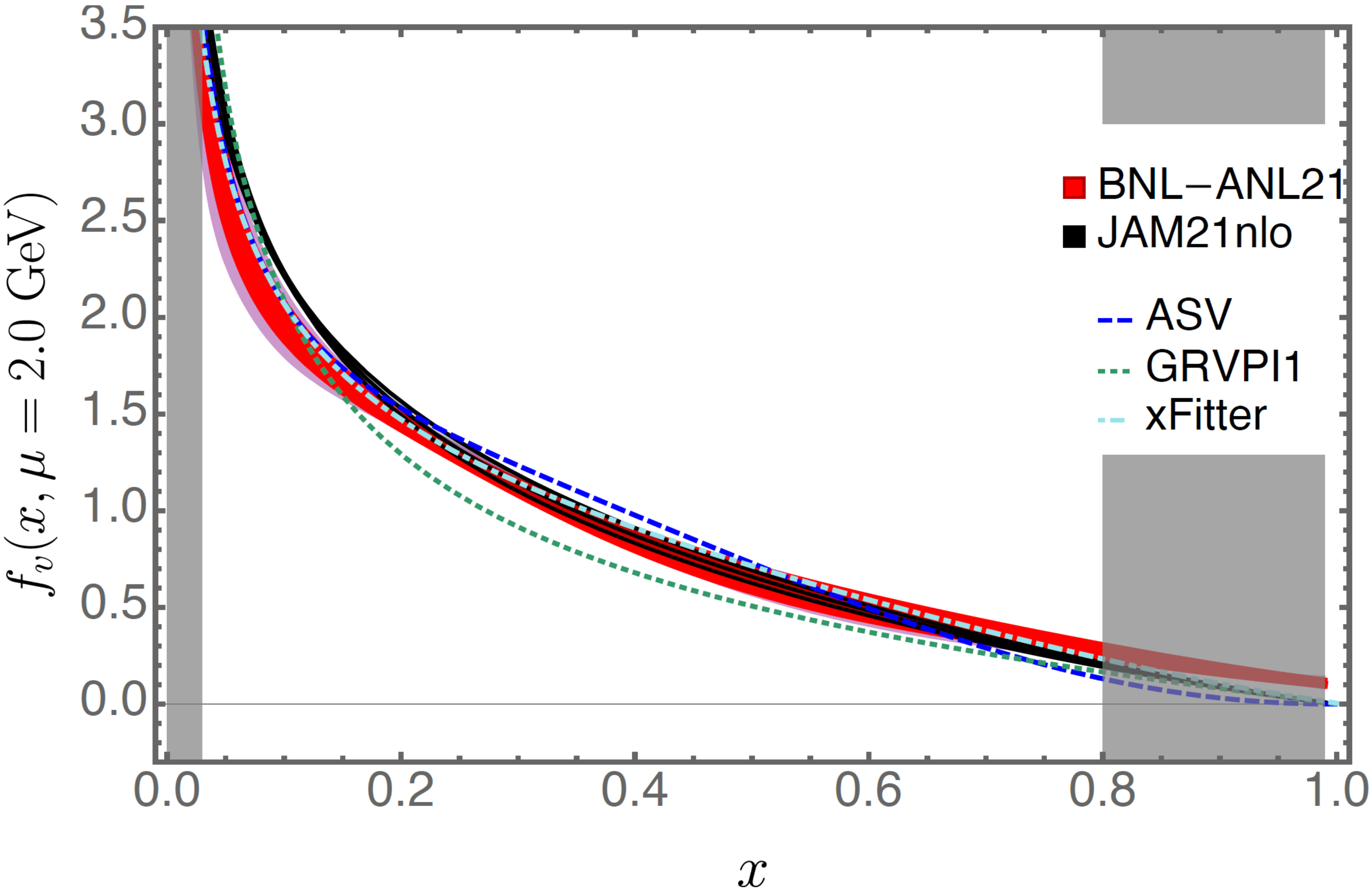
P^z = 1.45 GeV (γ=10)

1.94 GeV (γ=14)

2.42 GeV (γ=17)



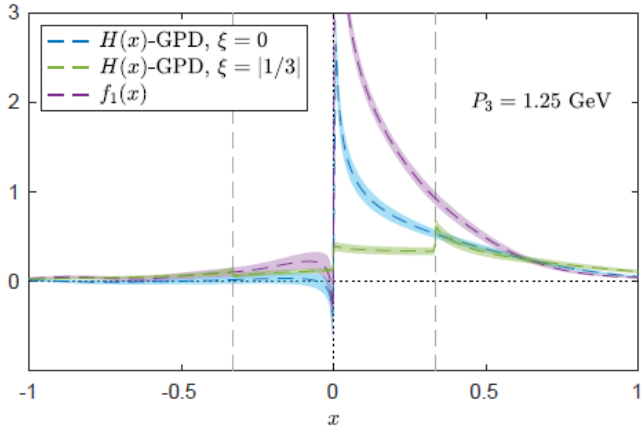
Comparison with lattice result



GPDs via LaMET

ETMC

PRL125 (2020)



MSU

[2008.12474](2008.12474) (2020)

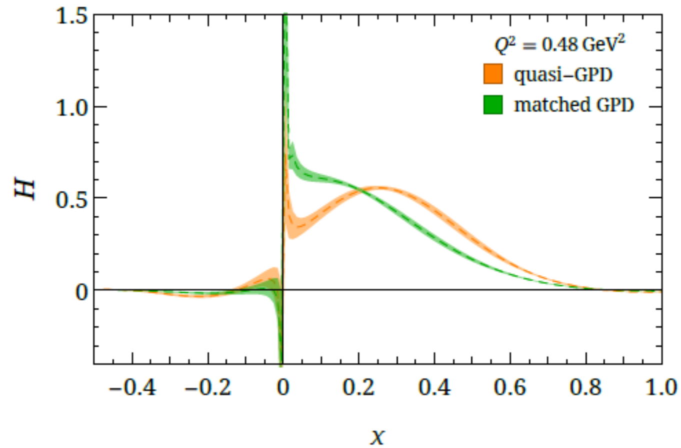


Figure 2: Examples of the first GPD data from lattice QCD. The left plot shows the matched GPD H at $\xi = 0$ and $|\xi| = 1/3$ with momentum transfer $Q^2 = 0.69$ GeV^2 [18]. The right one shows the quasi-GPD and matched GPD H at momentum transfer $Q^2 = 0.48$ GeV^2 and $\xi = 0$ [19].

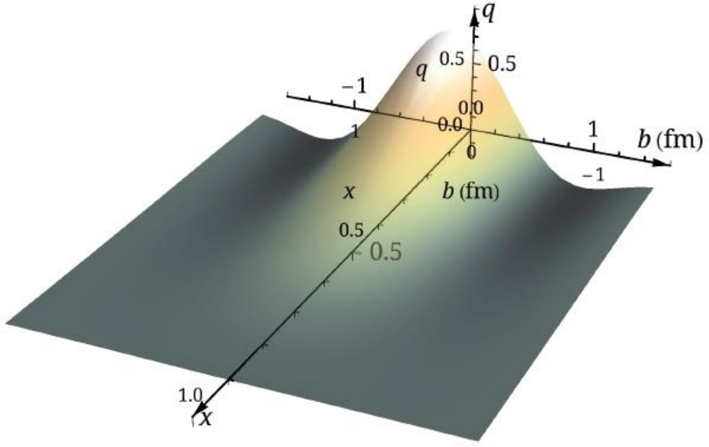
Quark transverse-plan distributions in the proton from lattice (H.W.Lin, PRL2021)

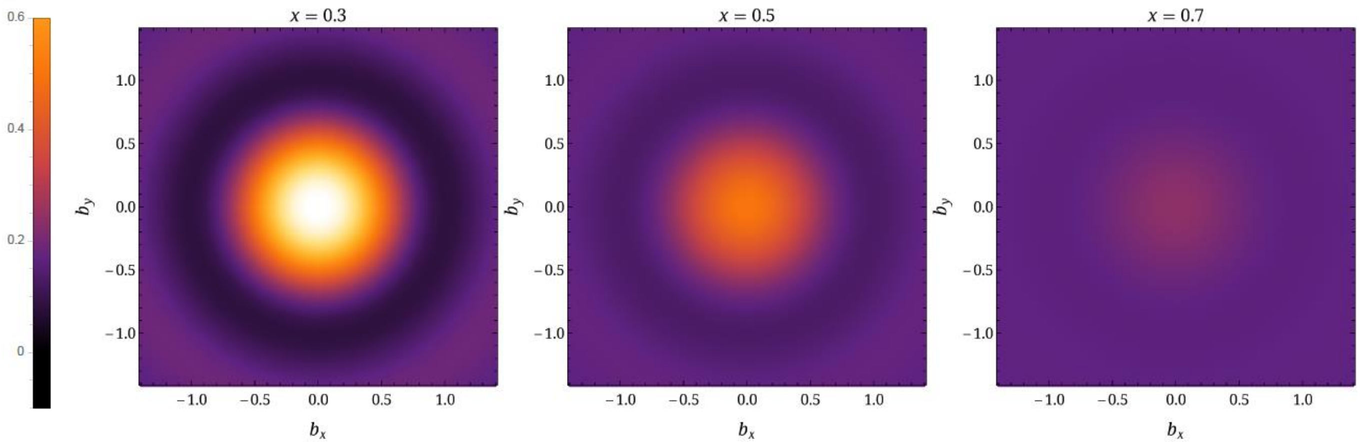
- MILC configurations

  a = 0.09 fm, m_π = 140 MeV

  Pz = 2.2 GeV

$$\mathrm{q}(x,b)=\int\frac{d\mathbf{q}}{(2\pi)^2}H(x,\xi=0,t=-\mathbf{q}^2)e^{i\mathbf{q}\cdot\mathbf{b}},$$





TMDPDF & Lattice calculations

- Started from A. Schafer et al., invariant functions, matching with continuum quantities?

  Hagler et al, Much et al, Yoon et al. PRD96,094508 (2017)…

- A number of LaMET formulations:

  Ji et al., PRD91,074009 (2015); PRD99,114006(2019)

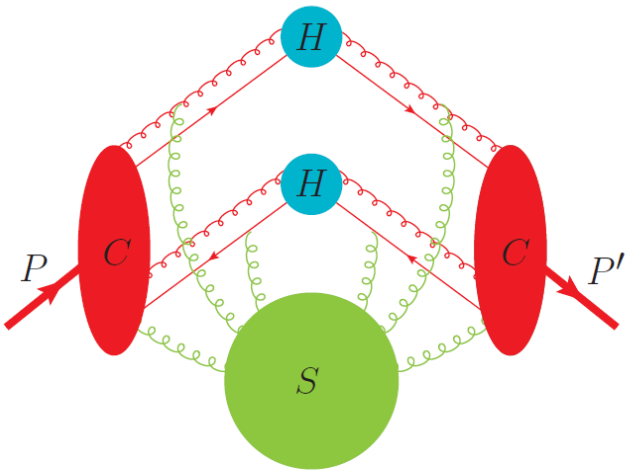
  Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019)

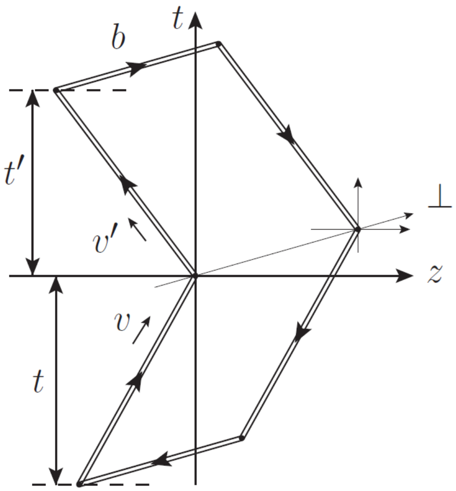
- Collins-Soper kernel can be calculated on lattice

  Ji et al., PRD91,074009 (2015); Ebert, Stewart and YZ, PRD 99(2019).M. Ebert, JHEP 03 (2020) 099

Soft Function

Ji, Liu, Liu, Nucl. Phys. B955 (2020)



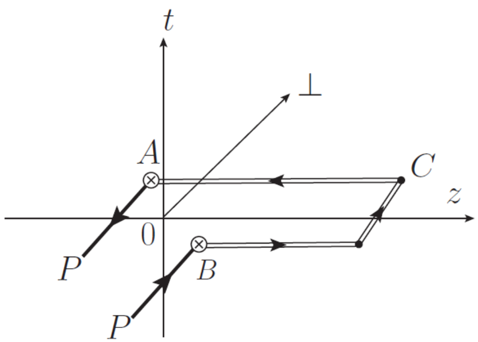


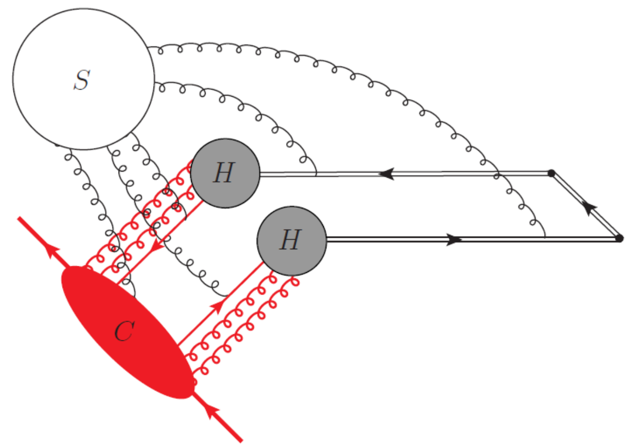
Factorization of form-factor of Light meson

Form-factors of heavy-quark pair

Quasi-TMDPDF Factorization

Ji, Liu, Liu, Phys.Lett.B 811 (2020)

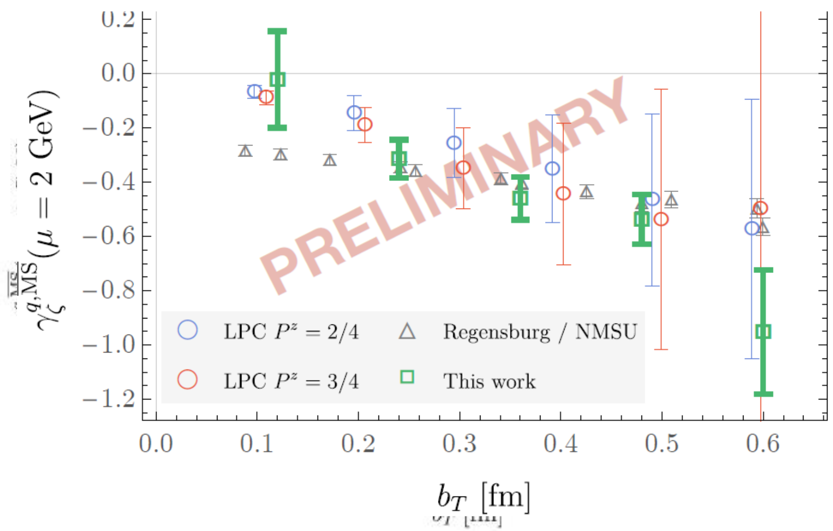


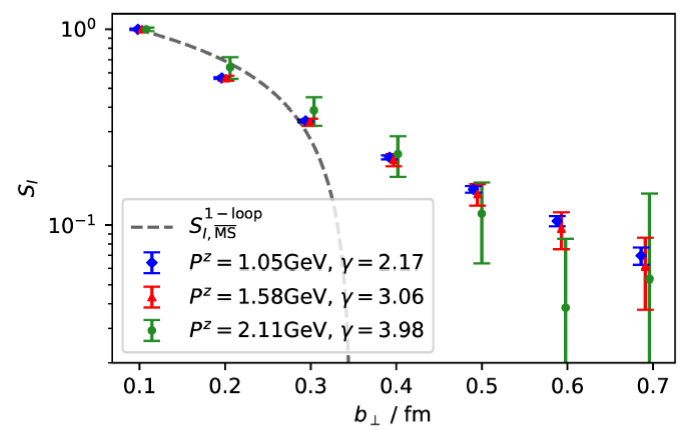


$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)} = H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\rm TMD}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu \frac{d}{d\mu} \ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\rm cusp} \ln \frac{\zeta_z}{\mu^2} + \gamma_C$$

Collins-Soper Kernel and Soft Factor





P. Shanahan et al, PRD 102 (2020)
& unpublished
Q.A.Zhang et al, PRL125 (2020)
M.Schlemmer, 2103.16991 (2021)

Q. A. Zhang et al, PRL125 (2020)
(LPC)

TMDPDF

- LPC (J.C.He et al, 2211.02340)

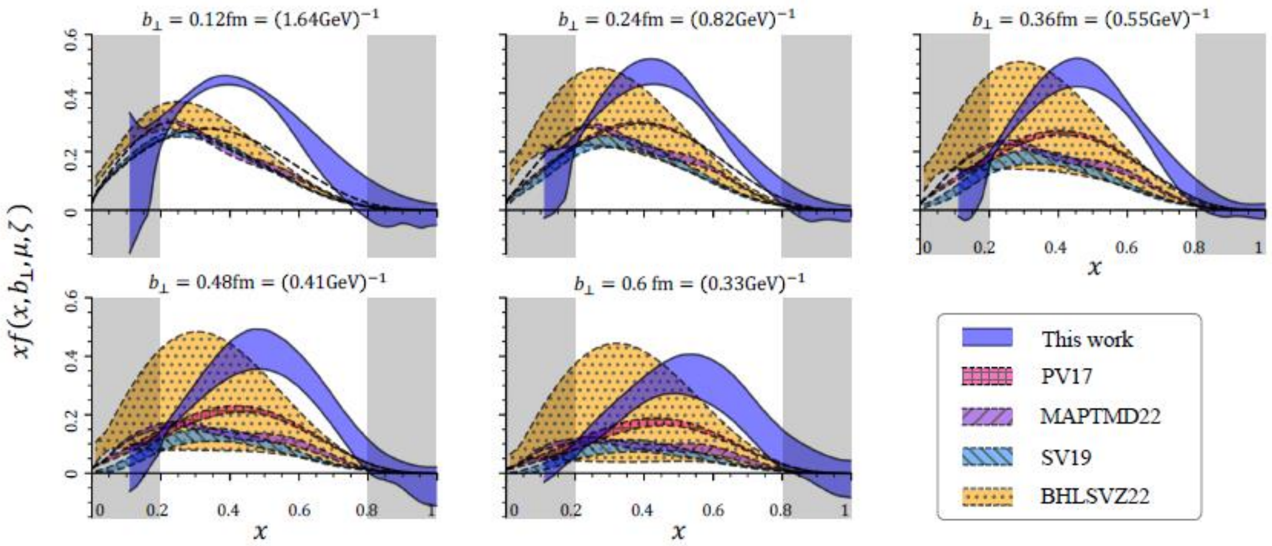


FIG. 5. Our final results for isovector unpolarized TMDPDFs $xf(x, b_\perp, \mu, \zeta)$ at renormalization scale $\mu = 2$ GeV and rapidity scale $\sqrt{\zeta} = 2$ GeV, extrapolated to physical pion mass 135 MeV and infinite momentum limit $P^z \to \infty$, compared with PV17 [6], MAPTMD22 [9], SV19 [7] and BHLSVZ22 [8] global fits (slashed bands). The colored bands denote our results with both statistical and systematic uncertainties, the shaded grey regions imply the endpoint regions where LaMET predictions are not reliable.

Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- However, there are a number of reasons this approach has not been very successful.
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji & Liu, to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

LPC result soon.

Transverse Momentum Dependent Wave Functions from Lattice QCD

(Lattice Parton Collaboration (LPC))

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We present a first lattice QCD calculation of the transverse momentum dependent wave functions (TMDWFs) in large-momentum effective theory. Numerical simulations are based on 2+1+1 flavors of highly improved staggered quarks action with lattice spacing a=0.121 fm from MILC Collaboration, and another 2 +1 flavor clover fermions and tree-level Symanzik gauge action configuration generated by CLS Collaboration with a=0.098 fm. We present the result for soft function that incorporates the one-loop perturbative contributions and a coherent normalization. Based on the obtained soft function, we simulate the equal-time quasi-TMDWFs on the lattice, and extract the physical TMDWFs. A comparison with the phenomenological parameterization is made and consistent behaviors between the two lattice ensembles and phenomenological model are found. Our studies provide crucial *ab initio* theory inputs for making precise predictions for exclusive processes under QCD factorization.

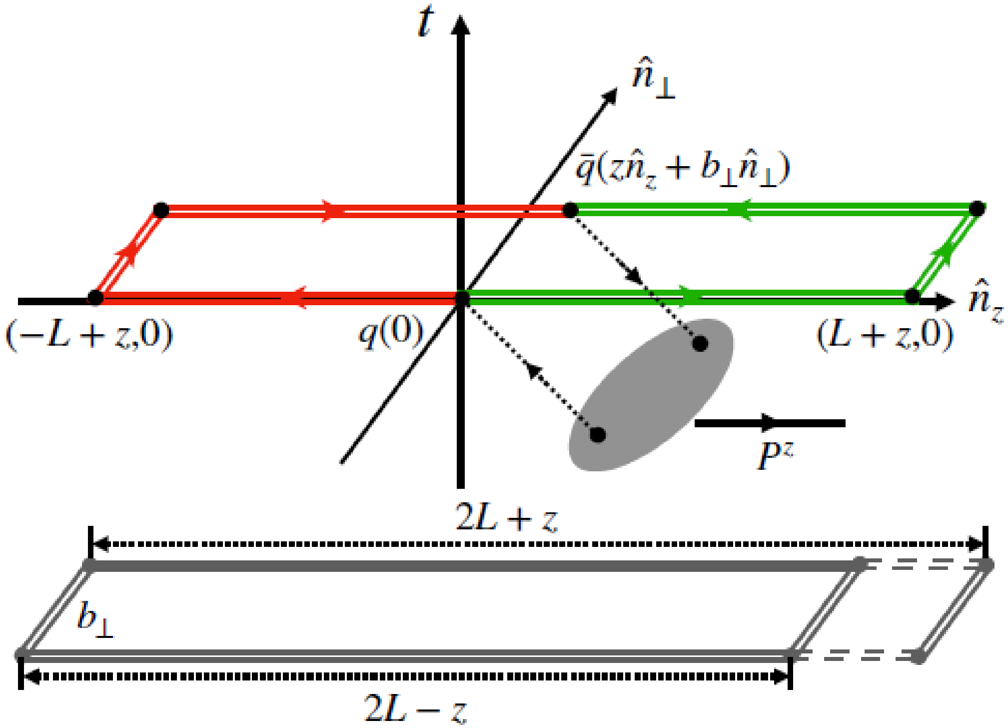


FIG. 1. Illustration of quasi-TMDWF in coordinate space with a staple-shaped gauge-link inside. As the green and red double lines represent the gauge-link in $\tilde{\Psi}^{+}(z, b_\perp, \mu, \zeta^z)$ and $\tilde{\Psi}^{-}(z, b_\perp, \mu, \zeta^z)$, a corresponding staple-shaped Wilson loop $Z_E(2L \pm z, b_\perp, \mu)$ is constructed to cancel the linear and cusp divergences.

TABLE I. The numerical simulation setup. On each ensemble, we put 8/4 source slices in time direction.

Ensemble	a(fm)	$L^3 \times T$	m_π^{sea}	m_π^{val}	Measure
a12m310	0.121	$24^3\times$ 64	310 MeV	670 MeV	1053×8
X650	0.098	$48^3\times$ 48	333 MeV	662 MeV	911×4

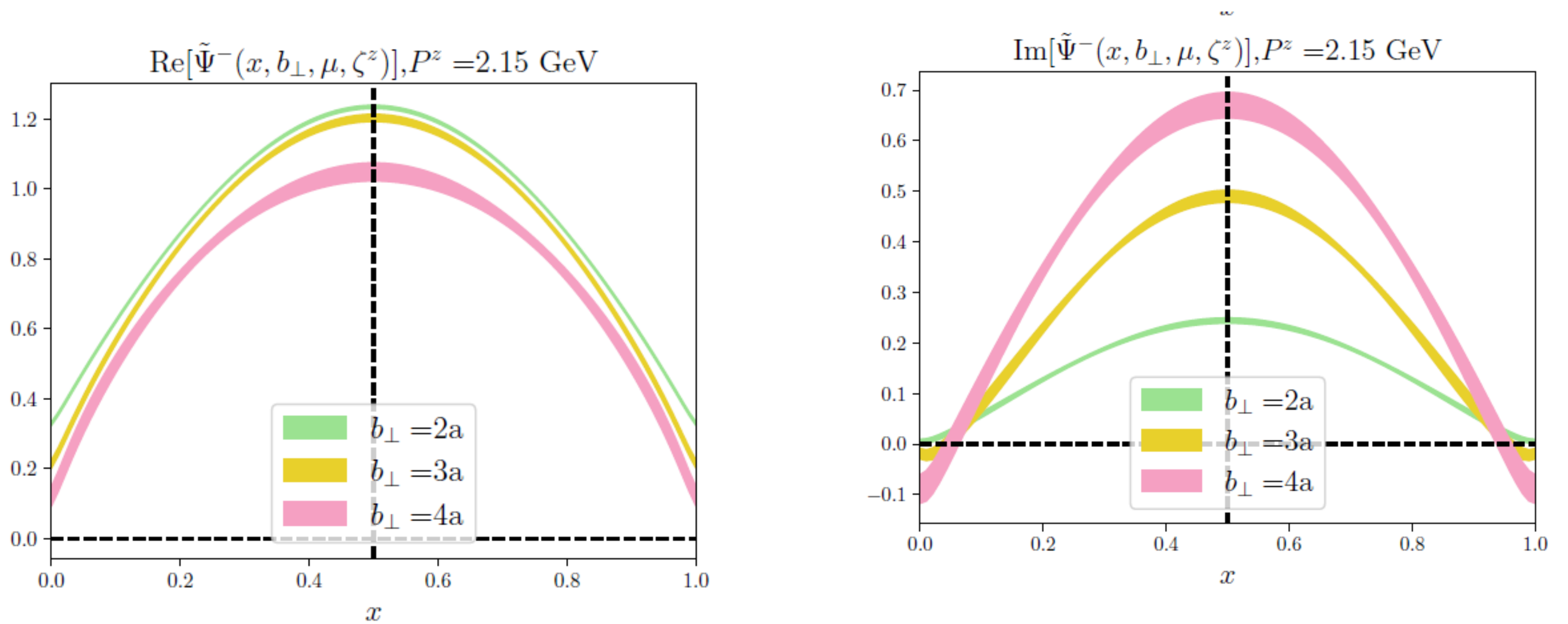


FIG. 2. The real part (upper panel) and the imaginary part (lower panel) of quasi-TMDWF in momentum space, with hadron momentum $P^z = 2.15$ GeV on MILC ensemble.

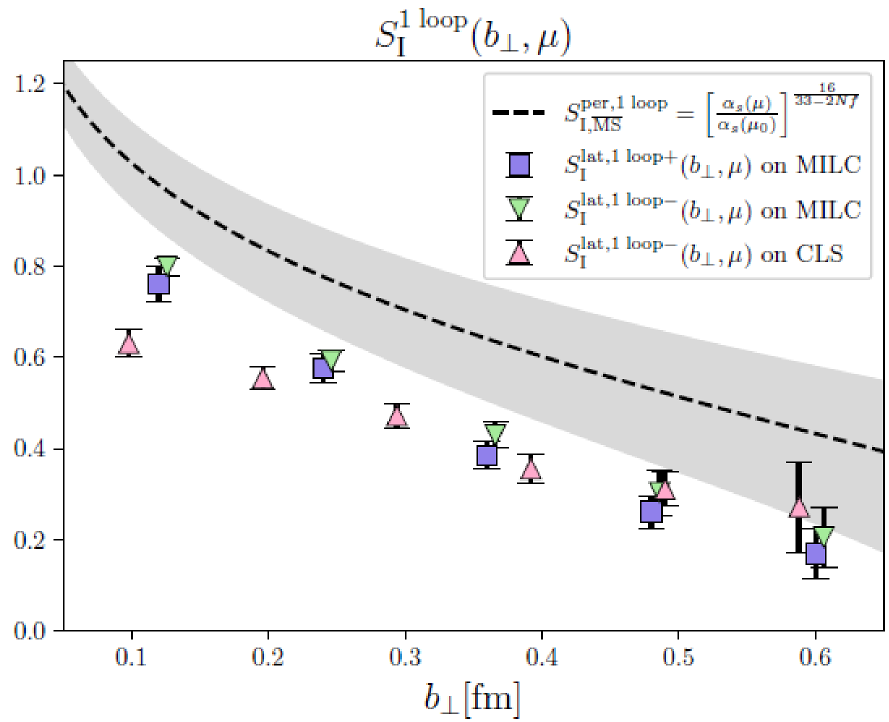
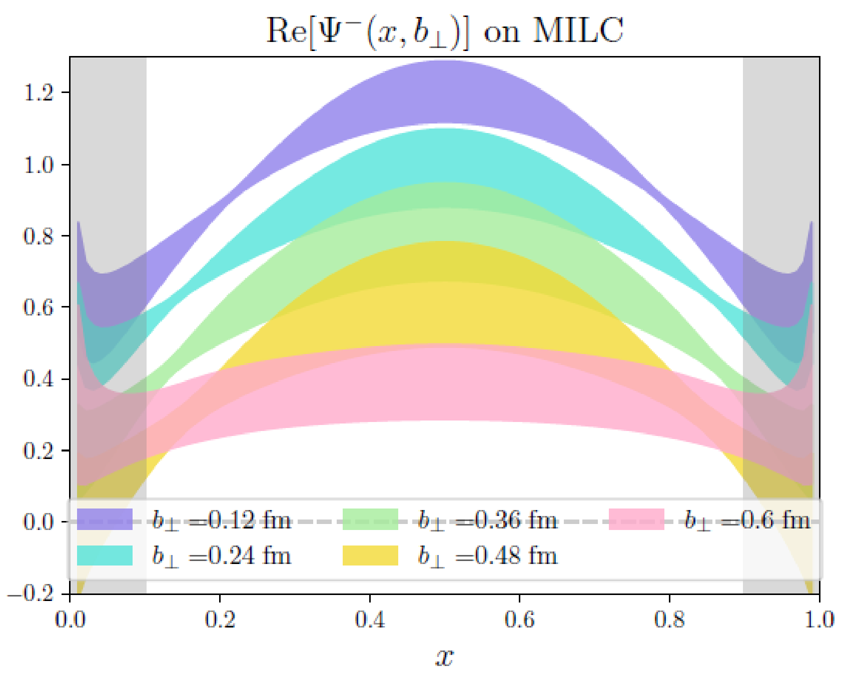
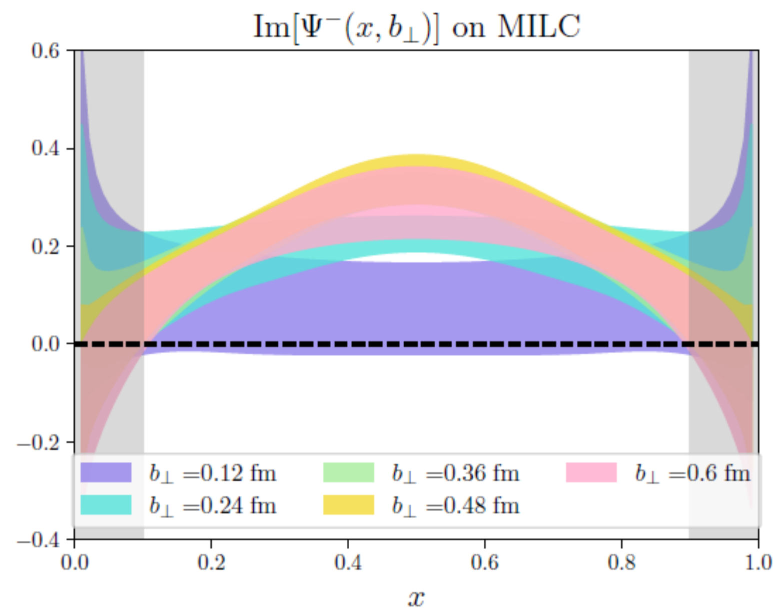


FIG. 3. The one-loop intrinsic soft function as a function of $b_\perp$. The grey band corresponds to the one-loop perturbative result in $\overline{\rm MS}$ scheme and the band is obtained by $\mu_0 = 1/b_\perp^*$ varying for the scale $b_\perp^* \in \left[1/\sqrt{2}, \sqrt{2}\right] b_\perp$. The label $\pm$ in $S^{\rm lat,1\ loop\pm}$ represents the lattice results extracted by $\tilde{\Psi}^\pm$.

Lowest fock state LFWF





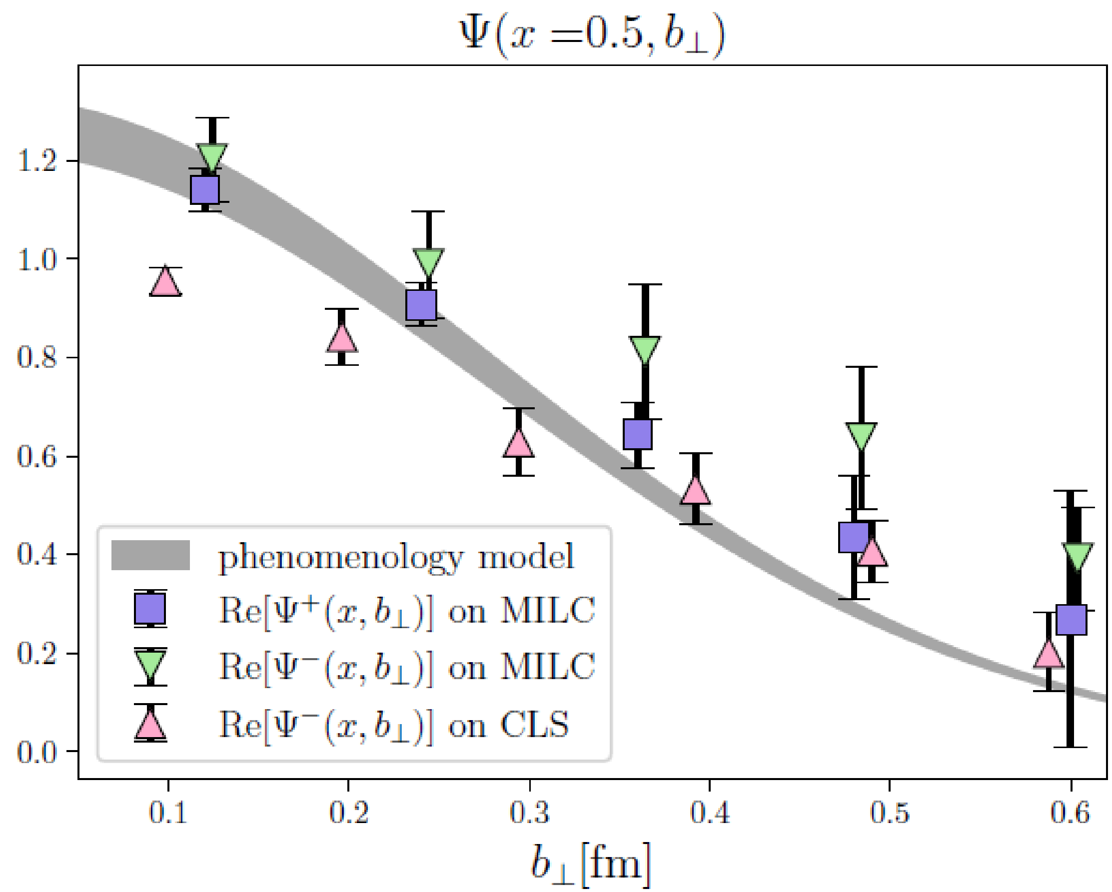


FIG. 5. Comparison of the transverse momentum distribution in our results with $\{\zeta, \mu\} = \{(6\ \mathrm{GeV})^2, 2\ \mathrm{GeV}\}$ and phenomenological model at $x = 0.5$ point.

Outlook

- LaMET is a systematic framework to calculate parton physics
- LaMET3.0 (~ 5% error?)
    - Improved non-pert renormalization at large z.
    - two-loop matching
    - P $\rightarrow$ 3 GeV
    - Singlet quark and gluon
- GPDs & TMDs & Wigner Functions & LFWF
    - More to be done!