Lecture 3:
What are partons? Three different pictures

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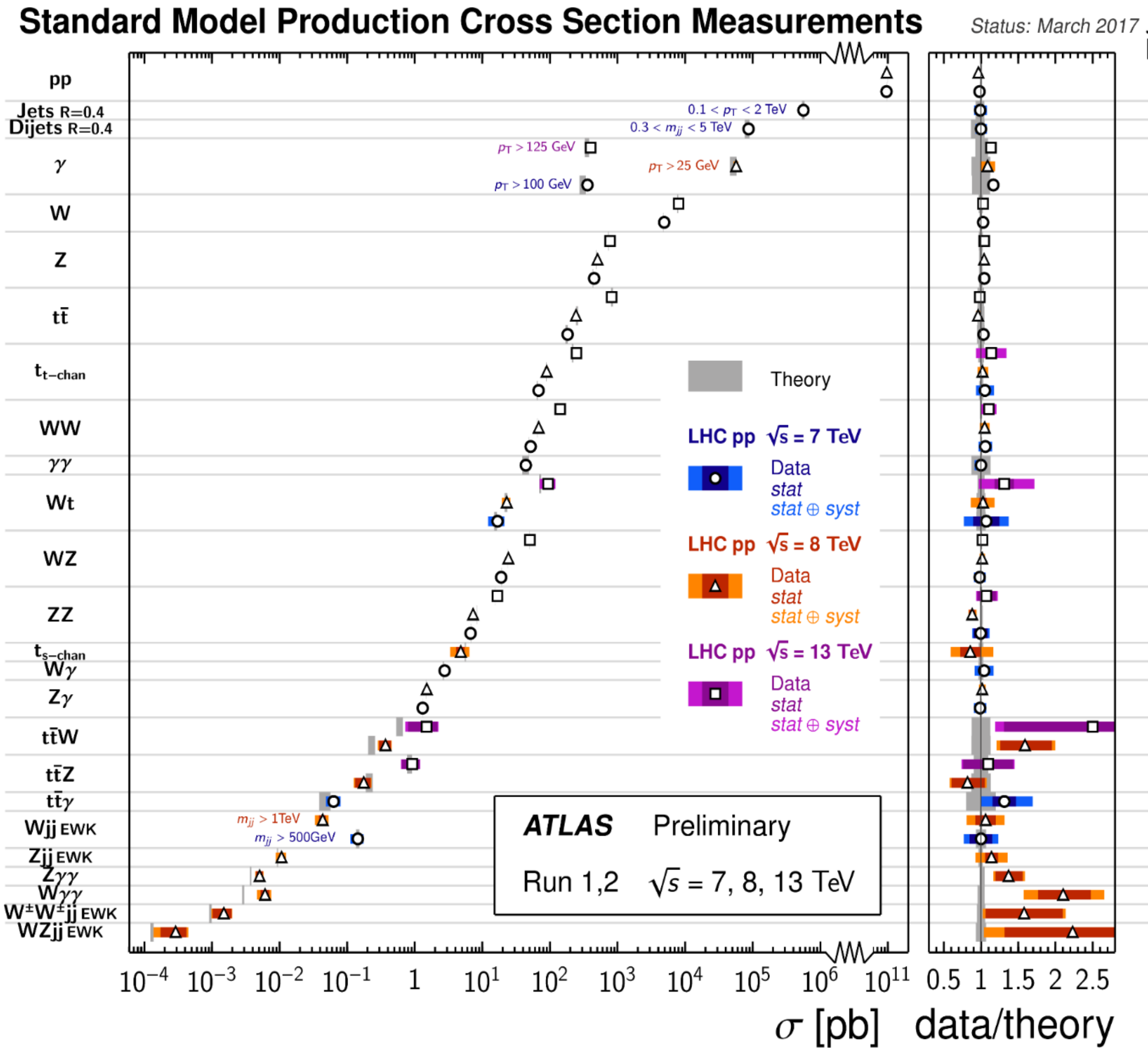
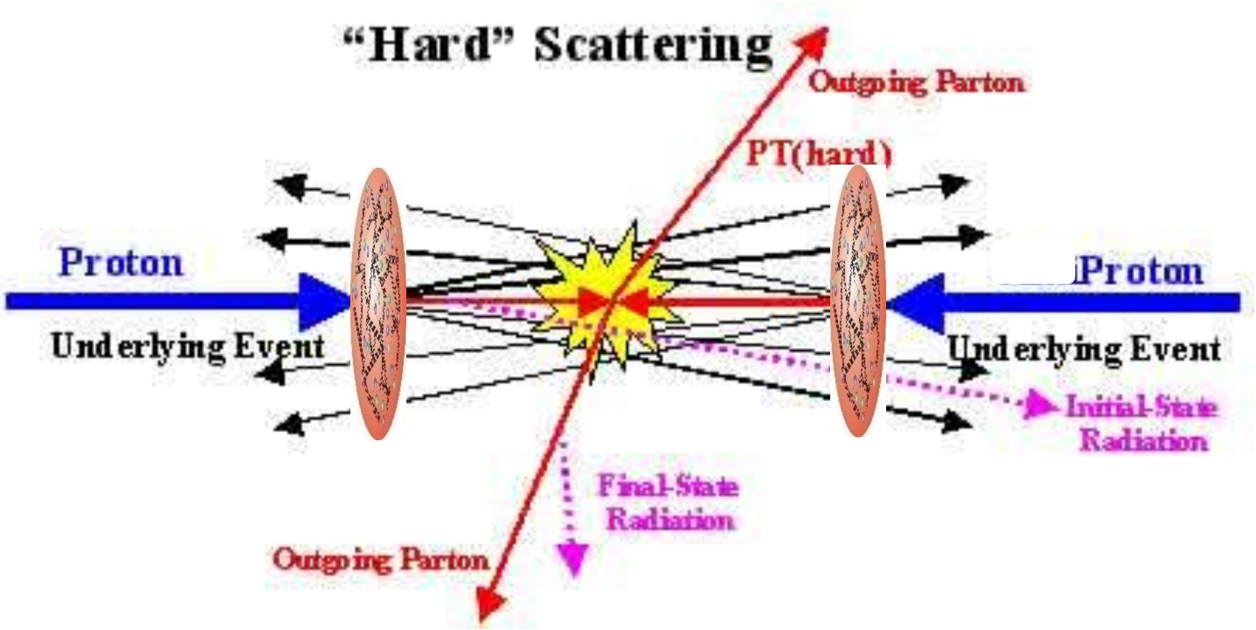
Standard model successes:

- The standard model itself has been hugely successful in explaining many physics phenomena
  - Electroweak processes
  - High-energy QCD processes

Perturbation theory works!   (LHC)



pp scattering at LHC

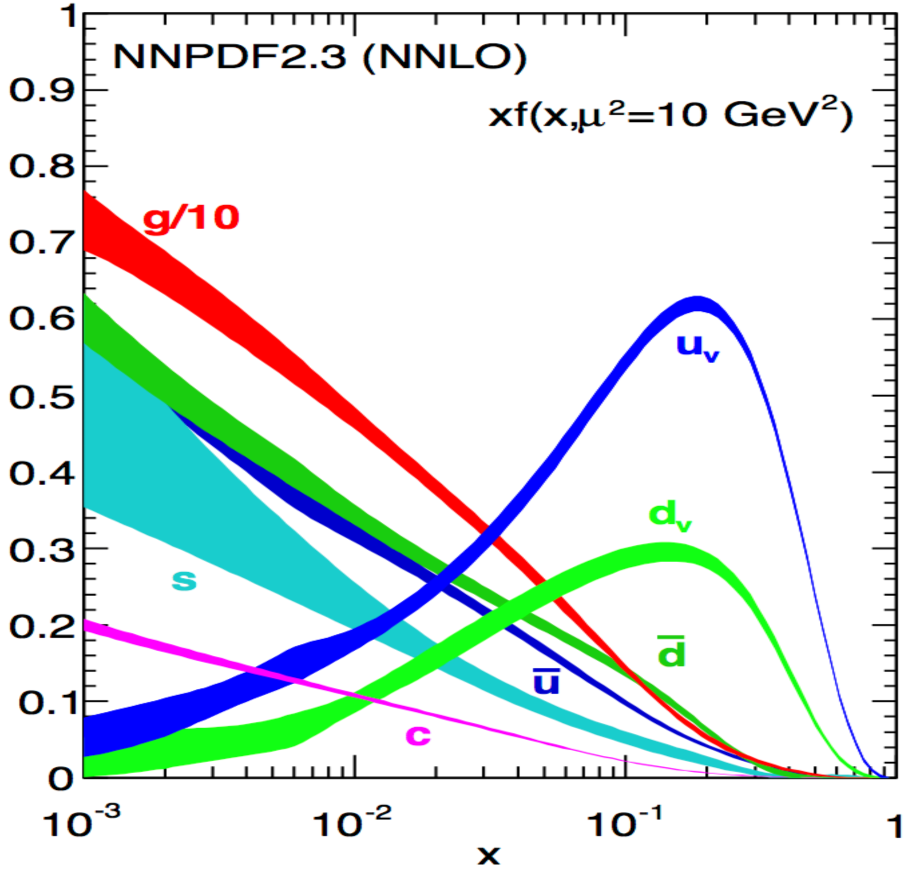


- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

Phenomenological PDFs

- Use experimental data (~50 yrs) to extract PDFs



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Main points

- Effective method (theory…, Effective QFT) is an important physis approach.
- Partons are special particles (quarks and gluons) which exist only in a hadron travelling exactly at the speed of light.
- Partons are “effective concepts”
- Three different representations of partons.
- The quarks and gluons in a high-energy proton are not partons, but quasi-partons.

Introduction to effective (quantum field) theories

Effective approach in physics

- Ignore unimportant factors, focusing on the most important ones.
- Idealized concepts
    - Point-like particles: no size
    - Frictionless surfaces: no friction
    - Ideal gases: no interactions
    - Ideal fluids: no viscosity
    - Absolute-zero temperature: $T=0$
    - Infinite potential well: $V=\infty$
    - Harmonic oscillators: no anharmonicity
    - …

Math methods: Taylor expansion

- A physical quantity f may depend important variable x, less important ones ε, δ, etc

$$f(x, \epsilon, \delta, \dots)$$

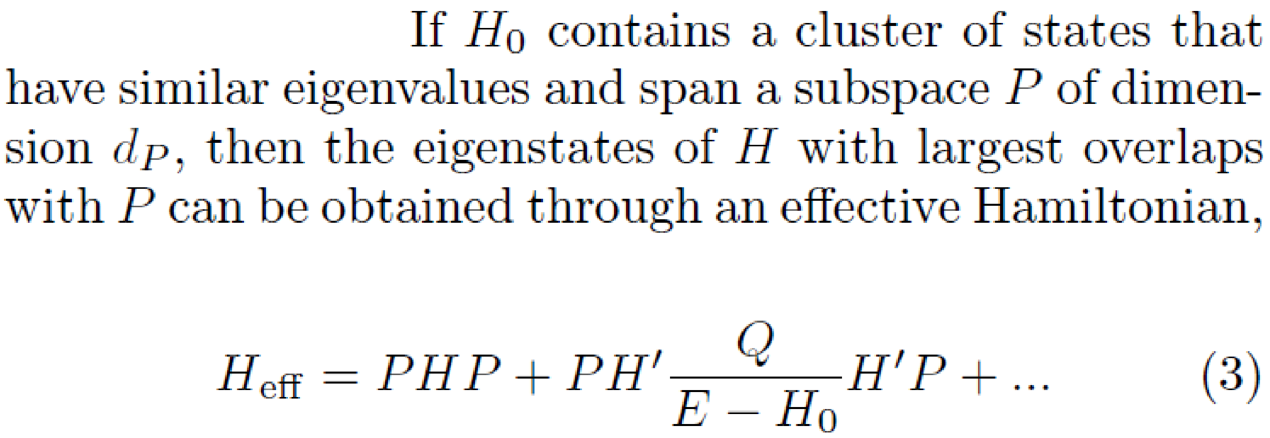
- One can simplify the problem by making Taylor expansion

$$f(x, \epsilon, \delta, \dots) = f(x, 0, 0, \dots) + \epsilon f_\epsilon(x, 0, 0, \dots) \\ + \delta f_\delta(x, 0, 0, \dots) + \dots\ .$$

- Most of time, we just care about the first term. But higher order terms can be calculated systematically. This is an effective theory.

More examples

- Multipole expansion in electrostatics: q, $\vec{q}$, Q_{ij}
- Virial expansion for the quation of state: p=nkT+…
- Perturbation theory in celestial mechanics: 3-body
- Perturbation theory in QM $H = H_0 + H'$



If H_0 contains a cluster of states that have similar eigenvalues and span a subspace P of dimension d_P, then the eigenstates of H with largest overlaps with P can be obtained through an effective Hamiltonian,

$$H_{\text{eff}} = PHP + PH'\frac{Q}{E - H_0}H'P + \ldots \qquad (3)$$

A few remarks

$$\begin{array}{cc} \left(\begin{array}{c|c} P & H' \\ \hline H' & Q \end{array}\right) & \begin{array}{c} E_p \\ E_Q \end{array} \end{array}$$

$$\underline{\Delta E = E_p - E_Q}$$

- The complementary space P = 1-Q has been summed or "integrated out"
- Expansion parameter is the energy ratio

$$\epsilon \;=\; ||H'||/\Delta E$$

where ΔE is the energy difference between P and Q spaces

- To calculate the observables using "effective wave functions", one needs to have "effective operators", O_{eff}

$$\langle \psi_{eff} | O_{eff} | \psi_{eff} \rangle = \langle \psi | O | \psi \rangle$$

Effective (quantum) field theory

- EFT refers to a theory in which an effective approach has been applied for QFT.
- Some well-know examples:
    - Renormalization is an EFT, thus all QFT are EFTs
    - Standard model is an EFT:
    - QCD perturbation theory is an EFT:
    - Chiral perturbation theory:
    - Lattice QCD:
    - Heavy quark effective theory (HQET):
    - Soft collinear effective theory (SCET):
    - ….

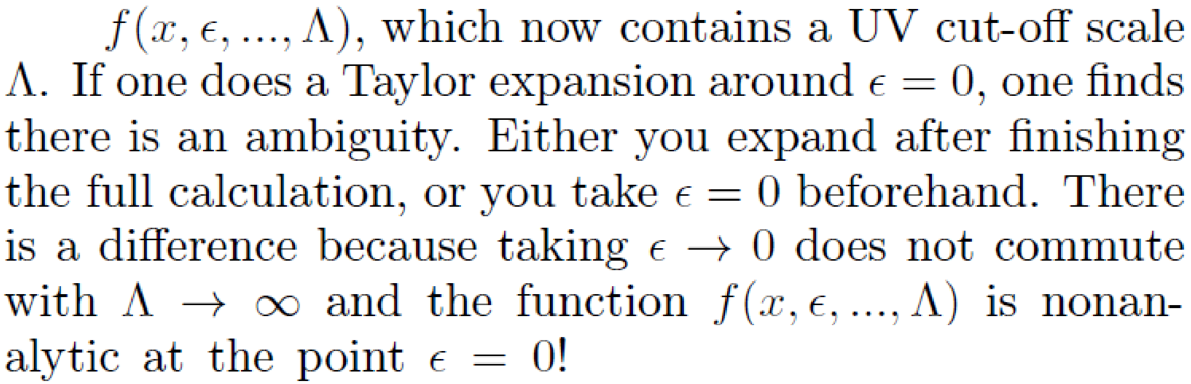
P-space: Model space, Q-space: Integrated out

Renormalization	P	Q
SM	P.	Q
pQCD	Q	P
χ.P.T.	P.	Q
lattice QCD	P.	Q
HQET	Q	P

New feature of EFT: UV divergences

Λ_{UV}

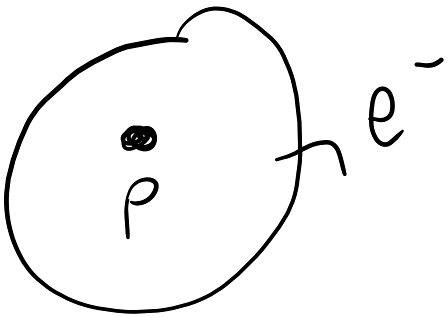
- Field theories have UV divergences, which are generally renormalized to make finite predictions.
- UV divergences make Taylor expansion more complicated.



$f(x, \epsilon, ..., \Lambda)$, which now contains a UV cut-off scale Λ. If one does a Taylor expansion around $\epsilon = 0$, one finds there is an ambiguity. Either you expand after finishing the full calculation, or you take $\epsilon = 0$ beforehand. There is a difference because taking $\epsilon \to 0$ does not commute with $\Lambda \to \infty$ and the function $f(x, \epsilon, ..., \Lambda)$ is nonanalytic at the point $\epsilon = 0$!

$\epsilon = 0$ is not a legitimate point for expansion!

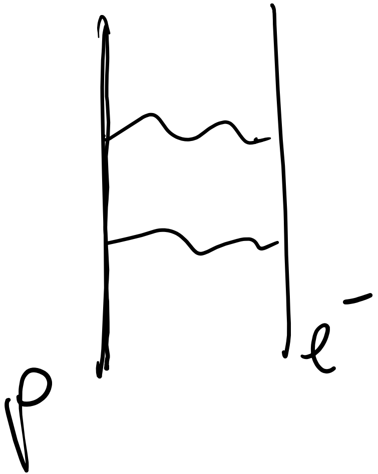
Spectrum of H-atom



proton mass is small effect, EFT-exp:

$$\hat{E}_n = E_n(m_p=\infty) + \frac{a_n}{m_p} + \frac{b_n}{m_p^2}$$

However, full calculation gives

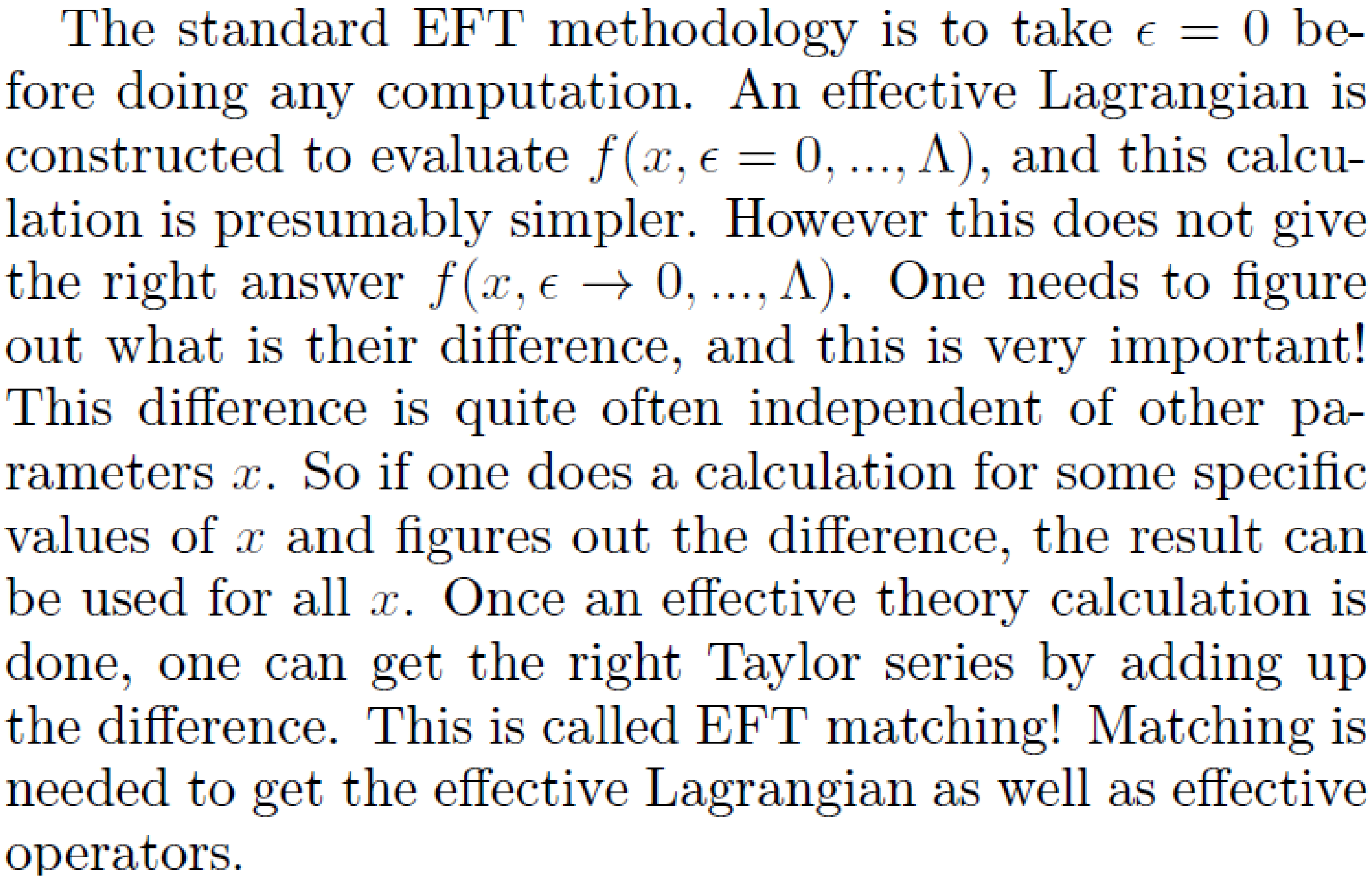


$$\longrightarrow \quad \delta E_n \sim \ln\frac{m_p}{m_e}$$

one cannot expand at $m_p=\infty$

Naive Taylor expansion does not work

Standard EFT approach: Matching



The standard EFT methodology is to take $\epsilon = 0$ before doing any computation. An effective Lagrangian is constructed to evaluate $f(x, \epsilon = 0, ..., \Lambda)$, and this calculation is presumably simpler. However this does not give the right answer $f(x, \epsilon \to 0, ..., \Lambda)$. One needs to figure out what is their difference, and this is very important! This difference is quite often independent of other parameters x. So if one does a calculation for some specific values of x and figures out the difference, the result can be used for all x. Once an effective theory calculation is done, one can get the right Taylor series by adding up the difference. This is called EFT matching! Matching is needed to get the effective Lagrangian as well as effective operators.

EFT Expansion for $m_p = \infty$

① First expand Lagrangian at $m_p = \infty$

all Feynman rules used have $m_{p=\infty}$

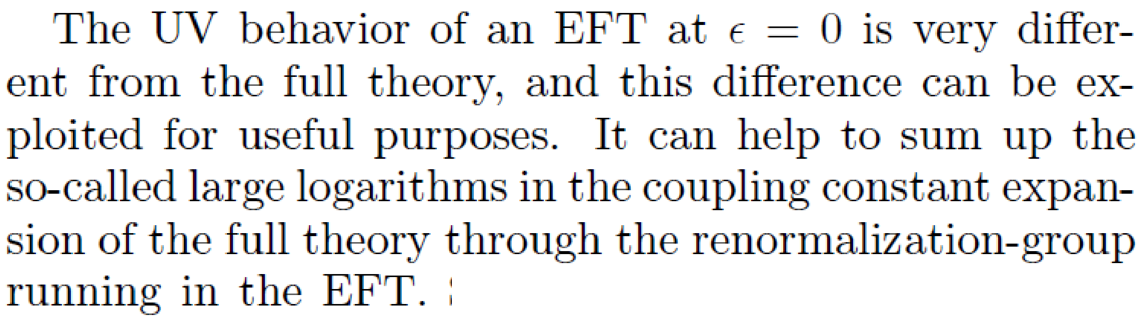
No m_p appear, However, UV physics is different

② Matching EFT with full theory

$$\Gamma_n^{\text{full}}(m_p) = Z\left(\frac{m_p}{\mu}\right)\Gamma_n^{EFT}(\mu)$$

$\Uparrow$ matching coefficient

Standard EFT approach: Running



The UV behavior of an EFT at $\epsilon = 0$ is very different from the full theory, and this difference can be exploited for useful purposes. It can help to sum up the so-called large logarithms in the coupling constant expansion of the full theory through the renormalization-group running in the EFT.

Matching and running are standard methods in EFT which have been exploited for various purposes

Large momentum effective theory uses EFT technique and makes parton physics calculable in Euclidean formulation of QCD, such as lattice gauge theory

Since E^{Full} is independent of

μ, $$\frac{d}{d\mu} Z\left(\frac{m_p}{\mu}\right) E_n^{EFT}(\mu) = 0$$

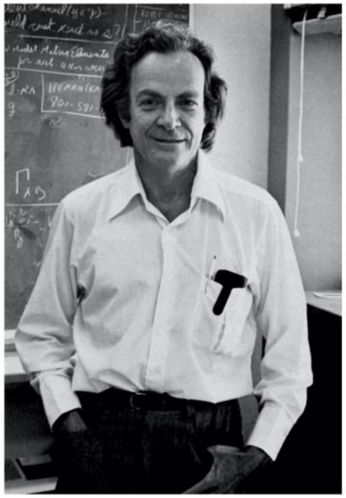
Renormalization group equation for $Z\left(\frac{m_p}{\mu}\right)$

Summing over all large <u>logs</u> $\left(\ln \frac{m_p}{m_e}\right)$

μ runs from $\underline{m_e \to m_p}$

Partons as an effective theory concept

Feynman’s parton model

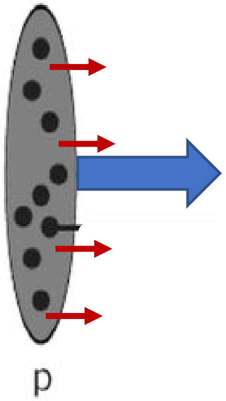


- When a high-energy proton travels at v → c, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)

- The proton may be considered as a collection of interaction-free particles: partons



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Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

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(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Parton distribution functions (PDF)

- Every parton has $k=\infty$, however,

$$x=k/p = \text{finite}, \in [0,1]$$

EFT feature

Not true in full theory

- Parton distribution function

$$f(x)$$

is the probability of finding parton in a proton, carrying x fraction of the momentum of the parent.

- PDF is a bound state property of the proton, essential to explain the results of high-energy collisions.

Partons in QFT are effective DOFs

- Partons are not just the quarks and gluons in the usual QCD lagrangian.
- Partons are a special type of IR collinear modes with momentum

$$k^{\mu} = \left(k^0, k^z, \vec{k}_{\perp}\right)$$

p - space

with $k^z \to \infty, k^0 \to \infty, k_{\perp} \sim \Lambda_{QCD}, k_{\mu}^2 \sim \Lambda_{QCD}^2$

In a proton with infinite mom.

Collins, Soper and Sterman, QCD factorization, 70'-80's

Bauer, Stewart et al, Soft-Collinear EFT, 00's

Integrate out all other modes

Parsons in light front theory

Weinberg's rules

(1933-2021)

- What does an object look like when travelling at infinite momentum or speed of light?
- S. Weinberg (scalar QFT)

  Dynamics at infinite momentum

  Phys. Rev. 150 (1966) 1313-1318

- All kinematic infinities can be removed from the calculations, resulting a set of rules for Hamiltonian perturbation theory ("old-fashioned p.t.")
- The result is similar to a "non-relativistic" theory.

EFT for partons

More Weinberg's rules… and a discovery

- L. Susskind, K. Bardakci, and M. B.Halpern,…
- S. J. Chang and S. K. Ma (1969)

Feynman rules and quantum electrodynamics at infinite momentum,

Phys.Rev. 180 (1969) 1506-1513

- J. Kogut and D. Soper (1969)

Parton EFT for QED

Quantum Electrodynamics in the Infinite Momentum Frame,
Phys.Rev.D 1 (1970) 2901-2913

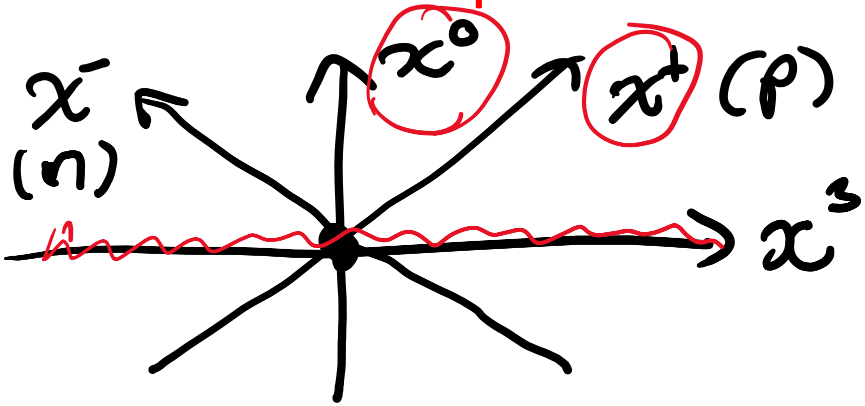
Chang and Ma's discovery

- All Weinberg's rules in the P=∞ limit can be obtained by quantizing the theory with "new coordinates"

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating x^+ as the new "time"

x^- as the new "space" dimension.



Dirac’s form of dynamics

- The Weinberg’s rules exactly correspond to what Dirac proposed in 1949.
- Paul A.M. Dirac,

Forms of Relativistic Dynamics,

Rev. Mod. Phys. 21 (1949) 392-399.

“Front form”

or Light-front quantization (LFQ)

Solve QCD in LFQ?

- All slow-moving stuff in zero-modes (vacuum).
- EFT has extra rapidity divergences which are not entirely UV nature.
- It is a strongly coupled problem!
- There is no demonstration that the weak coupling expansion actually works for QCD.

K. Wilson et. al. Phys. Rev. D49 (1994)

Hamiltonian is not known!

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PHYSICS TOPICS

- hadronic structure
- meson and baryon spectroscopy
- parton physics
- finite temperature and density QCD
- few- and many-body physics

METHODOLOGIES

- light-front field theories
- lattice field theory
- effective field theories
- phenomenological models
- present and future facilities

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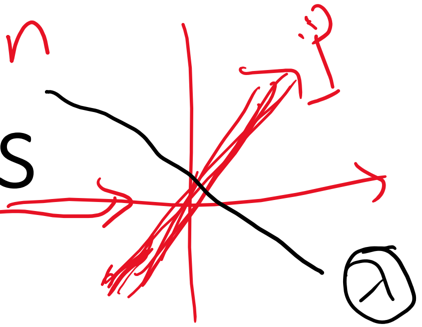
Partons in QCD factorization

Light-front collinear modes



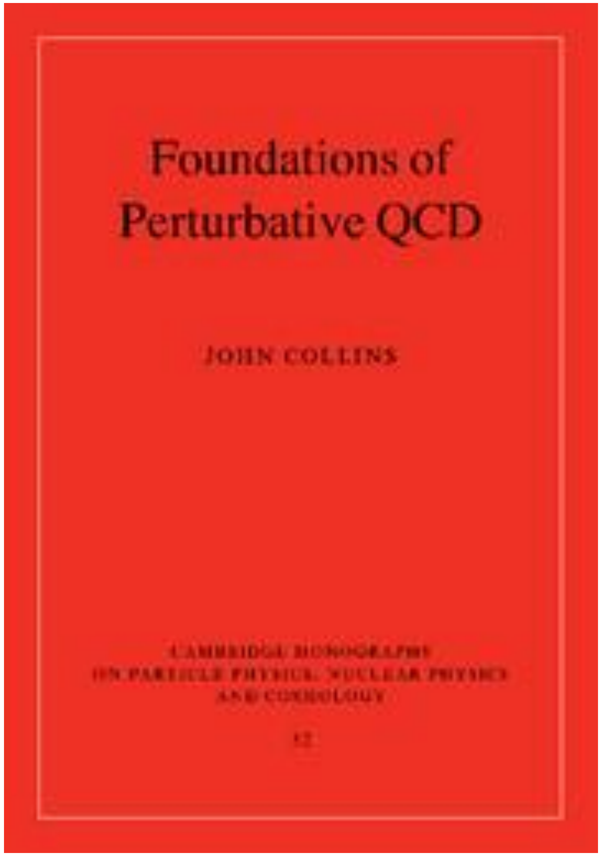
- In lagrangian formulation of parton physics, the partons are represented by collinear modes in QCD

$$\psi(\lambda n)\,,\ n^2 = 0$$

λ is the distance along the LF

- Parton physics is related to correlations of these fields along n with distance λ.

e.g. Soft-collinear Effective Theory (SCET) (Bauer, Stewart, et al)



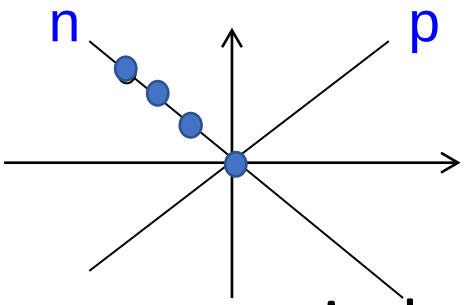
Foundations of Perturbative QCD

JOHN COLLINS

Partons as LF correlations

- Probes (operators) are light-cone correlations

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n})\phi_2(\lambda_2 n)\ldots\phi_k(\lambda_k n)$$



- The matrix elements are independent of hadron momentum, and they can be calculated in the states in the rest frame. → opposite to Feynman picture
- "Heisenberg picture"

2h

Real-time Monte Carlo in path integrals?

- Monte Carlo simulations have not been very successful with quantum real-time dynamics.

$$\exp(-iHt)$$

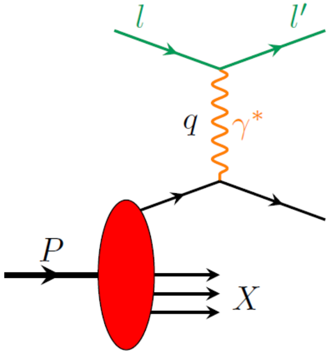
an oscillating phase factor!

- "Sign problem": Hubbard model for high Tc.
- Signals are exponentially small!
- Quantum computer?

Feynman parton in QFT & connection with other parton EFTs

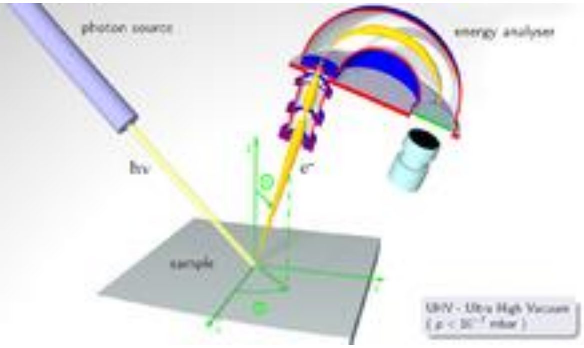
Origin of Parton Model

- Electron-proton deep-inelastic scattering (DIS)



- Knock out scattering in NR systems
  - e-scattering on atoms
  - ARPES in CM systems
  - Neutron scattering on liquid He

  …



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega|\hat{\psi}^+(\vec{r})\hat{\psi}(0)|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

"Equal time"

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \ \rightarrow \infty$

Feynman’s partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp \, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \rightarrow \infty$ limit,

$$f(k^z, P^z) \rightarrow_{P^z \rightarrow \infty} f(x) \quad with\; x = \frac{k^z}{P^z},$$

Or

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

↑ Taylor expansion in (M/p^z)

Correct in NR theory

Euclidean formulation of partons

- Calculate the Euclidean correlation

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z) \Gamma \psi(0) | P^z = \infty \rangle$$

$$\lambda \ = \ \lim_{P^z \to \infty, z \to 0} (z P^z).$$

- Parton distribution

$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Relations among parton formalisms

	States	operators	Time-signature	
LFQ	P=∞	LF correlators	Minkowski	X
SCEF	p=finite	LF correlators	Minkowski	x
Feynman	p=∞	Equal time	Euclidean	✓