Four Lectures on non-perturbative QCD, hadron structure and parton physics

6th Chilean School of High Energy Physics
Jan. 2023

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Outline of Lectures

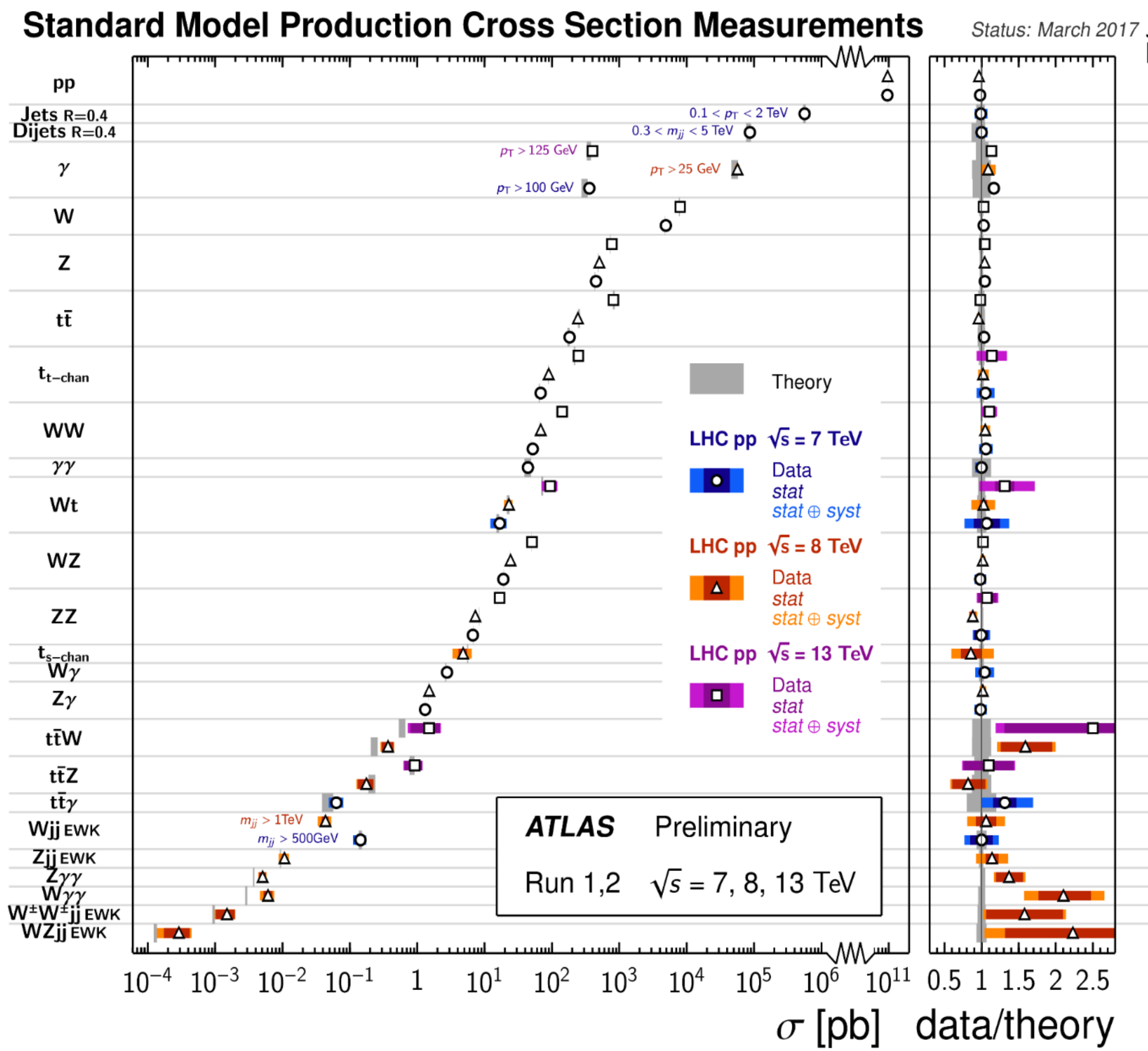
- Lecture 1: Why non-perturbative QCD is a challenge ?
- Lecture 2: An amateur's introduction to lattice QCD
- Lecture 3: What is parton physics?
- Lecture 4: Large momentum effective theory for calculating partons from first principles

Lecture 1: : Why non-perturbative QCD is a challenge ?

Standard model successes:

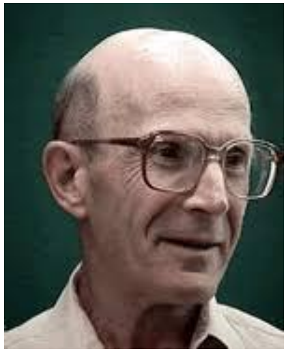
- The standard model itself has been hugely successful in explaining many physics phenomena
  - Electroweak processes
  - High-energy QCD processes

Perturbation theory works! (LHC)



Birth of QCD (1964-1973)

- Quantum Chromodynamics (QCD): A SU(3) quantum field theory of quarks and gluons



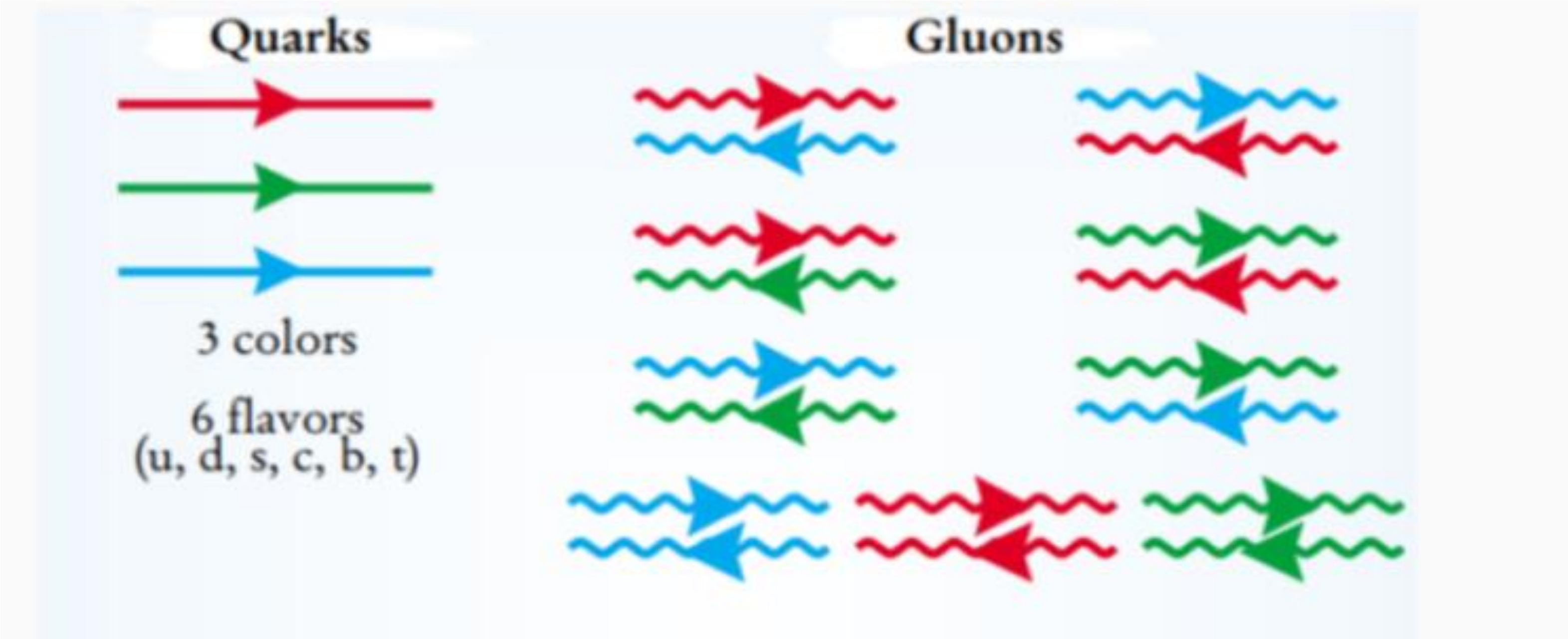






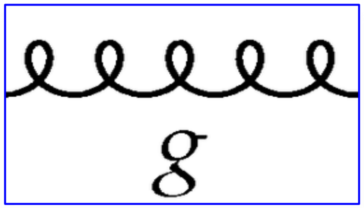
QCD: QFT of strong interactions

Color-charge SU(3) non-Abelian gauge theory



Protons & neutrons made of light up & down quarks

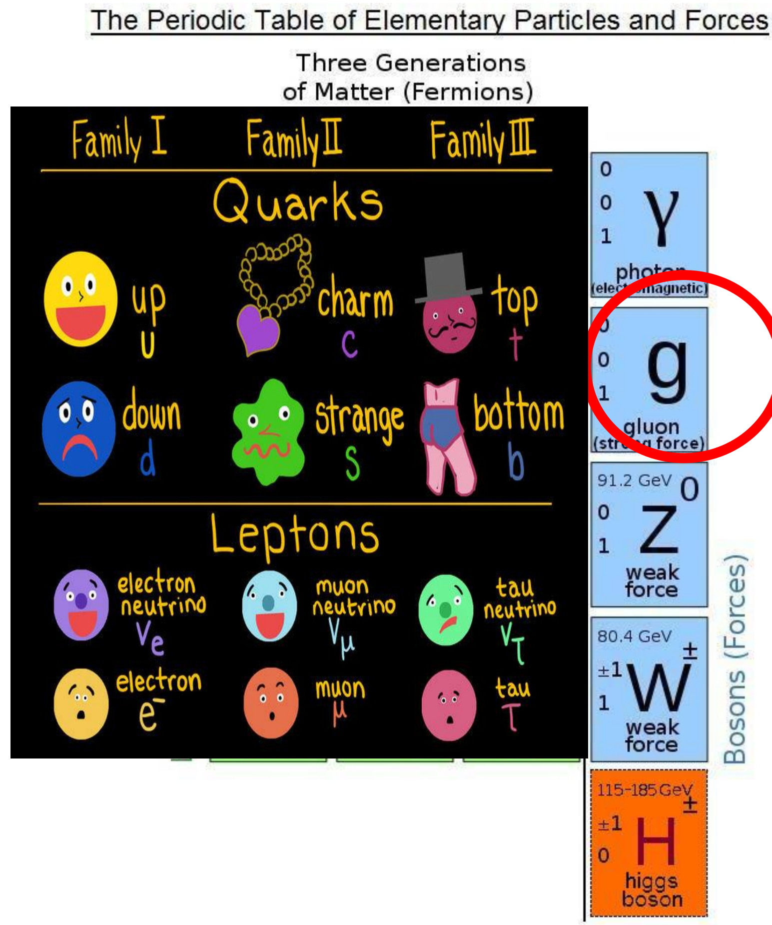
What is a gluon?



$m_g = 0, \;\; \text{spin} = 1,$

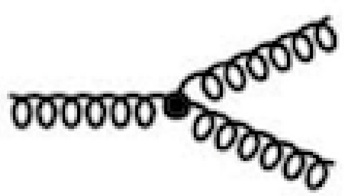
"color charge" a = 1, ..., 8

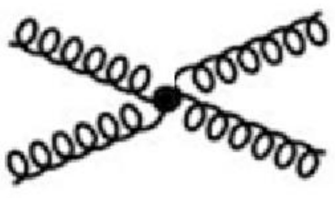
- Mediators of strong interactions (like γ in E&M)
- Conducting interactions between colored quarks in quantum chromodynamics (QCD)



“Charged” gluons and their non-linear self-interactions

- Unlike photon that has no electricity, the gluons carry color charges and therefore self-interact with strong coupling α_s





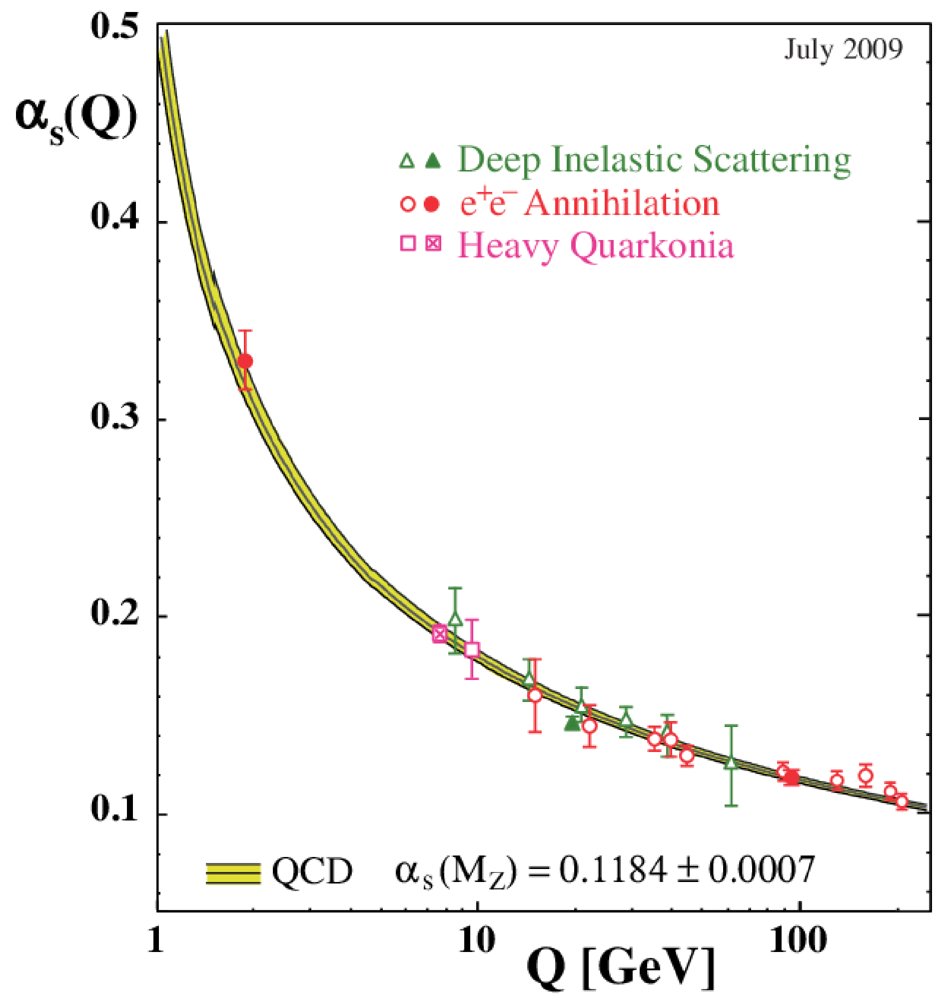
- Non-linear interactions lead to $\alpha_s(Q^2)$ running as the momentum scale Q^2 in an opposite way as in QED.

Running QCD coupling: $\alpha_s(Q^2)$

- The strong coupling becomes small at large momentum

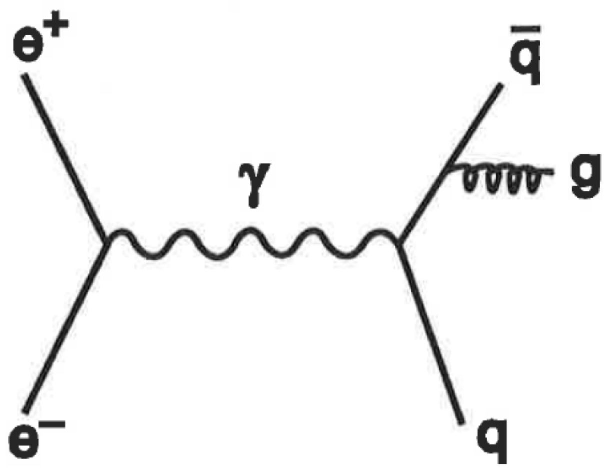
$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln Q^2/\Lambda^2_{QCD}}$$

Gluons can be seen as free particles at short distance or high energy!

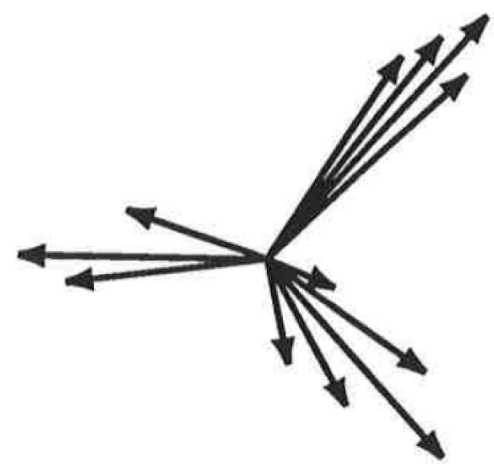


High-energy scattering

- Gluons can reveal themselves through high-energy scattering

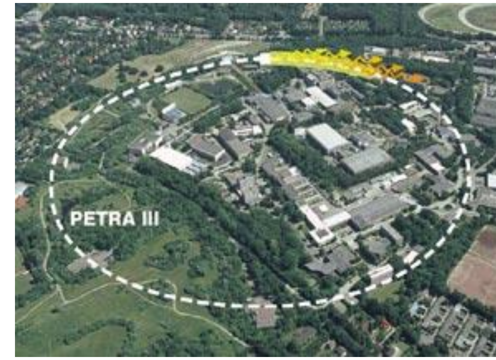


$e^+e^- \longrightarrow q\bar{q}g$

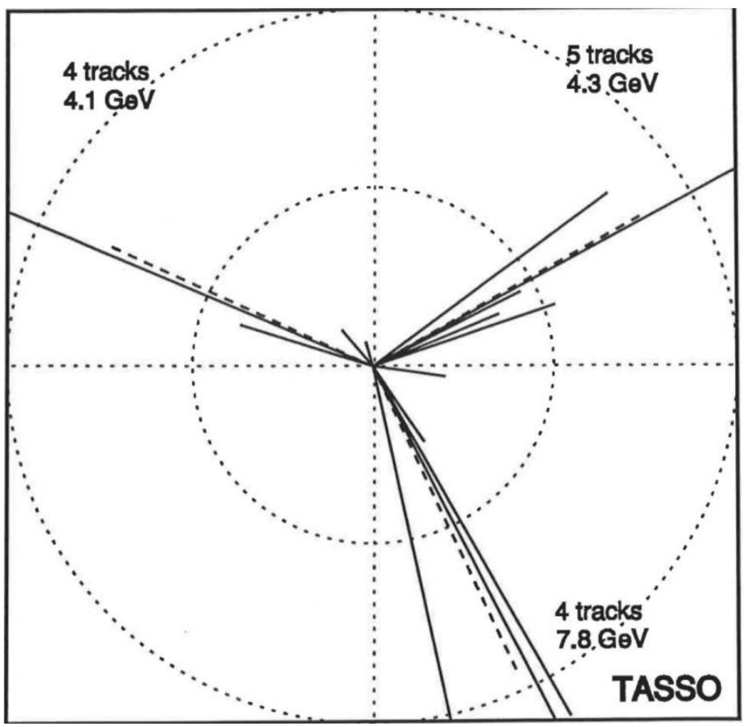


Three-Jet Events

Discovery of gluons



- In 1979, first **three-jet event** observed by TASSO collaboration at PETRA ($\sqrt{s} = 27.4$ GeV), Hamburg



1995

HIGH ENERGY AND PARTICLE PHYSICS PRIZE

of the

EUROPEAN PHYSICAL SOCIETY

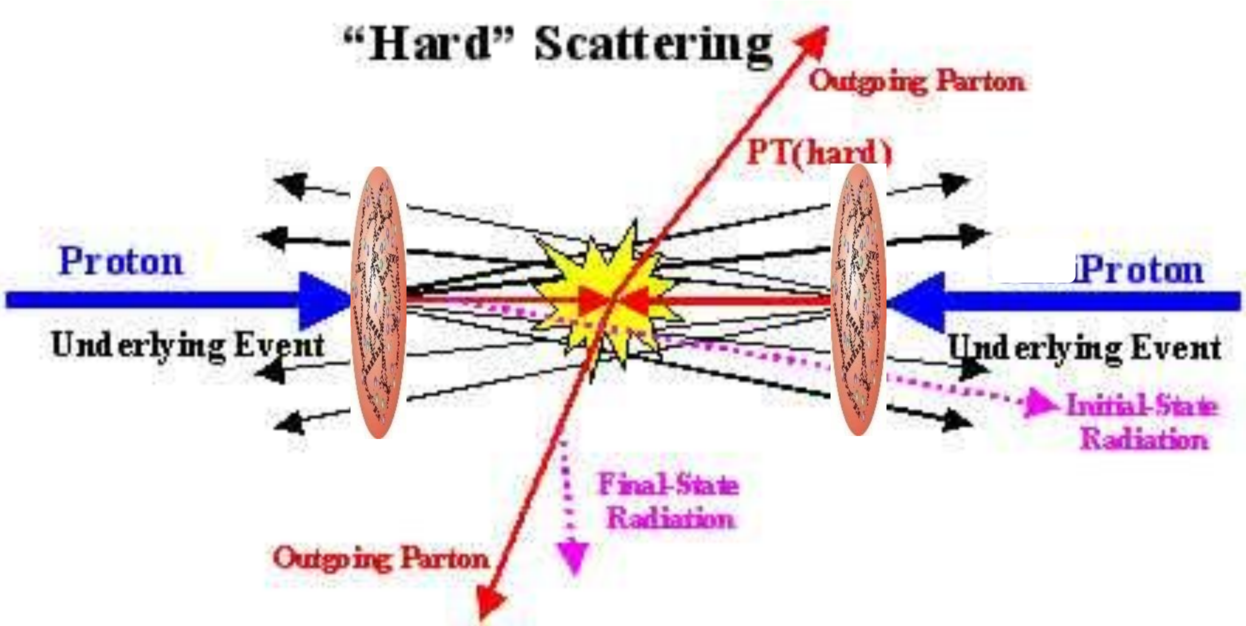
The 1995 High Energy and Particle Physics Prize of the European Physical Society is awarded to

Paul Söding
Björn Wiik
Günther Wolf
Sau Lan Wu

for the first evidence for three-jet events in e^+e^- collisions at PETRA.

Brussels, 27 July 1995

pp scattering at LHC

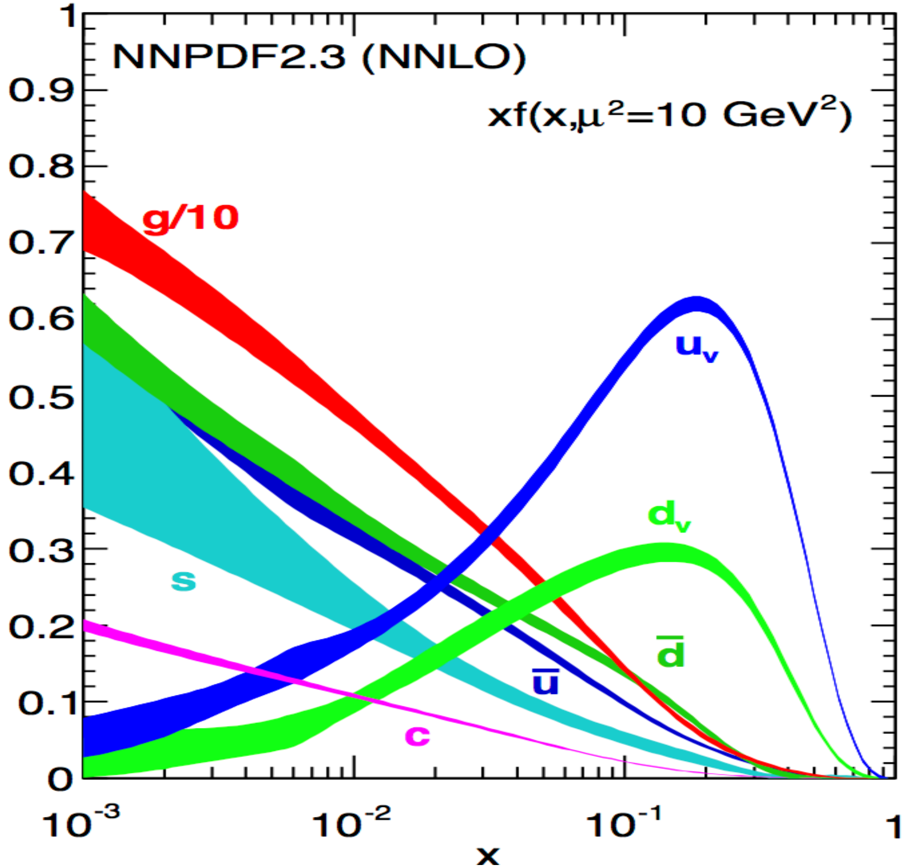


- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

Phenomenological PDFs

- Use experimental data (~50 yrs) to extract PDFs



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Structure of the proton

- To calculate PDFs from QCD, we need to understand the structure of the proton, a quantum mechanical bound state.
- Our experiences with bound states in QM are limited to mainly non-relativistic systems:

Atomic systems: electrons + Coulomb int

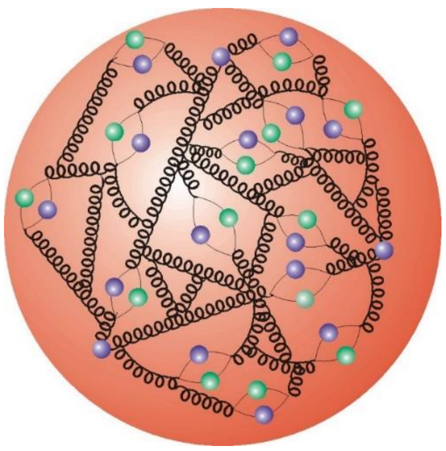
Nuclear systems: protons + neutrons

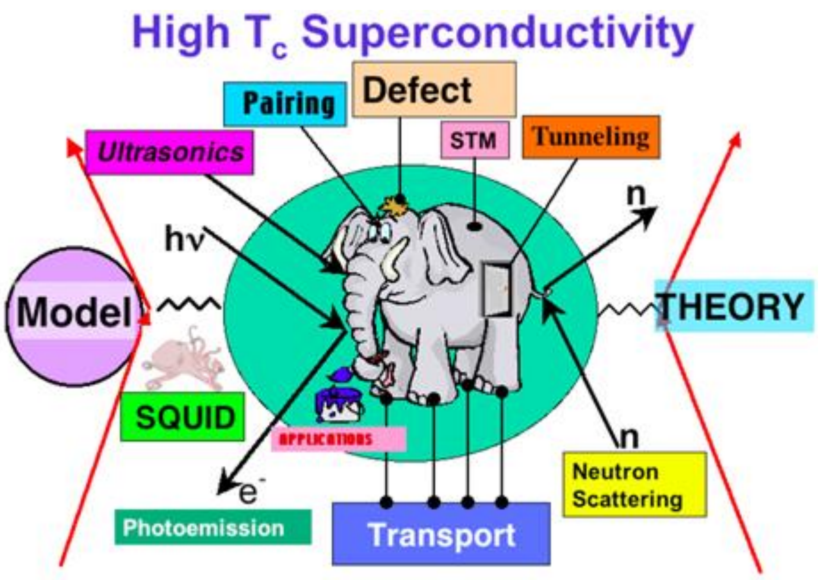
A large body of experience has been accumulated but not useful for the structure of proton.

QCD bound states

- However, it is an unprecedented challenge to understand how QCD works at low energy, where theory becomes non-perturbative:
  - Guts of Strong Interactions!
- Similar in nature to Condensed Matter Physics: the Lagrangian is known, but the solution is hard

High Tc, Hall effects, strongly coupled electron systems, etc

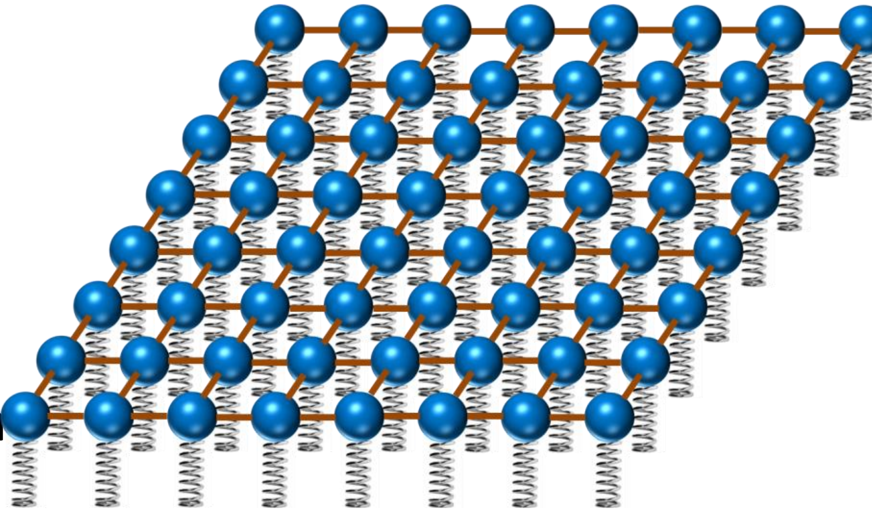




QCD (& SM) is a quantum field theory (QFT)

- QFT = QM + Relativity
- The fundamental degrees of freedoms are quantized fields (quarks + gluons)
- The fields fill in all space and time. Every point of space and time has a few quantum mechanical degrees of freedom (DOF), which are interacting with their neighboring DOFs (local QFT).
- Extremely complicated (infinite DOF) systems.....

Infinity is a problem



Two “easy” cases

- Non-relativistic limit:

  Masses of the particles are much larger than the kinetic and potential energies.

  Atomic and condensed matter physics are a non-relativistic limit of QED.

  Nuclei physics is the non-relativistic limit of QCD.

  Particle number is conserved. (For neutral systems, IR photon decouples)

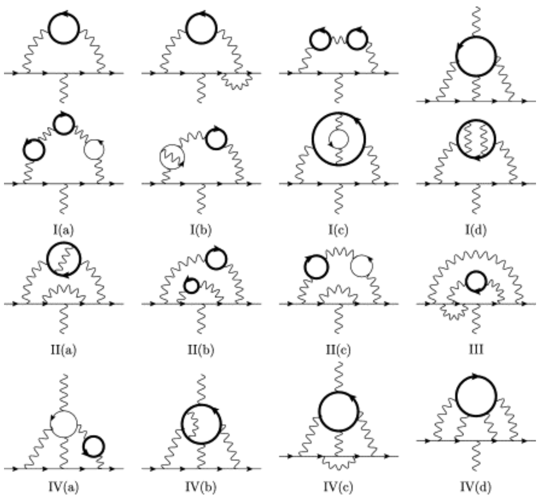
  QFT -> Hamiltonian dynamics

Two “easy” cases

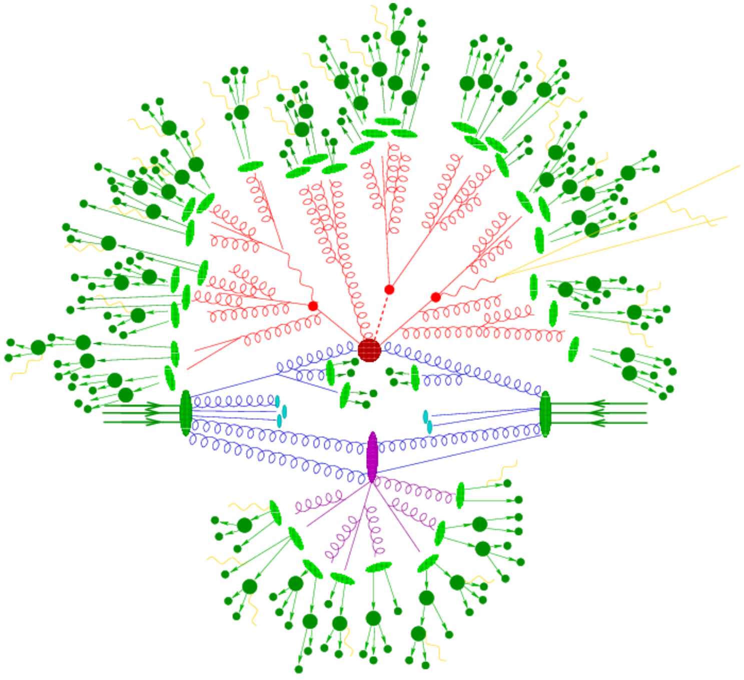
- Weak coupling limit

  In the week coupling limit, one can use perturbation theory developed by Feynman et al.

High-loop calculations for g-2



Parton Shower



Why strong QCD is so difficult?

- **Strongly coupled: Similar to NR electron systems**
  - Non-perturbative approximation methods must be devised beyond fermi liquid theory.
  - Ab initio numerical simulations
  - Effective degrees of freedom?
- **Extra difficulty: Relativity**
  1. Center-of-mass and internal motions coupled
  2. The QCD vacuum (ground state)
  3. Number of particles not fixed (going to infinity)

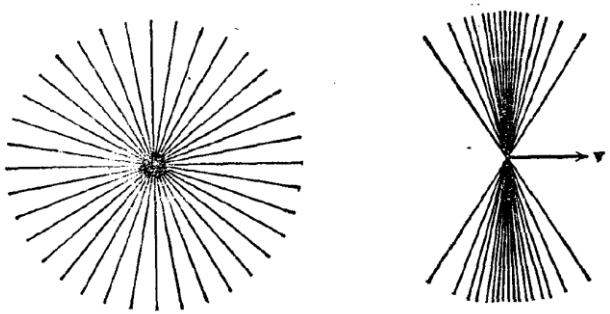
Relativity: internal states are frame-dependent

- The center-of-mass motion is part of the physics: the bound state has definite total momentum
- Because the boost operator is dynamical, the internal states are different at different momenta!

$$|p'\rangle = U(L)|p\rangle$$

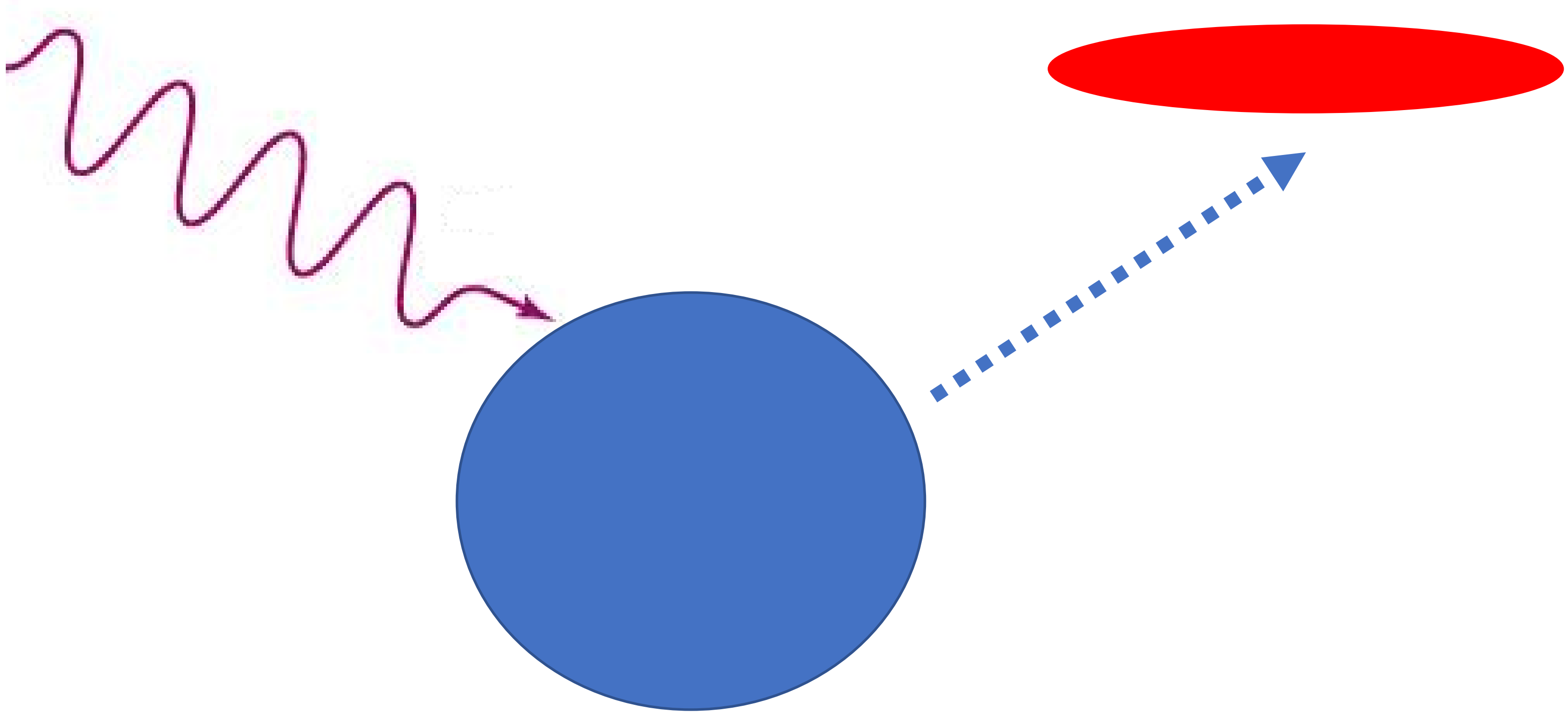
where $|p'\rangle$ is different from $|p\rangle$ dynamically!

  - The electromagnetic fields of a moving charge depends on its velocity or $\beta = v/c$



Elastic scattering: form factors

hadron states (wave functions) depending on the external momenta

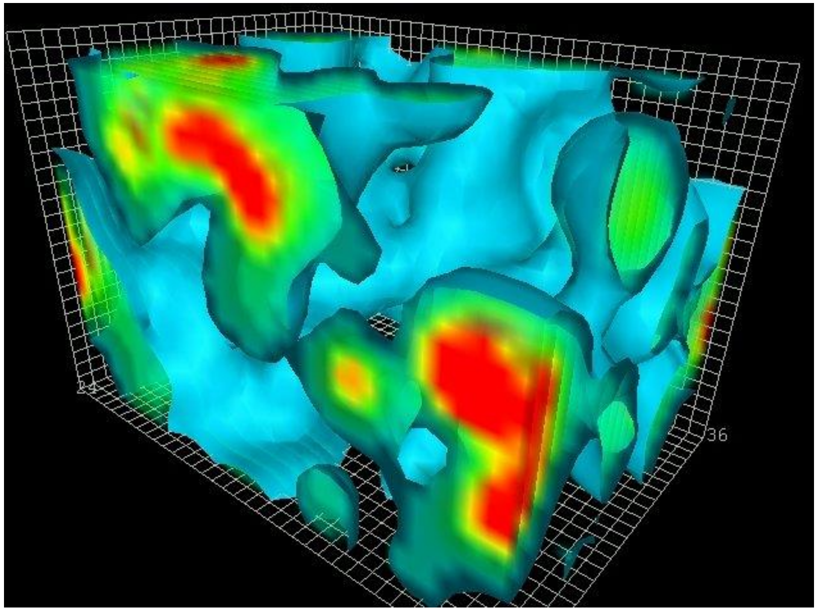


The QCD vacuum (ground state)

- Hadron systems are built upon the QCD vacuum which in itself is extraordinary complex
  - Similar to a strongly-interacting fermi sea in Condensed Matter Systems, where Landau's fermi liquid theory breaks down!
- And the hadron physics phenomena occur as complex excitations of this vacuum.
- It will be difficult to understand the excitations without understanding the ground state.

What does QCD vacuum look like?

- In semi-classical approximation, the vacuum is filled with interacting instantons/anti-instantons, a particular classical gluon configuration.



- The BPST instanton is an essentially non-perturbative classical solution of the Yang–Mills field equations
- $\overrightarrow{iE} = \vec{B}, or\ \overrightarrow{iE} = -\vec{B}$
- Instanton has zero energy density and zero angular momentum density

Vacuum properties: effective scalar field or gluon condensate

- Vacuum scalar field

$\phi = \langle 0 | F^2 | 0 \rangle$ ~density of instantons

The vacuum has a scalar field density which generates a dimension, Λ^4, which sets the strong interaction scale parameter.

- The real QCD physics is independent of this scale parameter, unless there is a new scale introduced, such as quark masses.
- This scalar field is similar to the Higgs field in electroweak theory.

Vacuum properties: chiral symmetry & spontaneous breaking

- When Nf massless quarks q_i are introduced, there is a chiral symmetry

$$U_L(N_f) \times U_R(N_f) = U_V(1) \times U_A(1) \times SU_L(N_f) \times SU_R(N_f)$$

acting on the spaces of $q_{iL}(x)$ and $q_{iR}(x)$

- In the QCD vacuum, the symmetry is broken by instantons

$$U_L(N_f) \times U_R(N_f) \rightarrow U_V(1) \times SU_V(N_f)$$

($U_A(1)$ symmetry is broken explicitly by instantons)

Smoking gun for chiral symmetry breaking

- Non-zero chiral condensate in the instanton vacuum

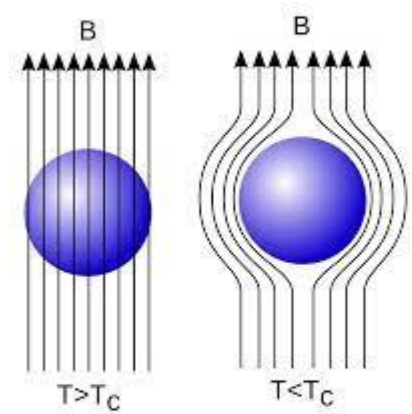
$$\langle 0|\bar{q}_L q_R|0\rangle \neq 0 \quad \text{(density of instantons)}$$

- Appearance of massless particles: Goldstone bosons as vacuum excitations.
- When quarks have small masses, we have pseudo-Goldstone bosons π, K which has mass-squared proportional to quark mass

$$m_\pi^2 \sim m_q$$

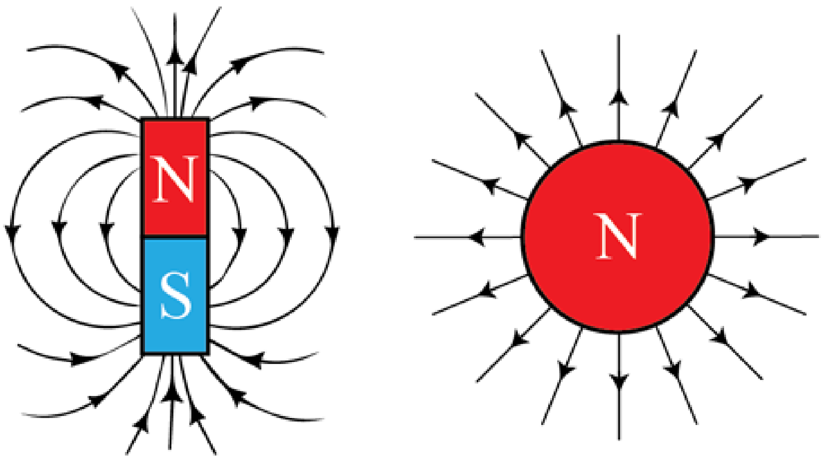
Color confinement

- An isolated colored charge has infinite energy in the QCD vacuum. Thus, colored states are not part of the QCD spectrum.
- The QCD vacuum expels the color electric field, just like the superconductor expels the magnetic field

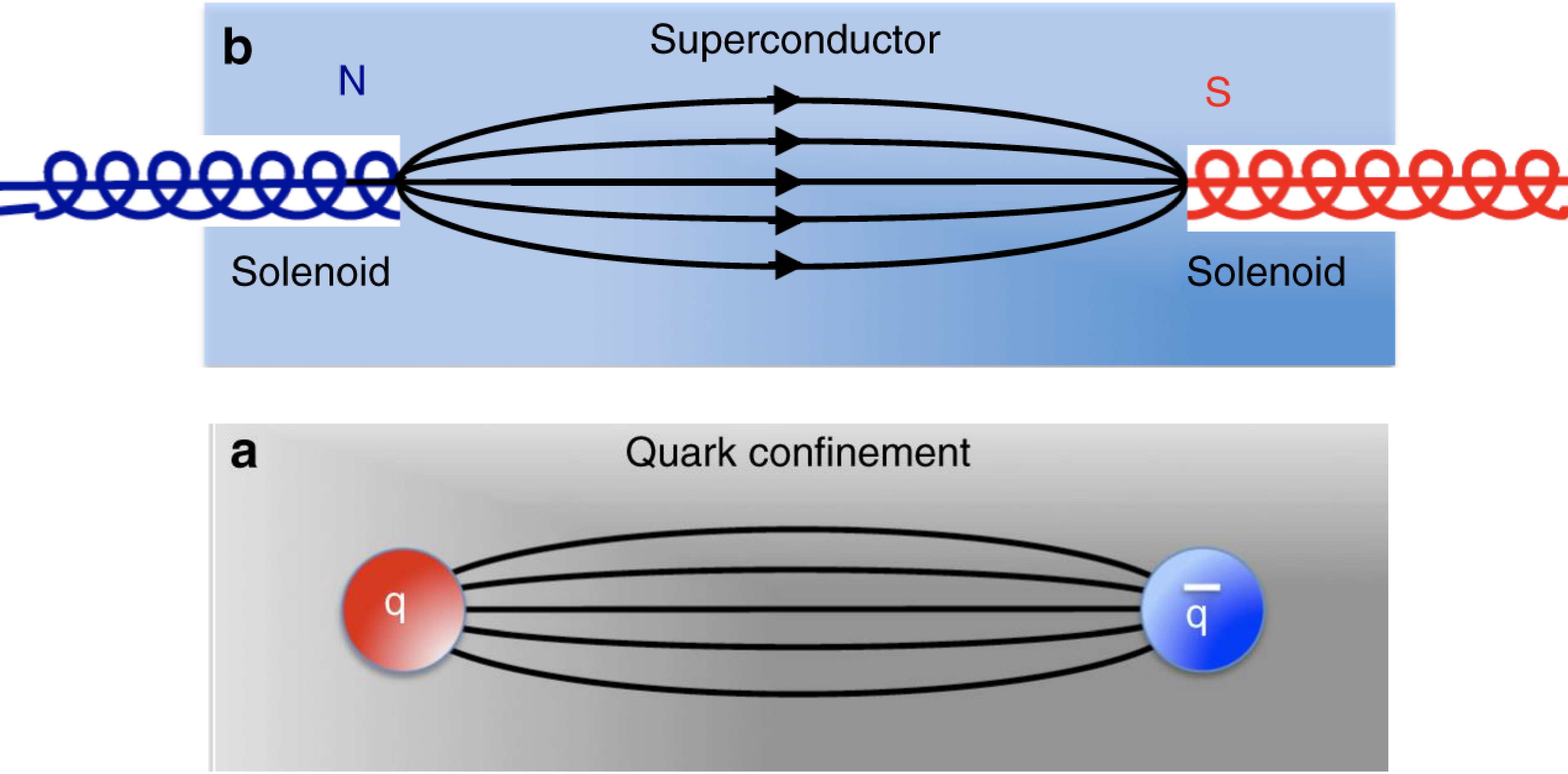


Color confinement

- Experiment: No free quarks and gluon have ever been detected directly.
- Theory: Hypothesis of Color Confinement:
  - No colored particles can exist in isolation
  - All strong interaction eigenstates are color neutral (singlet)



What happens when you pull a color neutral quark-antiquark pair apart?


b
Superconductor
N
S
Solenoid
Solenoid
a
Quark confinement
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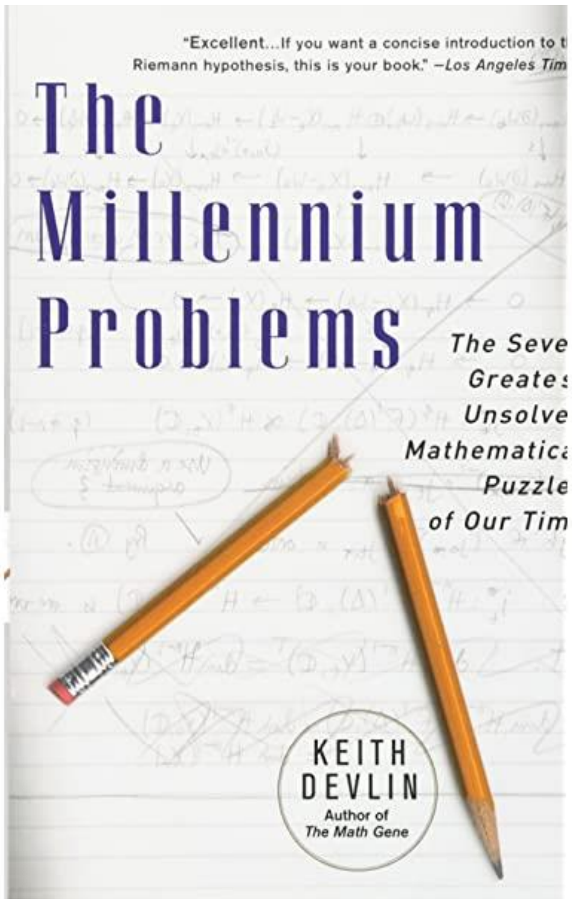
Millennium math problems

by Clay Math Institute



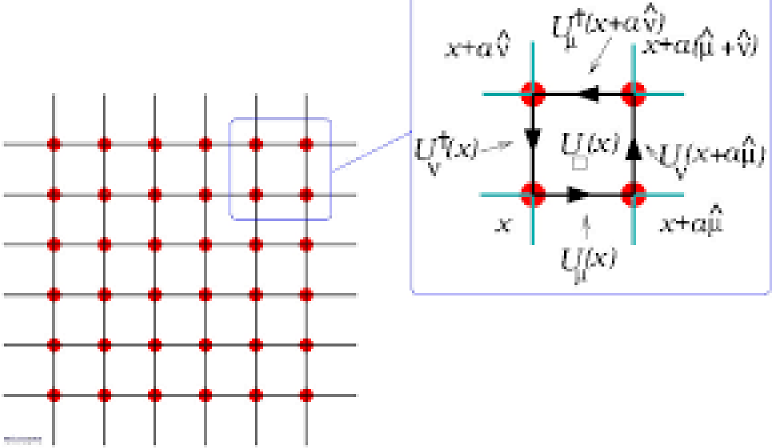
Hardest ways to make one million dollars:

1. Yang-Mills and Mass Gap
2. Riemann Hypothesis
3. P vs. NP Problem
4. Navier-Stokes Equation
5. Hodge Conjecture
6. ~~Poincare Conjecture~~
7. Birch and Swinnerton-Dyer Conjecture

Hadrons

- Is it possible to understand the structure of hadrons without understanding the QCD vacuum?
- Is it possible that there are effective degrees of freedom using which one can describe the hadrons?
  - Basis for model building (quark models, bag models)
  - QFT-> simple few body problem.
- In hadron physics, a universal effective description of hadrons has not been found
  - Existing ones are partially effective in limited domains.
  - We are forced to start from scratch

Ab initio calculations



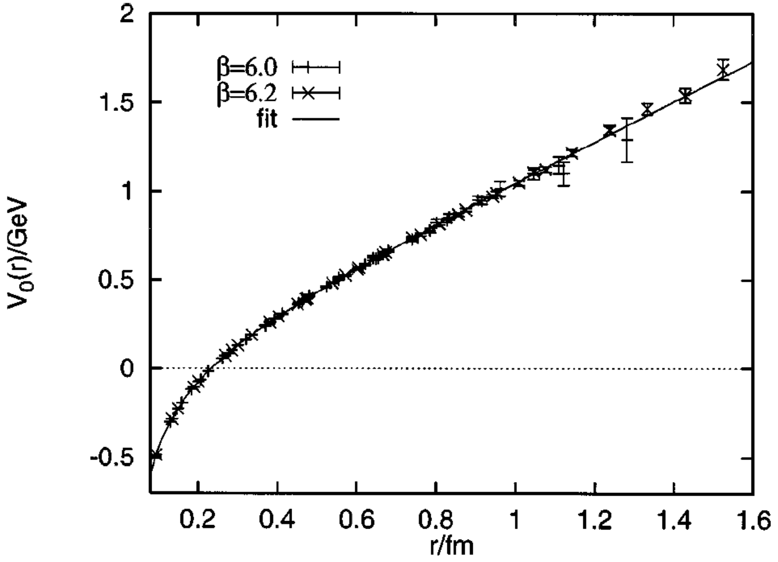
Lattice QCD

- The only first principles approach to solve QCD is lattice method, first proposed by Ken Wilson in 1974

- Formulated in Euclidean space-time, allowing calculating Eulidean correlation functions.
- The vacuum properties can be calculated.
- The certain properties of hadrons can be calculated through the Euclidean correlation functions.
- Methods of quantum fields, not particles!

Static potential between heavy quarks when there are no light quarks

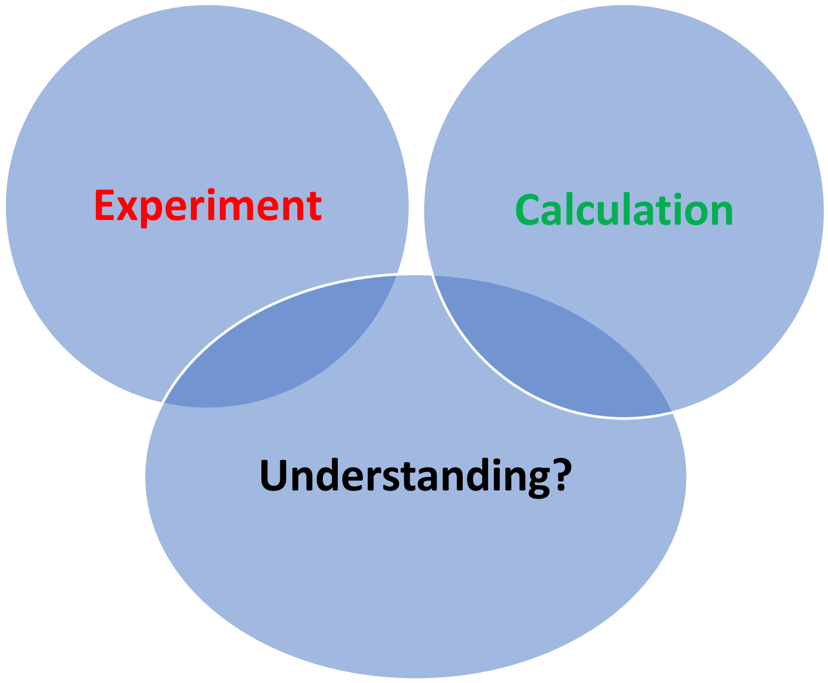
- G. Bali et al, PRD '97



Understanding proton structure in non-perturbative QCD

- Analytically solving QCD is a long shot (AdS/CFT is hardly a controlled approximation)
- Calculations using lattice QCD have made important progress, but is difficult to produce penetrating insight.
- What are the deep insights about the structure of the proton in nonperturbative QCD?
- This the most important reason for experimental facilities like Jlab and future EIC in the USA!

how does QCD (non-pert. QFT) work?



- We don’t have a simple language to explain empirical rules and strong interaction phenomenology.
  - Quark models
  - Instantons
  - …

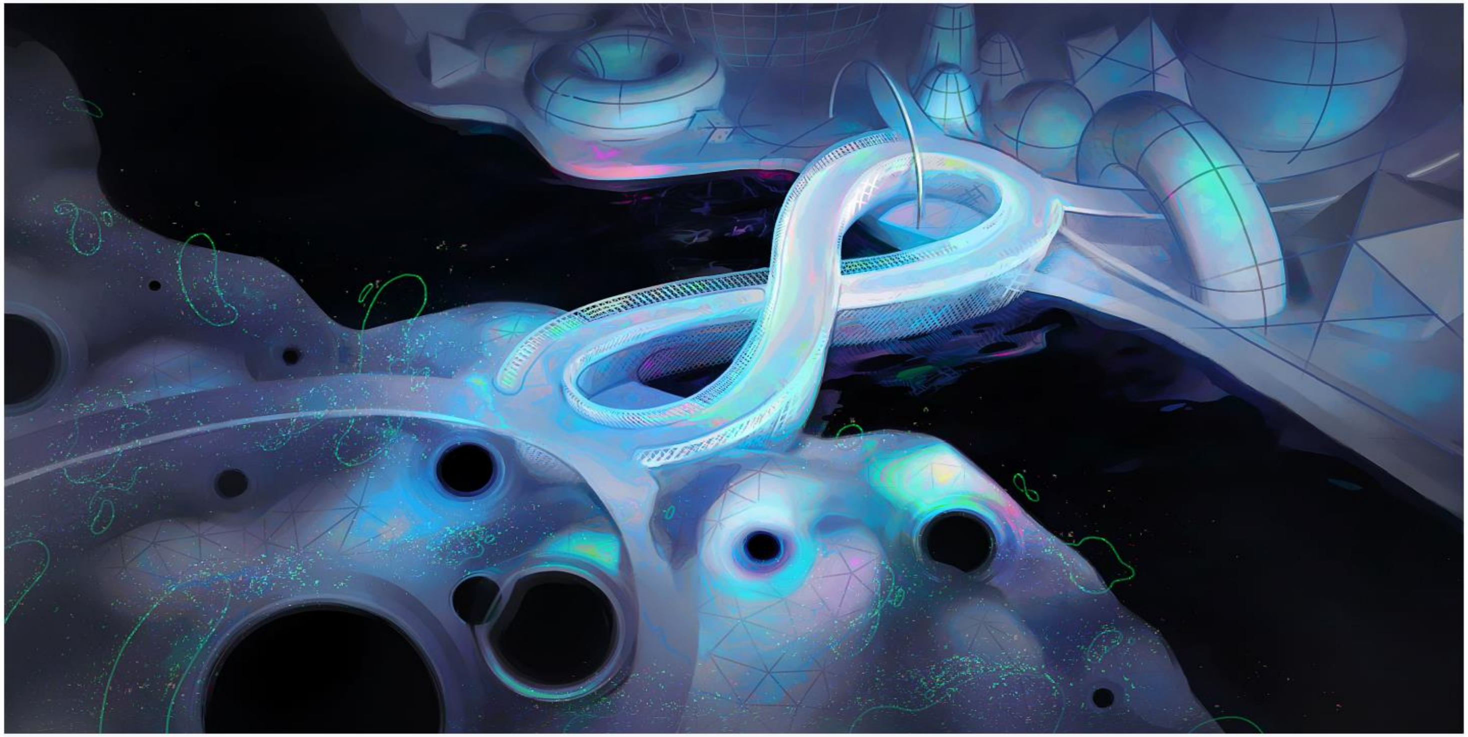
SERIES MATH MEETS QFT

The Mystery at the Heart of Physics That Only Math Can Solve

By KEVIN HARTNETT

June 10, 2021

The accelerating effort to understand the mathematics of quantum field theory will have profound consequences for both math and physics.



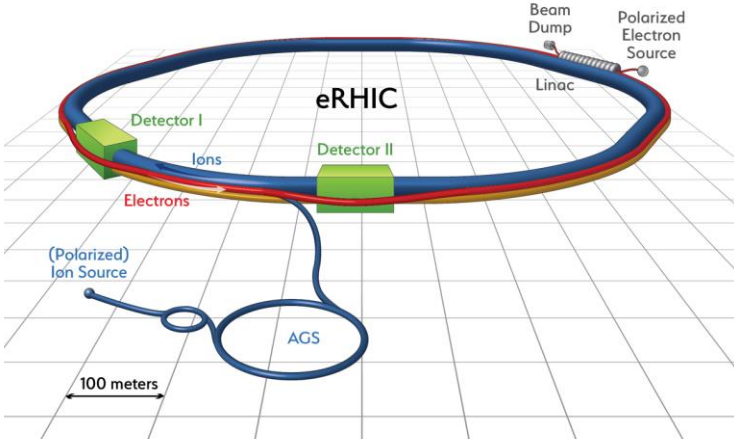
Olena Shmahalo/Quanta Magazine

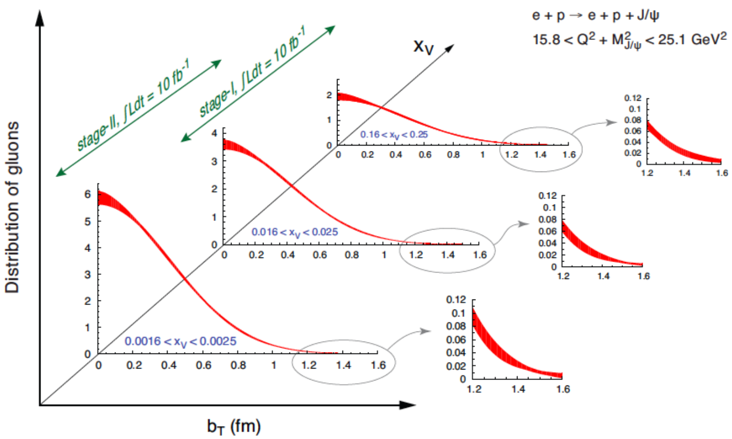
Over the past century, quantum field theory has proved to be the single most sweeping and successful physical theory ever invented. It is an umbrella term that encompasses many specific quantum field theories — the way "shape" covers specific examples like the square and the circle. The most prominent of these theories is known as the Standard Model, and it is this framework of physics that has been so successful.

Jlab 12 GeV facility

BNL Electron-Ion Collider:

Understanding the glue that binds us all





What is so special about EIC?

- High resolution (Q^2) ~ 10x: achieved through collider mode.

  In the past, most of the proton structure studies were made in fixed-target mode (SLAC, HERMES,CERN).

  For the gluon structure, this is essential.

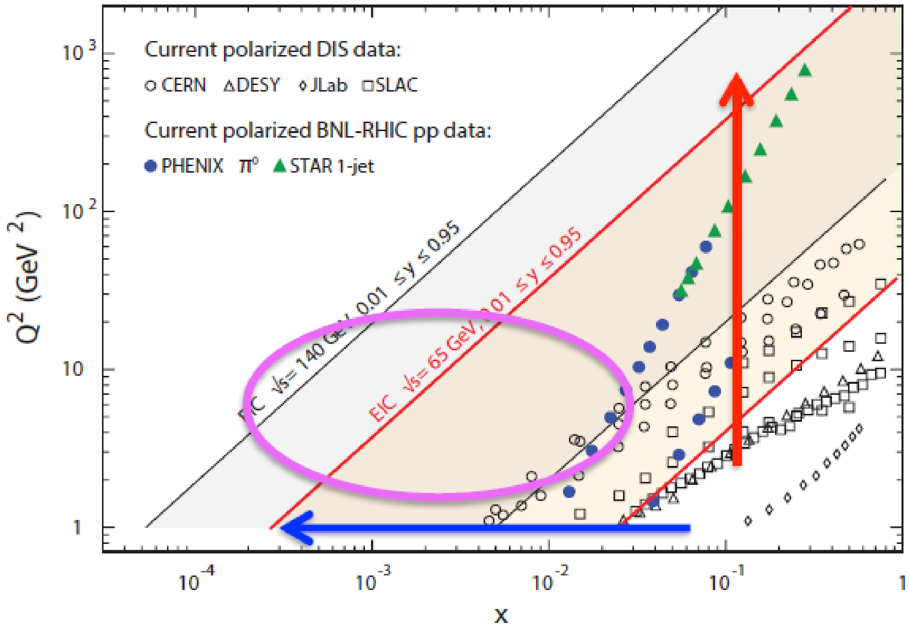
- High-luminosity:

  Necessary for semi-inclusive measurements

  for exclusive production

- Measurements of final states

US EIC – Kinematic reach & properties

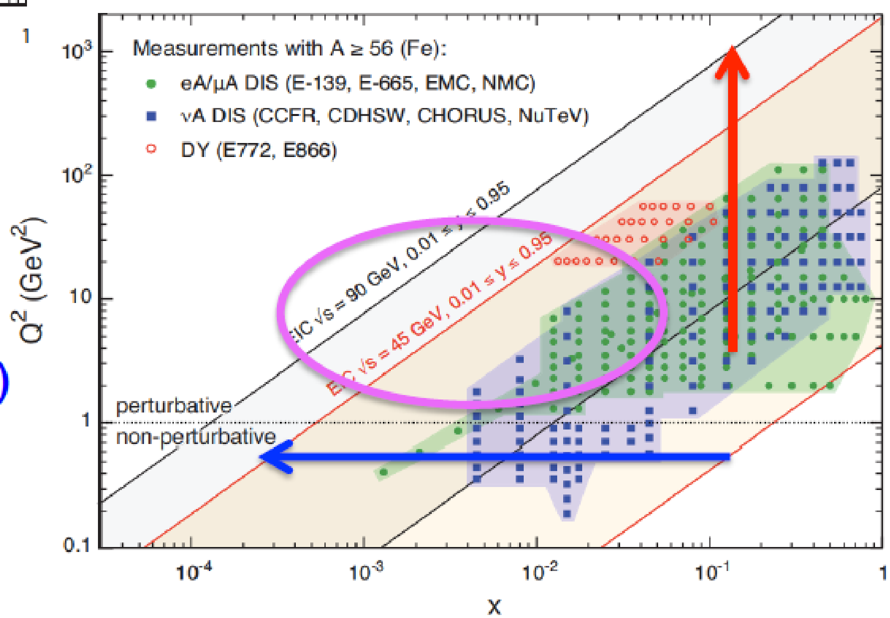


For e-N collisions at the EIC:

- ✓ Polarized beams: e, p, d/^{3}He
- ✓ Variable center of mass energy
- ✓ Wide Q^2 range → evolution
- ✓ Wide x range → spanning from valence to low-x physics
- ✓ 100-1K times of HERA Luminosity

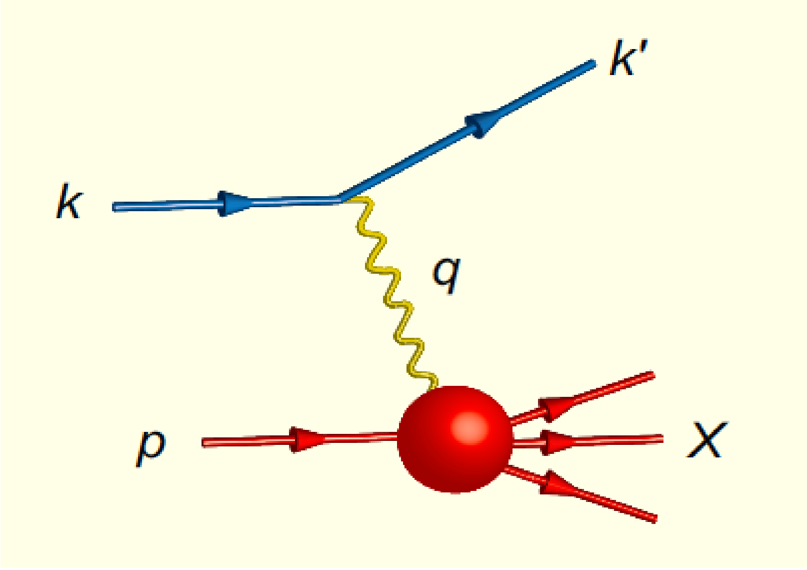
For e-A collisions at the EIC:

- ✓ Wide range in nuclei
- ✓ Variable center of mass energy
- ✓ Wide Q^2 range (evolution)
- ✓ Wide x region (high gluon densities)



Deep-inelastic scattering

- The quark is struck by the virtual photon, forming high-energy jets, separated from the remnant of the nucleon



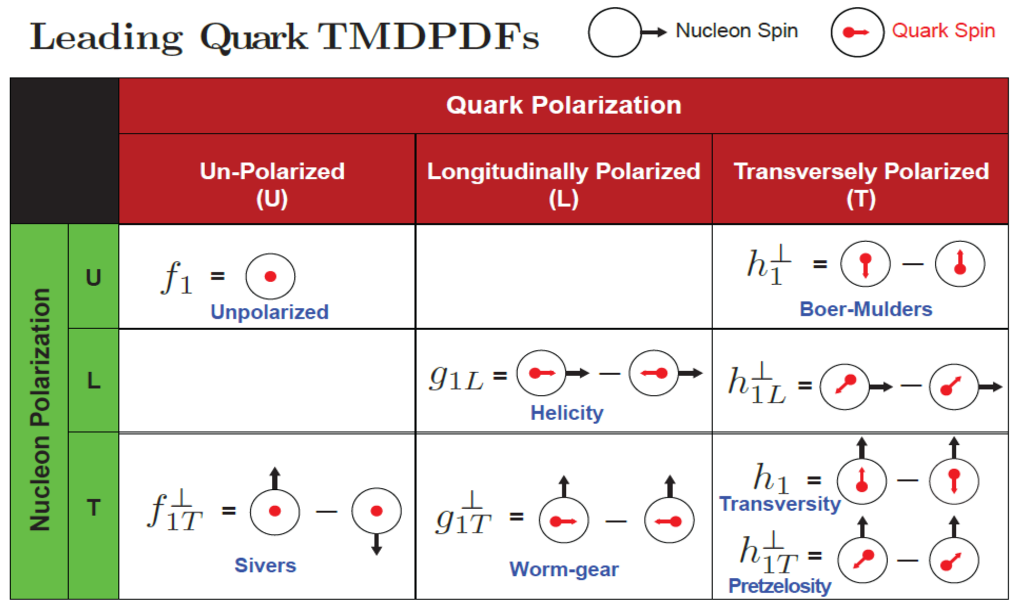
- Inclusive DIS
  - Parton distributions
- Semi-inclusive DIS, measure additional hadrons in final state
  - P_t-dependent parton distributions

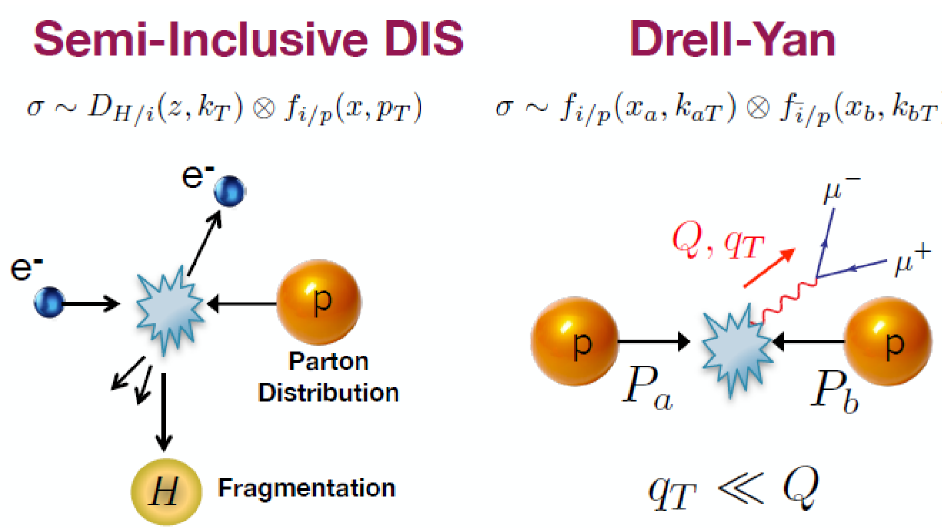
TMD distributio

- Partons have transverse momentum $\vec{k}_\perp$, from which one can define various TMD PDFs

  (TMD Handbook, 2022)

- TMD distributions can be measured in various high-energy scattering processes

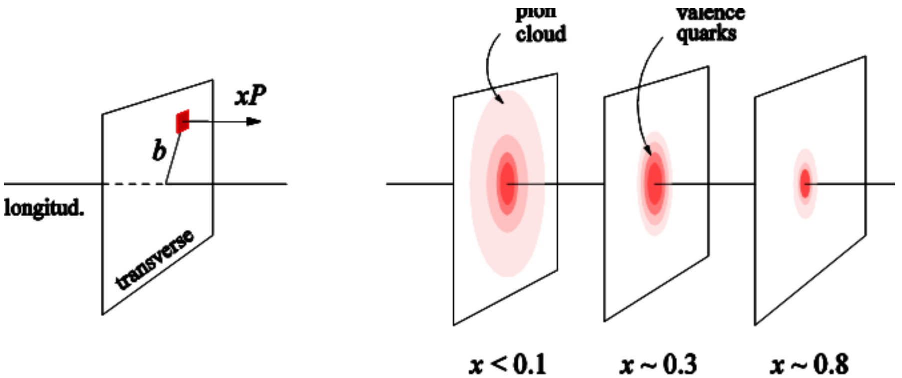


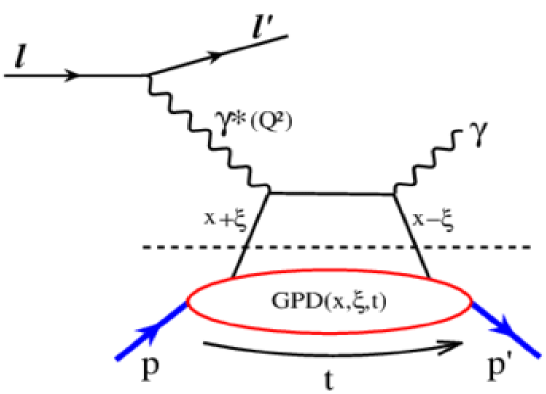


Transverse-plane parton distributions

- Parton distributions in transverse plane: $f(x, b_\perp)$
- A type of generalied parton distributions

  $H(x,\xi,t), E(x,\xi,t)$
- Such distributions can be measured in high-energy exclusive processes such as DVCS

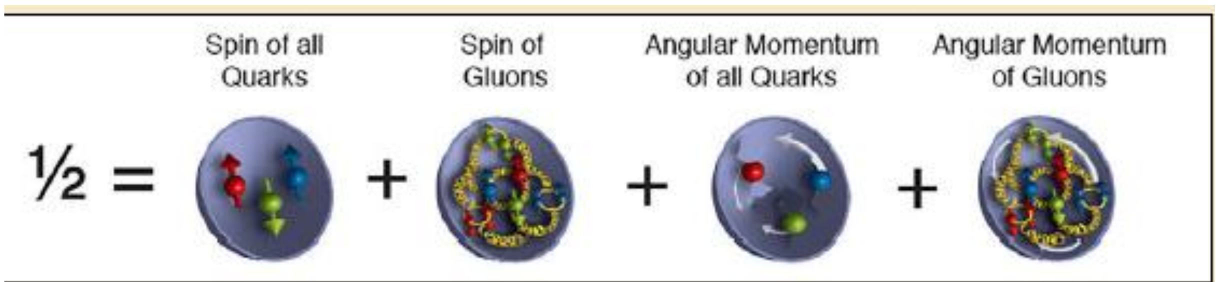




Deeply-virtual Compton scattering

Spin structure of the proton

- Where the spin of the nucleon come from?



- Orbital angular momentum can be constructed from the transverse distributions of partons

$$\vec{\mathrm{J}}_{\perp} \sim \vec{b} \times x\vec{P}$$

$$J \quad = \frac{1}{2} \int x dx [H(x, \xi, t) + E(x, \xi, t)] \Big|_{t=0}$$