Quarkonium transport in weakly and strongly coupled plasmas

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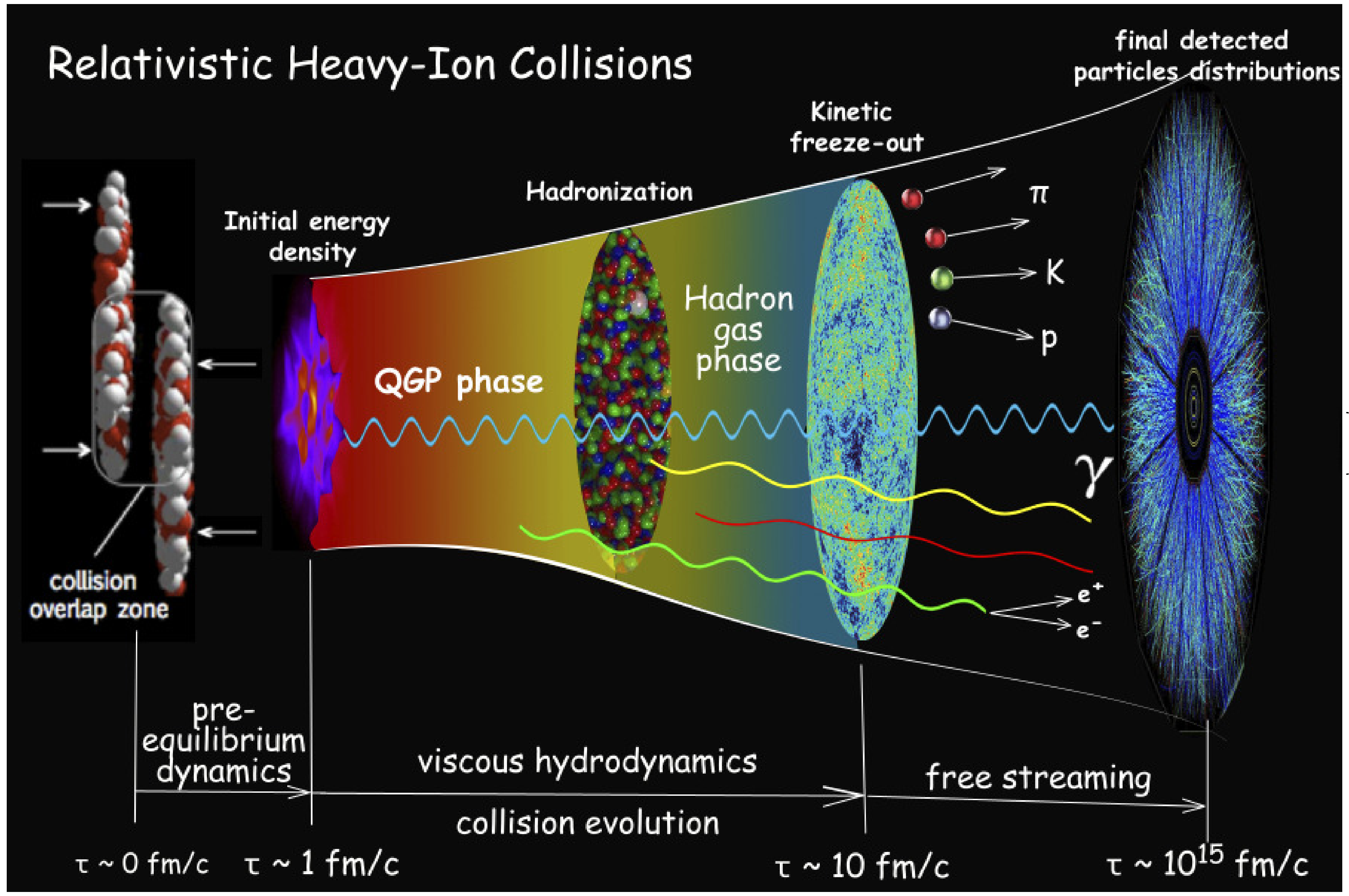
with Xiaojun Yao (UW) and Govert Nijs (MIT)
based on 2107.03945, 2205.04477, 2302.XXXXX

Quarkonium in Heavy-Ion Collisions

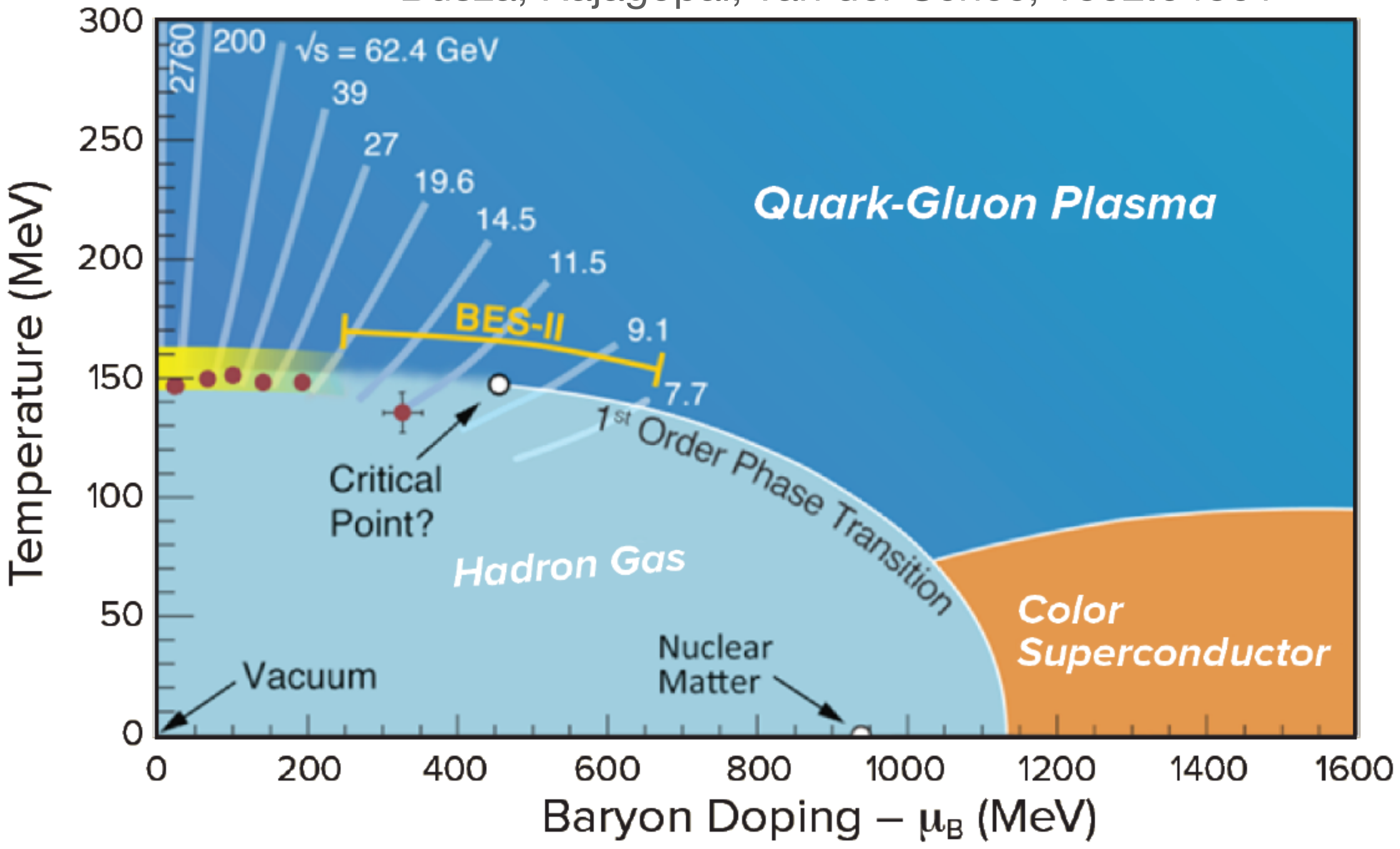
- Heavy quarks and quarkonia are amongst the most informative probes of the QGP (talks by Aichelin, Kabana, Kopeliovich, …).

- To interpret the full wealth of data, we need a precise theoretical understanding of heavy quarks in a thermal medium,
    - as single open heavy flavors, and
    - as pairs of heavy flavors that can bind into quarkonia.

credit: Paul Sorensen and Chun Shen, 1304.3634



Busza, Rajagopal, van der Schee, 1802.04801



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→ **This talk**

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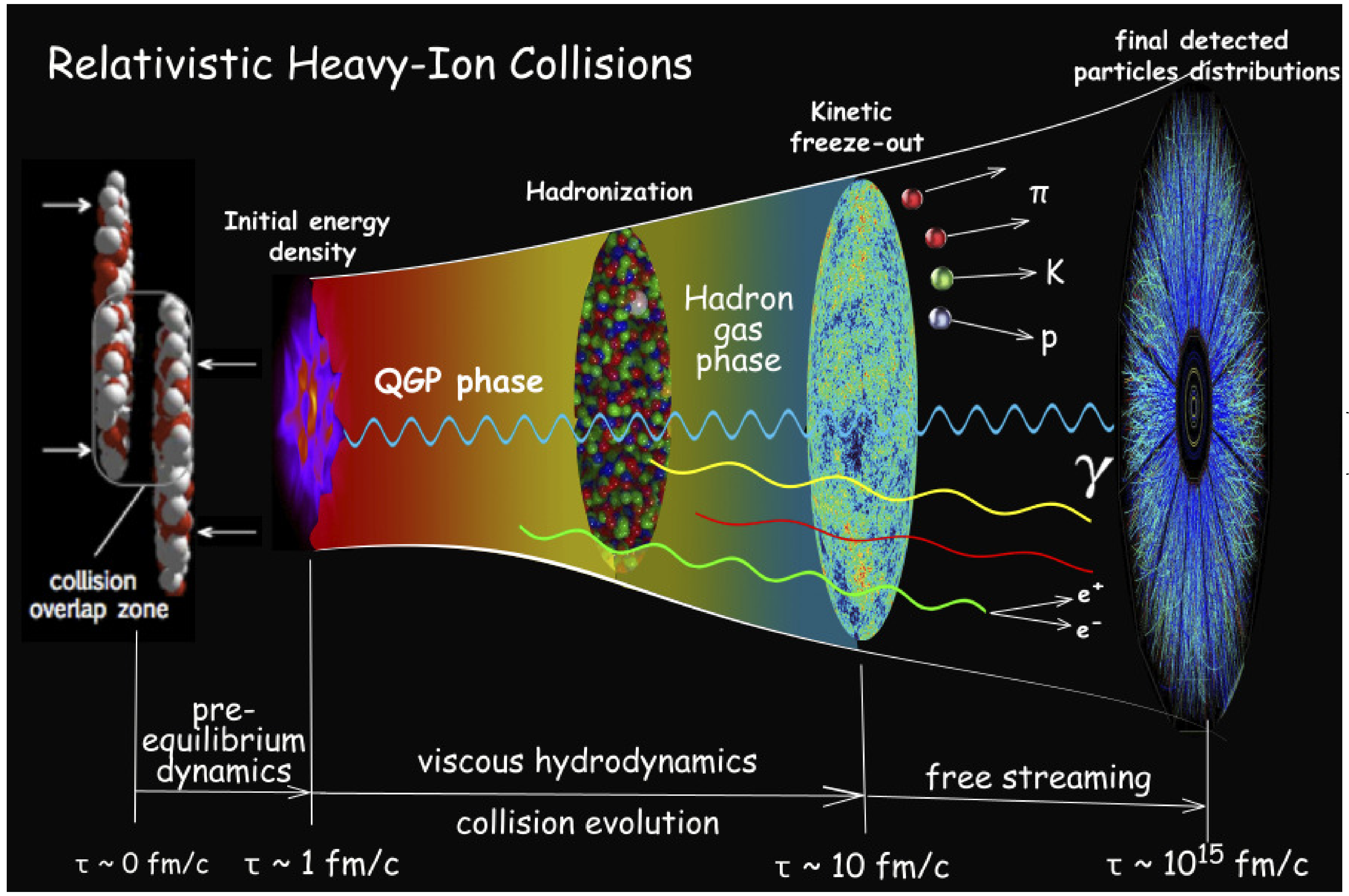

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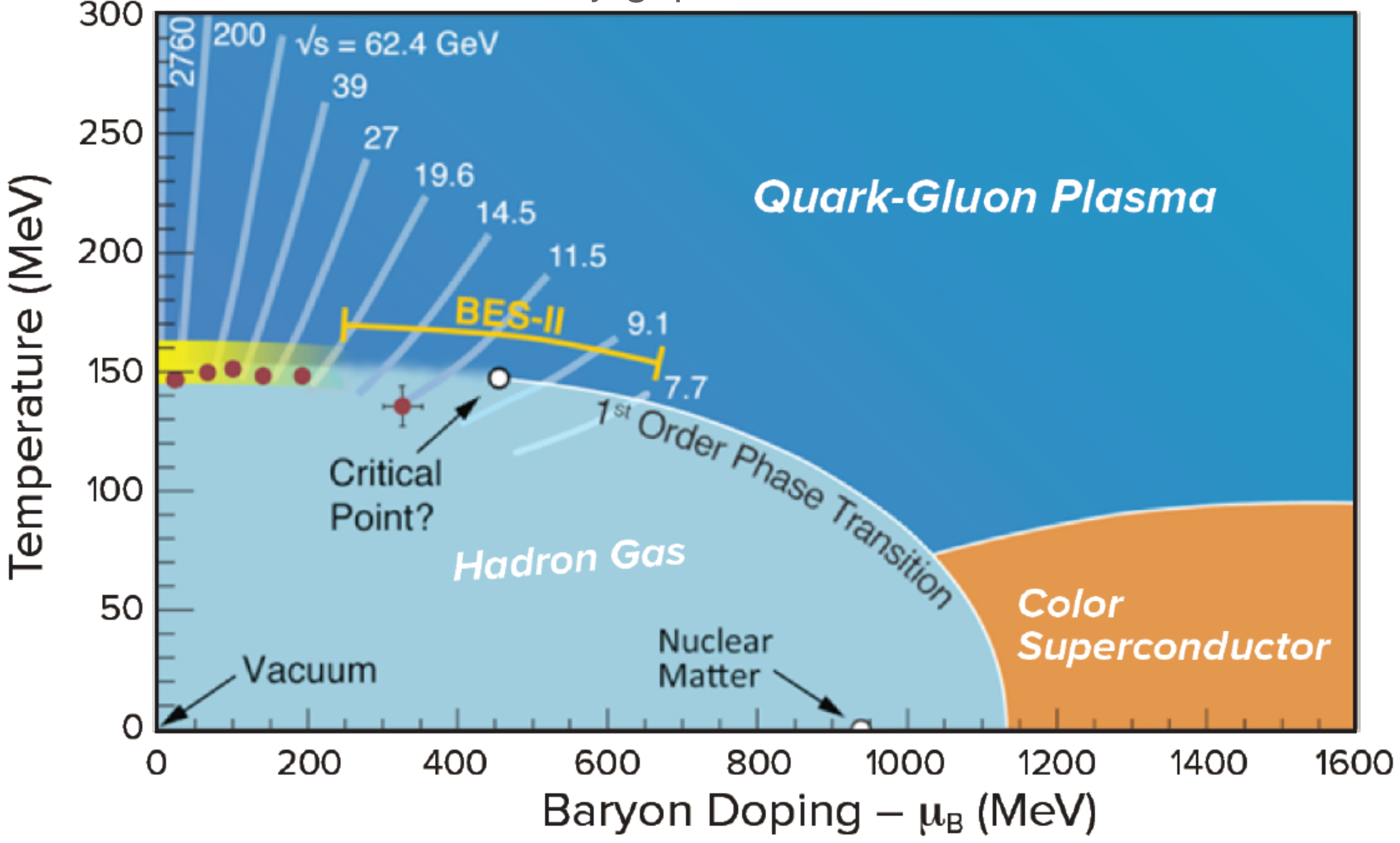

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This talk: $Q\bar{Q}$ comoving with the QGP

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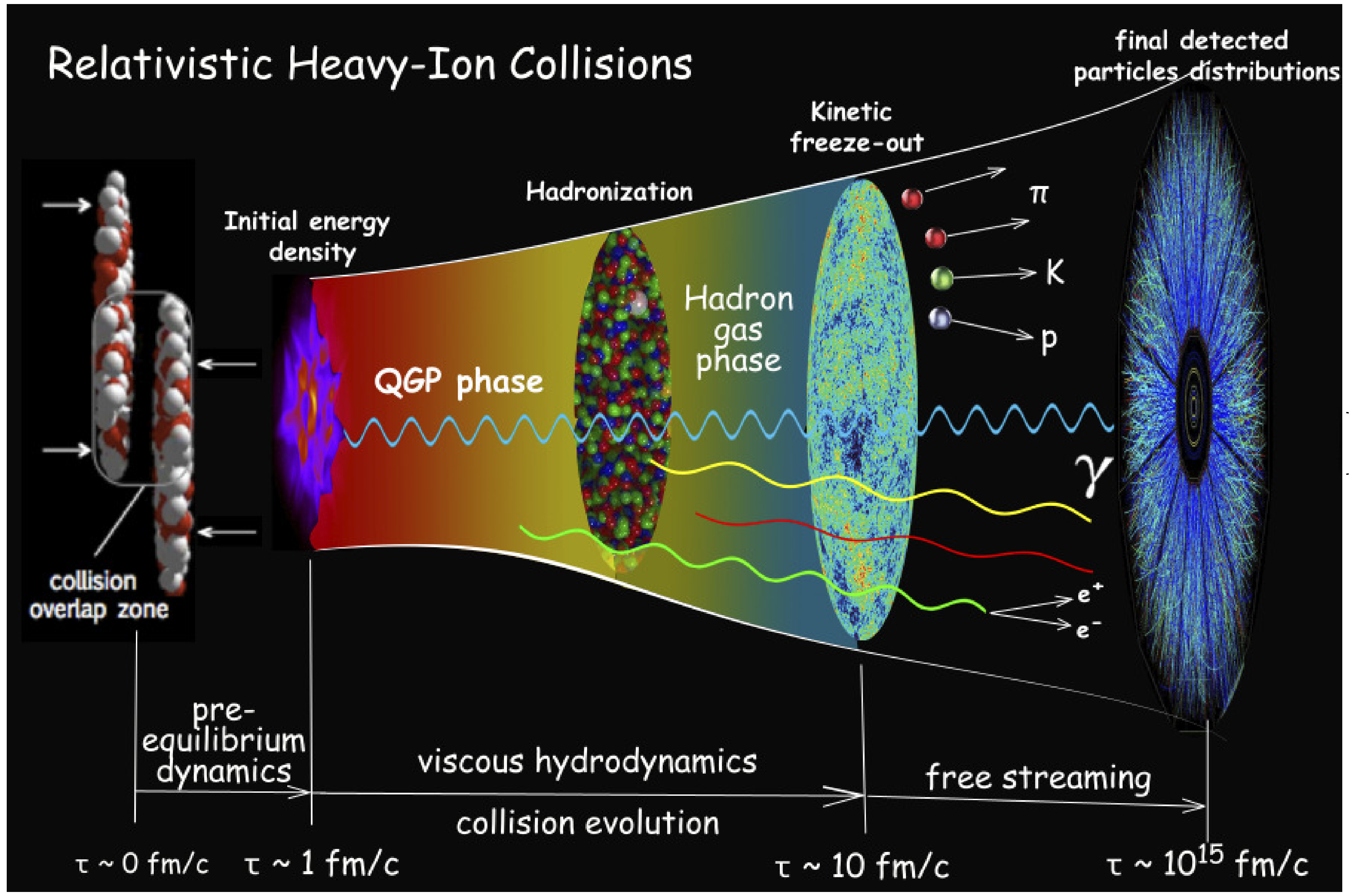

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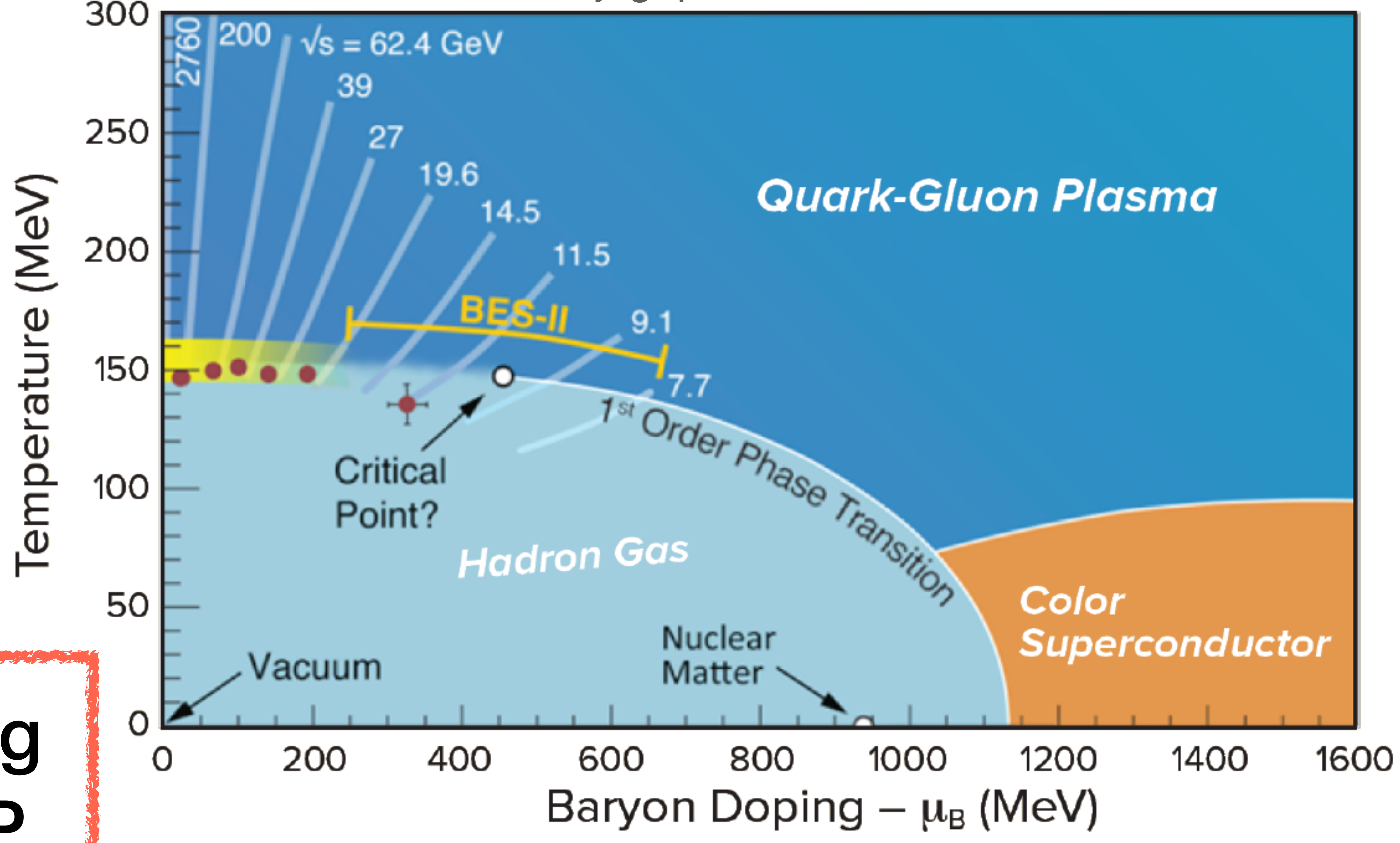
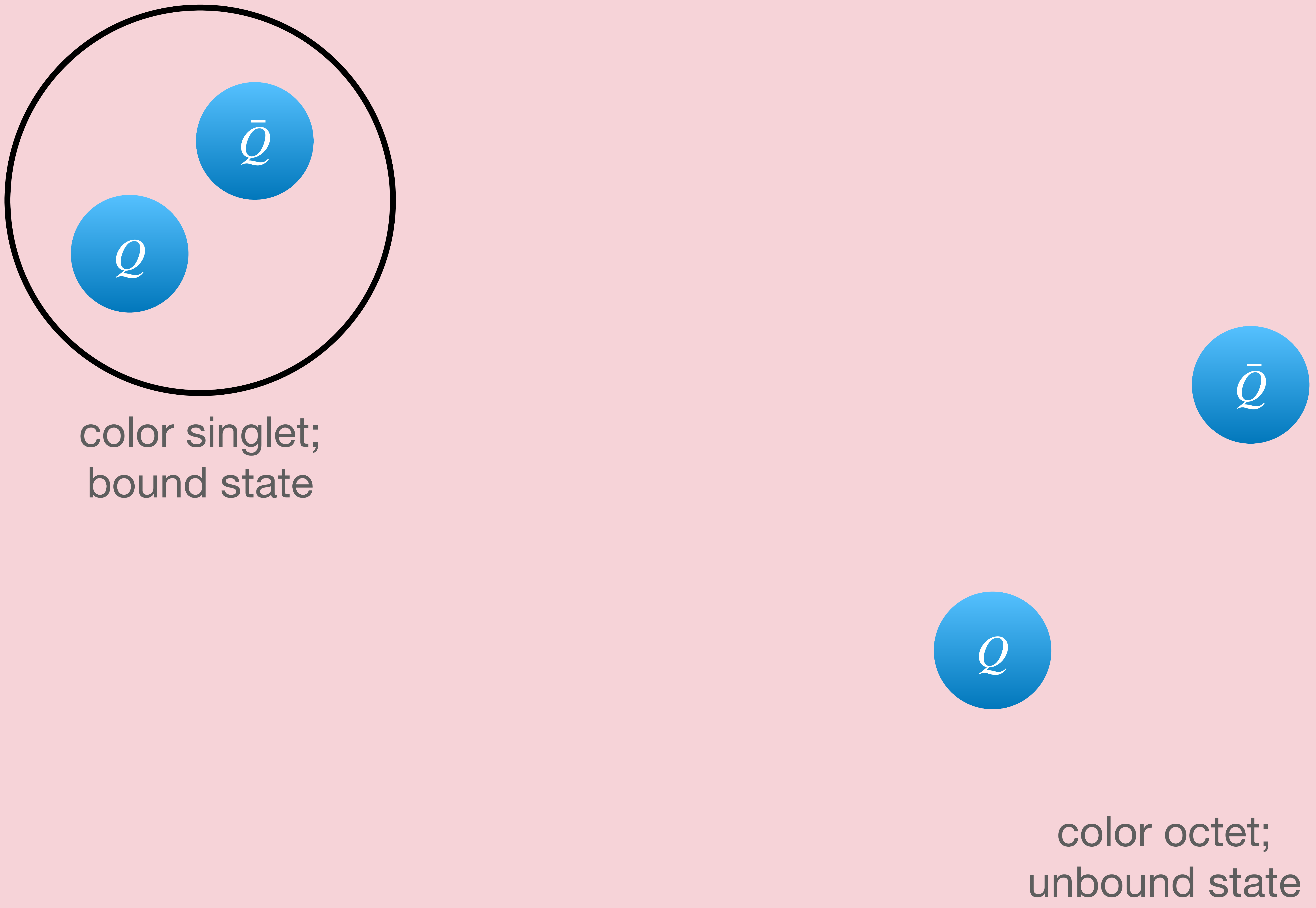


Quarkonium in medium

$M \gg Mv \gg Mv^2$

M: heavy quark mass
v: typical relative speed



Q: c or b quark
$\bar{Q}$: $\bar{c}$ or $\bar{b}$ quark

$T > 0$

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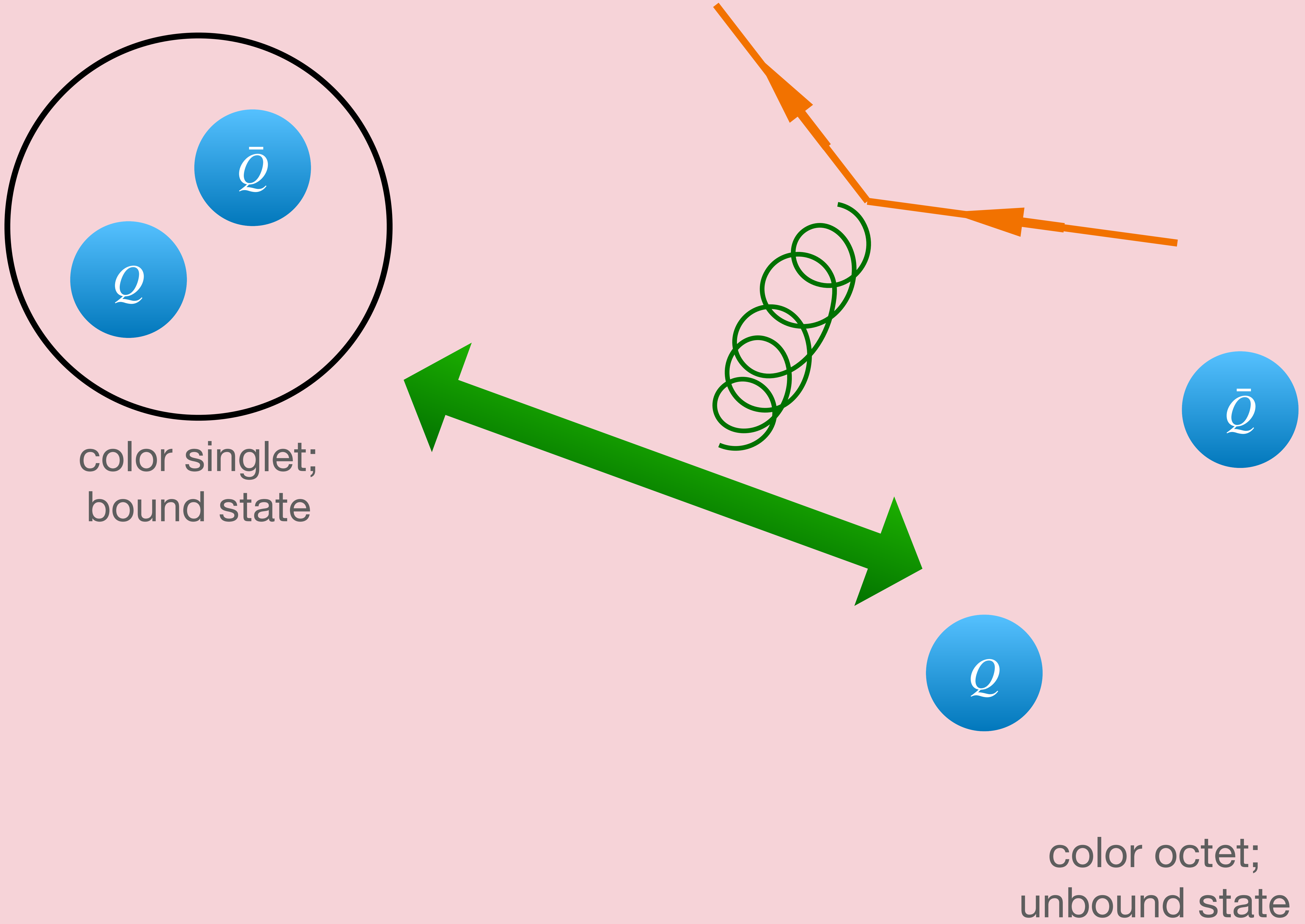

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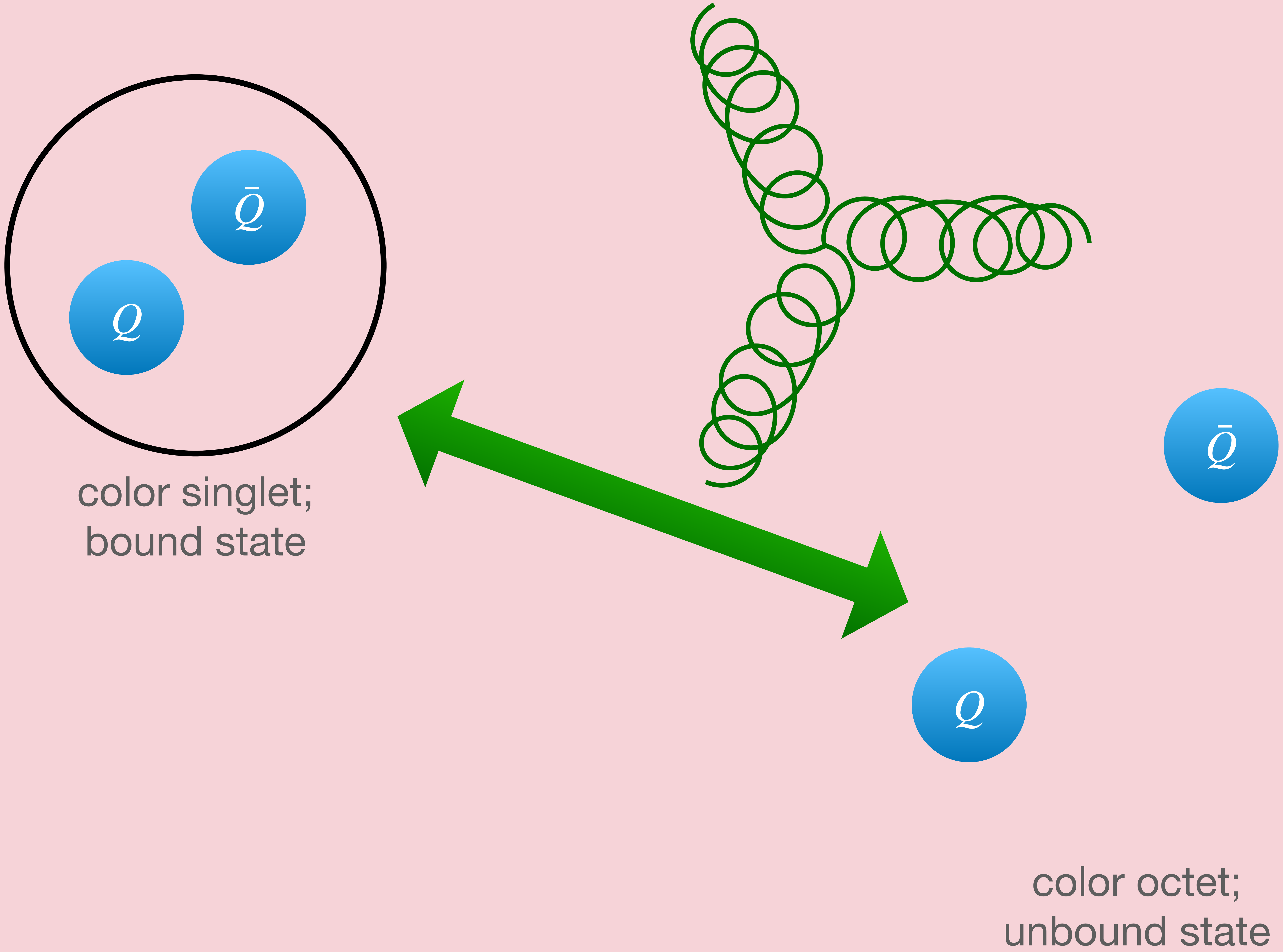

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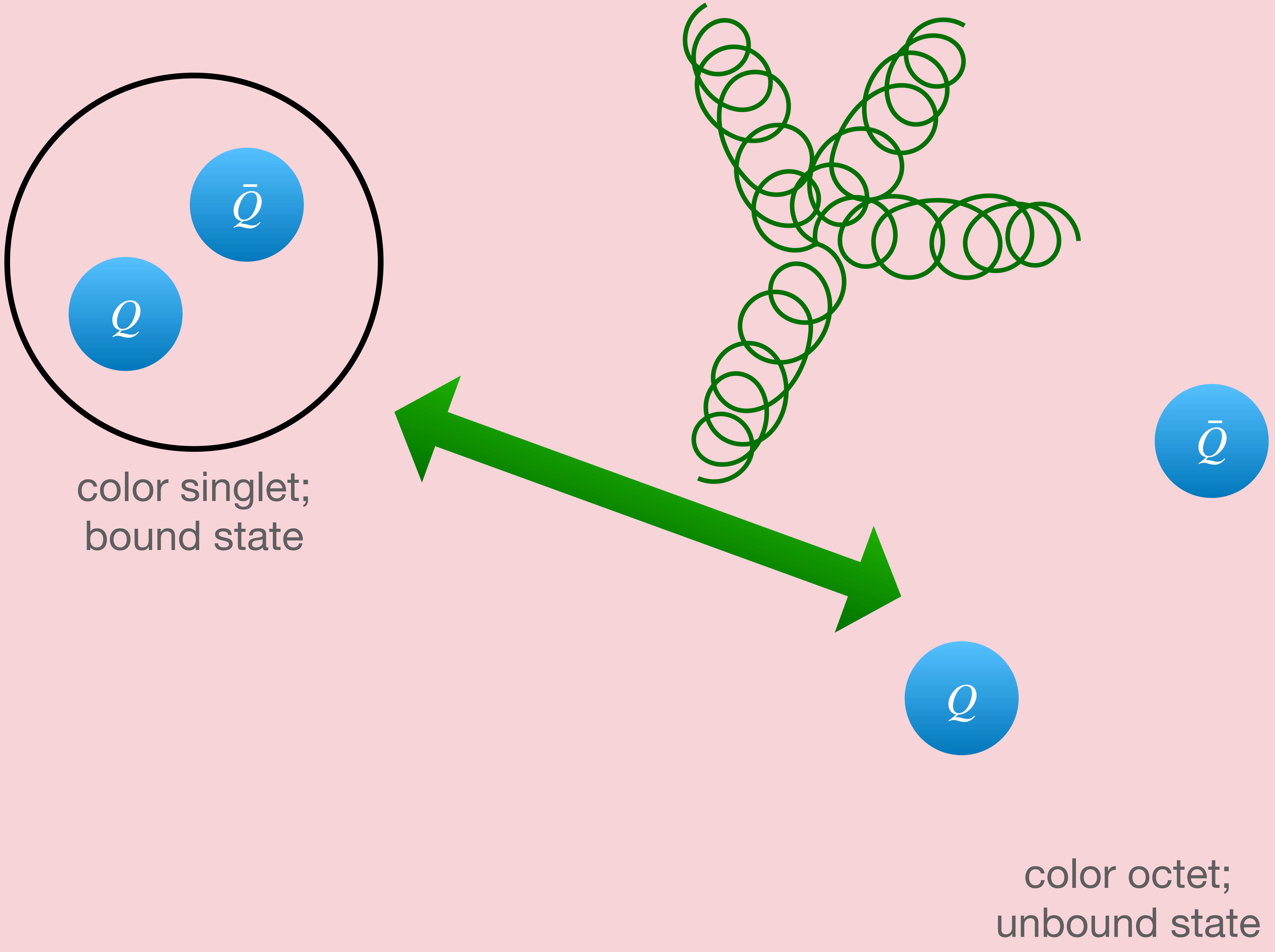

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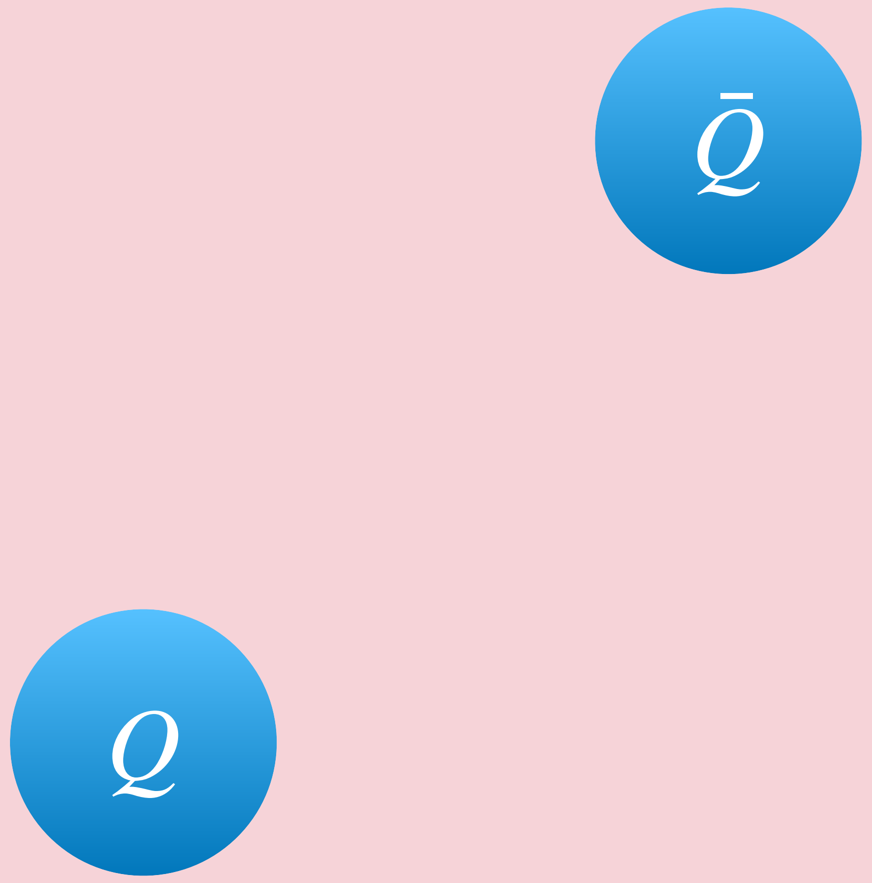
Quarkonium in medium

At high T, quarkonium "melts" because the medium screens the interactions between heavy quarks (Matsui & Satz 1986)

$$Q\bar{Q} \text{ melts if } r \sim \frac{1}{Mv} \gg \frac{1}{T}$$



color octet;
unbound state

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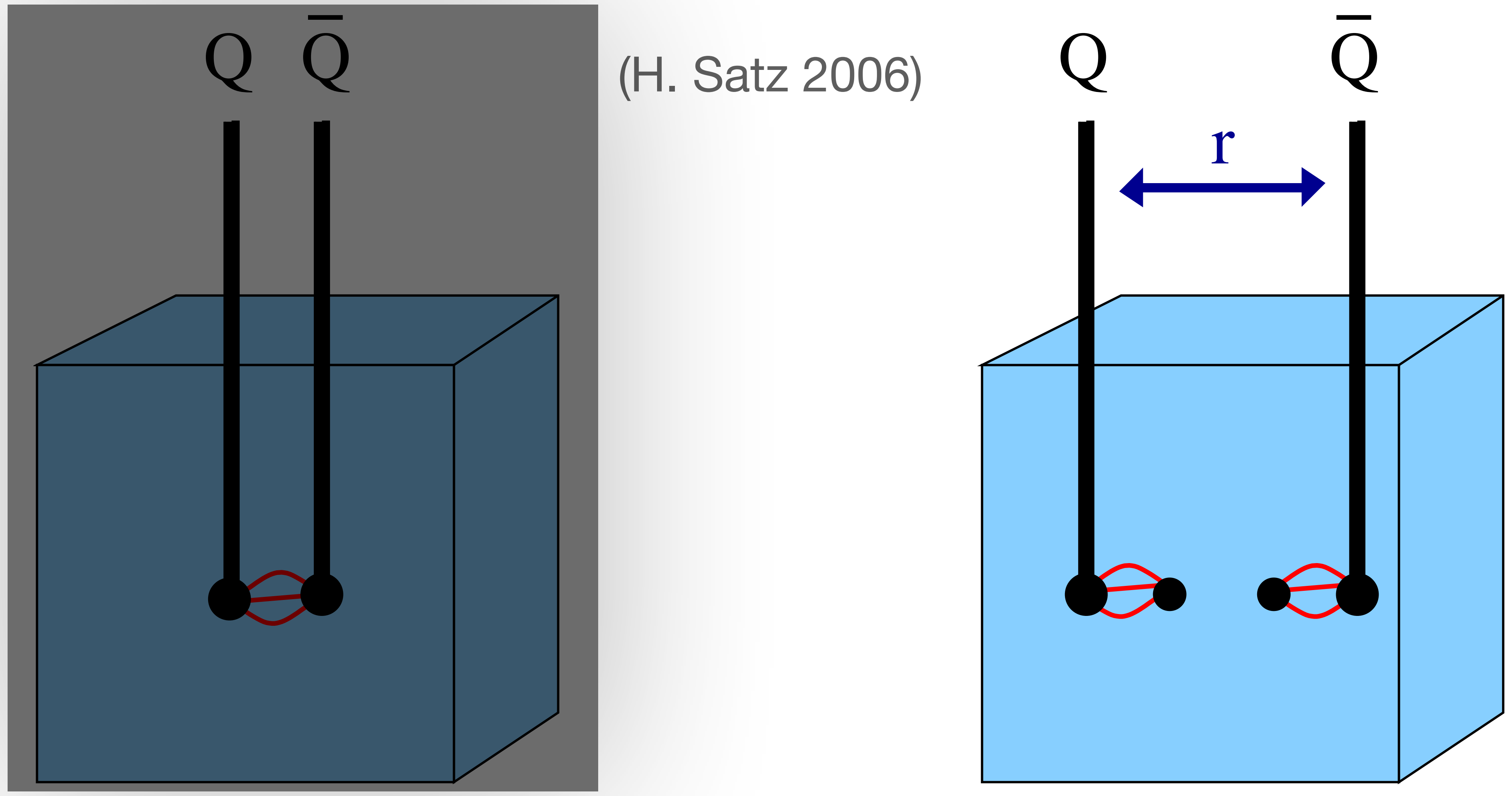

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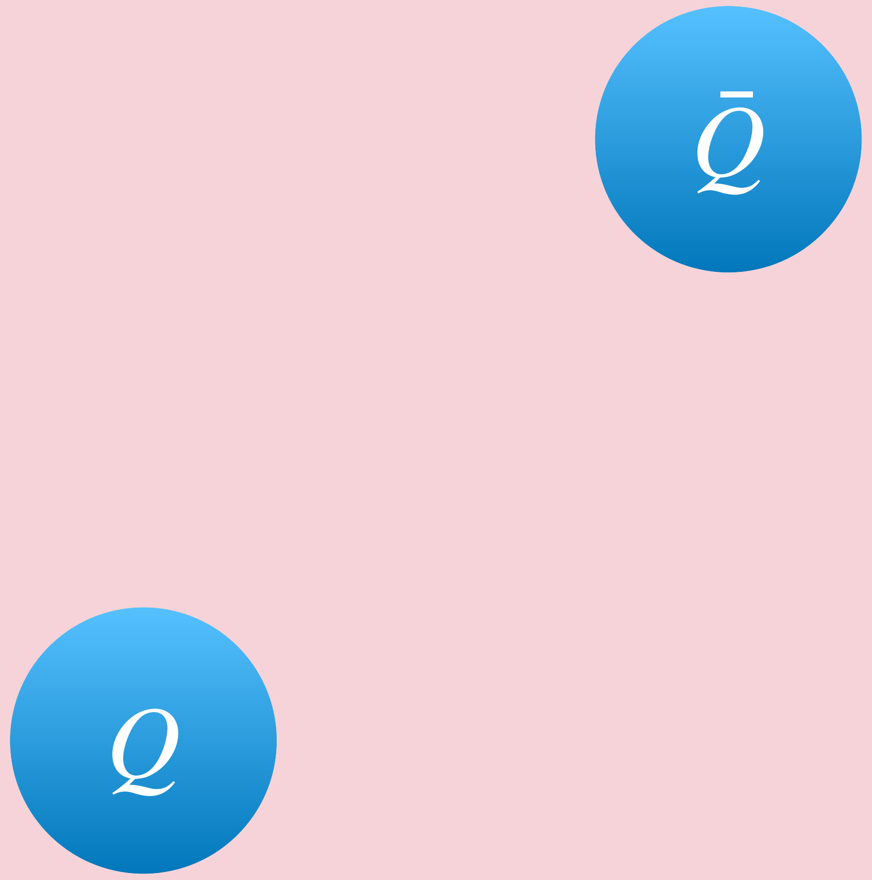

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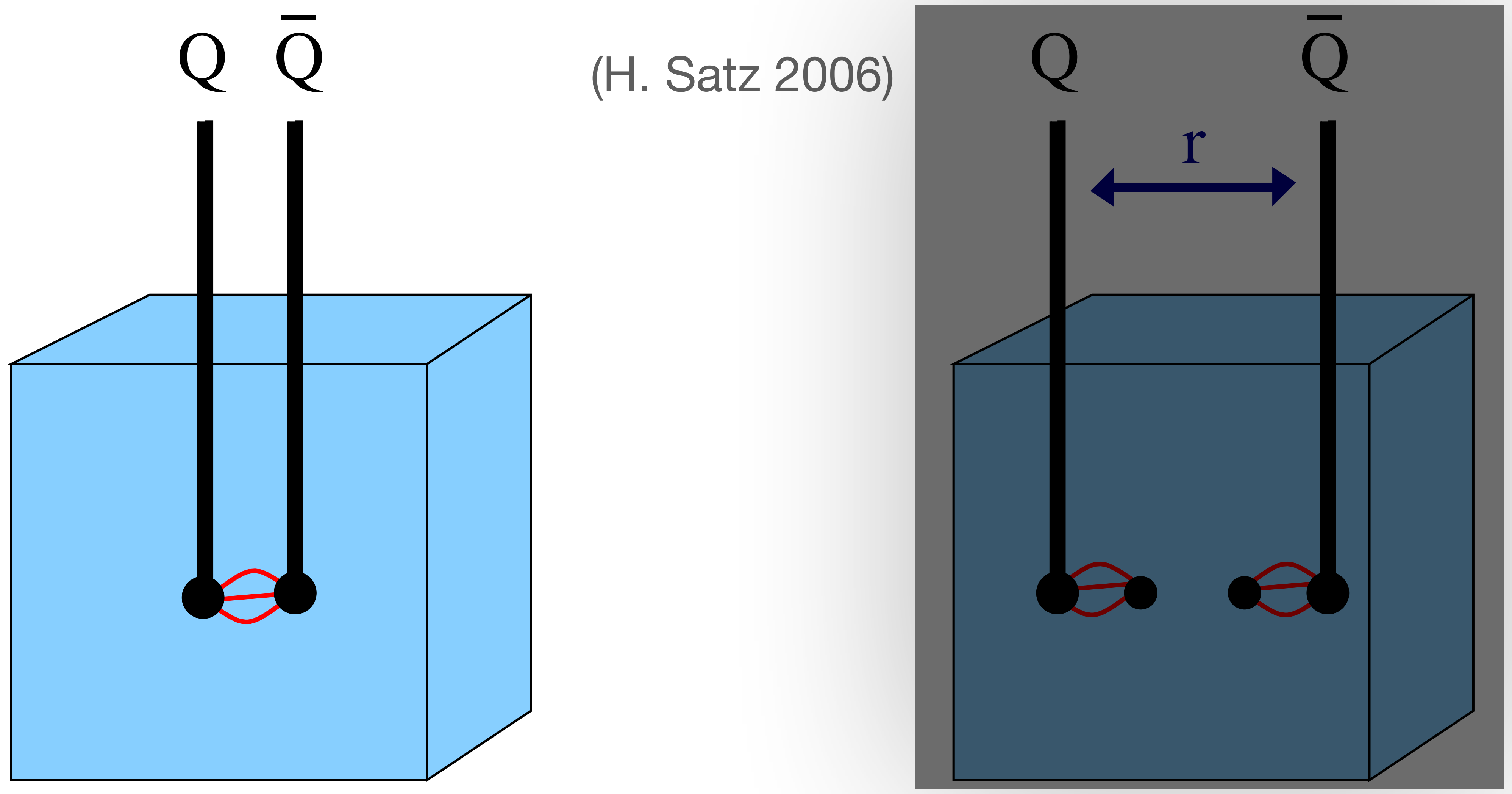
(H. Satz 2006)



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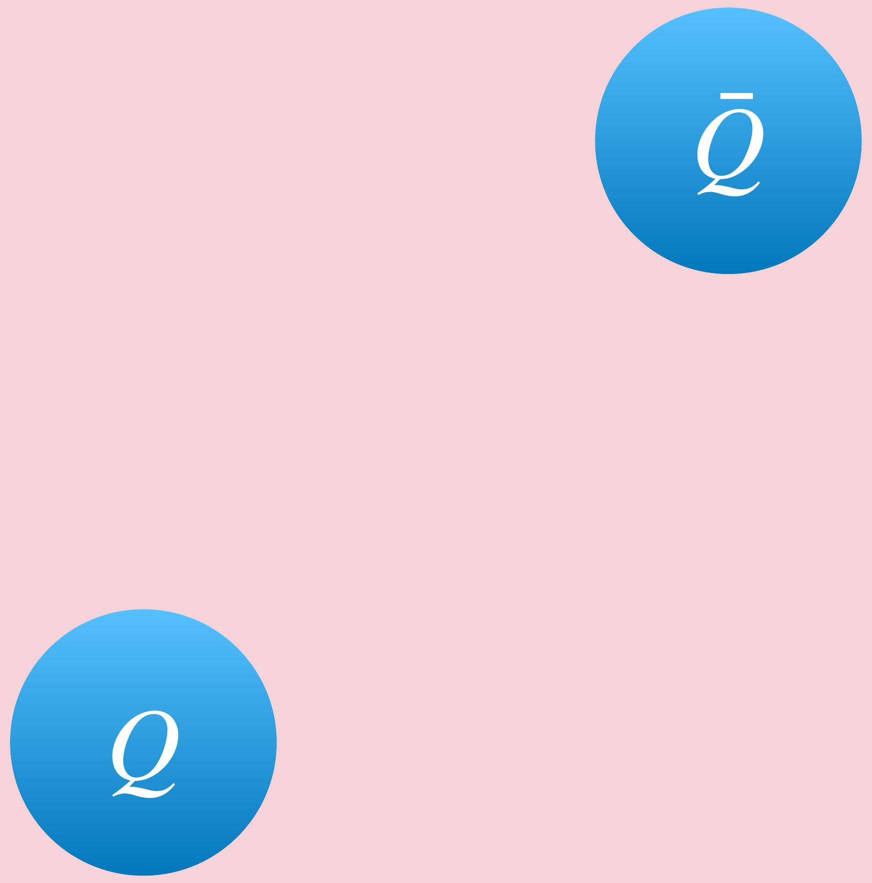

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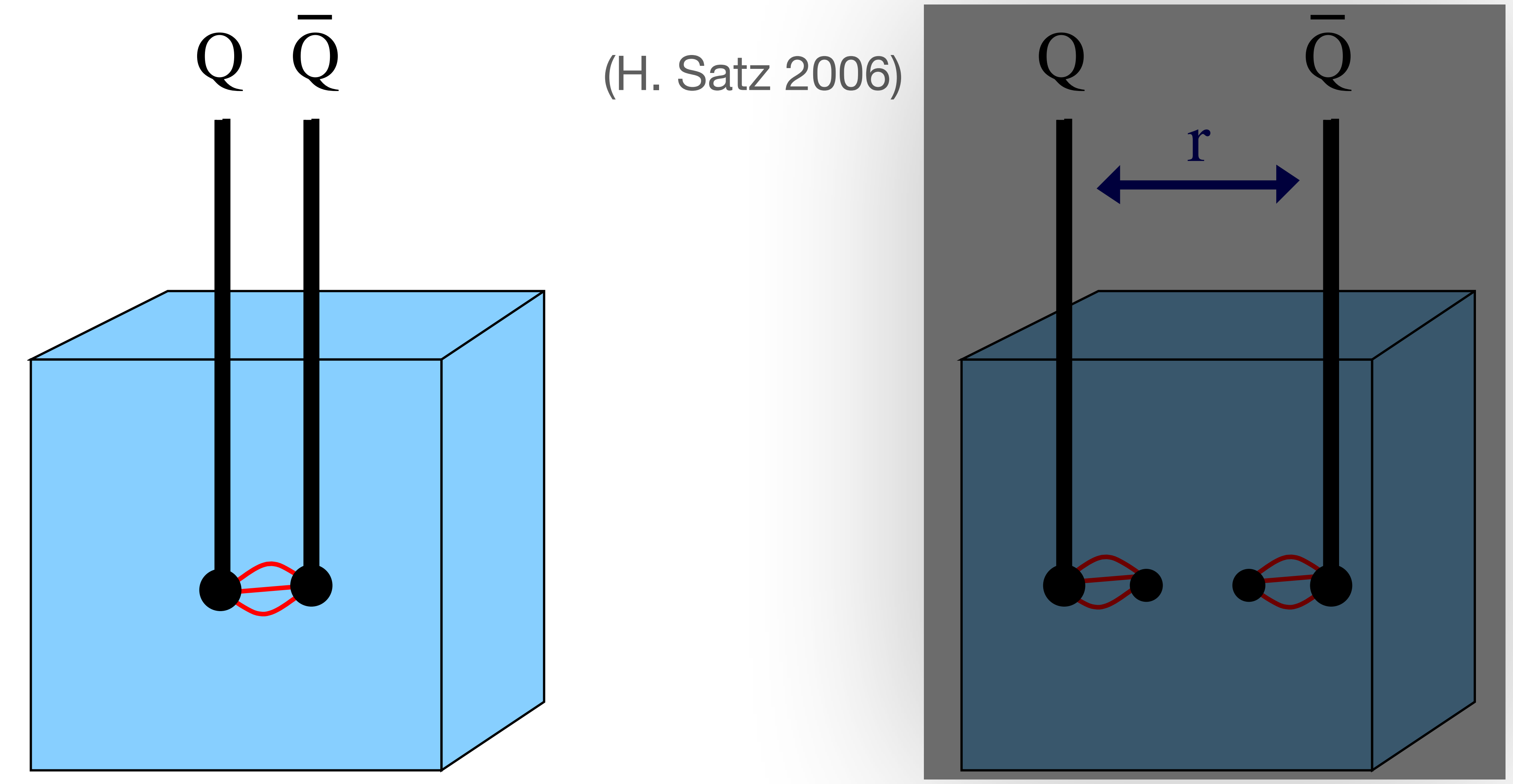
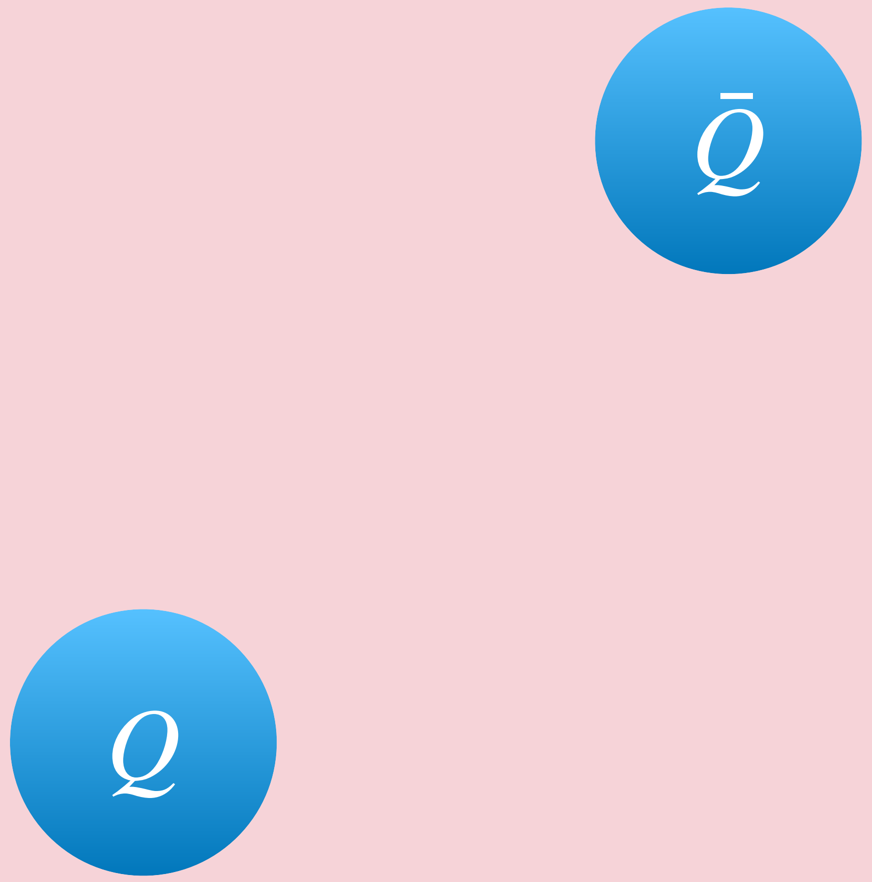


$Q\bar{Q}$ can bind if $r \sim \dfrac{1}{Mv} < \dfrac{1}{T}$

$\Longrightarrow$ most of quarkonium starts to form when $Mv \gtrsim T$



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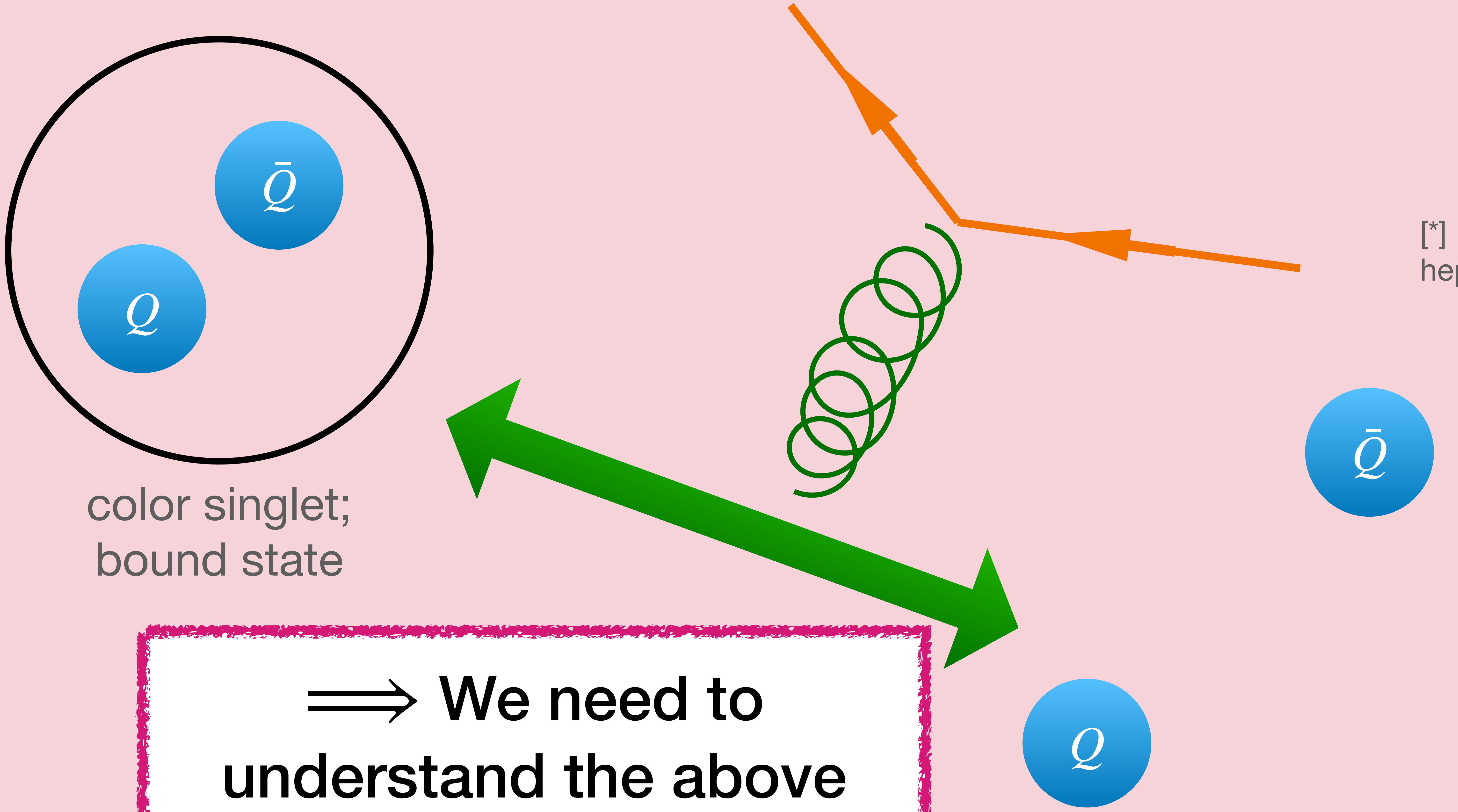

[*] N. Brambilla, A. Pineda, J. Soto. A. Vairo
hep-ph/9907240, hep-ph/0410047

$\Longrightarrow$ **We need to understand the above dynamics in the hierarchy**

$$Mv \gg T$$

$\Longrightarrow$ **pNRQCD [*]**

$T > 0$

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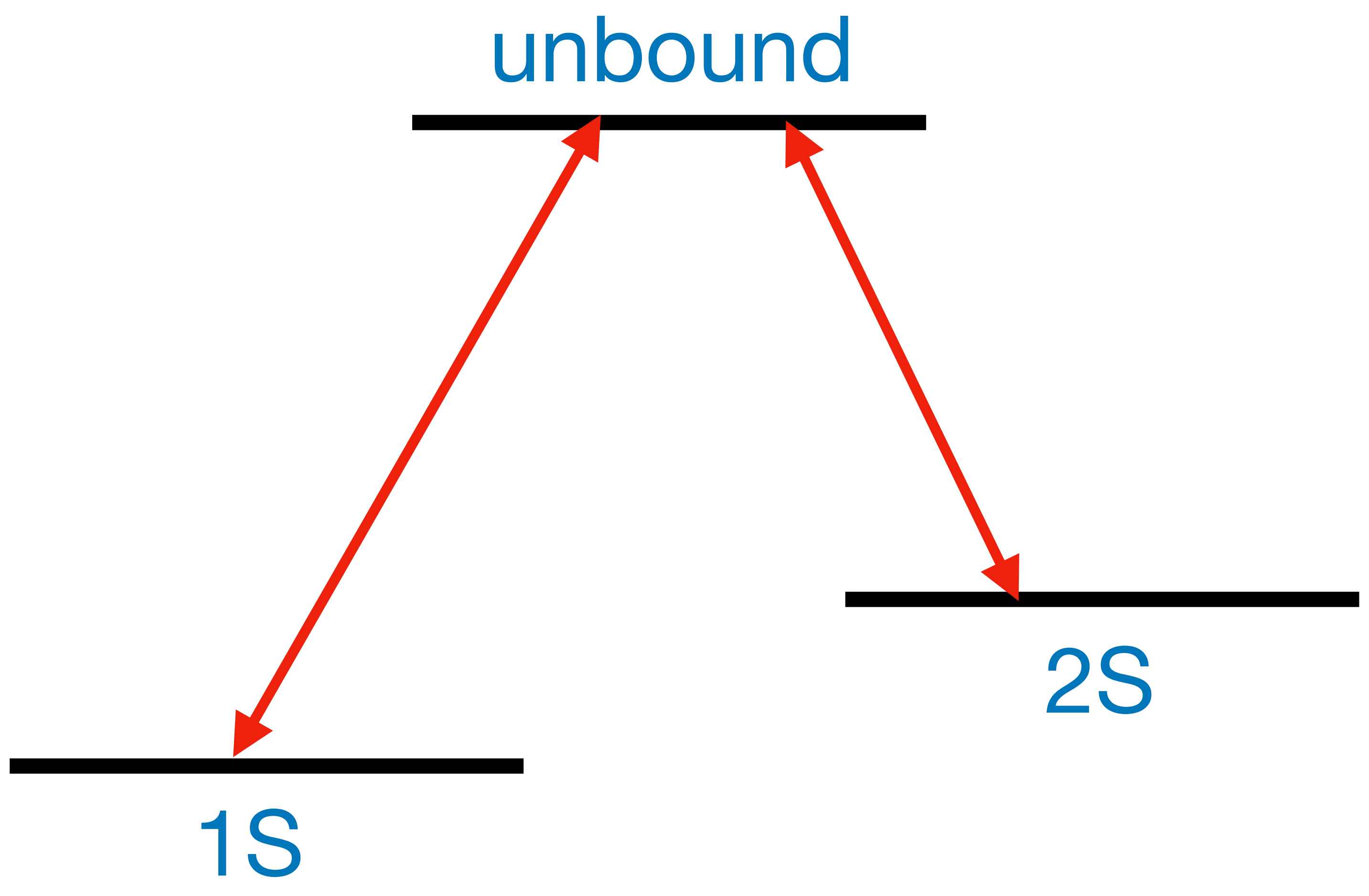
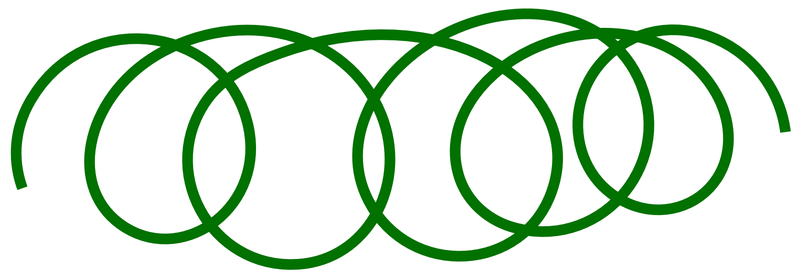
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\, quarks} + \mathscr{L}_{\rm gluon} + \int d^3r {\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+V_A\big(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\big) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

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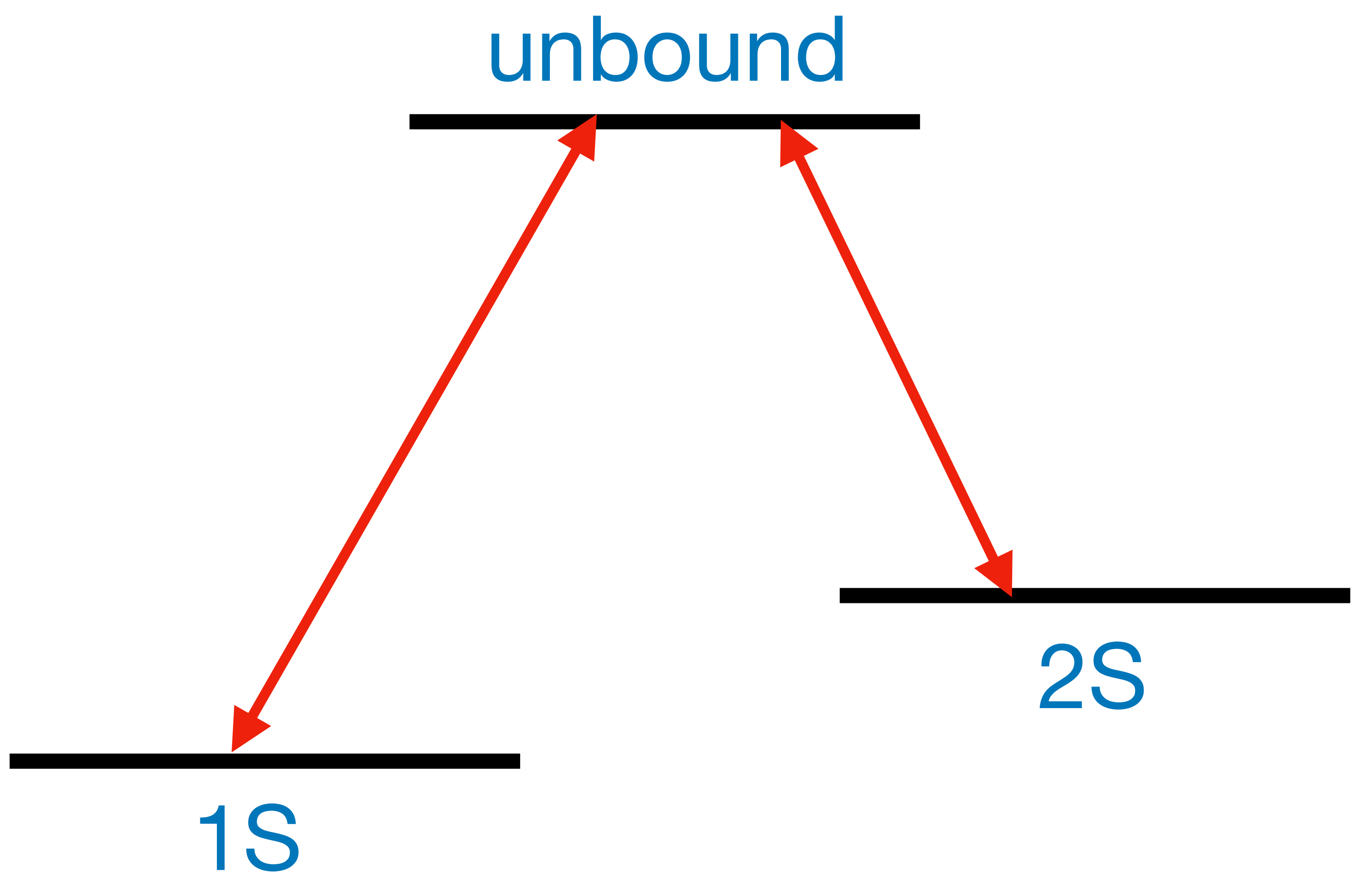

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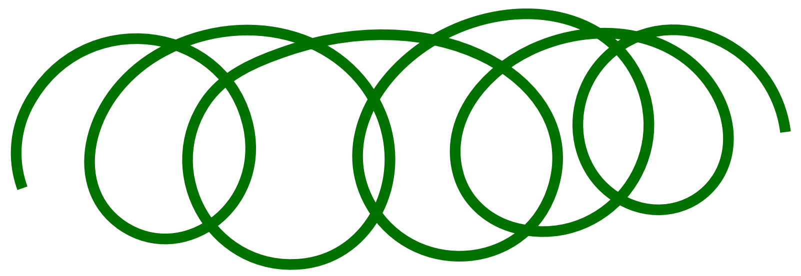

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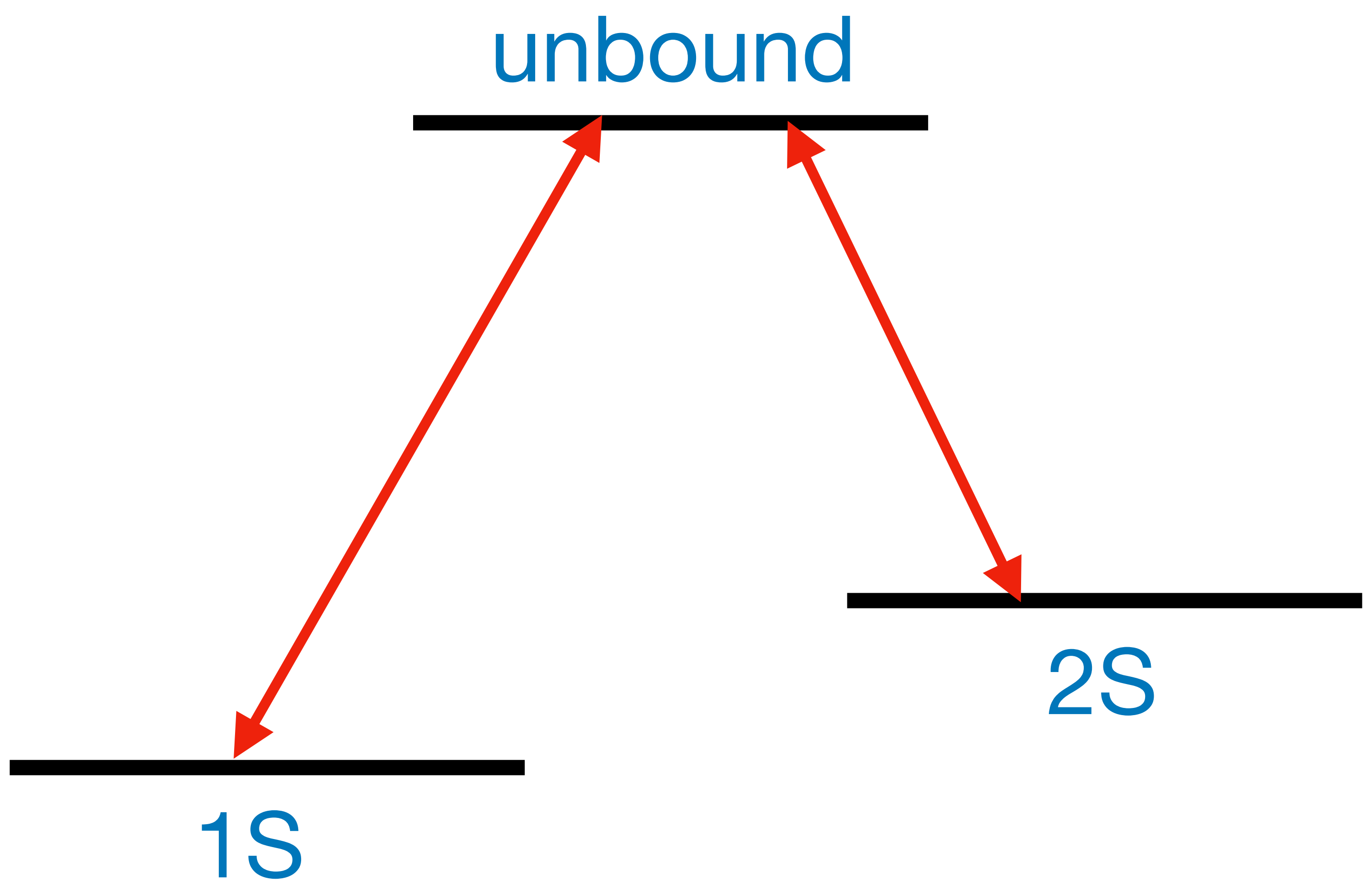

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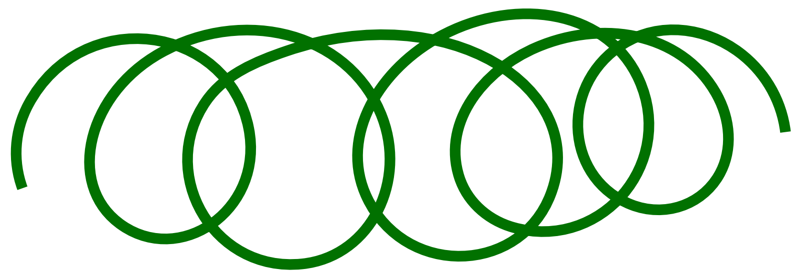

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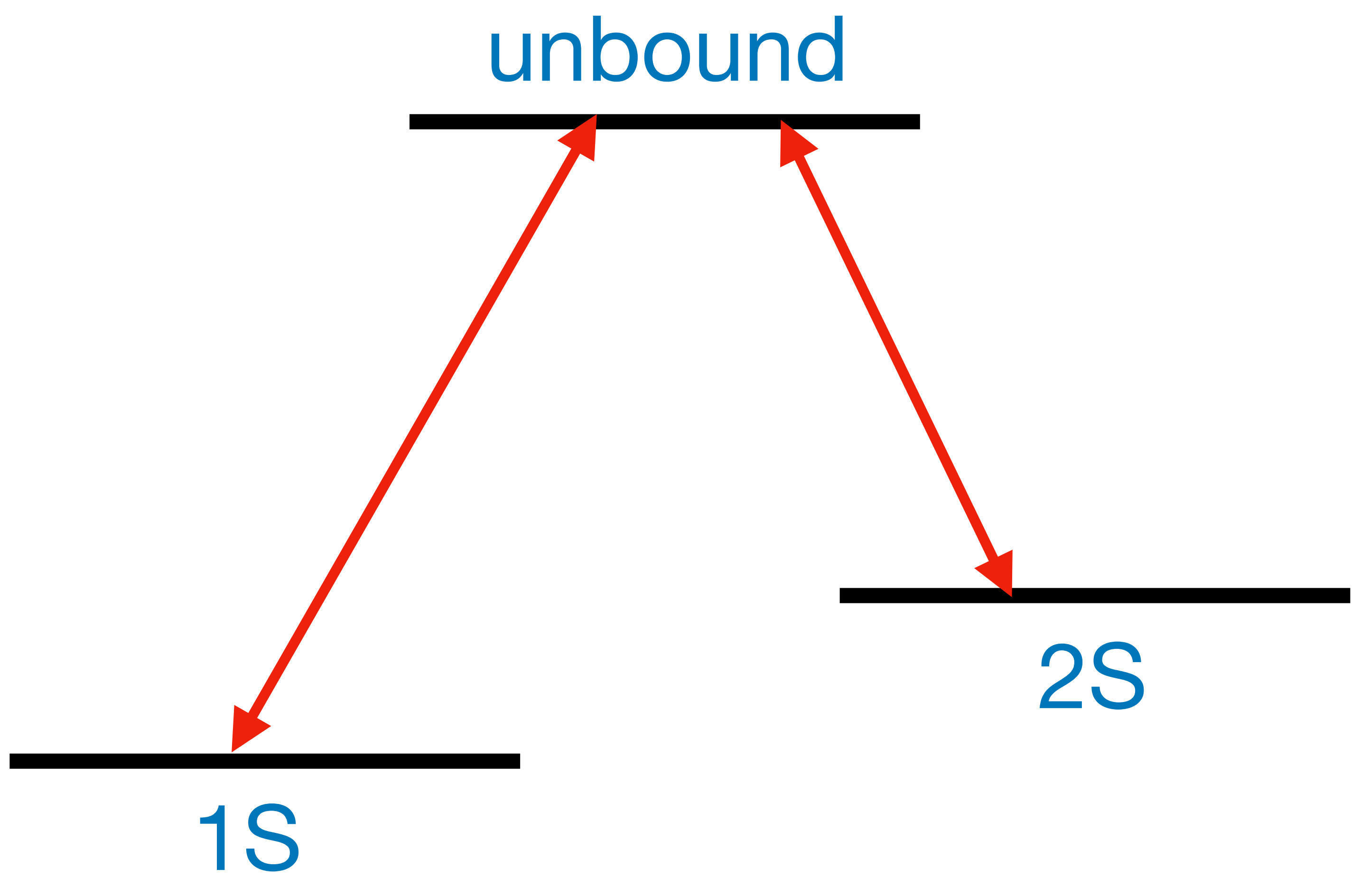

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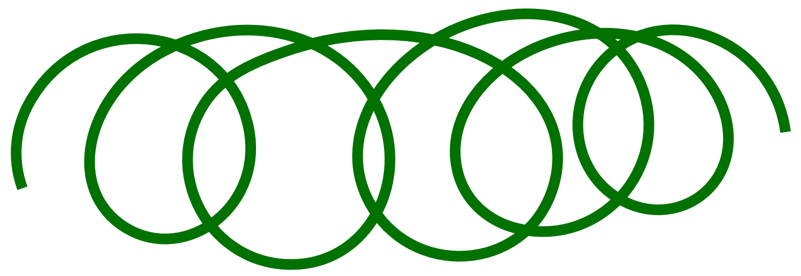

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Open quantum systems

“tracing/integrating out” the QGP

- Given an initial density matrix $\rho_{\text{tot}}(t=0)$, quarkonium coupled with the QGP evolves as

$$\rho_{\text{tot}}(t) = U(t)\rho_{\text{tot}}(t=0)U^{\dagger}(t).$$

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- Then, one derives an evolution equation for $\rho_S(t)$, assuming that at the initial time we have $\rho_{\text{tot}}(t=0) = \rho_S(t=0) \otimes e^{-H_{\text{QGP}}/T}/\mathcal{Z}_{\text{QGP}}$.

Open quantum systems

“tracing/integrating out” the QGP: semi-classic description

Unitary evolution of environment + subsystem

⇓ Trace out the environment degrees of freedom

OQS: ρ_S has non-unitary, time-irreversible evolution

⇓ Markovian approximation $\Longleftrightarrow$ weak coupling in H_I

OQS: Lindblad equation

⇓ Wigner transform: $f(\mathbf{x},\mathbf{k},t) \equiv \int_{k'} e^{i\mathbf{k}'\cdot\mathbf{x}} \left\langle \mathbf{k}+\frac{\mathbf{k}'}{2} \middle| \rho_S(t) \middle| \mathbf{k}-\frac{\mathbf{k}'}{2} \right\rangle$

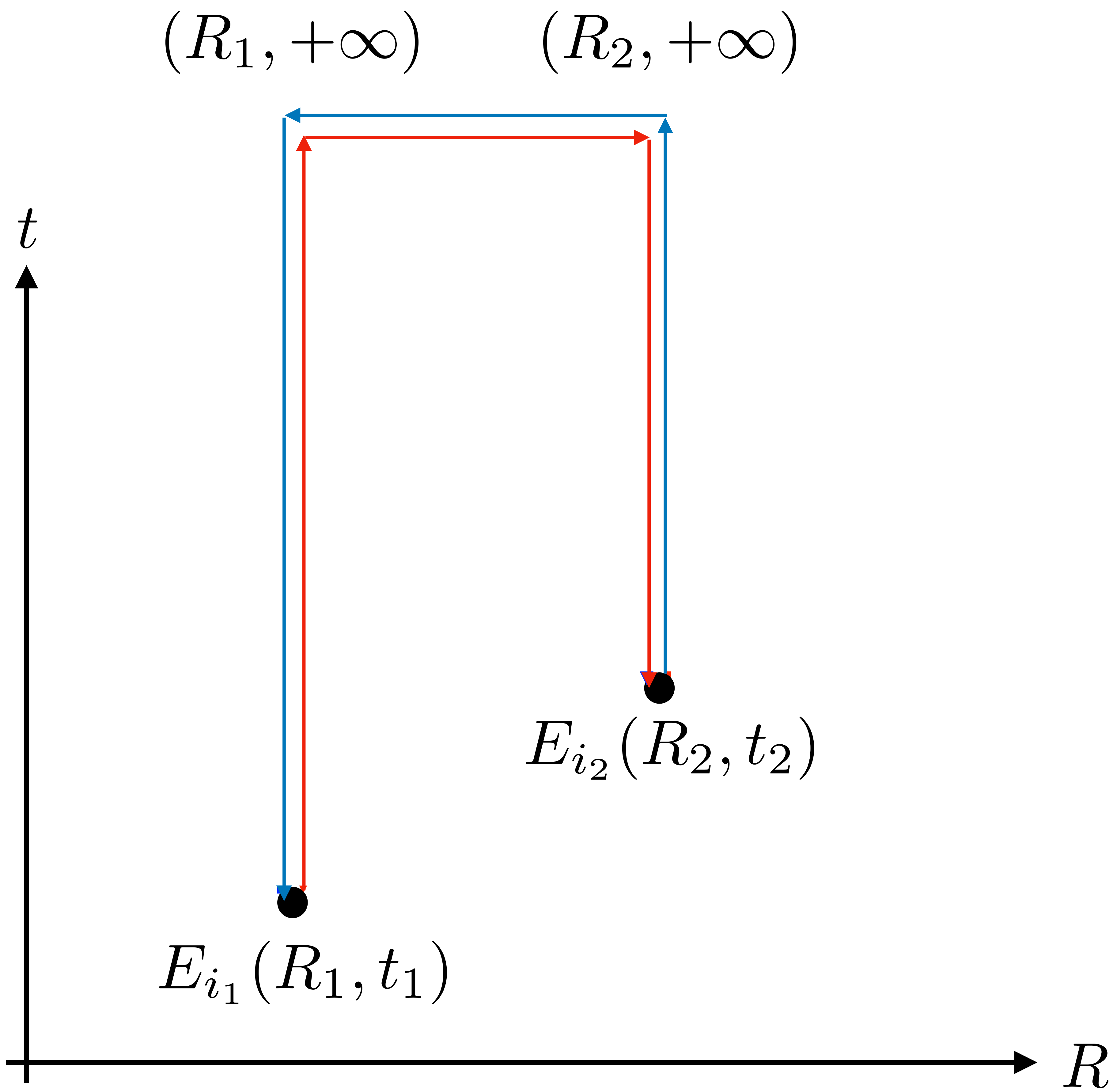
Semi-classic subsystem: Boltzmann/Fokker-Planck equation

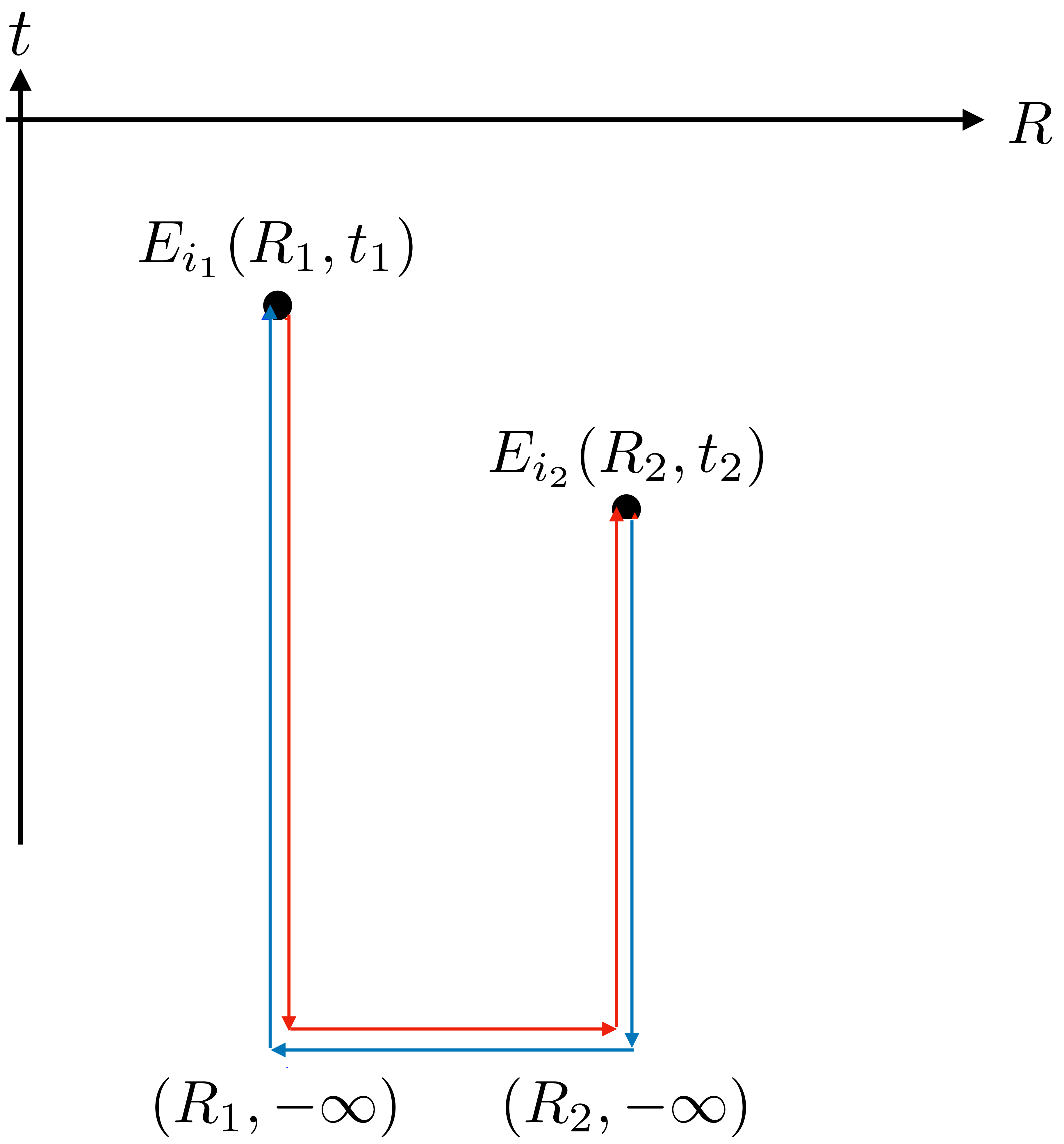
How does the QGP enter the dynamics?

QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2) \right)^a \left(E_{i_1}(\mathbf{R}_1, t_1) \mathscr{W}_{1'} \right)^a \right\rangle_T$$





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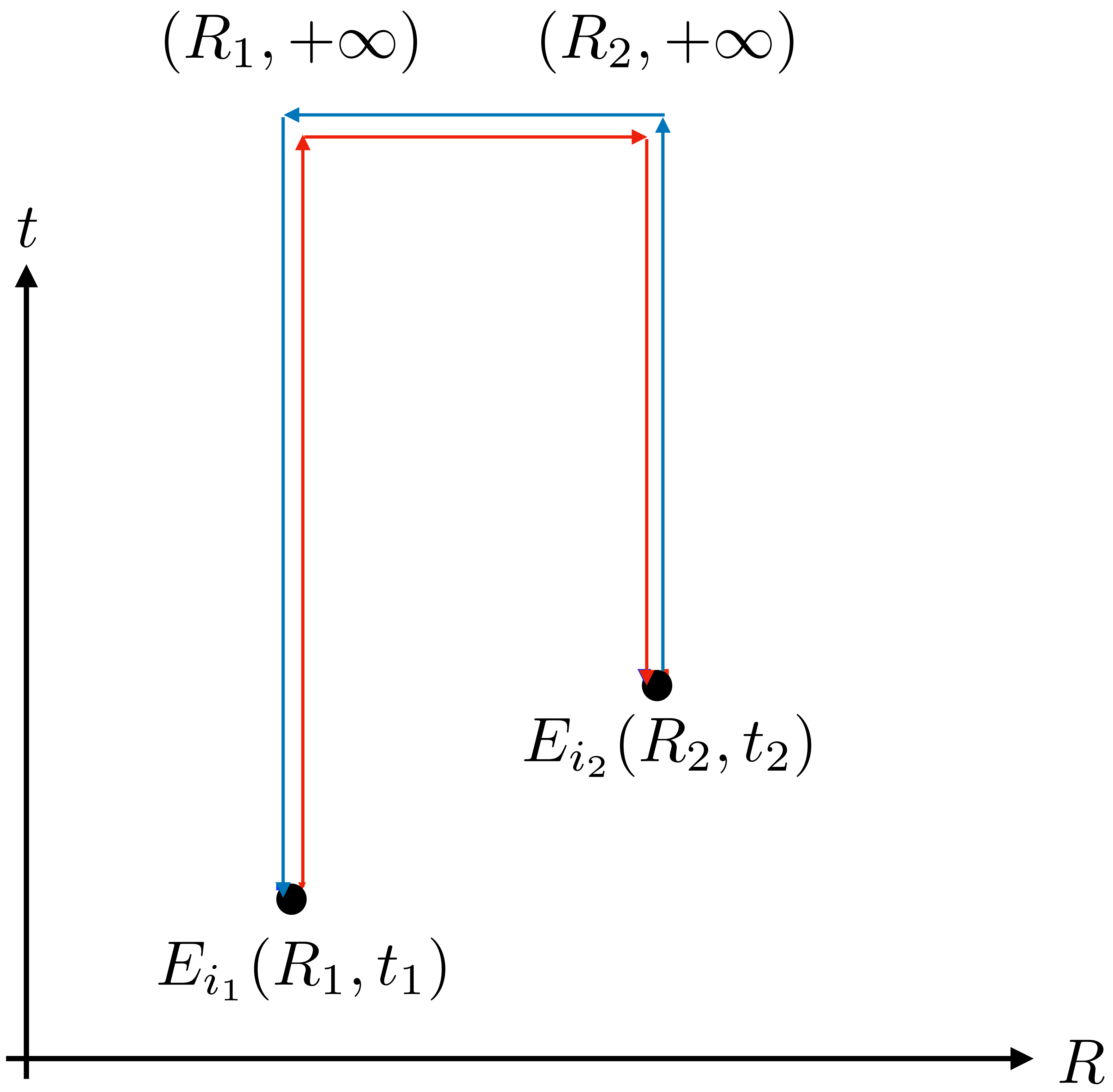

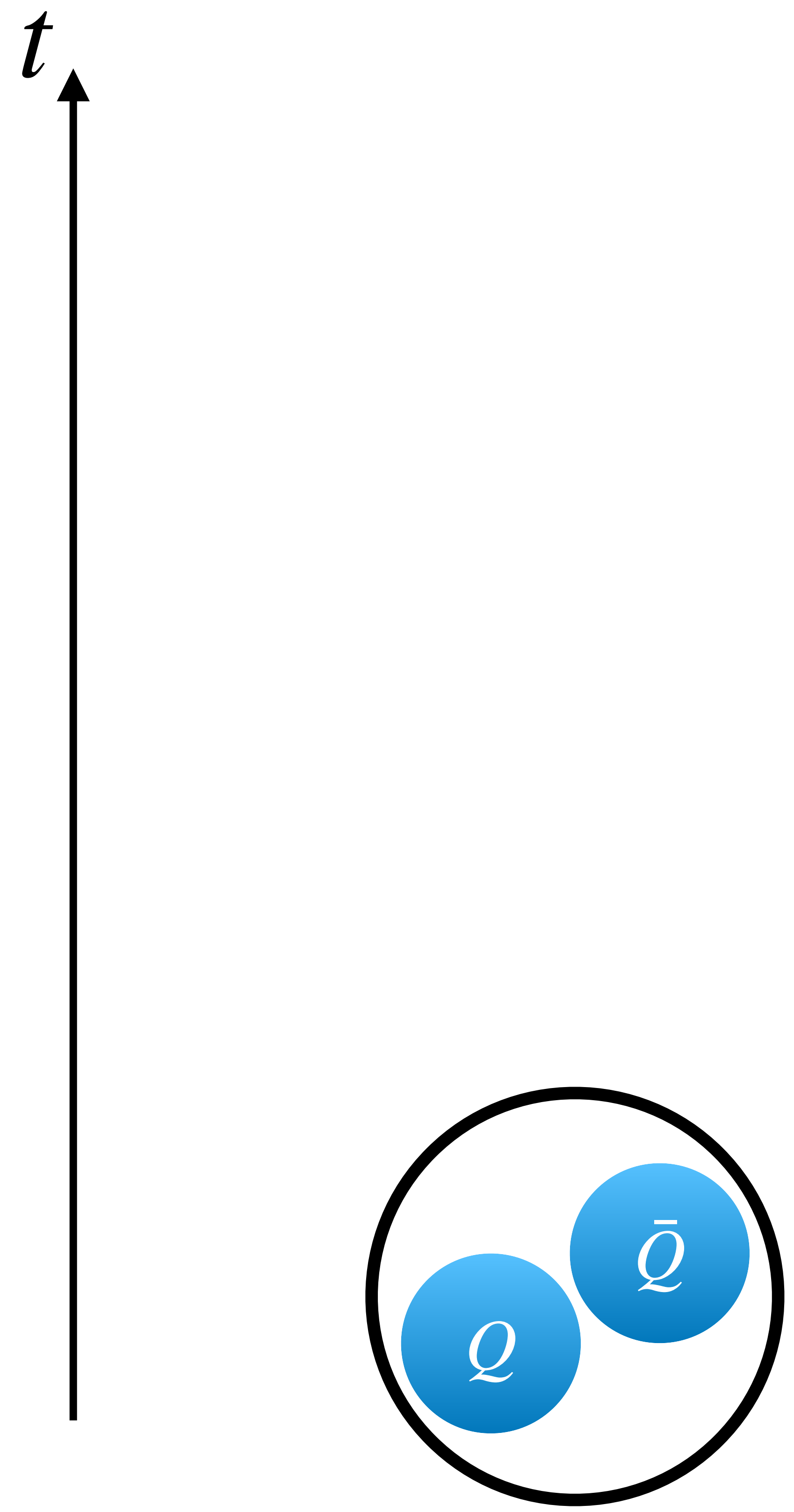

bound state:
color singlet

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See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

X. Yao and T. Mehen, hep-ph/2009.02408

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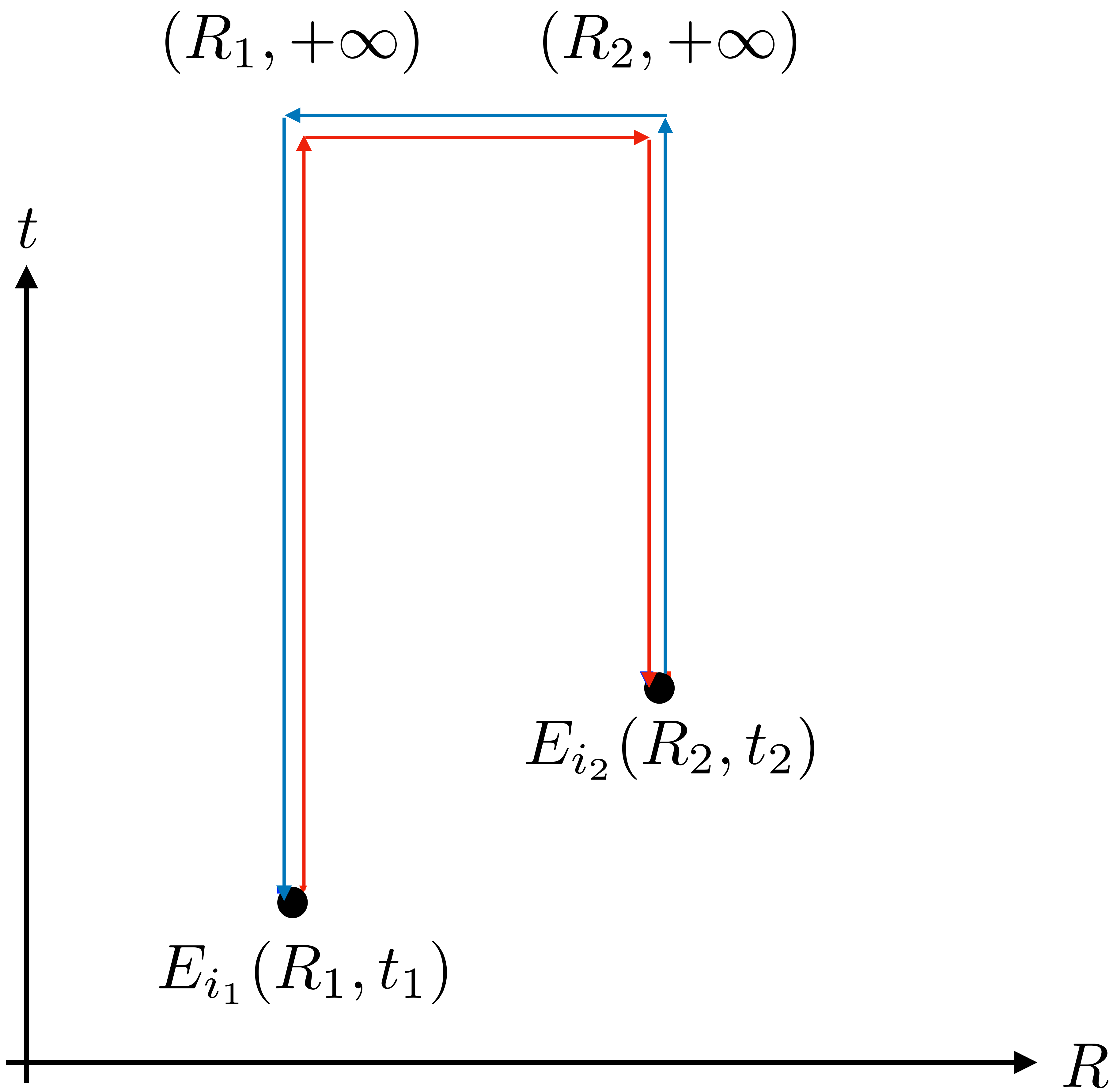

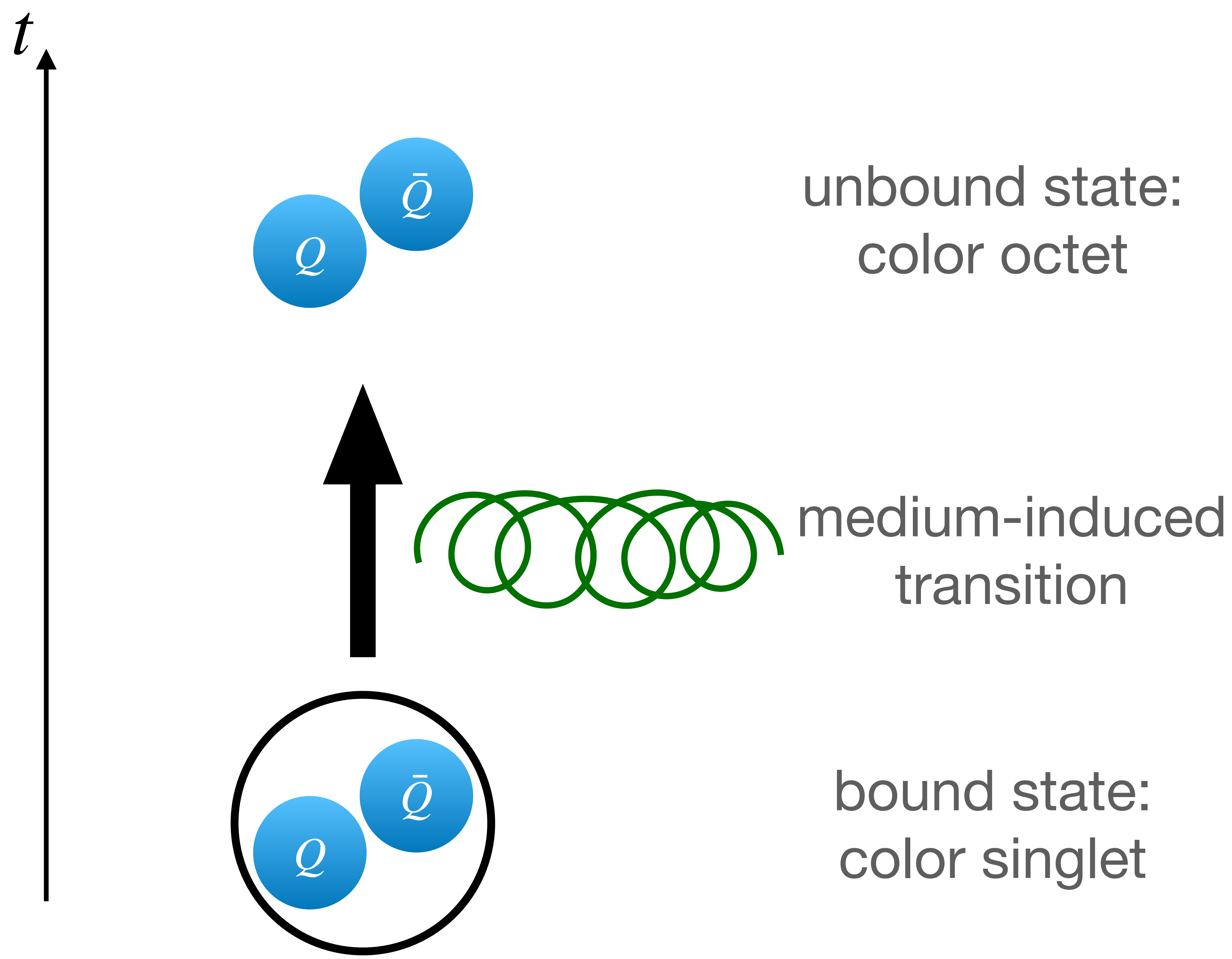

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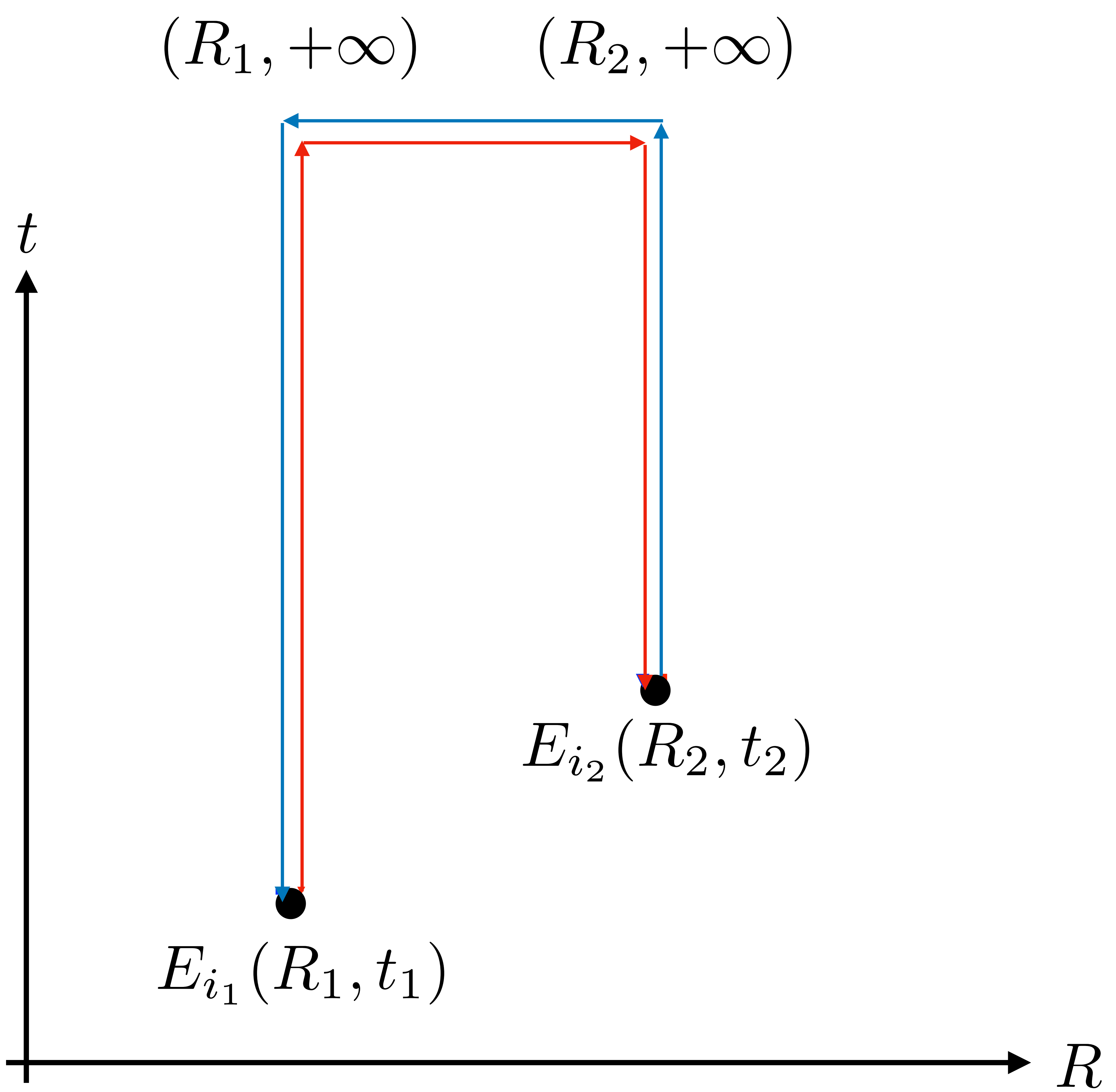

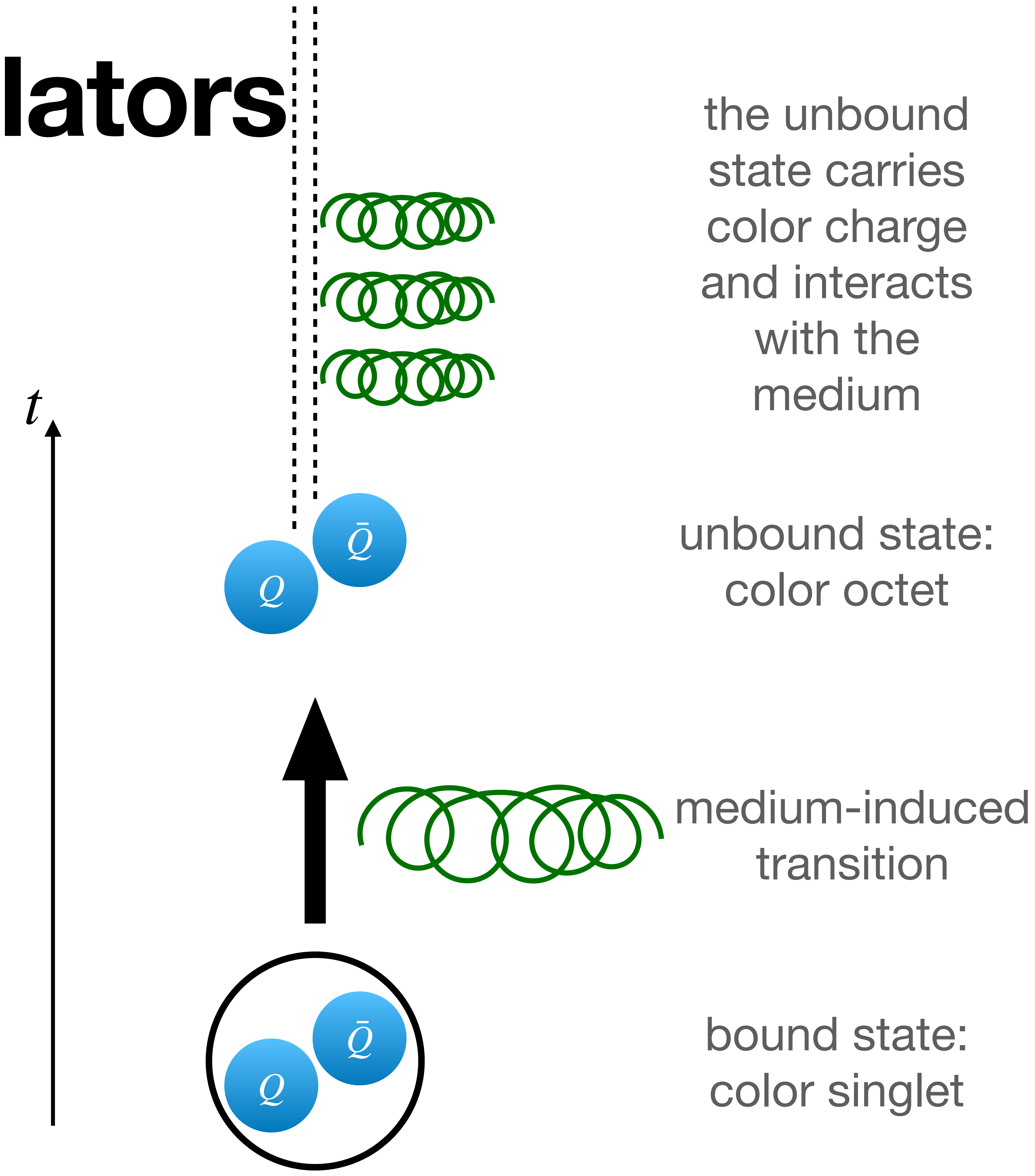

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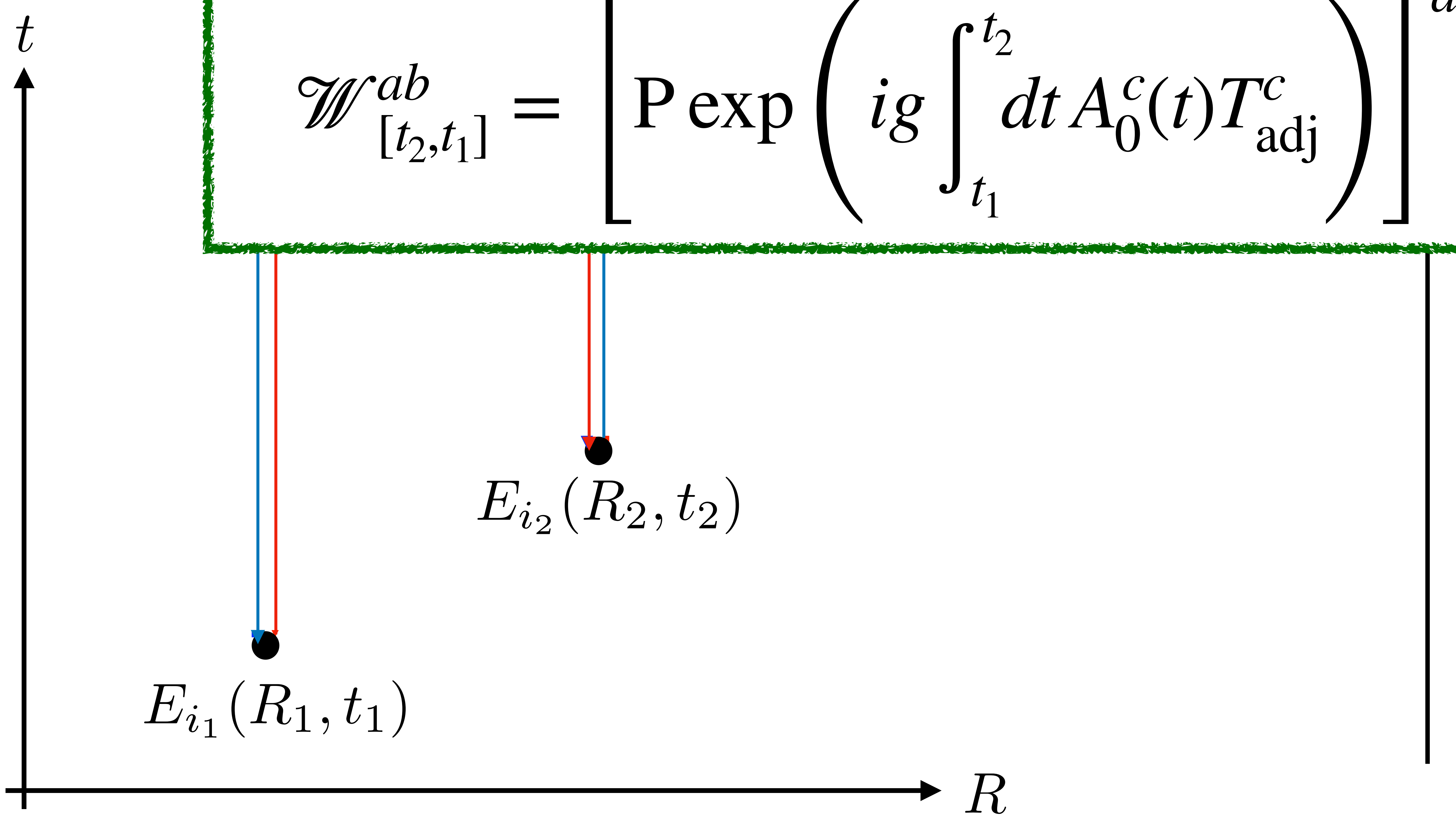
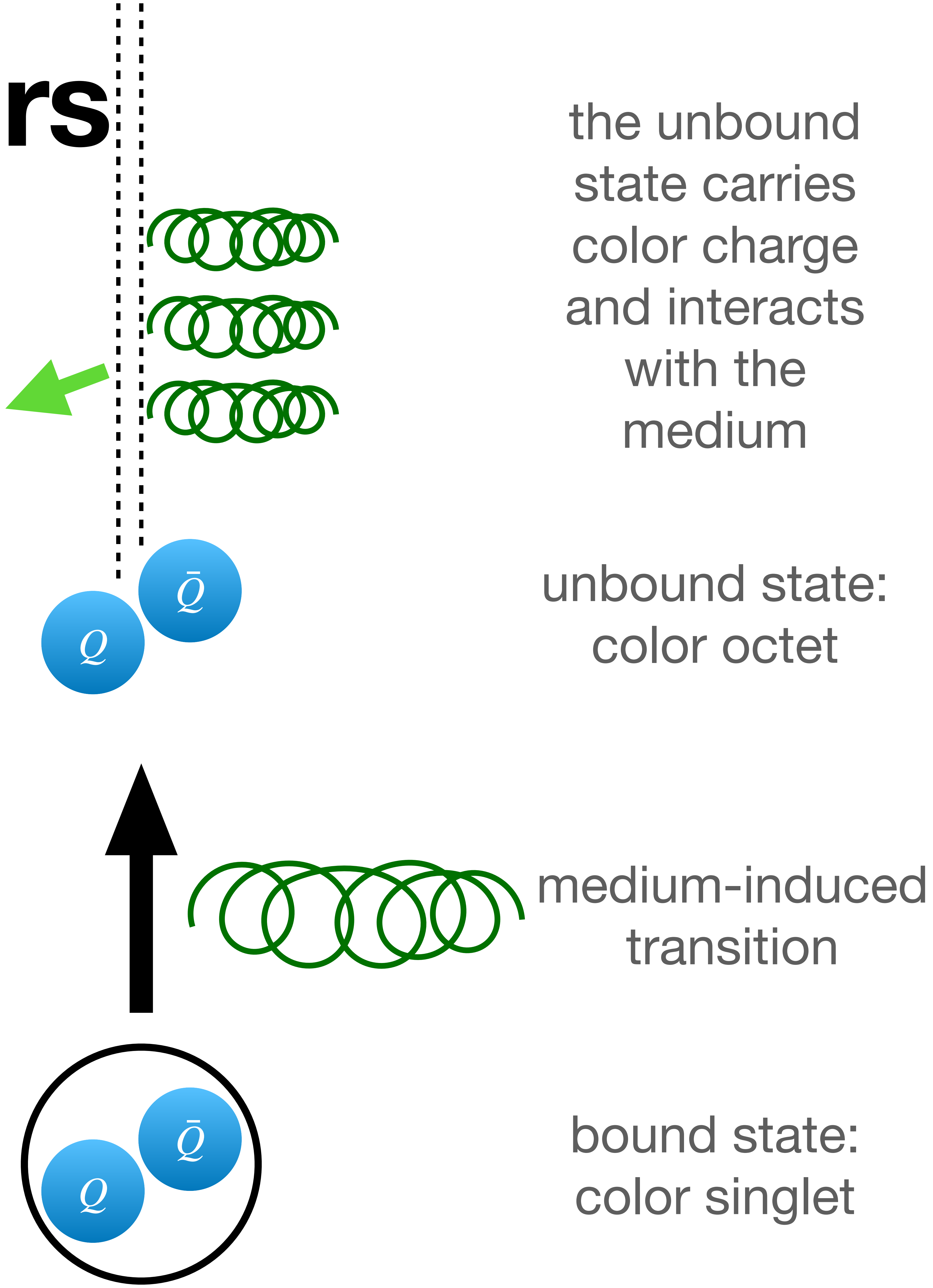
for quarkonia transport

Re-summing the one-gluon insertions along the heavy quark path generates a Wilson line:

$$\mathscr{W}^{ab}_{[t_2,t_1]} = \left[\mathrm{P}\exp\left(ig \int_{t_1}^{t_2} dt\, A_0^c(t) T^c_{\mathrm{adj}} \right) \right]^{ab}$$





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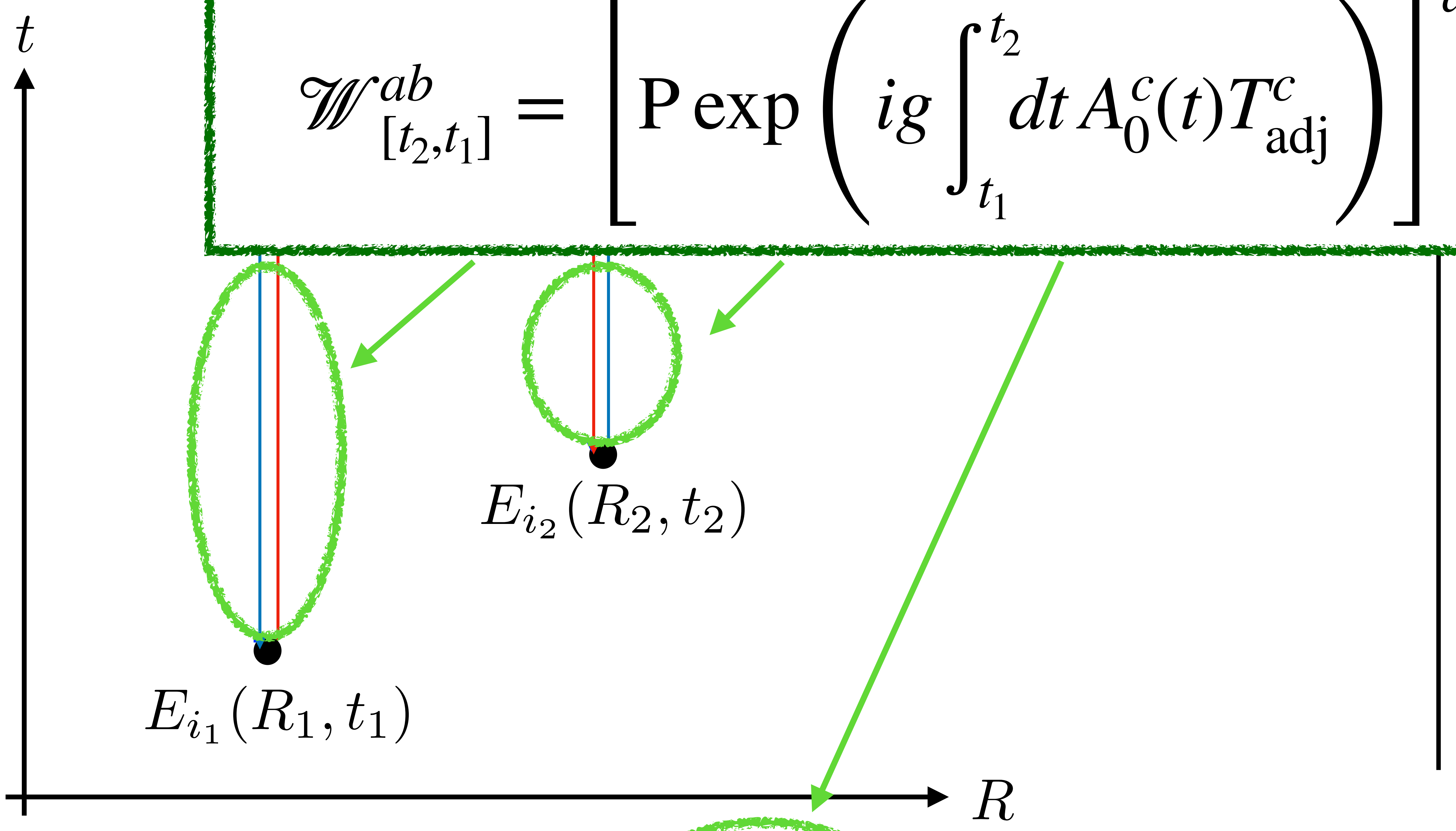

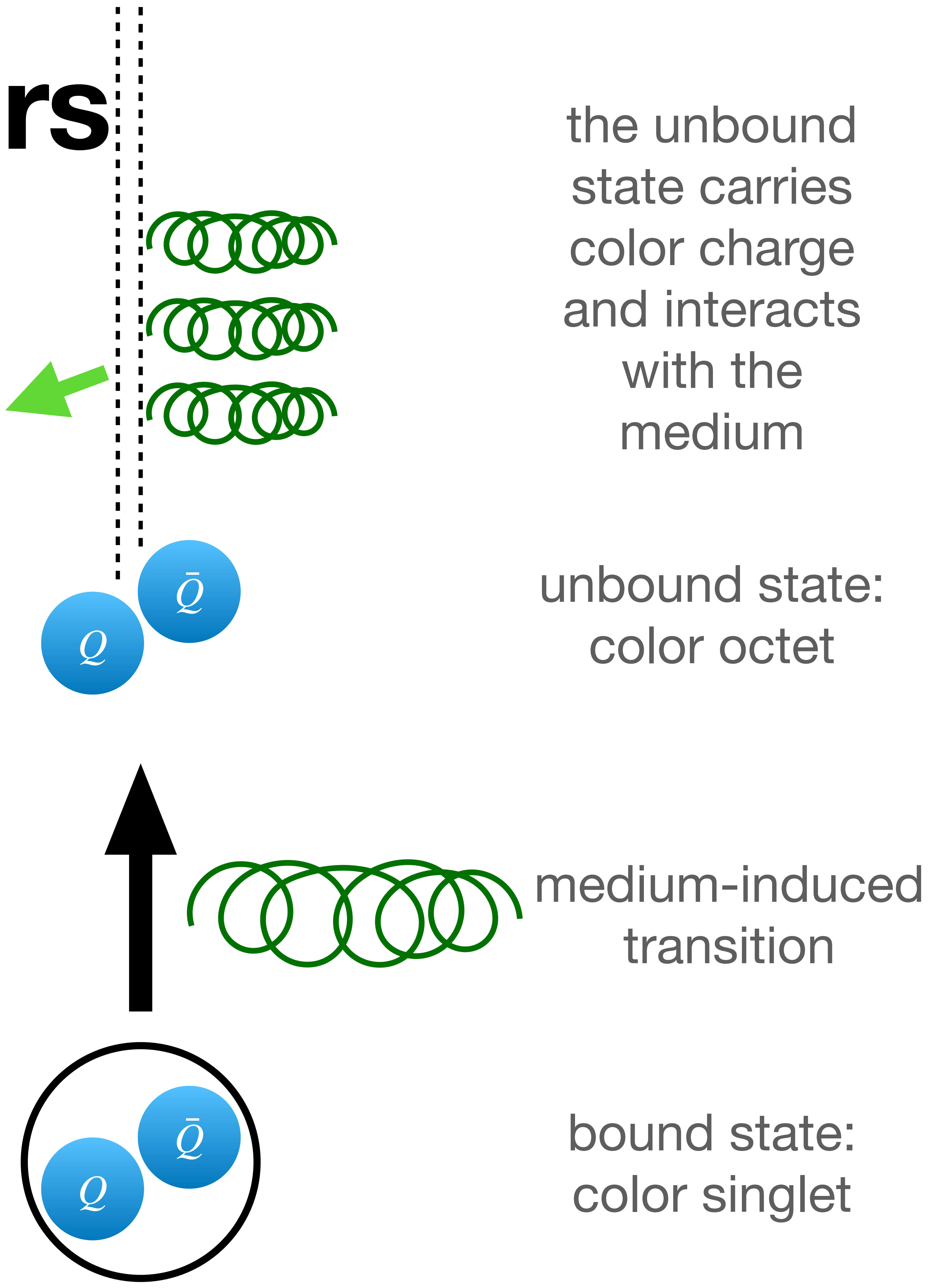

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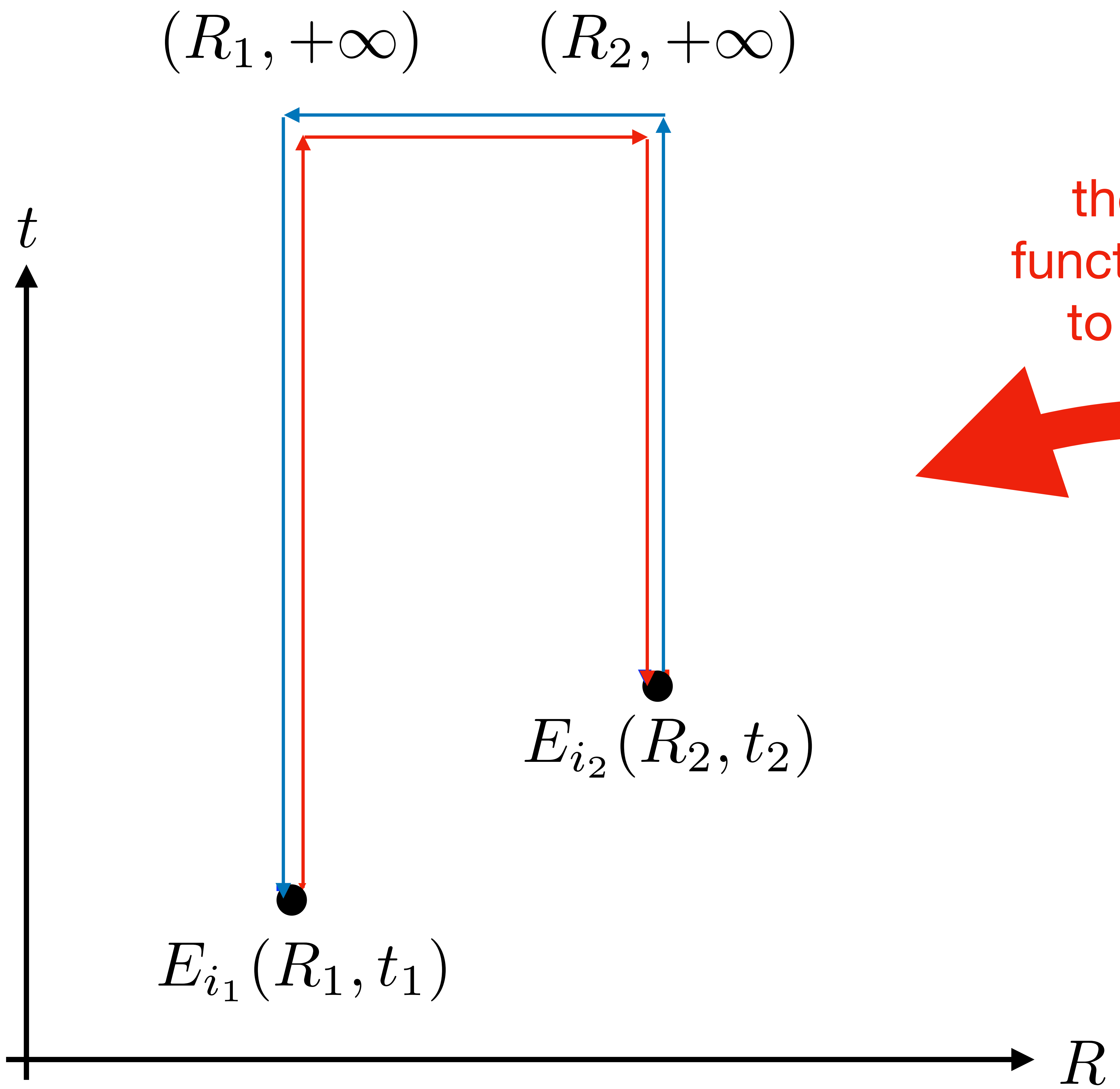
See also: N. Brambilla et al. hep-ph/1612.07248, hep-ph/1711.04515, hep-ph/2205.10289

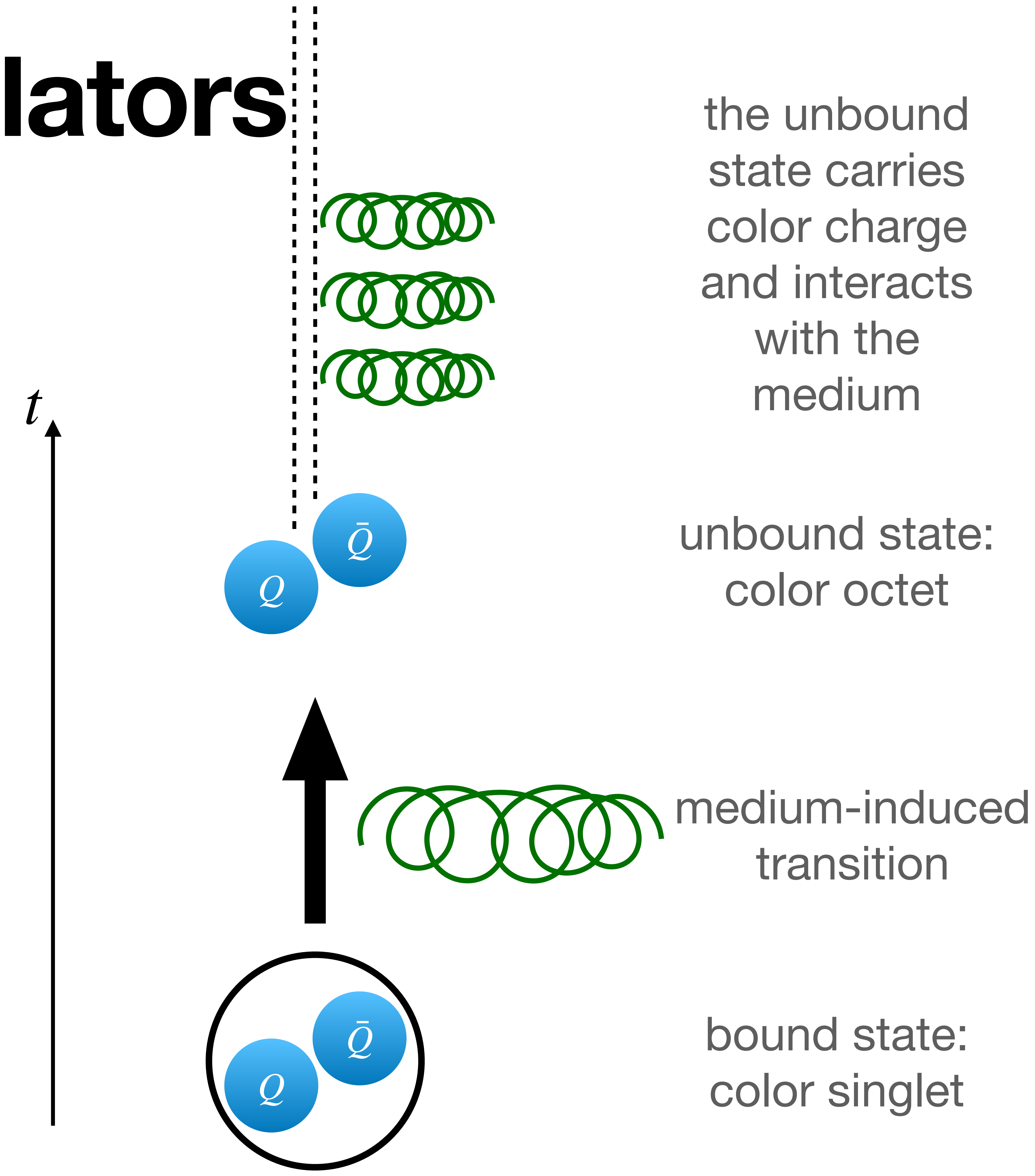
X. Yao and T. Mehen, hep-ph/2009.02408

QGP chromoelectric correlators

for quarkonia transport



the correlation function associated to this process



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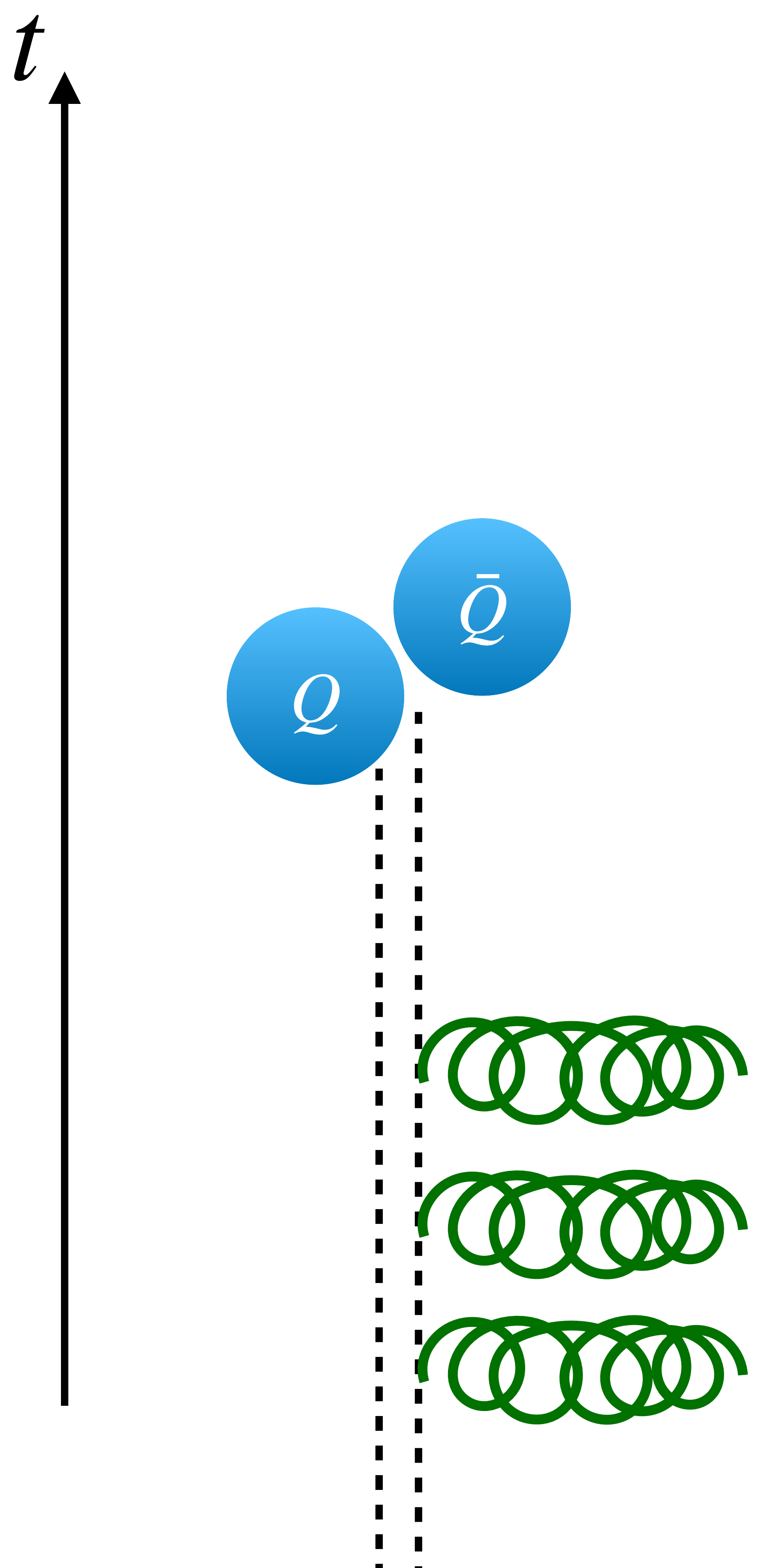
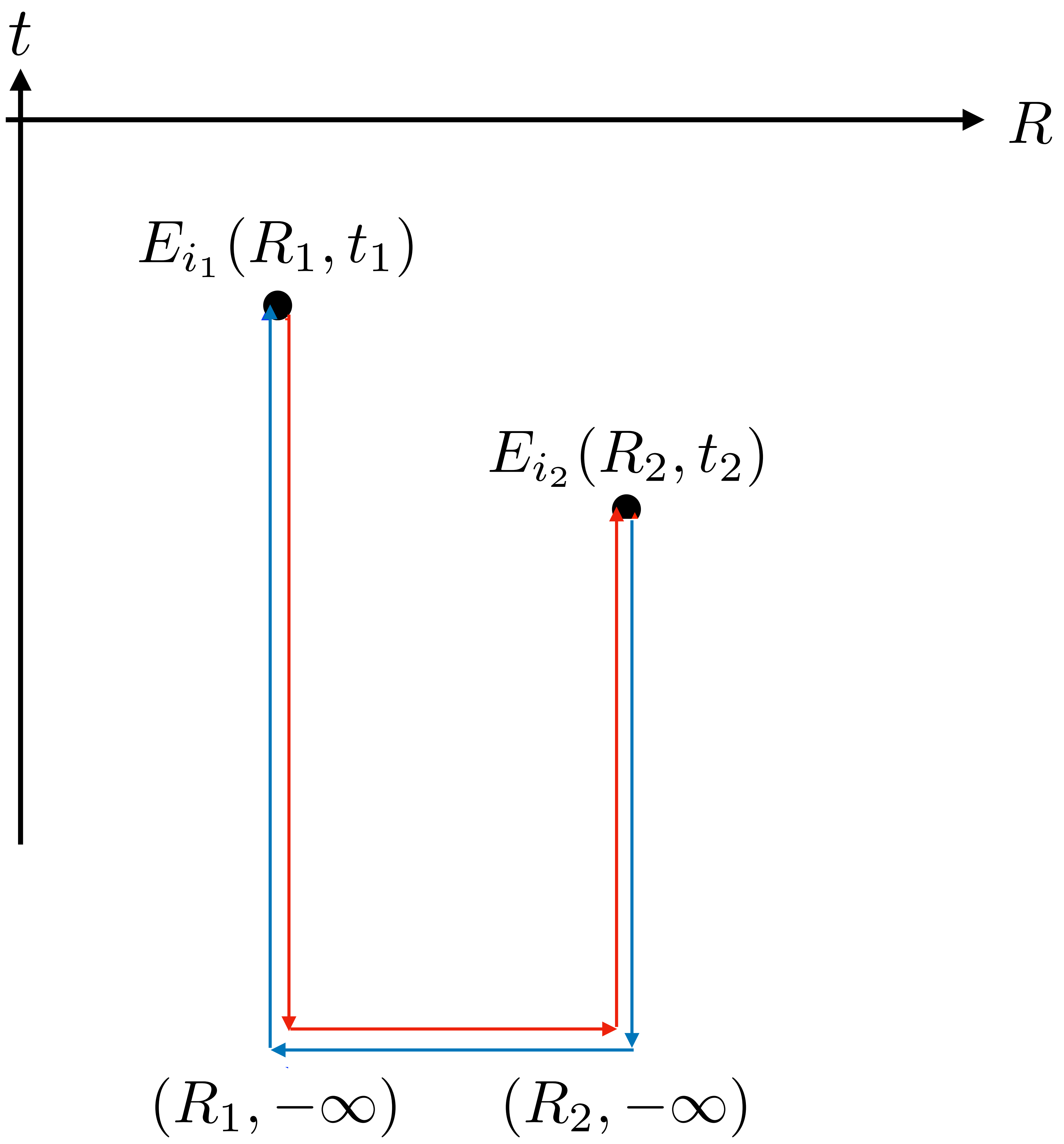
unbound state: color octet

the unbound state carries color charge and interacts with the medium





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medium-induced transition

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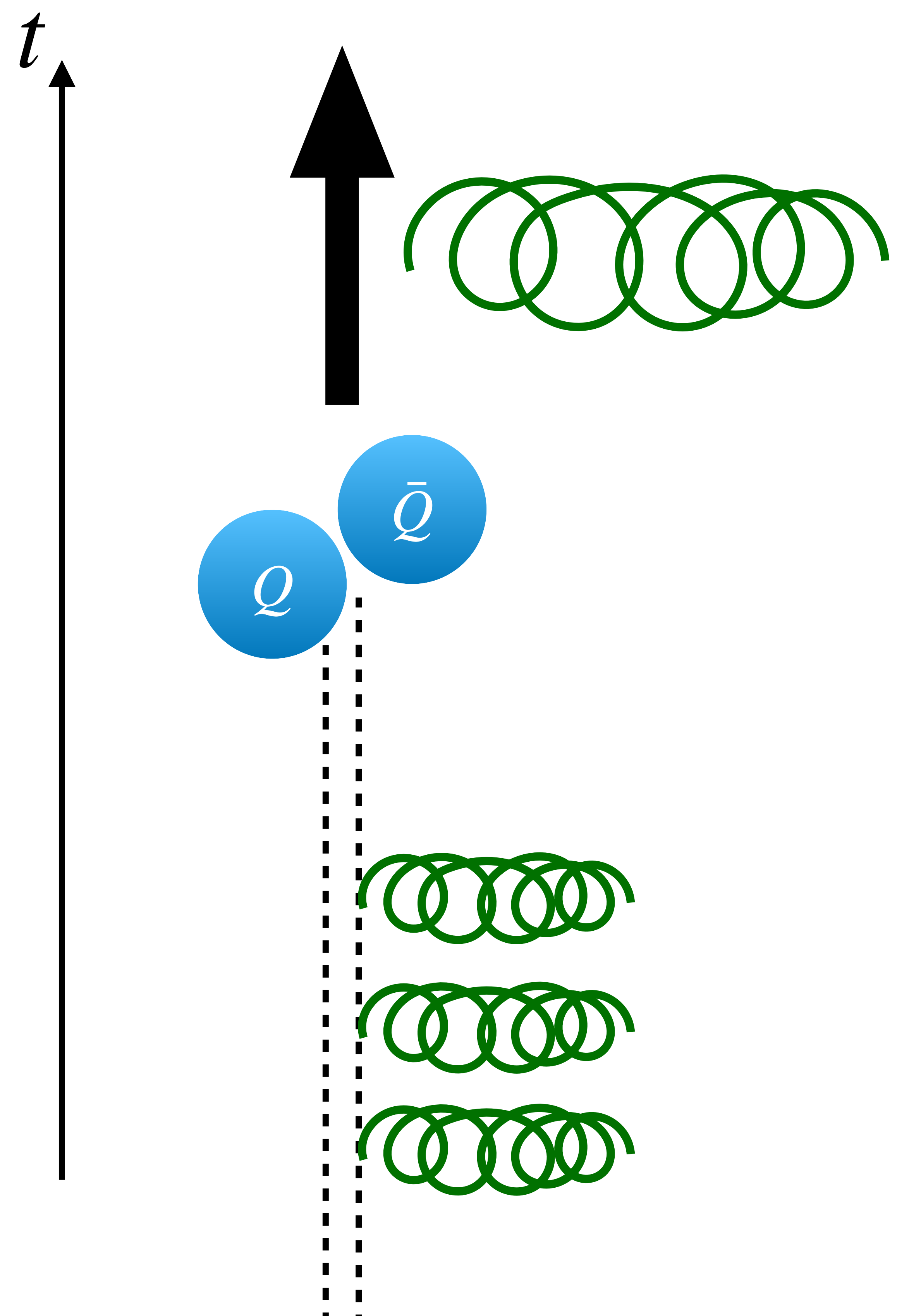

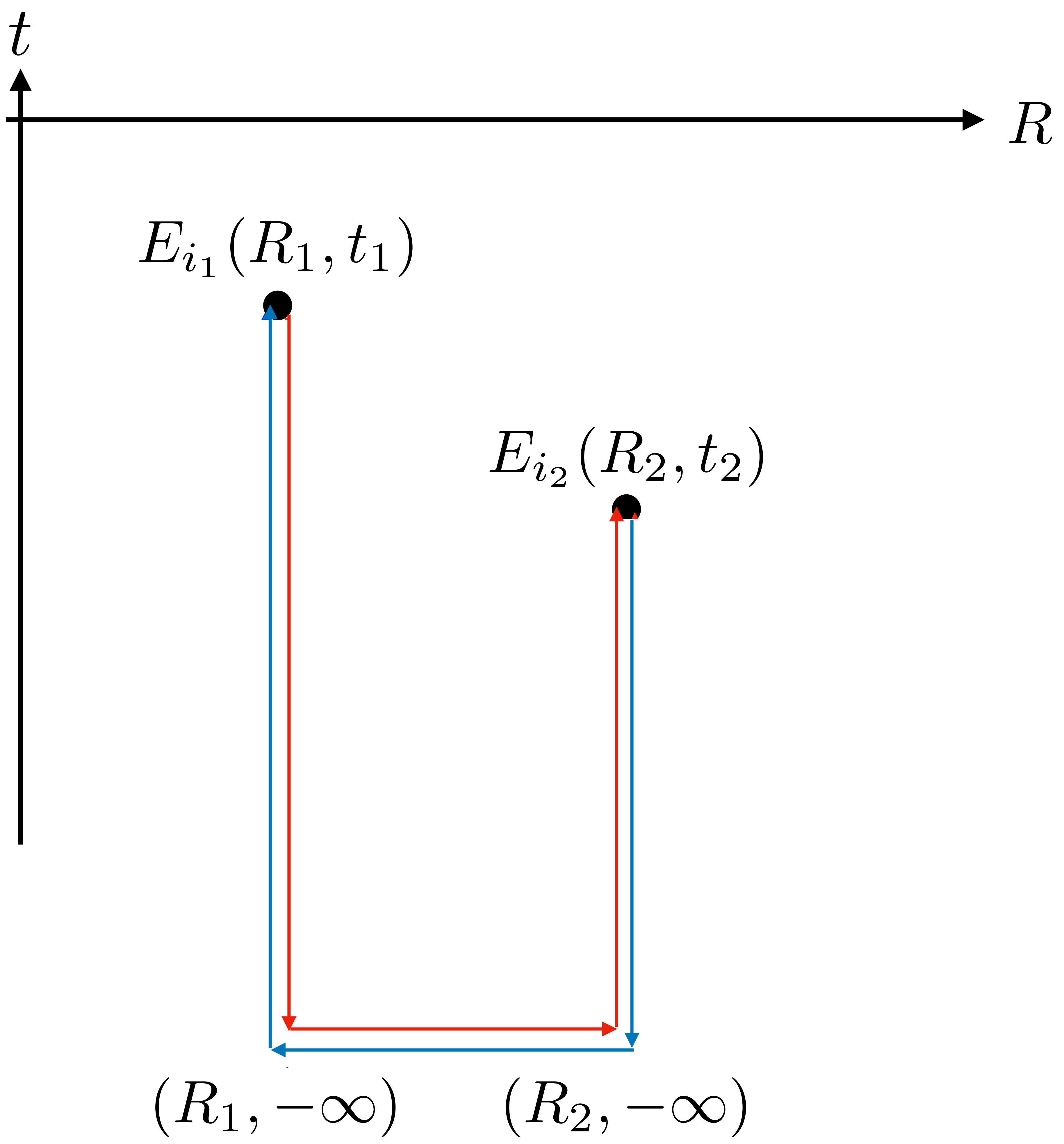

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color singlet

medium-induced
transition

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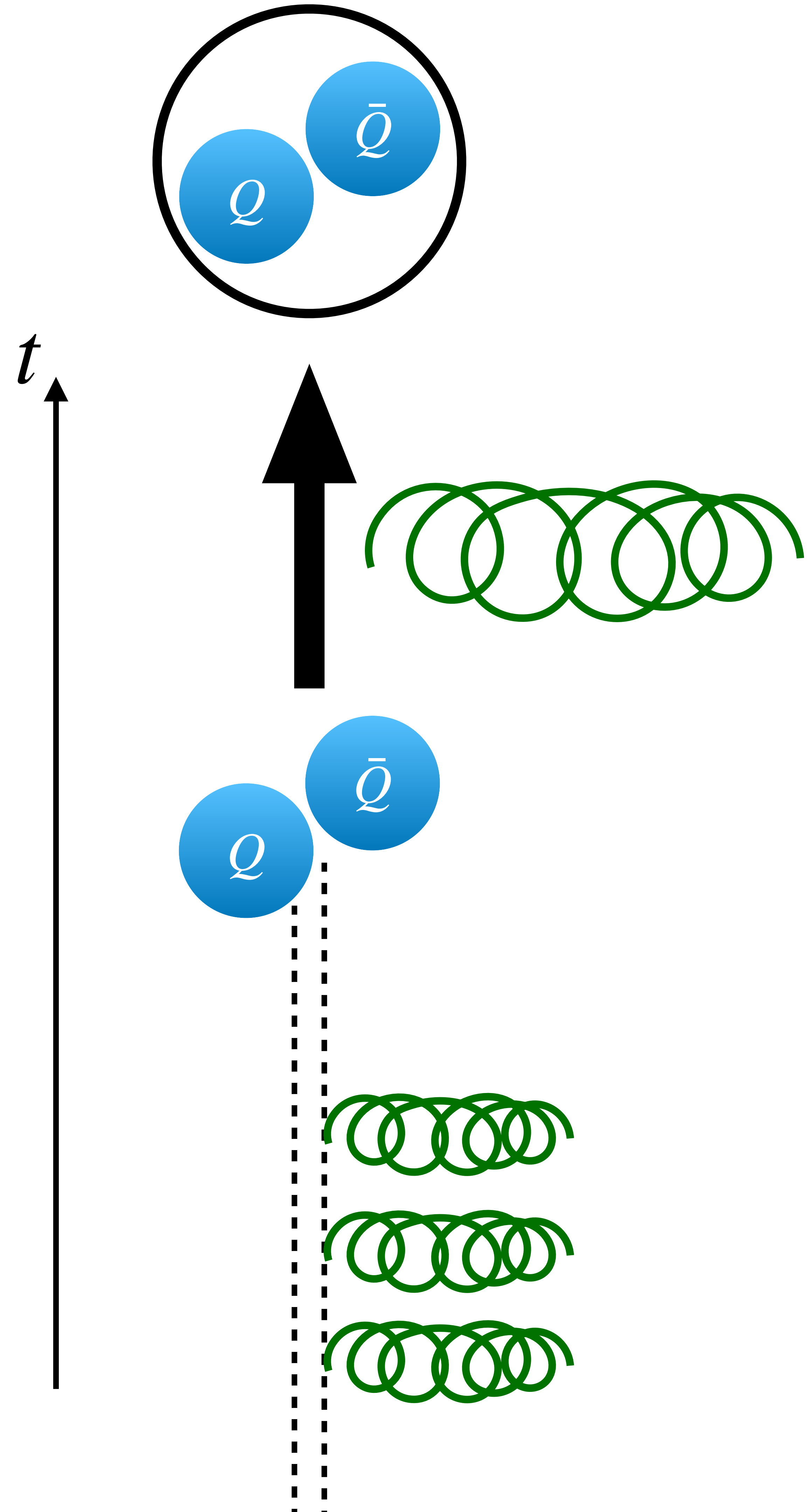

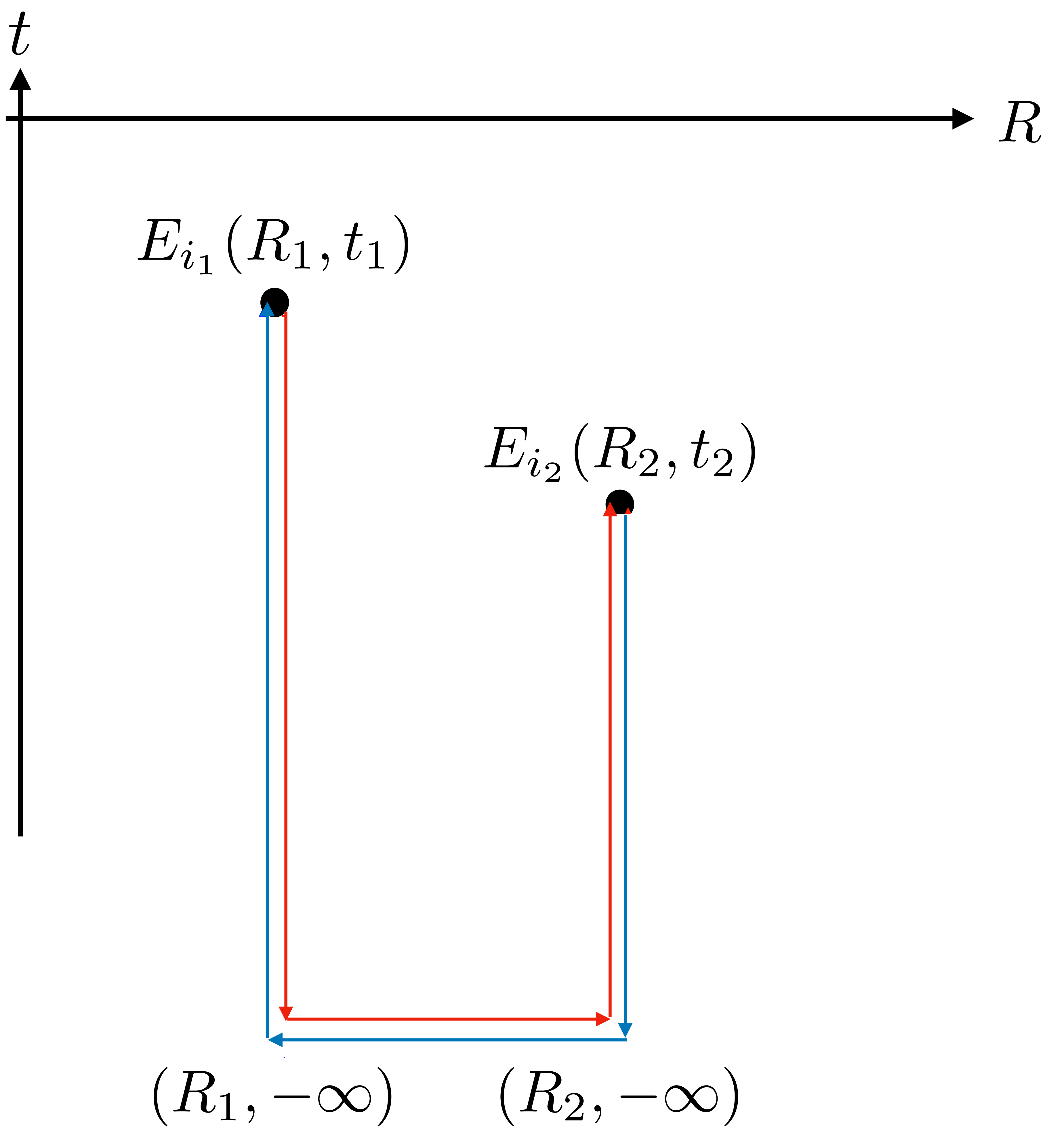

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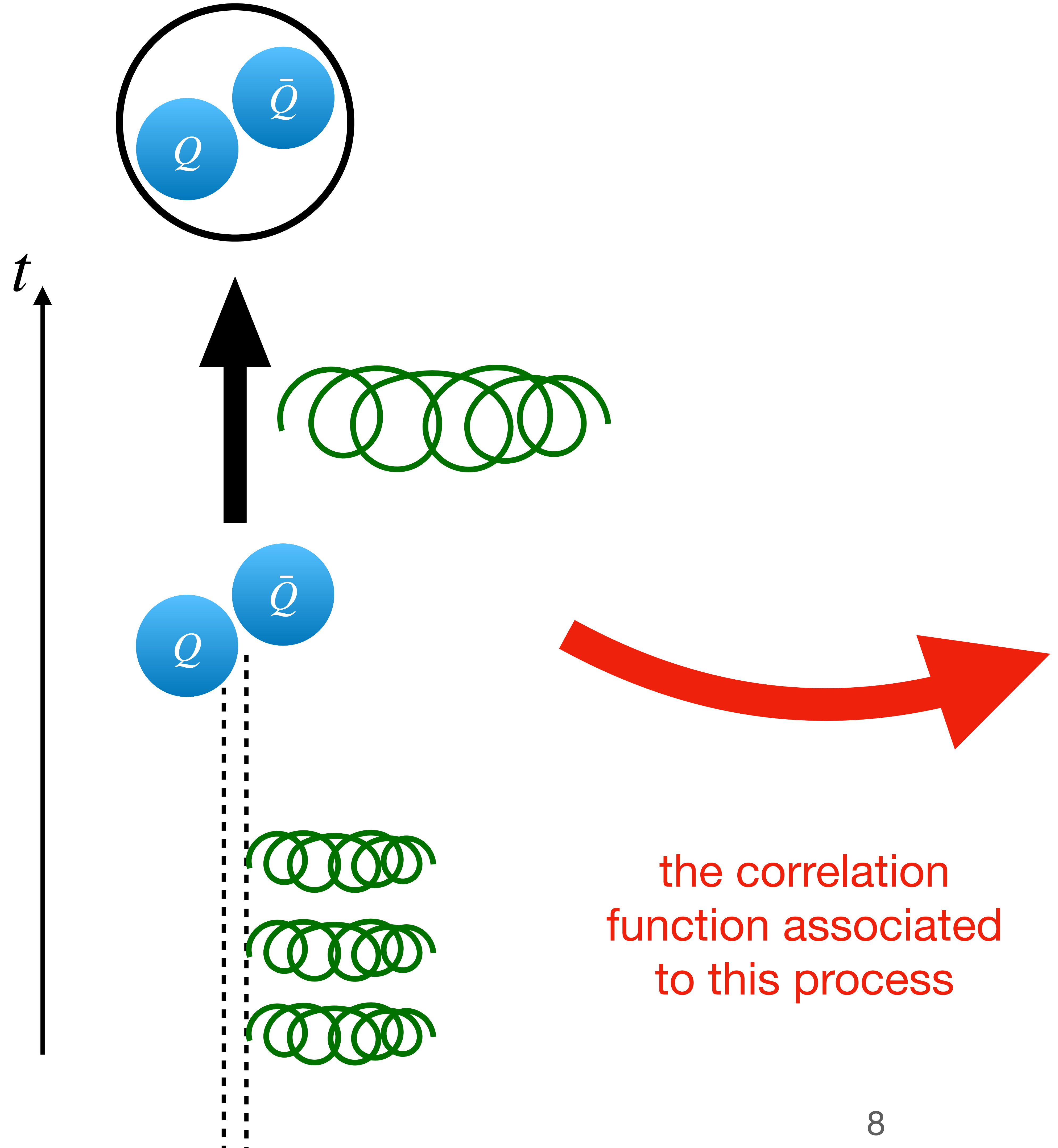

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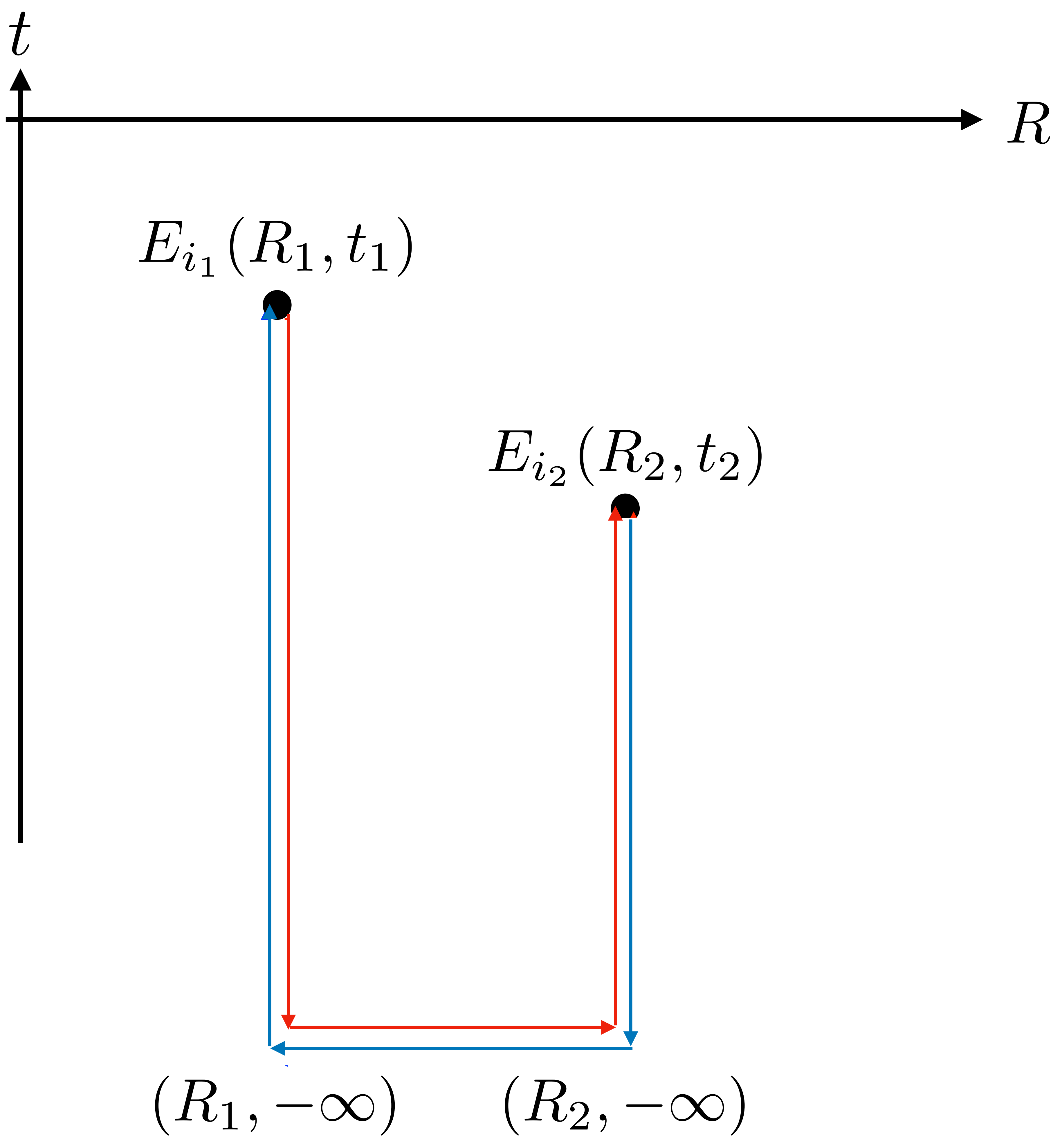

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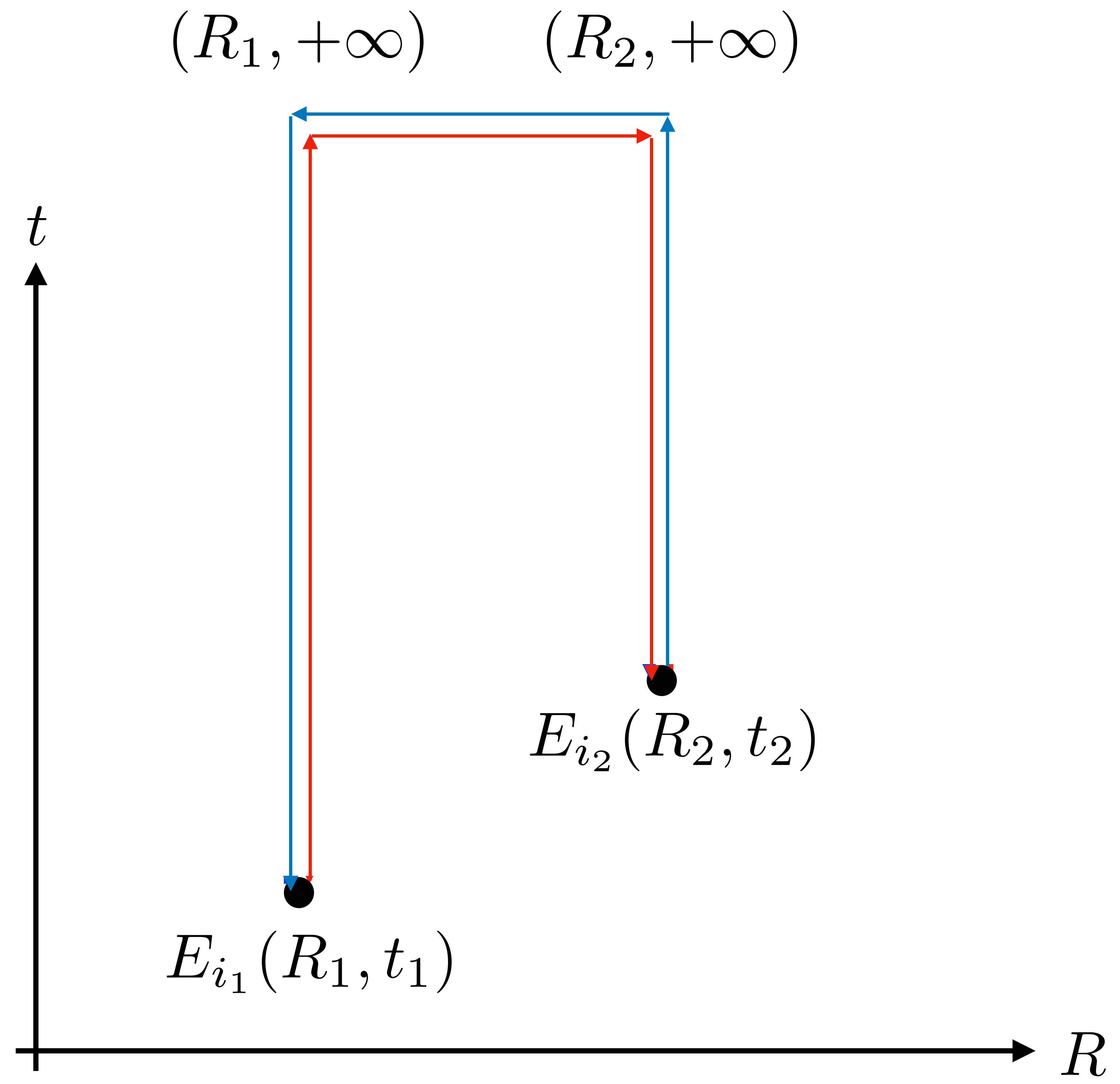

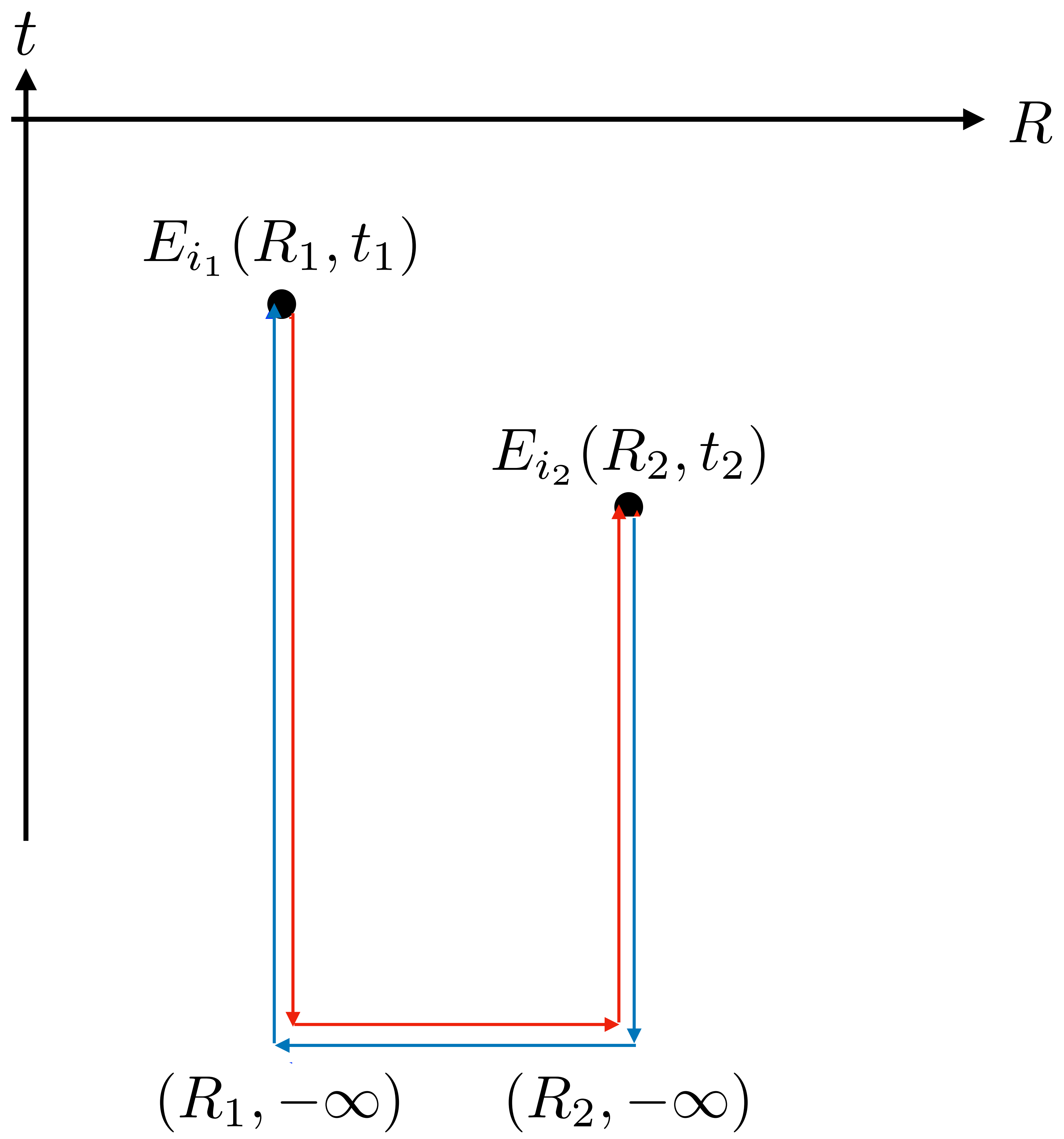

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Why are these correlators interesting?

These determine the dissociation and formation rates of quarkonia (in the quantum optical limit):

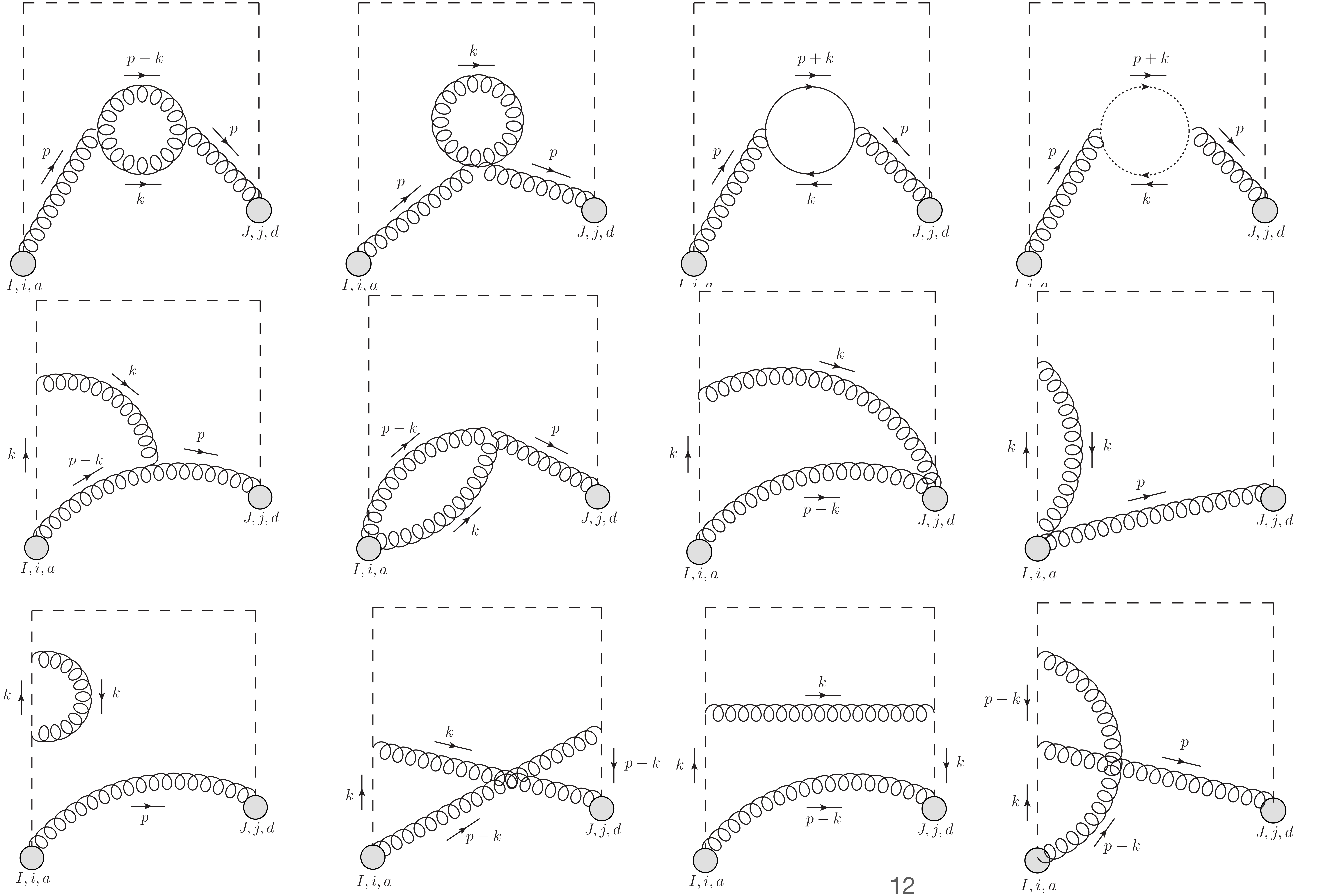
$$\Gamma^{\rm diss} \propto \int \frac{d^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2 [g_E^{++}]^{>}_{ii}\left(q^0 = E_{\mathscr{B}} - \frac{\mathbf{p}_{\rm rel}^2}{M}, \mathbf{q}\right),$$

$$\Gamma^{\rm form} \propto \int \frac{d^3\mathbf{p}_{\rm cm}}{(2\pi)^3}\frac{d^3\mathbf{p}_{\rm rel}}{(2\pi)^3}\frac{d^3\mathbf{q}}{(2\pi)^3} \,|\langle \psi_{\mathscr{B}}|\mathbf{r}|\Psi_{\mathbf{p}_{\rm rel}}\rangle|^2 [g_E^{--}]^{>}_{ii}\left(q^0 = \frac{\mathbf{p}_{\rm rel}^2}{M} - E_{\mathscr{B}}, \mathbf{q}\right)$$
$$\times f_{\mathscr{S}}(\mathbf{x}, \mathbf{p}_{\rm cm}, \mathbf{r} = 0, \mathbf{p}_{\rm rel}, t)\,.$$

So, let’s calculate

T. Binder, K. Mukaida, BS and X. Yao, hep-ph/2107.03945

Weakly coupled calculation in QCD



The real-time calculation proceeds by evaluating these diagrams (+ some permutations of them) on the Schwinger-Keldysh contour

The spectral function at NLO

It is simplest to write the integrated spectral function:

$$\varrho_E^{++}(p_0) = \frac{1}{2}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\delta^{ad}\delta_{ij}\left[\rho_E^{++}\right]_{ji}^{da}(p_0,\mathbf{p}) \,.$$

We found

$$g^2\varrho_E^{++}(p_0) = \frac{g^2(N_c^2-1)p_0^3}{(2\pi)^3}\left\{4\pi^2 + g^2\left[\left(\frac{11}{12}N_c - \frac{1}{3}N_f\right)\ln\left(\frac{\mu^2}{4p_0^2}\right) + \left(\frac{149}{36}+\frac{\pi^2}{3}\right)N_c - \frac{10}{9}N_f + F\left(\frac{p_0}{T}\right)\right]\right\}$$

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The spectral function at NLO
and a comparison with its heavy quark counterpart

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and the heavy quark counterpart is, with the same T-dependent function $F(p_0/T)$,

Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

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But they look so similar…

Heavy quark and quarkonia correlators

a small, yet consequential difference

The heavy quark diffusion coefficient can be defined from the real-time correlator J. Casalderrey-Solana and D. Teaney, hep-ph/0605199; see also A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

$$\left\langle \mathrm{Tr}_{\mathrm{color}} \left[U(-\infty, t) E_i(t) U(t,0) E_i(0) U(0, -\infty) \right] \right\rangle_T ,$$

whereas for quarkonia the relevant quantity is

$$T_F \left\langle E_i^a(t) \mathscr{W}^{ab}(t,0) E_i^b(0) \right\rangle_T .$$

Heavy quark and quarkonia correlators

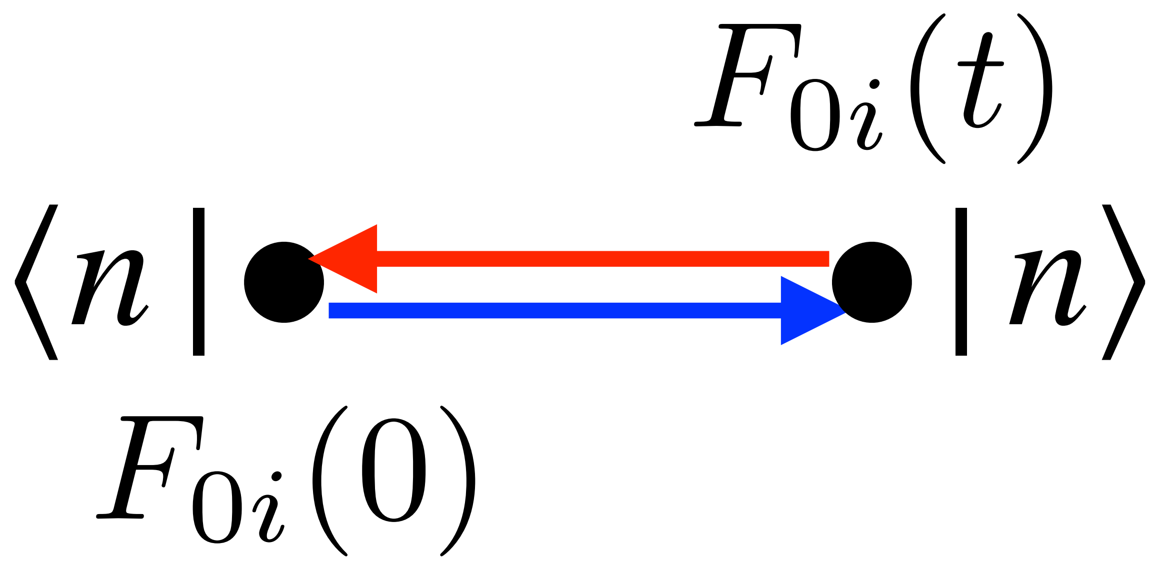
a small, yet consequential difference

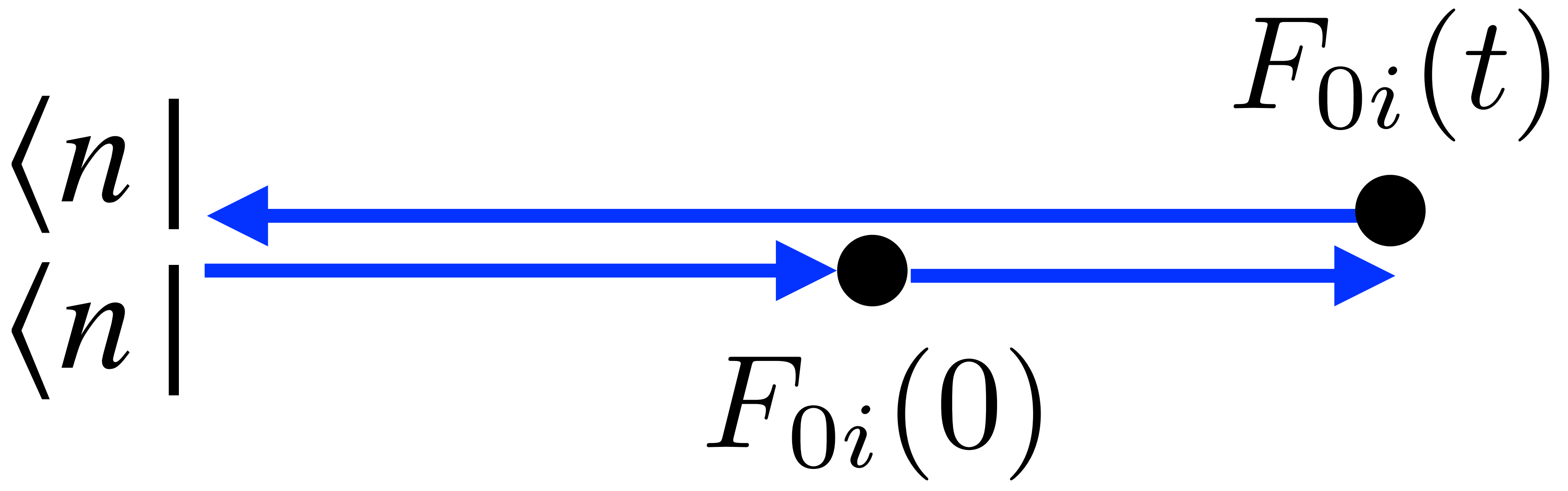
A. M. Eller, J. Ghiglieri and G. D. Moore, hep-ph/1903.08064

What we just found, and had been noticed even earlier by Eller, Ghiglieri and Moore, is simply stating that:

They compared M. Eidemuller and M. Jamin, hep-ph/9709419 with
Y. Burnier, M. Laine, J. Langelage and L. Mether, hep-ph/1006.0867

$$T_F \left\langle E_i^a(t)\mathscr{W}^{ab}(t,0)E_i^b(0)\right\rangle_T \neq \left\langle \mathrm{Tr}_{\mathrm{color}}\left[U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\right]\right\rangle_T$$





An axial gauge puzzle

an apparent (but not actual) inconsistency

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BS and X. Yao, hep-ph/2205.04477

An axial gauge puzzle

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We verified that this difference between the correlators is gauge invariant using an interpolating gauge condition:

$$G_M^a[A] = \frac{1}{\lambda} A_0^a(x) + \partial^\mu A_\mu^a(x)$$

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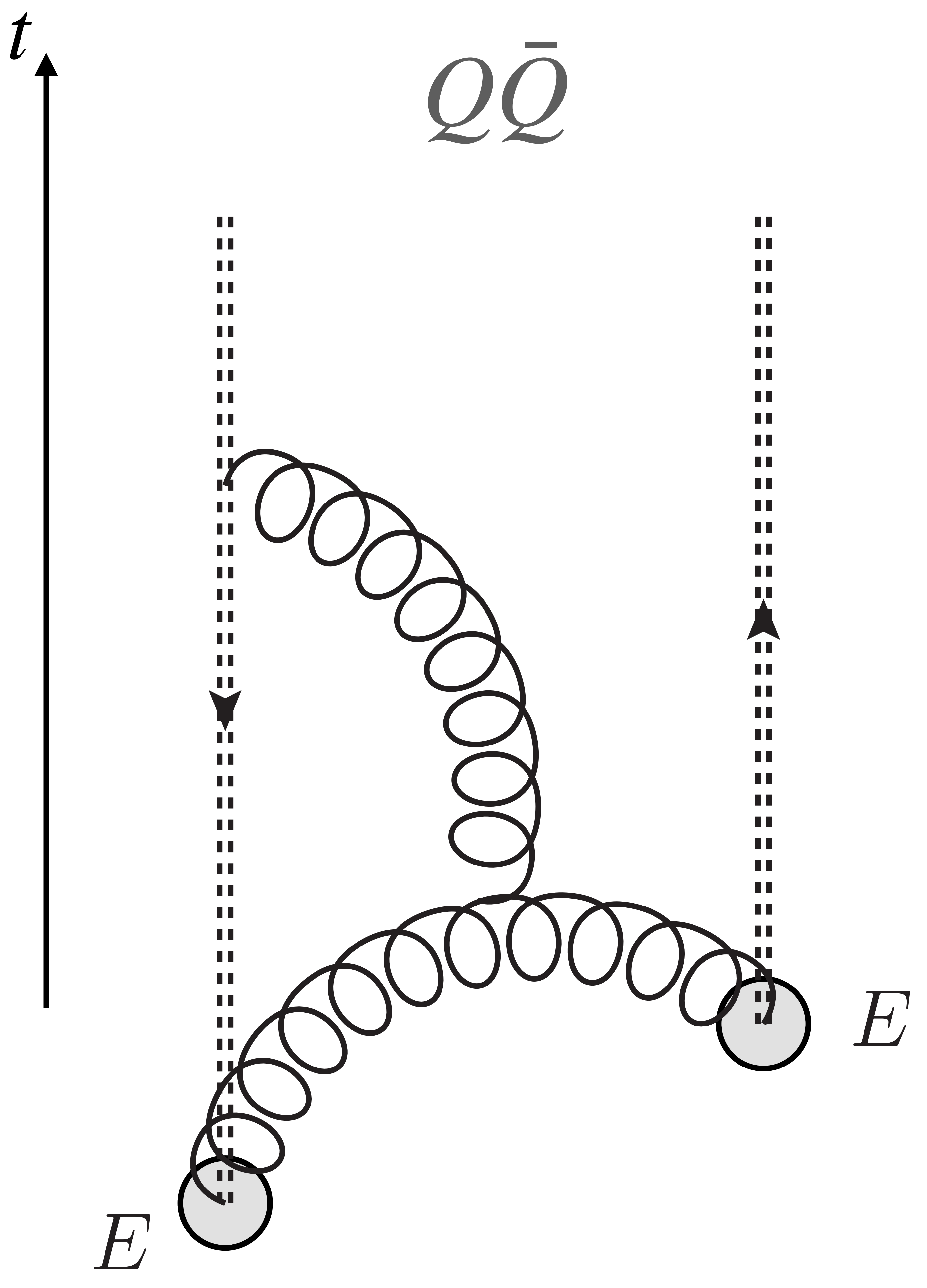
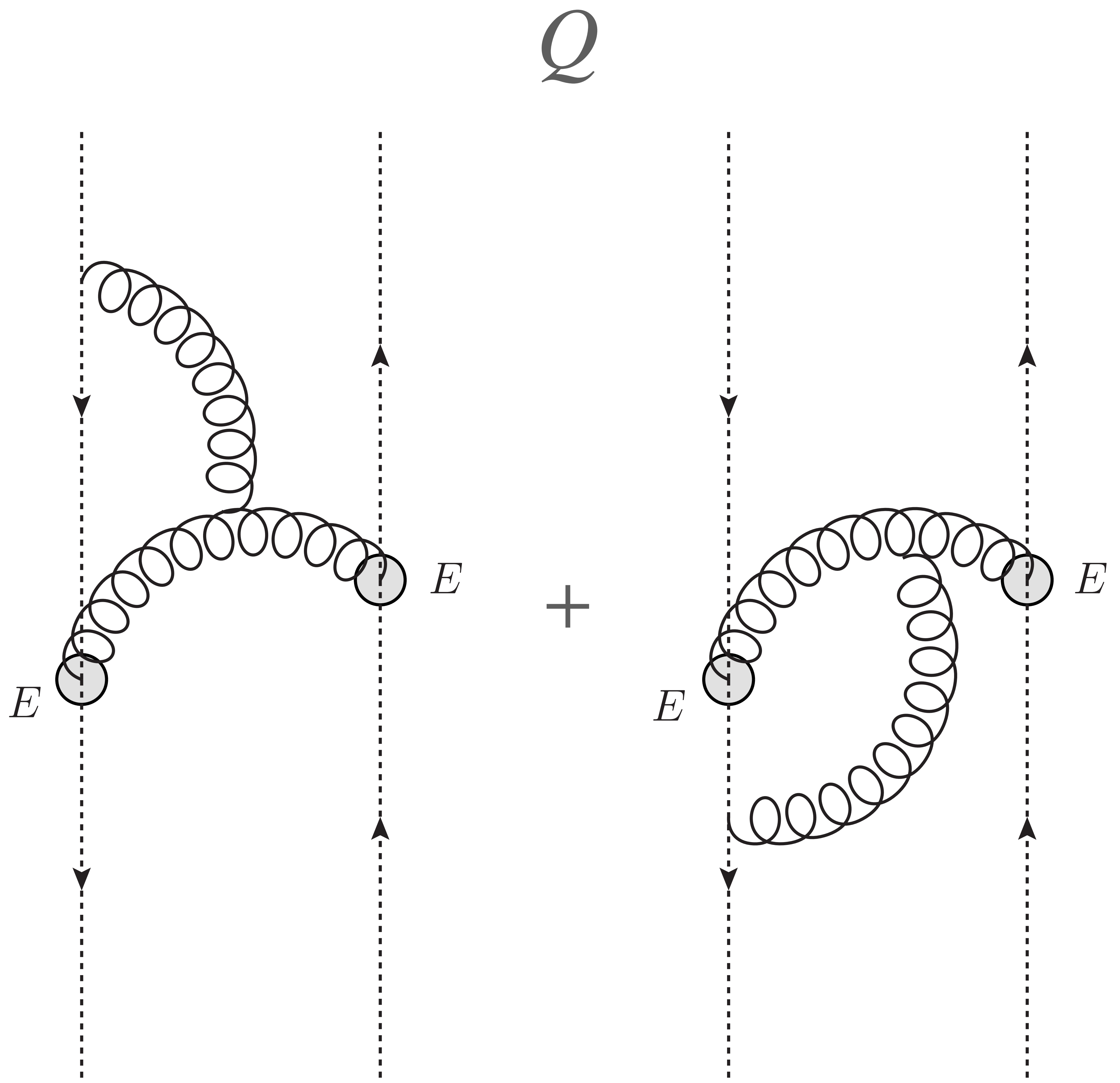
The difference in terms of diagrams

operator ordering is crucial!



Perturbatively, one can isolate the difference between the correlators to these diagrams.



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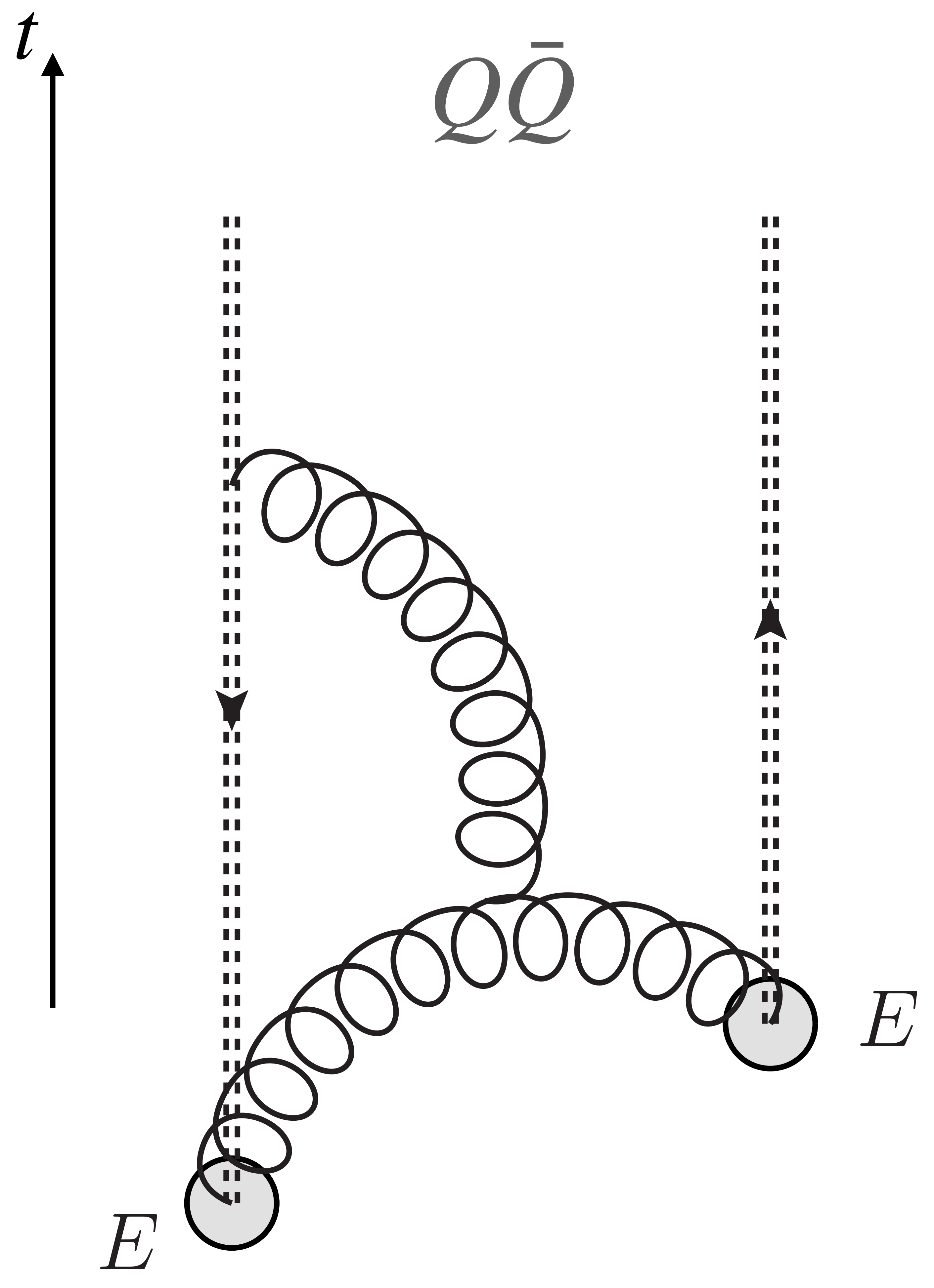

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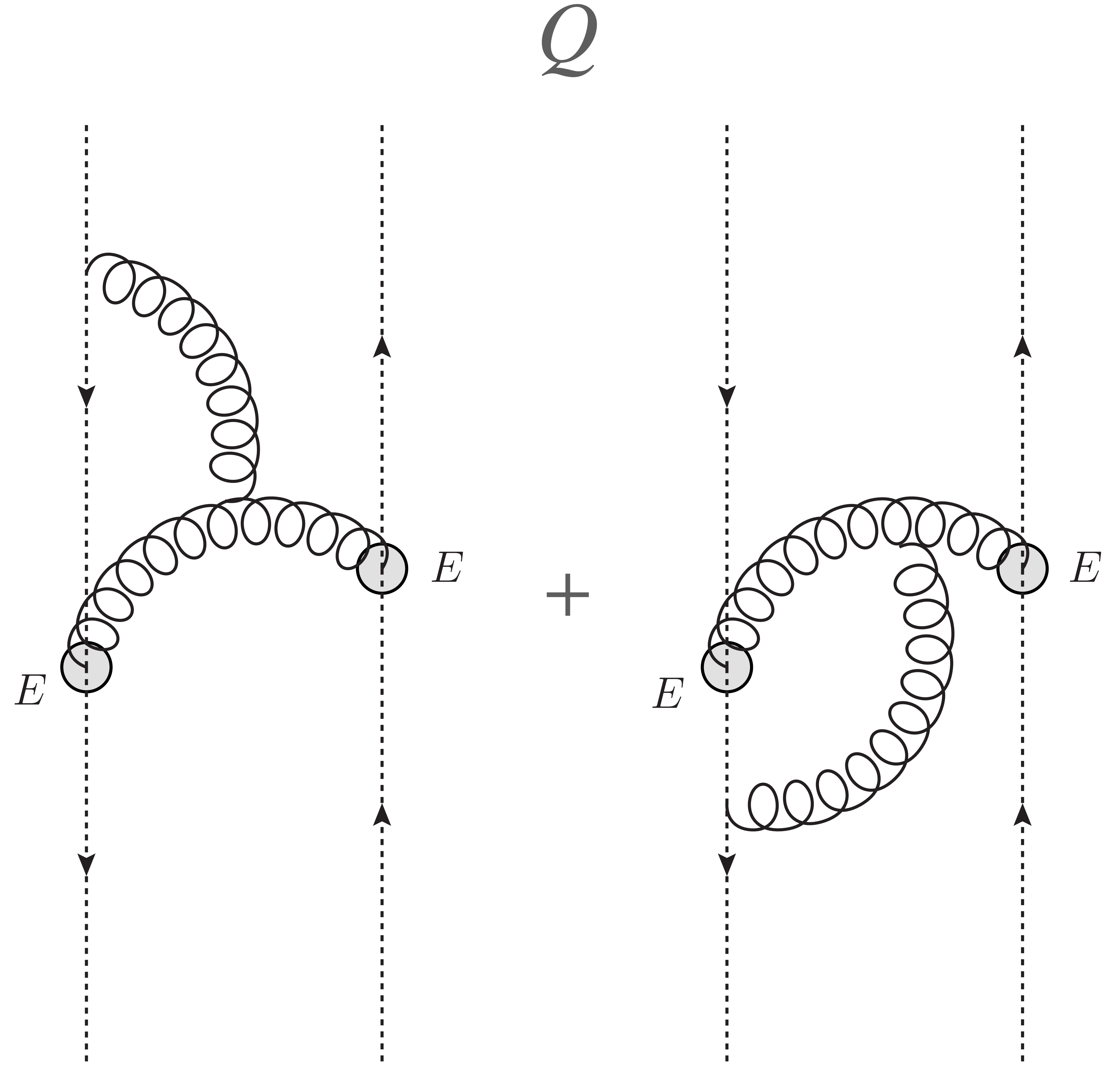
The difference is due to different operator orderings (different possible gluon insertions).



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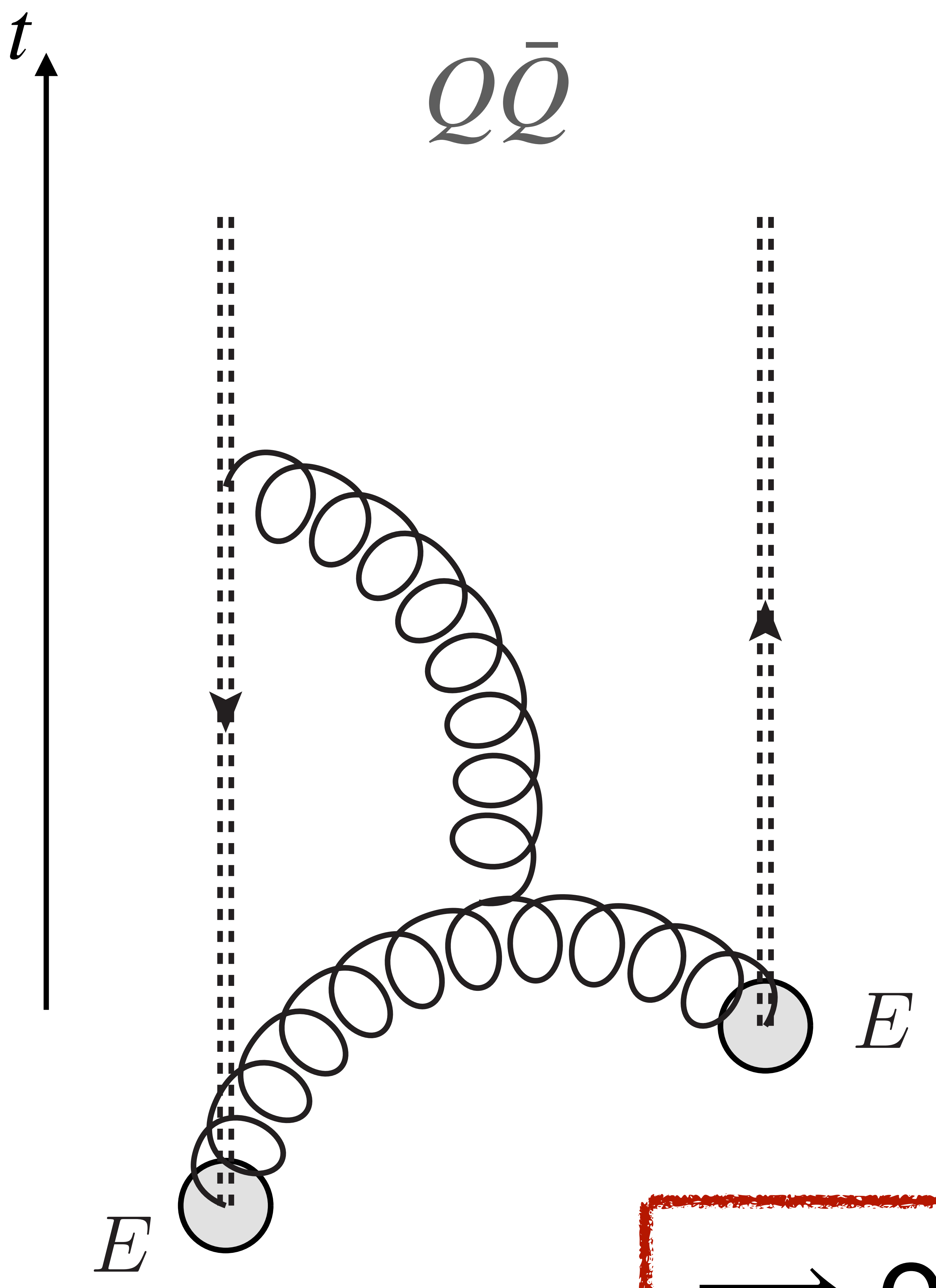

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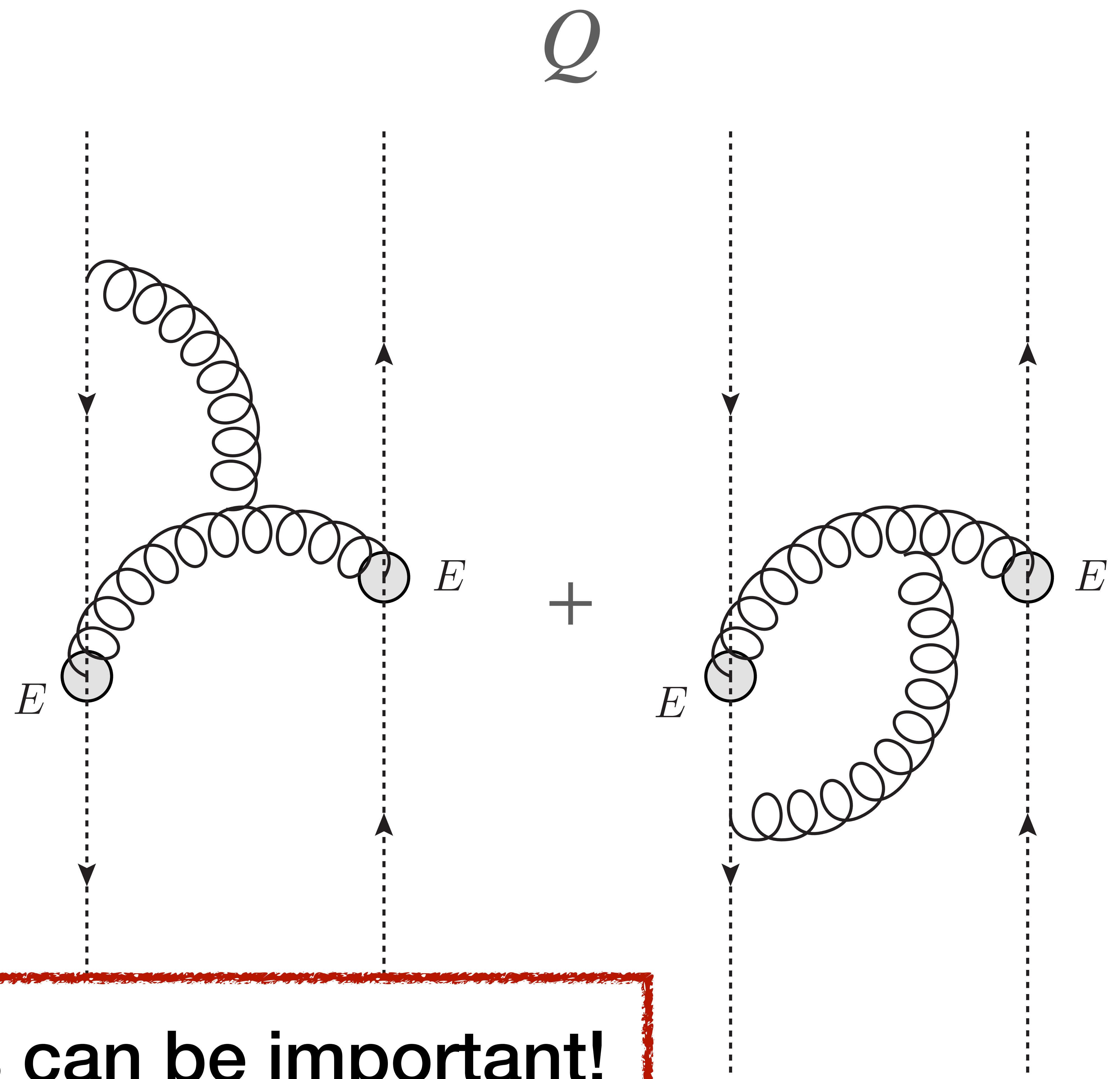

$\Longrightarrow$ Quantum color correlations can be important!

Can we calculate this difference non-perturbatively in QCD?

A Lattice QCD perspective

the imaginary time counterparts

- The heavy quark diffusion coefficient has been studied by evaluating the following correlation function (e.g., Altenkort et al. 2009.13553, Leino et al. 2212.10941):

$$G_{\text{fund}}(\tau) = -\frac{1}{3}\frac{\left\langle \text{ReTr}_c[U(\beta,\tau)\, gE_i(\tau)\, U(\tau,0)\, gE_i(0)]\right\rangle}{\left\langle \text{ReTr}_c[U(\beta,0)]\right\rangle} .$$

- The heavy quark diffusion coefficient is extracted by reconstructing the corresponding spectral function (Caron-Huot et al. 0901.1195):

$$G_{\text{fund}}(\tau) = \int_0^{+\infty} \frac{d\omega}{2\pi} \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)} \rho_{\text{fund}}(\omega) , \quad \kappa_{\text{fund}} = \lim_{\omega\to 0} \frac{T}{\omega} \rho_{\text{fund}}(\omega) .$$

- Main difficulty: it is a noisy observable to extract.

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- The [...] corre[...]

G_{f} [...] $)$.

However, the quarkonia correlator counterpart in imaginary time has received much less attention:

$$G_{\mathrm{adj}}(\tau) = \frac{T_F g^2}{3N_c} \left\langle E_i^a(\tau) W^{ab}(\tau,0) E_i^b(0) \right\rangle .$$

[ongoing work with P. Petreczky and X. Yao]

- Main difficulty: it is a noisy observable to extract.

So, we understand the weakly coupled limit in QCD, and are making progress on the lattice QCD formulation.

What about other tools at strong coupling?

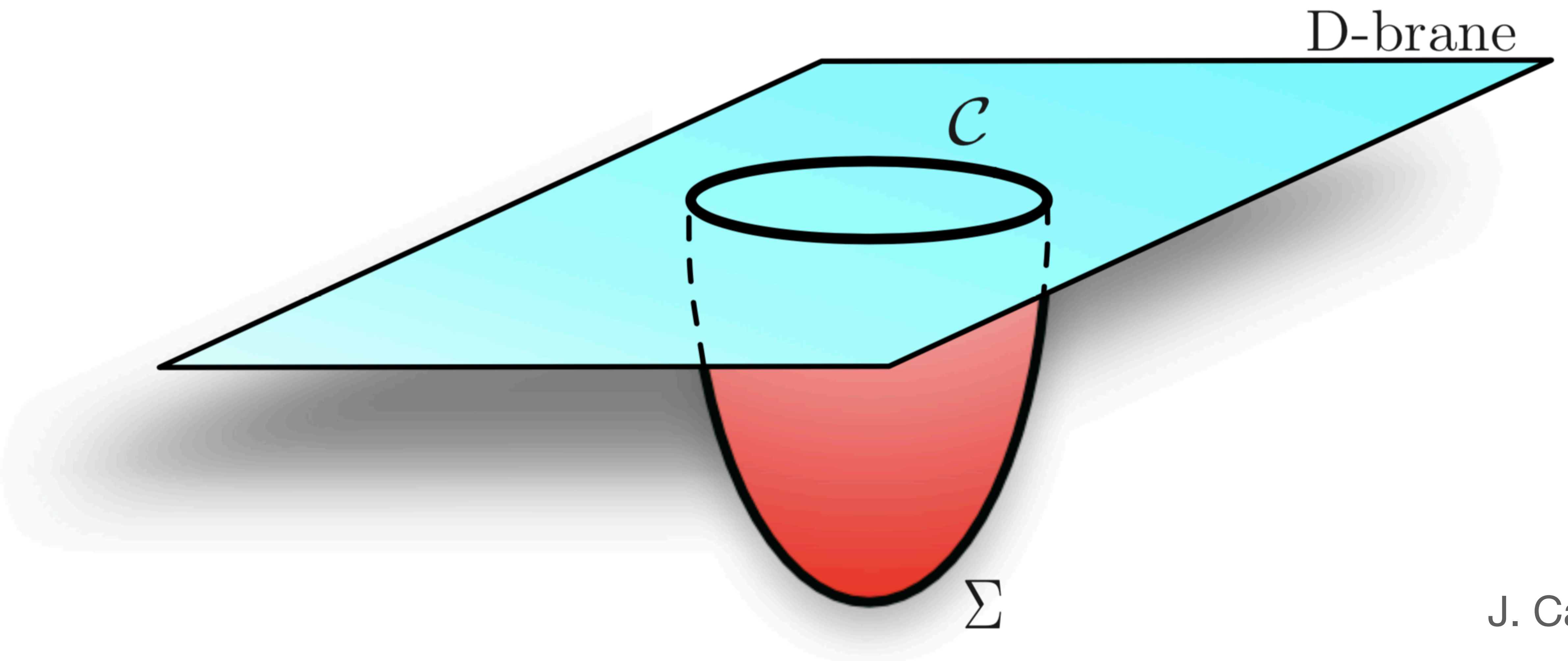
[**] J. Maldacena, hep-th/9711200

Wilson loops in AdS/CFT

setup

- The holographic duality provides a way to formulate the calculation of analogous correlators in strongly coupled theories. [**]
    - Wilson loops can be evaluated by solving classical equations of motion:

$$\langle W[\mathscr{C} = \partial\Sigma]\rangle_T = e^{iS_{\mathrm{NG}}[\Sigma]}$$



J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal and U. A. Wiedemann, hep-ph/1101.0618

Strongly coupled calculation in $\mathcal{N} = 4$ SYM

setup

- Field strength insertions along a Wilson loop can be generated by taking variations of the path $\mathcal{C}$:

$$\frac{\delta}{\delta f^{\mu}(s_2)} \frac{\delta}{\delta f^{\nu}(s_1)} W[\mathcal{C}_f] \bigg|_{f=0} = (ig)^2 \mathrm{Tr}_{\mathrm{color}} \Big[U_{[1,s_2]} F_{\mu\rho}(\gamma(s_2)) \dot{\gamma}^{\rho}(s_2) U_{[s_2,s_1]} F_{\nu\sigma}(\gamma(s_1)) \dot{\gamma}^{\sigma}(s_1) U_{[s_1,0]} \Big]$$

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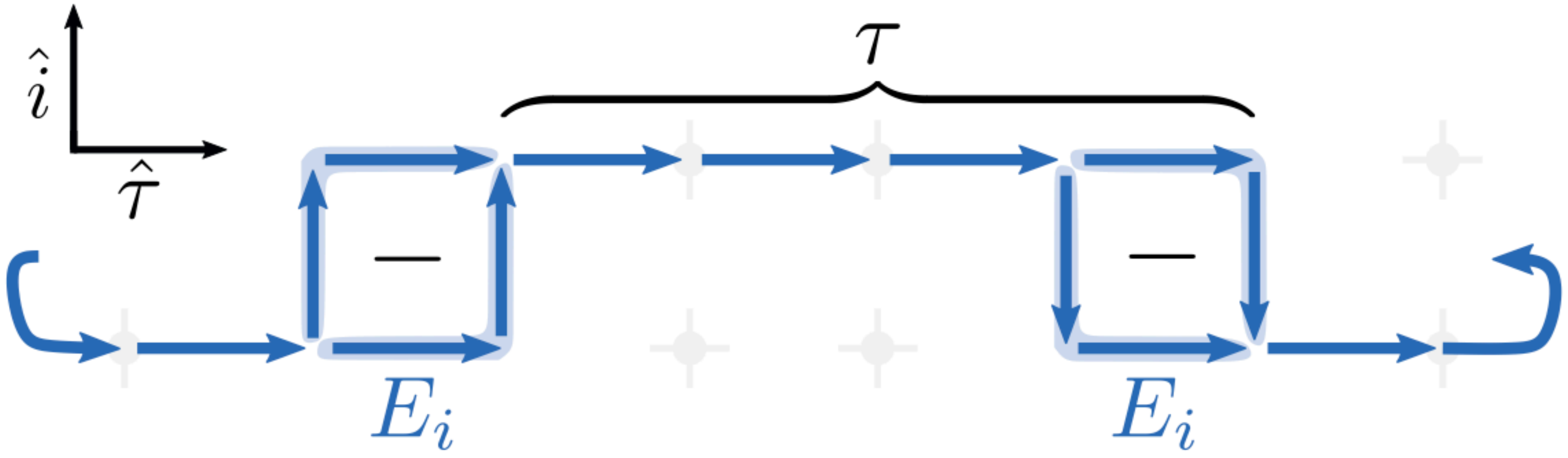
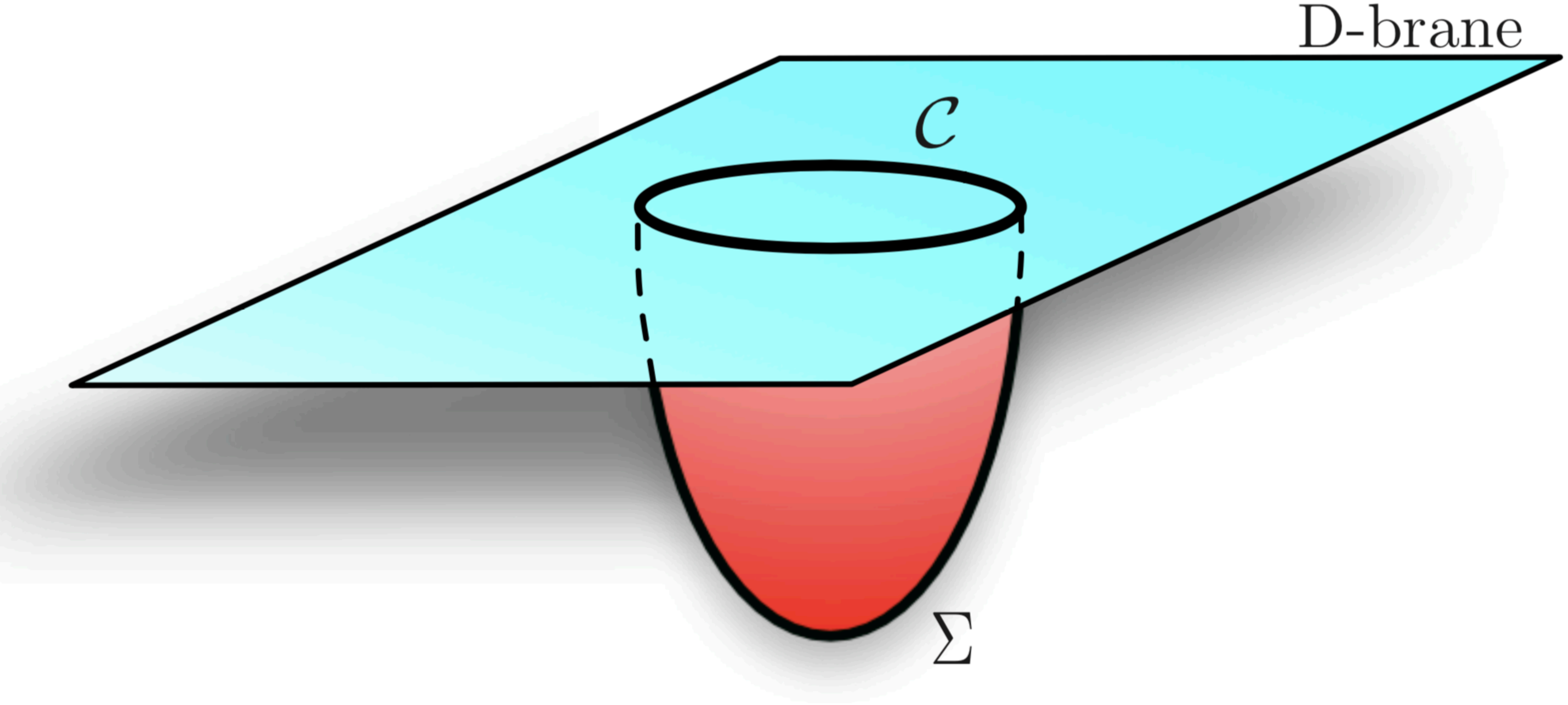
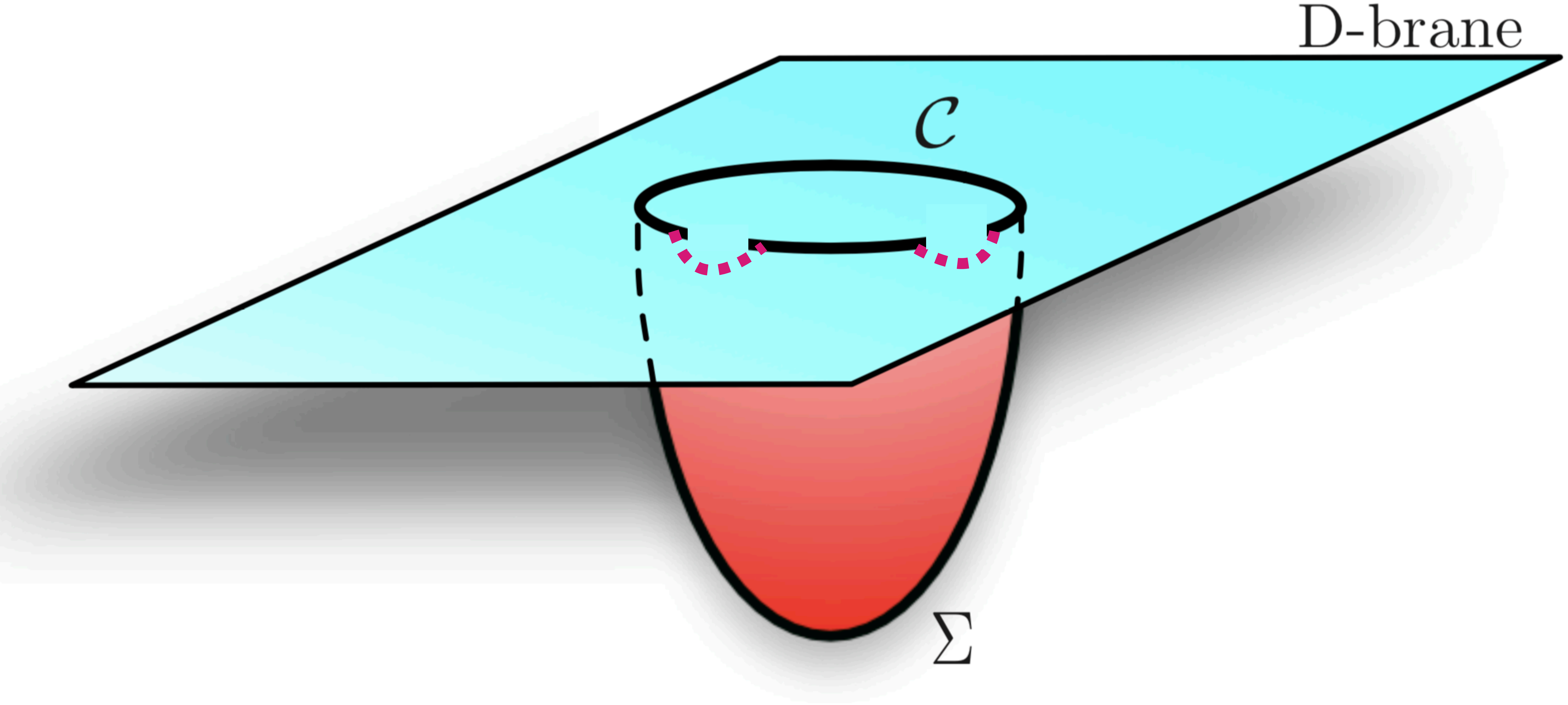
- Same in spirit as the lattice calculation of the heavy quark diffusion coefficient:




D-brane
$\mathcal{C}$
Σ


D-brane
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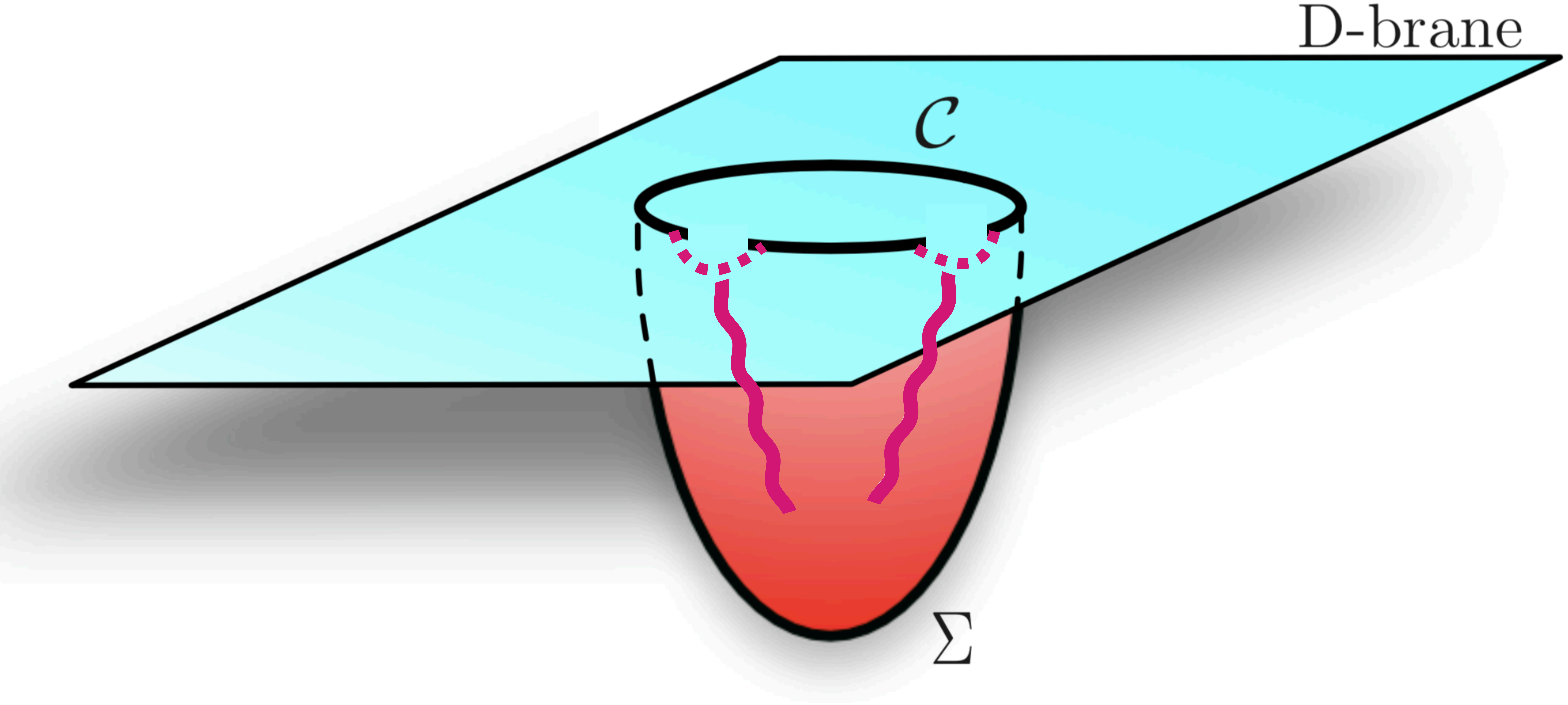
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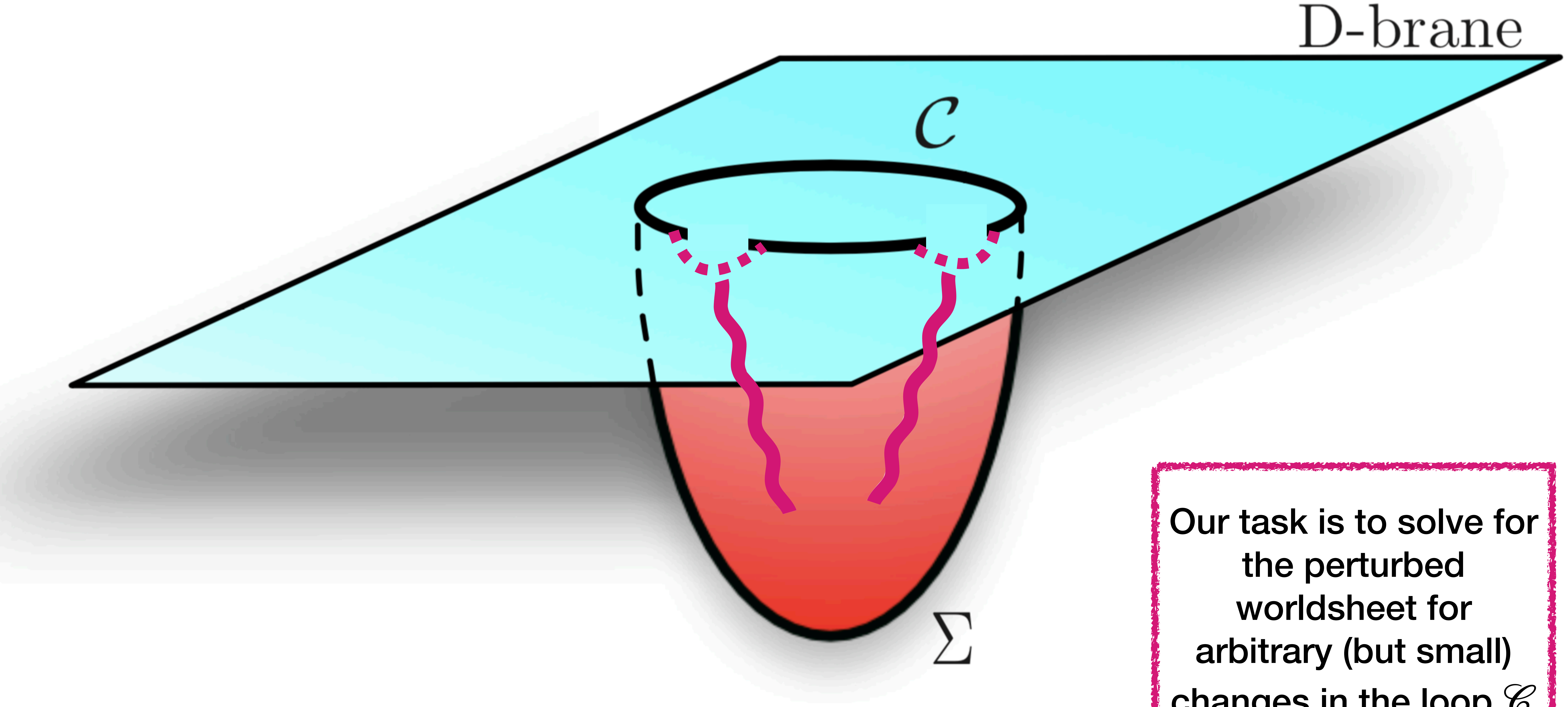


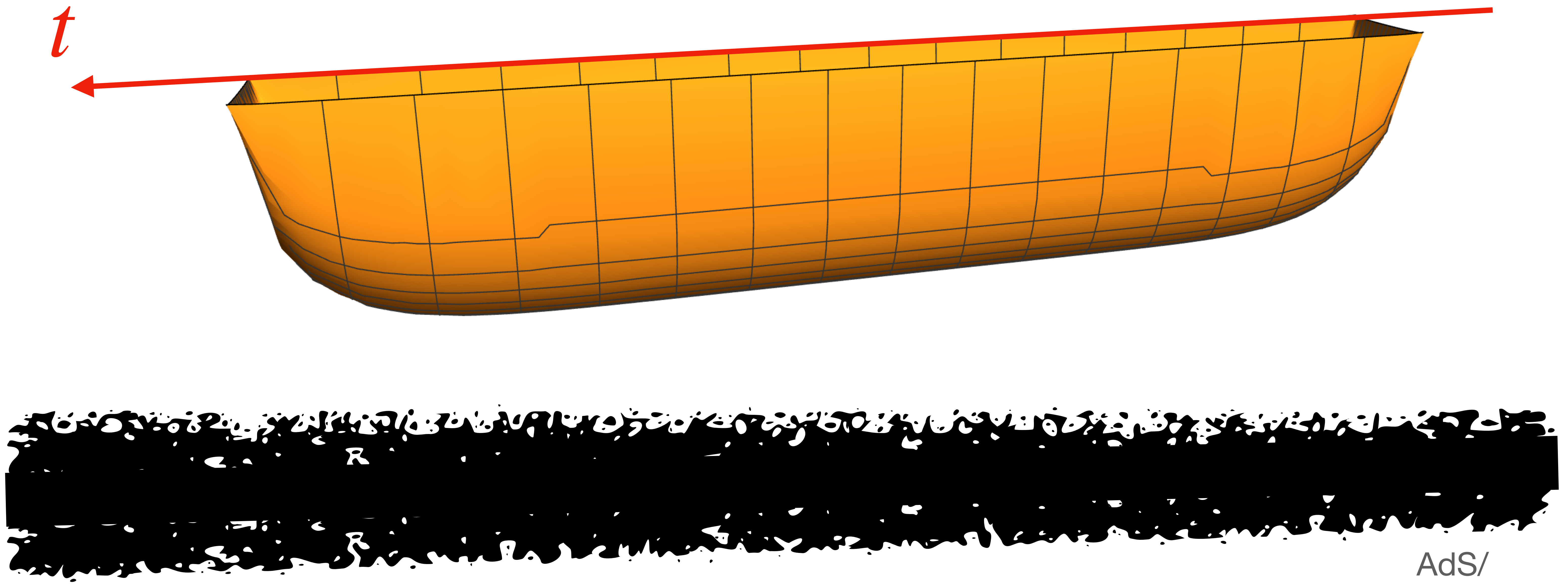
Our task is to solve for the perturbed worldsheet for arbitrary (but small) changes in the loop $\mathscr{C}$

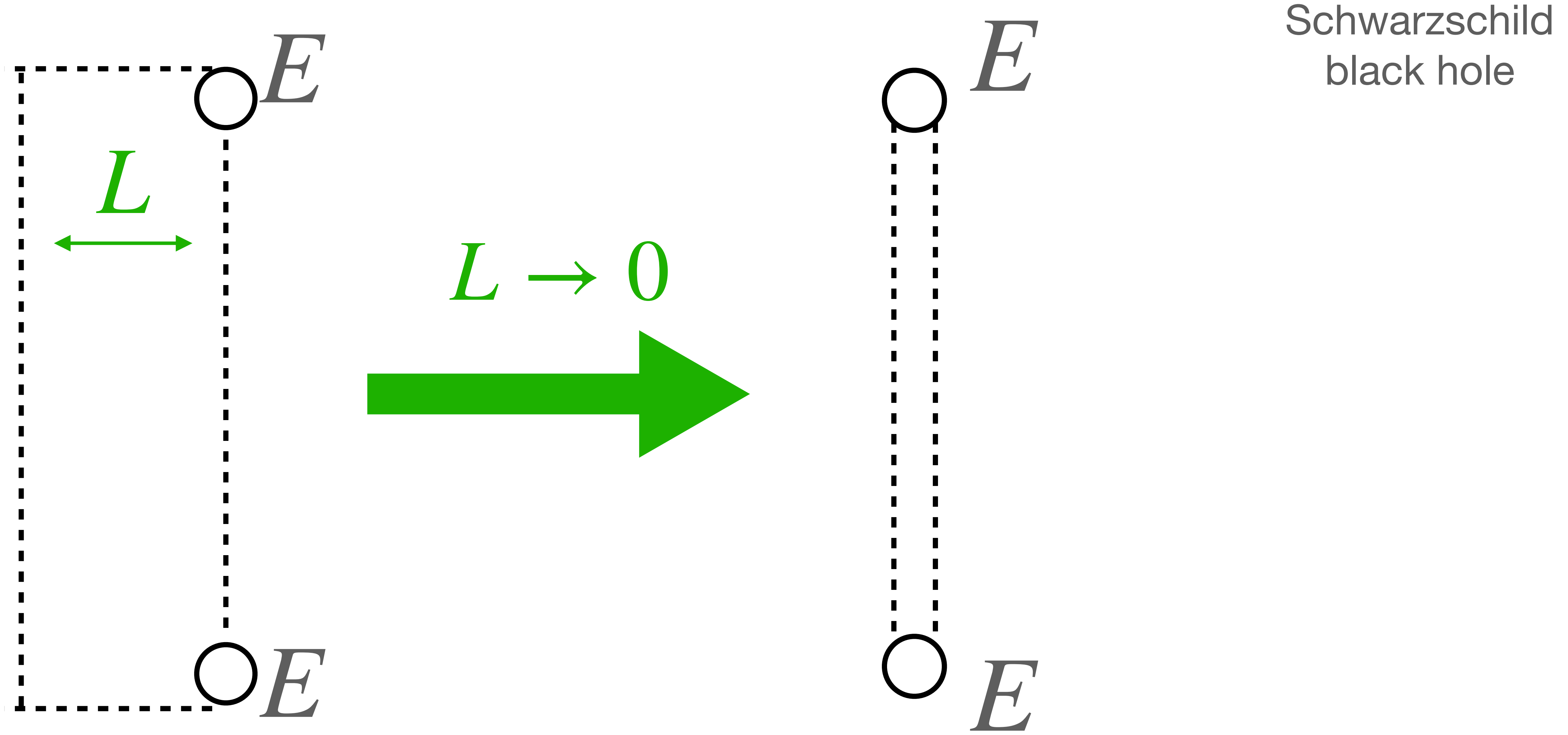
Quarkonia correlator in AdS/CFT

Quarkonium transport in AdS/CFT

Steps of the calculation:

1. Find the appropriate background solution



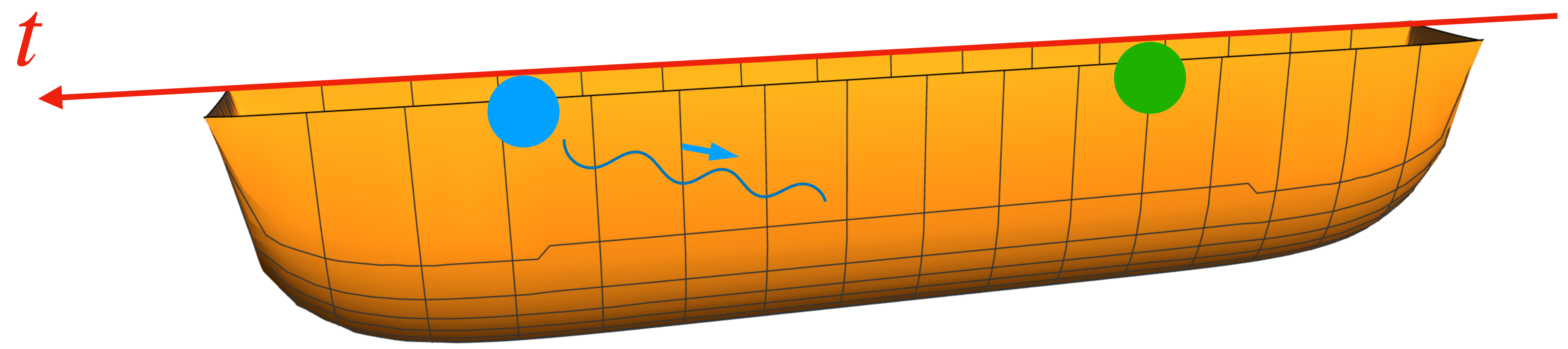


Quarkonium transport in AdS/CFT

Steps of the calculation:

1. Find the appropriate background solution

2. Introduce perturbations



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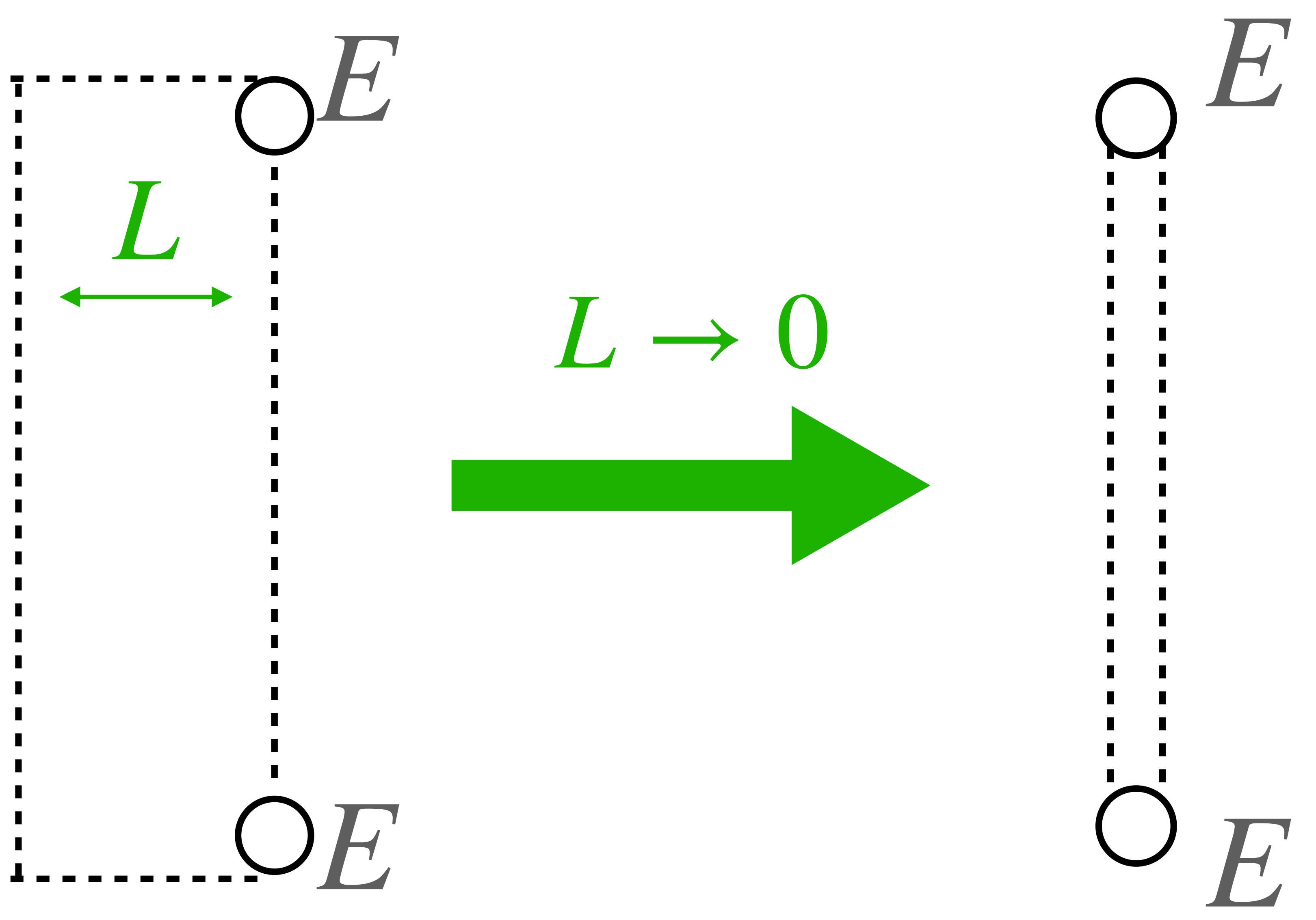
AdS/ Schwarzschild black hole



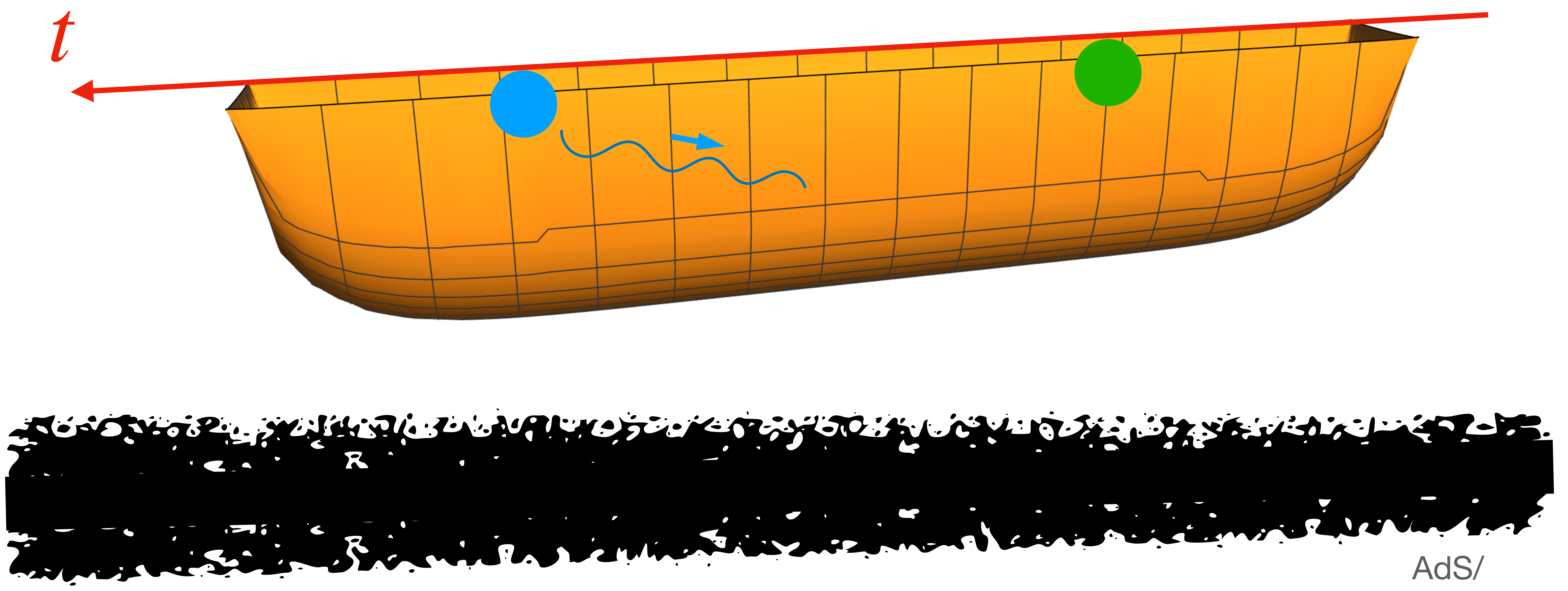
Quarkonium transport in AdS/CFT

Steps of the calculation:

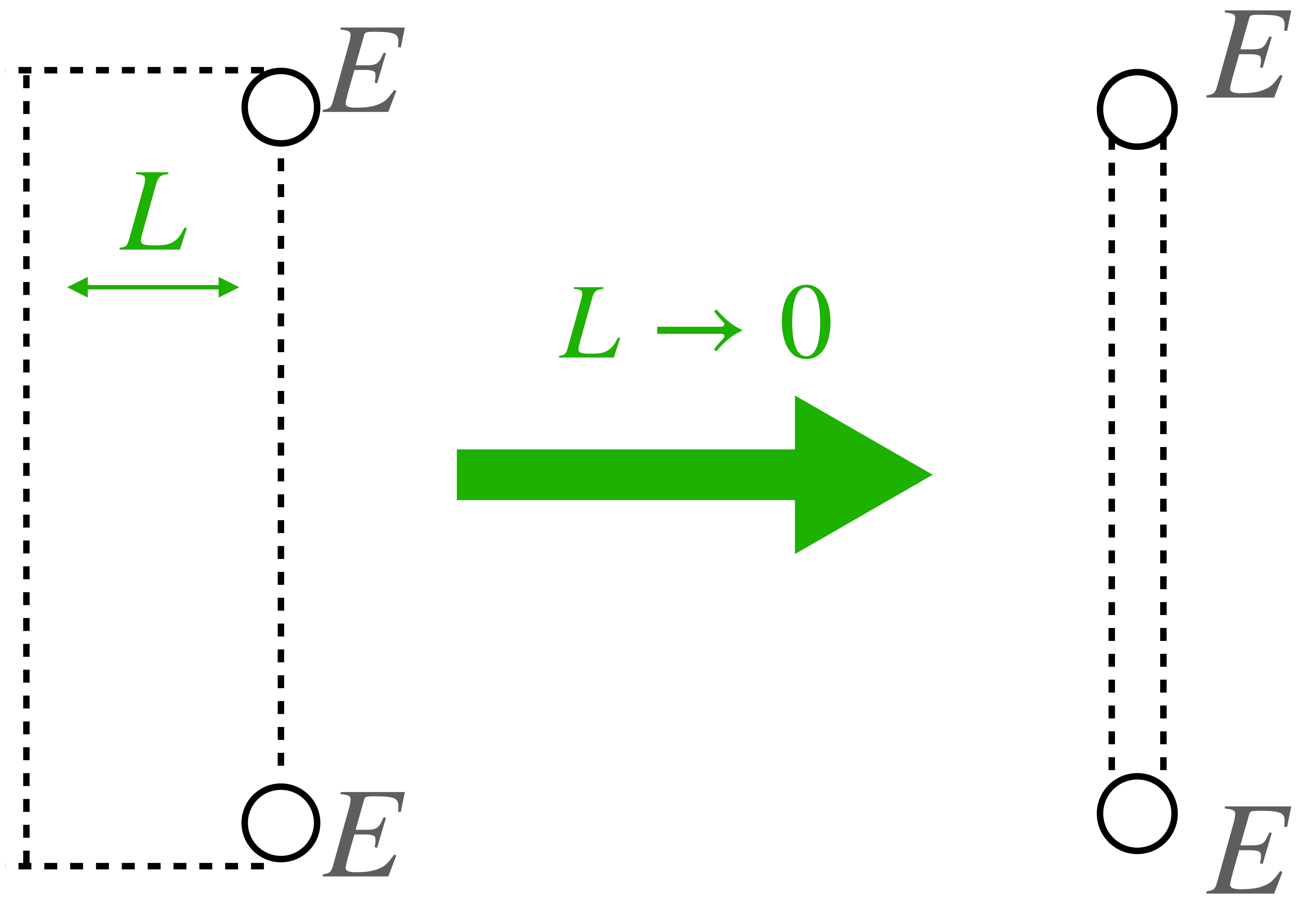
1. Find the appropriate background solution

2. Introduce perturbations

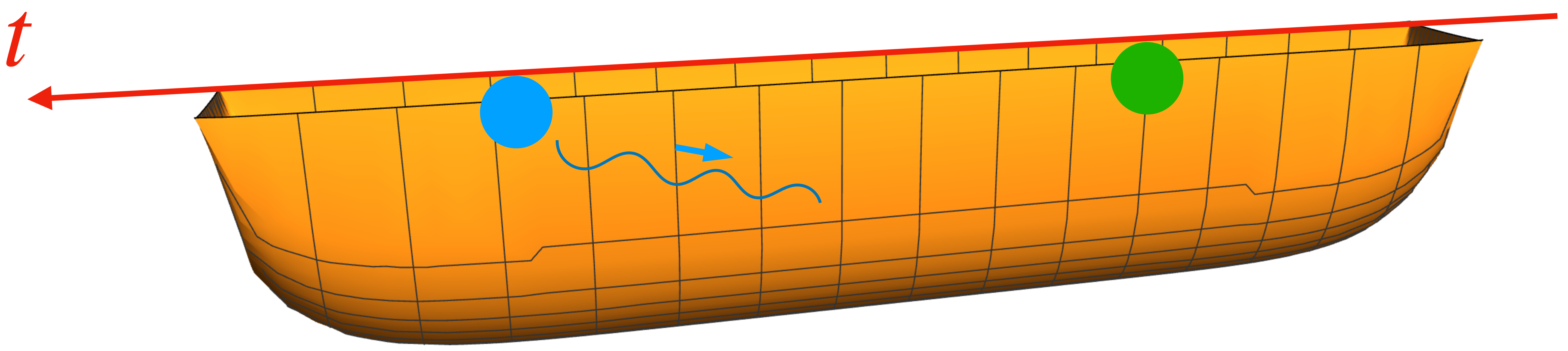
3. Evaluate the deformed Wilson loop and take derivatives



AdS/
Schwarzschild
black hole



Quarkonium transport in AdS/CFT



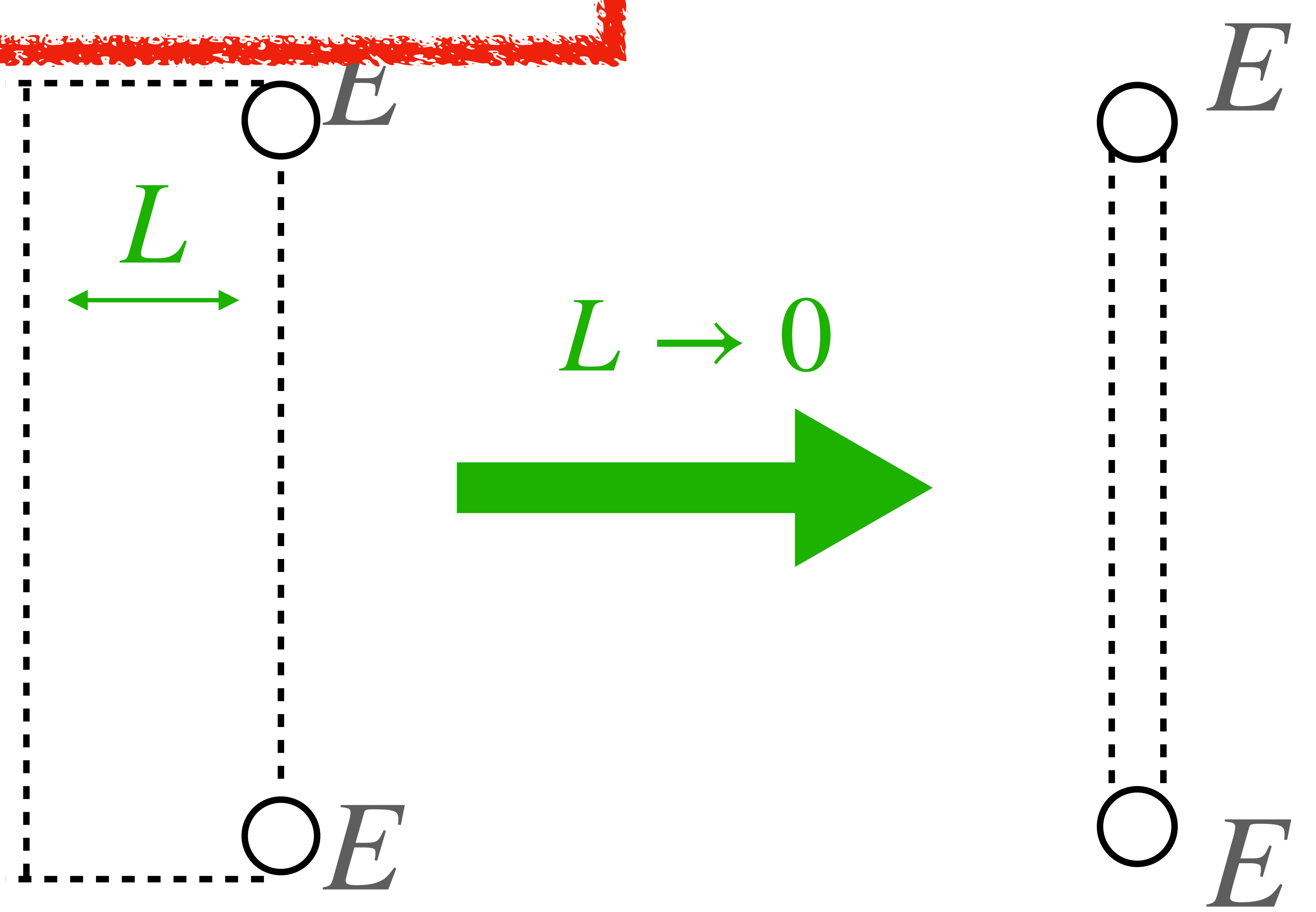
Steps of the calculation:

1. Find the appropriate background solution
2. Introduce perturbations
3. Evaluate the deformed Wilson loop and take derivatives

Ongoing calculation!

[redacted]

AdS/ Schwarzschild black hole



Summary and conclusions

- We have discussed how to calculate the chromoelectric correlators of the QGP that govern quarkonium transport

  A. at weak coupling in QCD

  B. on a discretized imaginary time lattice

  C. at strong coupling in $\mathcal{N} = 4$ SYM

- Next steps:
  - Generalize the calculations to include a boosted medium
  - Use them as input for quarkonia transport codes

- Stay tuned!

Summary and conclusions

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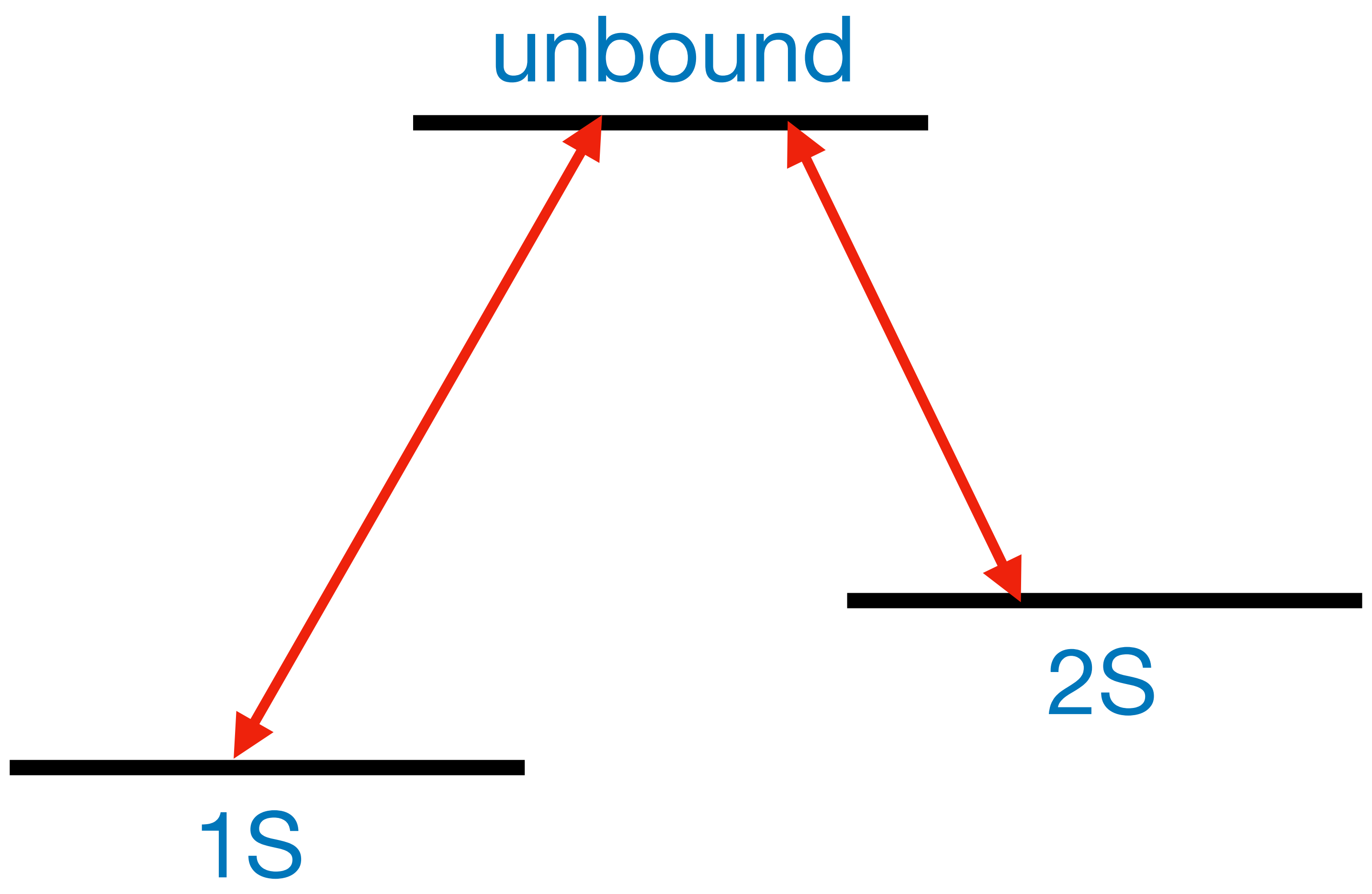
Thank you!

Extra slides

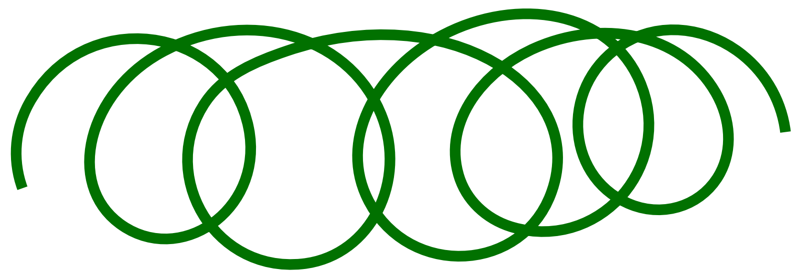
Time scales of quarkonia

Transitions between quarkonium energy levels (the system)



$$\frac{1}{\tau_S} \sim \Delta E_n \sim Mv^2$$

Interaction with the environment



$$\frac{1}{\tau_I} \sim \frac{H_{\rm int}^2}{T} \sim T\frac{T^2}{(Mv)^2}$$

QGP (the environment)

$$\frac{1}{\tau_E} \sim T$$

$$\mathscr{L}_{\rm pNRQCD} = \mathscr{L}_{\rm light\,quarks} + \mathscr{L}_{\rm gluon} + \int d^3r {\rm Tr}_{\rm color}\Big[S^\dagger(i\partial_0 - H_s)S + O^\dagger(iD_0 - H_o)O$$

$$+V_A\left(O^\dagger \mathbf{r}\cdot g\mathbf{E}S + {\rm h.c.}\right) + \frac{V_B}{2}O^\dagger\{\mathbf{r}\cdot g\mathbf{E}, O\} + \cdots\Big]$$

Lindblad equations for quarkonia at low T

quantum Brownian motion limit & quantum optical limit in pNRQCD

- After tracing out the QGP degrees of freedom, one gets a Lindblad-type equation:

$$\frac{\partial \rho}{\partial t} = -i[H_{\text{eff}}, \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2} \left\{ L_j^\dagger L_j, \rho \right\} \right)$$

- This can be done in two different limits within pNRQCD:

Quantum Brownian Motion:

$$\tau_I \gg \tau_E$$
$$\tau_S \gg \tau_E$$

relevant for $Mv \gg T \gg Mv^2$

Quantum Optical:

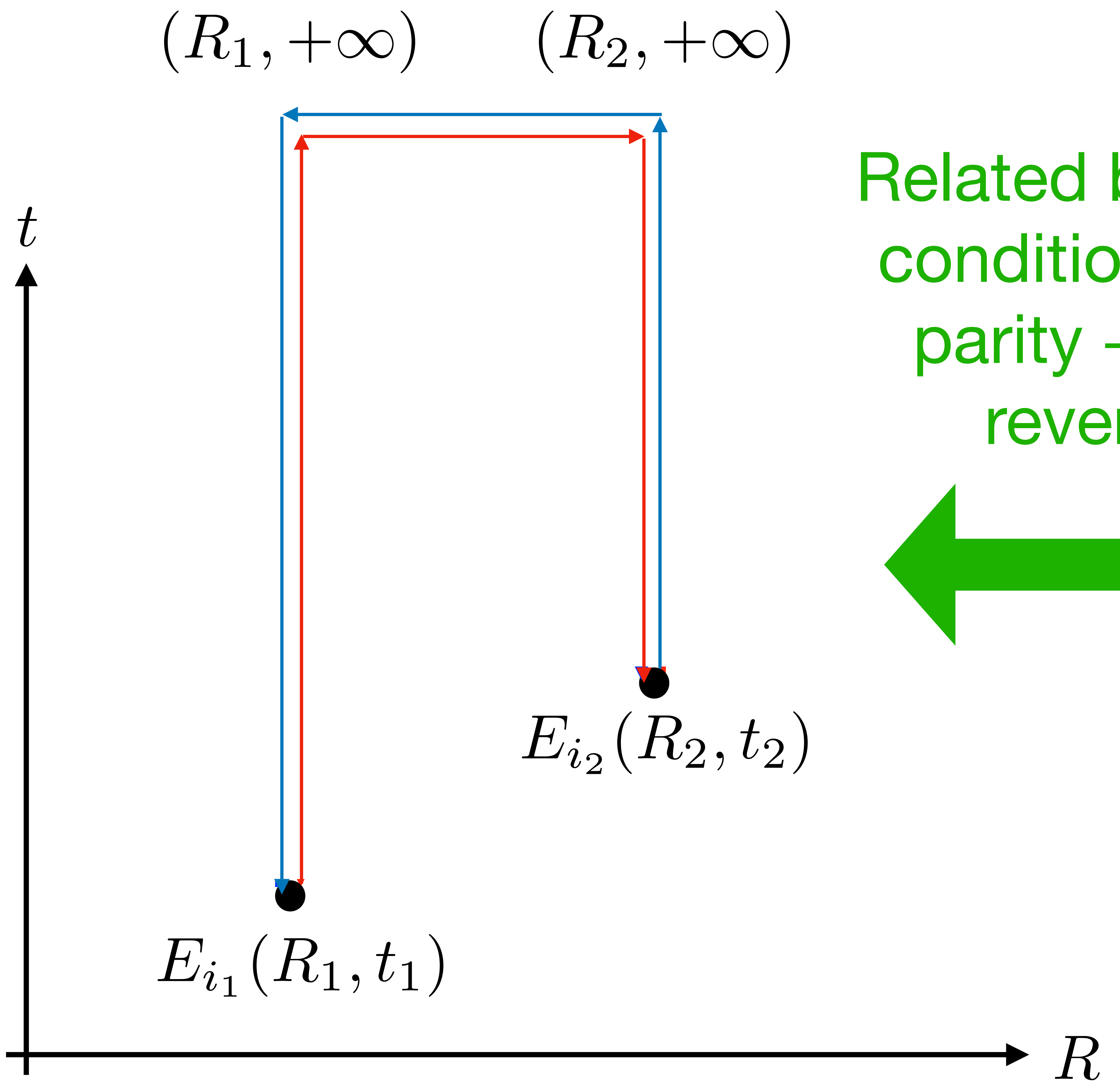
$$\tau_I \gg \tau_E$$
$$\tau_I \gg \tau_S$$

relevant for $Mv \gg Mv^2, T \gtrsim m_D$

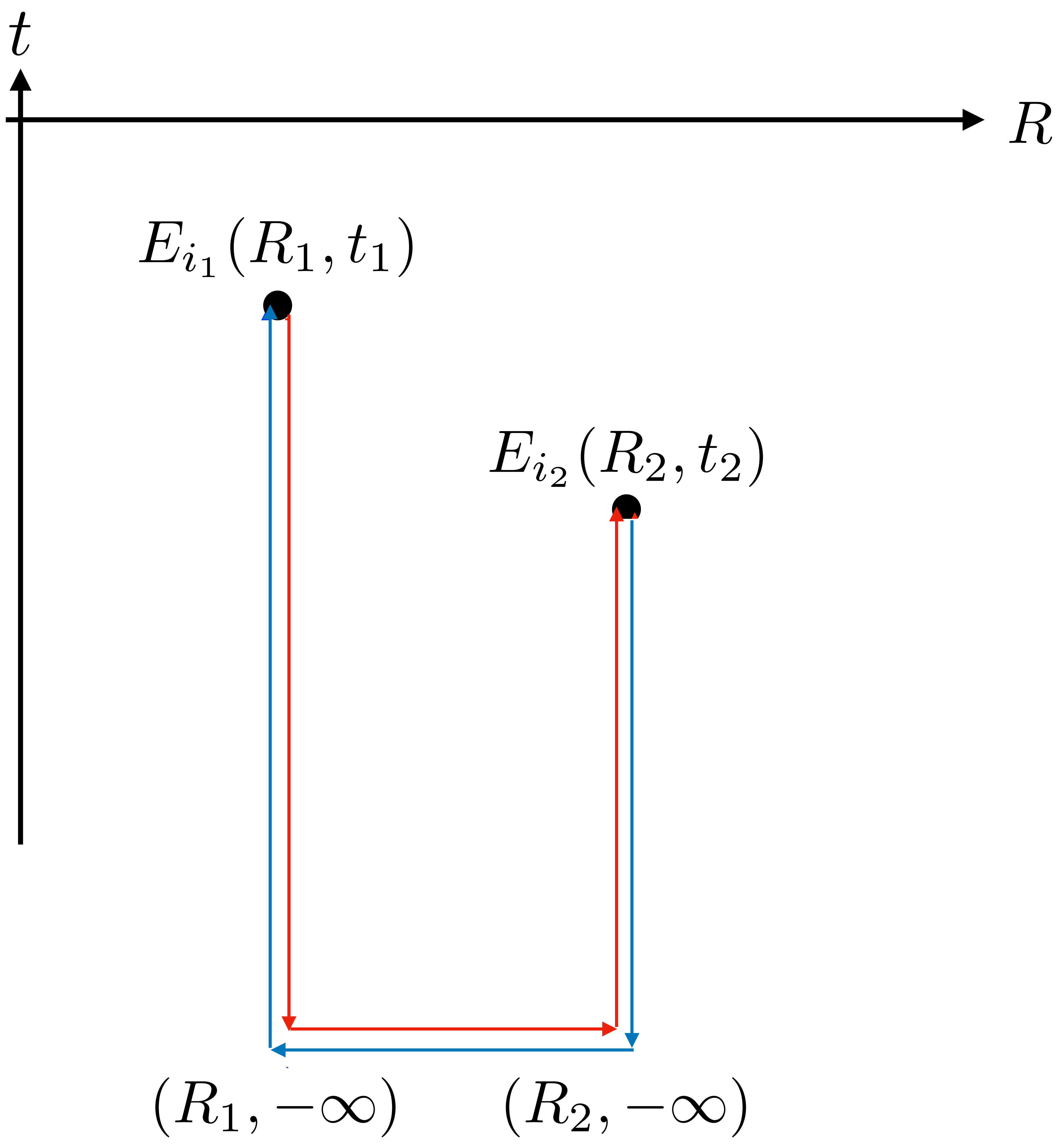
QGP chromoelectric correlators

for quarkonia transport

$$[g_E^{--}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(\mathscr{W}_{2'} E_{i_2}(\mathbf{R}_2, t_2)\right)^a \left(E_{i_1}(\mathbf{R}_1, t_1)\mathscr{W}_{1'}\right)^a \right\rangle_T$$



Related by KMS conditions and parity + time reversal



$$[g_E^{++}]^{>}_{i_2 i_1}(t_2, t_1, \mathbf{R}_2, \mathbf{R}_1) = \left\langle \left(E_{i_2}(\mathbf{R}_2, t_2)\mathscr{W}_2\right)^a \left(\mathscr{W}_1 E_{i_1}(\mathbf{R}_1, t_1)\right)^a \right\rangle_T$$

The correlators we discussed are also directly related to the correlators that define the transport coefficients in the quantum brownian motion limit:

$$\gamma \equiv \frac{g^2}{6N_c}\mathrm{Im}\int_{-\infty}^{\infty} ds\, \langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathcal{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,,$$

$$\kappa \equiv \frac{g^2}{6N_c}\mathrm{Re}\int_{-\infty}^{\infty} ds\, \langle \mathcal{T}\, E^{a,i}(s,\mathbf{0})\mathcal{W}^{ab}[(s,\mathbf{0}),(0,\mathbf{0})]E^{b,i}(0,\mathbf{0})\rangle\,.$$

The spectral function of quarkonia

symmetries and KMS relations

The KMS conjugates of the previous correlators are such that

$$[g_E^{++}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{++}]_{ji}^{<}(q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = e^{q^0/T}[g_E^{--}]_{ji}^{<}(q)\,,$$

and one can show that they are related by

$$[g_E^{++}]_{ji}^{>}(q) = [g_E^{--}]_{ji}^{<}(-q)\,, \quad [g_E^{--}]_{ji}^{>}(q) = [g_E^{++}]_{ji}^{<}(-q)\,.$$

The spectral functions $[\rho_E^{++/--}]_{ji}(q) = [g_E^{++/--}]_{ji}^{>}(q) - [g_E^{++/--}]_{ji}^{<}(q)$ are not necessarily odd under $q \leftrightarrow -q$. However, they do satisfy:

$$[\rho_E^{++}]_{ji}(q) = -\,[\rho_E^{--}]_{ji}(-q)\,.$$

How the calculation proceeds

what equations do we need to solve?

- The classical, unperturbed equations of motion from the Nambu-Goto action to determine Σ:

$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-\det\left(g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right)}\ .$$

- The classical, linearized equation of motion with perturbations in order to be able to calculate derivatives of $\langle W[\mathscr{C}_f]\rangle_T = e^{iS_{\rm NG}[\Sigma_f]}$:

$$S_{\rm NG}[\Sigma_f] = S_{\rm NG}[\Sigma] + \int dt_1 dt_2 \left.\frac{\delta^2 S_{\rm NG}[\Sigma_f]}{\delta f(t_1)\delta f(t_2)}\right|_{f=0} f(t_1)f(t_2) + O(f^3)\ .$$

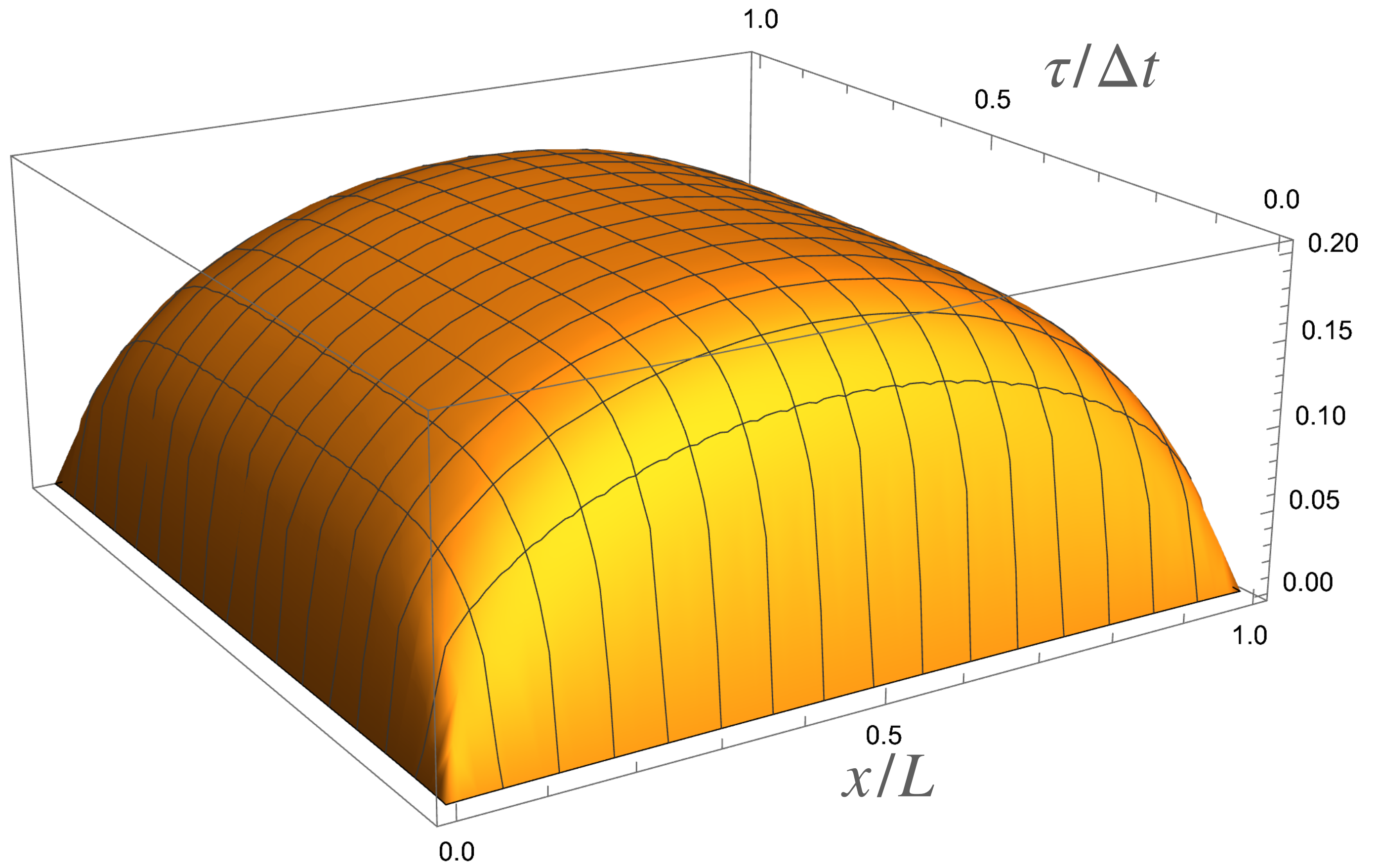
- In practice, the equations are only numerically stable in Euclidean signature, so we have to solve them and analytically continue back.

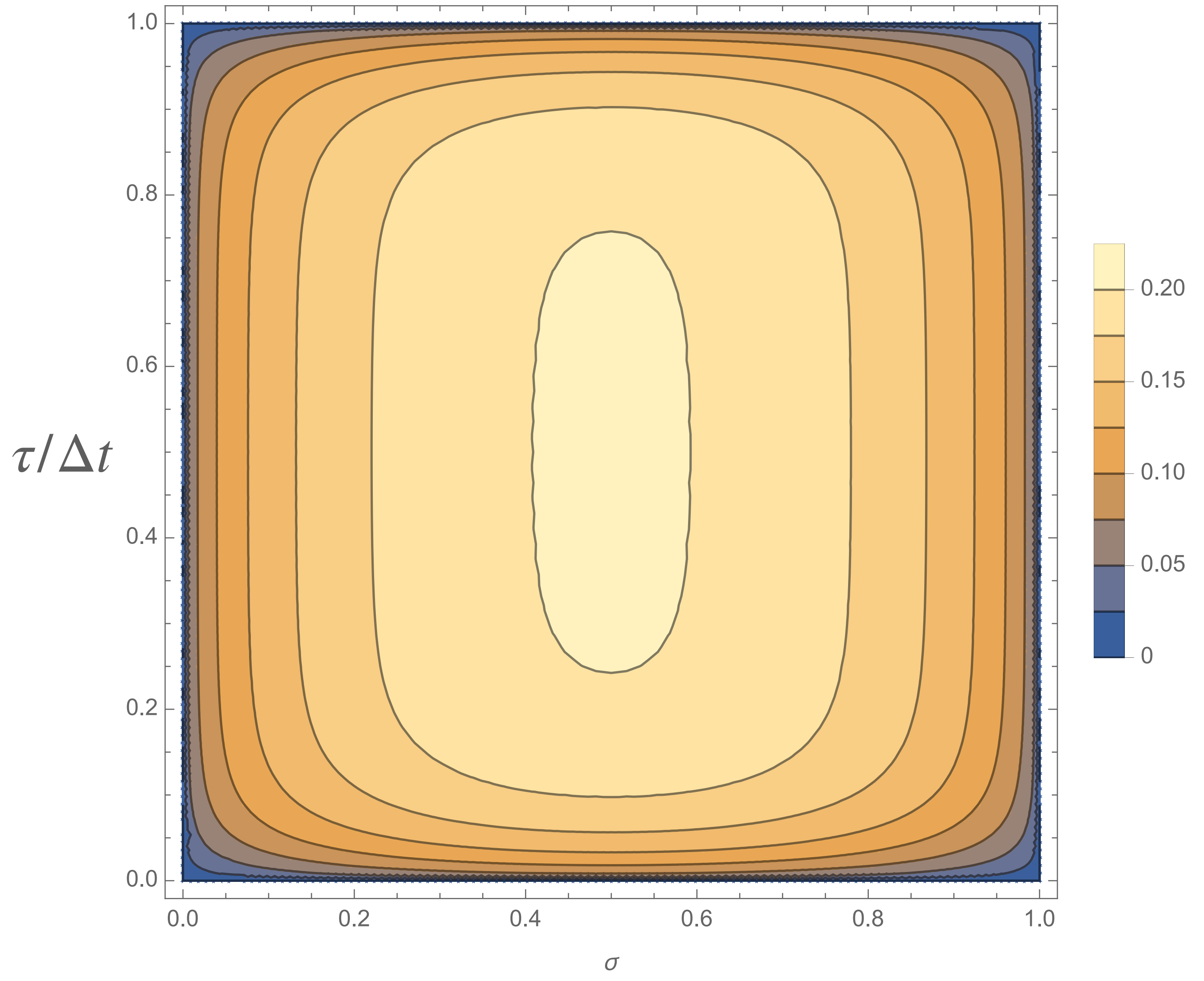
Extracting the EE correlator for quarkonia

the pipeline

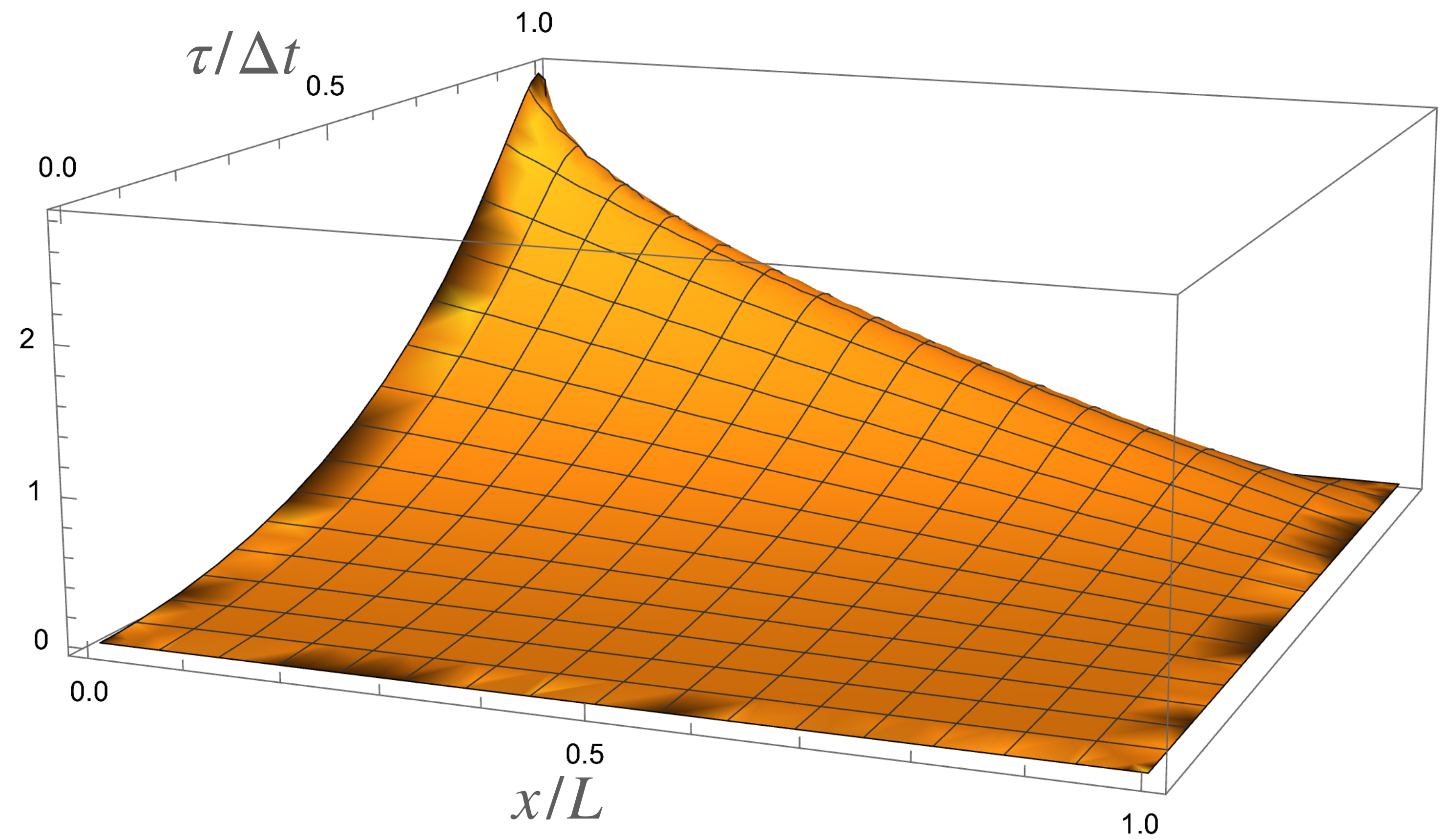
J.P. Boyd, "Chebyshev and Fourier Spectral Methods," Dover books on Mathematics (2001)

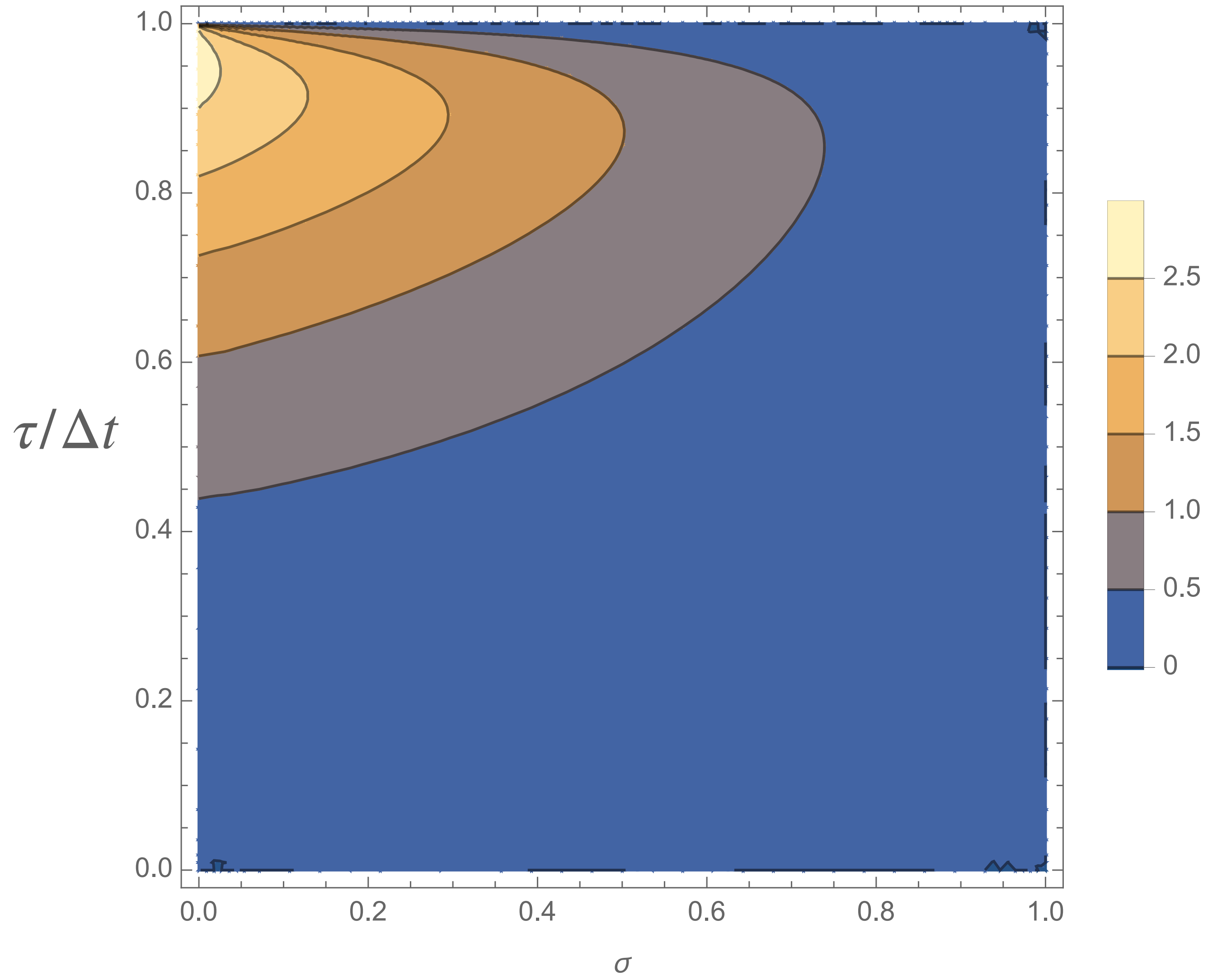
1) Solve for the background worldsheet solution:





2) Solve for the fluctuations with a source as a boundary condition:





3) Extrapolate in the limit $L \to 0$: