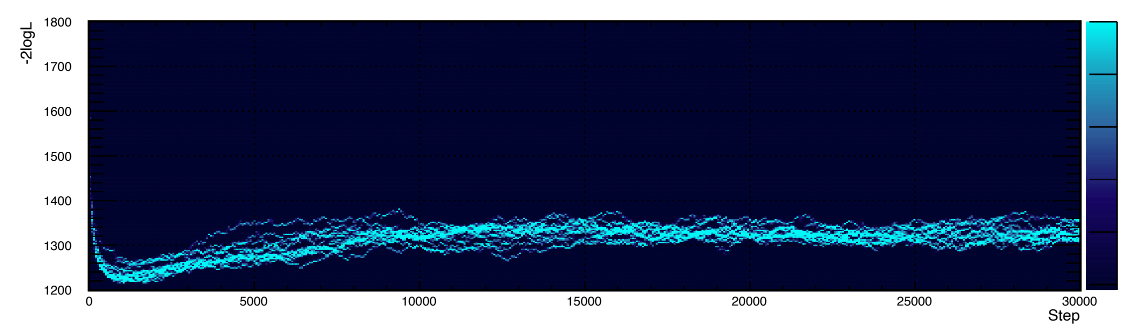
8th International Conference on High Energy Physics in the LHC Era
9-13 January 2023, Valparaiso, Chile

The MaCh3 Bayesian Oscillation Analysis framework of the T2K experiment



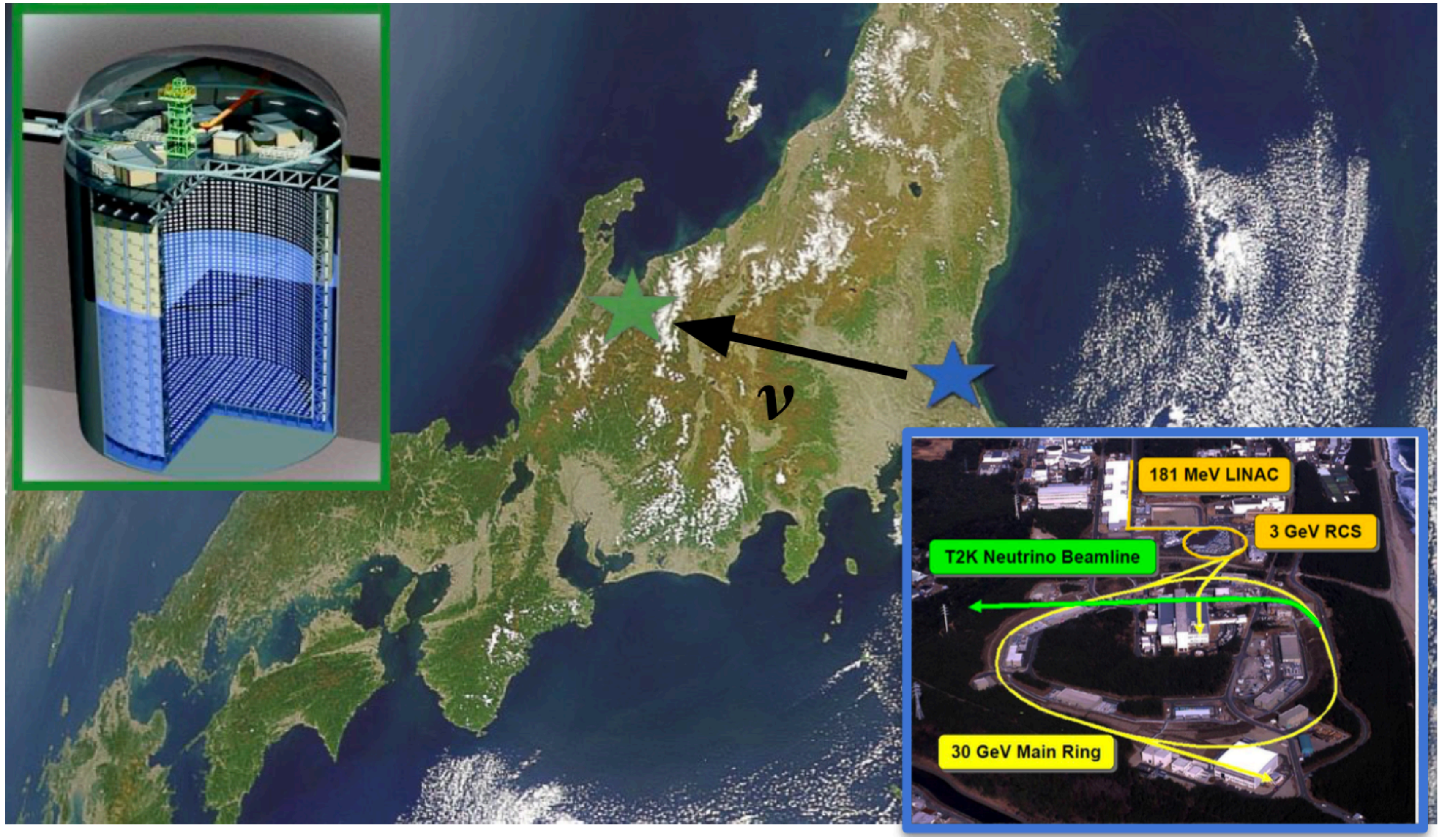
B. Radics
(on behalf of the T2K collaboration)

Tokai-to-Kamioka (T2K) experiment

- Tokai-to-Kamioka (T2K) is a long-baseline accelerator neutrino experiment.

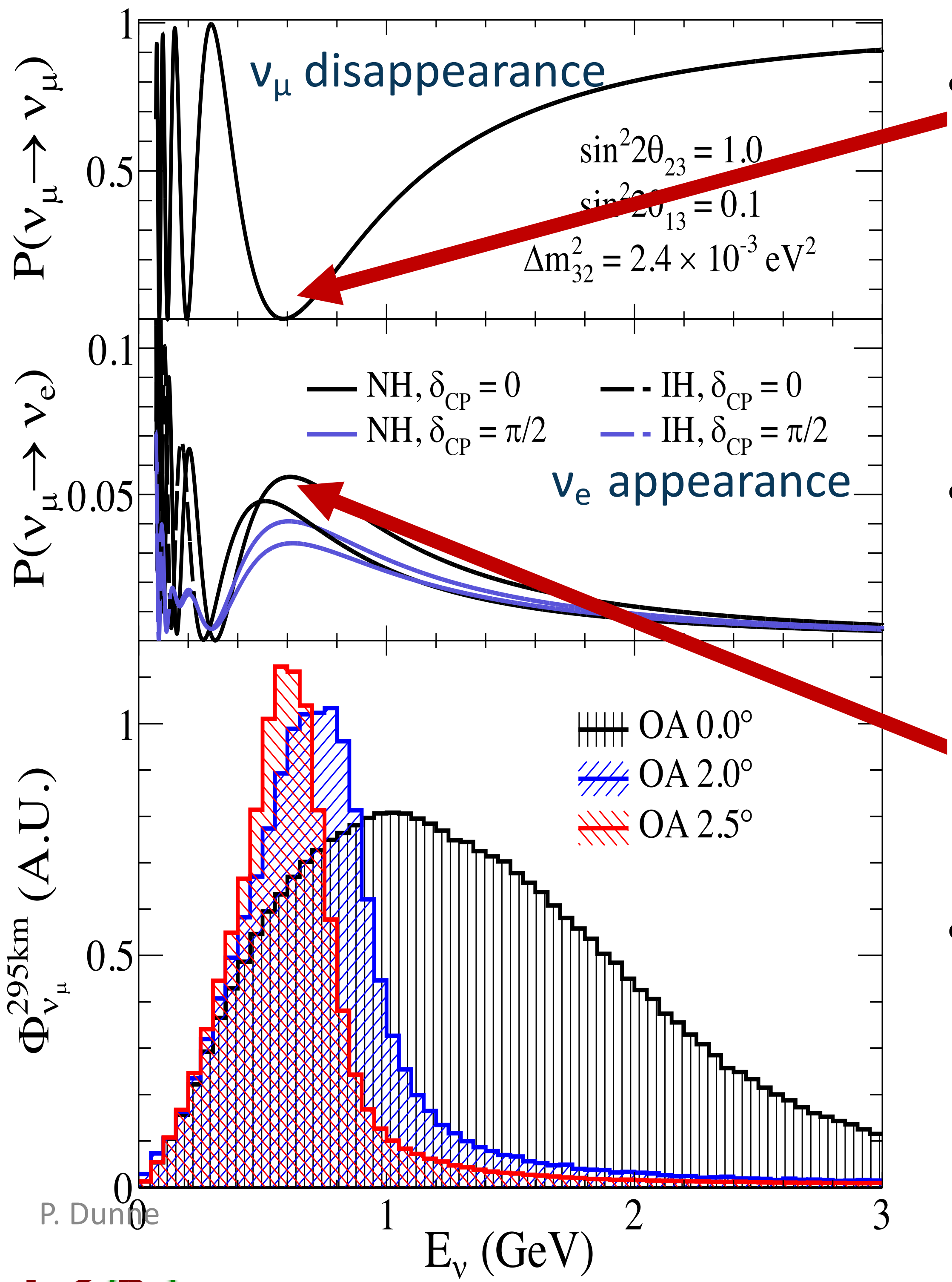
Physics goals:

- Precise measurement of PMNS neutrino flavour oscillation parameters: θ_{23} and Δm^2_{32}.
- Test of Charge-Parity-symmetry conservation through the PMNS parameter δ_{CP}
- Test of the 3-flavour oscillation model.



For latest T2K results, see <u>Akihiro Minamino's T2K Talk</u>

Sensitivity to neutrino flavour oscillation parameters



- Muon (anti)neutrino disappearance:
  - Location of dip determined by Δm^2_{32}
  - Depth of the dip given by $\sin^2(2\theta_{23})$
- Electron (anti)neutrino appearance:
  - Leading term depends on $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$ and Δm^2_{32}
  - Sub-leading δ_{CP} dependence
  - Matter effects give some dependence on mass hierarchy
- With the 295 km baseline the first oscillation maximum is at 0.6 GeV, a 2.5° off-axis neutrino beam flux can be focused to this energy

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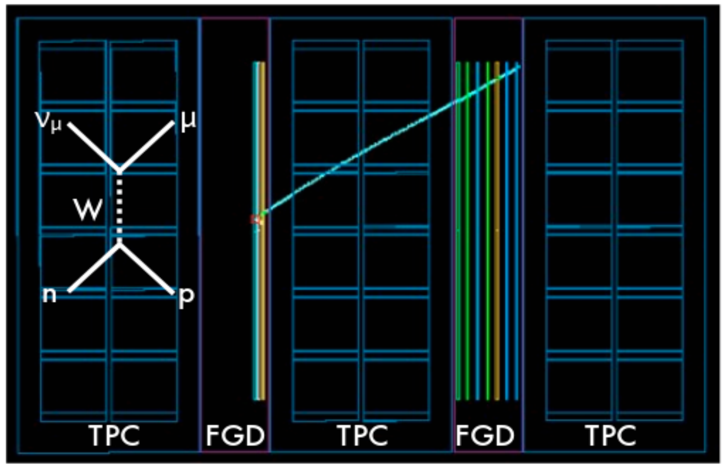
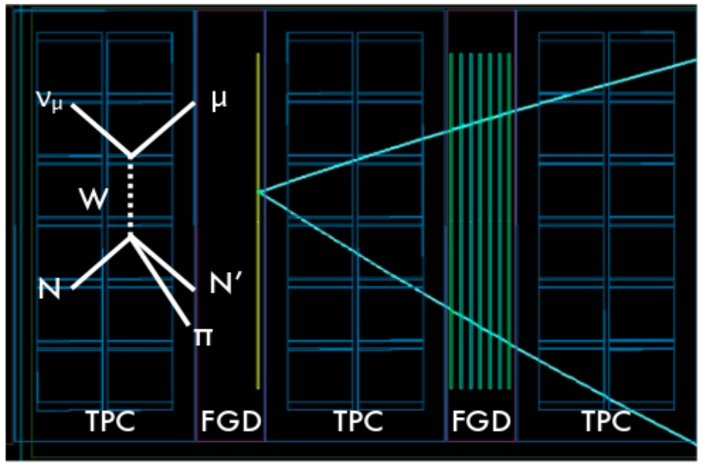
Near Detector (ND280): ν beam before flavour oscillations

- ND data constrains the **neutrino beam flux and interaction cross-section model** uncertainties.
- The (anti-)neutrino beam is sampled at the **Near Detector** via neutrino scattering events classified by the final-state: CC0π, CC1π, CCOther event samples and fit in final state kinematics.
  - Further splitting CC0π samples based on the presence of proton.
  - Separate π^0 events with a photon tag.

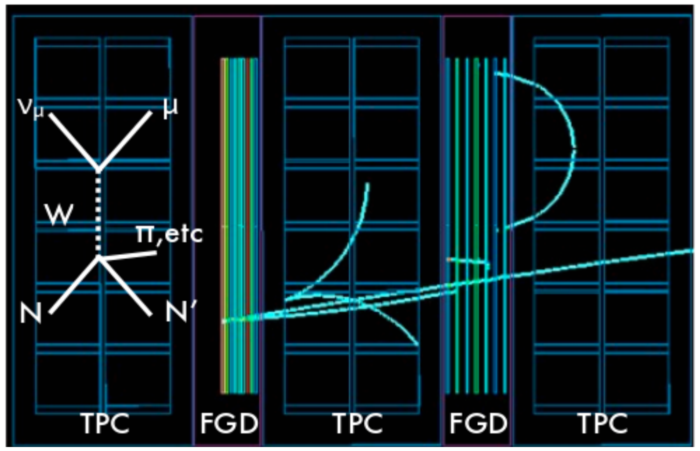
CC0π

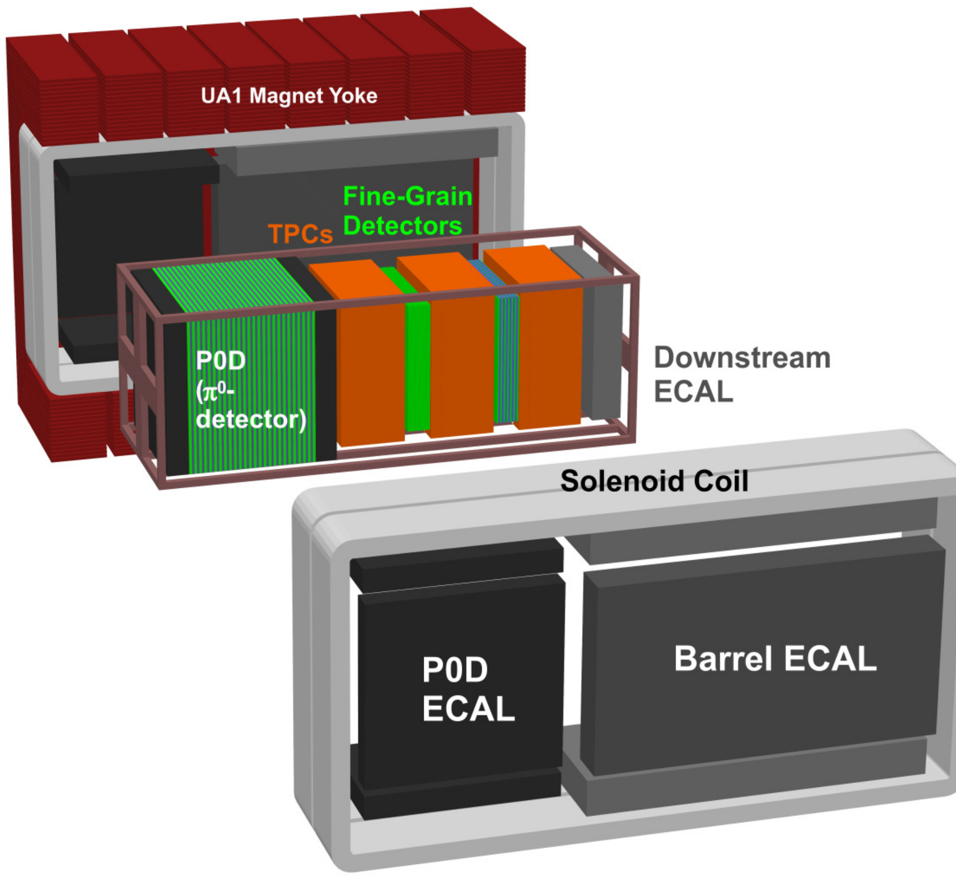


CC1π



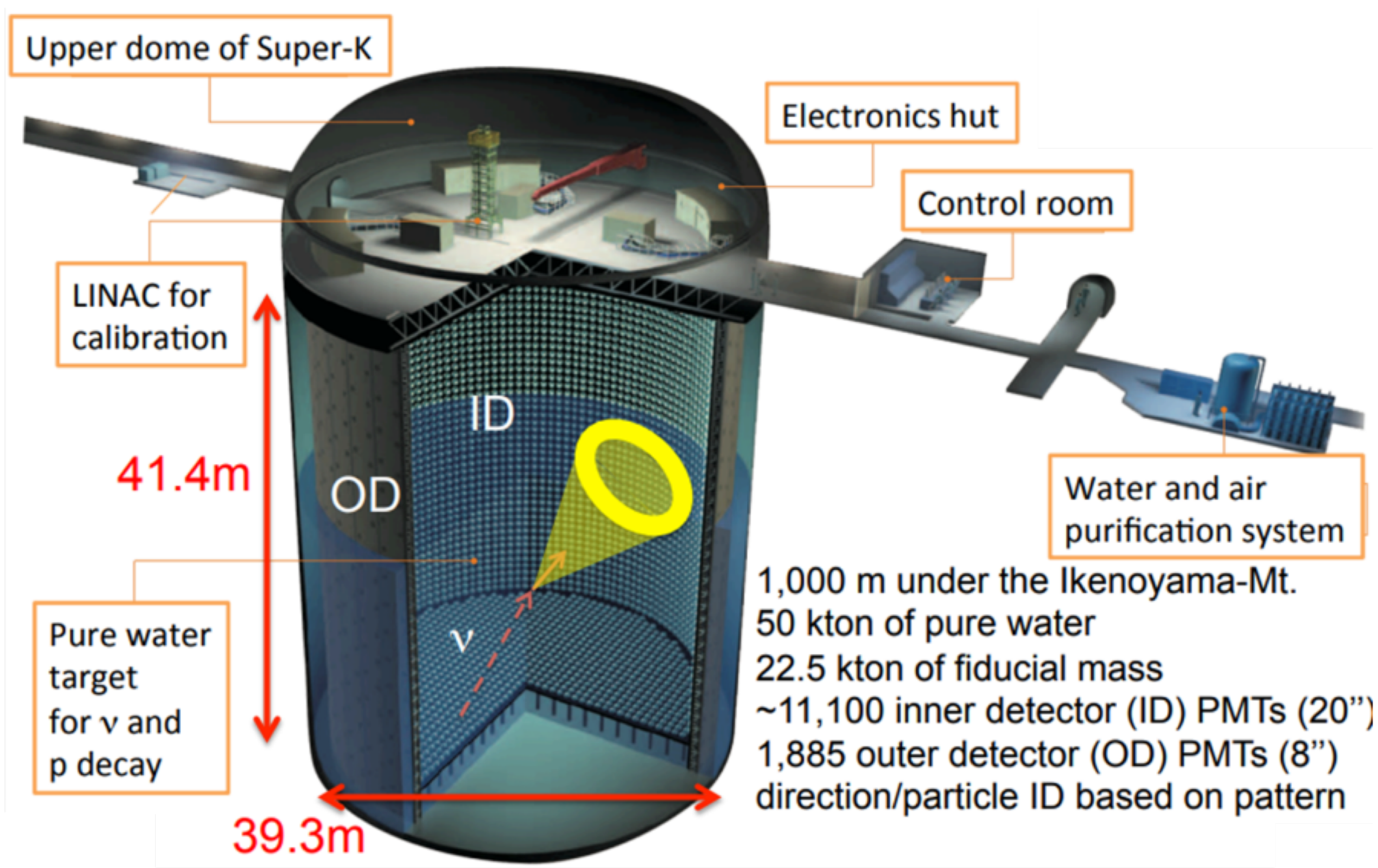
CCOther

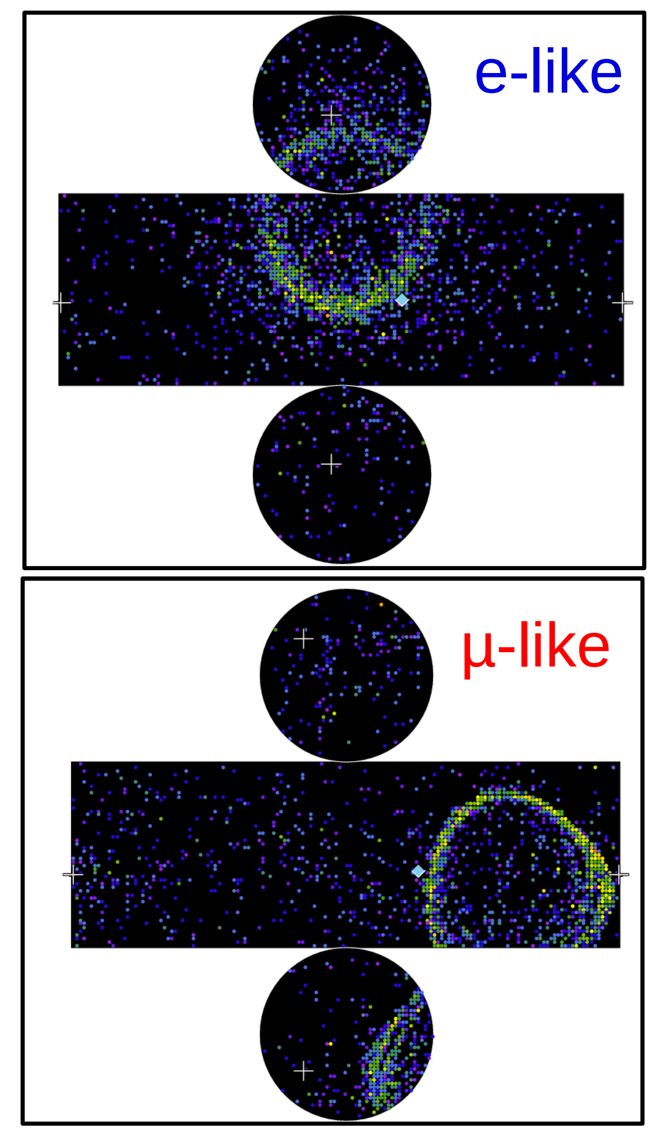




Far Detector (Super-Kamiokande): ν beam after flavour oscillations

- The (anti-)neutrino beam is sampled at the **Far Detector:** single ring: $\mu^{\pm}$ and $e^{\pm}$-like Cherenkov rings corresponding mainly to ν_μ and ν_e CC0π and CC1π interactions, and multiring selection: ν_μ CC1π sample.
- Event samples used from both neutrino-mode and anti-neutrino mode beams.

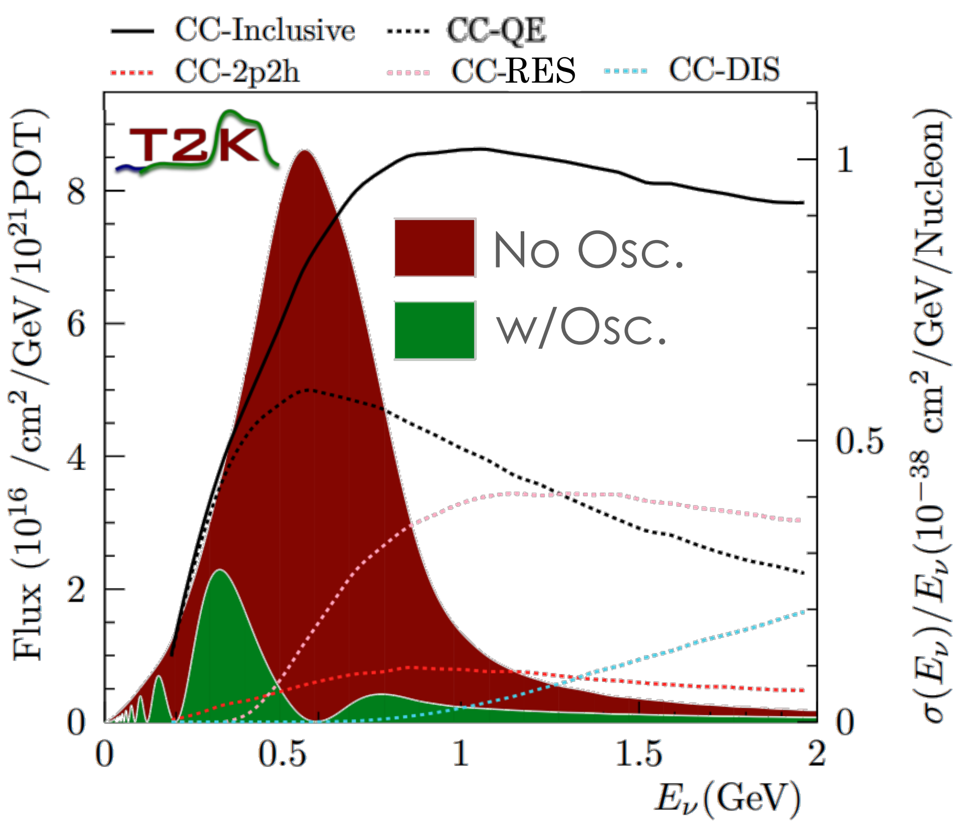


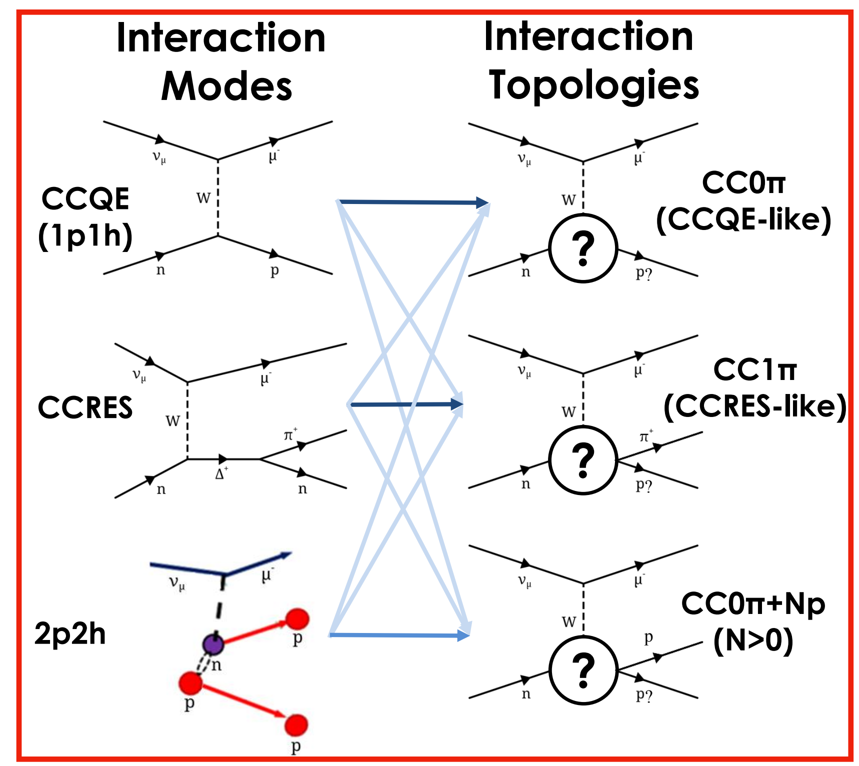


Neutrino Interactions at T2K

$$N_\ell(E_\nu) = P(\nu_\mu \to \nu_\ell)(E_\nu)\,\sigma(E_\nu)\,\Phi_\nu(E_\nu)\,\epsilon(E_\nu)$$

$N_\ell(E_\nu)$ = Event rate
$P(\nu_{\ell'} \to \nu_\ell)(E_\nu)$ = Oscillation probability
$\Phi_\nu(E_\nu)$ = Neutrino flux
$\epsilon(E_\nu)$ = Detector efficiency
$\sigma_\ell(E_\nu)$ = Interaction cross section

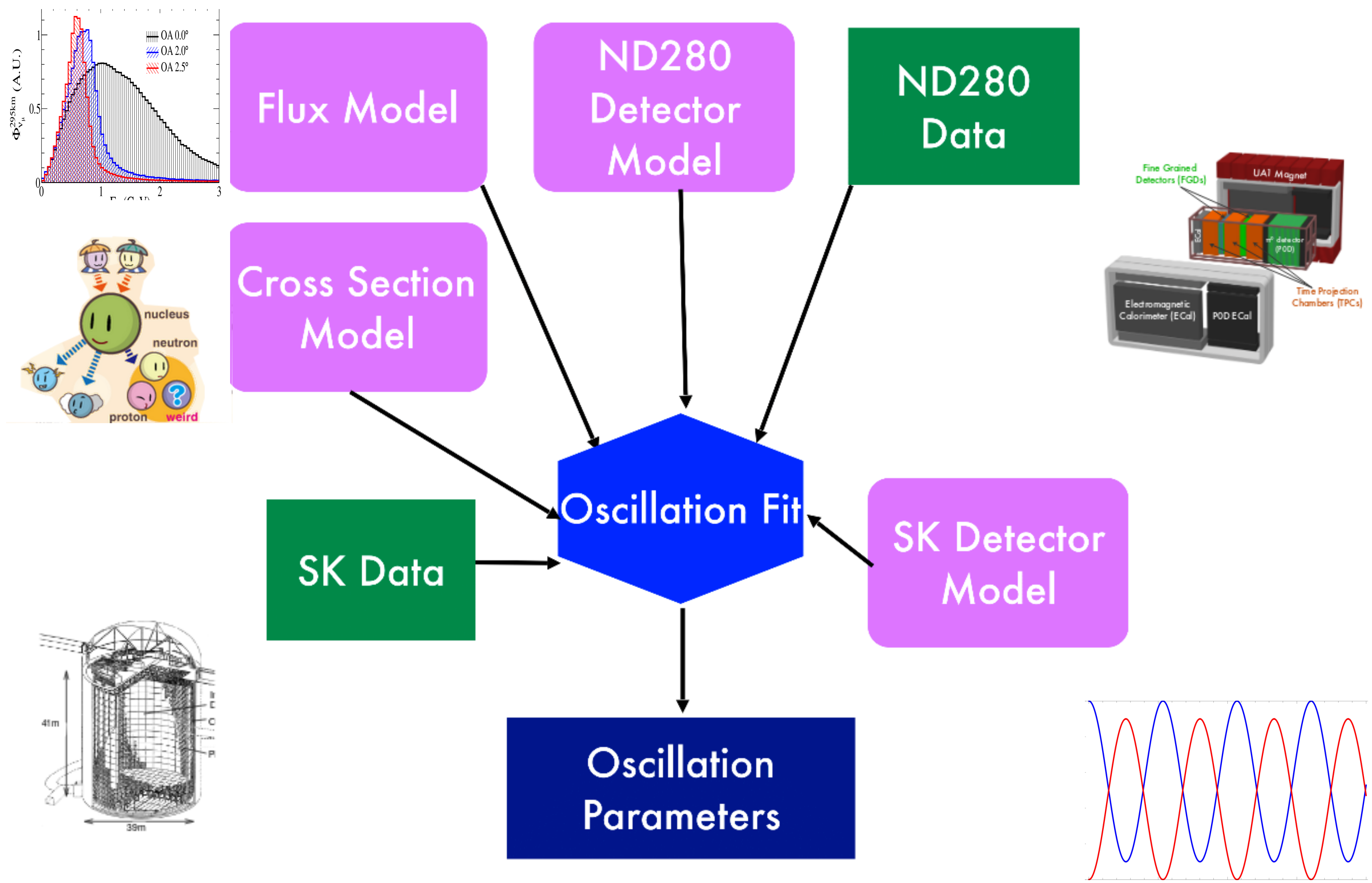




- Cannot separately measure $P(\nu_\mu \to \nu_x)(E_\nu)$, $\sigma(E_\nu)$, $\phi_\nu(E_\nu)$
- Cannot directly measure E_ν^{true}, cannot separately resolve interaction modes
- **Need a model-based analysis to infer parameters** and relate reconstructed quantities to true quantities

T2K joint oscillation fit overview

- **Model-based analysis**: statistical likelihood approach evaluates the compatibility between the observed data and MC predictions from a combined neutrino beam flux + interaction cross-section + detector + flavour oscillation model.
- T2K produces both frequentist and Bayesian results: cross-check each other.



Evaluating the Bayes-theorem with MaCh3

- The **MaCh3** analysis performs statistical inference using the Bayes-theorem.
- **Likelihood term** is
  - Likelihood ratio assuming Poisson statistics (Baker, Cousins 1984)
  - Terms for ND MC statistical fluctuation (Barlow, Beeston 1993)
- **Priors terms** are
  - Physics parameters or model constraints: multivariate “Gaussian” terms acting either in the model parameter space or in the observable final-state kinematic bins
- **Prior constraints:** ND fit, external data, detector systematics, etc.

Bayes-theorem:

Likelihood ↘ ↙ Prior

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}$$

Joint posterior probability dist:

$$P(D|\vec{\theta})P(\vec{\theta}) = \prod \mathcal{L}_{\text{Total}} = \prod \left(\mathcal{L}_{\text{Bins}} \times \mathcal{L}_{\text{Systematics}}\right)$$

$$\begin{aligned}
-2\log\mathcal{L}_{\text{Total}} =& -2\log\mathcal{L}_{\text{Stat}} - 2\log\mathcal{L}_{\text{Sys}} \\
=& 2\sum_{i}^{\text{Nbins}} N_i^{\text{MC}}(\vec{f},\vec{x},\vec{d}) - N_i^{\text{data}} + N_i^{\text{data}} \ln\left(\frac{N_i^{\text{data}}}{N_i^{\text{MC}}(\vec{f},\vec{x},\vec{d})}\right) + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
&+ \sum_{i}^{E_\nu bins} \sum_{j}^{E_\nu bins} \Delta\vec{f}_i \left(V_f^{-1}\right)_{i,j} \Delta\vec{f}_j \\
&+ \sum_{i}^{\text{xsecpars}} \sum_{j}^{\text{xsecpars}} \Delta\vec{x}_i \left(V_x^{-1}\right)_{i,j} \Delta\vec{x}_j \\
&+ \sum_{i}^{\text{detbins}} \sum_{j}^{\text{detbins}} \Delta\vec{d}_i \left(V_d^{-1}\right)_{i,j} \Delta\vec{d}_j
\end{aligned}$$

T2K

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The MaCh3 T2K joint (log) posterior

$$
-\ln(P(\vec{\theta}|D)) = \sum_{i}^{ND280bins} N_i^{ND,p}(\vec{f},\vec{x},\vec{d}) - N_i^{ND,d} + N_i^{ND,d} ln[N_i^{ND,d}/N_i^{ND,p}(\vec{f},\vec{x},\vec{d})] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}
$$

Data at ND280

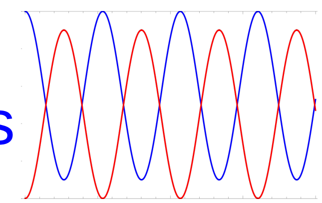
$$
+ \sum_{i}^{SKbins} N_i^{SK,p}(\vec{f},\vec{x},s\vec{kd},\vec{o}) - N_i^{SK,d} + N_i^{SK,d} ln[N_i^{SK,d}/N_i^{SK,p}(\vec{f},\vec{x},s\vec{kd},\vec{o})]
$$

Data at SK

What we want!!

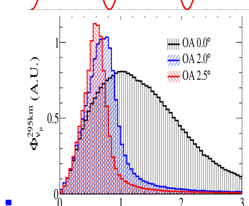
$$
+ \frac{1}{2}\sum_{i}^{osc}\sum_{j}^{osc} \Delta o_i (V_o^{-1})_{i,j} \Delta o_j
$$

Oscillation Parameters



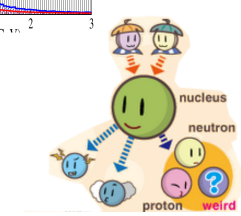
$$
+ \frac{1}{2}\sum_{i}^{flux}\sum_{j}^{flux} \Delta f_i (V_f^{-1})_{i,j} \Delta f_j
$$

Flux



$$
+ \frac{1}{2}\sum_{i}^{xsec}\sum_{j}^{xsec} \Delta x_i (V_x^{-1})_{i,j} \Delta x_j
$$

Interaction Model

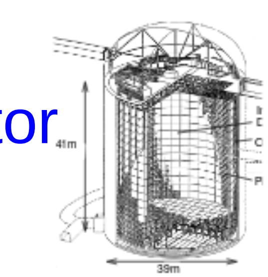


$$
+ \frac{1}{2}\sum_{i}^{nd280det}\sum_{j}^{nd280det} \Delta d_i (V_d^{-1})_{i,j} \Delta d_j
$$

ND280

$$
+ \frac{1}{2}\sum_{i}^{skdet}\sum_{j}^{skdet} \Delta skd_i (V_{skd}^{-1})_{i,j} \Delta skd_j
$$

SK Detector

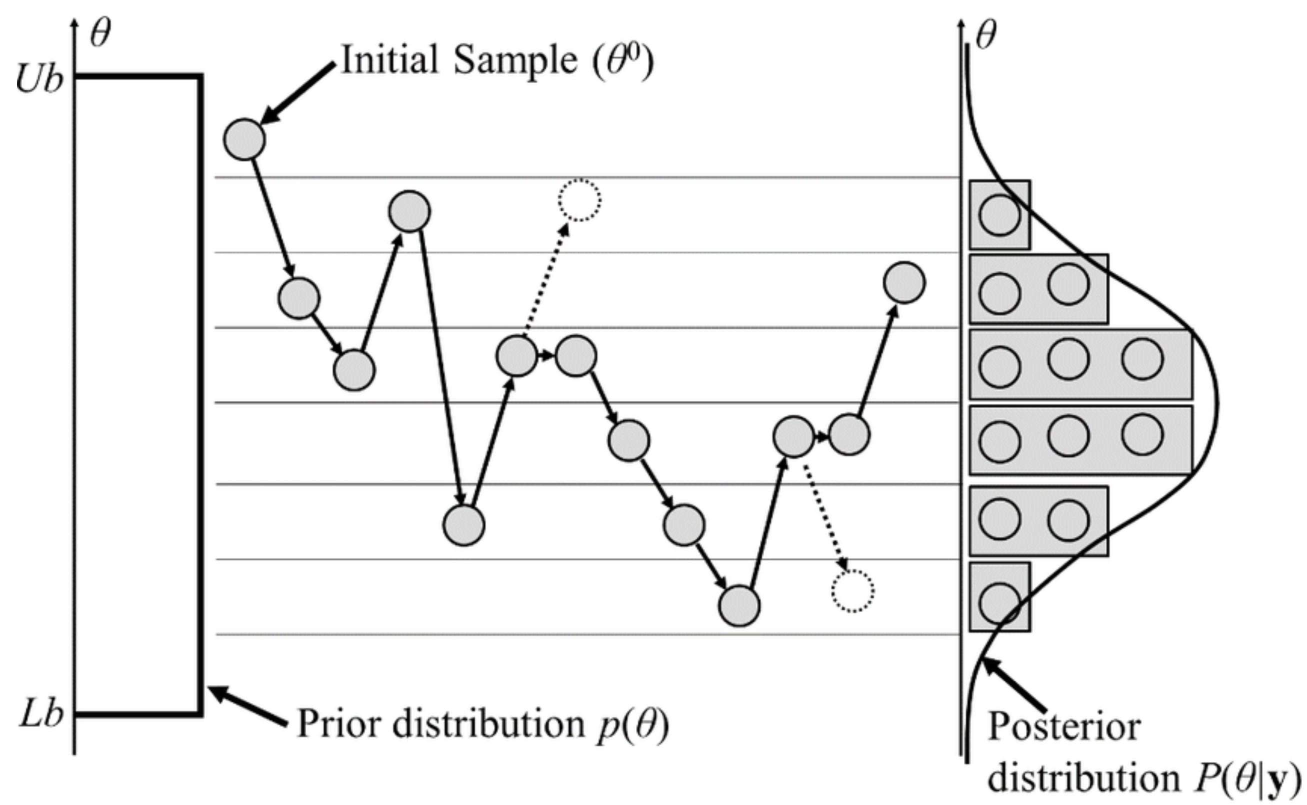


Priors

f: flux, x: cross section, d/skd: detectors, o: oscillation parameters
ND: Near Detector, SK: Super-Kamiokande Far Detector

MCMC exploration of the parameter space

- Joint oscillation fit uses multiple event samples simultaneously from the **Near (22)** and **Far Detector (6)**: large number of parameters (>800).
- Using **Markov Chain Monte Carlo** (MCMC) sampling from the joint probability distribution.
- MaCh3 uses the **Metropolis-Hastings** (MH) algorithm to perform the random sampling.
- Starting from random initial parameter values; after convergence the MC sequence approximates the joint posterior.

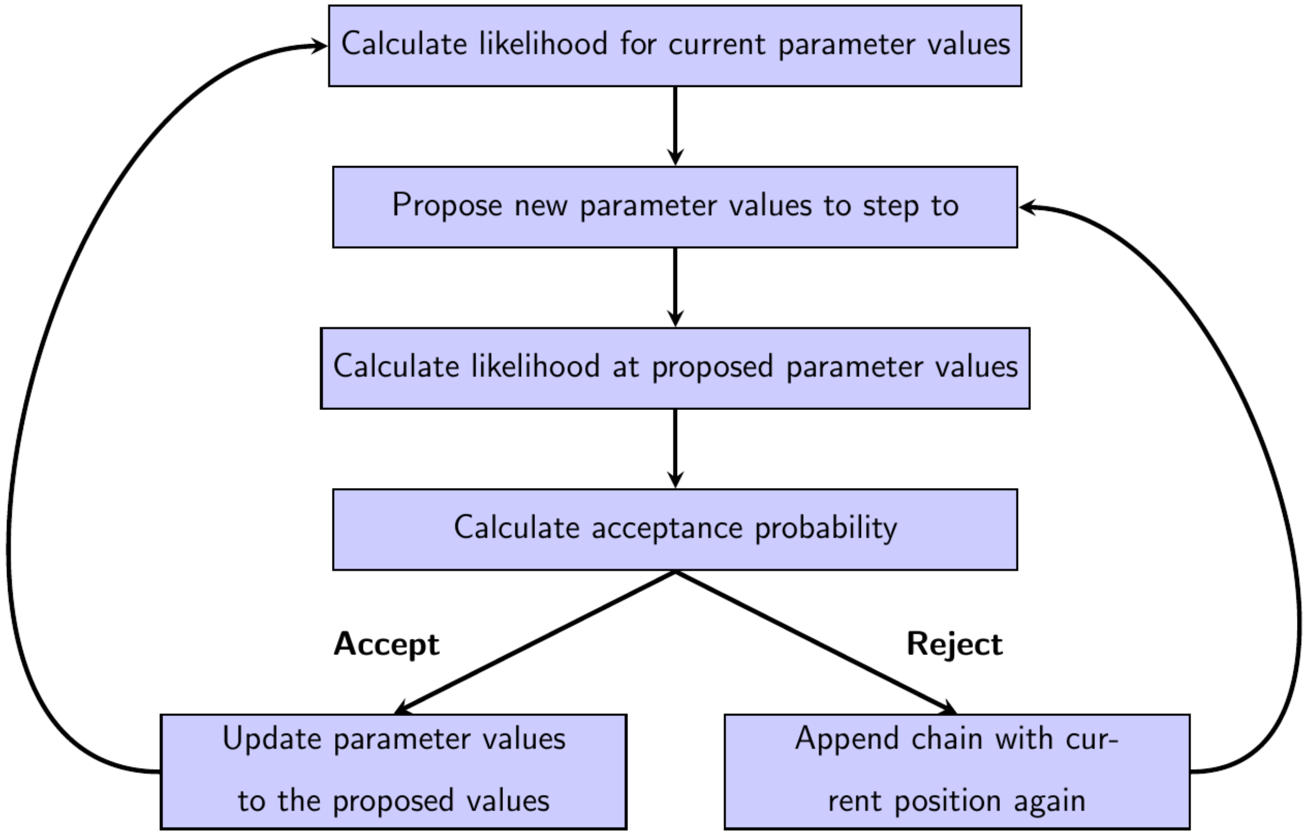


Metropolis-Hastings algorithm

- Metropolis-Hastings quasi-randomly walk in the parameter space, proposing new values to explore a distribution.
- A proposal function (Gaussian) suggests a step, which is taken based on the acceptance probability:

$$\alpha(\vec{x}_n, \vec{y}_n) = \min\left(1, \frac{P(\vec{y}_n|D)}{P(\vec{x}_n|D)}\right)$$

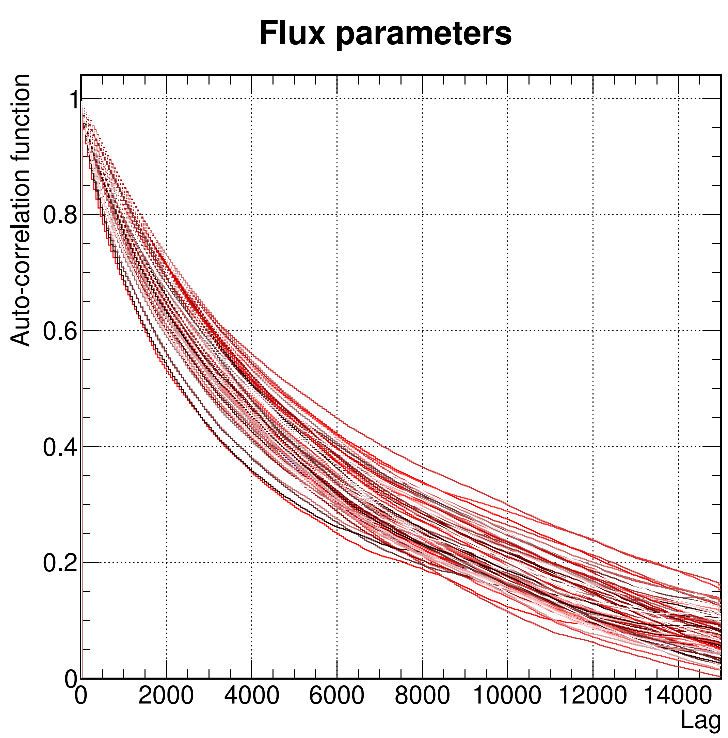
- Generate random number between 0-1 and accept the steps if α is greater!
- Always accept a step if probability is higher. Sometimes accept step if prob is lower.

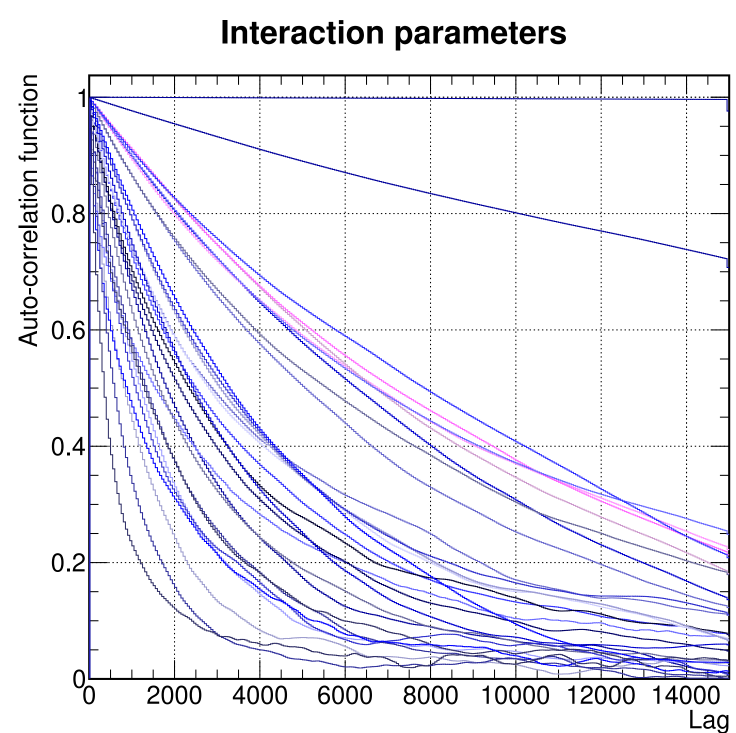


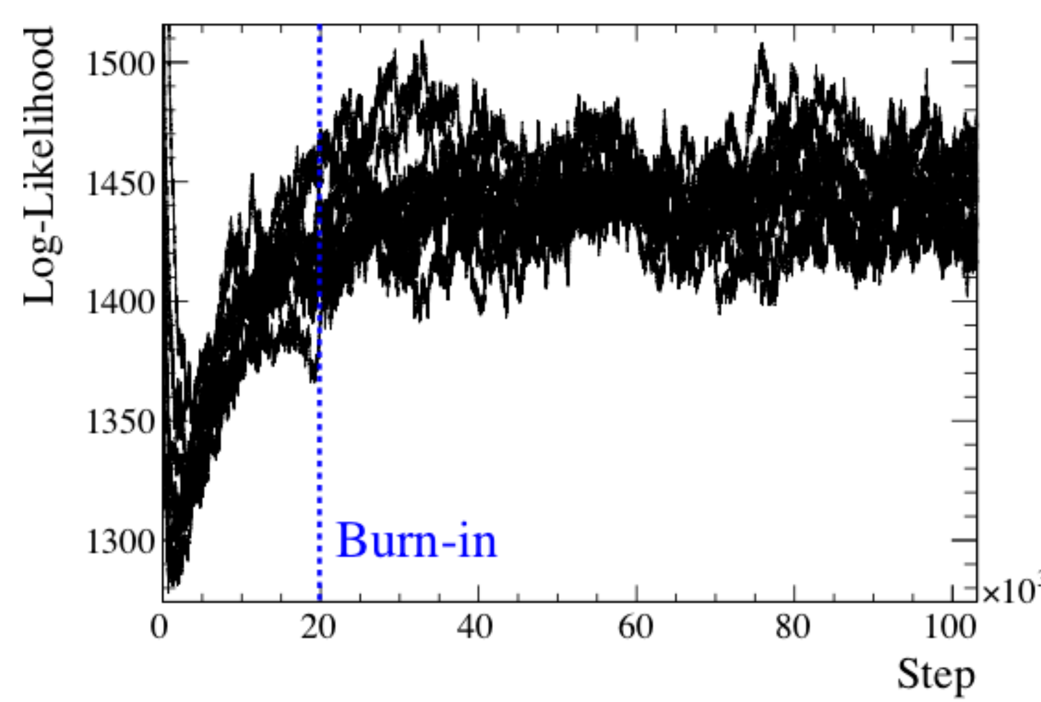
Markov Chain Monte Carlo diagnostics

- Aim: MCMC converges to the stationary target distribution, explores parameter space well.
- Control: step-size tuning. Small steps: poor parameter space exploration, highly correlated steps. Too large steps: leaving high-probability areas too quickly, poor exploration.
- Diagnostic tools:
    - Monitoring **acceptance probability** (literature probability: 0.234, Brooks et al. CRC Press, 2011)
    - **Autocorrelation** function r_k after lag k for a parameter Y_i at step i:
    - Typically aiming for auto-correlations less than 0.2 after a lag 10 000.
    - Between-chains and within-chain variance **convergence test**: $\hat{R}$-test (Gelman and Rubin, Statist. Sci. **7**, 457 (1992)) - starting to being used in MaCh3.

$$r_k = \frac{\sum_{i=1}^{N-k} (Y_i - \bar{Y})(Y_{i+k} - \bar{Y})}{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}$$

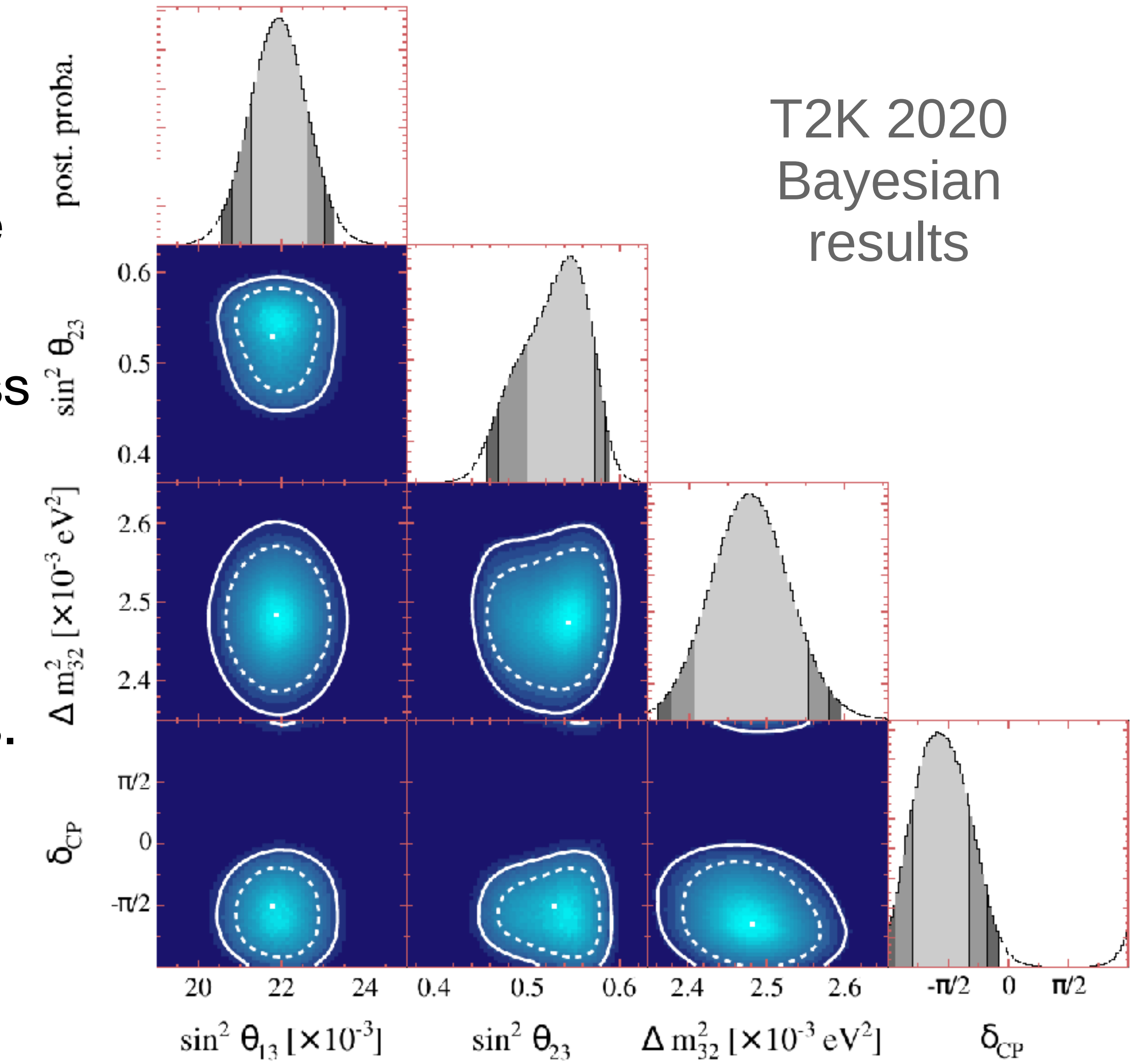






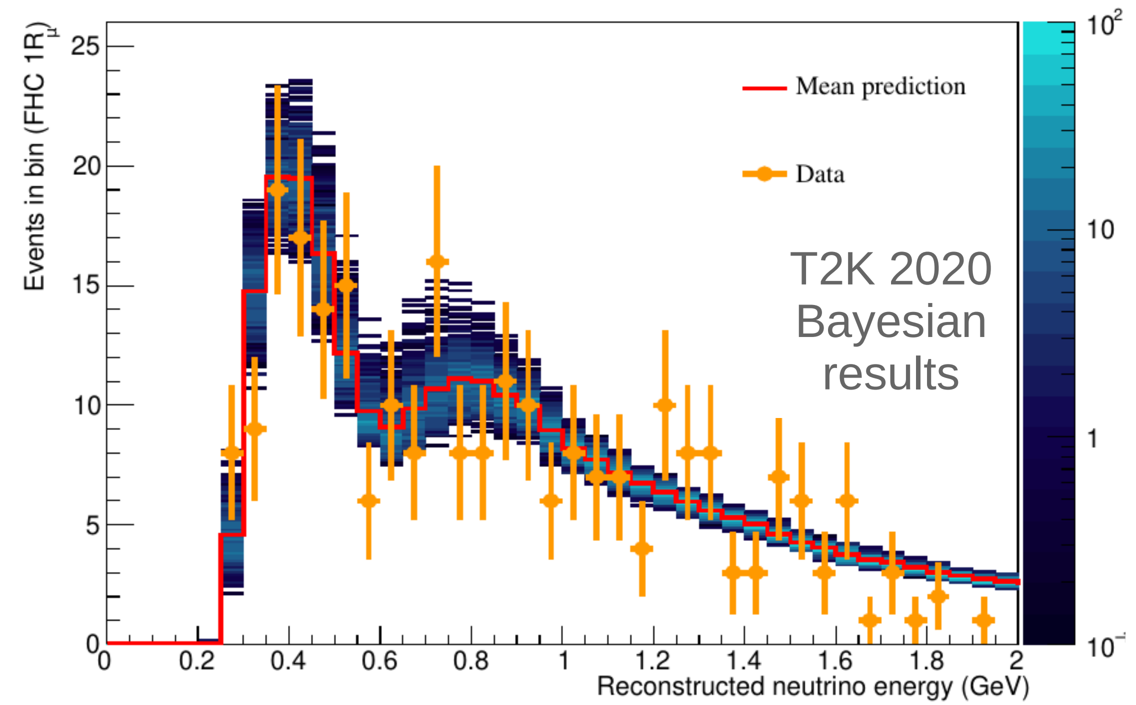
MaCh3 Joint posterior distribution

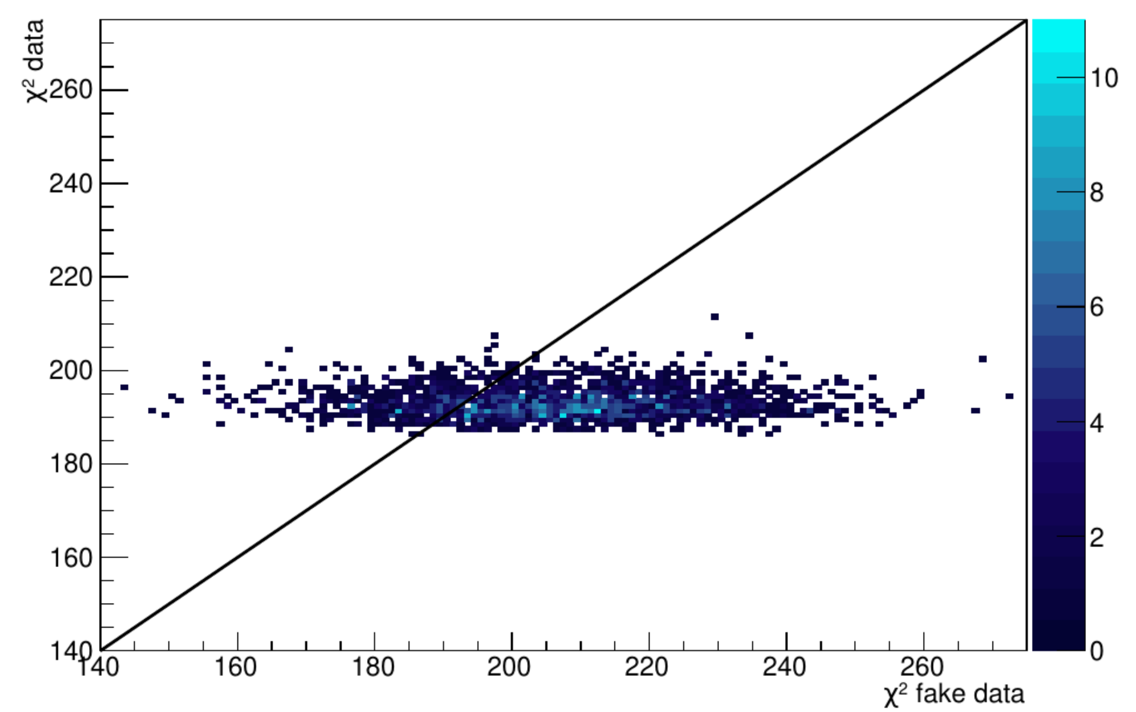
- Running each MCMC gives an N-dimensional posterior distribution (N ~ 800): ROOT TTree.
- In practice running multiple chain in parallel and use “hadd” of ROOT to combine chains.
- Simultaneously evaluating Normal and Inverted mass hierarchies by randomly “flipping” the sign of Δm^2_{32}.
- Marginalise out into 1D or 2D posterior distributions (Systematic or Oscillation parameters).
  - Can also marginalize over both mass hierarchies.
- Credible intervals/regions over any parameters are constructed using Highest Posterior Density (HPD).



Posterior-predictive checks, model comparison

- **Goodness-of-fit test** can be performed using Bayesian approach:
  - For each draw from the joint Posterior distribution a MC prediction and fake data set is generated.
  - MC prediction is compared to fake dataset and to real dataset by calculating a test statistics.
  - Fraction of throws for which data fits the MC prediction better gives the p-value:
  - $\chi^2(y_{fake}|\theta) \geq \chi^2(y_{data}|\theta)$
- **Model comparison**: **Bayes-factor**
  - Ratio of the marginal likelihoods of two hypotheses, or ratio of posterior probabilities for equal priors.
  - Ratio of number of steps in one hypothesis compared to other (e.g. Mass hierarchy).





Summary and outlook

Summary:

- T2K have been successfully performing oscillation fits using frequentist and Bayesian approaches: groups cross-check oscillation fit results.
- The MaCh3 Bayesian approach is used for the joint ND+FD fits in Oscillation Analysis.
- Many diagnostic tools used to check reliability of the results.
- T2K Nature results 2020: https://www.nature.com/articles/s41586-020-2177-0

Outlook:

- MaCh3 is now developed to be an experiment-independent neutrino analysis tool, and is being used by new groups:
    - T2K+SK (accelerator + atmospheric)
    - T2K+NOvA (accelerator combination)
    - DUNE, HyperK, etc. (future Long-Baseline neutrino experiments)
- New challenges for oscillation analyses and for neutrino physics.

MaCh3 reference: 10.5281/zenodo.6683821