8th International Conference on High Energy Physics in the LHC Era

Fermion mass and width in QED in a magnetic field

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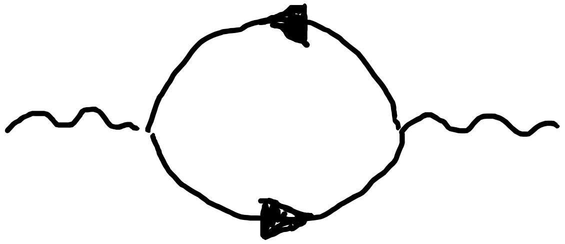
Fermion propagation in intense magnetic fields

- Magnetic fields influence the propagation of both charged **and** neutral particles (due to charged virtual fluctuations)



- Photon birrefringence (due to vacuum fluctuations): Breaking of Lorentz invariance leads to three polarization tensor structures

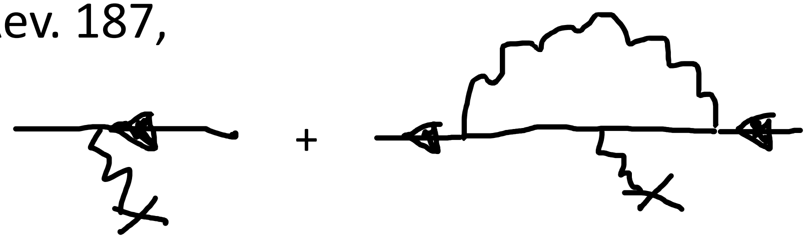
(A. Ayala, J. Castaño, M. Loewe and E. Muñoz, **Phys. Rev. D 101, 036016** (2020).

- The charged fermion propagator depends on the presence of a background magnetic field, as first noticed by Schwinger (Phys. Rev. 82, 664 (1951))

- Magnetic catalysis: Massless fermions may develop a finite magnetic mass, V. P. Gusynin et al., Nucl. Phys. B426, 249 (1996); Phys. Rev. D 53, 4747 (1995).

- Most results reported in the literature, concerning a fermion magnetic mass, are based on calculations restricted to the LLL, and find a **"double logarithm"** leading contribution (Tsai, Phys. Rev. D 10, 1342 (1974); Jancovici, Phys. Rev. 187, 2275 (1969); Machet, Int. J. Mod. Phys. 31, 1650071 (2016)).

$$m_B - m \sim \alpha\, m \left[\ln\left(|eB|/m^2\right)\right]^2$$



A semiclassical picture

"Classical Picture"

"The School of Athens" by Raphael (circa 1510)

"QFT Picture"

"Starry night" by V. van Gogh (1889)

Path-Integral formulation of QM

$$\langle x_f | e^{-it\hat{H}} | x_i \rangle = \int_{x(0)=x_i}^{x(t)=x_f} \mathcal{D}x(t) e^{iS[x(t)]} \sim [K(x_f, x_i, t)]\, e^{iS_{cl}[x(t)]}$$

The "classical" trajectory is the most probable one

Relativistic-covariant formulation of the EOM

$$\frac{dp^\mu}{d\tau} = \frac{q}{m} p_\nu F^{\nu\mu}$$

$$\gamma = \left(1 - \mathbf{v}^2/c^2\right)^{-1/2}$$

$$\frac{d}{dt}(\gamma m \mathbf{v}) = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$$

$$\frac{d\gamma}{dt} = \frac{q}{mc^2}\mathbf{v} \cdot \mathbf{E}$$

$$\mathbf{E} = 0$$

$$\frac{d\gamma}{dt} = 0 \to \gamma = \gamma_0$$

Decompose the velocity into mutually orthogonal components

$$\mathbf{v} = \mathbf{v}_\parallel + \mathbf{v}_\perp$$

$$\frac{d\mathbf{v}_\parallel}{dt} = 0 \longrightarrow \mathbf{v}_\parallel = \mathbf{v}_{\parallel,0}$$

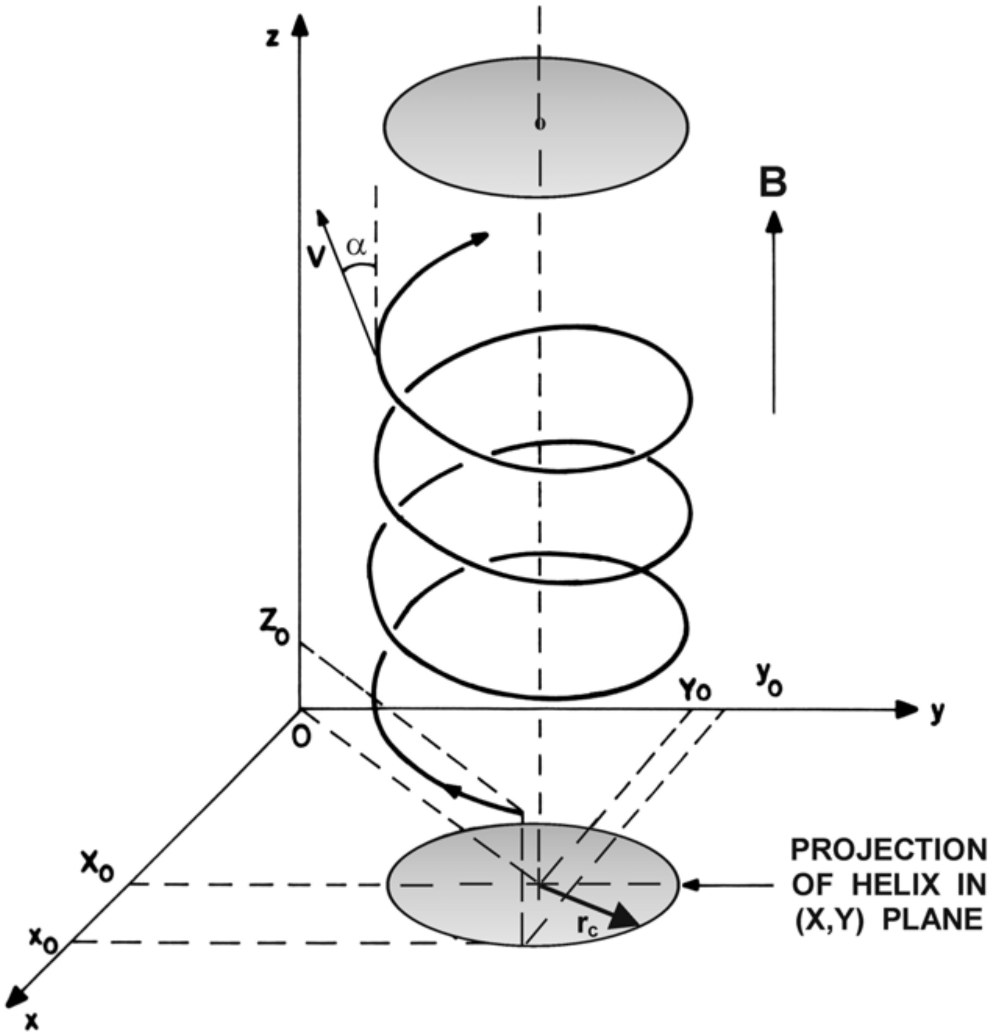
Cyclotron frequency

$$\boldsymbol{\Omega}_c = \frac{q\mathbf{B}}{\gamma_0\, mc}$$

$$\frac{d\mathbf{v}_\perp}{dt} = \mathbf{v}_\perp \times \boldsymbol{\Omega}_c$$

UCM in the plane perpendicular to the field

$$(x(t) - x_0)^2 + (y(t) - y_0)^2 = \frac{v_{\perp,0}^2}{\Omega_c^2} \equiv R_c^2$$



Radial-force: angular momentum is conserved along helical axis

$$\frac{dL_z}{dt} = 0$$

"Bohr-Sommerfeld quantization"

$$L_z = R_c p_\perp = m\gamma_0 \Omega_c R_c^2 \sim (2n+1)\hbar$$

The radius of the orbital helix is quantized

$$R_c = \left(\frac{(2n+1)\hbar}{m\gamma_0 \Omega_c} \right)^{1/2} = \sqrt{2n+1}\ l_B$$

Magnetic Landau radius

$$l_B \equiv \sqrt{\frac{\hbar c}{|qB|}}$$

Energy spectrum

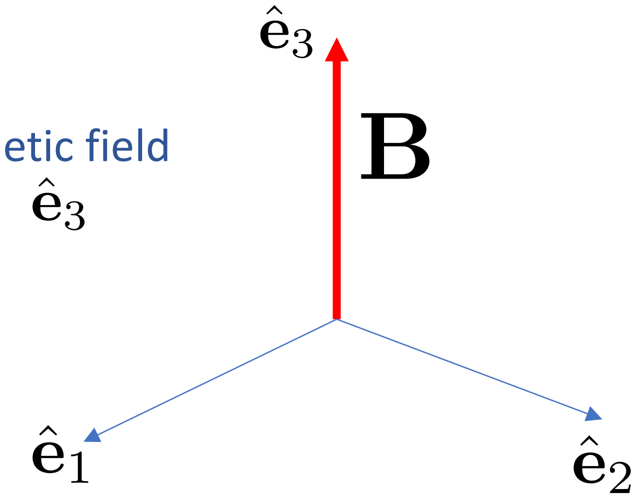
$$E = c\sqrt{\mathbf{p}_\parallel^2 + \mathbf{p}_\perp^2 + m^2c^2} = c\sqrt{\mathbf{p}_\parallel^2 + (2n+1)\frac{\hbar^2}{l_B^2} + m^2c^2}$$

Landau-level spectrum (almost exact except for spin)

$$E \sim c\sqrt{\mathbf{p}_\parallel^2 + (2n+1)\frac{\hbar|qB|}{c} + m^2c^2}$$

The quantum mechanical picture

Magnetic field along $\hat{\mathbf{e}}_3$



Gauge choice $$A_\mu = \frac{B}{2}(0, -x_2, x_1, 0)$$

Dirac´s equation $$\left[\gamma^\mu \left(i\partial_\mu - eA_\mu\right) + m\right]\psi(x) = 0$$

Look for eigenvectors and eigenstates $$\psi(x) = e^{iEt + ik_3x^3}\psi(\mathbf{x}_\perp)$$

Magnetic Landau-level spectrum $$E = \sqrt{k_3^2 + (2n + 1 - \alpha)|eB| + m^2} \qquad n \in \mathbb{N}_0, \quad \alpha = \pm 1$$

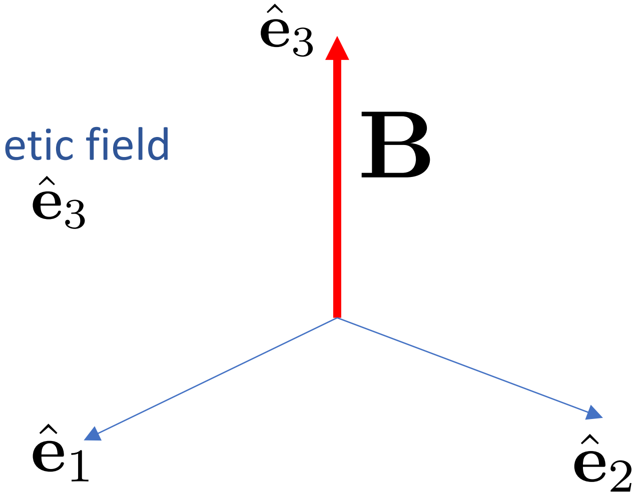
Split of the metric into two orthogonal subspaces $$g^{\mu\nu} = g_\parallel^{\mu\nu} + g_\perp^{\mu\nu} \quad \begin{cases} g_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, -1) \\ g_\perp^{\mu\nu} = \mathrm{diag}(0, -1, -1, 0) \end{cases}$$

For any 4-vector k, this implies the subsequent splitting of components

$$\not{k} = \not{k}_\parallel + \not{k}_\perp$$

$$k^2 = k_\parallel^2 - k_\perp^2 \Rightarrow \begin{cases} k_\perp^2 = k_1^2 + k_2^2 \\ k_\parallel^2 = k_0^2 - k_3^2 \end{cases}$$

The fermion propagator in a constant magnetic field background



Magnetic field along $\hat{\mathbf{e}}_3$

Schwinger, Phys. Rev. 82, 664 (1951)

Gauge choice $A_\mu = \frac{B}{2}(0, -x_2, x_1, 0)$

$$S_F(x, x') = \Phi(x, x') \int \frac{d^4 p}{(2\pi)^4} e^{-p\cdot(x-x')} S_F(p)$$

Translational invariant part (in Schwinger representation)

$$S_F(k) = -i \int_0^\infty \frac{d\tau}{\cos(eB\tau)} e^{i\tau\left(k_\parallel^2 - k_\perp^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + i\epsilon\right)} \left\{ \left[\cos(eB\tau) + i\gamma^1\gamma^2 \sin(eB\tau)\right](m + \not{k}_\parallel) + \frac{\not{k}_\perp}{\cos(eB\tau)} \right\}$$

Landau-level expansion

Miransky and Shovkovy, Phys. Rep. 576, 1 (2015)

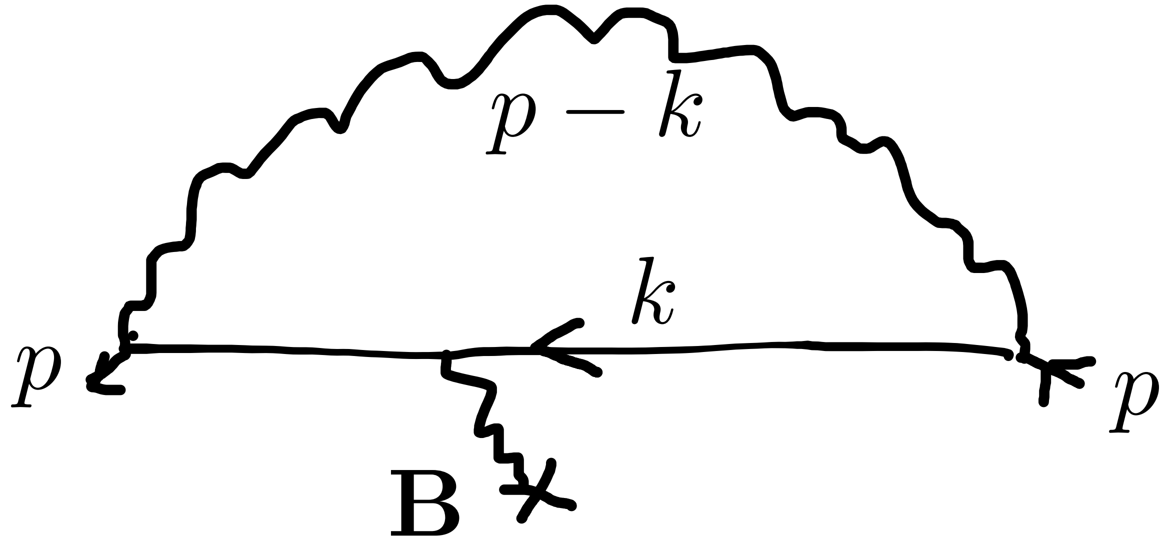
Gusynin, Miransky and Shovkovy, Nucl. Phys. B462, 249 (1996)

$$iS_F(k) = ie^{-\frac{\mathbf{k}_\perp^2}{|eB|}} \sum_{n=0}^{\infty} (-1)^n \frac{D_n(eB, k)}{k_\parallel^2 - m^2 - 2n|eB| + i\epsilon}$$

The Schwinger phase may break translational invariance

$$\Phi(x, x') = \exp\left\{ ie \int_x^{x'} d\xi^\mu \left[A_\mu + \frac{1}{2} F_{\mu\nu} (\xi - x')^\nu \right] \right\}$$

The self-energy diagram (at one loop)



$$-i\Sigma(p) = (-ie)^2 \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu i S_F(k) \gamma^\nu G_{\nu\mu}(p-k)$$

What about the Schwinger magnetic phase?

The Schwinger phase can be removed by a gauge transformation

$$\Phi(x,x') = \exp\left\{ i\,e \int_{x'}^{x} d\xi^\mu \left[A_\mu + \frac{1}{2} F_{\mu\nu} \left(\xi - x'\right)^\nu \right] \right\}$$

Path parametrization $\quad \xi^\mu = x'^\mu + t(x^\mu - x'^\mu), \quad \text{for } 0 < t < 1$

First, we remark that the second term vanishes

$$\int_{x'}^{x} d\xi^\mu F_{\mu\nu}(\xi)(\xi - x')^\nu = \frac{ie}{2} \int_0^1 (x - x')^\mu F_{\mu\nu} (x - x')^\nu dt = 0$$

$$F_{\mu\nu} = -F_{\nu\mu}$$

Original choice of gauge $\quad A_\mu = \frac{B}{2}(0, -x_2, x_1, 0)$

The first integral is given by

$$\int_{x'}^{x} d\xi^\mu A_\mu(\xi) = (x - x')^\mu A_\mu(x')$$

We can remove this term by introducing a gauge transformation

$$A_\mu(\xi) \to A'_\mu(\xi) = A_\mu(\xi) + \frac{\partial \alpha(\xi)}{\partial \xi^\mu}$$

With the choice

$$\alpha(\xi) = \frac{B}{2}(x'_2 \xi^1 - x'_1 \xi'_2)$$

Therefore, we obtain

$$\int_{x'}^{x} d\xi^\mu A'_\mu(\xi) = (x - x')^\mu A_\mu(x') + \alpha(x) - \alpha(x') = 0$$

and **the Schwinger phase is removed**

$$\Phi(x, x') = 0$$

Schwinger parametrization of the self-energy

Photon propagator

$$G_{\nu\mu}(p-k) = \frac{-ig_{\mu\nu}}{(p-k)^2 + i\epsilon} = -g_{\mu\nu} \int_0^\infty dx e^{ix[(p-k)^2 + i\epsilon]}$$

Fermion propagator

$$\gamma^\mu i S_F(k) \gamma_\mu = \int_0^\infty \frac{d\tau}{\cos(eB\tau)} e^{i\tau(k_\parallel^2 - k_\perp^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + i\epsilon)} \times \Big\{ 4m\cos(eB\tau) - 2\not{k}_\parallel \cos(eB\tau) - i\sin(eB\tau)\gamma^1\gamma^2\not{k}_\parallel - \frac{2\not{k}_\perp}{\cos(eB\tau)} \Big\}$$

Integrating out the internal k-momenta, we obtain the self-energy in terms of Schwinger parameters x, τ

$$\Sigma(p,B) = \frac{2e^2}{(4\pi)^2} \int_0^\infty \int_0^\infty \frac{dx d\tau}{(x+\tau)(x + \frac{\tan(eB\tau)}{eB})} \Bigg[2m - \frac{x}{x+\tau}\not{p}_\parallel - \frac{x\not{p}_\perp}{(x + \frac{\tan(eB\tau)}{eB})[\cos(eB\tau)]^2} - \frac{x\tan(eB\tau)}{x+\tau} i\gamma^1\gamma^2\not{p}_\parallel \Bigg] e^{i(xp^2 - \frac{x^2}{x+\tau}p_\parallel^2 + \frac{x^2 p_\perp^2}{x + \frac{\tan(eB\tau)}{eB}} - \tau m^2 + i\epsilon)}$$

Introducing dimensionless variables $\tau = \frac{s(1-y)}{m^2}, \quad x = \frac{sy}{m^2}, \quad \mathcal{B} = \frac{|eB|}{m^2}, \quad \rho^2_{\perp,\parallel} = \frac{p^2_{\perp,\parallel}}{m^2}$

Self energy

$$\Sigma(p,B) = \frac{2me^2}{(4\pi)^2}\int_0^\infty \frac{ds}{s}\int_0^1 dy[(A)+(B)-(C)]\, e^{is(\varphi(y,\rho,B)+i\epsilon)}$$

$$(A) = \frac{(2-y\not{p}_\parallel)\cos(\mathcal{B}s(1-y))}{y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}}$$

$$(B) = \frac{-y\not{p}_\perp}{[y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}]^2}$$

$$(C) = \frac{y\sin(\mathcal{B}s(1-y))}{y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}} \times i\gamma^1\gamma^2\mathrm{sign}(eB)\not{p}_\parallel$$

Magnetic field-dependent phase

$$\varphi(y,\rho,B) = y\rho^2 - y^2\rho^2_\parallel + \frac{y^2\cos(\mathcal{B}s(1-y))\rho^2_\perp}{y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}} - (1-y)$$

Renormalization conditions and counterterms

The renormalization conditions are imposed such that we recover the "free" fermion mass in the limit B -> 0

(I) $$\Sigma^{\text{ren}}(p,0)\big|_{\not{p}=m} = 0$$

(II) $$\frac{\partial}{\partial \not{p}}\Sigma^{\text{ren}}(p,0)\Big|_{\not{p}=m} = 0$$

$$\lim_{B\to 0}\varphi(y,\rho,B) \equiv \varphi(y,\rho,0) = (1-y)y(\not{\rho}^2-1)-(1-y)^2$$

$$\Sigma^{\text{ren}}(p,0) = \frac{2me^2}{(4\pi)^2}\int_0^\infty \frac{ds}{s}\int_0^1 dy e^{is(-(1-y)^2+i\epsilon)}\left[(2-y\not{\rho})e^{isy(1-y)(\not{\rho}^2-1)} + \text{c.t.}\right]$$

We thus determine the corresponding counterterms required to impose such conditions

$$\text{c.t.}_1 = -(2-y)$$

$$\text{c.t.}_2 = -(\not{\rho}-1)\left\{-\frac{y}{m} + 2is\frac{y(1-y)}{m}(2-y)\right\}$$

Renormalized self-energy at finite B

$$\Sigma^{\mathrm{ren}}(p,B) = \frac{2me^2}{(4\pi)^2}\int_0^\infty \frac{ds}{s}\int_0^1 dy e^{is(-(1-y)^2+i\epsilon)}$$
$$\times\left[((A)+(B)-(C))e^{is(\varphi(y,\rho,B)+(1-y)^2)}\right.$$
$$\left. -(2-y)-(\not{p}-1)\left\{-\frac{y}{m}+2is\frac{y(1-y)}{m}(2-y)\right\}\right]$$

$$(C) = \frac{y\sin(\mathcal{B}s(1-y))}{y\cos(\mathcal{B}s(1-y))+\frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}}\times i\gamma^1\gamma^2\mathrm{sign}(eB)\not{p}_{\parallel}$$

→ This part of the selfenergy is proportional to the spin operator

$$\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} = eB\, i\gamma^1\gamma^2$$

Spin projectors

$$\hat{O}^{(\pm)} = \frac{1}{2}(\mathbf{1}\pm i\gamma^1\gamma^2\mathrm{sign}(eB))$$

Definition of the fermion magnetic mass

The renormalization of the mass is, as usual, defined in terms of the shift of the pole of the propagator

$$\delta m_B = m_B - m = \Sigma^{\text{ren}}(p, B)\big|_{\not{p}_{\parallel}=m}$$

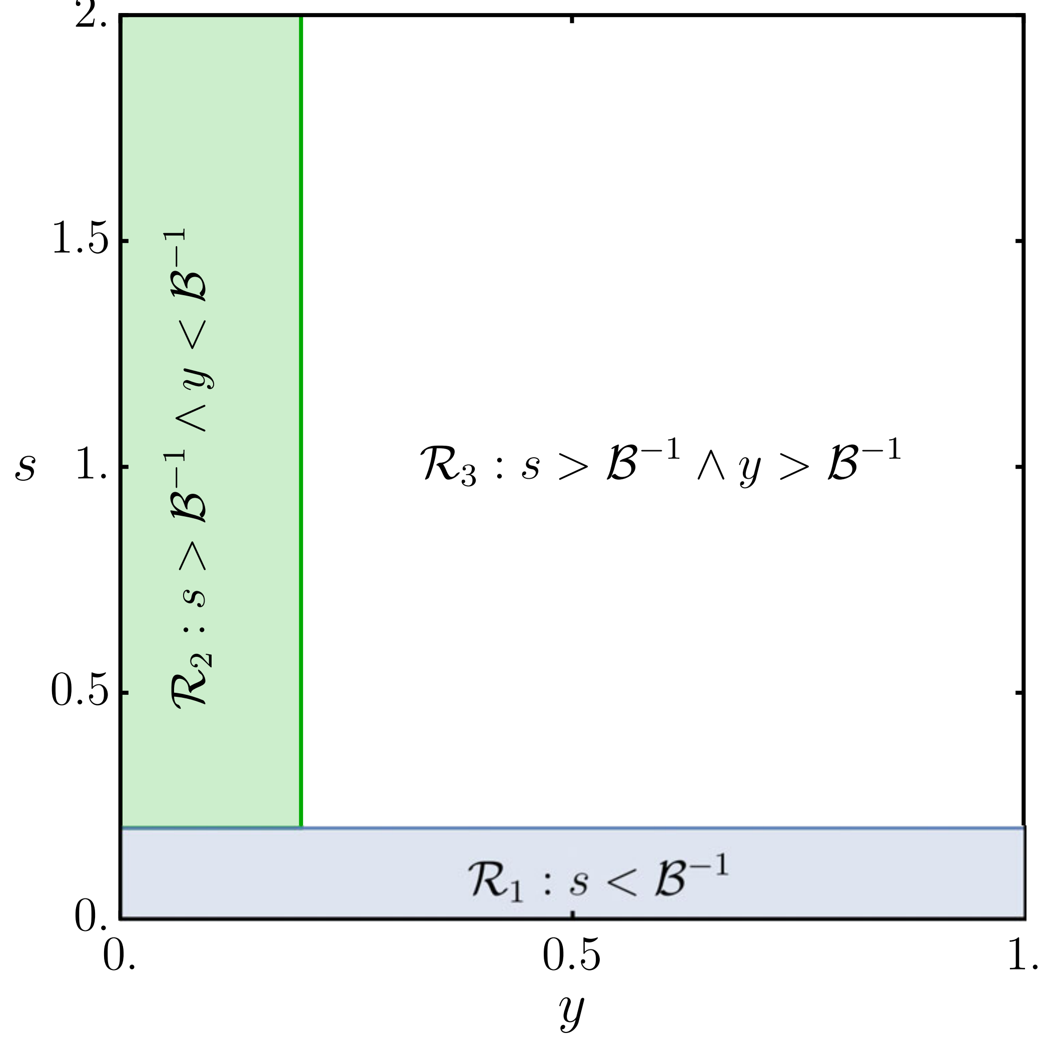
This shift is strictly magnetic,
and by construction it satisfies

$$\lim_{B\to 0} \delta m_B = 0$$

The presence of the spin-magnetic field interaction, expressed in terms of the projectors, determines a different mass shift for each spin component

$$\delta m_B = \hat{O}^{(+)} \delta m_B^{(+)} + \hat{O}^{(-)} \delta m_B^{(-)}$$

$$\delta m_B^{(\pm)} = \frac{2me^2}{(4\pi)^2}\int_0^1 dy \int_0^\infty \frac{ds}{s} e^{is(-(1-y)^2+i\epsilon)}\left[\frac{(2-y)\cos(\mathcal{B}s(1-y))}{y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}} - (2-y) \mp \frac{y\sin(\mathcal{B}s(1-y))}{y\cos(\mathcal{B}s(1-y)) + \frac{\sin(\mathcal{B}s(1-y))}{\mathcal{B}s}}\right]$$



Our strategy: Divide the integration domain into three separate regions

$$\int_0^1 dy \int_0^\infty ds = \underbrace{\int_0^1 dy \int_0^{\mathcal{B}^{-1}} ds}_{\mathcal{R}_1} + \underbrace{\int_0^{\mathcal{B}^{-1}} dy \int_{\mathcal{B}^{-1}}^\infty ds}_{\mathcal{R}_2} + \underbrace{\int_{\mathcal{B}^{-1}}^1 dy \int_{\mathcal{B}^{-1}}^\infty ds}_{\mathcal{R}_3}$$

$$\delta m_B^{(\pm)} = \delta m_B^{(\pm)}|_{\mathcal{R}_1} + \delta m_B^{(\pm)}|_{\mathcal{R}_2} + \delta m_B^{(\pm)}|_{\mathcal{R}_3}$$

We are interested in the high magnetic field regime where

$$\mathcal{B} \gg 1$$

The contribution arising from Region 1

$$\delta m_B^{(\pm)}|_{\mathcal{R}_1} = \frac{2me^2}{(4\pi)^2}\left(-\frac{157}{2016} + \frac{2041}{56700}\, i\mathcal{B}^{-1} \mp \left[\frac{91}{540} - \frac{257}{10080}\, i\mathcal{B}^{-1}\right]\right) + O(\mathcal{B}^{-2})$$

The contribution arising from Region 2

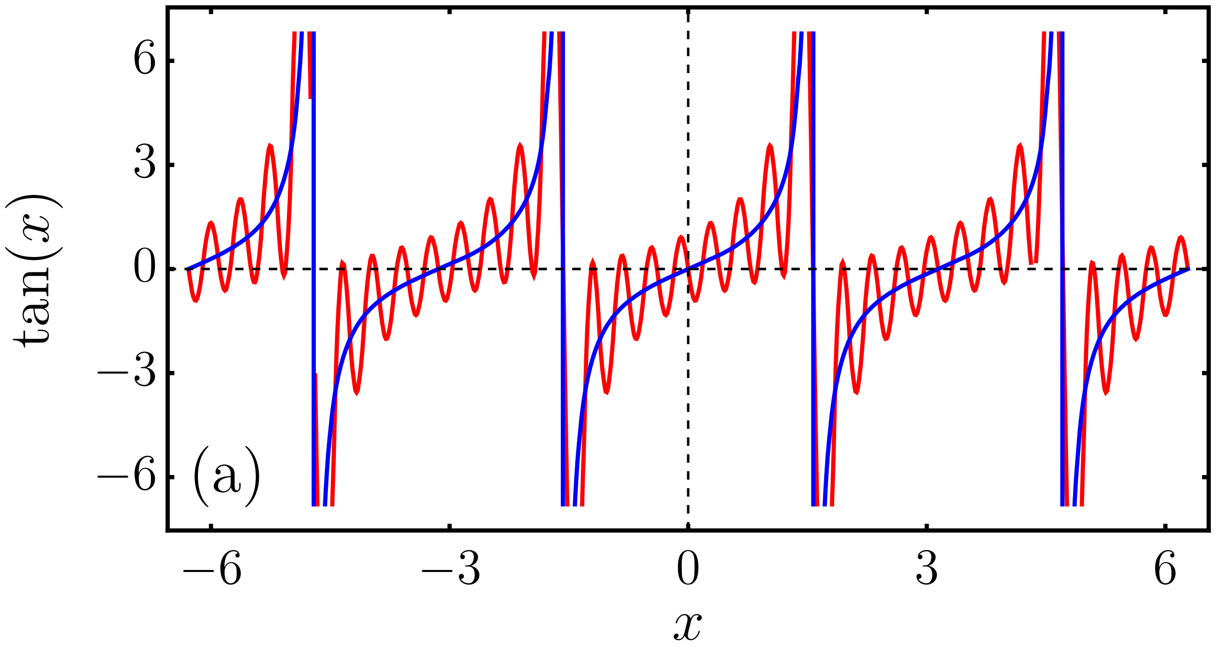
$$\delta m_B^{(\pm)}|_{\mathcal{R}_2} \sim \frac{2me^2}{(4\pi)^2}\left\{-1 - 2\mathcal{B}^{-1}\ln(\mathcal{B}) + \mathcal{B}^{-1}\left(2\gamma - 2\ln[|1-e^{2i}|] + i\left(\pi \pm \frac{1}{2}\right)\right)\right\} + O(\mathcal{B}^{-2})$$

The (dominant) contribution arising from Region 3, needs to be further analyzed…

$$\delta m_B^{(\pm)}|_{\mathcal{R}_3} = \frac{2me^2}{(4\pi)^2}\int_{\mathcal{B}^{-1}}^{1} dy \int_{\mathcal{B}^{-1}}^{\infty} \frac{ds}{s}\, e^{is(-(1-y)^2+i\epsilon)}\left[\frac{(2-y)(1-y)}{y} \mp \tan(\mathcal{B}s(1-y))\right]$$

An appropriate (periodic) series representation for the trigonometric tangent function

$$\tan(\mathcal{B}s(1-y)) = 2\sum_{n=1}^{\infty}(-1)^{n-1}\sin(2\mathcal{B}s(1-y))$$
$$= i\sum_{n=1}^{\infty}(-1)^{n}\left(e^{2in\mathcal{B}s(1-y)} - e^{-2in\mathcal{B}s(1-y)}\right)$$



Integrate term by term…

The LLL (n=0)

$$\int_{\mathcal{B}^{-1}}^{\infty}\frac{ds}{s}\,e^{is(-(1-y)^2+i\epsilon)} = \Gamma\left(0, i\,\frac{(1-y)^2}{\mathcal{B}}\right)$$

Incomplete Gamma function

$$\Gamma(0, iz) = -\gamma - \ln(iz) - \sum_{k=1}^{\infty}\frac{(-iz)^k}{k(k!)}$$

Higher LLs (n>0)

$$\int_{1/\mathcal{B}}^{\infty}\frac{ds}{s}\,e^{is(-(1-y)(1-y\pm 2n\mathcal{B})+i\epsilon)} = \Gamma\left(0, i\,\frac{(1-y)(1-y\pm 2n\mathcal{B})}{\mathcal{B}}\right)$$

The integral expression for the magnetic mass correction becomes

$$\delta m_B^{(\pm)}|_{\mathcal{R}_3} = \frac{2me^2}{(4\pi)^2}\int_{\mathcal{B}^{-1}}^{1} dy\left[\frac{(2-y)(1-y)}{y}\Gamma\left(0, i\frac{(1-y)^2}{\mathcal{B}}\right) \mp i\sum_{n=1}^{\infty}(-1)^n\left\{\Gamma\left(0, i\frac{(1-y)(1-y-2n\mathcal{B})}{\mathcal{B}}\right) - \Gamma\left(0, i\frac{(1-y)(1-y+2n\mathcal{B})}{\mathcal{B}}\right)\right\}\right]$$

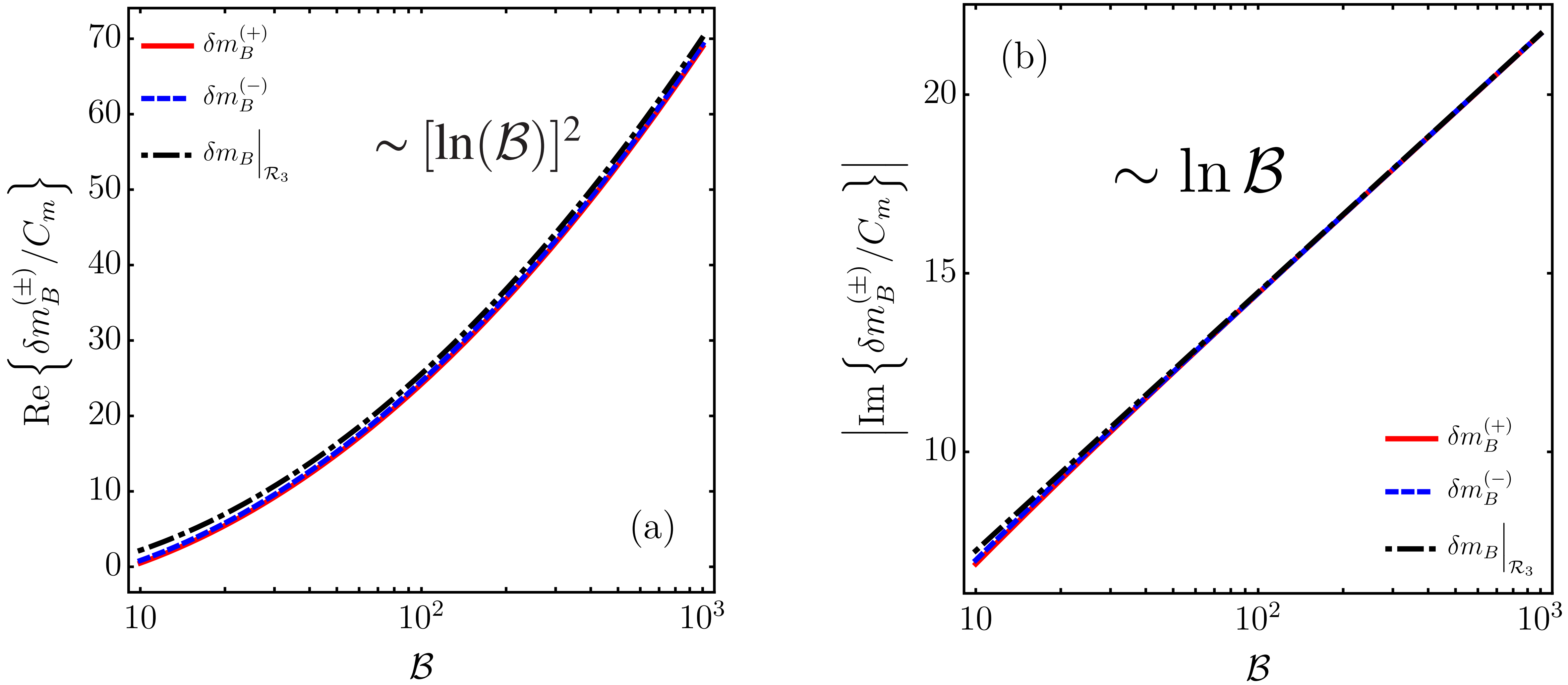
From the series representation of the incomplete Gamma function, the LLL (n = 0) contribution yields

$$\int_{\mathcal{B}^{-1}}^{1}\frac{dy}{y}(1-y)(2-y)\Gamma[0, i\mathcal{B}^{-1}(1-y)^2] = 2[\ln(\mathcal{B})]^2 - \left(2\gamma + \frac{5}{2} + i\pi\right)\ln(\mathcal{B}) + O(\mathcal{B}^0)$$

Therefore, the magnetic mass correction at the leading order is

$$\delta m_B^{(\pm)}|_{\mathcal{R}_3} = \frac{2me^2}{(4\pi)^2}\left\{2[\ln(\mathcal{B})]^2 - \left[2\gamma + \frac{5}{2} + i\pi\right]\ln(\mathcal{B})\right\} + O(\mathcal{B}^0)$$

Real and imaginary parts for both spin projections



$$C_m = 2me^2/(4\pi)^2$$

$$\delta m_B^{(\pm)} = \delta m_B^{(\pm)}|_{\mathcal{R}_1} + \delta m_B^{(\pm)}|_{\mathcal{R}_2} + \delta m_B^{(\pm)}|_{\mathcal{R}_3}$$

Analysis of our results

The inverse "dressed" propagator can be written, after Dyson's equation

$$[-iS_F(p)]^{-1} = \not{p} - m - \Sigma(p,B)$$

$$= (\hat{O}^{(+)} + \hat{O}^{(-)})(\not{p} - m) - \hat{O}^{(+)}\Sigma^{(+)}(p,B) - \hat{O}^{(-)}\Sigma^{(-)}(p,B)$$

$$= \hat{O}^{(+)}[-i\Delta_F^{(+)}(p)]^{-1} + \hat{O}^{(-)}[-i\Delta_F^{(-)}(p)]^{-1}$$

The dressed fermion propagator for each spin projection

$$\Delta_F^{(\pm)}(p) = \frac{i}{\not{p} - m - \Sigma^{(\pm)}(p,B) + i\epsilon}$$

The physical mass is renormalized by the real part of the selfenergy

$$m_B^{(\pm)} = m + \mathrm{Re}\Sigma^{(\pm)}(m,B) \qquad Re\,\Sigma^{(\pm)}(m,B) \sim \alpha\, m\left[\ln\left(|eB|/m^2\right)\right]^2$$

What is the role of the imaginary part? $\quad Im\,\Sigma^{(\pm)}(m,B) \sim -\alpha\, m \ln\left(|eB|/m^2\right)$

$$\Delta_F^{(\pm)}(p) = \frac{i}{\not{p} - m_B^{(\pm)} - i\mathrm{Im}\Sigma^{(\pm)}(m,B) + i\epsilon} \sim i\frac{\not{p} + m_B^{(\pm)} + i\mathrm{Im}\Sigma^{(\pm)}(m,B)}{p^2 - (m_B^{(\pm)})^2 - 2im_B^{(\pm)}\mathrm{Im}\Sigma^{(\pm)}(m,B)}$$

Breit-Wigner resonance

$$\Gamma^{(\pm)} = -2\mathrm{Im}\Sigma^{(\pm)}(m,B)$$

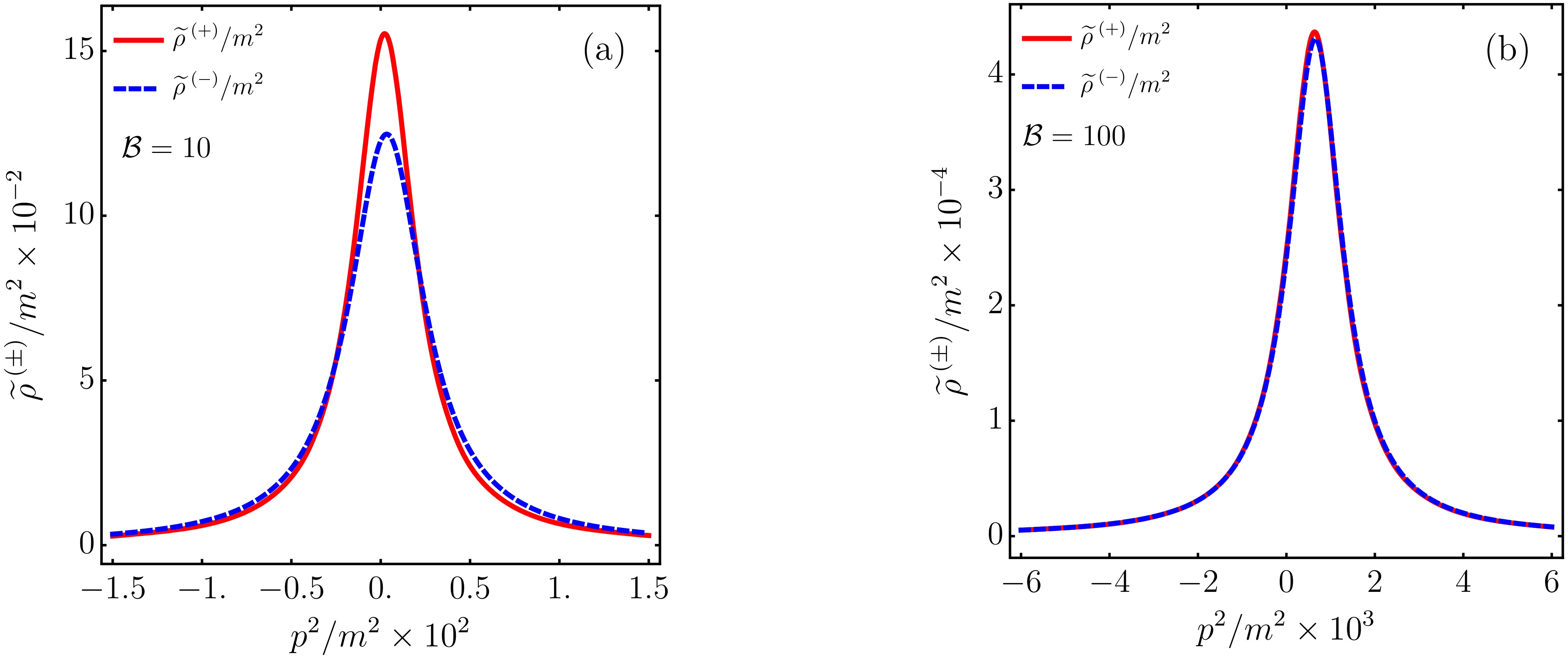
$$\frac{\Gamma^{(\pm)}}{m_B^{(\pm)}} = -\frac{2\mathrm{Im}\Sigma^{(\pm)}(m,B)}{m_B^{(\pm)}} \sim \frac{\ln(\mathcal{B})}{[\ln(\mathcal{B})]^2} \sim [\ln(\mathcal{B})]^{-1}$$

Relative spectral width of the resonance decays to zero as

$$\frac{|eB|}{m^2} = \mathcal{B} \to \infty$$

Spectral density is a Lorentzian

$$\tilde{\rho}(p^2) = -\frac{1}{\pi}\mathrm{Im}\left(\frac{1}{p^2 - (m_B^{(\pm)})^2 + im_B^{(\pm)}\Gamma^{(\pm)} + i\epsilon}\right) \sim \frac{m_B^{(\pm)}\Gamma^{(\pm)}/\pi}{(p^2 - (m_B^{(\pm)})^2)^2 + [m_B^{(\pm)}\Gamma^{(\pm)}]^2}$$



$$\tilde{\rho}(p^2) \sim \frac{\frac{1}{\pi}\frac{\Gamma^{\pm}}{m_B^{\pm}}}{\left(\frac{p^2}{(m_B^{\pm})^2}-1\right)^2+\left(\frac{\Gamma^{\pm}}{m_B^{\pm}}\right)^2} \xrightarrow[\mathcal{B}\to\infty]{} \delta\left[p^2-\left(m_B^{\pm}\right)^2\right]$$

Conclusions

- We revisited the problem of the fermion mass renormalization due to a "classical" magnetic field background in QED
- We find that the self-energy depends on the spin polarization (Zeeman-like coupling)
- The self-energy displays both a real and an imaginary part, that for large fields

$$Re\,\Sigma^{(\pm)}(m,B) \sim \alpha\, m\left[\ln\left(|eB|/m^2\right)\right]^2 \qquad Im\,\Sigma^{(\pm)}(m,B) \sim -\alpha\, m \ln\left(|eB|/m^2\right)$$

- The renormalized mass depends on the spin polarization, and is determined by the real part of the self-energy
- The imaginary part develops a spectral broadening due to the contribution of all the Landau levels in a Lorentzian distribution
- As the magnetic field grows very large, the relative spectral width decreases and the Lorentzian converges to a delta function, with a definite mass arising from the lowest Landau level n = 0

Thank you!

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