Constraints on Very Special Relativity from the Electron's gyromagnetic factor

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Koch, B., Muñoz, E., Santoni, A. (2022). Corrections to the gyromagnetic factor in very special relativity. Physical Review D, 106(9), 096009.

January 12, 2023

VSR is an original idea from Cohen and Glashow (2006).[1]

→ New Space-time fundamental symmetry:
Traslations + Special Lorentz's Subgroups

Motivations

- The addition of discrete symmetries, like P, CP or T enlarges the symmetry group to the full Poincarè group.

$$\text{Small } \textbf{CP VIOLATIONS} \Longleftrightarrow \text{Small } \textbf{VSR EFFECTS} \; .$$

- Alternative mechanism for **Neutrino masses**
- Same kinematical consequences of Special Relativity

[1] Andrew G Cohen and Sheldon L Glashow. "Very special relativity". Physical review letters, 97(2):021601, 2006.

$SIM(2)$ = Largest Lorentz's proper subgroup → 4 generators

$$T_1 = K_1 + J_2 \; ; \;\; T_2 = K_2 - J_1 \; ; \;\; J_3 \; ; \;\; K_3 \; ,$$

where $\vec{J}$ and $\vec{K}$ are the usual rotations and boosts generators.

$SIM(2)$ Advantages

- Only 1 invariant Tensor: $\eta_{\mu\nu}$
- Directly implies CPT invariance of the Quantum Theory

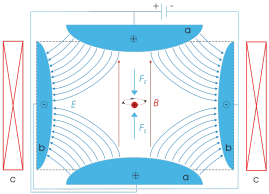
→ There exists a **preferred light-like spacetime direction**, labelled by vector n_μ, which is left almost invariant under $SIM(2)$

$$n_\mu \underset{SIM(2)}{\longrightarrow} e^{\phi} n_\mu \; , \tag{1}$$

New possibilities for **Lorentz-violating** Lagrangian terms!

At the moment, the **electron's $g-2$ factor** value is the most precise theoretical prediction ever verified by experiments.

Electron's **Pennings traps** experiments allows to calculate $a = \frac{g-2}{2}$ by measuring convenient differences of **energy eigenvalues** of the system



$$i.e.^{2} \quad a = \frac{E_0^- - E_1^+}{E_1^- - E_-^0 + \frac{3}{2}m\epsilon^2} \quad with \quad \epsilon \sim 10^{-9} \tag{2}$$

→ VSR corrections?

[2] Koch, B., Asenjo, F., Hojman, S. (2021). Almost relevant corrections for direct measurements of electron's g-factor. arXiv preprint arXiv:2110.05506.

Dirac Lagrangian in VSR framework

$$\mathcal{L}_f = \bar{\psi}\left(i\not{\partial} - m + i\frac{M^2}{2}\frac{\not{n}}{n\cdot\partial}\right)\psi \quad (3)$$

- where $\frac{1}{n\cdot\partial} = \int_0^\infty d\alpha e^{-\alpha n\cdot\partial}$
- *Dimensional Analysis* $\rightarrow [M] = mass$

"Squaring" the field equation $(i\not{\partial} - m + i\frac{M^2}{2}\frac{\not{n}}{n\cdot\partial})\psi = 0$ and using $n \cdot n = 0$ we get:

$$(\partial^2 + m_f^2)\psi = 0 \underset{Fourier}{\longrightarrow} (p^2 - m_f^2)\psi = 0 \;\; with \;\; m_f^2 \equiv m^2 + M^2 \quad (4)$$

where m_f is the **effective mass** of the spin 1/2 fermion in VSR

Dirac Equation in VSR with magnetic field $\vec{B}$

Adding a static and homogeneous **magnetic field** $\vec{B} = (0, 0, B)$

$$\left(i\not{D} - m + i\frac{M^2}{2}\frac{\not{n}}{n \cdot D} \right) \psi = 0 \quad \textit{with} \quad D_\mu = \partial_\mu + ieA_\mu \tag{5}$$

with the following **gauge choice**

$$A_\mu = (0, 0, Bx^1, 0) \rightarrow n \cdot A = 0 \tag{6}$$

Therefore, our eigenstates are

$$\psi(x) = e^{-iEt} e^{i\left(k^3x^3 + k^2x^2\right)} \begin{pmatrix} \varphi(x^1) \\ \chi(x^1) \end{pmatrix} . \tag{7}$$

For the case $\vec{B} \,||\, \vec{n}$

$$i \frac{M^2}{2} \frac{\not{n}}{n \cdot \partial} \rightarrow -\frac{M^2}{2} \frac{\gamma^0 - \gamma^3}{E - k^3} \,, \tag{8}$$

Solving for χ and defining

$$a(k^3, E) = \frac{1}{eB} \left[\left(E - \frac{M^2}{2(E - k^3)} \right)^2 - m^2 - \left(k^3 - \frac{M^2}{2(E - k^3)} \right)^2 \right] \tag{9}$$

we obtain an equation for $\varphi(x^1) = \begin{pmatrix} f^1(x^1) \\ f^2(x^1) \end{pmatrix}$

$$\left[-\partial_1^2 + \left(eBx^1 - k^2 \right)^2 - eB\sigma^3 - eB\, a(k_3, E) \right] \begin{pmatrix} f^1 \\ f^2 \end{pmatrix} = 0 \tag{10}$$

Defining $\xi = \sqrt{eB}\left(x^1 - \frac{k^2}{eB}\right)$,
we get to two uncoupled eigenvalues equations

$$\left[-\frac{d^2}{d\xi^2} + \xi^2 - \alpha\right] f_\alpha(\xi) = a(k^3, E) f_\alpha(\xi) \quad , \quad \alpha = \pm 1 \tag{11}$$

f_α = Hermite polynomials → implying the **quantization condition**

$$a(k^3, E) + \alpha = 2n + 1 \tag{12}$$

which translate to the **exact energy spectrum**

$$E_{\pm}^{(0)}(k^3, n, \alpha) = \pm\sqrt{eB(2n + 1 - \alpha) + (k^3)^2 + m_f^2} \tag{13}$$

→ No VSR modifications apart from $m_f^2 = m^2 + M^2$

Giving up on $\vec{B} \,||\, \vec{n}$, things get messier since

$$i\frac{M^2}{2}\not{N} \to -\frac{M^2}{2}\frac{\gamma^0 E - \gamma^1 \sin\theta - \gamma^3 \cos\theta}{E - k^3 \cos\theta - p^1 \sin\theta}\,. \tag{14}$$

As a result f^1 and f^2 now get mixed up $\to$ **perturbative treatment**

$$\Big[\underbrace{-\partial_\xi^2 + \xi^2 - \sigma^3}_{H_0} + \underbrace{\lambda\,\frac{\sin\theta}{P_\xi^2}(\sin\theta\,\sigma^3 - \cos\theta\,\sigma^1)}_{V}\Big]\begin{pmatrix} f^1 \\ f^2 \end{pmatrix} = a\begin{pmatrix} f^1 \\ f^2 \end{pmatrix} \tag{15}$$

with $P_\xi \equiv E - k^3\cos\theta + i\sqrt{eB}\sin\theta\partial_\xi$ and $\lambda \equiv \frac{M^2}{2}$

Perturbative Spectrum and VSR $g-$factor

The **unperturbed spectrum** was like

$$\left\{ \left|\vec{f}^{(0)}, 0, +1\right\rangle, \underbrace{\left[\left|\vec{f}^{(0)}, 0, -1\right\rangle, \left|\vec{f}^{(0)}, 1, +1\right\rangle\right]}_{degenerate}, \ldots, \underbrace{\left[\left|\vec{f}^{(0)}, n, -1\right\rangle, \left|\vec{f}^{(0)}, n+1, +1\right\rangle\right]}_{degenerate}, \ldots \right\}$$

Defining the perturbative parameters

- in $\mu \equiv M^2/m_f^2$
- in $\epsilon \equiv eB/m_f^2$

We get to the **perturbative correction**

$$\lambda a^{(1)}_{n,\alpha} = \alpha \frac{\mu \sin^2\theta}{2} \left(1 + 3(n + \frac{1}{2} + \delta_{\alpha,-1})\epsilon \sin^2\theta + \frac{n + \delta_{\alpha,-1}}{4} \epsilon \cos^2\theta\right) \tag{16}$$

New **perturbative energy spectrum**
+ Radiative corrections, i.e. $\frac{a}{2}\sigma_{\mu\nu}F^{\mu\nu}$

$$\begin{aligned}\frac{E_{n,\alpha}}{m_f} &= 1+\frac{\epsilon}{2}(2n+1-\alpha)+\frac{\epsilon}{4}\mu\alpha\sin^2\theta-\frac{\epsilon^2}{8}(2n+1-\alpha)^2-\frac{\epsilon}{2}\alpha a \\ &\quad -\frac{3}{8}\epsilon^2\alpha\mu\sin^2\theta\,(2n+1-\alpha)+\frac{3}{4}\epsilon^2\alpha\mu(n+\frac{1}{2}+\delta_{\alpha,-1})\sin^4\theta \\ &\quad +\frac{\epsilon^2}{16}\alpha\mu(n+\delta_{\alpha,-1})\sin^2\theta\cos^2\theta \end{aligned} \tag{17}$$

Let's remember the operative recipe to apply to our VSR spectrum

$$a_{VSR} = \frac{E_0^- - E_1^+}{E_1^- - E_-^0 + \frac{3}{2}m_f\epsilon^2} \tag{18}$$

Using our spectrum for a_{VSR} leads to a $g-$**factor discrepancy**

$$g_{VSR} - g = -\mu\left[1-\frac{11}{8}(2-\frac{34}{11}\sin^2\theta - a\cos^2\theta)\epsilon\right]\sin^2\theta \tag{19}$$

VSR constraint from $g - 2$

At this point we are ready to set constraints!

Neglecting Penning traps' electric field → $g_{exp} \sim g_{VSR}$
and considering

$$\begin{cases} g_{exp}/2 = 1.00115965218073(28) \\ g/2 = 1.001159652182032(720) \end{cases} \tag{20}$$

we get the following **VSR parameter's upperbound**[3]

$$\mu \lesssim 3 \times 10^{-12} \rightarrow M \lesssim 1\, eV \tag{21}$$

still allowing VSR to be the origin of neutrinos' masses.

[3] https://doi.org/10.1103/PhysRevD.106.096009

- We saw **VSR Dirac theory** with an external magnetic field

$$\left(i\not{D} - m + i\frac{M^2}{2}\frac{\not{n}}{n \cdot D} \right) \psi = 0 \tag{22}$$

and studied its consequences on the energy spectrum

- We found for the electronic **VSR mass** M the **upperlimit**

$$M \lesssim 1\, eV \tag{23}$$

not excluding VSR from neutrinos masses' mechanisms.

Upcoming Work

Extension to the muonic case, which is recently receiving huge attention due to increasing experimental discrepancy.

Thanks for your attention!