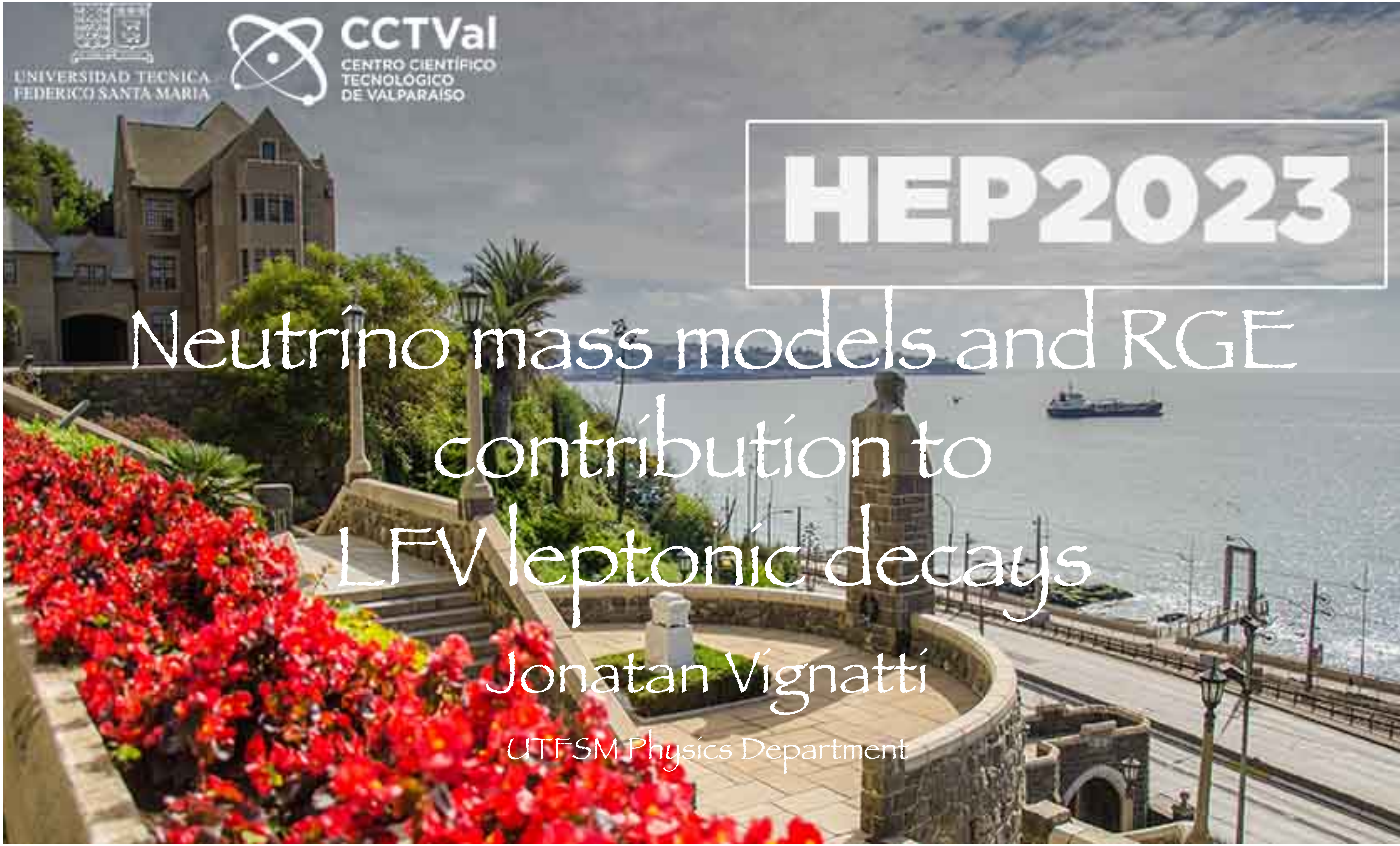

Neutrino mass models and RGE contribution to LFV leptonic decays

Jonatan Vignatti

UTFSM Physics Department

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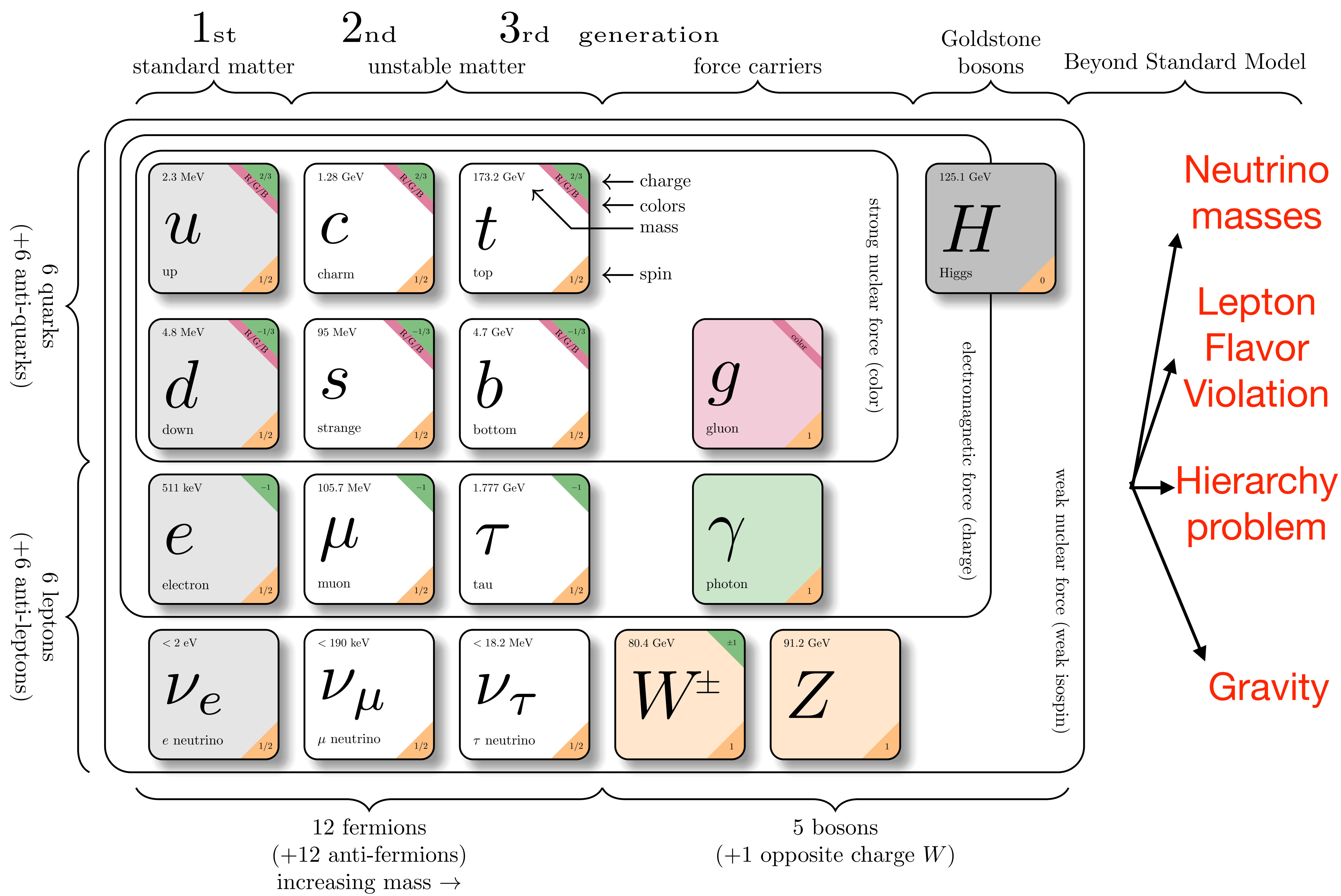
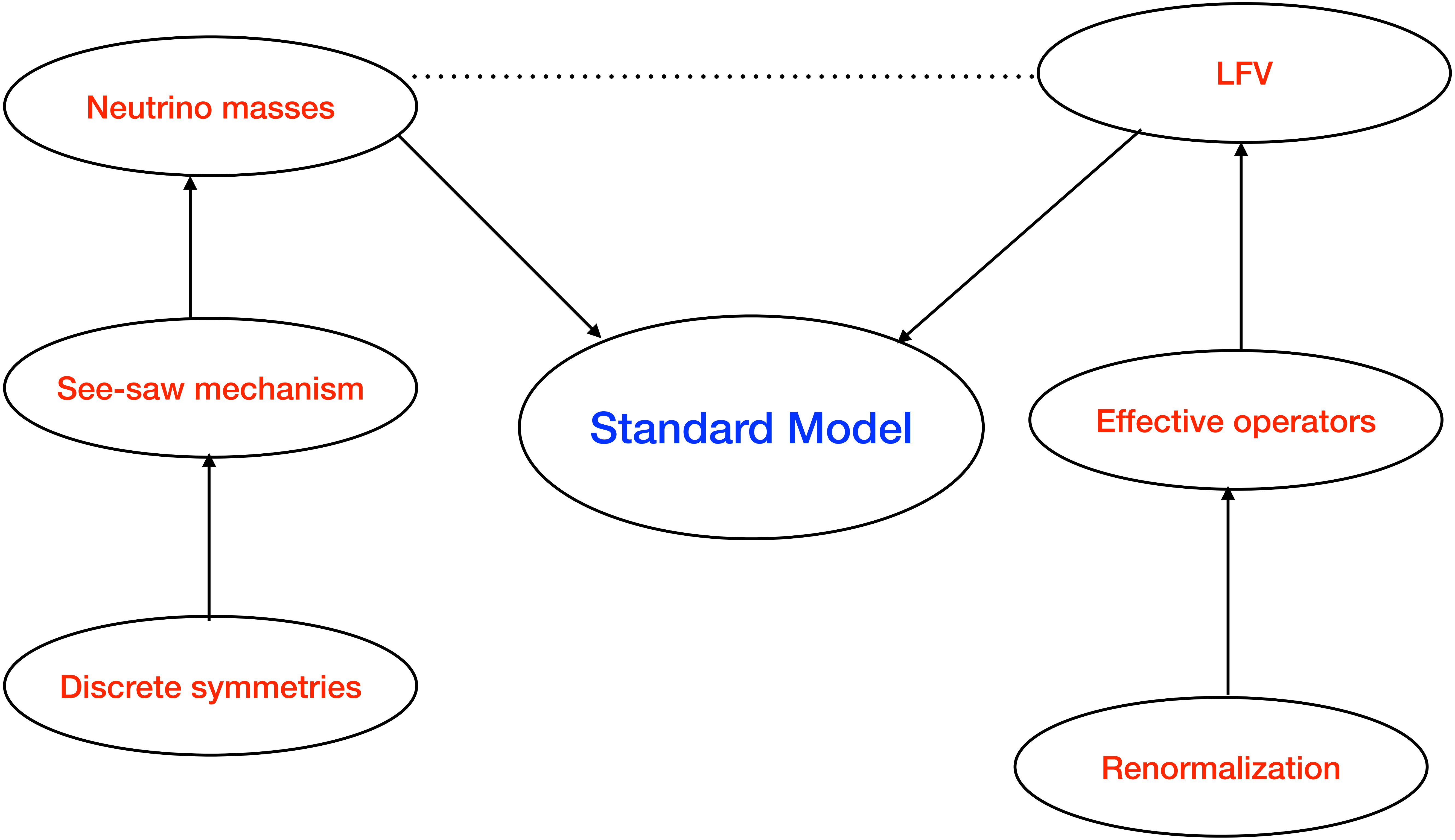
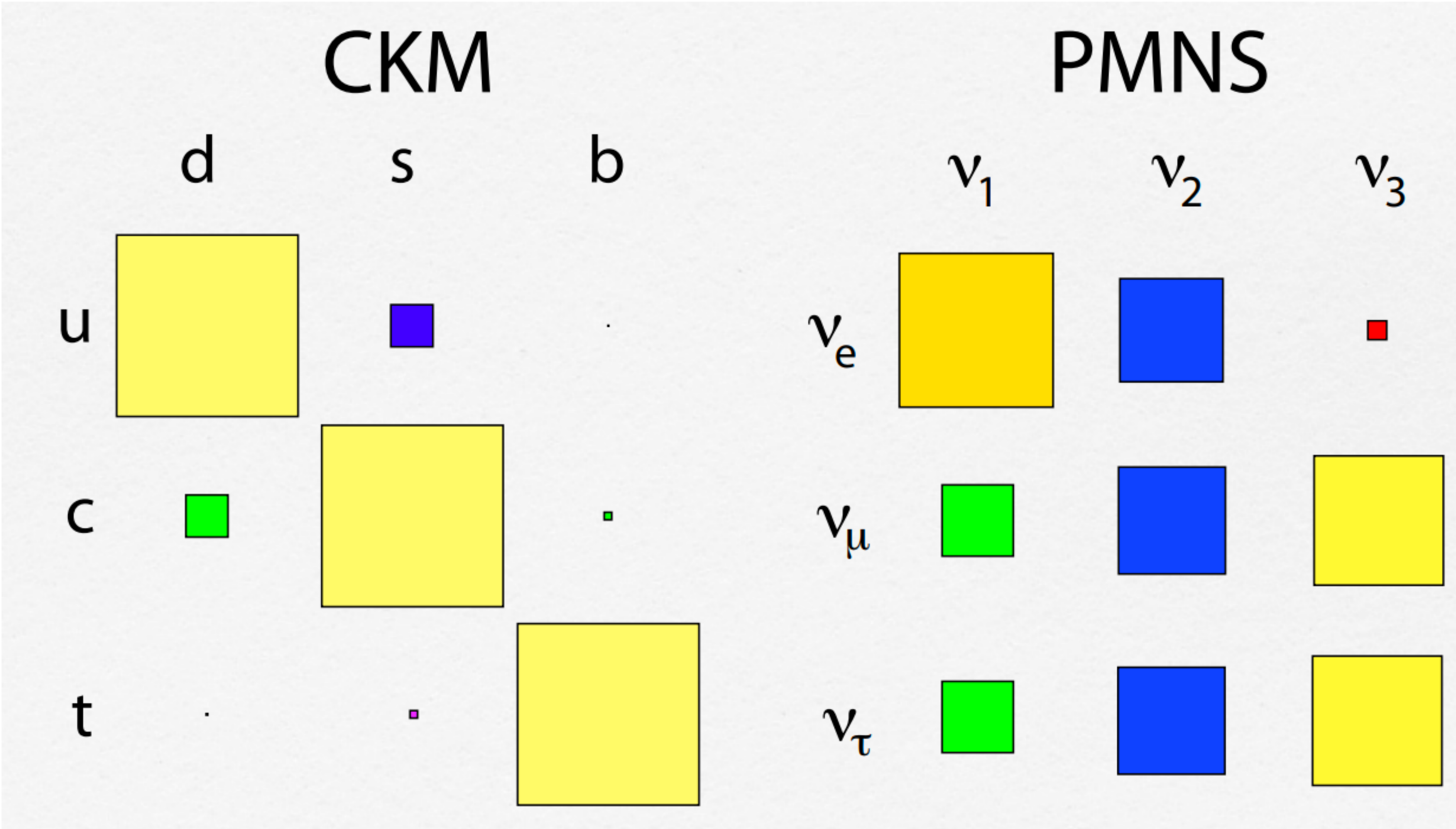
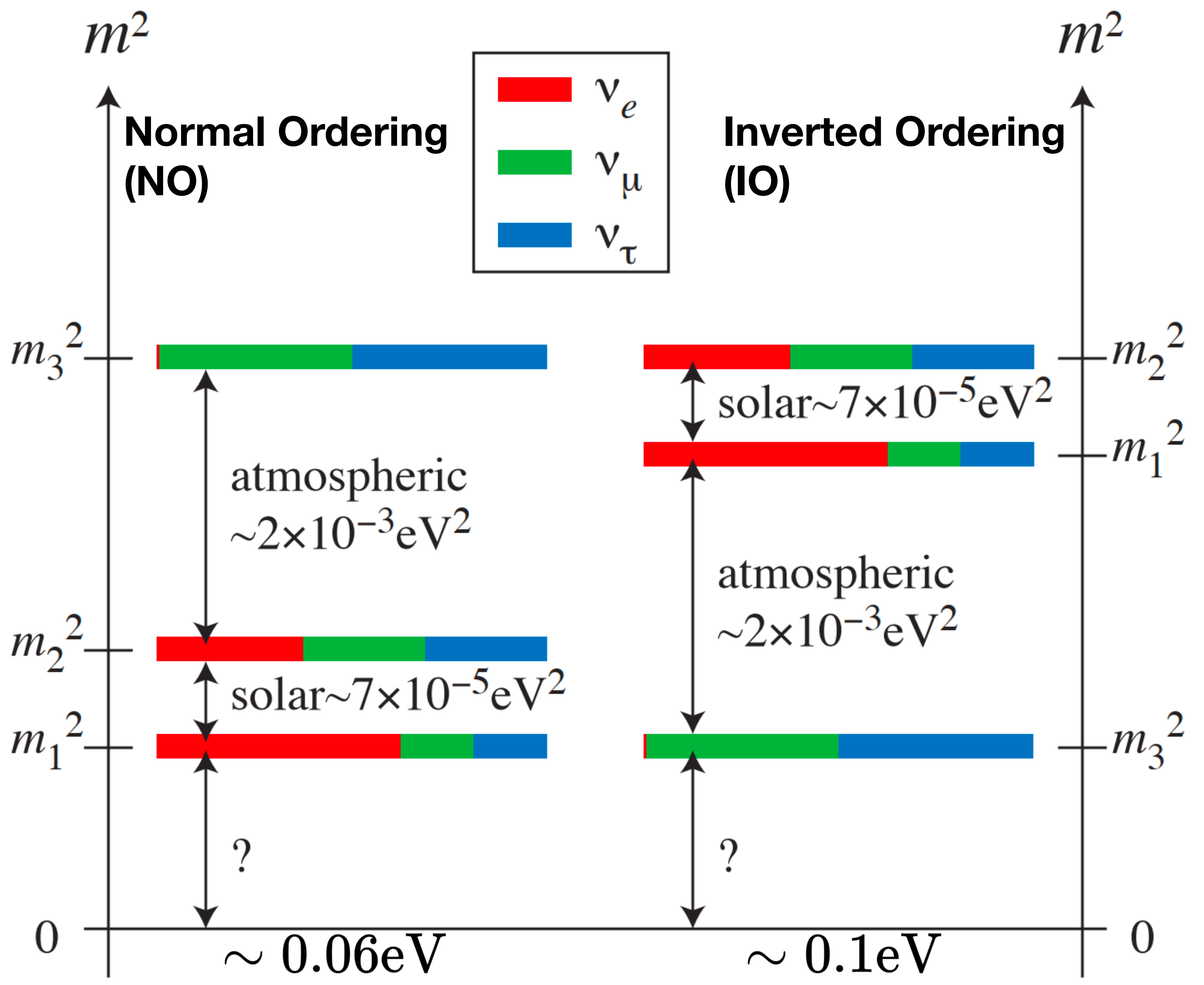

1st 2nd 3rd generation
standard matter
unstable matter
force carriers
Goldstone bosons
Beyond Standard Model
2.3 MeV 2/3 R/G/B u up 1/2
1.28 GeV 2/3 R/G/B c charm 1/2
173.2 GeV 2/3 R/G/B t top 1/2
charge
colors
mass
spin
125.1 GeV H Higgs 0
4.8 MeV −1/3 R/G/B d down 1/2
95 MeV −1/3 R/G/B s strange 1/2
4.7 GeV −1/3 R/G/B b bottom 1/2
color g gluon 1
511 keV −1 e electron 1/2
105.7 MeV −1 μ muon 1/2
1.777 GeV −1 τ tau 1/2
γ photon 1
< 2 eV ν_e e neutrino 1/2
< 190 keV ν_μ μ neutrino 1/2
< 18.2 MeV ν_τ τ neutrino 1/2
80.4 GeV ±1 W± 1
91.2 GeV Z 1
strong nuclear force (color)
electromagnetic force (charge)
weak nuclear force (weak isospin)
6 quarks (+6 anti-quarks)
6 leptons (+6 anti-leptons)
12 fermions (+12 anti-fermions) increasing mass →
5 bosons (+1 opposite charge W)
Neutrino masses
Lepton Flavor Violation
Hierarchy problem
Gravity

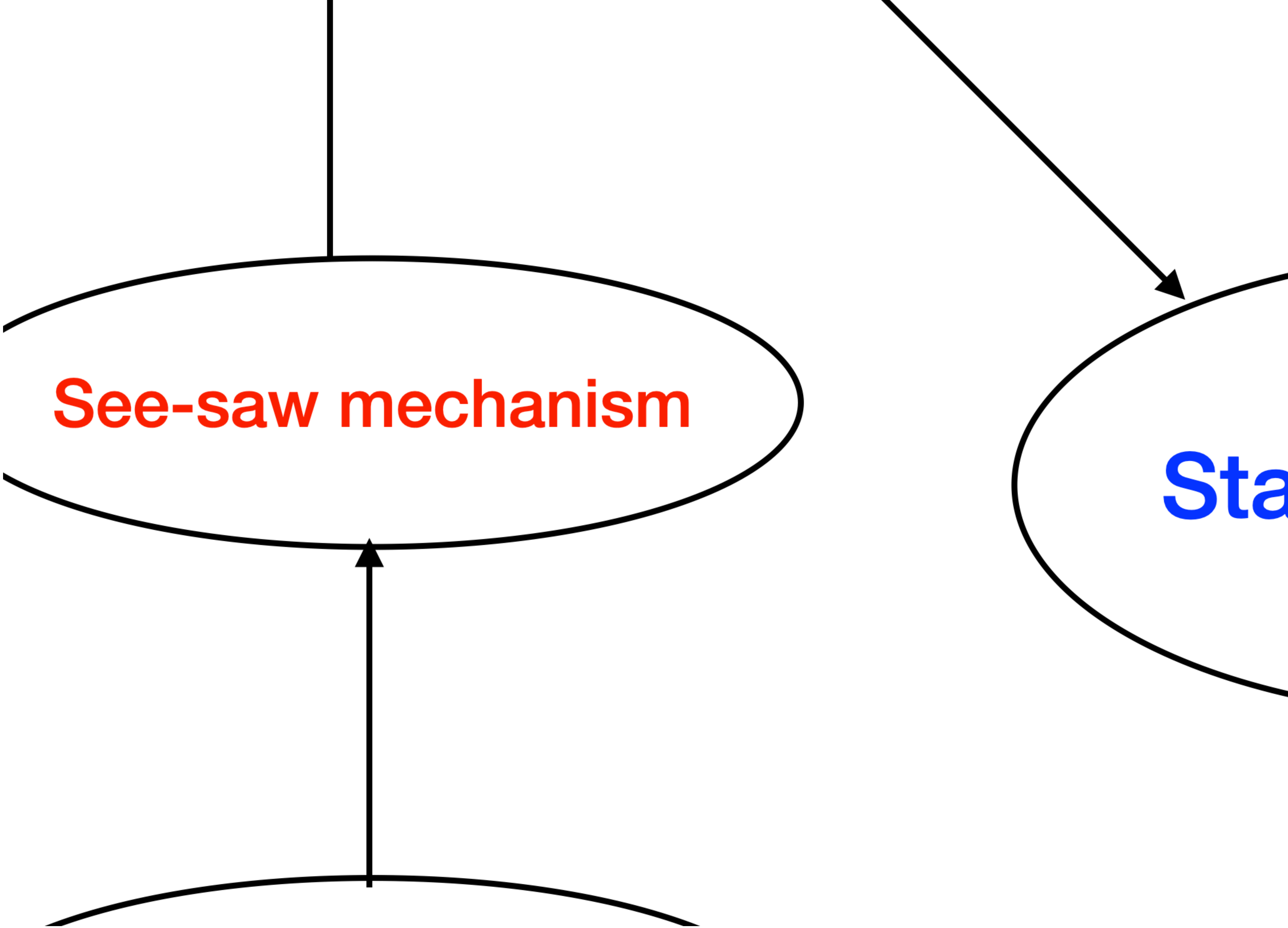

Neutrino masses
LFV
See-saw mechanism
Standard Model
Effective operators
Discrete symmetries
Renormalization

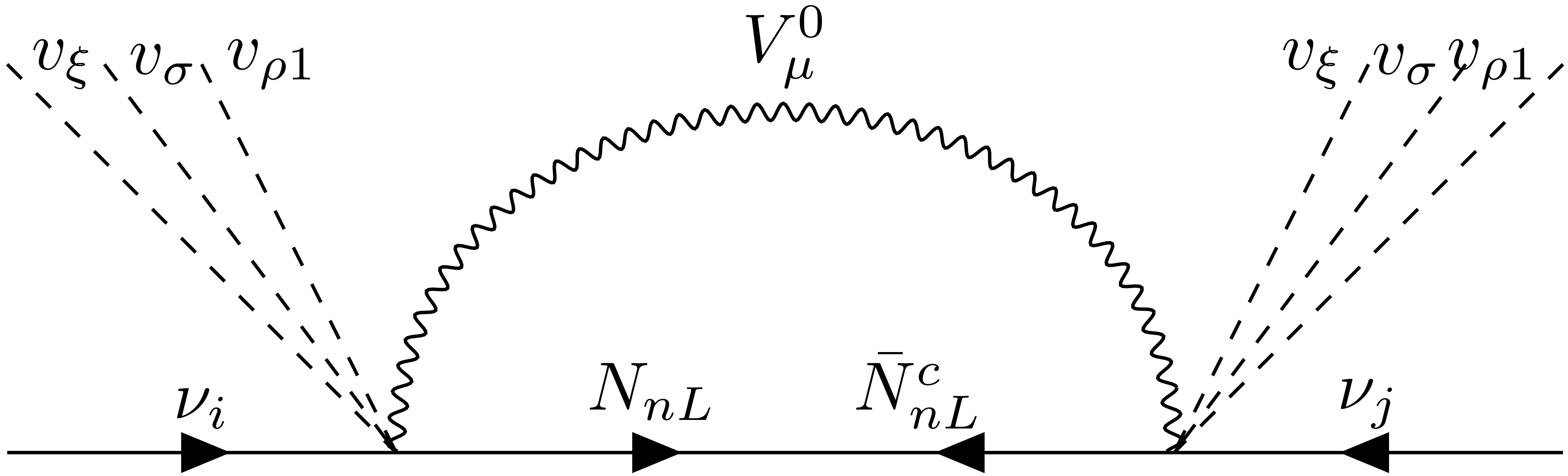


The sides of the squares represent the magnitude of the CKM and PMNS matrix elements.

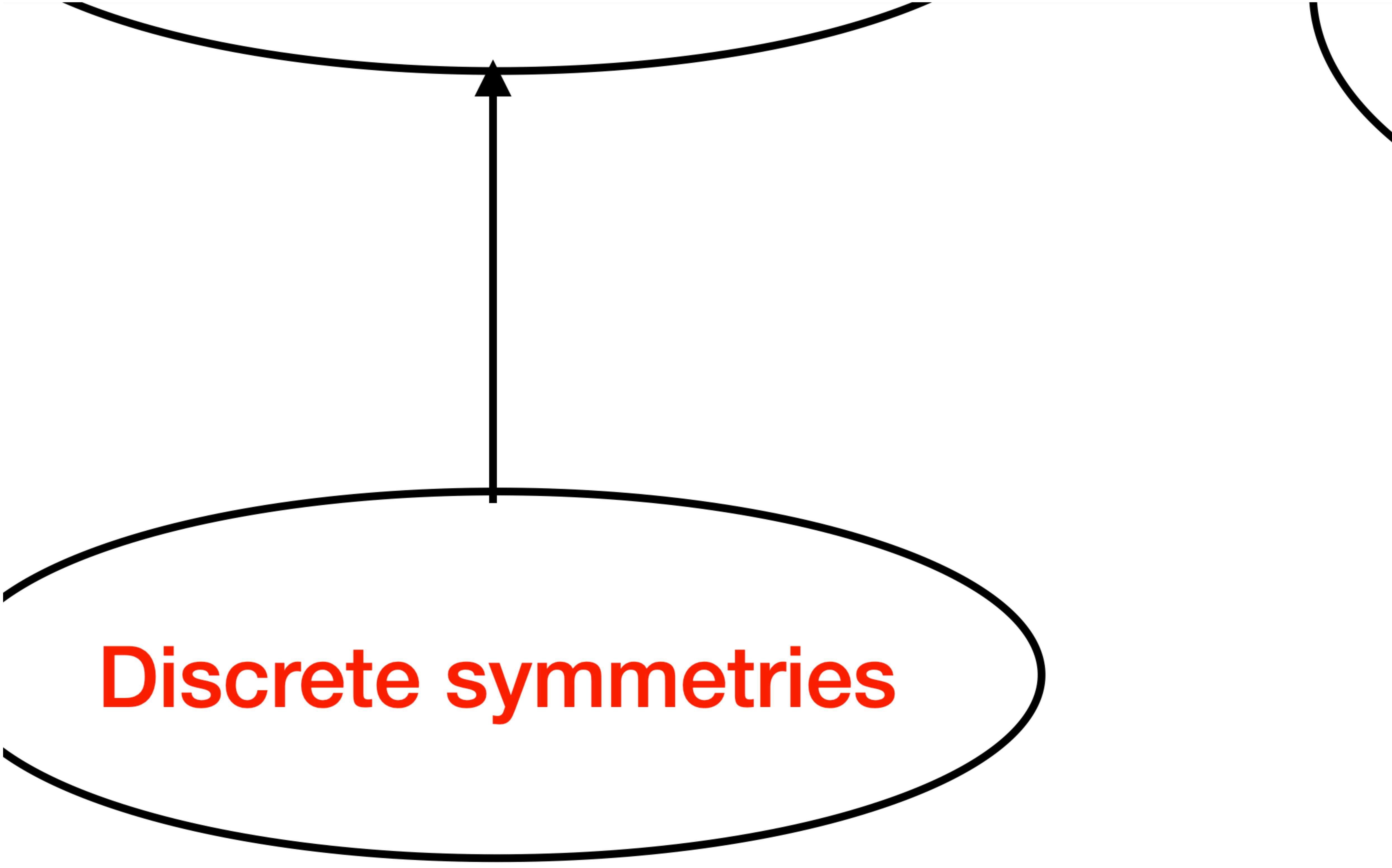


Ordering of the neutrino mass eigenstates. Colors represent the contribution of each flavor: electron, muon and tau are given in red, blue and green, respectively.


See-saw mechanism
Sta

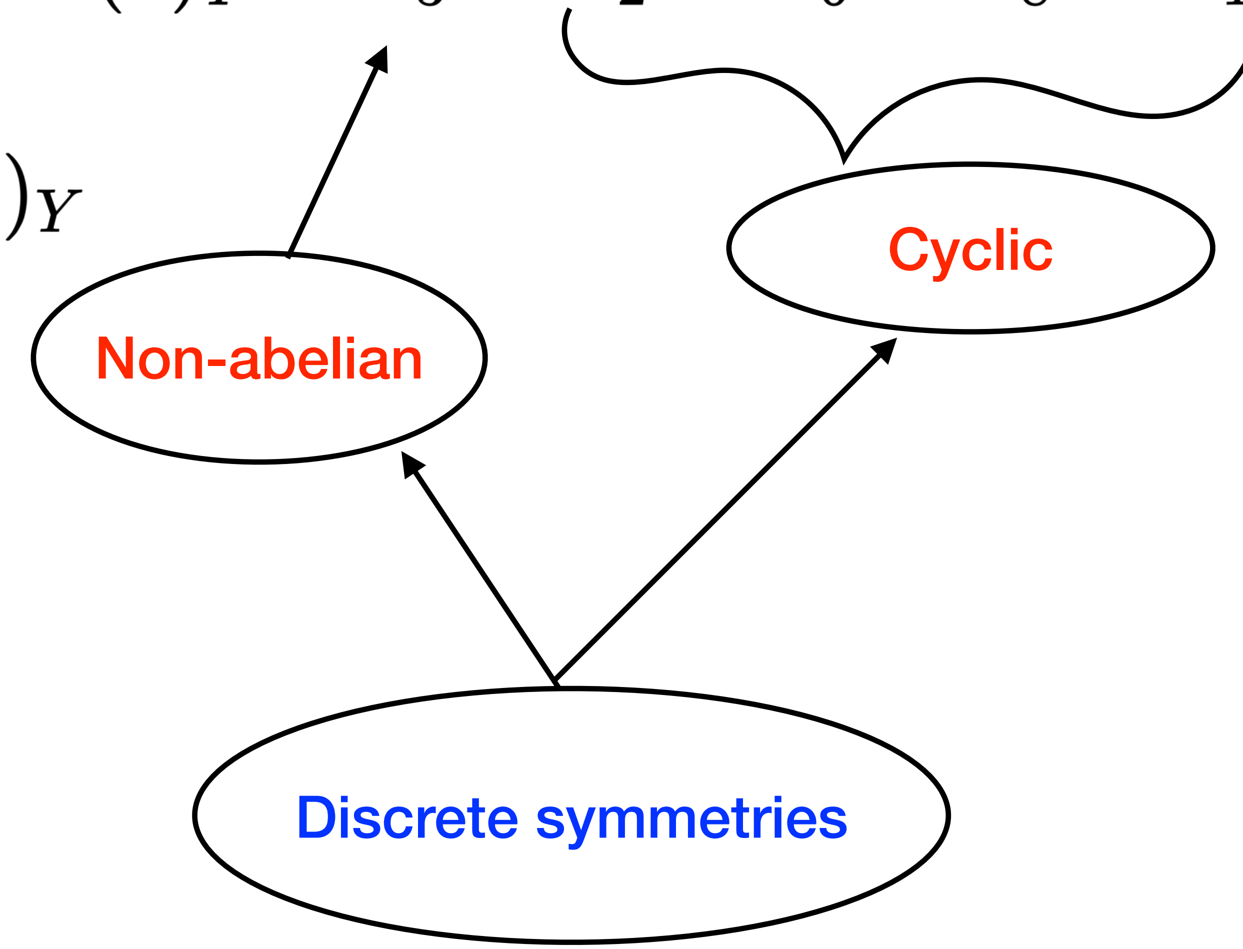


Radiative see-saw mechanism mediated by the neutral component of a doublet vector, transforming in the fundamental representation


Discrete symmetries

$$
\begin{aligned}
&\mathcal{G} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes S_3 \otimes Z_2 \otimes Z_6 \otimes Z_8 \otimes Z_{12} \\
&\qquad \downarrow \Lambda_{\rm int} \\
&SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
&\qquad \downarrow v \\
&SU(3)_C \otimes U(1)_Q
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes S_3 \otimes Z_2 \otimes Z_6 \otimes Z_8 \otimes Z_{12} \\
\downarrow \Lambda_{\text{int}} \\
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
\downarrow v \\
SU(3)_C \otimes U(1)_Q
\end{aligned}
$$



Flavons | Quark sector

	ϕ	φ	χ	ξ	η	σ	ρ_1	ρ_2	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}
S_3	1	1	2	2	1	1	1	$1'$	1	1	1	1	1	1	$1'$	$1'$	$1'$
Z_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Z_6	0	-2	0	-3	0	-3	0	0	0	0	0	0	0	0	3	3	3
Z_8	0	-1	0	0	-1	-2	0	0	-2	-1	0	2	1	0	2	1	0
Z_{12}	0	0	0	0	0	0	-3	-2	0	0	0	6	6	0	0	0	0

Flavons — Quark sector

	ϕ	φ	χ	ξ	η	σ	ρ_1	ρ_2	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}
S_3	1	1	2	2	1	1	1	$1'$	1	1	1	1	1	1	$1'$	$1'$	$1'$
Z_2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Z_6	0	-2	0	-3	0	-3	0	0	0	0	0	0	0	0	3	3	3
Z_8	0	-1	0	0	-1	-2	0	0	-2	-1	0	2	1	0	2	1	0
Z_{12}	0	0	0	0	0	0	-3	-2	0	0	0	6	6	0	0	0	0

$$\mathcal{L}_Y^{(q)} = y_{33}^{(u)}\bar{q}_{3L}\tilde{\phi}u_{3R} + y_{23}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{3R}\frac{\eta}{\Lambda} + y_{13}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{3R}\frac{\eta^2}{\Lambda^2}$$
$$+ y_{32}^{(u)}\bar{q}_{3L}\tilde{\phi}u_{2R}\frac{\eta\rho_1^2}{\Lambda^3} + y_{22}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{2R}\frac{\eta^2\rho_1^2}{\Lambda^4} + y_{12}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{2R}\frac{\eta^3\rho_1^2}{\Lambda^5}$$
$$+ y_{11}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{1R}\frac{\eta^2\rho_1^2}{\Lambda^4} + y_{21}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{1R}\frac{\eta^3\rho_1^2}{\Lambda^5} + y_{11}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{1R}\frac{\eta^4\rho_1^2}{\Lambda^6}$$
$$+ \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(d)}\bar{q}_{jL}\phi d_{kR}\frac{\eta^{6-j-k}(\xi\xi\xi)_{1'}}{\Lambda^{9-j-k}} + h.c,$$

$$\mathcal{L}_Y^{q} = y_{21}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{1R}\frac{\eta^3\rho_1^2}{\Lambda^5}$$

Z_8	1	0	2	-3	0
Z_{12}	0	0	6	0	-6

$$\mathcal{L}_Y^{(q)} = y_{33}^{(u)}\bar{q}_{3L}\tilde{\phi}u_{3R} + y_{23}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{3R}\frac{\eta}{\Lambda} + y_{13}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{3R}\frac{\eta^2}{\Lambda^2}$$
$$+ y_{32}^{(u)}\bar{q}_{3L}\tilde{\phi}u_{2R}\frac{\eta\rho_1^2}{\Lambda^3} + y_{22}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{2R}\frac{\eta^2\rho_1^2}{\Lambda^4} + y_{12}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{2R}\frac{\eta^3\rho_1^2}{\Lambda^5}$$
$$+ y_{11}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{1R}\frac{\eta^2\rho_1^2}{\Lambda^4} + y_{21}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{1R}\frac{\eta^3\rho_1^2}{\Lambda^5} + y_{11}^{(u)}\bar{q}_{1L}\tilde{\phi}u_{1R}\frac{\eta^4\rho_1^2}{\Lambda^6}$$
$$+ \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(d)}\bar{q}_{jL}\phi d_{kR}\frac{\eta^{6-j-k}(\xi\xi\xi)_{\mathbf{1}'}}{\Lambda^{9-j-k}} + h.c,$$

$$\mathcal{L}_Y^q = y_{21}^{(u)}\bar{q}_{2L}\tilde{\phi}u_{1R}\frac{\eta^3\rho_1^2}{\Lambda^5}$$

$$v_\eta \sim v_{\rho_1} \sim v_{\rho_2} \sim v_\varphi \sim v_\chi \sim v_\xi = v_\sigma = \Lambda_{\text{int}} = \lambda\Lambda$$

$$M_U = \begin{pmatrix} a_{11}^{(u)}\lambda^6 & a_{12}^{(u)}\lambda^5 & a_{13}^{(u)}\lambda^2 \\ a_{21}^{(u)}\lambda^5 & a_{22}^{(u)}\lambda^4 & a_{23}^{(u)}\lambda \\ a_{31}^{(u)}\lambda^4 & a_{32}^{(u)}\lambda^3 & a_{33}^{(u)} \end{pmatrix}\frac{v}{\sqrt{2}}$$

$$M_D = \begin{pmatrix} a_{11}^{(d)}\lambda^7 & a_{12}^{(d)}\lambda^6 & a_{13}^{(d)}\lambda^5 \\ a_{21}^{(d)}\lambda^6 & a_{22}^{(d)}\lambda^5 & a_{23}^{(d)}\lambda^4 \\ a_{31}^{(d)}\lambda^5 & a_{32}^{(d)}\lambda^4 & a_{33}^{(d)}\lambda^3 \end{pmatrix}\frac{v}{\sqrt{2}}$$

In order to simplify the analysis, we adopt the following scenario:

$$
\begin{aligned}
a_{12}^{(u)} &= a_{21}^{(u)}, \qquad a_{31}^{(u)} = a_{13}^{(u)}, \quad a_{32}^{(u)} = a_{23}^{(u)}, \\
a_{12}^{(d)} &= \left|a_{12}^{(d)}\right| \mathrm{e}^{-\mathrm{i}\tau_1}, \quad a_{21}^{(d)} =| a_{12}^{(d)} \mathrm{e}^{\mathrm{i}\tau_1}, \\
a_{13}^{(d)} &= \left|a_{13}^{(d)}\right| \mathrm{e}^{-\mathrm{i}\tau_2}, \quad a_{31}^{(d)} = \left|a_{13}^{(d)}\right| \mathrm{e}^{\mathrm{i}\tau_2}, \quad a_{23}^{(d)} = a_{32}^{(d)}.
\end{aligned}
$$

The quark mass spectrum, quark mixing parameters and CP violating phase obtained in our model are in very good agreement with the experimental data.

$$
\begin{aligned}
&a_{11}^{(u)} \simeq 0,58, \quad a_{22}^{(u)} \simeq 2,19, \quad a_{12}^{(u)} \simeq 0,67, \\
&a_{13}^{(u)} \simeq 0,80, \quad a_{23}^{(u)} \simeq 0,83, \quad a_{11}^{(d)} \simeq 1,96, \\
&a_{12}^{(d)} \simeq 0,53, \quad a_{13}^{(d)} \simeq 1,07, \quad a_{22}^{(d)} \simeq 1,93, \\
&a_{23}^{(d)} \simeq 1,36, \quad a_{33}^{(d)} \simeq 1,35, \quad \tau_1 \simeq 9,56^{\circ}, \quad \tau_2 \simeq 4,64^{\circ}.
\end{aligned}
$$

Results for quark masses

Observable	Model value	Experimental value
m_u(MeV)	1.44	$1.45^{+0.56}_{-0.45}$
m_c(MeV)	656	635 ± 86
m_t(GeV)	177.1	$172.1 \pm 0.6 \pm 0.9$
m_d(MeV)	2.9	$2.9^{+0.5}_{-0.4}$
m_s(MeV)	57.7	$57.7^{+16.8}_{-15.7}$
m_b(GeV)	2.82	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}$	0.225	0.225
$\sin\theta_{23}$	0.0412	0.0412
$\sin\theta_{13}$	0.00351	0.00351
δ	64°	68°

Flavons

	ϕ	φ	χ	ξ	η	σ	ρ_1	ρ_2
S_3	1	1	2	2	1	1	1	$1'$
Z_2	0	0	0	0	0	0	0	0
Z_6	0	-2	0	-3	0	-3	0	0
Z_8	0	-1	0	0	-1	-2	0	0
Z_{12}	0	0	0	0	0	0	-3	-2

Lepton sector

	l_{1L}	l_L	l_{1R}	l_{2R}	l_{3R}	N_{1L}	N_{2L}
S_3	$1'$	2	1	1	$1'$	1	1
Z_2	0	0	0	0	0	1	1
Z_6	-2	-2	-2	-2	-2	3	3
Z_8	-3	-1	-1	3	-1	4	4
Z_{12}	0	0	0	0	6	6	6

$$\mathcal{L}_Y^{(l)} = y_{11}^{(l)}\bar{l}_{1L}\phi l_{1R}\frac{\eta^2\rho_2^3\varphi^4}{\Lambda^9} + y_{22}^{(l)}\left(\bar{l}_L\phi\chi\right)_1 l_{2R}\frac{\eta^4}{\Lambda^5} + y_{13}^{(l)}\bar{l}_{1L}\phi l_{3R}\frac{\eta^2\rho_1^2}{\Lambda^4} + y_{23}^{(l)}\left(\bar{l}_L\phi\chi\right)_1 l_{3R}\frac{\rho_2^3}{\Lambda^4}$$
$$+ y_{33}^{(l)}\left(\bar{l}_L\phi\chi\right)_{1'} l_{3R}\frac{\rho_1^2}{\Lambda^3} + \frac{1}{2}\sum_{n=1}^{2} m_{N_n}\bar{N}_{nL}N_{nL}^C + h.c,$$

$$\mathcal{L}_Y^l = y_{22}^{(l)}\left(\bar{l}_L\phi\chi\right)_1 l_{2R}\frac{\eta^4}{\Lambda^5}$$

S_3	2	1	2	1	1
Z_6	+2	0	0	-2	0
Z_8	+1	0	0	3	-4

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1y_1 + x_2y_2)_{\mathbf{1}} + (x_1y_2 - x_2y_1)_{\mathbf{1}'} + \begin{pmatrix} x_2y_2 - x_1y_1 \\ x_1y_2 + x_2y_1 \end{pmatrix}_{\mathbf{2}}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2y' \\ x_1y' \end{pmatrix}_2, \quad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x'y')_{\mathbf{1}}.$$

$$\mathcal{L}_Y^{(l)} = y_{11}^{(l)} \bar{l}_{1L} \phi l_{1R} \frac{\eta^2 \rho_2^3 \varphi^4}{\Lambda^9} + y_{22}^{(l)} \left(\bar{l}_L \phi \chi\right)_1 l_{2R} \frac{\eta^4}{\Lambda^5} + y_{13}^{(l)} \bar{l}_{1L} \phi l_{3R} \frac{\eta^2 \rho_1^2}{\Lambda^4} + y_{23}^{(l)} \left(\bar{l}_L \phi \chi\right)_1 l_{3R} \frac{\rho_2^3}{\Lambda^4}$$

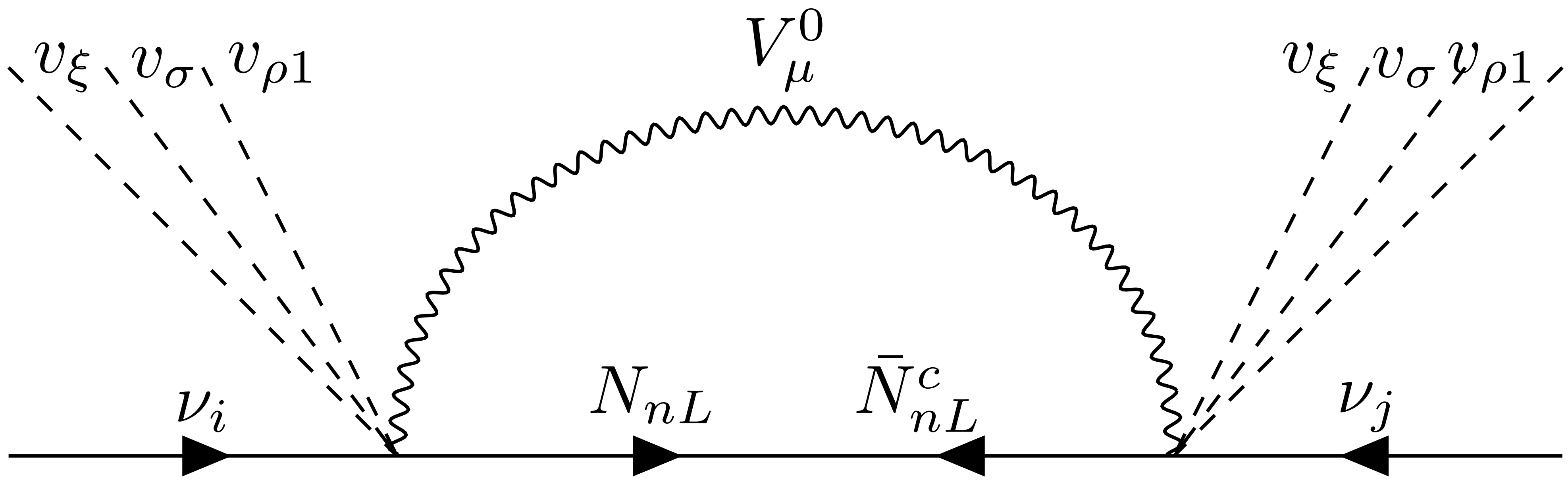
$$+ y_{33}^{(l)} \left(\bar{l}_L \phi \chi\right)_{1'} l_{3R} \frac{\rho_1^2}{\Lambda^3} + \frac{1}{2} \sum_{n=1}^{2} m_{N_n} \bar{N}_{nL} N_{nL}^C + h.c,$$

$$\mathcal{L}_Y^l = y_{22}^{(l)} \left(\bar{l}_L \phi \chi\right)_1 l_{2R} \frac{\eta^4}{\Lambda^5}$$

$$v_\eta \sim v_{\rho_1} \sim v_{\rho_2} \sim v_\varphi \sim v_\chi \sim v_\xi = v_\sigma = \Lambda_{\text{int}} = \lambda \Lambda$$

$$M_l = \frac{v}{\sqrt{2}} \begin{pmatrix} a_1 \lambda^9 & 0 & a_4 \lambda^4 \\ 0 & a_2 \lambda^5 & a_5 \lambda^4 \\ 0 & 0 & a_3 \lambda^3 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} Z & Y & \sqrt{2}Y \\ Y & X & \sqrt{2}X \\ \sqrt{2}Y & \sqrt{2}X & 2X \end{pmatrix}$$



$$V_\mu = \begin{pmatrix} V_\mu^+ \\ V_\mu^0 \end{pmatrix}$$

$$M_\nu = \begin{pmatrix} Z & Y & \sqrt{2}Y \\ Y & X & \sqrt{2}X \\ \sqrt{2}Y & \sqrt{2}X & 2X \end{pmatrix}$$

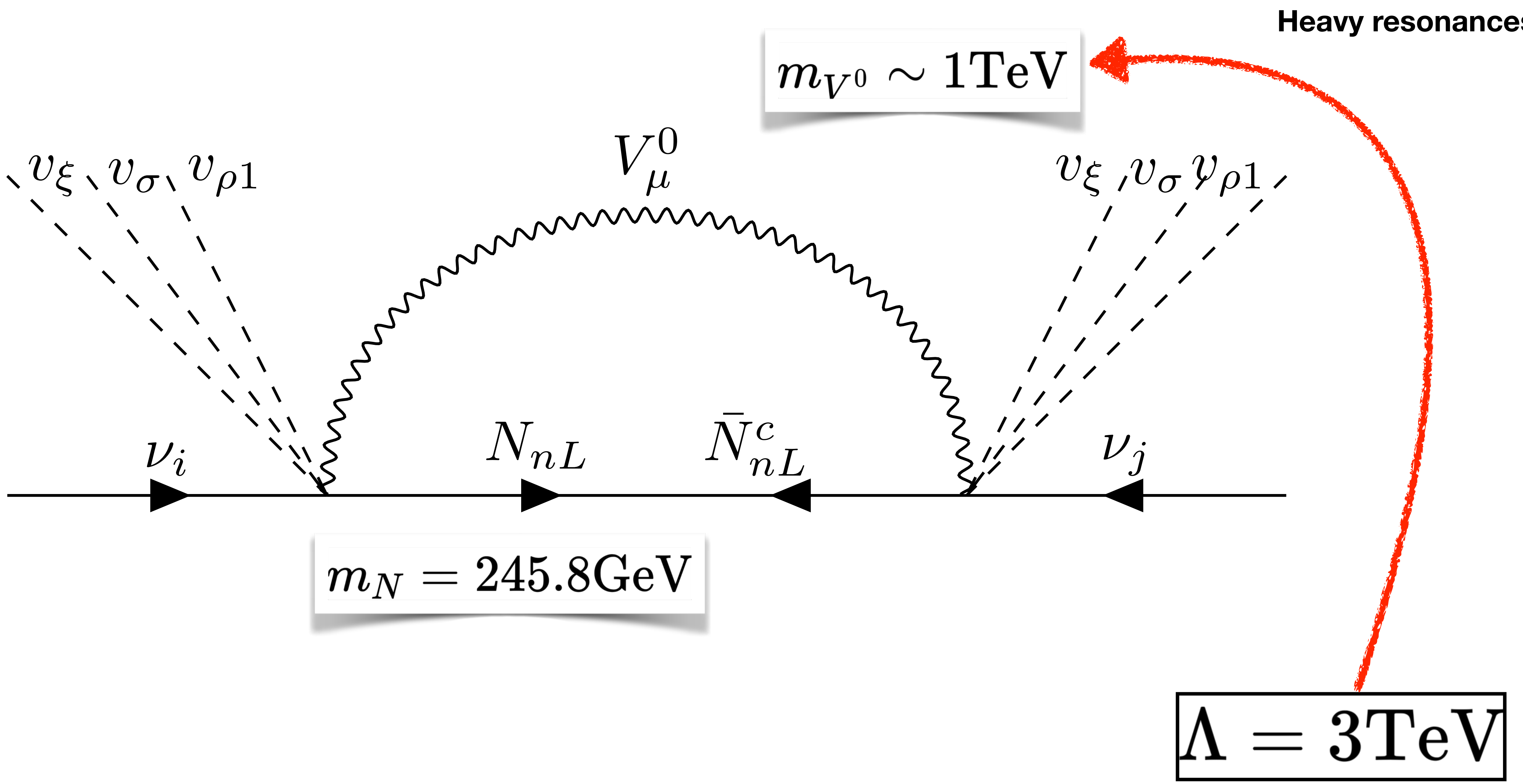
$$X \simeq \sum_{n=1}^{2} \left(y_{2n}^{(V)}\right)^2 \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n}$$

$$Y \simeq \sum_{n=1}^{2} \left(y_{1n}^{(V)}\right)\left(y_{2n}^{(V)}\right) \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n},$$

$$Z \simeq \sum_{n=1}^{2} \left(y_{1n}^{(V)}\right)^2 \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n}$$

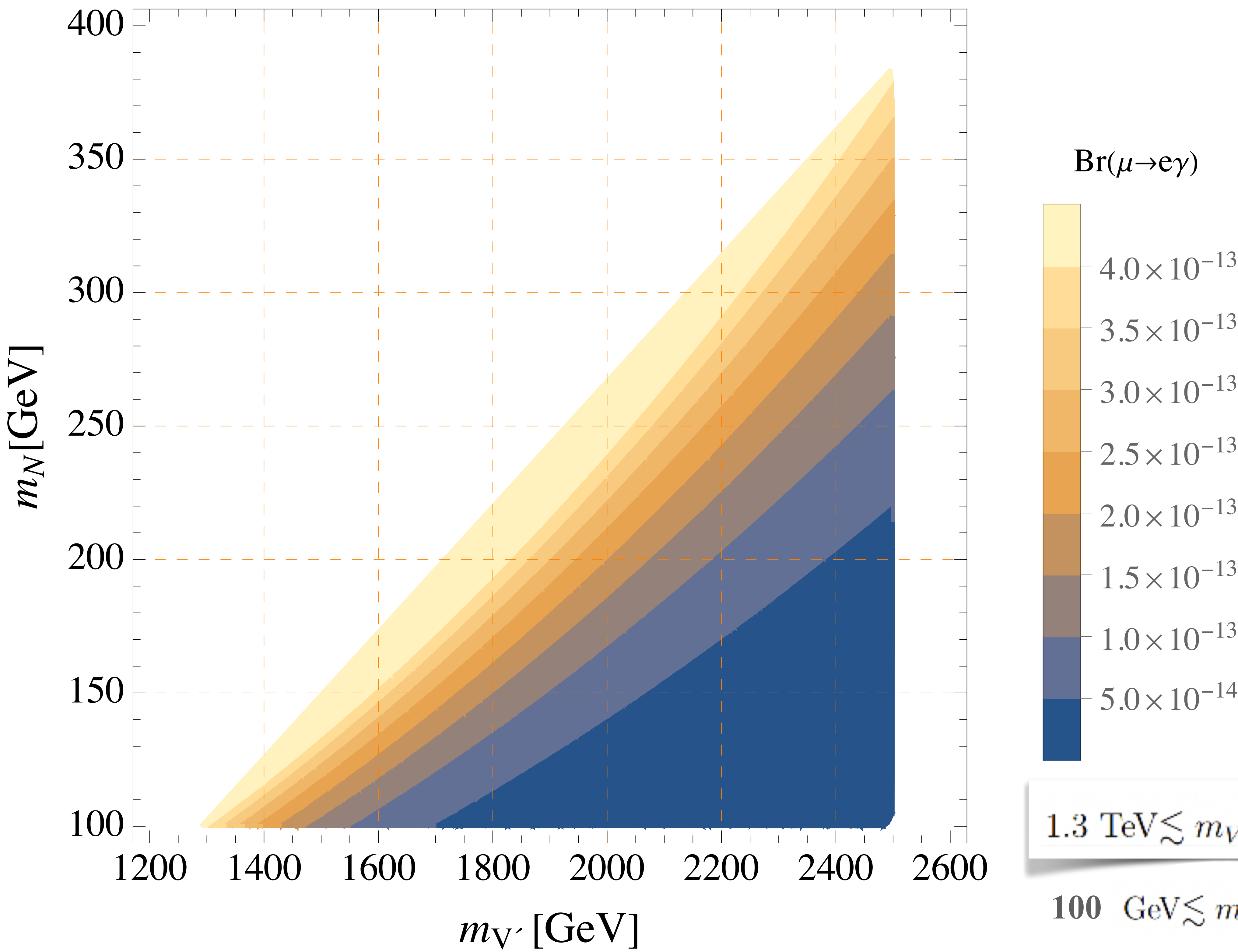
The parameters for Inverted ordering are reproduce with the following benchmark point.

$$a_1 \simeq 1.96168, \quad a_2 \simeq 1.03698, \quad a_3 \simeq 0.84294, \quad |a_4| \simeq 1.00752, \quad \arg(a_4) \simeq 218^\circ,$$
$$a_5 \simeq -0.597641, \quad X \simeq 16.5289\text{meV}, \quad Y \simeq -0.219701\text{meV},$$
$$Z \simeq 49.5616\text{meV},$$



Result from the model for Inverted hierarchy

Observable	Model value	Experimental value		
		1σ range	2σ range	3σ range
$m_e[\mathrm{MeV}]$	0.487	0.487	0.487	0.487
$m_\mu[\mathrm{MeV}]$	102.8	102.8 ± 0.0003	102.8 ± 0.0006	102.8 ± 0.0009
$m_\tau[\mathrm{GeV}]$	1.75	1.75 ± 0.0003	1.75 ± 0.0006	1.75 ± 0.0009
$m_1[\mathrm{meV}]$	49.19	$\cdots$	$\cdots$	$\cdots$
$m_2[\mathrm{meV}]$	49.96	$\cdots$	$\cdots$	$\cdots$
$m_3\left[\mathrm{meV}^2\right]$	0	$\cdots$	$\cdots$	$\cdots$
$\Delta m_{21}^2\left[10^{-5}\mathrm{eV}^2\right](\mathrm{IH})$	7.55	$7.55_{-0.16}^{+0.20}$	$7.20-7.94$	$7.05-8.14$
$\Delta m_{13}^2\left[10^{-3}\mathrm{eV}^2\right](\mathrm{IH})$	2.42	$2.42_{-0.04}^{+0.03}$	$2.34-2.47$	$2.31-2.51$
$\delta[^\circ](\mathrm{IH})$	309.719	281_{-27}^{+23}	$229-328$	$202-349$
$\sin^2\theta_{12}/10^{-1}(\mathrm{IH})$	3.20	$3.20_{-0.16}^{+0.20}$	$2.89-3.59$	$2.73-3.79$
$\sin^2\theta_{23}/10^{-1}(\mathrm{IH})$	5.33	$5.51_{-0.30}^{+0.18}$	$4.91-5.84$	$4.53-5.98$
$\sin^2\theta_{13}/10^{-2}(\mathrm{IH})$	2.248	$2.220_{-0.076}^{+0.074}$	$2.07-2.36$	$1.99-2.44$

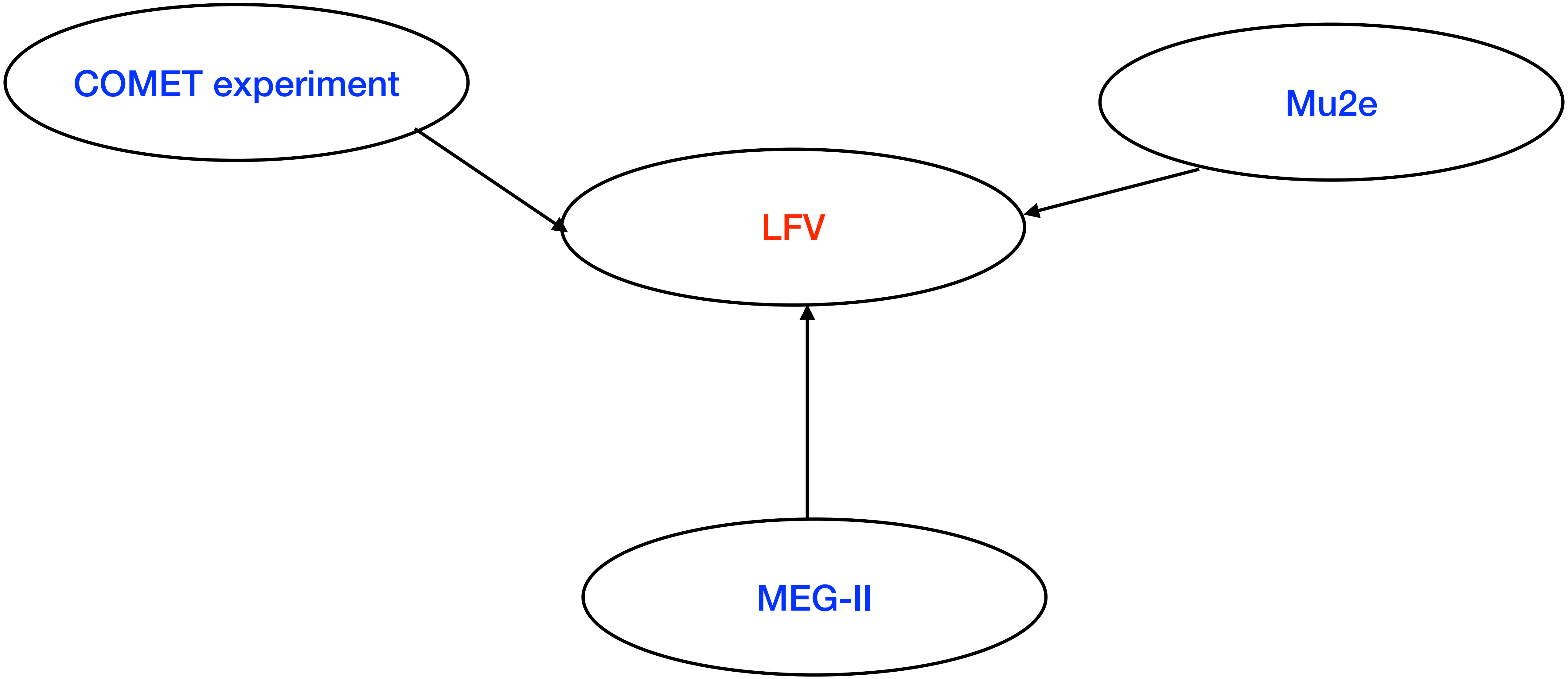


Experimental bounds

1.3 TeV $\lesssim m_{V^\pm} \lesssim$ 2.5 TeV

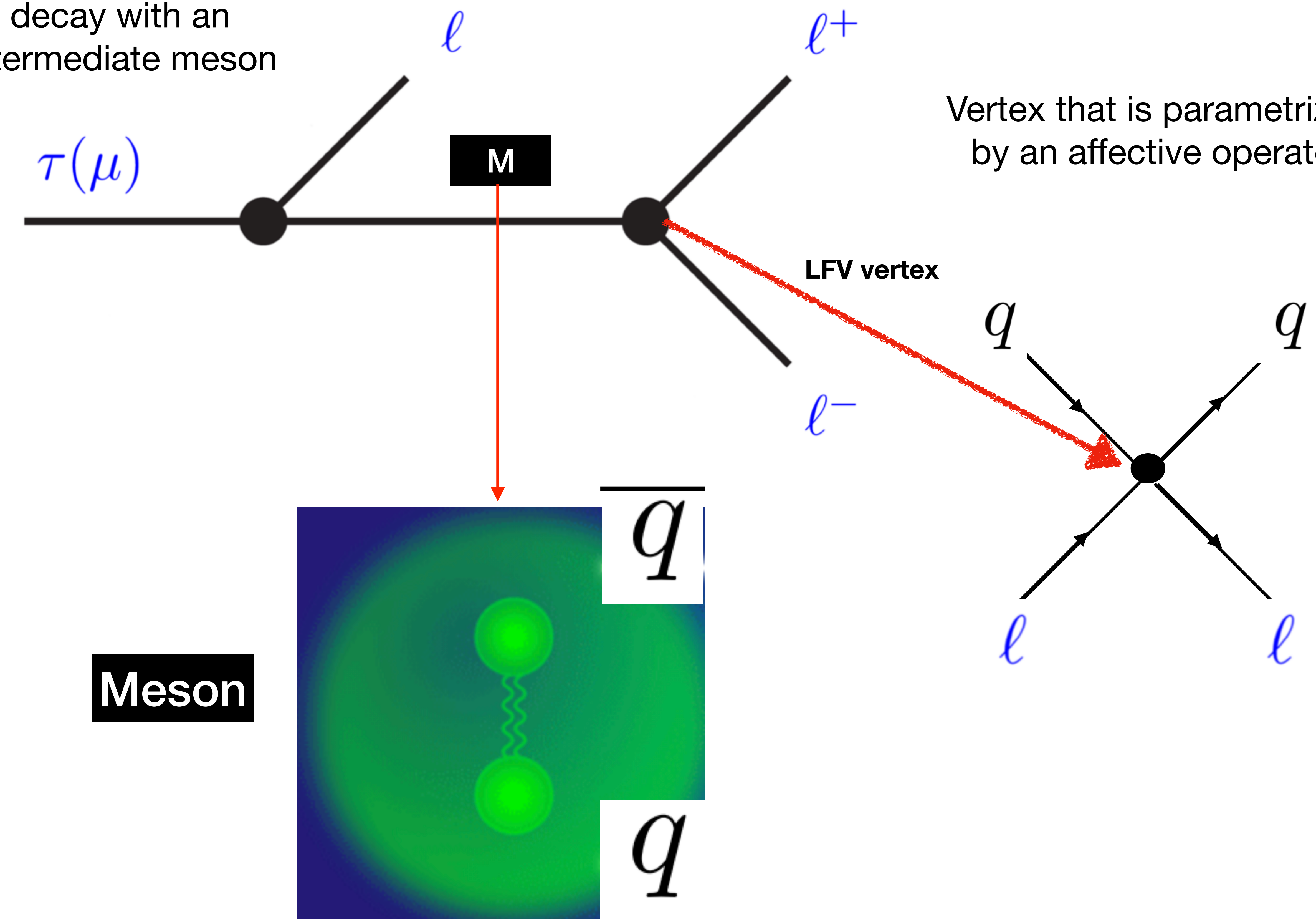
100 GeV $\lesssim m_N \lesssim$ 380 GeV

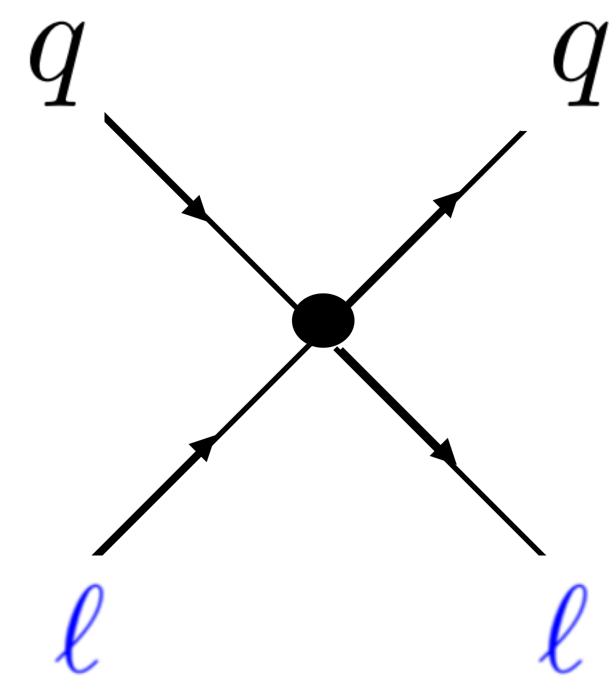
Allowed parameter space in the mV ± – mN plane consistent with the LFV constraints.


COMET experiment
Mu2e
LFV
MEG-II

Three-body LFV decay with an intermediate meson

Vertex that is parametrized by an affective operator



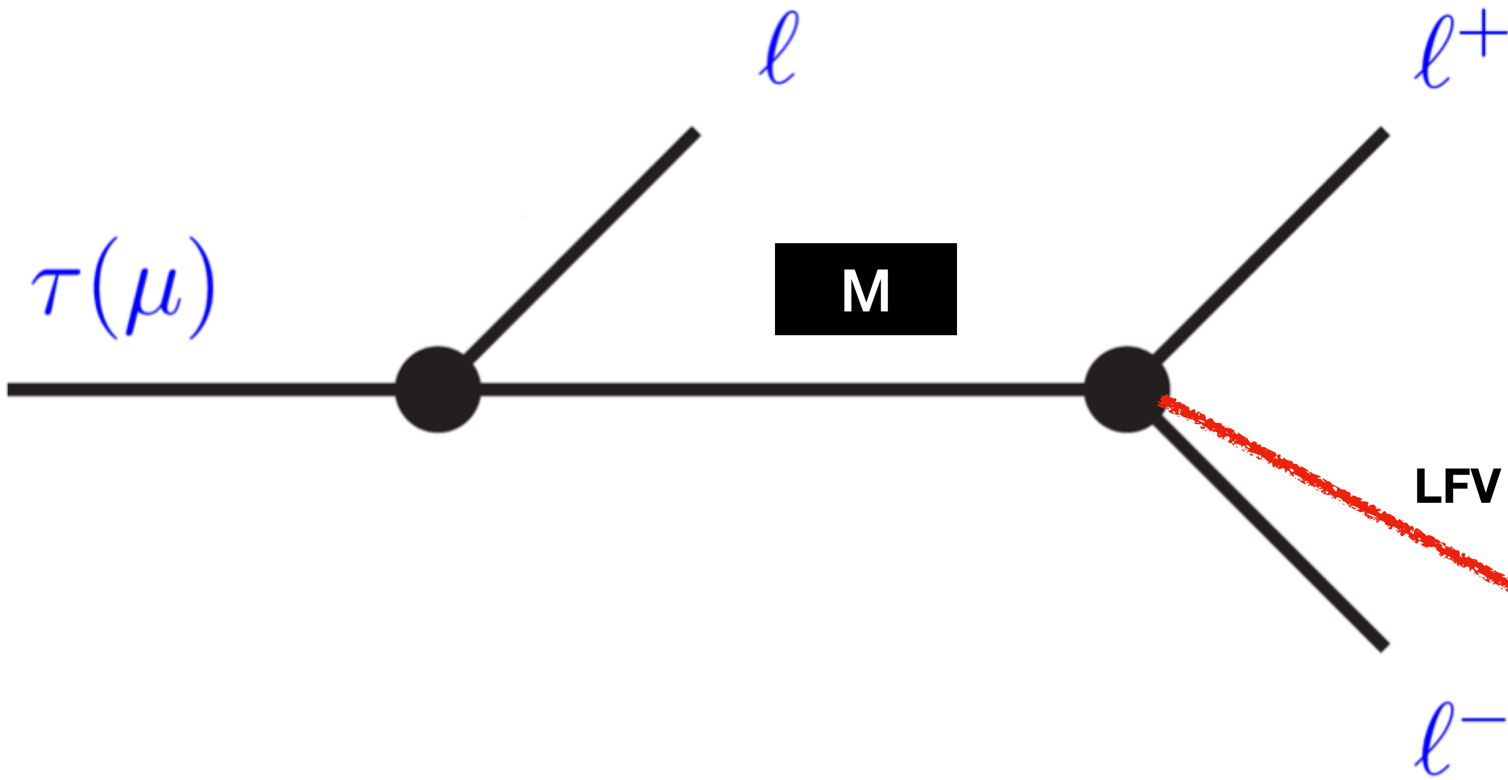


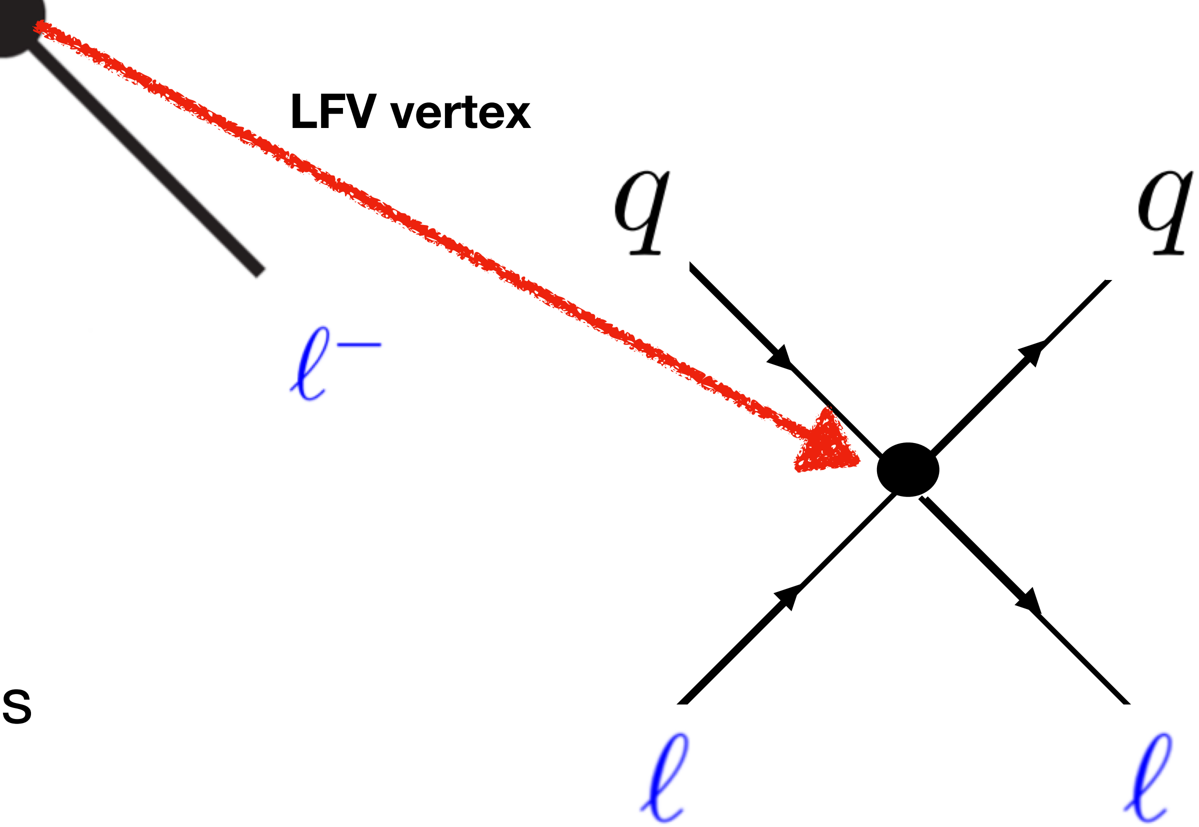
Effective operator in chiral base

$$\mathcal{O}^{(q)XY}_{1,\ell_1\ell_2} = 4\left(\bar{\ell}_1 P_X \ell_2\right)\left(\bar{q} P_Y q\right)$$

$$\mathcal{O}^{(q)XX}_{2,\ell_1\ell_2} = 4\left(\bar{\ell}_1 \sigma^{\mu\nu} P_X \ell_2\right)\left(\bar{q} \sigma_{\mu\nu} P_X q\right)$$

$$\mathcal{O}^{(q)XY}_{3,\ell_1\ell_2} = 4\left(\bar{\ell}_1 \gamma^{\mu} P_X \ell_2\right)\left(\bar{q} \gamma_{\mu} P_Y q\right)$$



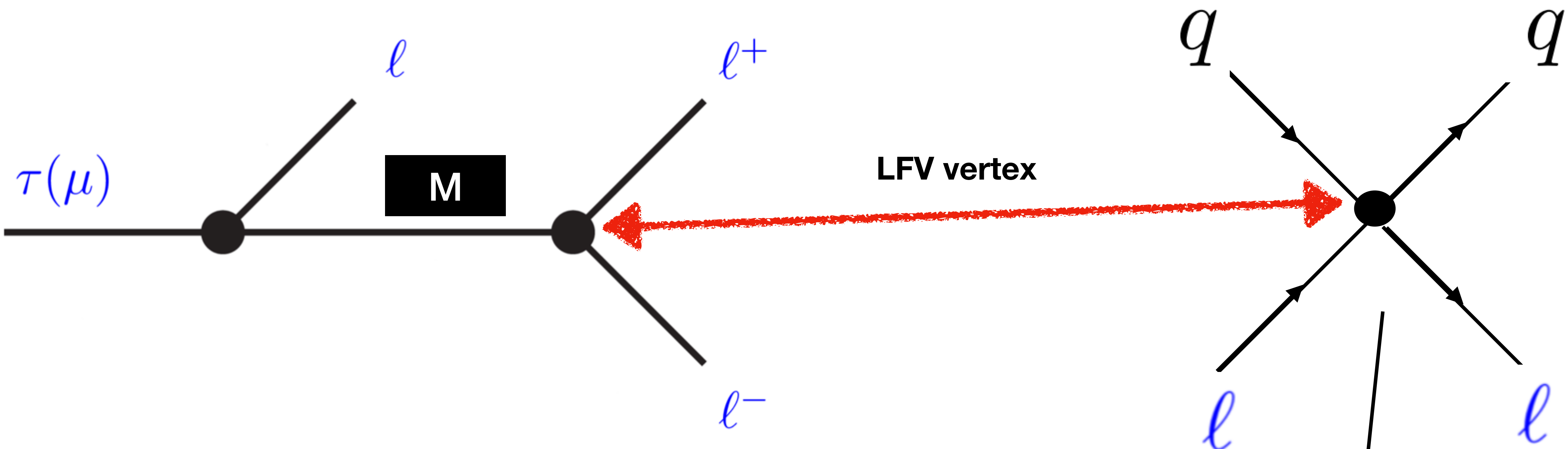


$$\tau(\mu) \to 3\ell$$

The experimental upper limit for this process

$$\mathrm{Br}\left(\tau^- \to \mathrm{e}^-\mathrm{e}^+\mathrm{e}^-\right) < 2{,}7 \times 10^{-8}$$

Let us see the Lagrangian for each process



$$\langle \ell_1^+ \ell_2^- | \mathcal{L}_{\ell M} | M \rangle \approx \langle \ell_1^+ \ell_2^- | \mathcal{L}_{\ell q} | M \rangle$$

$$\begin{aligned}
\mathcal{L}_{\ell M} = {} & V_\mu \left(g^{(V)}_{V\ell_1\ell_2} [\bar\ell_1\gamma^\mu\ell_2] + g^{(A)}_{V\ell_1\ell_2} [\bar\ell_1\gamma^\mu\gamma_5\ell_2] \right) \\
& + A_\mu \left(g^{(V)}_{A\ell_1\ell_2} [\bar\ell_1\gamma^\mu\ell_2] + g^{(A)}_{A\ell_1\ell_2} [\bar\ell_1\gamma^\mu\gamma_5\ell_2] \right) \\
& + \frac{g^{(T)}_{V\ell_1\ell_2}}{M_V} F^V_{\mu\nu} [\bar\ell_1\sigma^{\mu\nu}\ell_2] + \frac{g^{(T)}_{A\ell_1\ell_2}}{M_A} F^A_{\mu\nu} [\bar\ell_1\sigma^{\mu\nu}\gamma^5\ell_2] \\
& + S \left(g^{(S)}_{S\ell_1\ell_2} [\bar\ell_1\ell_2] + g^{(P)}_{S\ell_1\ell_2} [\bar\ell_1\gamma_5\ell_2] \right) + P \left(i g^{(S)}_{P\ell_1\ell_2} [\bar\ell_1\ell_2] + i g^{(P)}_{P\ell_1\ell_2} [\bar\ell_1\gamma_5\ell_2] \right) \\
& + \frac{\partial_\mu P}{M_P} \left(g^{(V)}_{P\ell_1\ell_2} [\bar\ell_1\gamma^\mu\ell_2] + g^{(A)}_{P\ell_1\ell_2} [\bar\ell_1\gamma^\mu\gamma^5\ell_2] \right) + \text{H.c.}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\ell q} = {} & \frac{1}{\Lambda^2} \Big[\bar\ell_1 \left(C^{(q_1q_2)SS}_{\ell_1\ell_2} + C^{(q_1q_2)PS}_{\ell_1\ell_2} \gamma^5 \right) \ell_2 \cdot \overline{q_1} q_2 \\
& + \bar\ell_1 \left(C^{(q_1q_2)SP}_{\ell_1\ell_2} + C^{(q_1q_2)PP}_{\ell_1} \gamma^5 \right) \ell_2 \cdot \bar q_1 \gamma_5 q_2 \\
& + \bar\ell_1 \left(C^{(q_1q_2)VV}_{\ell_1\ell_2} \gamma^\mu + C^{(q_1q_2)AV}_{\ell_1\ell_2} \gamma^\mu\gamma^5 \right) \ell_2 \cdot \overline{q_1} \gamma_\mu q_2 \\
& + \bar\ell_1 \left(C^{(q_1q_2)VA}_{\ell_1\ell_2} \gamma^\mu + C^{(q_1q_2)AA}_{\ell_1\ell_2} \gamma^\mu\gamma^5 \right) \ell_2 \cdot \overline{q_1} \gamma_\mu \gamma_5 q_2 \\
& + C^{(q_1q_2)TT}_{\ell_1\ell_2} \bar\ell_1 \sigma^{\mu\nu} \ell_2 \cdot \overline{q_1} \sigma_{\mu\nu} q_2 \Big] + \text{H.c.}
\end{aligned}$$

We use the on-shell matching condition for the relation between quark-lepton and meson-lepton couplings.

$C^{(q_1 q_2)AB}_{\ell_1 \ell_2}$ and g_M

$$\langle \ell_1^+ \ell_2^- | \mathcal{L}_{\ell M} | M \rangle \approx \langle \ell_1^+ \ell_2^- | \mathcal{L}_{\ell q} | M \rangle$$

and for this example we are interested in Vector coupling for J/ψ

$$g^{(V/A)}_{J/\psi \ell_1 \ell_2} = \frac{M^2_{J/\psi}}{\Lambda^2} f_{J/\psi} C^{(c)VV/AV}_{\ell_1 \ell_2}$$

$$\mathrm{Br}\left(\tau^- \to \ell^- e^+ e^-\right) = \frac{\Gamma(V \to \tau\ell)\Gamma\left(V \to e^+ e^-\right)}{\Gamma\left(W \to e\bar{\nu}_e\right)^2} \frac{\Gamma\left(\mu^- \to e^- \bar{\nu}_e \nu_\mu\right)}{\Gamma(\tau \to All)} \times \left(\frac{M_W}{M_V}\right)^6 \left(\frac{M_\tau}{M_\mu}\right)^5$$

$$\begin{aligned}\Gamma(V \to \ell_1^+ \ell_2^-) = \frac{|\vec{p}_\ell|}{6\pi} \Bigg[& \left(1 - \frac{m_-^2}{M_M^2}\right)\left(1 + \frac{m_+^2}{2M_M^2}\right)\left(\alpha_{(q)\ell_1\ell_2}^{(V/A)} C_{\ell_1\ell_2}^{(q)VV}\right)^2 \\ & + \left(1 - \frac{m_+^2}{M_M^2}\right)\left(1 + \frac{m_-^2}{2M_M^2}\right)\left(\alpha_{(q)\ell_1\ell_2}^{(V/A)} C_{\ell_1\ell_2}^{(q)AV}\right)^2 \\ & + 2\left(1 - \frac{m_-^2}{M_M^2}\right)\left(1 + \frac{2m_+^2}{M_M^2}\right)\left(\alpha_{(q)\ell_1\ell_2}^{(T)} C_{\ell_1\ell_2}^{(q)TT}\right)^2 \\ & - 6\frac{m_+}{M_M}\left(1 - \frac{m_-^2}{M_M^2}\right)\left(\alpha_{(q)\ell_1\ell_2}^{(V/A)}\right)^2 \left(C_{\ell_1\ell_2}^{(q)VV} C_{\ell_1\ell_2}^{(q)AV}\right)\Bigg]\end{aligned}$$

Revisiting LFV quark-lepton Lagrangian

$$\mathcal{L}_{\ell q} = \frac{1}{\Lambda^2}\Big[\bar{\ell}_1\left(C^{(q_1q_2)SS}_{\ell_1\ell_2} + C^{(q_1q_2)PS}_{\ell_1\ell_2}\gamma^5\right)\ell_2 \cdot \overline{q_1}q_2$$
$$+\bar{\ell}_1\left(C^{(q_1q_2)SP}_{\ell_1\ell_2} + C^{(q_1q_2)PP}_{\ell_1}\gamma^5\right)\ell_2 \cdot \bar{q}_1\gamma_5 q_2$$
$$+\bar{\ell}_1\left(C^{(q_1q_2)VV}_{\ell_1\ell_2}\gamma^\mu + C^{(q_1q_2)AV}_{\ell_1\ell_2}\gamma^\mu\gamma^5\right)\ell_2 \cdot \overline{q_1}\gamma_\mu q_2$$
$$+\bar{\ell}_1\left(C^{(q_1q_2)VA}_{\ell_1\ell_2}\gamma^\mu + C^{(q_1q_2)AA}_{\ell_1\ell_2}\gamma^\mu\gamma^5\right)\ell_2 \cdot \overline{q_1}\gamma_\mu\gamma_5 q_2$$
$$+C^{(q_1q_2)TT}_{\ell_1\ell_2}\bar{\ell}_1\sigma^{\mu\nu}\ell_2 \cdot \overline{q_1}\sigma_{\mu\nu}q_2\Big] + \text{H.c.}$$

$$\mathcal{L}_{\ell q} = \frac{1}{\Lambda^2}\sum_{(IJ)} C^{\Gamma_I\Gamma_J}_{if,\ell_1\ell_2}\left[\bar{\ell}_1\Gamma_I\ell_2\right]\cdot\left[\bar{q}_f\Gamma_J q_i\right] + \text{H.c.}$$

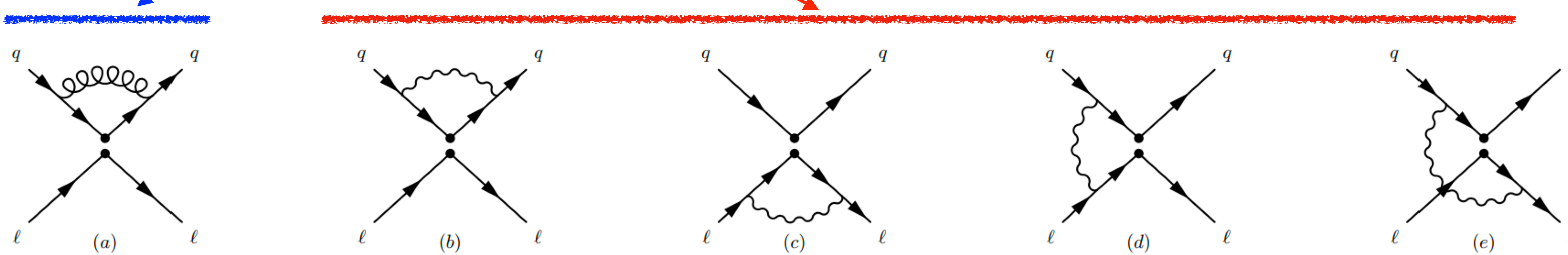
Effective LFV quark-lepton in quiral base operators

$$\mathcal{L}_{lq} = \frac{1}{\Lambda^2}\sum_{i,XY} C^{(q)XY}_{i,\ell_1\ell_2}(\mu)\cdot\mathcal{O}^{(q)XY}_{i,\ell_1\ell_2}(\mu) + \text{H.c.}$$

$$\mathcal{O}^{(q)XY}_{1,\ell_1\ell_2} = 4\left(\bar{\ell}_1 P_X \ell_2\right)\left(\bar{q}P_Y q\right)$$
$$\mathcal{O}^{(q)XX}_{2,\ell_1\ell_2} = 4\left(\bar{\ell}_1\sigma^{\mu\nu}P_X\ell_2\right)\left(\bar{q}\sigma_{\mu\nu}P_X q\right)$$
$$\mathcal{O}^{(q)XY}_{3,\ell_1\ell_2} = 4\left(\bar{\ell}_1\gamma^\mu P_X\ell_2\right)\left(\bar{q}\gamma_\mu P_Y q\right)$$

The general form of the corrected operator matrix elements is given

$$\langle \mathcal{O}_i \rangle^{(0)} = \left[\delta_{ij} + \frac{\alpha_s}{4\pi} b^s_{ij} \left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{-p^2} \right) \right) + \frac{\alpha_{em}}{4\pi} b^{em}_{ij} \left(\frac{1}{\epsilon} + \ln\left(\frac{\mu^2}{-p^2} \right) \right) \right] \langle \mathcal{O}_j \rangle_{\text{tree}}$$



This would be the renormalized operator: $\langle \mathcal{O}_i \rangle^{(0)} = Z_q^{-1} Z_\ell^{-1} \mathcal{Z}_q^{-1} Z_{ij} \langle \mathcal{O}_j \rangle$

Requiring the cancelation of the singularities, one finds the renormalization constant

$$\tilde{Z}^{s}_{ij}\left(\alpha_s, \alpha_{em}\right)$$

$$Z_{ij} = \delta_{ij} + \frac{\alpha_s}{4\pi}\left(b^{s}_{ij} - C_F \delta_{ij}\right)\frac{1}{\epsilon} + \frac{\alpha_{em}}{4\pi}\left(b^{em}_{ij} - \left(Q_q^2 + Q_\ell^2\right)\delta_{ij}\right)\frac{1}{\epsilon} + O\left(\alpha_s^2, \alpha_{em}\alpha_s, \alpha_{em}^2\right)$$

$$\tilde{Z}^{em}_{ij}\left(\alpha_s, \alpha_{em}\right)$$

anomalous dimension

$$\gamma_{ij}\left(\alpha_s, \alpha_{em}\right) = -2\alpha_s \frac{\partial \tilde{Z}^{em}_{ij}\left(\alpha_s, \alpha_{em}\right)}{\partial \alpha_s} - 2\alpha_{em} \frac{\partial \tilde{Z}^{s}_{ij}\left(\alpha_s, \alpha_{em}\right)}{\partial \alpha_{em}}$$

$$\hat{\gamma}^{em,LL/RR} = \begin{pmatrix} -6\left(Q_\ell^2 + Q_q^2\right) & -2Q_\ell Q_q & 0 \\ -96 Q_\ell Q_q & 2\left(Q_\ell^2 + Q_q^2\right) & 0 \\ 0 & 0 & 12 Q_\ell Q_q \end{pmatrix}$$

Renormalization group equations (RGE) for the WC's

$$\frac{d\vec{C}(\mu)}{d\log\mu} = \hat{\gamma}^T \vec{C}(\mu)$$

These matrix elements will induce mixing between Wilson coefficients

$i \neq j \neq 0$

$$C_i(\mu) = \sum_j U_{ij}(\mu, \Lambda) C_j(\Lambda) \leq C_i^{exp}$$

$$\left| C^{(q_1 q_2)AB}_{\ell_1 \ell_2} \right| \left(\frac{1\mathrm{GeV}}{\Lambda} \right)^2 = 4\pi \left(\frac{1\mathrm{GeV}}{\Lambda^{(q_1 q_2)AB}_{\ell_1 \ell_2}} \right)^2$$

$\Lambda^{(q)}_{\mu e}$	Without QED ⊗ QCD [TeV]	With QED ⊗ QCD ($\Lambda = 1$ TeV) [TeV]	With QED ⊗ QCD ($\Lambda = 10$ TeV) [TeV]
$\Lambda^{(0)VV/AV}_{\mu e}$	5.1×10^3	5.1×10^3	5.1×10^3
$\Lambda^{(0)AA/VA}_{\mu e}$	0.38	4.6×10^2	5.3×10^2
$\Lambda^{(0)SS}_{\mu e}$	1.8	23	27
$\Lambda^{(0)PS}_{\mu e}$	1.8	2.7	2.8
$\Lambda^{(0)PP}_{\mu e}$	5.4	23	27
$\Lambda^{(0)SP}_{\mu e}$	5.4	7.8	8.3
$\Lambda^{(0)TT}_{\mu e}$	5.8×10^2	5.1×10^2	5.0×10^2
$\Lambda^{(3)VV/AV}_{\mu e}$	5.4×10^2	5.4×10^2	5.4×10^2
$\Lambda^{(3)AA/VA}_{\mu e}$	2.2	8.0×10^2	9.2×10^2
$\Lambda^{(3)SS}_{\mu e}$	0.45	40	47
$\Lambda^{(3)PS}_{\mu e}$	0.45	0.65	0.70
$\Lambda^{(3)PP}_{\mu e}$	8.0	40	47
$\Lambda^{(3)SP}_{\mu e}$	7.9	11	12
$\Lambda^{(3)TT}_{\mu e}$	61	54	53

CONCLUSIONS

- We used model building, and built two predictive and viable models that accommodates the values of the experimental neutrino parameters. The first model we built was for the Inverted Ordering (IO) of neutrinos, the second one contemplates a model for Normal Ordering (NO), which is favored experimentally.

- We have generated the light active neutrinos mass by radiative see-saw, mediated by the neutral component of a doublet vector, in the fundamental representation.

- We found that the vector mass is around 1 TeV, giving that we consider the cutoff of 3 TeV, because the heavy neutrino mass near the 250 GeV.

- We solved the renormalization group equations, using effective operators and we improve the upper limits in the lepton violating three body decay, with a meson exchanged.

- The most important finding is that the evolution operator matrix mixed up the Wilson coefficients, so this induced new limits, in the upper existing limit in this set of three body leptonic decays, and improve the limits known, in the literature, up to two orders of magnitud.


תודה
Dankie
Gracias
Спасибо
شكراً
Merci
Takk
Köszönjük
Terima kasih
Grazie
Dziękujemy
Dėkojame
Ďakujeme
Vielen Dank
Paldies
Kiitos
Täname teid
谢谢
Thank You
Tak
感謝您
Obrigado
Teşekkür Ederiz
감사합니다
Σας Ευχαριστούμ
ขอบคุณ
Bedankt
Děkujeme vám
ありがとうございます
Tack

Back up slides

$$X \simeq \sum_{n=1}^{2} \left(y_{2n}^{(V)}\right)^2 \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n}$$

$$Y \simeq \sum_{n=1}^{2} \left(y_{1n}^{(V)}\right)\left(y_{2n}^{(V)}\right) \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n},$$

$$Z \simeq \sum_{n=1}^{2} \left(y_{1n}^{(V)}\right)^2 \lambda^{16} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) m_{N_n}$$

$$\begin{aligned} f(m_{\mathrm{Re}\,V^0}, m_{\mathrm{Im}\,V^0}, m_{N_n}) = \frac{1}{16\pi^2} \Bigg\{ & \frac{\Lambda^2}{m_{\mathrm{Re}\,V^0}^2} - \frac{\Lambda^2}{m_{\mathrm{Im}\,V^0}^2} + \frac{m_{\mathrm{Re}\,V^0}^2}{m_{\mathrm{Re}\,V^0}^2 - m_{N_n}^2} \ln\left(\frac{m_{\mathrm{Re}\,V^0}^2}{m_{N_n}^2}\right) \\ & - \frac{m_{\mathrm{Im}\,V^0}^2}{m_{\mathrm{Im}\,V^0}^2 - m_{N_n}^2} \ln\left(\frac{m_{\mathrm{Im}\,V^0}^2}{m_{N_n}^2}\right) \\ & + \left(\frac{m_{N_n}^4}{m_{\mathrm{Re}\,V^0}^2\left(m_{\mathrm{Re}\,V^0}^2 - m_{N_n}^2\right)} - \frac{m_{N_n}^4}{m_{\mathrm{Im}\,V^0}^2\left(m_{\mathrm{Im}\,V^0}^2 - m_{N_n}^2\right)}\right) \ln\left(\frac{\Lambda^2 + m_{N_n}^2}{m_{N_n}^2}\right) \Bigg\} \end{aligned}$$

$$\hat{b}^{em,LL/RR} = \begin{pmatrix} 4(Q_\ell^2 + Q_q^2) & Q_q Q_\ell & 0 \\ 48 Q_\ell Q_q & 0 & 0 \\ 0 & 0 & Q_\ell^2 - 6 Q_\ell Q_q + Q_q^2 \end{pmatrix}$$

$$\gamma_{ij}^{em} = -2\left(b_{ij}^{em} - \left(Q_q^2 + Q_\ell^2\right)\delta_{ij}\right)$$

$$C_{\ell_1\ell_2}^{(0/3)\Gamma_i\Gamma_J} = C_{\ell_1\ell_2}^{(u)\Gamma_i\Gamma_J} \pm C_{\ell_1\ell_2}^{(d)\Gamma_i\Gamma_J}$$

$$\langle\chi\rangle = v_\chi\,(1,0)\,, \qquad \langle\xi\rangle = v_\xi\left(1,\sqrt{2}\right).$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_2 = (x_1y_1 + x_2y_2)_{\mathbf{1}} + (x_1y_2 - x_2y_1)_{\mathbf{1}'} + \begin{pmatrix} x_2y_2 - x_1y_1 \\ x_1y_2 + x_2y_1 \end{pmatrix}_{\mathbf{2}}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2y' \\ x_1y' \end{pmatrix}_2, \quad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x'y')_{\mathbf{1}}.$$

$$V_\mu = \begin{pmatrix} V_\mu{}^+ \\ V_\mu{}^0 \end{pmatrix} = \begin{pmatrix} v_\mu{}^+ \\ \frac{V_\mu^1 + i v_\mu^2}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L} = \lambda \bar{L} \gamma^\mu V_\mu e_R$$

$$\mathcal{L} = -\frac{1}{2}\left(D_\mu V_\nu - D_\nu V_\mu\right)^\dagger \left(D^\mu V^\nu - D^\nu V^\mu\right) + M_V^2 V_\mu^\dagger V^\mu + \lambda_2\left(\phi^\dagger\phi\right)\left(V_\mu^\dagger V^\mu\right) + \lambda_3\left(\phi^\dagger V_\mu\right)\left(V^{\mu\dagger}\phi\right) + \lambda_4\left(\phi^\dagger V_\mu\right)\left(\phi^\dagger V^\mu\right) + \alpha_1 \phi^\dagger D_\mu V^\mu + \alpha_2\left(V_\mu^\dagger V^\mu\right)\left(V_\nu^\dagger V^\nu\right) + \alpha_3\left(V_\mu^\dagger V^\nu\right)\left(V_\nu^\dagger V^\mu\right) + \mathrm{i} g \kappa_1 V_\mu^\dagger W^{\mu\nu} V_\nu + \mathrm{i}\frac{g'}{2}\kappa_2 V_\mu^\dagger B^{\mu\nu} V_\nu + \text{ h.c.}$$

$$\mathcal{L} = \lambda \bar{\psi}\left(1+\gamma^5\right)\gamma^\mu V_\mu \frac{1}{2}\left(1+\gamma^5\right) e = \lambda \bar{\psi}\frac{1}{4}(1+\gamma^5)(1-\gamma^5)\gamma^\mu V_\mu e$$

$$\mathcal{L} = \lambda \bar{L} \gamma^\mu V_\mu N_L$$

$$\mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}$$

$$\Downarrow \Lambda_{int}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\Downarrow v$$

$$SU(3)_C \times U(1)_Q$$

$$\boxed{S_3 \times Z_2 \times Z_6 \times Z_8 \times Z_{12}}$$

IO

$$V_\mu \sim (\mathbf{1}, 1, 0, 0, 0)$$

$$-\mathcal{L}_{VlN} = \sum_{n=1}^{2} y_{1n}^{(V)} \bar{l}_{1L} \gamma^\mu V_\mu N_{nL} \frac{\sigma \eta^4 \rho_1^2 \varphi}{\Lambda^8} + \sum_{n=1}^{2} y_{2n}^{(V)} \bar{l}_L \gamma^\mu V_\mu N_{nL} \frac{\xi \eta^4 \rho_1^2 \varphi}{\Lambda^8}$$

$$\boxed{S_3 \times Z_3 \times Z_4 \times Z_5 \times Z_8}$$

NO

$$V_\mu \sim (\mathbf{1}, 0, 2, 0, 0)$$

$$m_{\mathrm{Im}V^0} = 1.5\mathrm{TeV}, \qquad m_{\mathrm{Re}V^0} = 1.6\mathrm{TeV}, \qquad m_N = 159\mathrm{MeV}.$$

NO

Observable	Model Value	Experimental value		
		1σ range	2σ range	3σ range
m_e [MeV]	0.487	0.487	0.487	0.487
m_μ [MeV]	102.8	102.8 ± 0.0003	102.8 ± 0.0006	102.8 ± 0.0009
m_τ [GeV]	1.75	1.75 ± 0.0003	1.75 ± 0.0006	1.75 ± 0.0009
m_1 $[meV]$	0	$\cdots$	$\cdots$	$\cdots$
m_2 $[meV]$	8.67	$\cdots$	$\cdots$	$\cdots$
m_3 $[meV]$	50	$\cdots$	$\cdots$	$\cdots$
Δm_{21}^2 $[10^{-5}\, eV^2]$	7.55	$7.55^{+0.20}_{-0.16}$	$7.20 - 7.94$	$7.05 - 8.14$
Δm_{31}^2 $[10^{-3}\, eV^2]$	2.50	2.50 ± 0.03	$2.44 - 2.57$	$2.41 - 2.60$
$\sin^2(\theta_{12})/10^{-1}$	3.20	$3.20^{+0.20}_{-0.16}$	$2.89 - 3.59$	$2.73 - 3.79$
$\sin^2(\theta_{23})/10^{-1}$	5.47	$5.47^{+0.20}_{-0.30}$	$4.67 - 5.83$	$4.45 - 5.99$
$\sin^2(\theta_{13})/10^{-2}$	2.160	$2.160^{+0.083}_{-0.069}$	$2.03 - 2.34$	$1.96 - 2.41$
δ_{CP}	218°	$218^{+38^\circ}_{-27^\circ}$	$182^\circ - 315^\circ$	$157^\circ - 349^\circ$

$$Br\left(l_i \to l_j \gamma\right) = \frac{\alpha_W^3 s_W^2 m_{l_i}^5}{256\pi^2 m_{V^\pm}^4 \Gamma_i} \left| \sum_{n=1}^{2} G\left(\frac{m_{N_n}^2}{m_{V^\pm}^2}\right) \right|^2 ,$$

$$G(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^2} - \frac{3x^3}{2(1-x)^4} \ln x$$

$1.3 \text{ TeV} \lesssim m_{V^\pm} \lesssim 2.5 \text{ TeV}$

$100 \text{ GeV} \lesssim m_N \lesssim 380 \text{ GeV}$

MUON g – 2

2.1 TeV

$1.3\ \text{TeV} \lesssim m_{V^\pm} \lesssim 2.5\ \text{TeV}$

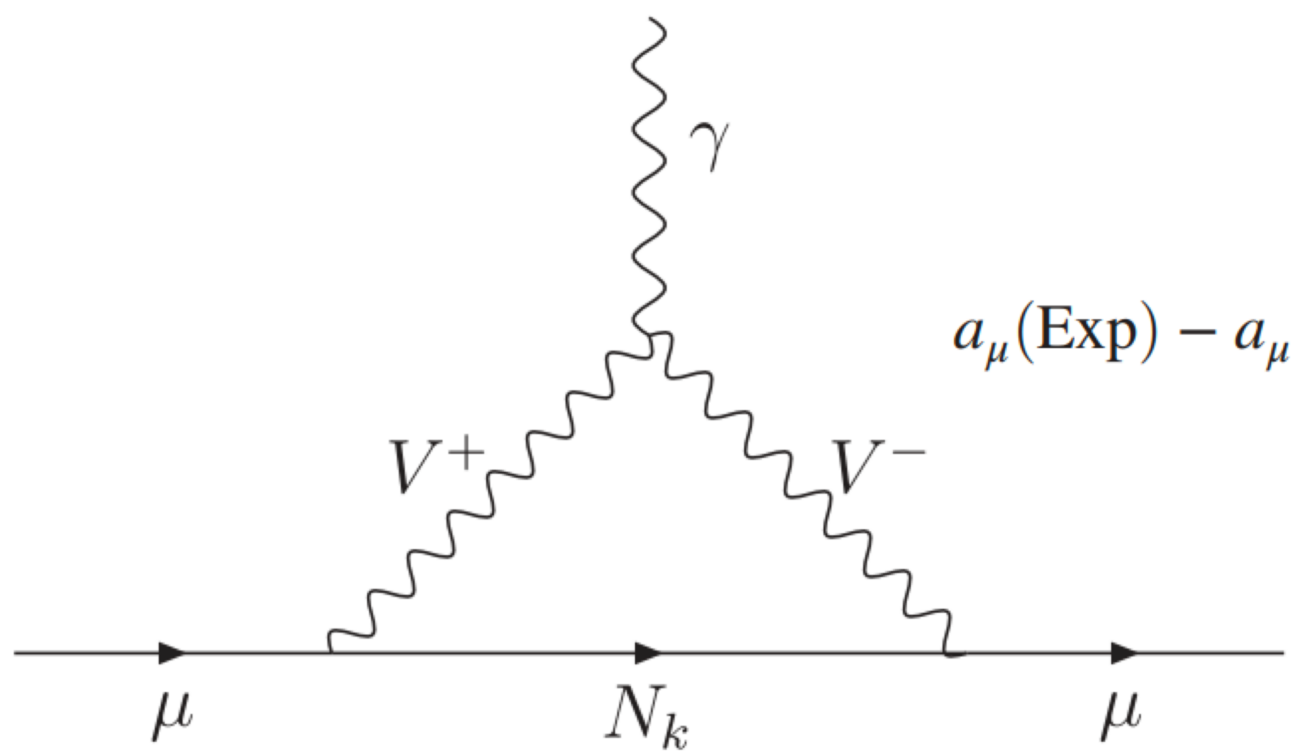
$100\ \text{GeV} \lesssim m_N \lesssim 380\ \text{GeV}$

$\sum_k |h_{\mu k}|^2/4\pi \simeq 1$

$$\Delta a_\mu = \frac{1}{8\pi^2}\frac{m_\mu^2}{m_{V^-}^2}\sum_k |h_{\mu k}|^2 \times \int_0^1 dx x \frac{x(1+x)m_{V^-}^2 + (1-x)(1-\frac{x}{2})M_k^2}{x m_{V^-}^2 + (1-x)M_k^2}$$

$m_{V^-} \gg M_k.$ → **1**

$$\Delta a_\mu = 250 \times 10^{-11}$$



$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

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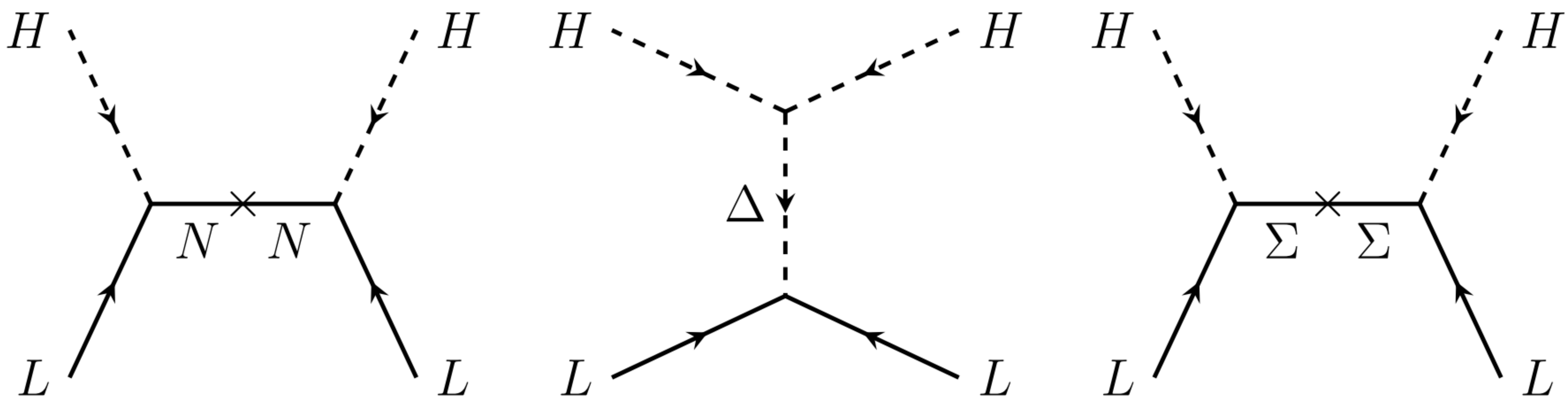

Type I see-saw

Type II see-saw

Type III see-saw

Seesaw neutrino mass mechanisms.