

S₃ AS A MODULAR SYMMETRY: CONSEQUENCES IN THE QUARK AND HIGGS SECTORS

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PLB788 (2018); EPJC81 (2021); arXiv:23???.????

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UTFSM, Valparaíso

HOW TO GO BSM?

- Many ways to go BSM
- Usually: add symmetries, add particles, add interactions
- All of the above
- Messy...
- I will concentrate on masses and mixings
- And the possibility of dark matter (and perhaps leptogenesis...)



SOME ASPECTS OF THE FLAVOUR PROBLEM

- ▶ Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- ▶ Neutrino masses unknown, only difference of squared masses.
- ▶ Type of hierarchy (normal or inverted) also unknown
- ▶ Higgs sector under study

- ▶ Quark mixing angles

$$\theta_{12} \approx 13.0^\circ$$

$$\theta_{23} \approx 2.4^\circ$$

$$\theta_{13} \approx 0.2^\circ$$

- ▶ Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$

$$\Theta_{23} \approx 48.6^\circ$$

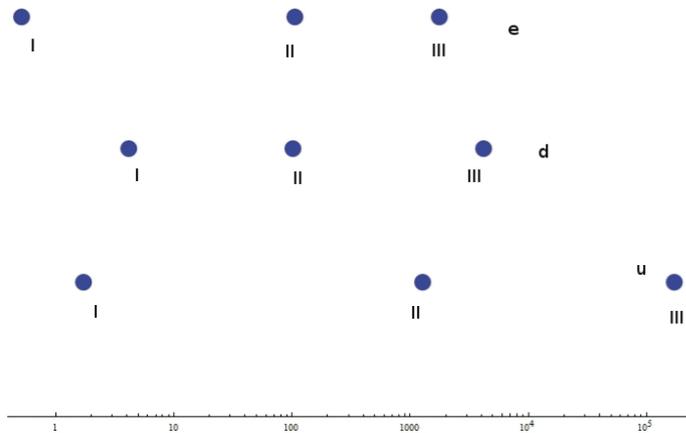
$$\Theta_{13} \approx 8.6^\circ$$

- ▶ Small mixing in quarks, large mixing in neutrinos.
Very different
- ▶ Is there an underlying symmetry?

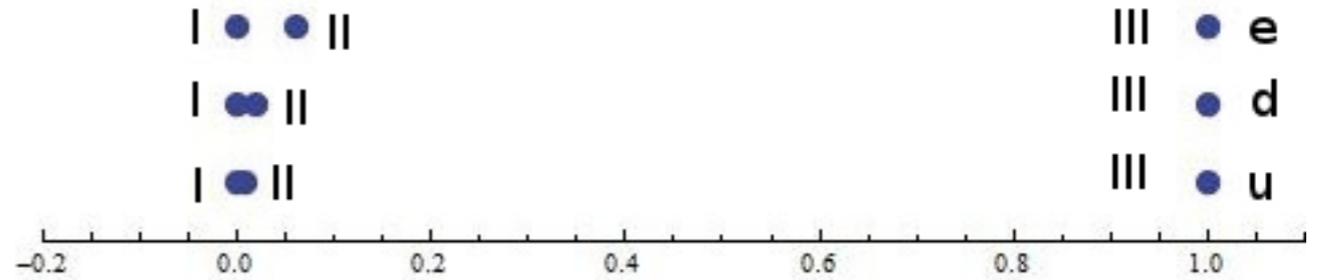


HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs
- Look at low energy phenomenology
- At some point they should intersect...
- In here:
 - Find the smallest flavour symmetry suggested by data
 - Explore how generally it can be applied (universally)
 - Follow it to the end
 - Compare it with the data



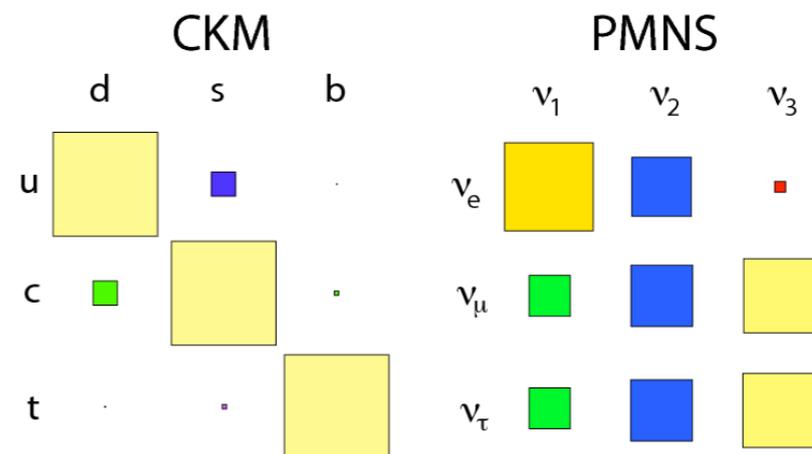
Plot of mass ratios



Logarithmic plot of quark masses

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

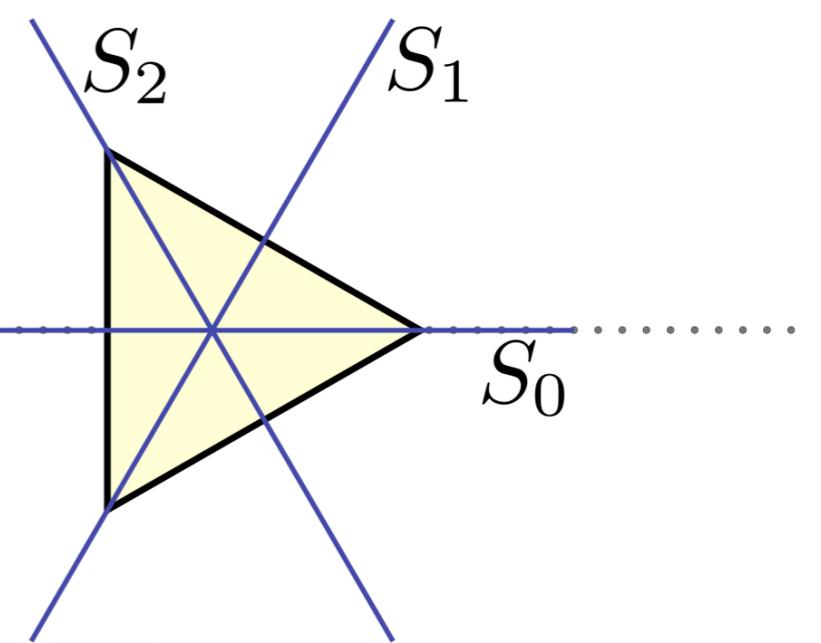
Suggests a $2 \oplus 1$ structure



3HDM

- Without symmetry \implies 54 real parameters in potential
- Complemented with additional symmetry(ies)
- Studies started in the 70's, hope to find global symmetry that explains the mass and mixing patterns
- The first symmetries to be added were the permutational groups S_3 and S_4
- Different modern versions of these models exist

3HDM WITH S3



- Low-energy model
- Extend the concept of flavour to the Higgs sector by adding two more eW doublets
- Add symmetry: permutation symmetry of three objects, symmetry operations (reflections and rotations) that leave an equilateral triangle invariant
- **3HDM with symmetry S3:**
8 couplings in the Higgs potential

A sample of S3 models

S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)

E. Derman, Phys. Rev. D19, 317 (1979)

D. Wyler, Phys. Rev. D19, 330 (1979)

R. Yahalom, Phys. Rev. D29, 536 (1984)

Y. Koide, Phys. Rev. D60, 077301 (1999)

A. Mondragon et al, Phys. Rev. D59, 093009, (1999)

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J. Kubo et al, Phys. Rev. D70, 036007 (2004)

S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)

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S. Chen et al, Phys. Rev. D70, 073008 (2004)

T. Teshima et al, Phys.Rev. D84 (2011)
016003 Phys.Rev. D85 105013 (2012)

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H.B. Benaoum, Phys. RevD.87.073010 (2013)

E. Ma and B. Melic, arXiv:1303.6928

F. Gonzalez Canales, A. &M Mondragon, U. Saldaña, L. Velasco, arXiv:1304.6644

R. Jora et al, Int.J.Mod.Phys. A28 (2013),1350028

A. E. Cárcamo Hernández, E. Cataño Mur, R. Martinez, Phys.Rev. D90 (2014) no.7, 073001

A.E. Cárcamo, I. de Medeiros E. Schumacheet, Phys.Rev. D93 (2016) no.1, 016003

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D Das, P Pal, Pays Rev D98 (2018)

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O. Felix-Beltran, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)

A. Dicus, S Ge, W Repko, Phys. Rev D82 (2010)

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)

D. Meloni, JHEP 1205 (2012) 124

S. Dev et al, Phys.Lett. B708 (2012) 284-289

S. Zhou, Phys.Lett. B704 (2011) 291-295

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

E. Ma and B. Melic, Phys.Lett. B725 (2013)

E. Barradas et al, 2014

P. Das et al, PhyrRev D89 (2014,) 2016

ZZ Zhing, D Zhang JHEP 03 2019)

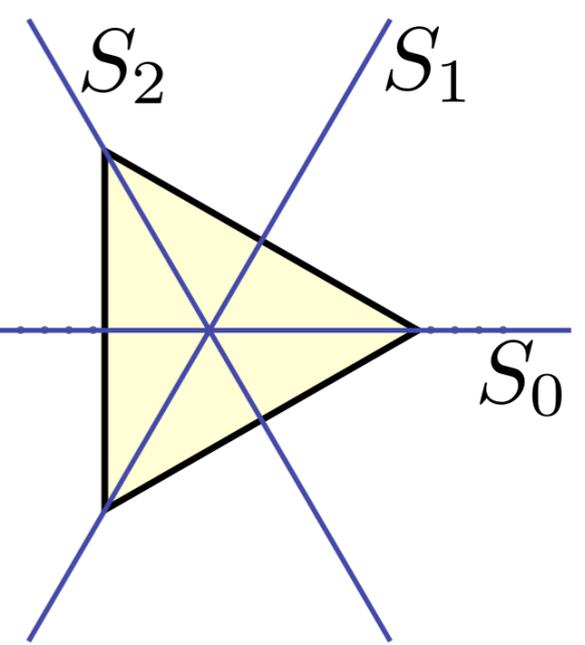
S Pramanick, Phys Rev D100 (2019)

M. Gómez-Bock, A. Pérez, MM, EPJC81 (2021)

*Just a sample, there are many more...
I apologize for those not included*

S3

- Smallest non-Abelian discrete group
- Has irreducible representations, 2, 1_S and 1_A
- We add three right-handed neutrinos to implement the see-saw mechanism
- We apply the symmetry “universally” to quarks, leptons and Higgs-es
 - First two families in the doublet
 - Third family in symmetric singlet
- Three sectors related, we treat them simultaneously



PREDICTIONS, ADVANTAGES?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- CKM has NNI and Fritzsch textures
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Reactor mixing angle
 $\theta_{13} \neq 0$
- Some FCNCs suppressed by symmetry
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs
→ residual symmetry of a more fundamental one?
- Lots of Higgses:
3 neutral, 4 charged,
2 pseudoscalars
- Further predictions will come from Higgs sector:
decays, branching ratios

FERMION MASSES

- The Lagrangian of the model

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

- The general form of the fermion mass matrices in the symmetry adapted basis is

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}.$$

where $m_{1,3} = Y_{1,3}v_3$ and $m_{1,2,4,5} = Y_{1,2,4,5}(v_1 \text{ or } v_2)$

QUARKS

without taking into account minimization conditions

3HDM: $G_{SM} \otimes S_3$

	ψ_L^f	ψ_R^f	Mass matrix	Possible mass textures	
A	$\mathbf{2}, 1_S$	$\mathbf{2}, 1_S$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_6^f \\ \mu_4^f & \mu_1^f - \mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & \mu_2^f s c (3 - t^2) & 0 \\ \mu_2^f s c (3 - t^2) & -2\mu_2^f c^2 (1 - 3t^2) & \mu_7^f / c \\ 0 & \mu_7^{f*} / c & \mu_3^f - \mu_1^f - \mu_2^f c^2 (1 - 3t^2) \end{pmatrix}$	
A'				$\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_9^f & \mu_3^f - \mu_1^f \end{pmatrix}$	NNI
B	$\mathbf{2}, 1_A$	$\mathbf{2}, 1_A$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_7^f \\ \mu_4^f & \mu_1^f - \mu_2^f & -\mu_6^f \\ -\mu_9^f & \mu_8^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & -\mu_4^f c^2 (1 - 3t^2) & 0 \\ -\mu_4^f c^2 (1 - 3t^2) & 2\mu_4^f s c (3 - t^2) & -\mu_6^f / c \\ 0 & -\mu_6^{f*} / c & \mu_3^f - \mu_1^f + \mu_4^f s c (3 - t^2) \end{pmatrix}$	
B'				$\begin{pmatrix} 0 & -2\mu_4^f & 0 \\ -2\mu_4^f & 0 & -2\mu_6^f \\ 0 & 2\mu_8^f & \mu_3^f - \mu_1^f \end{pmatrix}$	NNI

Table 2: Mass matrices in S_3 family models with three Higgs $SU(2)_L$ doublets: H_1 and H_2 , which occupy the S_3 irreducible representation $\mathbf{2}$, and H_S , which transforms as 1_S for the cases when both the left- and right-handed fermion fields are in the same assignment. The mass matrices shown here follow a normal ordering of their mass eigenvalues (m_1^f, m_2^f, m_3^f) . We have denoted $s = \sin \theta$, $c = \cos \theta$ and $t = \tan \theta$. The third column of this table corresponds to the general case, while the fourth column to a case where we have rotated the matrix to a basis where the elements $(1, 1)$, $(1, 3)$ and $(3, 1)$ vanish. The primed cases, A' or B' , are particular cases of the unprimed ones, A or B , with $\theta = \pi/6$ or $\theta = \pi/3$, respectively.

Mass matrices reproduce the NNI or the Fritzsch forms (rotation + shift)

HIGGS SECTOR – TESTS FOR THE MODEL

General Potential:

$$\begin{aligned}
 V = & \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right) + \mu_0^2 \left(H_s^\dagger H_s \right) + a \left(H_s^\dagger H_s \right)^2 + b \left(H_s^\dagger H_s \right) \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right) \\
 & + c \left(H_1^\dagger H_1 + H_2^\dagger H_2 \right)^2 + d \left(H_1^\dagger H_2 - H_2^\dagger H_1 \right)^2 + e f_{ijk} \left(\left(H_s^\dagger H_i \right) \left(H_j^\dagger H_k \right) + h.c. \right) \\
 & + f \left\{ \left(H_s^\dagger H_1 \right) \left(H_1^\dagger H_s \right) + \left(H_s^\dagger H_2 \right) \left(H_2^\dagger H_s \right) \right\} + g \left\{ \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1 \right)^2 \right\} \\
 & + h \left\{ \left(H_s^\dagger H_1 \right) \left(H_s^\dagger H_1 \right) + \left(H_s^\dagger H_2 \right) \left(H_s^\dagger H_2 \right) + \left(H_1^\dagger H_s \right) \left(H_1^\dagger H_s \right) + \left(H_2^\dagger H_s \right) \left(H_2^\dagger H_s \right) \right\} \quad (1)
 \end{aligned}$$

Derman and Tsao (1979); Sugawara and Pawasa (1978); Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009), Das and Dey (2014), Barradas et al (2014), Costa, OGREID, Osland and Rebelo (2016), etc

➤ The minimum of potential can be parameterised in spherical coordinates, two angles and v

➤ **Minimisation fixes** $v_1^2 = 3v_2^2$

➤ $e = 0$ massless scalar, residual continuous S_2 symmetry

➤ Conditions for normal vacuum already studied, also for CP breaking ones

Felix-Beltrán, Rodríguez-Jáuregui, M.M (2007); Barradas et al (2015); Costa et al (2016)

$$v_1 = v \cos \varphi \sin \theta, \quad v_2 = v \sin \varphi \sin \theta, \quad v_3 = v \cos \theta.$$

$$\begin{aligned}
 \tan \varphi = 1/\sqrt{3} & \Rightarrow \sin \varphi = \frac{1}{2} \quad \& \quad \cos \varphi = \frac{\sqrt{3}}{2} \\
 \tan \theta = \frac{2v_2}{v_3} & \Rightarrow \sin \theta = \frac{2v_2}{v} \quad \& \quad \cos \theta = \frac{v_3}{v}
 \end{aligned}$$

STABILITY CONDITIONS

$$\begin{aligned} \lambda_8 &> 0 \\ \lambda_1 + \lambda_3 &> 0 \\ \lambda_5 &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\ \lambda_5 + \lambda_6 - 2|\lambda_7| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\ \lambda_1 - \lambda_2 &> 0 \\ \lambda_1 + \lambda_3 + |2\lambda_4| + \lambda_5 + 2\lambda_7 + \lambda_8 &> 0 \\ \lambda_{13} &> 0 \\ \lambda_{10} &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\ \lambda_{10} + \lambda_{11} - 2|\lambda_{12}| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\ \lambda_{14} &> -2\sqrt{\lambda_8\lambda_{13}}. \end{aligned}$$

Das and Dey (2014)

UNITARITY CONDITIONS

$$\begin{aligned} a_1^\pm &= (\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2}) \\ &\pm \sqrt{(\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2})^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5 + \lambda_6}{2}) - \lambda_4^2]} \\ a_2^\pm &= (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8) \\ &\pm \sqrt{(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - 2\lambda_7^2]} \\ a_3^\pm &= (\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8) \\ &\pm \sqrt{(\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - \frac{\lambda_6^2}{2}]} \\ a_4^\pm &= (\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7) \\ &\pm \sqrt{(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7)^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5}{2} + \lambda_7) - \lambda_4^2]} \\ a_5^\pm &= (5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8) \\ &\pm \sqrt{(5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8)^2 - 4[3\lambda_8(5\lambda_1 - \lambda_2 + 2\lambda_3) - \frac{1}{2}(2\lambda_5 + \lambda_6)^2]} \\ a_6^\pm &= (\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) \pm ((\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7)^2 - \\ &4[(\lambda_1 + \lambda_2 + 4\lambda_3)(\frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) - 9\lambda_4^2])^{1/2} \end{aligned}$$

$$\begin{aligned} b_1 &= \lambda_5 + 2\lambda_6 - \lambda_7 \\ b_2 &= \lambda_5 - 2\lambda_7 \\ b_3 &= 2(\lambda_1 - 5\lambda_1 - 2\lambda_3) \\ b_4 &= 2(\lambda_1 - \lambda_1 - 2\lambda_3) \\ b_5 &= 2(\lambda_1 + \lambda_1 - 2\lambda_3) \\ b_6 &= \lambda_5 - \lambda_6. \end{aligned}$$

HIGGS MASSES

- After electroweak symmetry breaking (Higgs mechanism) we are left with **9 massive particles**

doesn't couple to gauge bosons: Z2 symmetry massless when $e=0$, S2 symmetry

$$m_{h_0}^2 = -9ev^2 \sin \theta \cos \theta$$

$$m_{H_1, H_2}^2 = (M_a^2 + M_c^2) \pm \sqrt{(M_a^2 - M_c^2)^2 + (M_b^2)^2}$$

$$M_a^2 = \left[2(c + g)v^2 \sin^2 \theta + \frac{3}{2}ev^2 \sin \theta \cos \theta \right]$$

$$M_b^2 = \left[3ev^2 \sin^2 \theta + 2(b + f + 2h)v^2 \sin \theta \cos \theta \right]$$

$$M_c^2 = 2av^2 \cos^2 \theta - \frac{ev^2 \tan \theta \sin^2 \theta}{2}$$

H1 or H2 can be the SM Higgs boson

$$m_{A_1}^2 = -v^2 \left[2(d + g) \sin^2 \theta + 5e \cos \theta \sin \theta + 2h \cos^2 \theta \right]$$

$$m_{A_2}^2 = -v^2 (e \tan \theta + 2h)$$

Das and Dey (2014)

Barradas, Félix, González (2014)

Gómez-Bock, MM, Perez-Martínez (2022)

$$m_{H_1^\pm}^2 = -v^2 \left[5e \sin \theta \cos \theta + (f + h) \cos^2 \theta + 2g \sin^2 \theta \right]$$

$$m_{H_2^\pm}^2 = -v^2 \left[e \tan \theta + (f + h) \right]$$

RESIDUAL Z2 SYMMETRY

- After eW symmetry breaking, S3 breaks -> residual Z2 symmetry

Das and Dey (2014), Ivanov (2017)

- h_0 decoupled from gauge bosons

- There are 2 “alignment” limits 🙄

- H2 is the SM Higgs → H1 decoupled from gauge bosons

- H1 is the SM Higgs → H2 decoupled from gauge bosons

$m_{H2} < m_{H1}$

- Z2 parity:

h_0, A_1, H_{1^\pm} parity -1,

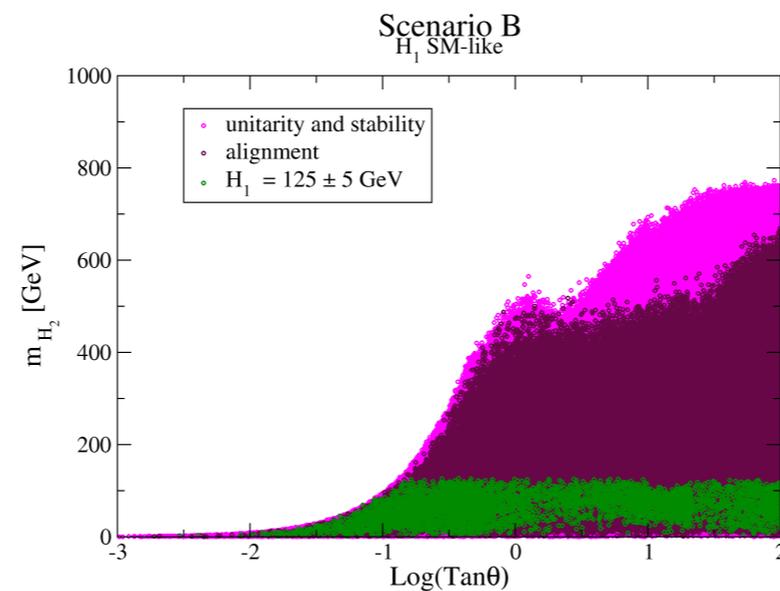
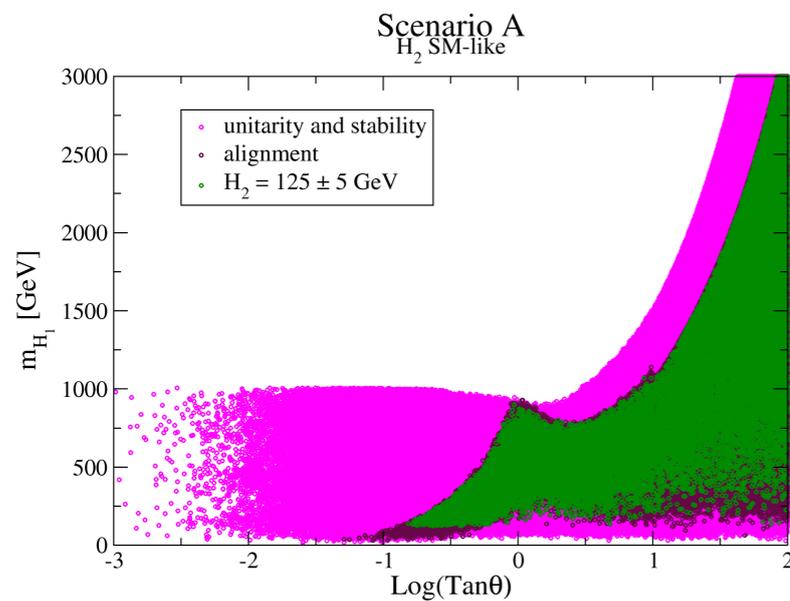
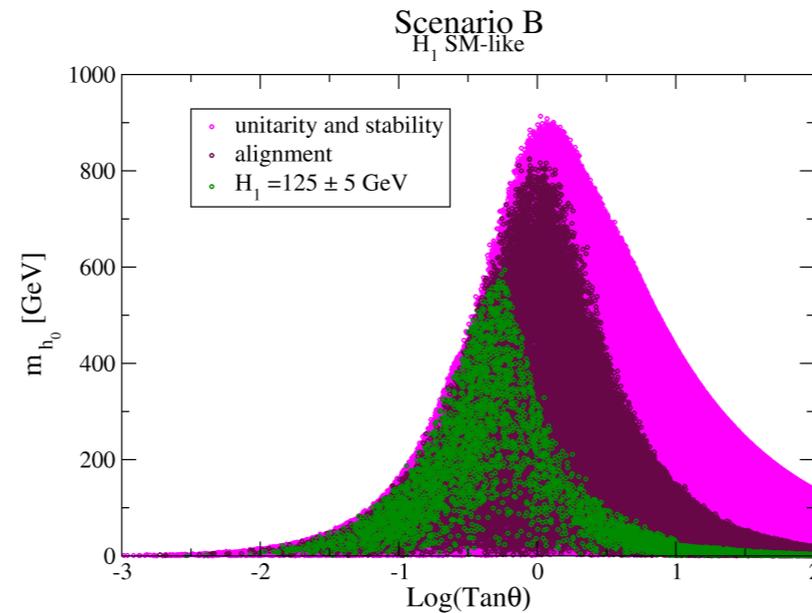
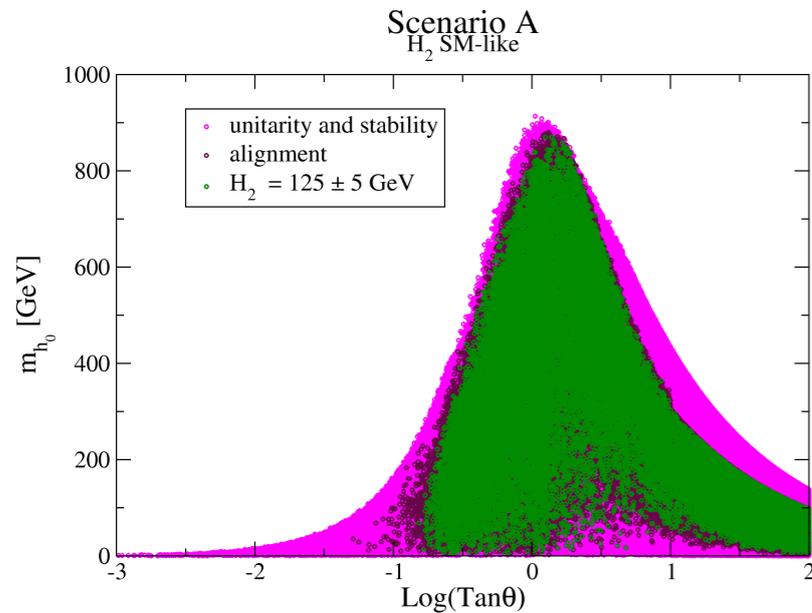
H_1, H_2 parity +1

H_{2^\pm}, A_2 parity +1

Das and Dey (2014)

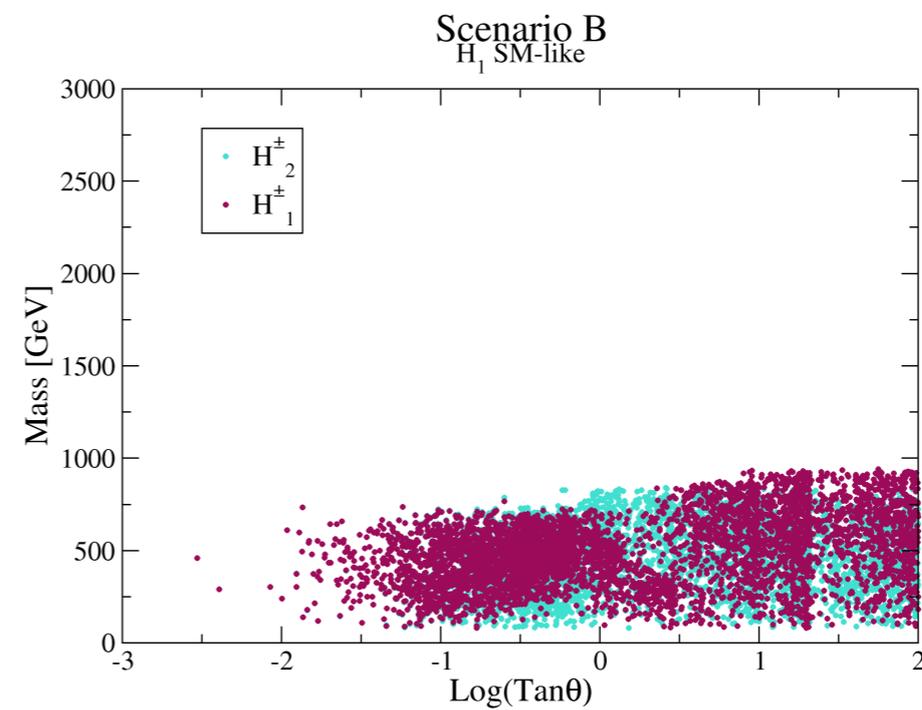
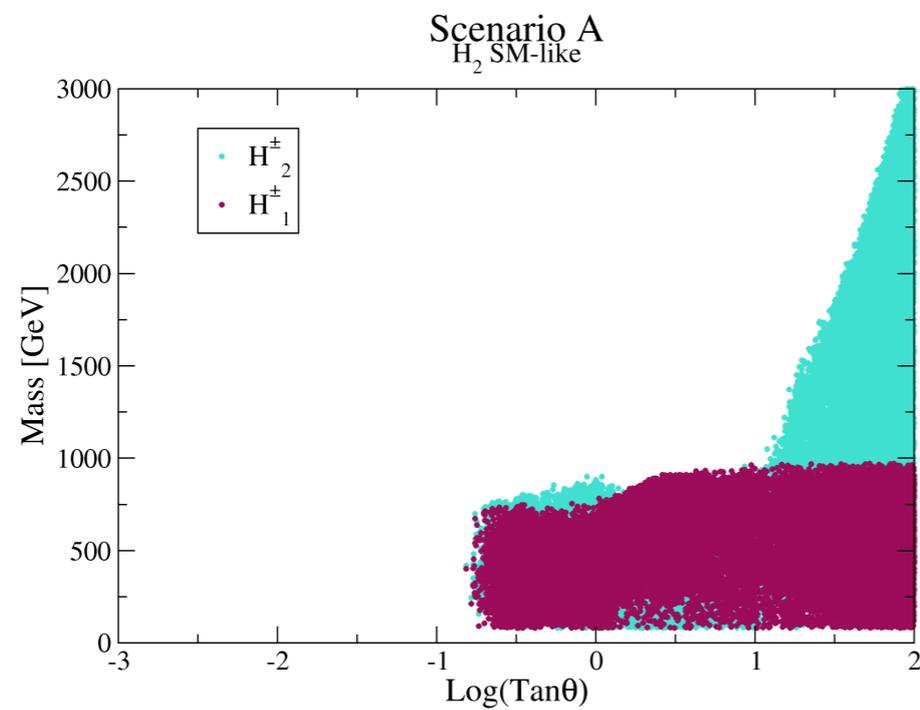
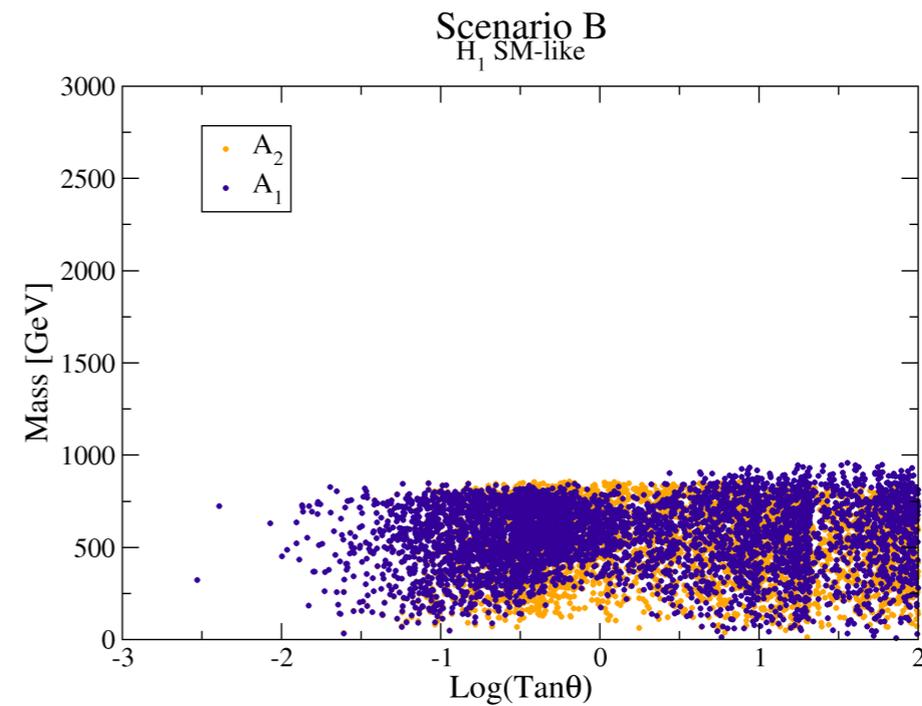
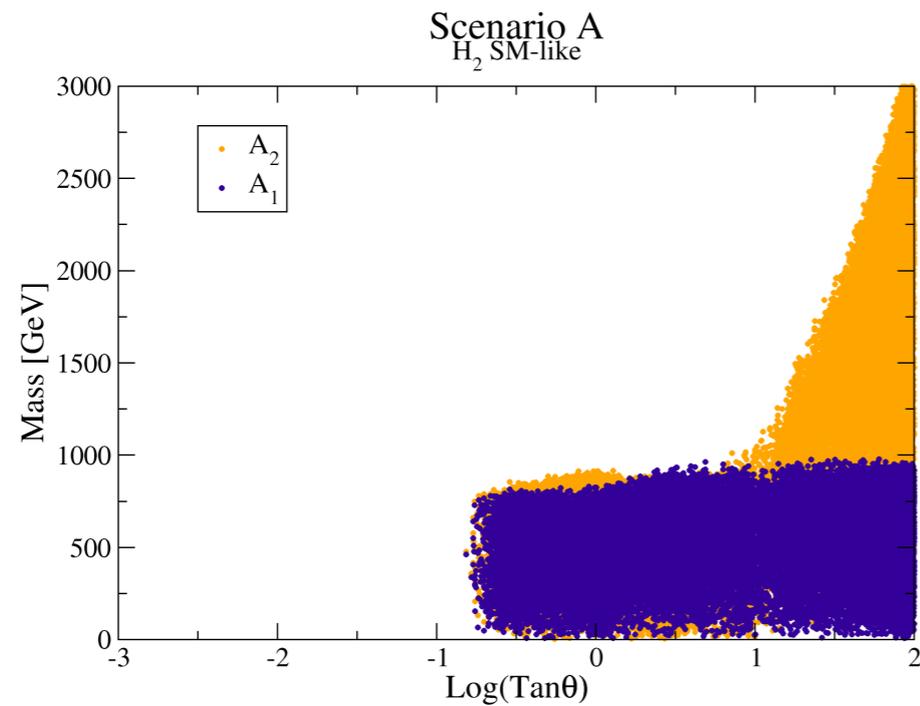
- This forbids certain couplings

NEUTRAL SCALAR MASSES



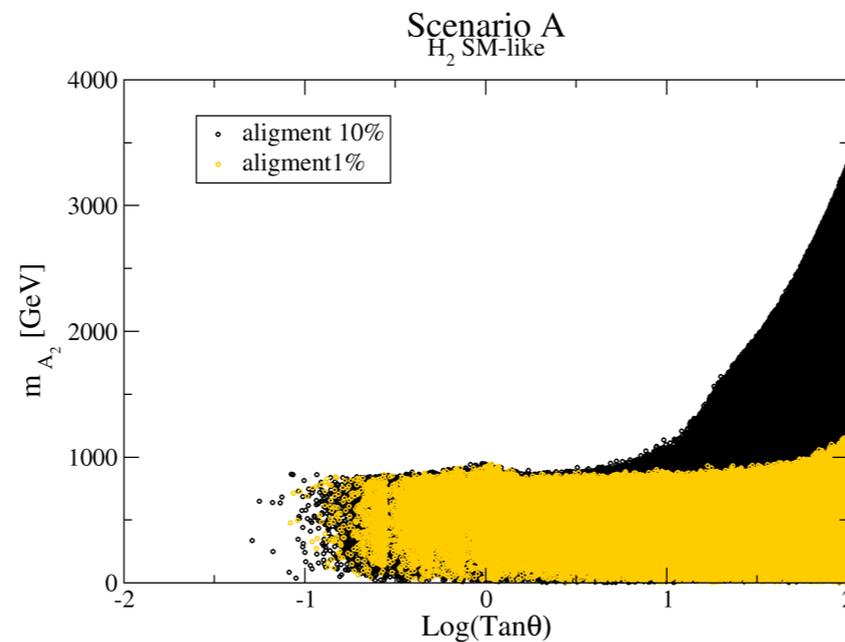
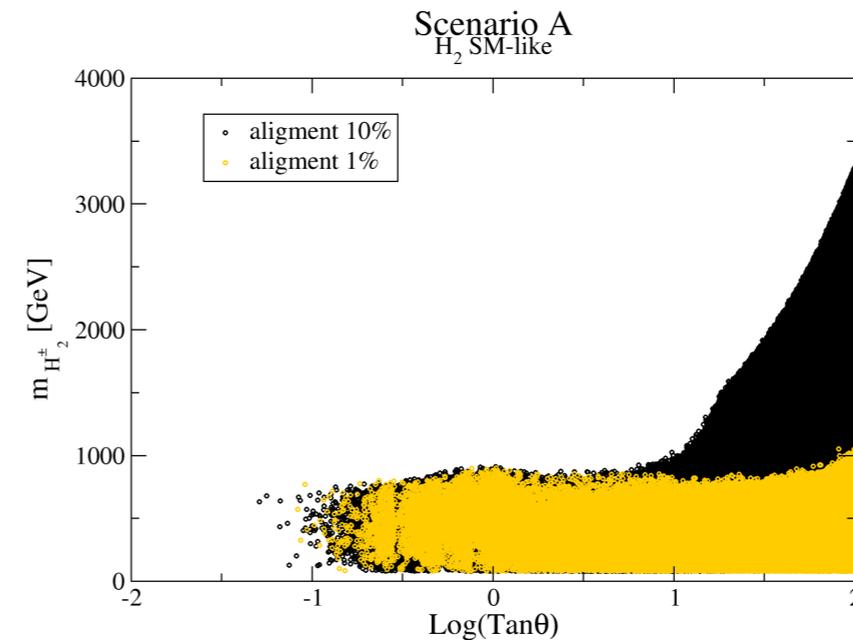
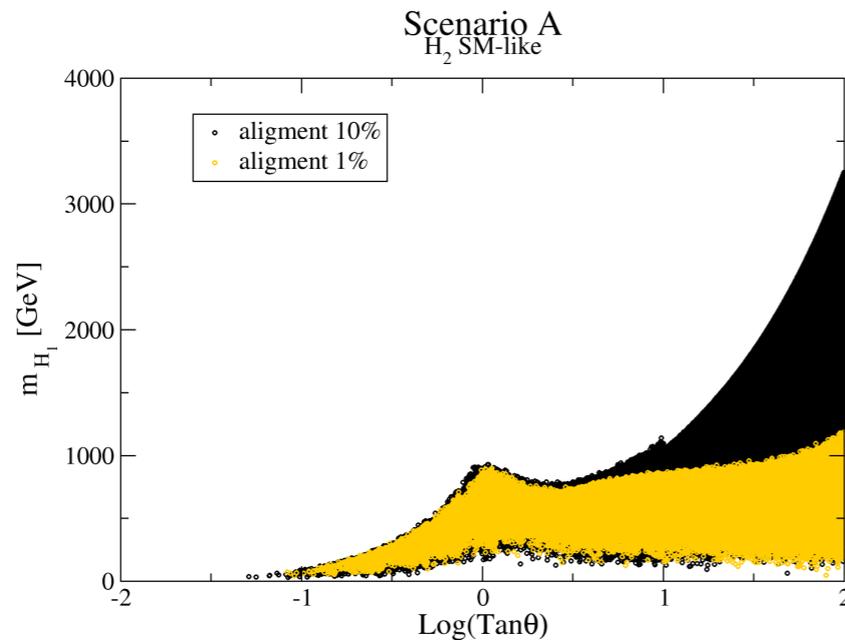
- **Magenta** satisfy stability and unitarity bounds
- **Maroon** satisfy alignment limit at 10%
→ upper bound to the scalar masses
consistent with Das & Day (2014)
- **Green** restricted to
A: mH₂ = 125 ± 5 GeV
B: mH₁ = 125 ± 5 GeV

PSEUDO SCALARS AND CHARGED SCALARS



*Points shown pass all constraints
we assume conservative limit $m_{H^\pm} > 80$ GeV*

SCENARIO A AT 1% AND 10%



- **Black** alignment limit at 10%
- **Yellow** satisfy alignment limit at 1% on $(\alpha-\theta)$

- Experimental limit at 10%
- \rightarrow upper bound to the scalar masses
- Other masses not affected
- Scenario B not affected

MASSES — TREE LEVEL — ALIGNMENT LIMITS

➤ Scenario A, H2 SM Higgs

➤ Upper bound for masses

$$m_{h0} \approx 900 \text{ GeV}, \quad m_{H1} \approx 3 \text{ TeV}$$

$$m_{A1} \approx 1 \text{ TeV}, \quad m_{A2} \approx 3 \text{ TeV}$$

$$m_{H1} \approx 1 \text{ TeV}, \quad m_{H2} \approx 3 \text{ TeV}$$

➤ Taking $(\alpha-\theta)$ 1% lowers $m_{H1}, m_{A2}, m_{H2} \approx 1 \text{ TeV}$

➤ Scenario B, H1 SM Higgs

➤ Upper bound for masses

$$m_{h0} \approx 600 \text{ GeV}, \quad m_{H1} \approx 120 \text{ GeV (by construction)}$$

$$m_{A1}, m_{A2}, m_{H1}, m_{H2} \approx 1 \text{ TeV}$$

➤ Both scenarios allow for a neutral scalar lighter than SM Higgs **$h0$ in A, $H2$ in B**

➤ Some of scalar masses are almost degenerate → good for oblique parameters

TRILINEAR HIGGS-GAUGE COUPLINGS

- In the exact alignments limits only H2 (H1) has couplings to the gauge bosons

$\frac{\cos(\alpha - \theta)}{H_1 W^+ W^-}$	$\frac{\sin(\alpha - \theta)}{H_2 W^+ W^-}$
$H_1 Z Z$	$H_2 Z Z$
$Z A_2 H_2$	$Z A_2 H_1$
$W^\pm H_2^\mp H_2$	$W^\pm H_2^\mp H_1$
$Z W^\pm H_2^\mp H_2$	$Z W^\pm H_2^\mp H_1$
$\gamma W^\pm H_2^\mp H_2$	$\gamma W^\pm H_2^\mp H_1$

- h0 has no trilinear gauge couplings, only:

In accordance with Z2 symmetry

$$Z A_1 h_0, Z W^\pm H_1^\mp h_0, W^\pm H_1^\mp h_0 \text{ y } \gamma W^\pm H_1^\mp h_0$$

- h0 has no Yukawa couplings: Dark Matter candidate!

SCALAR-GAUGE COUPLINGS

$$g_{h_0 W^\pm W^\mp} = 0, \quad g_{h_0 Z Z} = 0;$$

$$g_{H_1 W^\pm W^\mp} = \frac{2M_W^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 W^\pm W^\mp} = \frac{2M_W^2 \sin(\alpha - \theta) g^{\mu\nu}}{v};$$

$$g_{H_1 Z Z} = \frac{M_Z^2 \cos(\alpha - \theta) g^{\mu\nu}}{v}, \quad g_{H_2 Z Z} = \frac{M_Z^2 \sin(\alpha - \theta) g^{\mu\nu}}{v};$$

$$g_{h_0 h_0 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{h_0 h_0 Z Z} = \frac{M_Z^2 g^{\mu\nu}}{2v^2};$$

$$g_{H_1 H_1 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2}, \quad g_{H_2 H_2 W^\pm W^\mp} = \frac{M_W^2 g^{\mu\nu}}{v^2};$$

$$g_{H_1 H_1 Z Z} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}, \quad g_{H_2 H_2 Z Z} = \frac{M_Z^2 g^{\mu\nu}}{2v^2}.$$

*Differs from Barradas et al,
consistent with Z2 symmetry*

SCALAR-SCALAR COUPLINGS

$$g_{h_0 h_0 h_0} = 0,$$

$$g_{H_2 H_2 H_2} = -\frac{1}{v s_{2\theta}} \left[m_{h_0}^2 \frac{c_{\alpha-\theta}^3}{9c_\theta^2} + m_{H_2}^2 (c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta) \right],$$

$$g_{H_1 H_1 H_1} = \frac{1}{v s_{2\theta}} \left[m_{h_0}^2 \frac{s_{\alpha-\theta}^3}{9c_\theta^2} - m_{H_1}^2 (c_\alpha^2 s_{\alpha-\theta} - s_\alpha c_\theta) \right],$$

$$g_{h_0 h_0 H_1} = \frac{1}{v s_{2\theta}} (m_{h_0}^2 s_{\alpha+\theta} + m_{H_1}^2 s_\alpha c_\theta),$$

$$g_{h_0 h_0 H_2} = -\frac{1}{v s_{2\theta}} (m_{h_0}^2 c_{\alpha+\theta} + m_{H_2}^2 c_\alpha c_\theta),$$

$$g_{H_1 H_1 H_2} = -\frac{s_{\alpha-\theta}}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2} \right) + m_{H_1}^2 s_{2\alpha} + \frac{m_{H_2}^2 s_{2\alpha}}{2} \right),$$

$$g_{H_1 H_2 H_2} = \frac{c_{\alpha-\theta}}{v s_{2\theta}} \left(m_{h_0}^2 \left(\frac{s_{2(\alpha-\theta)}}{6c_\theta^2} \right) + \frac{m_{H_1}^2 s_{2\alpha}}{2} + m_{H_2}^2 s_{2\alpha} \right),$$

$$g_{h_0 h_0 h_0 h_0} = \frac{1}{24v^2 s_\theta^2} \left(m_{h_0}^2 + 3m_{H_1}^2 s_\alpha^2 + 3m_{H_2}^2 c_\alpha^2 \right),$$

$$g_{H_1 H_1 H_1 H_1} = \frac{1}{2v^2 s_{2\theta}^2} \left(m_{h_0}^2 s_{\alpha-\theta}^3 \frac{(s_{\alpha-\theta} + 2s_{\alpha+\theta})}{9c_\theta^2} + m_{H_1}^2 (s_\alpha^2 s_{\alpha-\theta} + c_\alpha s_\theta)^2 + m_{H_2}^2 \frac{s_{2\alpha}^2 s_{\alpha-\theta}^2}{4} \right),$$

$$g_{H_2 H_2 H_2 H_2} = \frac{1}{2v^2 s_{2\theta}^2} \left(m_{h_0}^2 c_{\alpha-\theta}^3 \frac{(c_{\alpha-\theta} + 2c_{\alpha+\theta})}{9c_\theta^2} + m_{H_1}^2 \frac{s_{2\alpha}^2 c_{\alpha-\theta}^2}{4} + m_{H_2}^2 (c_\alpha^2 c_{\alpha-\theta} - s_\alpha s_\theta)^2 \right).$$

EXACT ALIGNMENT LIMIT A

- In the exact alignment limit A (SM Higgs the lightest scalar)

$$\sin(\alpha - \theta) = 1, \cos(\alpha - \theta) = 0.$$

- “Our” SM Higgs trilinear and quartic couplings reduce exactly to SM real ones

$$g_{H_2 H_2 H_2} = \frac{1}{v s_{2\theta}} [m_{H_2}^2 s_\alpha s_\theta] = \frac{1}{2v} \frac{s_\alpha}{c_\theta} m_{H_2}^2 = \frac{m_{H_2}^2}{2v} \equiv \lambda_{SM}.$$

$$g_{H_1 H_1 H_1} = \frac{1}{v s_{2\theta}} \left[\frac{1}{9c_\theta^2} m_{h_0}^2 - s_\theta^2 m_{H_1}^2 \right] = \frac{1}{v s_{2\theta} c_\theta^2} \left[\frac{1}{9} m_{h_0}^2 - \frac{1}{2} s_{2\theta} m_{H_1}^2 \right].$$

$$g_{H_2 H_2 H_2 H_2} = \frac{1}{2v^2 s_{2\theta}^2} m_{H_2}^2 (-s_\theta^3 c_\theta - c_\theta^3 s_\theta)^2 = \frac{m_{H_2}^2}{8v^2}.$$

$$g_{H_2 H_2 h_0 h_0} = \frac{1}{v^2 s_{2\theta}} \left(\frac{1}{6} m_{h_0}^2 3s_{2\theta} + \frac{1}{4} m_{H_2}^2 s_{2\theta} \right) = \frac{1}{4v^2} (2m_{h_0}^2 + m_{H_2}^2).$$

LIMITS ON MASSES — TREE LEVEL

- Some couplings depend only on masses in alignment limit
- Allows to put lower bounds on these masses, through the absence of corresponding decays

$$g_{H_2 h_0 h_0} = \frac{1}{2v}(m_{H_2}^2 + 2m_{h_0}^2), \quad g_{H_2 A_1 A_1} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_1}^2), \quad g_{H_2 A_2 A_2} = \frac{1}{2v}(m_{H_2}^2 + 2m_{A_2}^2),$$
$$g_{H_2 H_1^\pm H_1^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_1^\pm}^2), \quad g_{H_2 H_2^\pm H_2^\mp} = \frac{1}{v}(m_{H_2}^2 + 2m_{H_2^\pm}^2), \quad g_{H_2 H_2 H_2 H_1} = g_{H_1 H_1 H_1 H_2} = 0.$$

- Sets a limit for all scalar masses (other than H1 and H2) at tree level of

$$m_{H_i} \gtrsim 63 \text{ GeV}$$

ALIGNMENT NOT EXACT — LIMITS ON PARAMETERS

➤ Higgs-gauge couplings have been determined with 5% precision $\rightarrow \kappa_\lambda$ scaling factor

➤ $-1.8 < \kappa_\lambda < 9.2$

Degrassi, Di Micco, Giardino, Rossi (2021)

➤ If the alignment limit is not exact we can parameterize deviations from SM

$$g_{H_2H_2H_2} \equiv \lambda_{SM}\kappa_\lambda = \frac{m_{H_2}^2}{2v} \left[(1 + 2\delta^2)\sqrt{1 - \delta^2} + \delta^3(\tan\theta - \cot\theta) - \frac{m_{h_0}^2}{m_{H_2}^2} \frac{\delta^3}{9s_\theta c_\theta^3} \right]$$

$$\cos(\alpha - \theta) = \cos\left(\frac{\pi}{2} - \epsilon\right) = \sin\epsilon \equiv \delta,$$

➤ The max value for m_{h_0} sets constraints on $\tan\theta$
e.g. for $\delta \sim 0.1 \rightarrow \tan\theta \leq 15$

FORM OF ONE-LOOP CORRECTIONS TO MASSES

$$\Sigma^\phi(s) + \Sigma^V(s) = \begin{pmatrix} \Sigma_{h_0}^{\phi,V}(s) & 0 & 0 \\ 0 & \Sigma_{H_1}^{\phi,V}(s) & \Sigma_{H_1 H_2}^{\phi,V}(s) \\ 0 & \Sigma_{H_2 H_1}^{\phi,V}(s) & \Sigma_{H_2}^{\phi,V}(s) \end{pmatrix}$$

$$\begin{aligned} \Sigma_{H_n}^{\phi,V} &= \sum_i \frac{g_{H_n H_n \phi_i^0 \phi_i^0}}{16\pi^2} A0(m_{\phi_i^0}^2) + \sum_{i,j} \frac{g_{H_n \phi_i^0 \phi_j^0}^2}{8\pi^2} B0(p^2, m_{\phi_i^0}^2, m_{\phi_j^0}^2) + \sum_k \frac{g_{H_n \phi_k^\pm \phi_k^\mp}^2}{8\pi^2} B0(p^2, m_{\phi_k^\pm}^2, m_{\phi_k^\mp}^2) \\ &+ \sum_i \frac{g_{H_n H_n V_i V_i}}{16\pi^2} A0(m_{V_i}^2) + \sum_i \frac{g_{H_n V_i V_i}^2}{8\pi^2} B0(p^2, m_{V_i}^2, m_{V_i}^2), \end{aligned}$$

with $n = 1, 2$.[‡] For the mixing term H_{12} we get

$$\begin{aligned} \Sigma_{H_1 H_2}^{\phi,V} &= \sum_i \frac{g_{H_1 H_2 \phi_i^0 \phi_i^0}}{16\pi^2} A0(m_{\phi_i^0}^2) + \sum_{i,j} \frac{g_{H_1 \phi_i^0 \phi_j^0} g_{H_2 \phi_i^0 \phi_j^0}}{8\pi^2} B0(p^2, m_{\phi_i^0}^2, m_{\phi_j^0}^2) \\ &+ \sum_k \frac{g_{H_1 \phi_k^\pm \phi_k^\mp} g_{H_2 \phi_k^\pm \phi_k^\mp}}{8\pi^2} B0(p^2, m_{\phi_k^\pm}^2, m_{\phi_k^\mp}^2) + \sum_i \frac{g_{H_1 V_i V_i} g_{H_2 V_i V_i}}{8\pi^2} B0(p^2, m_{V_i}^2, m_{V_i}^2) \\ &+ \sum_k \frac{g_{H_1 \phi_k^\pm W^\mp} g_{H_2 \phi_l^\pm W^\mp}}{8\pi^2} B0(p^2, m_{\phi_l^\pm}^2, m_W^2), \end{aligned}$$

where $\phi_{i(i)}^0 = h_0, H_1, H_2, A_1, A_2, G^0$, $\phi_k^\pm = H_{1,2}^\pm, G^\pm$ and $V_i = W^\pm, Z^0$.

ONE-LOOP POSSIBILITIES...

- Check for benchmarks where off-diagonal terms vanish, i.e. loop contributions extremely small (gauge and Higgs only)

Scalar benchmarks	Masses (GeV)	$\tan \theta$
light spectrum	$m_{h_0} = 80, m_{H_1} = 200, m_{A_{1,2}} = 80, m_{H_{1,2}^\pm} = 100$	1
heavy spectrum	$m_{h_0} = 800, m_{H_1} = 800, m_{A_{1,2}} = 800, m_{H_{1,2}^\pm} = 800$	2.1



Table 2: Parameter values in scenario A that make the one-loop mixing parameter vanish, $\Sigma_{H_1 H_2}^\phi = 0$, taking into account only the scalar and gauge contributions.

- For N-Higgs doublet models: oblique parameters OK in compact almost degenerate spectrum Grimus et al (2008); Cárcamo et al (2015)
- You can also fix m_{H_SM} mass as finite at tree level and renormalize the rest (on-shell ran)

Work in progress

IN YUKAWA SECTOR

- The Higgs Z_2 symmetry will lead to zeroes in the CKM and PMNS matrices 😱

Das, Dey, Pal (2015), Ivanov (2017)

- To recover the good features of the symmetry:

- Add S_3 singlet

Brown, Deshpande, Sugawara, Pakwasa (1984)

- Break very softly the S_3 symmetry with mass terms, recover original structure

e.g., Kubo, Okada, Sakamaki (2004), Das, Dey, Pal (2015)

- Consider CP violation

Costa, OGREID, OSLAND, REBELO (2014, 2021)

- Higher order interactions

- Second B-L sector at higher scale with some interaction

Gómez-Izquierdo, MM (2018)

- Combinations of the above: all introduce more parameters

OR MAKE IT MODULAR

will it help?

MODULAR SYMMETRIES

- Related to moduli spaces, geometric spaces: solutions of geometric classification problems. Objects are identified (isomorphic) if they are the same geometrically.
- Using modular symmetries as flavor symmetries:
 - Inspiration from supersymmetric theories, initially with extra dimensions Feruglio, Altarelli (2006-2022); Petcov et al (2019, 2021, 2022)
 - Magnetized branes, superstring theories Cremades et al (2004); Kobayashi et al (2018)
 - Superstring compactifications, especially from orbifold compactifications e.g. Kobayashi et al (2018, 2019); Chen, Ramos-Sánchez, Ratz (2022)
- Usually applied in supersymmetric models, but now also in non-supersymmetric models e.g. Nomura, Okada et al, (2019,2020)

MODULAR GROUP

- Projective special linear group of 2x2 matrices and determinant; linear fractional transformations of upper half of complex plane

$$\Gamma = SL_2(\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}.$$

The transformation γ over a parameter τ

$$\gamma(\tau) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\tau) \rightarrow \frac{a\tau + b}{c\tau + d}. \quad \gamma \in \Gamma$$

- Modular forms of weight k , functions that transform under Γ with weight k

$$f(\gamma\tau) = (c\tau + d)^k f(\tau)$$

GAMMA AND POLYGONS

- Isomorphism between some finite modular groups and some groups associated to polygons (invariance under rotations and reflections)

$$\Gamma_2 \simeq S_3$$

$$\Gamma_3 \simeq A_4$$

$$\Gamma_4 \simeq S_4$$

$$\Gamma_5 \simeq A_5$$

- Yukawa couplings expressed in terms of modular forms, i.e. functions of a complex scalar field

$$Y(\alpha, \beta, \gamma|\tau) = \frac{d}{d\tau} \left(\alpha \log \eta \left(\frac{\tau}{2} \right) + \beta \log \eta \left(\frac{\tau + 1}{2} \right) + \gamma \log \eta (2\tau) \right)$$

- Fermions and scalar fields transform with a weight

$$\phi \rightarrow (c\tau + d)^{k_\phi} \phi,$$

S3 MODULAR SYMMETRY

- We will impose a modular S_3 or Γ_2 to a non-supersymmetric Lagrangian

$$SU(3)_C \times SU_L(2) \times U_y(1) \times \Gamma_2$$

- 3HDM, 3 ν_R , quarks and leptons:

first two generations in a doublet

third generation in a singlet

same for 3 Higgses: 2 of them in a doublet, third in a singlet

- We assign specific modular weights (again, some *liberty* there...) to get a NNI texture

THE ASSIGNMENT FOR THE MODEL

- We assign the fields the following weights

	(Q_1, Q_2)	(q_1, q_2)	Q_3	q_3	(H_1, H_2)	H_s	$(Y_1^{(2,4)}(\tau), Y_2^{(2,4)}(\tau))$	$Y_s^{(4)}(\tau)$
$SU(2)$	2	1	2	1	2	2	1	1
S_3	2	2	1	1	2	1	2	1
k	-2	-2	0	0	0	0	(2, 4)	4

Table 2: charges, assignments, and modular weights of $SU(2)$ and S_3 . The superscript (2,4) on the modular forms indicates that they are of modular weight 2 or 4. The subscript s indicates the symmetric singlet of the modular form of weight 4.

- The Yukawa part of the Lagrangian is

$$\begin{aligned}
 \mathcal{L}_y^{(u)} &= C_1 \bar{Q} \otimes u \otimes \tilde{H} \otimes Y^{(4)} + C_2 \bar{Q} \otimes u \otimes \tilde{H} \otimes Y_s^{(4)} + C_3 \bar{Q} \otimes u \otimes \tilde{H}_s \otimes Y^{(4)} \\
 &+ C_4 \bar{Q} \otimes u \otimes \tilde{H}_s \otimes Y_s^{(4)} + C_5 \bar{Q} \otimes u_{3R} \otimes \tilde{H} \otimes Y^{(2)} + C_6 \bar{Q} \otimes u_{3R} \otimes \tilde{H}_s \otimes Y^{(2)} \\
 &+ C_7 \bar{Q}_3 \otimes u \otimes \tilde{H} \otimes Y^{(2)} + C_8 \bar{Q}_3 \otimes u \otimes \tilde{H}_s \otimes Y^{(2)} + C_9 \bar{Q}_3 \otimes u_{3R} \otimes \tilde{H}_s + \text{h.c.}
 \end{aligned}$$

ELEMENTS OF MASS MATRIX

➤ The elements of the quark mass matrix are now

$$M_{11}^{(u)} = (\alpha + \gamma)v_1Y_1^{(4)} + (\alpha - \gamma)v_2Y_2^{(4)} + C_2v_2Y_s^{(4)} + C_3v_sY_2^{(4)} + C_4v_sY_s^{(4)}$$

$$M_{12}^{(u)} = (\beta + \gamma)v_2Y_1^{(4)} + (\gamma - \beta)v_1Y_2^{(4)} + C_2v_1Y_s^{(4)} + C_3v_sY_1^{(4)}$$

$$M_{13}^{(u)} = C_5(v_2Y_1^{(2)} + v_1Y_2^{(2)}) + C_6v_sY_1^{(2)}$$

$$M_{21}^{(u)} = (\beta + \gamma)v_1Y_2^{(4)} + (\gamma - \beta)v_2Y_1^{(4)} + C_2v_1Y_s^{(4)} + C_3v_sY_1^{(4)}$$

$$M_{22}^{(u)} = (\alpha + \gamma)v_2Y_2^{(4)} + (\alpha - \gamma)v_1Y_1^{(4)} - C_2v_2Y_s^{(4)} - C_3v_sY_2^{(4)} + C_4v_sY_s^{(4)}$$

$$M_{23}^{(u)} = C_5(v_1Y_1^{(2)} - v_2Y_2^{(2)}) + C_6v_sY_2^{(2)}$$

$$M_{31}^{(u)} = C_7(v_2Y_1^{(2)} + v_1Y_2^{(2)}) + C_8v_sY_1^{(2)}$$

$$M_{32}^{(u)} = C_7(v_1Y_1^{(2)} - v_2Y_2^{(2)}) + C_8v_sY_2^{(2)}$$

$$M_{33}^{(u)} = C_9v_s,$$

Lots of free parameters!! $\alpha, \beta, \gamma, v_2, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ y τ

WHAT CAN WE DO?

- A lot of freedom! too many parameters...
- Can we do something about it?
- But, look at the symmetries — geometry, of the problem
- In the symmetry points parameters are identified or related:
only few parameters remain
- This way: possible to explain mixings, S_4 and A_5 studied
Novichkov, Penedo, Petcov (2021)
- S_3 studied too, but so far without exploiting these symmetric points
Kobayashi et al (2019,2020)

MODULAR SYMMETRIC POINTS

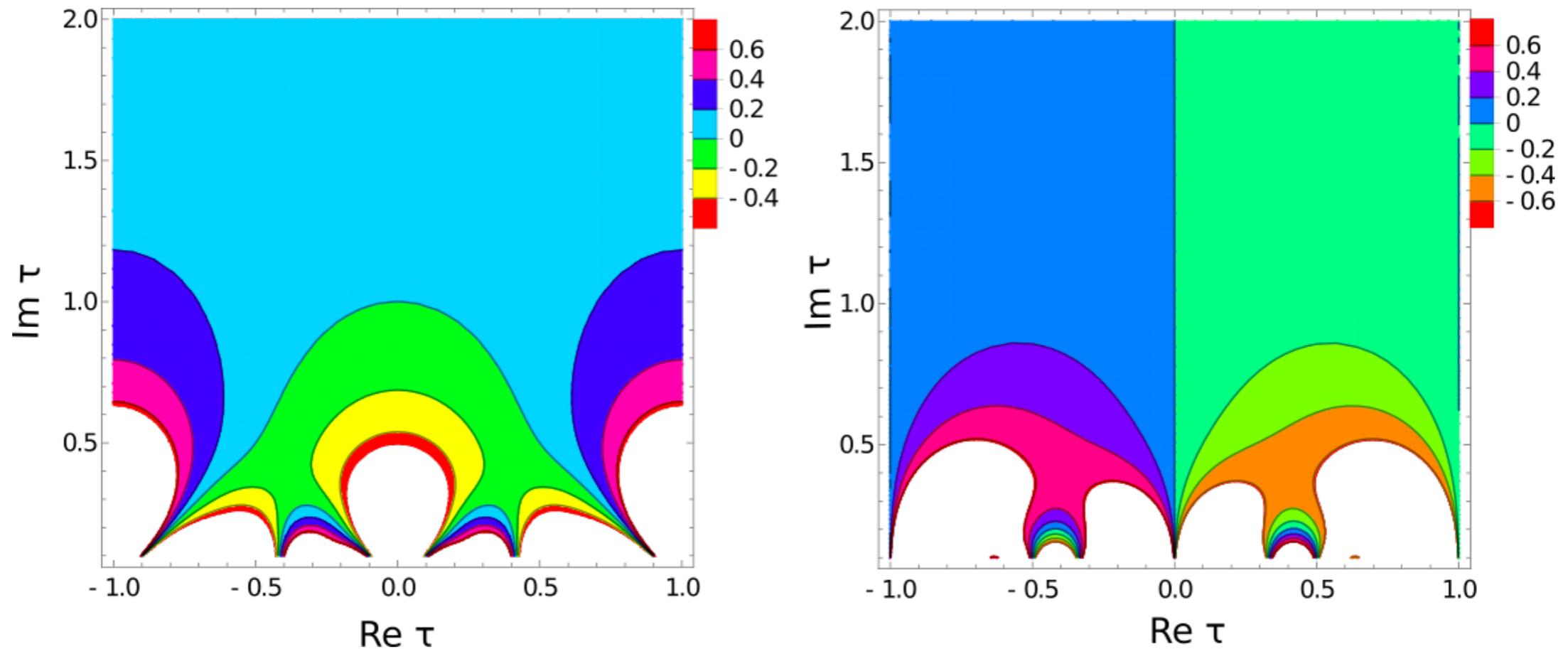


Figure 3: Real (left) and imaginary (right) part of the given expression in M_{13} y M_{31} , that is, $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau)$. It is observed that $Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau) = 0$, for both its real and imaginary parts, at the point $\tau = i$, which guarantees that $M_{13} = M_{31} = 0$.

LAGRANGIAN AND FREE PARAMETERS SO FAR

- We want a NNI of the form, which is known to reproduce the VCKM (not always possible)

$$\begin{pmatrix} 0 & a & 0 \\ a^* & b & c \\ 0 & c^* & d \end{pmatrix}$$

- Rotations to get the zeroes \rightarrow conditions on parameters
- Minimisation condition $v_1^2 = 3v_2^2$
- Modular symmetric points

$$Y_2^{(2)}(\tau) - \sqrt{3}Y_1^{(2)}(\tau) = 0, \quad \tau = i$$

REPARAMETERIZATION

- Rewrite mass matrices in polar form, real matrix multiplied by phase matrix
- Use three matrix invariants: trace, determinant, and the trace of the square matrix

$$\bar{M}^{(u)} = \begin{pmatrix} 0 & |C| & 0 \\ |C| & C'_4 & |C'_5| \\ 0 & |C'_5| & C'_9 \end{pmatrix}$$

$$P_f = \text{diag}(1, e^{i\phi_1}, e^{i(\phi_1 - \phi_2)})$$

$$|C| = \sqrt{\frac{\tilde{\sigma}_1 \tilde{\sigma}_2}{C'_9}} \quad \tilde{\sigma}_i = m_i/m_3$$

$$C'_4 = (\tilde{\sigma}_1 - \tilde{\sigma}_2 + 1 - C'_9)$$

$$|C'_5| = \sqrt{\frac{(1 - C'_9)(C'_9 - \tilde{\sigma}_1)(C'_9 + \tilde{\sigma}_2)}{C'_9}}$$

$C'_{9u}, C'_{9d}, \phi_{1u}, \phi_{2u}, \phi_{1d} \text{ and } \phi_{2d}.$

V_{CKM} MATRIX

- Assuming the NNI form and a hierarchical structure for the mass matrices u and d , we can reparameterize them in terms of mass ratios $\tilde{\sigma}_i = m_i/m_3$
- Exact analytical expression for the V_{CKM} corresponding to the symmetry S_3 with the NNI structure
- Without loss of generality we can fix the values of 2 phases

$$\phi_{1d} = \phi_{2d} = 0$$

- Now only 4 free parameters to fit the V_{CKM}
- We perform a χ^2 analysis to find the numerical values of our parameters

$$\begin{aligned}
V_{ud}^{th} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_s \xi_1^u \xi_1^d}{\mathcal{D}_{1u} \mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d}{\mathcal{D}_{1u} \mathcal{D}_{1d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \xi_1^u \xi_1^d + \sqrt{\delta_u \delta_d} \xi_2^u \xi_2^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{us}^{th} &= -\sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \xi_1^u \xi_2^d}{\mathcal{D}_{1u} \mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s}{\mathcal{D}_{1u} \mathcal{D}_{2d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \xi_1^u \xi_2^d + \sqrt{\delta_u \delta_d} \xi_2^u \xi_1^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{ub}^{th} &= \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d \tilde{\sigma}_s \delta_d \xi_1^u}{\mathcal{D}_{1u} \mathcal{D}_{3d}}} + \sqrt{\frac{\tilde{\sigma}_u}{\mathcal{D}_{1u} \mathcal{D}_{3d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \delta_d \xi_1^u - \sqrt{\delta_u} \xi_2^u \xi_1^d \xi_2^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cd}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_s \xi_2^u \xi_1^d}{\mathcal{D}_{2u} \mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_d}{\mathcal{D}_{2u} \mathcal{D}_{1d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \xi_2^u \xi_1^d + \sqrt{\delta_u \delta_d} \xi_1^u \xi_2^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cs}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d \xi_2^u \xi_2^d}{\mathcal{D}_{2u} \mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_c \tilde{\sigma}_s}{\mathcal{D}_{2u} \mathcal{D}_{2d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \xi_2^u \xi_2^d + \sqrt{\delta_u \delta_d} \xi_1^u \xi_1^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{cb}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_d \tilde{\sigma}_s \delta_d \xi_2^u}{\mathcal{D}_{2u} \mathcal{D}_{3d}}} + \sqrt{\frac{\tilde{\sigma}_c}{\mathcal{D}_{2u} \mathcal{D}_{3d}}} \left(\sqrt{(1 - \delta_u)(1 - \delta_d)} \delta_d \xi_2^u - \sqrt{\delta_u} \xi_1^u \xi_1^d \xi_2^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{td}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_s \delta_u \xi_1^d}{\mathcal{D}_{3u} \mathcal{D}_{1d}}} + \sqrt{\frac{\tilde{\sigma}_d}{\mathcal{D}_{3u} \mathcal{D}_{1d}}} \left(\sqrt{\delta_u (1 - \delta_u)(1 - \delta_d)} \xi_1^d - \sqrt{\delta_d} \xi_1^u \xi_2^d \xi_2^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{ts}^{th} &= -\sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_d \delta_u \xi_2^d}{\mathcal{D}_{3u} \mathcal{D}_{2d}}} + \sqrt{\frac{\tilde{\sigma}_s}{\mathcal{D}_{3u} \mathcal{D}_{2d}}} \left(\sqrt{\delta_u (1 - \delta_u)(1 - \delta_d)} \xi_2^d - \sqrt{\delta_d} \xi_1^u \xi_2^d \xi_1^d e^{i\phi_2} \right) e^{i\phi_1}, \\
V_{tb}^{th} &= \sqrt{\frac{\tilde{\sigma}_u \tilde{\sigma}_c \tilde{\sigma}_d \tilde{\sigma}_s \delta_u \delta_d}{\mathcal{D}_{3u} \mathcal{D}_{3d}}} + \left(\sqrt{\frac{\xi_1^u \xi_2^u \xi_1^d \xi_2^d}{\mathcal{D}_{3u} \mathcal{D}_{3d}}} + \sqrt{\frac{\delta_u \delta_d (1 - \delta_u)(1 - \delta_d)}{\mathcal{D}_{3u} \mathcal{D}_{3d}}} e^{i\phi_2} \right) e^{i\phi_1}.
\end{aligned}$$

$$\begin{aligned}
\delta_{u,d} &= 1 - C'_{9u,d} \\
\xi_1^{u,d} &= 1 - \tilde{\sigma}_{u,d} - \delta_{u,d}, \\
\xi_2^{u,d} &= 1 + \tilde{\sigma}_{c,s} - \delta_{u,d}, \\
\mathcal{D}_{1(u,d)} &= (1 - \delta_{u,d})(\tilde{\sigma}_{u,d} + \tilde{\sigma}_{c,s})(1 - \tilde{\sigma}_{u,d}), \\
\mathcal{D}_{2(u,d)} &= (1 - \delta_{u,d})(\tilde{\sigma}_{u,d} + \tilde{\sigma}_{c,s})(1 + \tilde{\sigma}_{c,s}), \\
\mathcal{D}_{3(u,d)} &= (1 - \delta_{u,d})(1 - \tilde{\sigma}_{u,d})(1 + \tilde{\sigma}_{c,s}).
\end{aligned}$$

VCKM FIT

- Excellent fit (too excellent...overfitted?)
- Probably we have correlations among parameters → one too many?
- Analytical expression successful

	Center value and error
$\tilde{\sigma}_u$	7.032×10^{-6}
$\tilde{\sigma}_d$	9.44×10^{-4}
$\tilde{\sigma}_s$	0.0190 ± 0.00046
$\tilde{\sigma}_c$	0.00375 ± 0.00023

	Values in the fit
C'_{9u}	0.816393
C'_{9d}	0.828604
ϕ_{1u}	1.63797
ϕ_{1d}	0
ϕ_{2u}	0.0981477
ϕ_{2d}	0
χ^2	0.00070



$$V_{CKM}^{th} = \begin{pmatrix} 0.97435 & 0.2250 & 0.00369 \\ 0.22486 & 0.97349 & 0.04182 \\ 0.00857 & 0.04110 & 0.999118 \end{pmatrix}$$

$$\mathcal{J}^{th} = 3.07 \times 10^{-5}.$$

CONCLUSIONS

- S_3 is a small symmetry that goes a long way
- S_3 -3H models consistent with CKM and PMNS
 $\theta_{13} \neq 0$ naturally
Possible to calculate all neutrino masses and mixings
- In Higgs sector:
 - masses bounded from above and below
 - trilinear and quartic Higgs coupling are SM ones in alignment limits
 - Possible to have light “semi-invisible” Higgs in both scenarios,
with different signals/characteristics
 - Possible to have one or more DM candidates

CONCLUSIONS - HIGGS

- Regions of parameter space that pass all Higgs bounds:
Extra Higgses sufficiently decoupled or inert possible
 $\tan\theta$ small solutions appear both in Higgs and DM sectors
Good DM candidate(s)
 - h_0 as DM candidate
 - possible to add R-handed neutrino as DM
 - 4th inert Higgs as DM ✓
- Vacuum much more complicated than in SM, all checks necessary:
Need to add one-loop corrections
- Leptogenesis possible
- Above all:
Consistent with known physics
New predictions
Testable

CONCLUSIONS – MODULAR

- Using modular S_3 we get a very good fit for the VCKM with few parameters, no zeroes
- Exact analytical expressions with the given assignment and hierarchical structure
- One parameter less than in previous similar analysis by exploiting the natural symmetries of the system
- All the good features of the Higgs sector remain by assigning zero modular weights to the fields and couplings
- Simultaneous study of Higgs, fermionic sector and DM shows model is self-consistent

THANKS!

GOING UP?

- You can embed the model (or a version of it) in a SUSY model with Q6 symmetry
- Grand Unified SU(5) x Q6 model already studied, preserves the nice features of S3 in quarks and leptons. Mixing angles in good agreement with experiment, both hierarchies allowed.

J.C. Gómez-Izquierdo, F. González-Canales, M.M. (2014)

Neutrino masses: add singlets or non-renormalizable interactions or radiatively

- Possible to have different assignments of Q6 in leptonic sector \implies **breaking of mu-tau symmetry**

J.C. Gómez-Izquierdo, M.M. (2017)

- Flavour structure in trilinear soft SUSY breaking terms \rightarrow LFV $\tau \rightarrow \mu + \gamma$, g-2 contributions through LFV in leptonic sector

F. Flores-Báez, M. Gómez-Bock, M.M. (2018)

- Non-SUSY B-L model with S3, also breaking of mu-tau symmetry

J.C. Gómez-Izquierdo, M.M. (2019)