QED Fermions in a noisy magnetic field backgound

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This talk is based on the article:

Jorge David Castaño-Yepes, M. Loewe, Enrique Muñoz, Juan Cristóbal Rojas, and R. Zamora; “QED Fermions in a noisy magnetic field backgound”, arXiv: 2211.16985 [hep-th]

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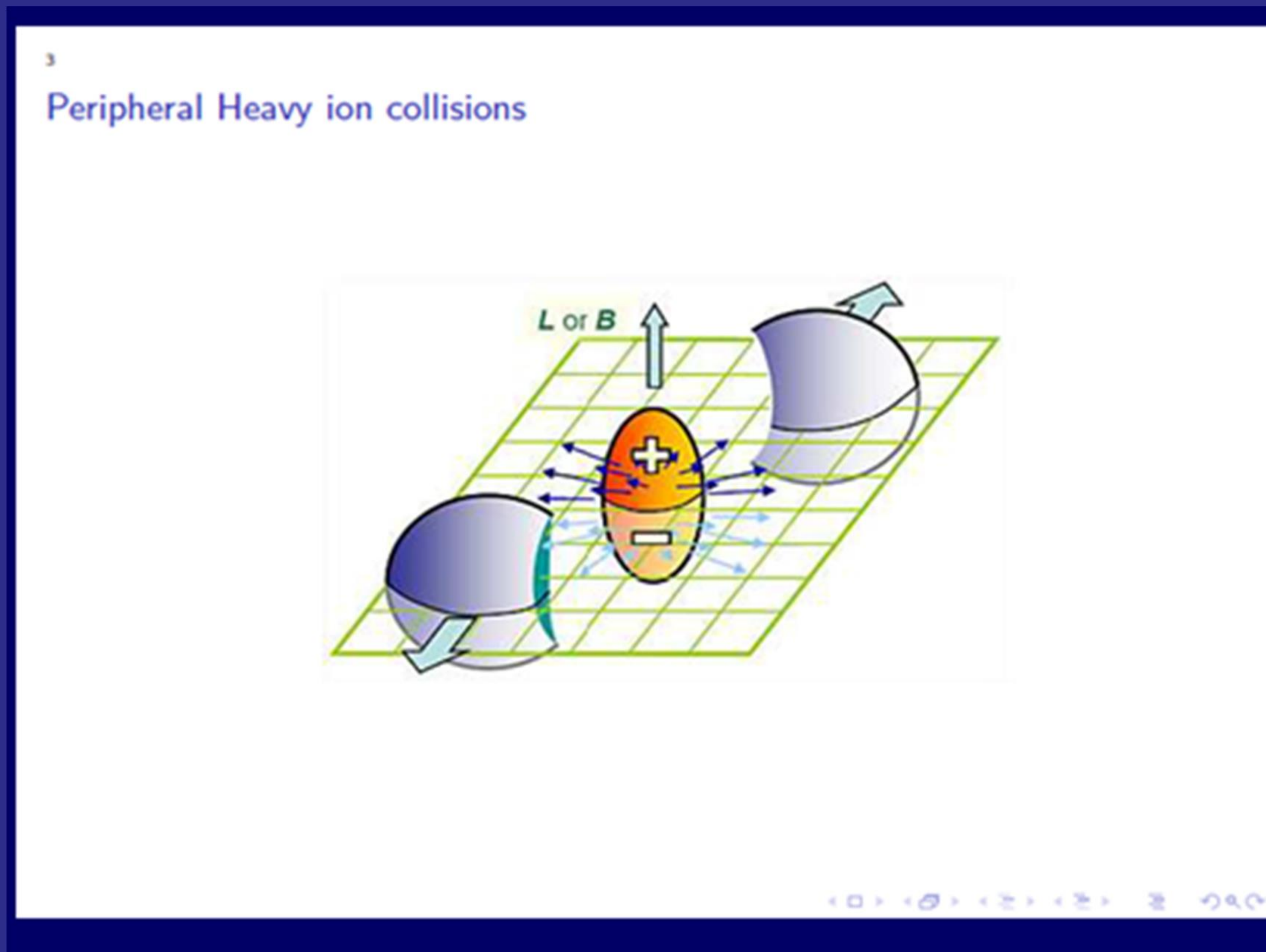
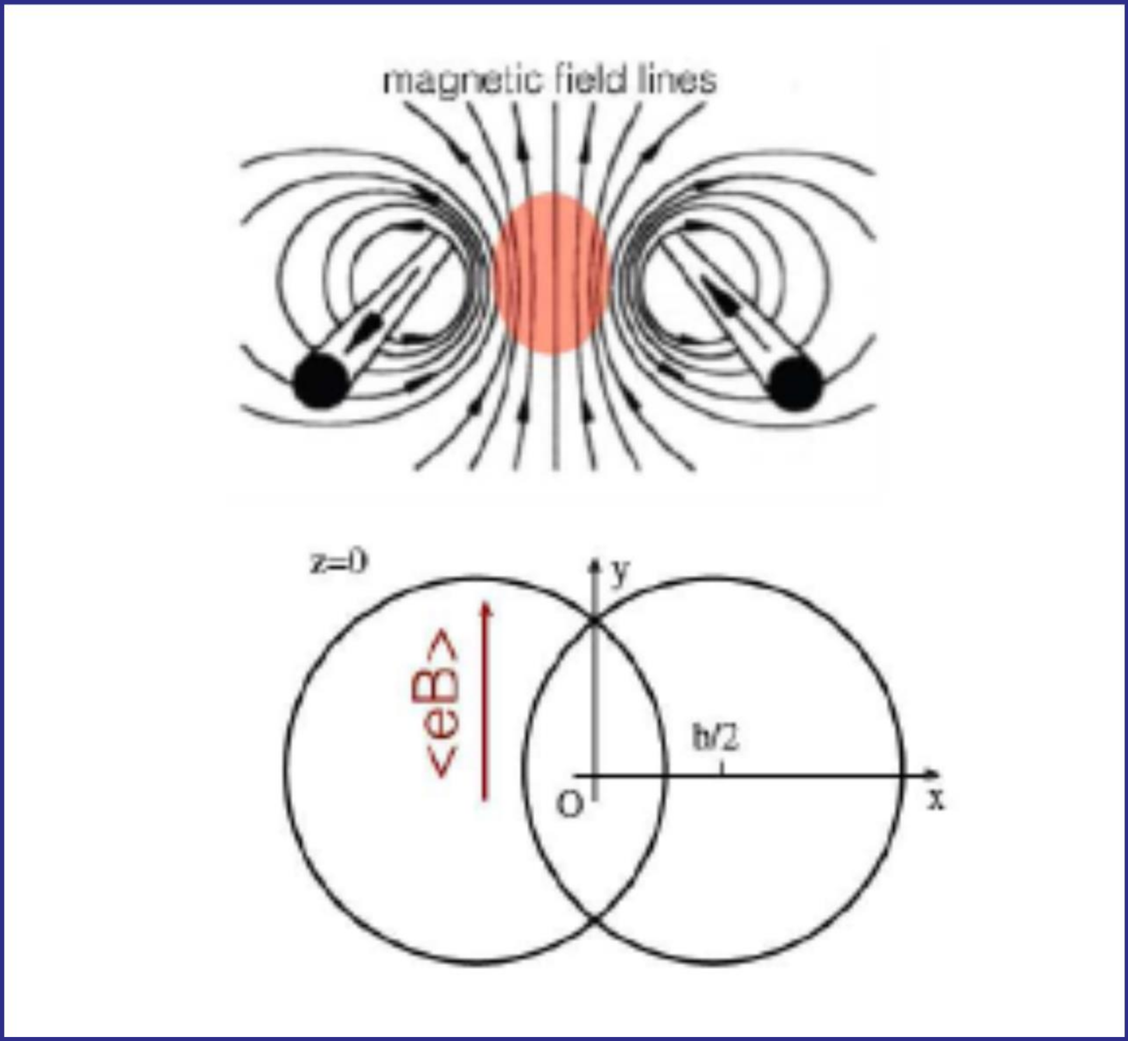
Magnetic Fields in perpheral heavy ion collisions

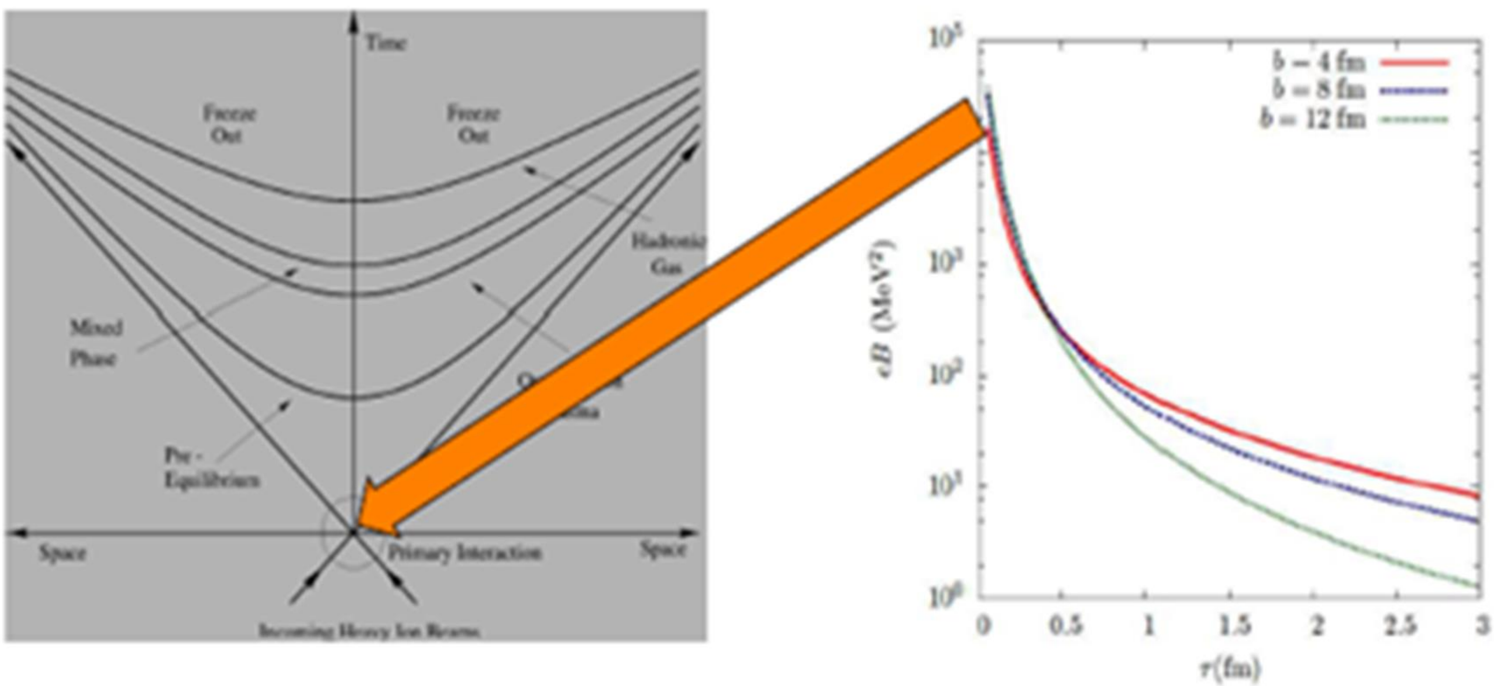


Huge Magnetic fields are produced in perhipheral heavy-ion collisions



Time evolution of a uniform magnetic field in a heavy-ion collision

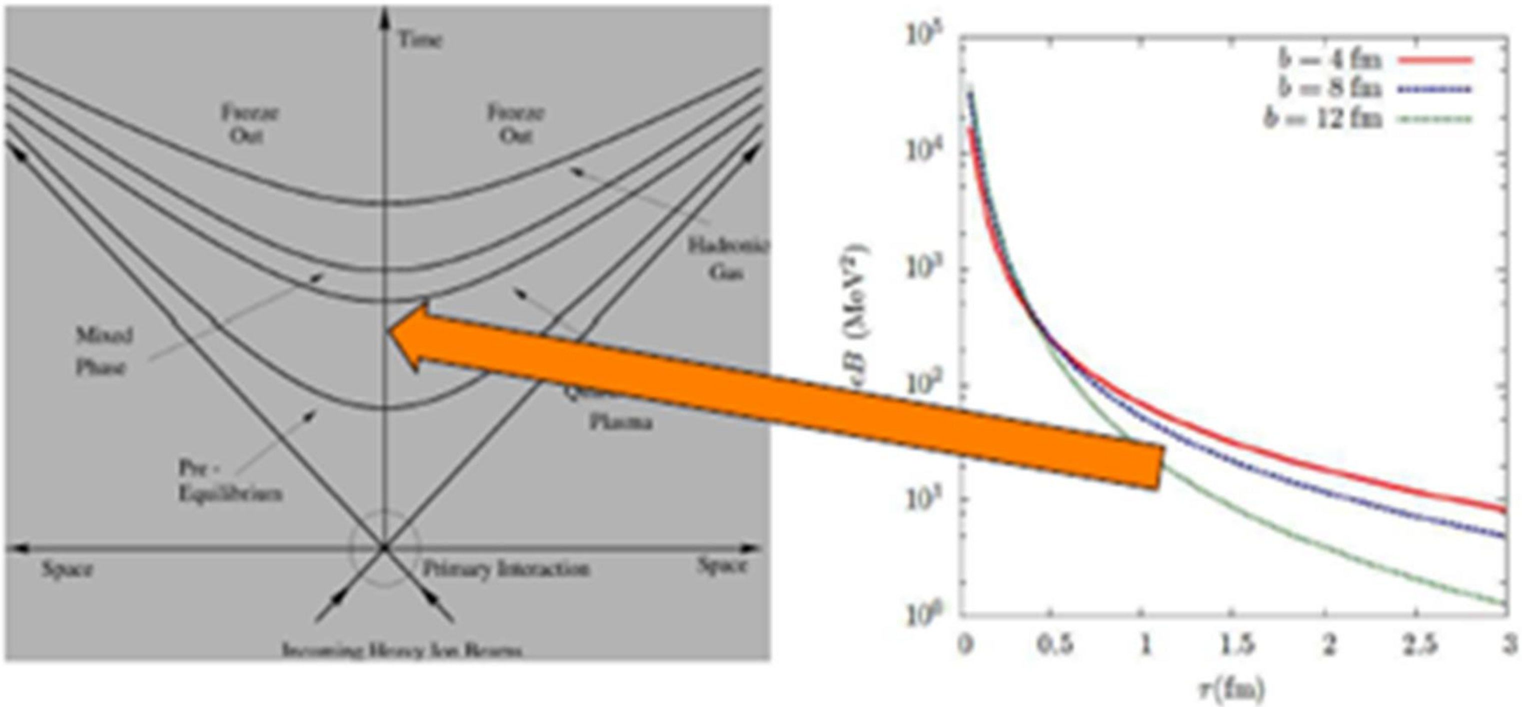
- Very intense field at early collision times



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

Time evolution of a constant B field in a heavy-ion collision

- Magnetic field rapidly decreasing function of collision time



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

Effects of a constant and classical magnetic field background have been studied since the seminal work by J. Schwinger (Phys. Rev. 82, 664, 1951).

In all these studies, the background magnetic field is idealized as static and uniform: spatial anisotropies or fluctuations in its magnitude were normally disregarded.
Here we want to discuss the effects of magnetic fluctuations, respect to a finite backgound magnetic field, over the renormalization of the fermion propagator in QED.

The replica method will be used (M. Kardar, G. Parisi and Y. C. Zhang, P.R.L. 56, 889. 1986)

The Model: QED in the presence of a classical and static magnetic field possesing random spatial fluctuations

$$A^\mu(x) \to A^\mu(x) + A^\mu_{\mathrm{BG}}(x) + \delta A^\mu_{\mathrm{BG}}(\mathbf{x})$$

We consider a static white noise spatial fluctuation with respect to the mean value, i.e.

$$\langle \delta A^j_{\mathrm{BG}}(\mathbf{x}) \delta A^k_{\mathrm{BG}}(\mathbf{x}') \rangle = \Delta_B \delta_{j,k} \delta^3(\mathbf{x} - \mathbf{x}'),$$
$$\langle \delta A^\mu_{\mathrm{BG}}(\mathbf{x}) \rangle = 0.$$

As it is well known, these statistical properties are represented by a Gaussian functional distribution

$$dP\left[\delta A^{\mu}_{\mathrm{BG}}\right] = \mathcal{N} e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]^2}{2\Delta_B}} \mathcal{D}\left[\delta A^{\mu}_{\mathrm{BG}}(\mathbf{x})\right]$$

In this way, we have the following decomposition

$$\mathcal{L} = \mathcal{L}_{\mathrm{FBG}} + \mathcal{L}_{\mathrm{NBG}},$$

$$\mathcal{L}_{\mathrm{FBG}} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\mathrm{BG}} - e A\!\!\!/ - m\right)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

$$\mathcal{L}_{\mathrm{NBG}} = \bar{\psi}\left(-e\delta A\!\!\!/_{\mathrm{BG}}\right)\psi.$$

The generating functional in absence of sources is given by

$$Z[A] = \int \mathcal{D}[\bar{\psi}, \psi] e^{\mathrm{i} \int d^4x [\mathcal{L}_{\text{FBG}} + \mathcal{L}_{\text{NBG}}]}.$$

The physics of the system will be determined by a statistical average over the magnetic fluctuations of $\overline{\ln Z}.$

This is achieved in terms of the Replica Trick introduced by Parisi and coworkers

$$\overline{\ln Z[A]} = \lim_{n \to 0} \frac{\overline{Z^n[A]} - 1}{n}$$

The statistical average is given in terms of the Gaussian functional measure.

Z^n is obtained by the incorporation of replica components for the fermion fields.

$$\psi(x) \to \psi^a(x) \qquad 1 \leq a \leq n.$$

$$\overline{Z^n[A]} = \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] \int \mathcal{D}\left[\delta A^{\mu}_{\rm BG}\right] e^{-\int d^3x \frac{\left[\delta A^{\mu}_{\rm BG}(\mathbf{x})\right]^2}{2\Delta_B}} \times e^{\mathrm{i}\int d^4x \sum_{a=1}^{n}\left(\mathcal{L}_{\rm FBG}[\bar{\psi}^a,\psi^a]+\mathcal{L}_{DBG}[\bar{\psi}^a,\psi^a]\right)}$$

$$= \int \prod_{a=1}^{n} \mathcal{D}[\bar{\psi}^a, \psi^a] e^{\mathrm{i}\bar{S}[\bar{\psi}^a,\psi^a;A]}$$

After perfoming the integral over the magnetic fluctuations

$$\bar{S}\left[\bar{\psi^a}, \psi^a; A\right] = \int d^4x \left(\sum_a \bar{\psi}^a \left(\mathrm{i}\partial\!\!\!/ - e A\!\!\!/_{\mathrm{BG}} - e A\!\!\!/ - m \right) \psi^a - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$
$$+ \mathrm{i}\frac{e^2 \Delta_B}{2} \int d^4x \int d^4y \sum_{a,b} \sum_{j=1}^{3} \bar{\psi}^a(x) \gamma^j \psi^a(x) \bar{\psi}^b(y) \gamma_j \psi^b(y) \delta^3(\mathbf{x} - \mathbf{y}).$$

It remind us the NJL-action.

We have ended ap with an effective interacting theory with an instantaneous local interaction proportional to Δ_B

The “free” part of the action corresponds to fermions in the presence of the average background classical field A^{μ}_{BG} (x).

We choose the background to represent a constant magnetic field in the Z direction

$$\mathbf{B} = \hat{e}_3 B,$$

And use the symmetric gauge

$$A^{\mu}_{\rm BG}(x) = \frac{1}{2}(0, -Bx^2, Bx^1, 0).$$

We know the Schwinger proper time representation of the “free-Fermion Propagator dressed by the background field. Now, some technicalities…

$$[S_F(k)]_{a,b} = -i\delta_{a,b} \int_0^\infty \frac{d\tau}{\cos(eB\tau)} e^{i\tau\left(k_\parallel^2 - \mathbf{k}_\perp^2 \frac{\tan(eB\tau)}{eB\tau} - m^2 + i\epsilon\right)} \times \left\{ \left[\cos(eB\tau) + i\gamma^1\gamma^2 \sin(eB\tau)\right](m + \not{k}_\parallel) + \frac{\not{k}_\perp}{\cos(eB\tau)} \right\}$$

Which is diagonal in the replica indeces

It is natural to decompose $g^{\mu\nu} = g_\parallel^{\mu\nu} + g_\perp^{\mu\nu}$ such that

$$g_\parallel^{\mu\nu} = \mathrm{diag}(1, 0, 0, -1),$$
$$g_\perp^{\mu\nu} = \mathrm{diag}(0, -1, -1, 0),$$

$$\not{k} = \not{k}_\perp + \not{k}_\parallel$$

$$k^2 = k_\parallel^2 - \mathbf{k}_\perp^2$$

$$k_\parallel^2 = k_0^2 - k_3^2,$$

It is interesting to notice that Schwinger´s propagator can be written as

$$
\begin{aligned}
[S_{\mathrm{F}}(k)]_{a,b} &= -\mathrm{i}\delta_{a,b}\left[(m+\not{k})\,\mathcal{A}_1\right.\\
&\left.+(\mathrm{i}eB)\mathrm{i}\gamma^1\gamma^2\left(m+\not{k}_{\parallel}\right)\frac{\partial\mathcal{A}_1}{\partial\mathbf{k}_{\perp}^2}+(\mathrm{i}eB)^2\,\not{k}_{\perp}\frac{\partial^2\mathcal{A}_1}{\partial(\mathbf{k}_{\perp}^2)^2}\right]\\
&= -\mathrm{i}\delta_{a,b}\left[\left(m+\not{k}_{\parallel}\right)\mathcal{A}_1+\mathrm{i}\gamma^1\gamma^2\left(m+\not{k}_{\parallel}\right)\mathcal{A}_2+\mathcal{A}_3\not{k}_{\perp}\right]
\end{aligned}
$$

where

$$
\mathcal{A}_1(k,B)=\int_0^\infty d\tau e^{\mathrm{i}\tau\left(k_{\parallel}^2-m^2+\mathrm{i}\epsilon\right)-\mathrm{i}\frac{\mathbf{k}_{\perp}^2}{eB}\tan(eB\tau)},
$$

You may check that

$$\lim_{B\to 0} \mathcal{A}_1(k,B) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\epsilon} \equiv \frac{\mathrm{i}}{\mathcal{D}_0(k)}$$

$$\mathcal{D}_0(k) = k^2 - m^2 + \mathrm{i}\epsilon,$$

$$\mathcal{A}_2(k,B) \equiv \int_0^\infty d\tau \;\tan(eB\tau) e^{\mathrm{i}\tau\left(k_\parallel^2 - t_B(\tau)\mathbf{k}_\perp^2 - m^2 + \mathrm{i}\epsilon\right)}$$

$$= \mathrm{i}eB \frac{\partial \mathcal{A}_1}{\partial(\mathbf{k}_\perp^2)}$$

$$\mathcal{A}_3(k,B) \equiv \int_0^\infty \frac{d\tau}{\cos^2(eB\tau)} e^{\mathrm{i}\tau\left(k_\parallel^2 - t_B(\tau)\mathbf{k}_\perp^2 - m^2 + \mathrm{i}\epsilon\right)}$$

$$= \mathcal{A}_1 + (\mathrm{i}eB)^2 \frac{\partial^2 \mathcal{A}_1}{\partial(\mathbf{k}_\perp^2)^2}$$

Last technicality…..(so far). It is not difficult to see that the inverse propagator can be written as

$$\hat{S}_{\mathrm{F}}^{-1}(k) = \frac{\mathrm{i}}{\mathcal{D}(k)} \left[\left(m - \not{k}_{\parallel} \right) \mathcal{A}_1 - \mathrm{i}\gamma^1\gamma^2 \left(m - \not{k}_{\parallel} \right) \mathcal{A}_2 - \mathcal{A}_3 \not{k}_{\perp} \right]$$

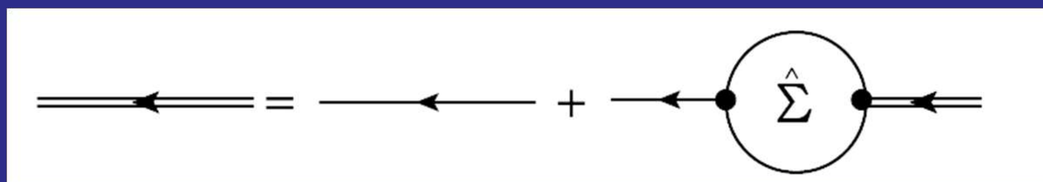
with

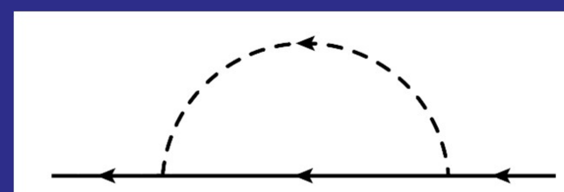
$$\mathcal{D}(k) = \mathcal{A}_3^2 \mathbf{k}_{\perp}^2 - \left(\mathcal{A}_1^2 - \mathcal{A}_2^2 \right) \left(k_{\parallel}^2 - m^2 \right)$$

Everything is expressed in terms of the master integral $A_1(k,B)$. The idea is to explore the effects of the magnetic fluctuations as a perturbative expansion In terms of the Δ_B parameter.

We will have "self-energy" corrections Σ to the propagator as well as corrections to the 4-point function Γ.

The Schwinger-Dyson equation for the dressed propagator (double line) in terms of the “free” propagator (Schwinger propagator) corresponds to





The lowest order contribution in terms $\Delta = e^2\Delta_B$

$$\hat{\Sigma}_{\Delta}(q) = (\mathrm{i}\Delta) \int \frac{d^3p}{(2\pi)^3} \gamma^j \hat{S}_F(p+q; p_0 = 0)\gamma_j$$

$$\hat{\Sigma}_\Delta(q) = \frac{i(\mathrm{i}\Delta)}{(2\pi)^3} \int d^3p \left\{3\left(\gamma^0 q_0 - m\right) \mathcal{A}_1(q_0, p_3; \mathbf{p}_\perp) \right.$$
$$\left. + \mathrm{i}\gamma^1\gamma^2 \left(m - q_0\gamma^0\right) (\mathrm{i}eB) \frac{\partial}{\partial \mathbf{p}_\perp^2} \mathcal{A}_1(q_0, p_3; \mathbf{p}_\perp)\right\}$$

It turns out that

$$\hat{\Sigma}_\Delta(q) = \frac{\mathrm{i}(\mathrm{i}\Delta)}{(2\pi)^3} \left[3\left(\gamma^0 q_0 - m\right) \tilde{\mathcal{A}}_1(q_0) \right.$$
$$\left. -\mathrm{i}\gamma^1\gamma^2 (\mathrm{i}\pi eB) \left(m - q_0\gamma^0\right) \tilde{\mathcal{A}}_2(q_0)\right]$$

$$\tilde{\mathcal{A}}_1(q_0) \equiv \int d^3p \mathcal{A}_1(q_0, p_3; \mathbf{p}_\perp),$$
$$\tilde{\mathcal{A}}_2(q_0) \equiv \int_{-\infty}^{+\infty} dp_3 \mathcal{A}_1(q_0, p_3; \mathbf{p}_\perp = 0)$$

Playing around, i.e. inserting the first order in Δ for the self-energy into the Schwinger-Dyson equation we obtain the dressed inverse propagator at first order in Δ

$$\hat{S}_{\Delta}^{-1}(k) = \hat{S}_{\mathrm{F}}^{-1}(k) - \hat{\Sigma}_{\Delta}$$

The expression is quite long. When comparing the dressed inverse propagator with the inverse of the Schwinger propagator, it is possible to identify renormalization constants.

Let us define by z, z_3 and m´ the renormalization factors for the wave function, the charge and the mass, respectively

These factors are given by

$$z = 1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3} \frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)} \mathcal{D}(q),$$

$$z_3 = \frac{1 - \frac{\mathrm{i}\pi(\mathrm{i}\Delta)(eB)}{(2\pi)^3} \frac{\widetilde{\mathcal{A}}_2(q_0)}{\mathcal{A}_2(q)} \mathcal{D}(q)}{1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3} \frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)} \mathcal{D}(q)},$$

$$m' = m,$$

We can define

A noise dependent index of refraction

$$\frac{v'}{c} = z^{-1} = \left(1 + \frac{3\mathrm{i}\Delta}{(2\pi)^3} \frac{\widetilde{\mathcal{A}}_1(q_0)}{\mathcal{A}_1(q)} \mathcal{D}(q)\right)^{-1}$$

We obtained these factors from a comparison between

$$\hat{S}_{\Delta}^{-1}(q) = \mathrm{i}\left[\frac{m\mathcal{A}_1(q)}{\mathcal{D}(q)} + \frac{3m(\mathrm{i}\Delta)}{(2\pi)^3}\widetilde{\mathcal{A}}_1(q_0)\right] - \mathrm{i}\left[\frac{\mathcal{A}_1(q)}{\mathcal{D}(q)} + \frac{3(\mathrm{i}\Delta)}{(2\pi)^3}\widetilde{\mathcal{A}}_1(q_0)\right](q_0\gamma^0) - \mathrm{i}\left[\frac{m\mathcal{A}_2(q)}{\mathcal{D}(q)} - \mathrm{i}\frac{m\pi(\mathrm{i}\Delta)(eB)}{(2\pi)^3}\widetilde{\mathcal{A}}_2(q_0)\right](\mathrm{i}\gamma^1\gamma^2) + \mathrm{i}\left[\frac{\mathcal{A}_2(q)}{\mathcal{D}(q)} - \frac{\mathrm{i}\pi(\mathrm{i}\Delta)(eB)}{(2\pi)^3}\widetilde{\mathcal{A}}_2(q_0)\right](\mathrm{i}\gamma^1\gamma^2 q_0\gamma^0) - \mathrm{i}\frac{\mathcal{A}_1(q)}{\mathcal{D}(q)}(q_3\gamma^3) + \mathrm{i}\frac{\mathcal{A}_2}{\mathcal{D}(q)}(\mathrm{i}\gamma^1\gamma^2 q_3\gamma^3) - \mathrm{i}\frac{\mathcal{A}_3}{\mathcal{D}(q)}\not{q}_{\perp}.$$

And Schwinger´s propagator

$$\hat{S}_{\mathrm{F}}^{-1}(k) = \frac{\mathrm{i}}{\mathcal{D}(k)}\left[\left(m - \not{k}_{\parallel}\right)\mathcal{A}_1 - \mathrm{i}\gamma^1\gamma^2\left(m - \not{k}_{\parallel}\right)\mathcal{A}_2 - \mathcal{A}_3\not{k}_{\perp}\right],$$

The factor z_3 will be associated to the tensor structures involving the spin-magnetic field interactions

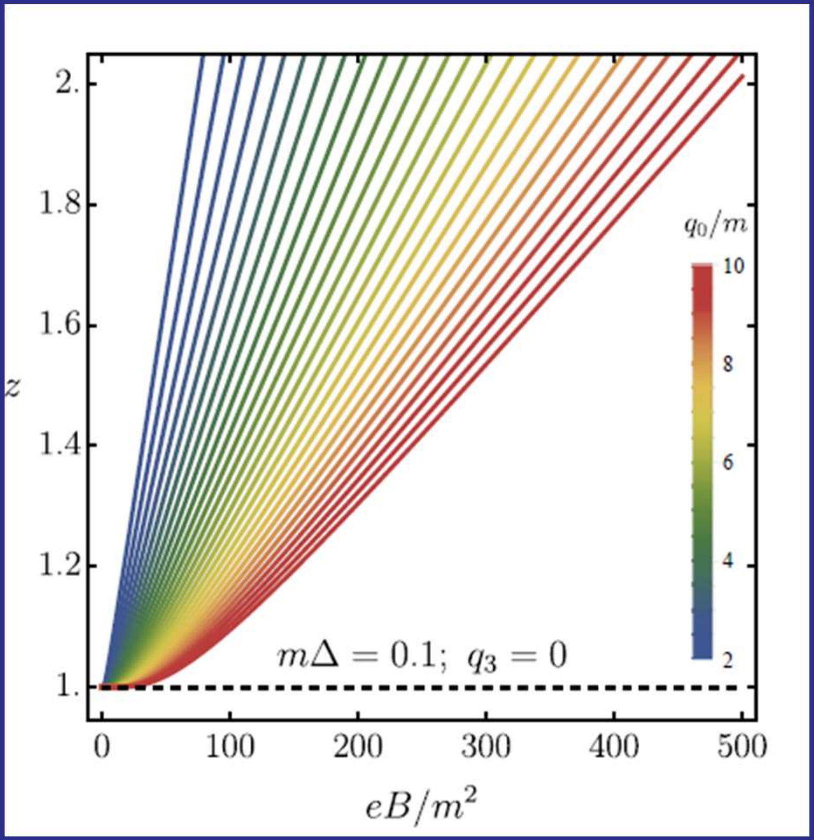
$$e\sigma_{\mu\nu}F^{\mu\nu}_{\mathrm{BG}} = i\gamma_1\gamma_2 eB$$

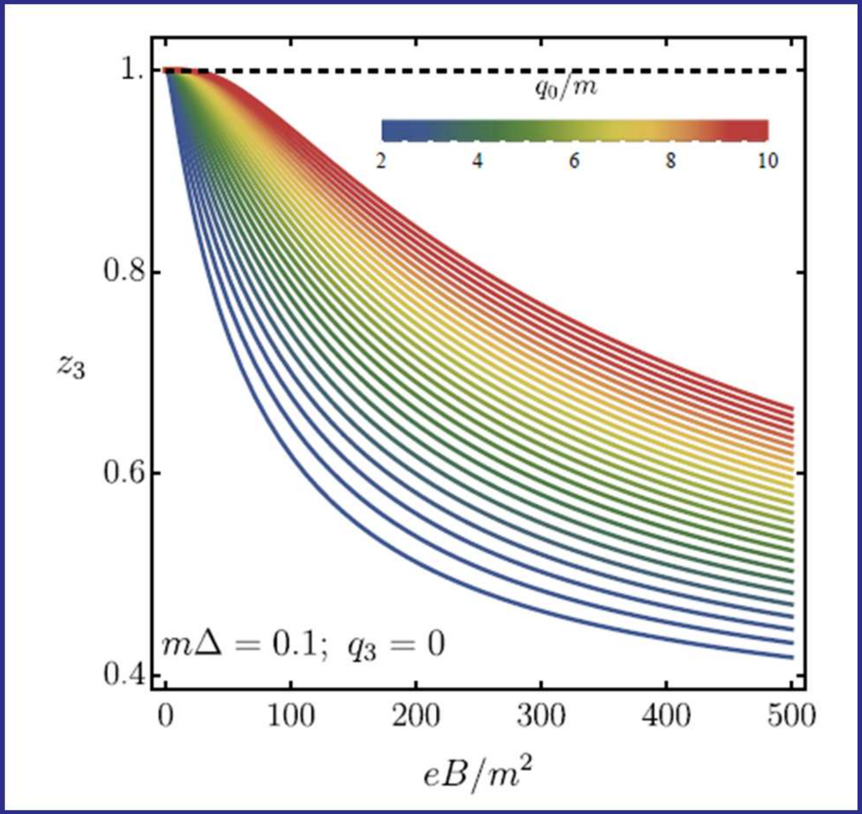
it can be shown that the magnetic noise dressed propagator is given by

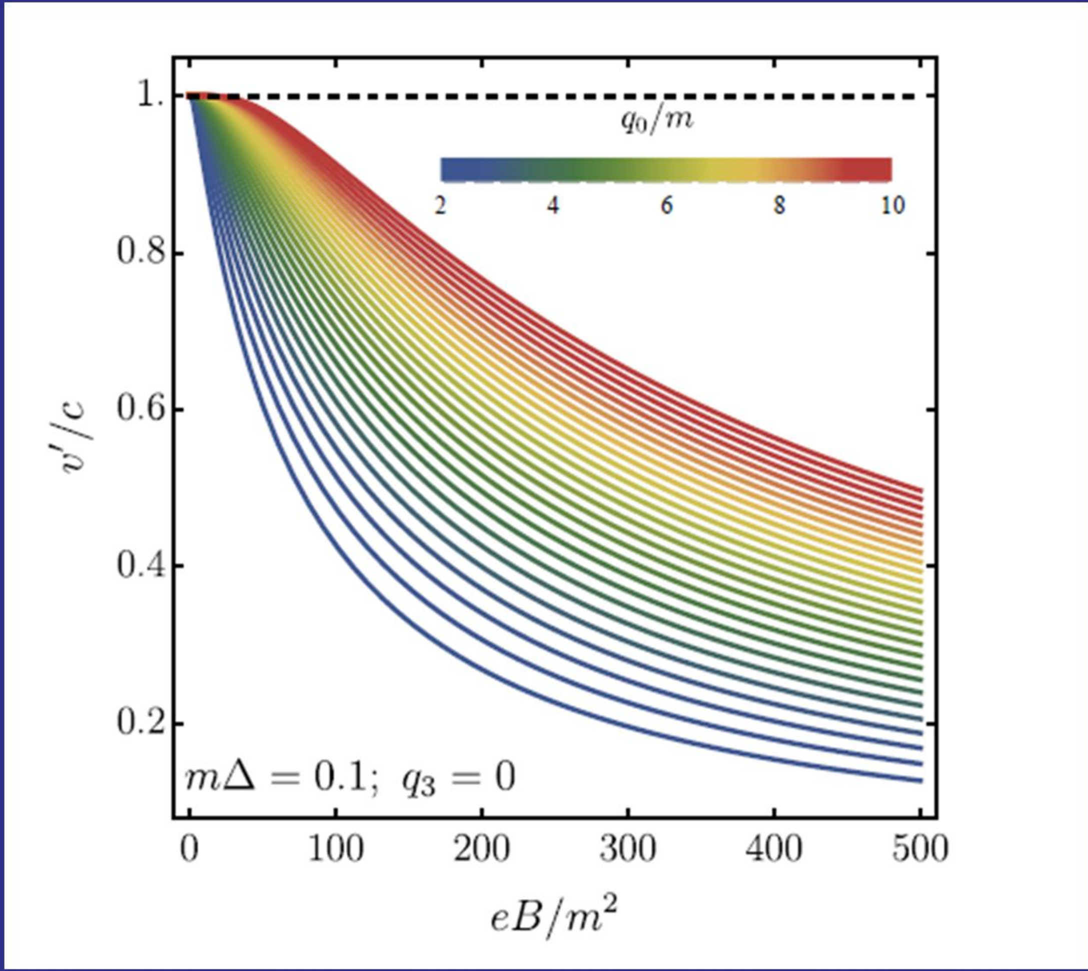
$$S_\Delta(q) = -\mathrm{i}z^{-1}\frac{\mathcal{D}(q)}{\tilde{\mathcal{D}}(q)}\left[\left(m+\tilde{\not q}_{\parallel}\right)\mathcal{A}_1(q) + \mathrm{i}z_3\gamma^1\gamma^2\left(m+\tilde{\not q}_{\parallel}\right)\mathcal{A}_2(q) + \mathcal{A}_3(q)\tilde{\not q}_{\perp}\right]$$

where $\tilde{q}^\mu = (q^0, z^{-1}\mathbf{q})$

Results The calculation were done for very small, intermediate and ultrahigh magnetic field intensities

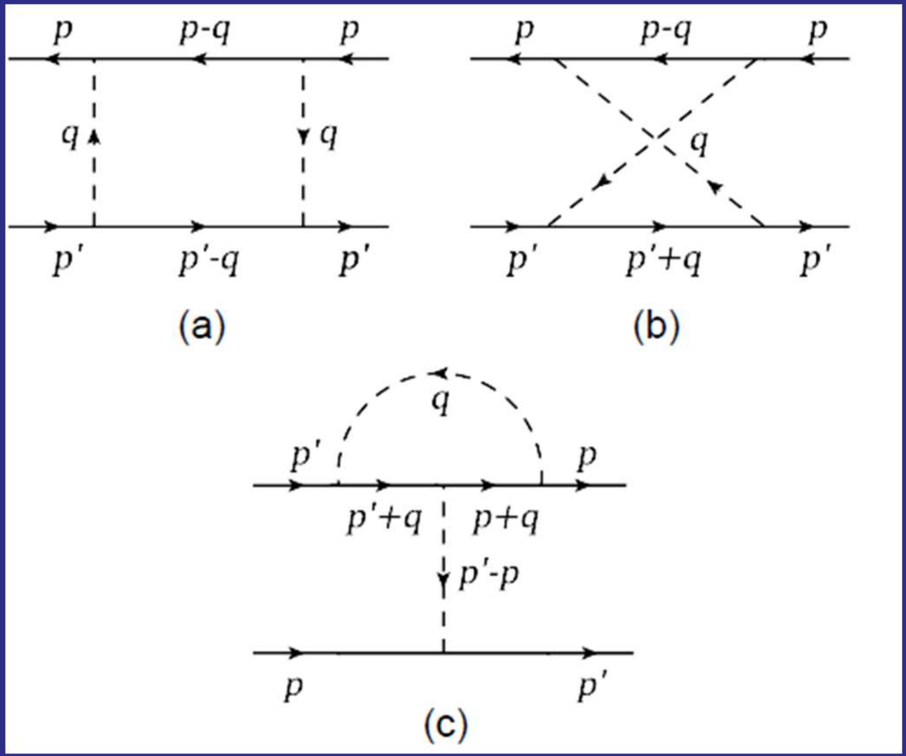






q_0/m
2
4
6
8
10
1.
0.8
0.6
0.4
0.2
v'/c
$m\Delta = 0.1;\ q_3 = 0$
0
100
200
300
400
500
eB/m^2

We also calculated the vertex correction. At the lowest order (Δ^2) we have the following contributions



$$\hat{\Gamma} = \tilde{\Delta}(\bar{\psi}\gamma^i\psi)(\bar{\psi}\gamma^i\psi) + \text{other tensor structures.}$$

$$\hat{\Gamma}_{(a)} = \int \frac{d^3q}{(2\pi)^3} \gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j,$$

$$\hat{\Gamma}_{(b)} = \int \frac{d^3q}{(2\pi)^3} \gamma^i S_F(p-q)\gamma^j \otimes \gamma_i S_F(p'+q)\gamma_j,$$

$$\hat{\Gamma}_{(c)} = \int \frac{d^3q}{(2\pi)^3} \gamma^i S_F(p+q)\gamma^j \otimes \gamma_i S_F(p'-q)\gamma_j.$$

Introducing

$$\hat{\Gamma}^{(\lambda,\sigma)} = \int \frac{d^3q}{(2\pi)^3} \gamma^i S_F(p+\lambda q)\gamma^j \otimes \gamma_i S_F(p'+\sigma q)\gamma_j$$

Where λ, σ = ± 1

$$\hat{\Gamma} = 2\hat{\Gamma}^{(-,-)} + 2\hat{\Gamma}^{(-,+)} + 4\hat{\Gamma}^{(+,-)}$$

The renormalized coefficient $\tilde{\Delta}$ is given by

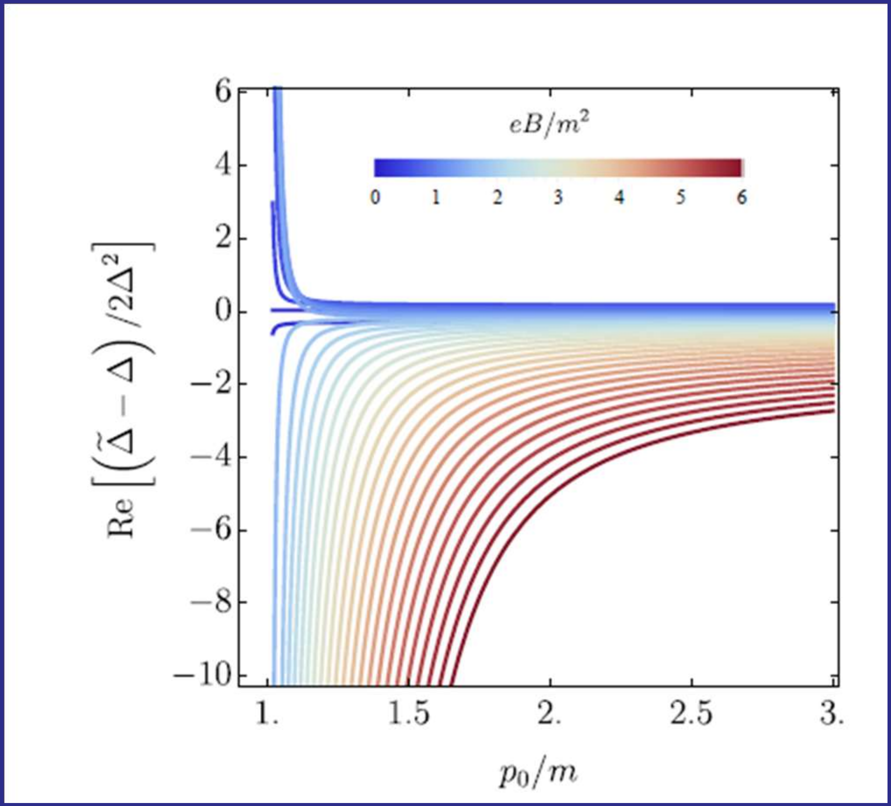
$$\tilde{\Delta} = \Delta + 2\Delta^2 \Big(\mathcal{J}_2^{(-,-)} + \mathcal{J}_2^{(-,+)} + 2\mathcal{J}_2^{(+,-)} + \left(1-\partial_x^2\right)\left(1-\partial_y^2\right)\mathcal{J}_3^{(-,-)} + \left(1-\partial_x^2\right)\left(1-\partial_y^2\right)\mathcal{J}_3^{(-,+)} + 2\left(1-\partial_x^2\right)\left(1-\partial_y^2\right)\mathcal{J}_3^{(+,-)} \Big)$$

$$\mathcal{J}_1^{(\lambda,\sigma)}(p,p') \equiv \int \frac{d^3q}{(2\pi)^3} \mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q),$$

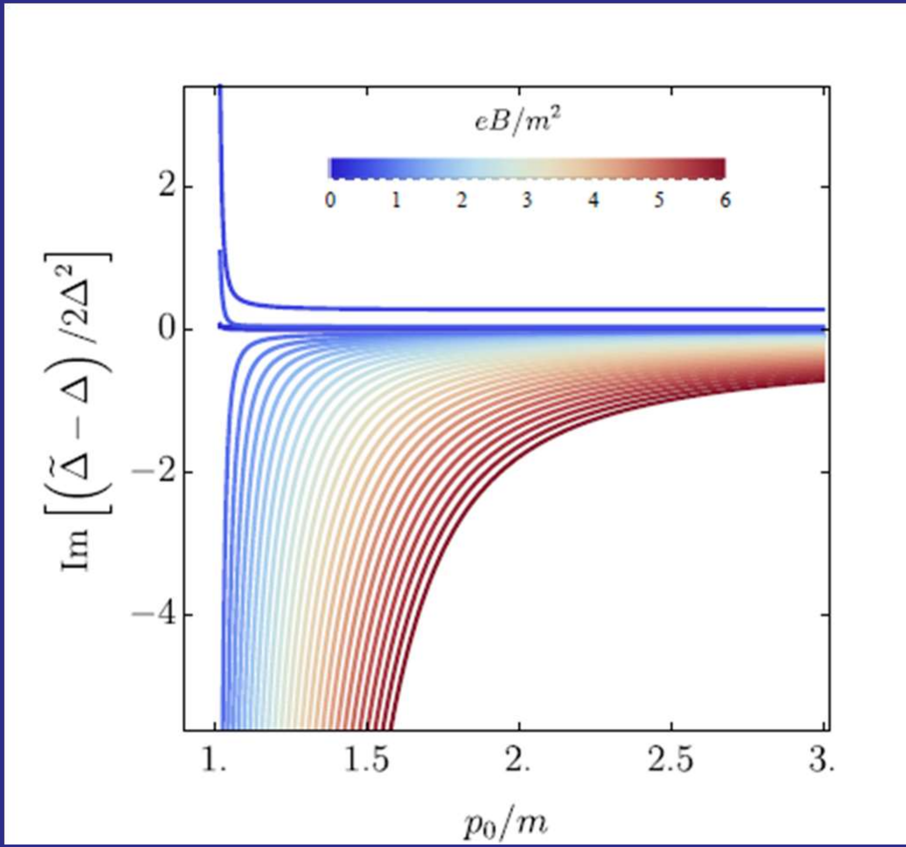
$$\mathcal{J}_2^{(\lambda,\sigma)}(p,p') \equiv \int \frac{d^3q}{(2\pi)^3} q_{\|}^2 \mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q)$$

$$\mathcal{J}_3^{(\lambda,\sigma)}(p,p') \equiv \int \frac{d^3q}{(2\pi)^3} \mathbf{q}_{\perp}^2 \mathcal{A}_1(p+\lambda q)\mathcal{A}_1(p'+\sigma q).$$

It developes real and imaginary components.

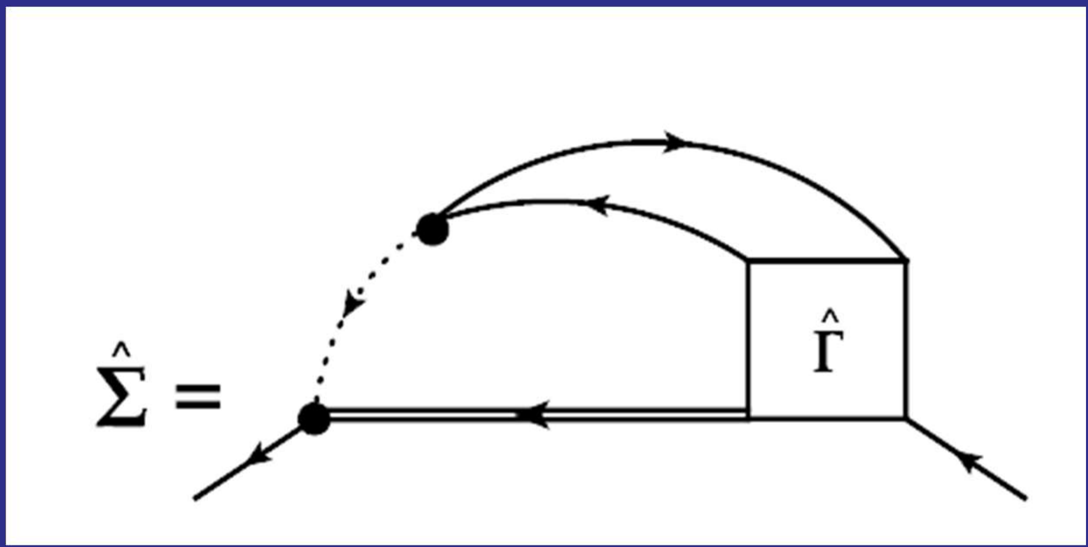


6
4
2
0
−2
−4
−6
−8
−10
1.
1.5
2.
2.5
3.
p_0/m
$\mathrm{Re}\left[\left(\tilde{\Delta}-\Delta\right)/2\Delta^2\right]$
eB/m^2
0
1
2
3
4
5
6



2
0
−2
−4
1.
1.5
2.
2.5
3.
p_0/m
$\mathrm{Im}\left[\left(\tilde{\Delta}-\Delta\right)/2\Delta^2\right]$
eB/m^2
0
1
2
3
4
5
6

An interesting comment: all diagrams are encoded in the skeleton diagram representing The self energy for the effective interaction. The dashed line is the disorder-induced interaction Δ_B while the the box $\hat{\Gamma}$ represents the 4-point vertex function



Conclusions

We have studied the effects of the quenched white noise spatial fluctuations in an otherwise uniform backkground magnetic field over properties of the QED fermion propagator.

Up to first order in Δ the propagator retains its form, representing renormalized quasiparticles with the same mass but propagating in the magnetized medium with an index of refraction 1/z and an effective charge z_3e..

The effects become negligible at high energies.

The renormalized coupling also receive contributions from the magnetic fluctuations

In a more realistic senario, magnetic fluctuations could play a relevant role on the behavior of the QCD phase diagram, in particular on the posiction of the critical end point

Thank You!!