Monopoles, Strings & Gravitational Waves

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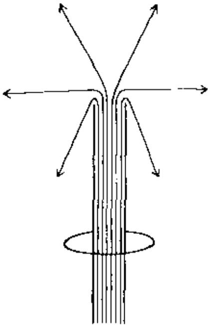
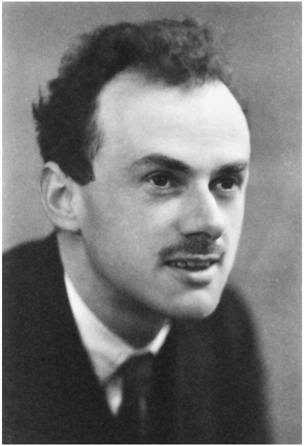
Dirac Monopole (1931)



Annu. Rev. Nucl. Part. Sci. 1984.34:461-530



$$\frac{eg}{4\pi} = \frac{n}{2}$$

Dirac Monopole (1931)

- $A_\theta^U = \frac{g}{4\pi}(1 - cos\theta)$, upper hemisphere $(0 \leq \theta \leq \pi/2)$
- $A_\theta^L = -\frac{g}{4\pi}(1 + cos\theta)$, lower hemisphere $(\pi/2 \leq \theta \leq \pi)$
- A^UandA^L are connected by gauge transformation at $\theta = \pi/2$.

$$A_\theta^U(\theta = \pi/2) - A_\theta^L(\theta = \pi/2) = \frac{2g}{4\pi} = \frac{1}{ie}(\partial_\theta \Omega)\Omega^{-1},$$

  where $\Omega(\theta) = exp[(i2eg\theta)/4\pi]$

- For $\Omega(\theta)$ to be single-valued, $\frac{eg}{4\pi} = \frac{n}{2}$

t’Hooft-Polyakov Monopole (Toy Model)

- Scalar triplet ϕ^a in the adjoint representation of $SU(2)$ breaks $SU(2) \to U(1)_{em}$.
- We can choose the identity map or ”hedgehog” configuration such that $\lim_{r\to\infty} \phi^a(\vec{x}) = v\hat{r}^a$.
- To ensure a finite energy solution, we require $D_\mu \phi^a(x) = 0$ at the boundary.
- Ansatz for the Higgs and gauge fields,

$$\phi^a(\vec{x}) = vf(r)\hat{r}^a,$$

$$A_i^a(\vec{x}) = a(r)\frac{\varepsilon_{aij}\hat{r}^j}{er}.$$

- Monopole mass $M \sim \frac{M_w}{\alpha}$, core size $\sim M_w^{-1}$.
- Magnetic charge $\frac{4\pi}{e}$ (two units of Dirac charge).

Magnetic Monopoles in Unified Theories

- Grand Unified Theories based on gauge groups $SU(5)$, $SO(10)$ and E_6 predict the existence of a topologically stable superheavy magnetic monopole that carries one quantum $(2\pi/e)$ of Dirac magnetic charge.

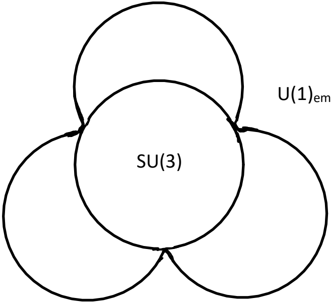
  This charge is compatible with the Dirac quantization condition because the monopole also carries color magnetic charge which is screened beyond Λ_{QCD}^{-1}.
- If the symmetry breaking to the Standard Model (SM) proceeds via some intermediate step (s), lighter monopoles may appear that carry two or more quanta of the Dirac charge.
- If the SM is embedded in gauge symmetries such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ or $SU(3)_c \times SU(3)_L \times SU(3)_R$, and if we don't require gauge coupling unification, significantly lighter monopoles are predicted, which opens up the possibility of producing them in high energy colliders.

Magnetic Monopoles in Unified Theories

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Example :

- SU(5) $\rightarrow$ SM (3-2-1)
  Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;



monopole mass $\sim \frac{M_G}{\alpha_G}$

$SU(5)$ Monopole

$$\text{SU(5)} \underset{\text{24-plet Higgs}}{\longrightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\text{SU(3)}_c \times SU(2)_L \times U(1)_Y \underset{\text{5-plet Higgs}}{\longrightarrow} SU(3)_c \times U(1)$$

$$Q_{em} = \begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$SU(5)$ Monopole

- A 2π rotation with Q_{em} yields:

$$\text{diag}\left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}, 1, 1\right)$$

- Next, we perform a $\frac{2\pi}{3}$ rotation with

$$Q_{color} = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0\right)$$

  $\rightarrow$ return to identity element.

- The monopole carries one unit of Dirac magnetic charge and also color magnetic charge.

$SU(5)$ Monopole

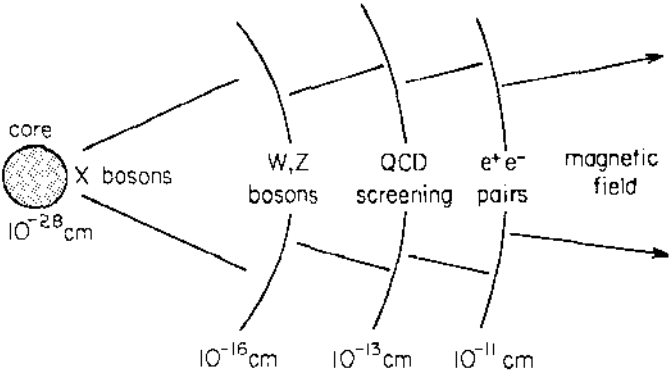


Figure 1 **Structure of a grand unified monopole.**

Annu. Rev. Nucl. Part. Sci. 1984.34:461-530

[redacted]

SO(10)

Usually broken via one or more intermediate steps to the SM

- $G = SO(10)/\text{Spin}(10)$
- $H = SU(3)_c \times U(1)_{e.m.}$
- $\Pi_2(G/H) \cong \Pi_1(H) \Rightarrow$ Monopoles
- $\Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow$ Cosmic Strings (provided $G \to H$ breaking uses only tensor representations)
- $\mathbb{Z}_2 \subset \mathbb{Z}_4$ (center of $SO(10)$)
  [T. Kibble, G. Lazarides, Q.S., PLB, 1982]
- Intermediate scale monopoles and cosmic strings may survive inflation.
- Recent work suggests that this Z_2 symmetry can yield plausible cold dark matter candidates.
  [Mario Kadastik, Kristjan Kannike, and Martti Raidal Phys. Rev. D 81 (2010), 015002; Yann Mambrini, Natsumi Nagata, Keith A. Olive, Jeremi Quevillon, and Jiaming Zheng Phys.Rev. D91 (2015) no.9, 095010 ; Sofiane M. Boucenna, Martin B. Krauss, Enrico Nardi Phys.Lett. B755 (2016) 168-17]

- $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

  Electric charge is quantized with the smallest permissible charge being $\pm(e/6)$; Lightest monopole carries two units of Dirac magnetic charge;

- SO(10) $\rightarrow$ 4-2-2 $\rightarrow$ 3-2-1

  Two sets of monopoles: First breaking produces monopoles with a single unit of Dirac charge.
  Second breaking yields monopoles with two Dirac units.

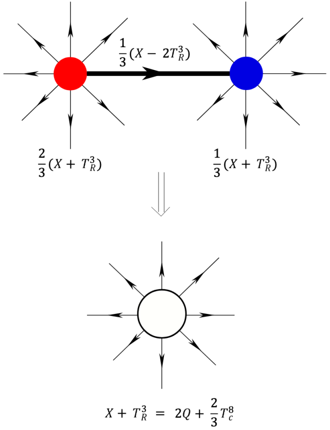
- E_6 breaking to the SM can yield intermediate mass monopoles carrying three units of Dirac charge.

- $E_6 \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow$
  $SU(3)_c \otimes SU(2)_L \otimes U(1)_{em}$

  The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics $\&$ cosmology.

'Schwinger' Monopole

$$SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \tag{1}$$



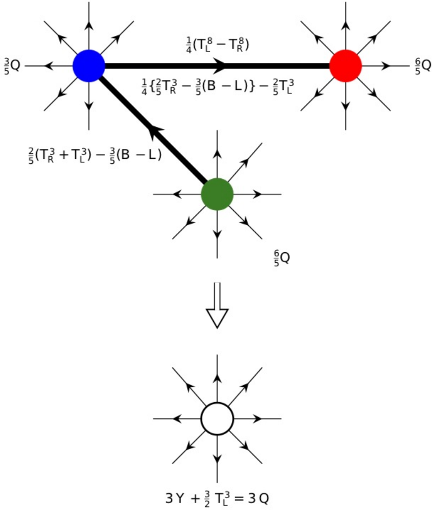
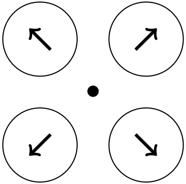


Figure 1: Emergence of the topologically stable triply charged monopole from the symmetry breaking $G \to SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{em}$. An $SU(2)_R$ (green) monopole is connected by a flux tube to an $SU(3)_L$ (blue) monopole which, in turn, is connected to an $SU(3)_R$ (red) monopole by a superconducting flux tube. The constituent monopoles are pulled together to form the triply charged monopole. The fluxes along the tubes and around the monopoles are indicated.

Primordial Monopoles

They are produced via the Kibble Mechanism as $G \to H$:



Center of monopole has G symmetry $\langle\phi\rangle = 0$

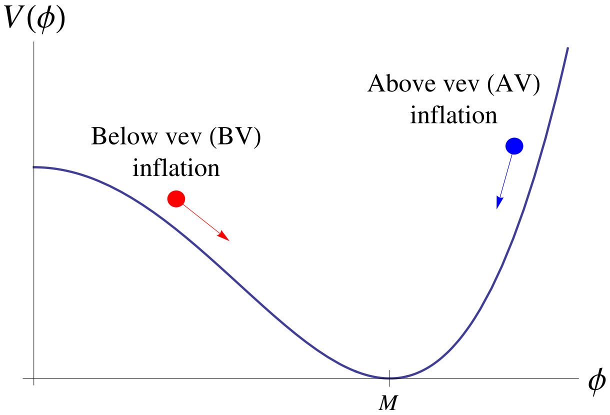
Initial no. density $\propto T_c^{-3}$. With big bang cosmology such numbers are unacceptable.

$r_{in} = \frac{N_m}{N_\gamma} \sim 10^{-2}$.

$\Rightarrow$ Primordial Monopole Problem (Zeldovich & Khlopov, Preskill)

(Need Inflation)

Inflation with a CW Higgs Potential

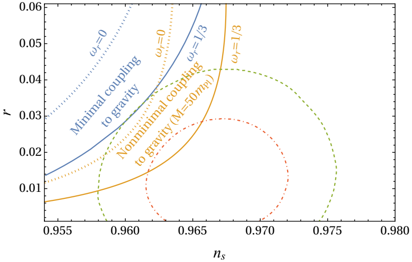


Inflation with GUT-singlet ϕ

- Inflation driven by Coleman-Weinberg potential with GUT-singlet ϕ

$$V(\phi) = A\phi^4 \left[\log\left(\frac{\phi}{M}\right) - \frac{1}{4}\right] + V_0.$$

- Minimal and non-minimal coupling to gravity: $(1+\xi(\phi^2 - M^2))R$.
- $|\xi| \sim 10^{-3}$ for below VEV inflation.



Evolution of Intermediate-mass Monopoles

Number density at production, $\xi = \min\left(H^{-1}, m_{\text{eff}}^{-1}\right)$

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$Y_M \simeq \frac{\frac{\xi^{-3}}{10}\ \exp(-3N_M)\ \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}$$

Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Monopole Searches in Colliders

- Gauge symmetries such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ and $SU(3)_c \times SU(3)_L \times SU(3)_R$ are not truly unified without additional assumptions.
  However, electric charge is quantized in these models, and it's plausible that their symmetry breaking scale lies well below the GUT scale.
- If the scale is $\sim$ few TeV or so, the corresponding monopoles may be accessible in HE colliders.
- Monopoles carry two and three quanta of Dirac magnetic charges (respectively).
- In addition, we may find exotic states that are color singlets but carry fractional electric charges, $\pm e/2$ ($\pm e/3$).

Monopole Searches in Colliders

High Energy Physics - Experiment

[Submitted on 22 Jun 2021]

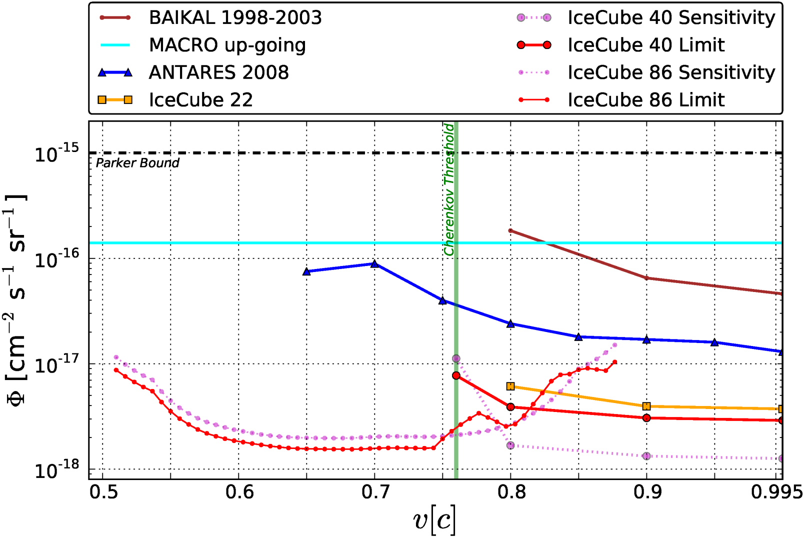
First experimental search for production of magnetic monopoles via the Schwinger mechanism

B. Acharya, J. Alexandre, P. Benes, B. Bergmann, S. Bertolucci, A. Bevan, H. Branza, P.Burian, M. Campbell, Y. M. Cho, M. de Montigny, A. De Roeck, J.R. Ellis, M. El Sawy, M. Fairbairn, D. Felea, M. Frank, O. Gould, J. Hays, A. M. Hirt, D.L.J. Ho, P.Q. Hung, J. Janecek, M. Kalliokoski, A. Korzenev, D. H. Lacarrère, C. Leroy, G. Levi, A. Lionti, A. Maulik, A. Margiotta, N. Mauri, N. E. Mavromatos, P. Mermod, L. Millward, V. A. Mitsou, I. Ostrovskiy, P.-P. Ouimet, J. Papavassiliou, B. Parker, L. Patrizii, G. E. Păvălaş, J. L. Pinfold, L. A. Popa, V. Popa, M. Pozzato, S. Pospisil, A. Rajantie, R. Ruiz de Austri, Z. Sahnoun, M. Sakellariadou, A. Santra, S. Sarkar, G. Semenoff, A. Shaa, G. Sirri, K. Sliwa, R. Soluk, M. Spurio, M. Staelens, M. Suk, M. Tenti, V. Togo, J. A. Tuszyński, A. Upreti, V. Vento, O. Vives

Schwinger showed that electrically-charged particles can be produced in a strong electric field by quantum tunnelling through the Coulomb barrier. By electromagnetic duality, if magnetic monopoles (MMs) exist, they would be produced by the same mechanism in a sufficiently strong magnetic field. Unique advantages of the Schwinger mechanism are that its rate can be calculated using semiclassical techniques without relying on perturbation theory, and the finite MM size and strong MM-photon coupling are expected to enhance their production. Pb-Pb heavy-ion collisions at the LHC produce the strongest known magnetic fields in the current Universe, and this article presents the first search for MM production by the Schwinger mechanism. It was conducted by the MoEDAL experiment during the 5.02 TeV/nucleon heavy-ion run at the LHC in November 2018, during which the MoEDAL trapping detectors (MMTs) were exposed to 0.235 nb^{-1} of Pb-Pb collisions. The MMTs were scanned for the presence of magnetic charge using a SQUID magnetometer. MMs with Dirac charges $1g_D \leq g \leq 3g_D$ and masses up to 75 GeV/c^2 were excluded by the analysis. This provides the first lower mass limit for finite-size MMs from a collider search and significantly extends previous mass bounds.

Subjects: **High Energy Physics - Experiment (hep-ex)**; High Energy Physics - Phenomenology (hep-ph)
Cite as: arXiv:2106.11933 **[hep-ex]**
(or arXiv:2106.11933v1 **[hep-ex]** for this version)

Relativistic Monopoles at IceCube



Source: IceCube Collaboration, Eur. Phys. J. C (2016) 76:133

$SU(2)_L \times U(1)_Y$

- Consider (Nambu)

$$H = \frac{v_D}{\sqrt{2}} \begin{pmatrix} cos(\theta/2) \\ sin(\theta/2)e^{i\phi} \end{pmatrix},$$

  such that $H^\dagger \sigma_i H = \frac{v_D^2}{2}\frac{x_i}{r}$
- Monopole is accompanied by a string (Z-flux) along the negative x_3-axis where $\theta = \pi$.
- If the neutrino is covariantly transported around the string its wavefunction acquires an Aharanov-Bohm phase $exp(iQ_Z^\nu \Phi_Z)$

  $\longrightarrow$ Q_Z^ν is $SU(2)_L$ Z-charge
  $= \frac{e}{sin(\theta_W)cos(\theta_W)}(T_L^3/2 - Qsin^2(\theta_W))$
  $\longrightarrow$ Φ_Z is the Z-flux
  $\Longrightarrow$ $\Phi_Z = (4\pi/e)sin(\theta_W)cos(\theta_W)$
- Similarly $\Phi_{em} = (4\pi/e)sin^2(\theta_W)$
- In GUTs, $sin^2(\theta_W) = \frac{3}{8}$

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- Ignoring the flux tube, following Nambu, the monopole mass is $\sim 700\ GeV$.

  $$\rho_{str} \sim 2 \times 10^{-2}\ GeV^{-1} \quad , \quad \mu_{str} \sim 3 \times 10^{5}\ GeV^{2}$$

  Dumbbell mass $\sim 5 - 6\ TeV$
- These topological structures also appear in GUTs such as $SU(5)$, $SO(10)$ and SUSY extensions.

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- Consider the $SU(5)$ couplings $5^\dagger \times 24 \times 5$ and $5^\dagger \times 24^2 \times 5$
- After electroweak breaking the heavy $SU(2)_L$ triplet scalar $(Y = 0)$ in $\underset{\sim}{24}$ acquires an induced $VEV \propto \frac{\langle H \rangle}{M_T} \langle H \rangle$
- Ignoring EW breaking by H, $SU(2)_L \underset{\langle \phi \rangle}{\rightarrow} U(1)_L$ yields a monopole, with magnetic flux corresponding to a 2π rotation around T_L^3
- Reintroducing $\langle H \rangle$, such that $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, this electroweak monopole ceases to be topologically stable

$$Q = \frac{T_L^3}{2} + Y \text{ is unbroken;}$$

$$\mathcal{B} = \frac{T_L^3}{2} - \frac{3Y}{5} \text{ is broken.}$$

Electroweak Monopole and Magnetic (Nambu) Dumbbell

- The monopole with one unit of charge along T_L^3 carries a Coulomb magnetic charge of $\frac{3}{4}\left(\frac{2\pi}{e}\right)$, and is attached to a Z-flux tube.
- A monopole and an antimonopole are expected to pair up and form a magnetic dumbbell (Nambu) connected by this flux tube (1977/1978).

Combining Nambu and Dirac Monopoles in the SM

- For a $U(1)_Y$ Dirac monopole in the Standard Model, Dirac quantization condition gives,

$$m_Y = \frac{2\pi}{y} n = \frac{12\pi}{g'} n, \;\; n \in \mathcal{Z},$$

- Consider Nambu monopole in the symmetry breaking $SU(2)_L \to U(1)_L$.

$$m_L = \frac{2\pi}{g/2} n' = \frac{4\pi}{g} n', \;\; n' \in \mathcal{Z},$$

- The net Z and A magnetic charges on a conglomerate of n $U(1)_Y$ and n' $SU(2)_L$ monopoles are

$$\begin{aligned} m_{Y+L,Z} &= \frac{4\pi n'}{g} \cos\theta_w - \frac{12\pi n}{g'} \sin\theta_w, \\ m_{Y+L,A} &= \frac{4\pi n'}{g} \sin\theta_w + \frac{12\pi n}{g'} \cos\theta_w \end{aligned}$$

Combining Nambu and Dirac Monopoles in the SM

- This configuration should not have any net Z magnetic charge because Z magnetic fields are confined once the electroweak symmetry is broken. Any net Z flux would form a string that would confine the monopole conglomerate to an anti-conglomerate. Thus, we require

$$\frac{4\pi n'}{g}\cos\theta_w - \frac{12\pi n}{g'}\sin\theta_w = 0.$$

  the above constraint gives

$$n' = 3n,$$

  and so the conglomerate should contain three times as many Nambu monopoles as the Dirac Y-monopoles.
- The electromagnetic magnetic charge on the conglomerate is

$$m_{Y+L,A} = \frac{12\pi}{e} n$$

Combining Nambu and Dirac Monopoles in the SM

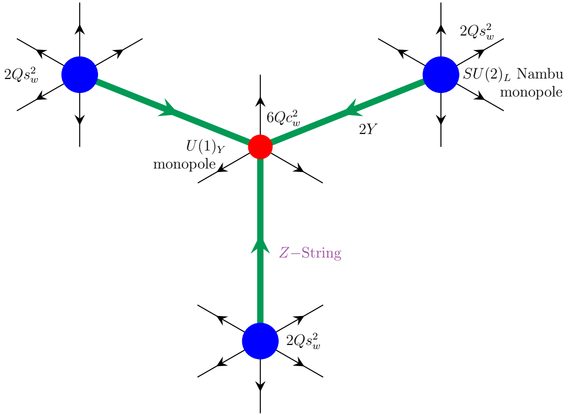


Figure: A purely $U(1)_Y$ monopole (red color) with winding number six from the breaking $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$ has a core of size M_{GUT}^{-1} and mass $\sim 10\ M_{GUT}$. It merges following electroweak breaking with three $SU(2)_L$ (Nambu) monopoles to yield a purely electromagnetic monopole that carries six quanta $(12\pi/e)$ of Dirac magnetic charge.

Colored $U(1)_Y$ Dirac Monopole

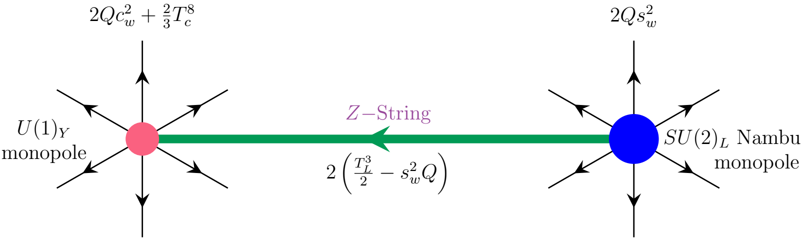


Figure: The Y-monopole emits EM and Z flux as well as color magnetic flux. The combined system made up of this monopole and the $SU(2)_L$ Nambu monopole carries two units $(2 \times 2\pi/e)$ of EM charge in addition to the color charge, compatible with the Dirac quantization condition.

Colored $U(1)_Y$ Dirac Monopole

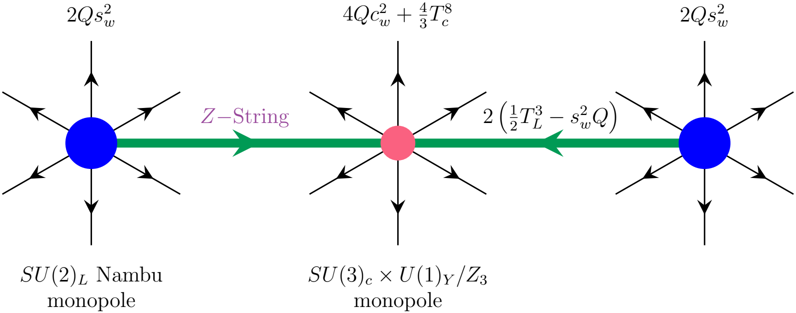


Figure: This conglomerate consists of two Nambu monopoles and a colored $U(1)_Y$ monopole.

Strings and Domain Walls

- Other topological structures such as strings, domain walls and composites can also appear in these theories.
- Cosmic strings may emit gravitational waves, which has attracted a great amount of attention in recent years.
- Strings associated with $U(1)_{PQ}$ symmetry breaking emit axion dark matter.
- Strings may be superconducting (Witten), which turns out to be the case for axion strings (Lazarides, QS; Callan, Harvey).
- Walls bounded by strings ($SO(10)$);

  Recently discovered in superfluid He3.

Walls Bounded by Strings

- Consider the breaking chain

$$\begin{array}{ccc} SO(10) & \xrightarrow{\underset{\sim}{54}} & SU(4)_c \times SU(2)_L \times SU(2)_R \\ & & \Big\downarrow \underset{\sim}{126} \\ & & SU(3)_c \times SU(2)_L \times U(1)_Y . \end{array}$$

- The first step leaves unbroken the discrete symmetry ‘C’ (also known as ‘D’) that interchanges left and right, and conjugates the representations.
- The $\underset{\sim}{126}$ vev breaks ‘C’ which produces domain walls
- Thus we end up with walls bounded by strings.
  Similar structures also arise in axion models.

Walls Bounded by Strings [J. Makinen, V. Dmitriev, et al. (Nature, 2019)]

HQVs in the PdB phase

KIBBLE-LAZARIDES-SHAFI (KLS) WALL or WALL BOUNDED BY STRINGS

Composite defect suggested in the context of phase transitions in the early Universe:

Phase 1 (Normal) ⇒ Phase 2 (Polar) ⇒ Phase 3 (PdB)

Kibble *et al*, PRD **26**, 435 (1982)

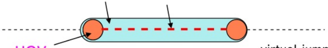
Phase 2: Strings (HQVs) become topologically stable

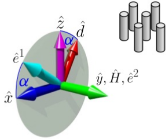
Phase 3: Strings (HQVs) are not protected by topology — walls emerge

Polar-distorted B phase:

$$A_{\mu i} = e^{i\phi}(\Delta_\parallel \hat{d}_\mu \hat{z}_i + \Delta_{\perp 1}\hat{e}^1_\mu \hat{x}_i + \Delta_{\perp 2}\hat{e}^2_\mu \hat{y}_i)$$

$$|\Delta_{\perp 1}| = |\Delta_{\perp 2}| = q|\Delta_\parallel|, \quad q < 1$$





$$\hat{\mathbf{d}} = \hat{\mathbf{x}}\cos\alpha - \hat{\mathbf{z}}\sin\alpha$$

$$\hat{\mathbf{e}}^1 = \hat{\mathbf{z}}\cos\alpha + \hat{\mathbf{x}}\sin\alpha$$

$$\hat{\mathbf{e}}^2 = \hat{\mathbf{y}} \parallel \mathbf{H}$$

Mäkinen *et al*, Nature Comm. **10**, 237 (2019)

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]
[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]
[Buchmüller, Domcke and Schmitz]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\,(\Phi\,\overline{\Phi} - M^2)$$

$S =$ gauge singlet superfield, $(\Phi\,,\overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,,\overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\,\overline{\Phi} \to \Phi\,\overline{\Phi},\;\; S \to e^{i\alpha}\,S,\;\; W \to e^{i\alpha}\,W$$

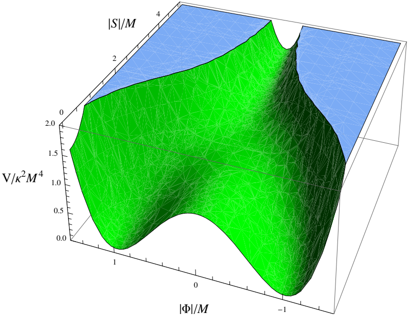
SUSY Higgs (Hybrid) Inflation

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M, \ \langle S\rangle = 0$$



SUSY Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_{\pm}^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\text{1loop}} = \frac{1}{64\pi^2}\text{Str}[\mathcal{M}^4(S)(\ln\frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4 + 1\right)\ln\frac{\left(x^4 - 1\right)}{x^4} + 2x^2\ln\frac{x^2+1}{x^2-1} + 2\ln\frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

SUSY Higgs (Hybrid) Inflation

Tree level + radiative corrections + minimal Kähler potential yield:

$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

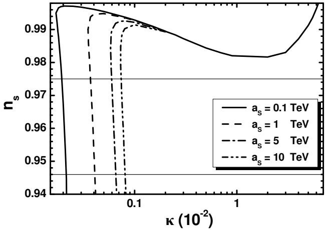
$\delta T/T$ proportional to M^2/M_p^2, where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model. In practice, $M \approx (1-5) \times 10^{15}$ GeV

Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

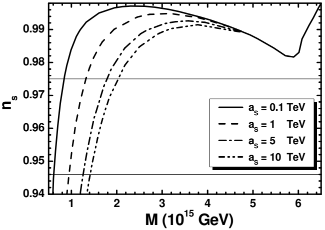
- include soft SUSY breaking terms, especially a linear term in S;
- employ non-minimal Kähler potential.

SUSY Higgs (Hybrid) Inflation

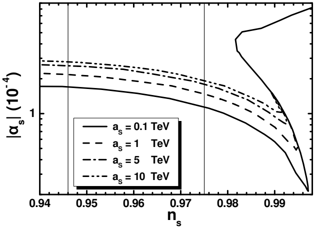
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



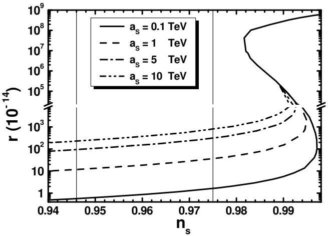
(a)



(b)



(a)

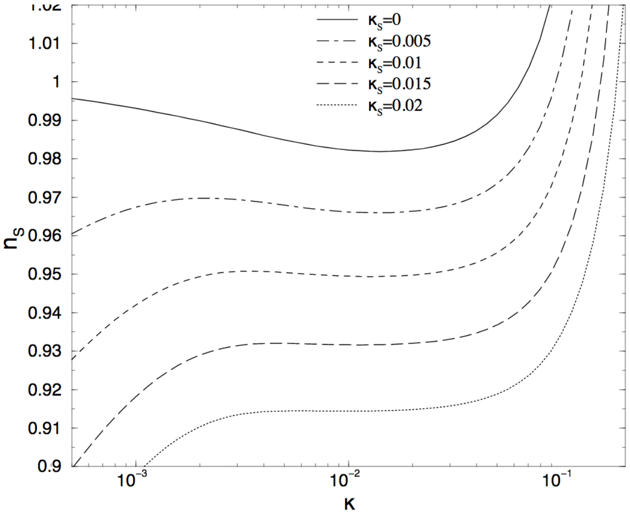


(b)

SUSY Higgs (Hybrid) Inflation

- $K \supset \kappa_s (S^\dagger S)^2$

[M. Bastero-Gil, S. F. King and Q. Shafi, 2006]



Susy Hybrid Inflation

- Some examples of gauge groups G such that

$$G \xrightarrow{\langle\Phi\rangle\neq 0} H \supseteq SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

where

$G = SM \times U(1)_{B-L}$, (cosmic strings)

$G = SU(5)$, $(\Phi = \overline{\Phi} = 24)$, (monopoles)

$G = SU(5) \times U(1)$, $(\Phi = 10)$, (Flipped $SU(5)$)

$G = SU(4)_c \times SU(2)_L \times SU(2)_R$, $(\Phi = (\overline{4}, 1, 2))$, (monopoles)

$G = SO(10)$, $(\Phi = 16)$ (monopoles)

(Non-minimal) Sugra Hybrid Inflation

[M. Bastero-Gil, S. F. King, Q. Shafi 2006; M. Rehman, V. N. Senoguz, Q. Shafi 2006]

- The superpotential is given by

$$W = \kappa S \left[\Phi\overline{\Phi} - M^2\right]$$

- The Kähler potential can be expanded as

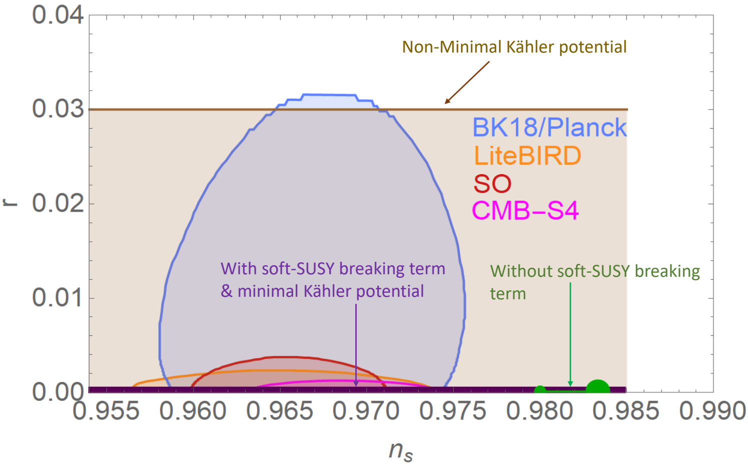
$$\begin{aligned} K &= |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 \\ &+ \kappa_S \frac{|S|^4}{4\,m_P^2} + \kappa_\Phi \frac{|\Phi|^4}{4\,m_P^2} + \kappa_{\overline{\Phi}} \frac{|\overline{\Phi}|^4}{4\,m_P^2} \\ &+ \kappa_{S\Phi} \frac{|S|^2|\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}} \frac{|S|^2|\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}} \frac{|\Phi|^2|\overline{\Phi}|^2}{m_P^2} \\ &+ \kappa_{SS} \frac{|S|^6}{6\,m_P^4} + \cdots \end{aligned}$$

- Now including all other corrections potential takes the following form

$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{S}{m_P}\right)^2 + \frac{\gamma_S}{2} \left(\frac{S}{m_P}\right)^4\right) + V_{1-loop} + V_{soft}$$

where, $\gamma_S = 1 - \frac{7\kappa_S}{2} - 2\kappa_S^2 - 3\kappa_{SS}$.

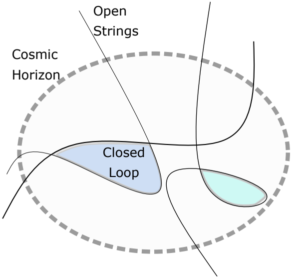
Results


Non-Minimal Kähler potential
BK18/Planck
LiteBIRD
SO
CMB−S4
With soft-SUSY breaking term
& minimal Kähler potential
Without soft-SUSY breaking
term
0.04
0.03
0.02
0.01
0.00
r
0.955
0.960
0.965
0.970
0.975
0.980
0.985
0.990
n_s

Cosmic Strings from SO(10)

Cosmic Strings arise during symmetry breaking of $G \to H$ if $\pi_1(G/H)$ is non-trivial. Consider

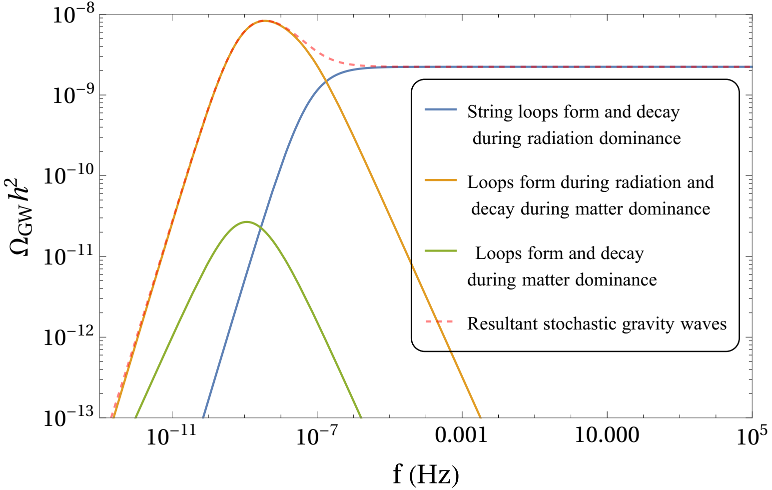
$SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$ Mass per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$. The strength of string gravity is determined by the dimensionless parameter $G\mu \ll 1$.



Stochastic Gravitational Waves from Strings

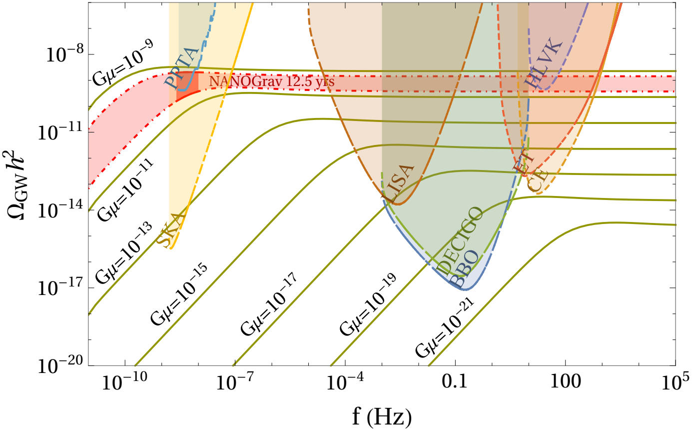
- Unresolved GWs bursts from string loops at different cosmic era produces the stochastic background.
- Loops that are formed and decay during radiation produce a plateau in the spectrum in the high frequency regime.
- Loops that are produced during radiation dominance but decay during matter dominance generate a sharply peaked spectrum at lower frequencies.
- Loops that are produced and decay during matter domination also generate a sharply peaked spectrum which, however, is overshadowed by the previous case.

Stochastic Gravitational Wave Background



Sousa, Avelino, Guedes, PRD 101 (2020) 10, 103508

GWs without Inflation and Observational Prospects



- Stringent constraint from PPTA: $G\mu \lesssim 10^{-11}$.
- Provisional GWs signal in NANOGrav: $G\mu \sim 10^{-10}$.

Evolution of Strings in Inflationary Cosmology

- The mean inter-string distance at cosmic time t (temp $= T$):

$$d_{\text{str}} = p\,\xi(\phi_I)\;\exp(N_{\text{str}})\;\left(\frac{t_r}{\tau}\right)^{\frac{2}{3}}\;\frac{T_r}{T}$$

Inter-string seperation at production $\xi = \min\left(H^{-1}, m_{\text{eff}}^{-1}\right)$

Expansion during Inflation

Expansion during Inflaton oscillations

Expansion after reheating

- The string network re-enters the post-inflationary horizon at cosmic time t_F if

$$d_{\text{str}}(t_F) = d_{\text{hor}}(t_F)$$

$$\text{with}\;\; d_{\text{hor}}(t_F) = \begin{cases} 2t_F & \text{(radiation dominance)} \\ 3t_F & \text{(matter domination).} \end{cases}$$

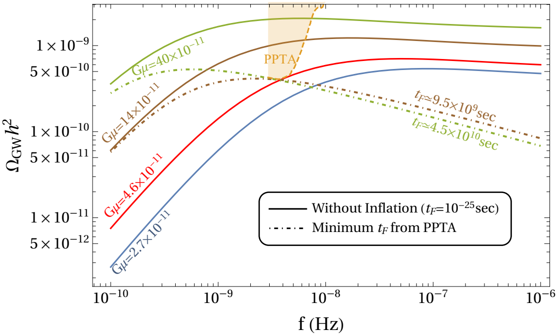
Chakrabortty, Lazarides, Maji, Shafi JHEP 02 (2021) 114

String Loops and Gravitational Waves

- After horizon re-entry the strings inter-commute and form loops at any subsequent time t_i.
- Loops of initial length $l_i = \alpha t_i$ decay via emission of gravity waves.
- The redshifted frequency of a normal mode k, emitted at time $\tilde{t}$, as observed today, is given by

$$f = \frac{a(\tilde{t})}{a(t_0)} \frac{2k}{\alpha t_i - \Gamma G\mu(\tilde{t} - t_i)}, \quad \text{with} \quad k = 1, 2, 3, \ldots$$

Inflation, GWs and PPTA bound



- Partially inflated strings re-enter horizon at time t_F in post-inflationary universe and decay via emission of GWs.
- Modified GWs spectra from 'diluted' strings can satisfy the PPTA bound.

Lazarides, Maji, Shafi, PRD 104 (2021) 9, 095004

String Loops and Gravitational Waves

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Thank You