Exclusive photoproduction of quarkonia pairs as a probe of gluon GPDs

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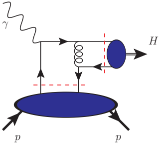
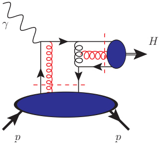
Foreword

Nucleon in QCD: sophisticated dynamical system
—Relativistic Quantum Mechanical systems, grand canonical ensemble
—Strongly interacting: chiral symmetry breaking, dynamical masses and interaction vertices ...
Theoretical description:
—Challenging for strongly coupled systems, effective models ...

Phenomenological studies:
—Based on factorization (separation of amplitude or cross-section) onto hadron- and process-dependent parts



—Require high energies, invariant masses:
⇒Avoid soft final-state interactions
⇒Suppress contributions of multiparton states (higher twist)



—Light-cone description (quantization), effectively $P \to \infty$ frame

(Generalized) partonic distributions: theoretical aspects

−Classification standardized since ∼2010 [PDG 2022, Sec 18.6]

− Leading twist-2 (dominant in many processes):

$$\int \frac{dz}{2\pi} e^{ix\bar{P}^+ z} \left\langle P' \left| \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \, e^{i\int d\zeta n\cdot A} \psi\left(\frac{z}{2}\right) \psi \right| P \right\rangle = \bar{U}\left(P'\right) \mathcal{F}^{(\Gamma)} U(P)$$

Γ	$\mathcal{F}^{(\Gamma)}$
γ^+	$H\gamma^+ + E\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}$
$\gamma^+\gamma_5$	$\tilde{H}\gamma^+\gamma_5 + \tilde{E}\frac{\gamma_5\Delta^+}{2m}$

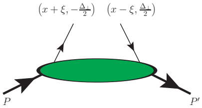
Γ	$\mathcal{F}^{(\Gamma)}$
$i\sigma^{+i}$	$H_T i\sigma^{+i} + \tilde{H}_T\frac{\bar{P}^+\Delta^i - \bar{P}^i\Delta^+}{m^2} +$ $+E_T\frac{\gamma^+\Delta^i - \gamma^i\Delta^+}{2m} + \tilde{E}_T\frac{\gamma^+\bar{P}^i - \gamma^i\bar{P}^+}{m}$

$$\bar{P} \equiv (P + P')/2 \qquad\qquad \Delta \equiv P' - P$$

*GPDs are different for each flavour, depend on 4 variables: x, ξ, t, μ^2

**Dependence on μ^2 ⇒DGLAP

**Dependence on x, ξ ⇒positivity, polynomiality constraints



⇒Challenge for modelling (“dimensionality curse”)

*For gluons use operators $G^{+\alpha}G^{+}_{\alpha}$, $G^{+\alpha}\tilde{G}^{+}_{\alpha}$, $\mathbb{S}G^{+i}G^{+j}$ in left-hand side

Might be reinterpreted in helicity basis, as Lorentz invariant decomposition of hadron-parton amplitude

Why do GPDs matter ?

Many physical observables are constructed from bilinear partonic operators:
−Energy-momentum tensor (≈enegry density, distribution of forces, ...):

$$T^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{2}\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi + \eta^{\mu\nu}\bar{\psi}\left(i\hat{D} - m\right)\psi$$

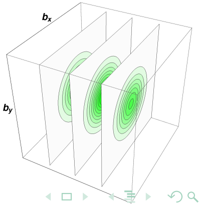
−Angular momentum density:

$$M^{\mu\nu\rho} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi + \frac{1}{2}\bar{\psi}\gamma^\mu x^{[\nu}iD^{\nu]}\psi - 2\mathrm{Tr}\left[F^{\mu\alpha}x^{[\nu}F^{\rho]}_{\alpha}\right] - x^{[\nu}g^{\rho]\mu}\mathcal{L}_{\mathrm{QCD}}$$

−Baryonic/electromagentic currents:

$$J^\mu_{\mathrm{baryonic}} = \bar{\psi}\gamma^\mu\psi, \qquad J^\mu_{\mathrm{em}} = \bar{\psi}\gamma^\mu\hat{Q}\psi$$

⇒ Moments of GPDs contain information about contribution of each parton flavour to local energy/charge density, distribution of forces/pressure, etc. Effectively "3D tomography" of the hadron.



What do we know about GPDs in 2023?

– **Experimental constraints on GPDs:**

*Special limits (PDF, form factors)

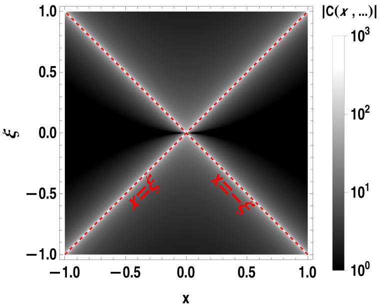
*2 → 2 **processes** (DVCS, DVMP, TCS, WACS, ...) [Monday talk of Stepanyan]

**Amplitude is a convolution of GPD with process-dependent coef. function:

$\mathcal{A} = \int dx\, C\,(x, \xi)\, H\,(x, \xi, ...)$

**Predominantly sensitive to GPDs at $x = \pm\xi$ boundary

**Deconvolution seems impossible (especially when NLO effects in C are taken into account)



⇓

Extraction of GPDs inevitably relies on modelling (and need multichannel analysis to constrain them better)

Current situation:

–For quark sector there is some qualitative understanding, phenomenological parametrizations (GK, KM, ...)

What do we know about *gluon* GPDs ?

– For gluon GPDs uncertanties are much larger:
  *Don't interact directly with leptons.

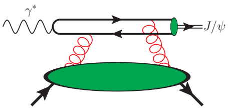
  *Show up only via higher order (NLO) corrections in many observables

  *6 of 8 GPDs are unknown, yet contribute to physical observables, e.g.:

$$J_g = \frac{1}{2}\int_0^1 dx\, x\,(H_g(x,\xi) + E_g(x,\xi))$$

Best constraints from exclusive quarkonia production:
*No sizeable "intrinsic" charm, bottom GPDs

*Light quark GPDs only via NLO, strongly suppressed



*As for DVMP, coef. function sensitive to GPDs on $x = \pm\xi$ line.

New tool for tomography: $2 \to 3$ processes

Process:

$$\gamma^{(*)} + p \to h_1 + h_2 + p$$

States h_1, h_2 are light hadrons or photons, many possibilities studied in the literature:

$-\gamma\pi,\ \gamma\rho$ [2212.00655, 2212.01034, JHEP 11 (2018) 179; 02 (2017) 054]

$\gamma\gamma$ [JHEP 08 (2022) 103; PRD **101**, 114027; **96**, 074008]

$\gamma\gamma^* \to \gamma\bar{\ell}\ell$ [Phys. Rev. D 103 (2021) 114002]

$\pi\rho$ [Phys.Lett.B 688 (2010) 154-167]

Main benefit:

–Can vary independently kinematics of h_1, h_2 to probe GPDs at $x \neq \xi$

Cost:

–Cross-section significantly smaller than for $2 \to 2$ processes, requires high luminosity

Our suggestion:

–Exclusive photoproduction of quarkonia pairs:

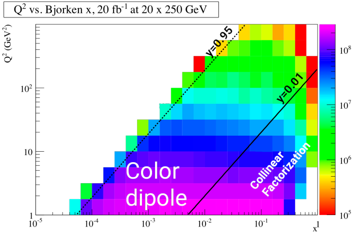
$$\gamma^{(*)} + p \to M_1 + M_2 + p$$

* Focus on quarkonia with opposite C-parity (e.g. $J/\psi\ \eta_c$), largest cross-section
* Predominantly sensitive to gluon GPDs H_g, E_g, no direct (LO) contributions from light quarks

Kinematics choice: Electron Ion Collider

Typical values of variables ξ, x_B

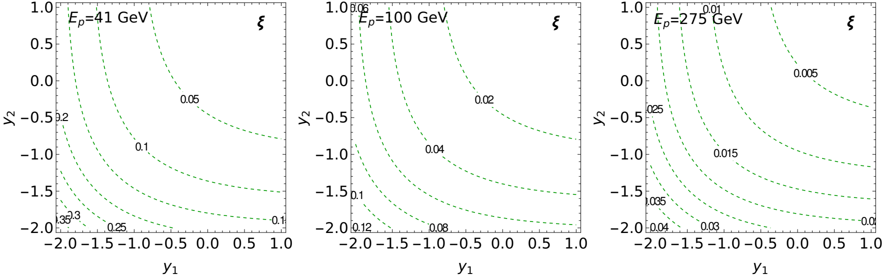
$$x_B \approx \frac{Q^2 + M_{12}^2}{Q^2 + W^2}, \qquad \xi = \frac{x_B}{2 - x_B}.$$



▷ Accessible kinematics (x_B, Q^2) depends on choice of electron-proton energy E_e, E_p

▷ Dominant: $Q^2 \approx 0$, $x_B, \xi \in \left(10^{-4}, 1\right)$

- Low-energy EIC runs to avoid $x_B, \xi \ll 1$ region (large NLO, saturation)



*Dashed lines: contours $\xi = \mathrm{const}$; E_p is the proton energy

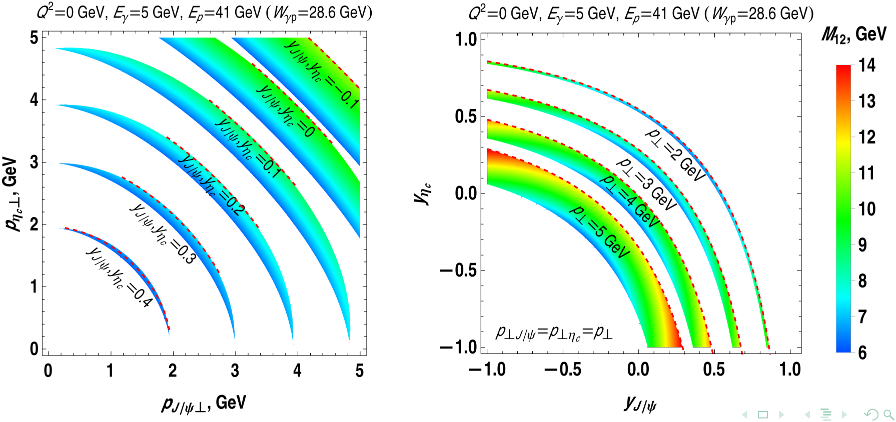
*y_1, y_2 are quarkonia rapidities in lab frame (positive in direction of electron)

Comment on kinematics

► Conventional choice: fixed Q^2, x_B (same as fixed invariant energy W of $\gamma^* p$)

$$\frac{d\sigma_{ep\to eM_1M_2p}}{d\ln x_B\,dQ^2\,d\Omega_h} = \frac{\alpha_{\rm em}}{\pi\,Q^2}\left[(1-y)\,\frac{d\sigma^{(L)}_{\gamma p\to M_1M_2p}}{d\Omega_h} + \left(1-y+\frac{y^2}{2}\right)\frac{d\sigma^{(T)}_{\gamma p\to M_1M_2p}}{d\Omega_h}\right],$$

▷ Not very convenient: quarkonia kinematical variables $y_1, p_{\perp 1},\ y_2, p_{\perp 2}$ are bound by energy-momentum conservation, onshellness of recoil proton, only certain domains (bands) are allowed:



Comment on kinematics (II)

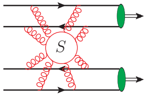
- Our choice: work with $Q^2, y_1, \boldsymbol{p}_{1\perp}, y_2, \boldsymbol{p}_{2\perp}$; fix invariant energy W of $\gamma^* p$ (and corresponding x_B) from energy-mometum conservation

$$\frac{d\sigma_{ep\to eM_1 M_2 p}}{dQ^2\, d\Omega_h} = \frac{\alpha_{\rm em}}{4\pi\, Q^2}\left[(1-y)\,\frac{d\bar{\sigma}^{(L)}_{\gamma p\to M_1 M_2 p}}{d\Omega_h} + \left(1-y+\frac{y^2}{2}\right)\frac{d\bar{\sigma}^{(T)}_{\gamma p\to M_1 M_2 p}}{d\Omega_h}\right],$$

$$d\bar{\sigma}^{(L,T)}_{\gamma p\to M_1 M_2 p} = \frac{dy_1\, dp_{1\perp}^2\, dy_2\, dp_{2\perp}^2\, d\phi_{12}\left|\mathcal{A}^{(L,T)}_{\gamma p\to M_1 M_2 p}\right|^2}{4\,(2\pi)^4\sqrt{\left(W_0^2+Q^2-m_N^2\right)^2+4Q^2 m_N^2}}$$

  - ▷No kinematic constraints on $y_1, \boldsymbol{p}_{1\perp}, y_2, \boldsymbol{p}_{2\perp}$
  - ▷Keep explicit symmetry of kinematic variables w.r.t. permutation of quarkonia $1 \leftrightarrow 2$ (neglect $M_{J/\psi} \neq M_{\eta_c}$)

- We consider that $Q \sim M_{J/\psi} \sim M_{\eta_c} \sim W_{\gamma p}$ are large scales
  - Since $M_{12}^2 \gtrsim \left(M_{J/\psi} + M_{\eta_c}\right)^2 \sim 36\ \mathrm{GeV}^2$ and cross-section is suppressed at large Q as $\lesssim 1/Q^6$, "classical" Bjorken limit $Q \gg M_{J/\psi}, M_{\eta_c}$ is difficult to study experimentally
  - Production at central rapidities, rapidity gaps from γ^*, p
  - Constraint on relative momentum of quarkonia $p_{\rm rel} \gtrsim 1\,\mathrm{GeV}$, to exclude possible soft final state interactions



Evaluations in collinear factorization framework

Evaluation is straightforward, amplitude (squared):

$$\sum_{\text{spins}} \left| \mathcal{A}^{(\mathfrak{a})}_{\gamma p \to M_1 M_2 p} \right|^2 = \frac{1}{(2-x_B)^2} \left[4\,(1-x_B) \left(\mathcal{H}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - x_B^2 \left(\mathcal{H}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} + \mathcal{E}_{\mathfrak{a}} \mathcal{H}^*_{\mathfrak{a}} + \right.\right.$$

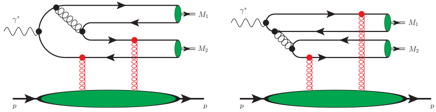
$$\left.\left. + \tilde{\mathcal{H}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} + \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{H}}^*_{\mathfrak{a}} \right) - \left(x_B^2 + (2-x_B)^2 \frac{t}{4m_N^2} \right) \mathcal{E}_{\mathfrak{a}} \mathcal{E}^*_{\mathfrak{a}} - x_B^2 \frac{t}{4m_N^2} \tilde{\mathcal{E}}_{\mathfrak{a}} \tilde{\mathcal{E}}^*_{\mathfrak{a}} \right],$$

$$\left\{ \mathcal{H}_{\mathfrak{a}},\, \mathcal{E}_{\mathfrak{a}},\, \tilde{\mathcal{H}}_{\mathfrak{a}},\, \tilde{\mathcal{E}}_{\mathfrak{a}} \right\} = \int dx\, dz_1\, dz_2\, C_{\mathfrak{a}}(x,\, z_1,\, z_2,\, y_1,\, y_2) \left\{ H_g,\, E_g,\, \tilde{H}_g,\, \tilde{E}_g \right\} \Phi_\eta(z_1)\, \Phi_{J/\psi}(z_2),$$

- Disregard transversity gluon GPDs (not known, should be small)
- Disregard internal motion of quarks, formally $\mathcal{O}(\alpha_s(m_Q)) \ll 1$

$$\Phi_\eta(z) \sim \Phi_{J/\psi}(z) \sim \delta\left(z - \frac{1}{2} \right)$$

Evaluation of coefficient function:



Summation over all possible gluon attachments is implied

– Two production mechanisms for $J/\psi\, \eta_c$

– Virtuality of (black colored) gluon is $\sim M_{12}^2/4$ in the left diag., $\sim M_1^2/4$, $M_2^2/4$ in the right, so use of perturbative treatment is justified.

Results for coefficient function

$$\left\{\mathcal{H}_{\mathfrak{a}},\, \mathcal{E}_{\mathfrak{a}},\, \tilde{\mathcal{H}}_{\mathfrak{a}},\, \tilde{\mathcal{E}}_{\mathfrak{a}}\right\} \sim \int dx\, C_{\mathfrak{a}}\left(x,\, \frac{1}{2},\, \frac{1}{2},\, y_1,\, y_2\right) \left\{H_g,\, E_g,\, \tilde{H}_g,\, \tilde{E}_g\right\},$$

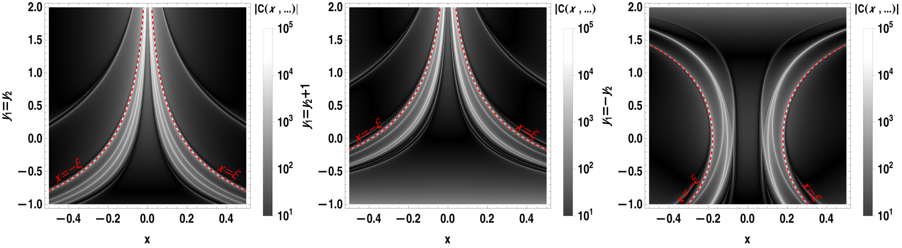
- Structure function $C_{\mathfrak{a}}(x)$:

$$C_{\mathfrak{a}}\left(x,\, \frac{1}{2},\, \frac{1}{2},\, y_1,\, y_2\right) \sim$$

$$\sim \sum_{\ell} \frac{\mathcal{P}_{\ell}(x)}{\prod_{k=1}^{n_\ell}\left(x - x_k^{(\ell)} + i0\right)}$$

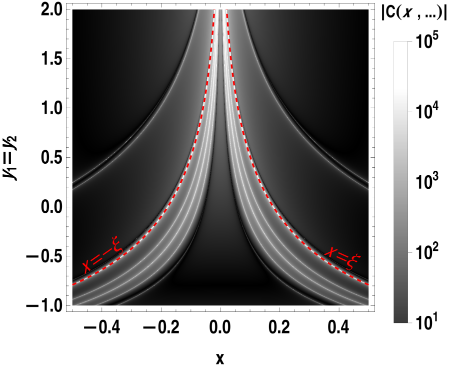
where $\mathcal{P}_{\ell}(x)$ are finite for $|x| < 1$

- – Each term might have up to 3 poles $x_k^{(\ell)}$ in the integration region $|x| < 1$
- – Position of poles depends on kinematics $(y_1,\, y_2,\, Q^2/m_Q^2)$
- – Poles do NOT overlap for $m_Q \neq 0$, so integrals exist in Principal Value sense



- Density plot of coefficient function. Regions near poles (white lines) give the dominant contribution in convolution

Results for coefficient function



Density plot of coefficient function. Regions near poles (white lines) give the dominant contribution in convolution

►Location of poles for $Q = 0$, $y_1 = y_2$:

$$|x_k| = \left\{ \xi\left(1 - \frac{2}{3}\frac{1}{1+\xi}\right), \xi\left(1 - \frac{1}{2}\frac{1}{1+\xi}\right), \xi\left(1 - \frac{1}{3}\frac{1}{1+\xi}\right), \xi,\ 3\xi\left(1 + \frac{1}{6}\frac{1}{1+\xi}\right) \right\}$$

Compare DVCS, DVMP: dominant contribution from $|x_k| = \xi$.

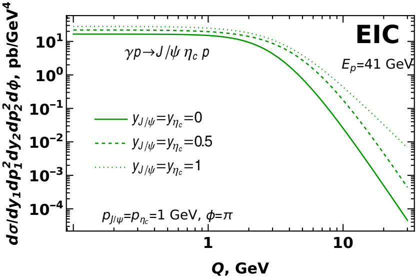
►In general expression for $C_a\left(x, \frac{1}{2}, \frac{1}{2}, y_1, y_2\right)$ is lengthy, deconvolution is impossible

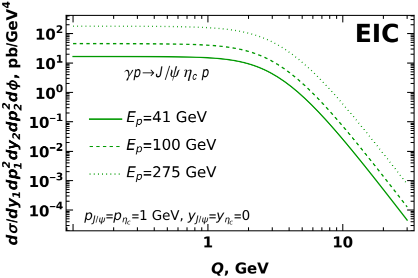
– Coeff. function sensitive to behaviour of GPDs outside "classical" $|x| \approx \xi$ line, might be used to test/constrain existing phenomenological models of gluon GPDs

Results for Q^2-dependence

► Use Kroll-Goloskokov GPD for gluons

▷ In $ep \to ep\,\eta_c\,J/\psi$ cross-section there is $\sim 1/Q^2$ from leptonic part, so consider instead cross-section of the $\gamma^* p \to p\,\eta_c\,J/\psi$ subprocess:





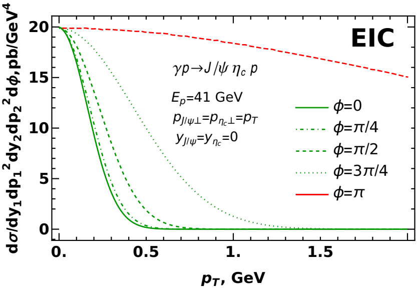
► The Q^2-dependence is controlled by

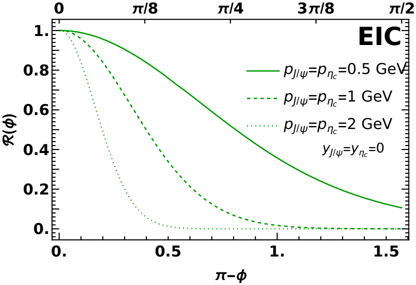
$$M_{12} = \sqrt{\left(p_{J/\psi} + p_{\eta_c}\right)^2} \gtrsim \left(M_{J/\psi} + M_{\eta_c}\right)$$

−very mild dependence for $Q^2 \lesssim M_{12}^2$

− $d\sigma \sim 1/Q^6$ for $Q^2 \gg M_{12}^2$

Results for p_T, ϕ-dependence





– The observed dependence largely reflects dependence of GPD on invariant momentum transfer

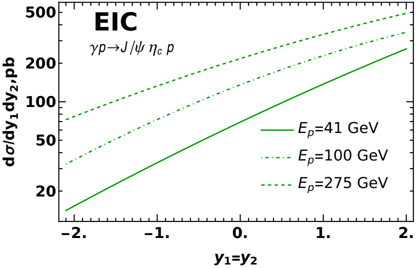
$$t = \Delta^2 = -\frac{4\xi^2 m_N^2 + \left(\boldsymbol{p}_1^\perp + \boldsymbol{p}_2^\perp\right)^2}{1-\xi^2} = -\frac{4\xi^2 m_N^2 + p_{1\perp}^2 + p_{2\perp}^2 + 2p_{1\perp}p_{2\perp}\cos\phi}{1-\xi^2}$$

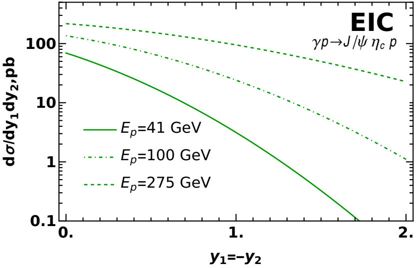
– Implemented in KG: $H_g(x,\xi,t) \sim e^{Bt}$

– "Residual" dependence on p_T at $\phi=\pi$ and $p_{1\perp} = p_{2\perp}$ is due to "kinematical higher twists" (via $M_{1\perp}, M_{2\perp}, M_{12}$ which depend on $p_\perp$). If disregard these "kinematical higher twists", the dependence is flat.

– For $p_{1\perp} = p_{2\perp} = p_\perp \gtrsim Q, M_{J/\psi}$(wide angle kinematics) even for $\phi=\pi$ expect that p_T dependence $\sim 1/p_T^6$, akin to Q-dependence for $Q \gg M_{J/\psi}$

Results for rapidity dependence



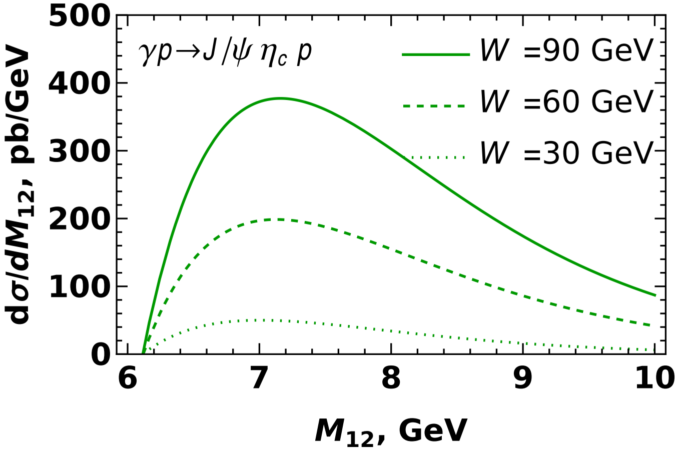


▷

– For $y_1 = y_2$ increase of rapidity implies:
  - *Larger invariant energy W
  - *Smaller x_B, ξ
  - *Larger cross-section due to growth of $H_g(x, \xi, t)$ at small x

– For $y_1 = -y_2$ increase of rapidity implies:
  - *Larger longitudinal recoil to proton Δ_L
  - *Larger values of $|t_{\min}|$, $|t| = |\Delta^2|$
  - *Suppression of cross-section due to $\sim e^{Bt}$ behaviour of $H_g(x, \xi, t)$

Results for invariant mass dependence



−Pronounced peak at $M_{12} \approx 7\,\mathrm{GeV}$

**Small relative momentum of quarkonia, $p_{\rm rel} \lesssim 2-3\,\mathrm{GeV}$

Summary

Exclusive production of heavy quarkonia pairs might be used as a new probe of the gluon GPDs:

- Unpolarized cross-section gets dominant contribution from GPD H_g, E_g
  - Sensitive to behaviour outside $x = \pm\xi$ line
  - Can vary independently rapidities of produced quarkonia to extract x, ξ dependence

- The cross-section is large enough for experimental studies, at least for charmonia
  - On par with $\gamma^{(*)} p \to \gamma\pi^0\, p$, $\gamma^{(*)} p \to \gamma\rho^0\, p$ suggested by other authors