The renormalization scale-setting problem in QCD

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Abstract

The purpose of this work is to present an optimal way to adjust the renormalization scale via the Principle of Maximum Conformality, and to show some PMC applications: Hadronic decay of the W boson, and Determination of heavy quark on-shell mass via the perturbative relation to its running mass.

I. Introduction

An observable in perturbative QCD for a physical process in its most general form can be expressed as

$$\mathcal{O}_p=\sum_{i=1}^{n}\left(\sum_{j=0}^{i-1}c_{i,j}\left(\mu_r,Q\right)\left[N_f\right]^j\right)\left[\frac{\alpha_s\left(\mu_r\right)}{4\pi}\right]^{p+i-1} \quad (1)$$

where:

- $\alpha_s\left(\mu_r\right)$ is running coupling.
- μ_r is the renormalization scale.
- N_f the number of active flavors (UV-divergent diagrams).
- p is the associated power α_s at the Bohr level.

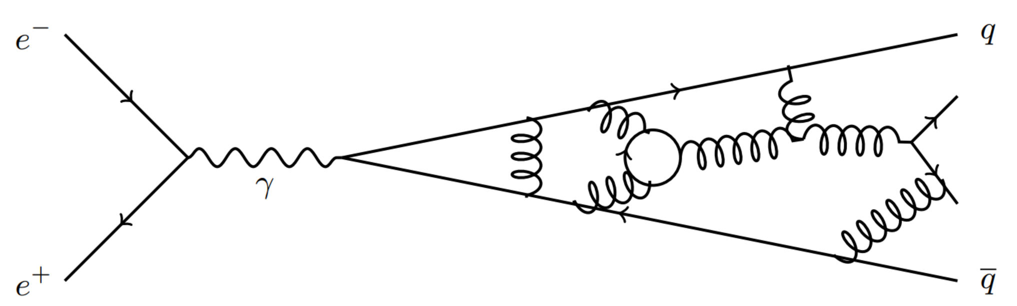

Fig 1. Schematic process electron-positron annihilation into hadrons.

► **Example**: In the study of the scattering process of a $e^-e^+\to$ Hadrons, it can be expressed as the following series:

$$R_{e^+e^-}(\mu_r)=\frac{\sigma\left(e^+e^-\to \text{hadrons}\right)}{\sigma\left(e^+e^-\to\mu^+\mu^-\right)}=r_0+r_1\alpha_s\left(\mu_r\right)+r_2\alpha_s^2\left(\mu_r\right)+\cdots \quad (2)$$

where r_0 and r_1 are schema independent, but the following coefficients r_i are scala-scheme dependent

What is the correct renormalization scale for the process?

► In an infinite order, there is no scale-scheme renormalization dependency, i.e. Any scale-scheme should deliver the same prediction

Renormalization Group Invariance (RGI)

RGI shows that if the effective coupling $a\left(\tau_{\mathcal{R}},\{c_i^{\mathcal{R}}\}\right)$ corresponds to a physical observable, the result from calculating in any scheme should be independent of any other scale $\tau_{\mathcal{S}}$ any other scheme parameters $\{c_j^{\mathcal{S}}\}$; i.e.

$$\frac{\partial}{\partial\tau_{\mathcal{S}}}a^{\mathcal{R}}\left(\tau_{R},\{r_i^{\mathcal{R}}\}\right)=0 \quad \text{(Scale invariance)} \quad (3)$$

$$\frac{\partial}{\partial c_j^{\mathcal{S}}}a^{\mathcal{R}}\left(\tau_{R},\{r_i^{\mathcal{R}}\}\right)=0 \quad \text{(Scheme invariance)} \quad (4)$$

with $\tau_{\mathcal{R}}=\left(\beta_0^2/\beta_1\right)\ln\mu_r^2$, and $a^{\mathcal{R}}=\left(\beta_1/\beta_0\right)\alpha_s/\left(4\pi\right)$.

II. The convencional scale setting (CSS)

But at fixed order, how to set the scale?

- Guess. $\mu_r\sim Q$ the momentum transferred, with the idea of eliminating the long Log

$$r_n=a+N_f\left(b+c\,\ln\frac{\mu_r^2}{Q^2}\right)+\cdots+N_f^{n-1}\left(\cdots+f\ln^{n-1}\frac{\mu_r^2}{Q^2}\right)+\cdots \quad (5)$$

- The scale remains fixed in all orders of the perturbation
- Measurement uncertainties: $\mu_r\in[Q/2,2Q]$

► For the example above, in the CSS, it presents a high dependence on the scale, as illustrated in Figure 2.

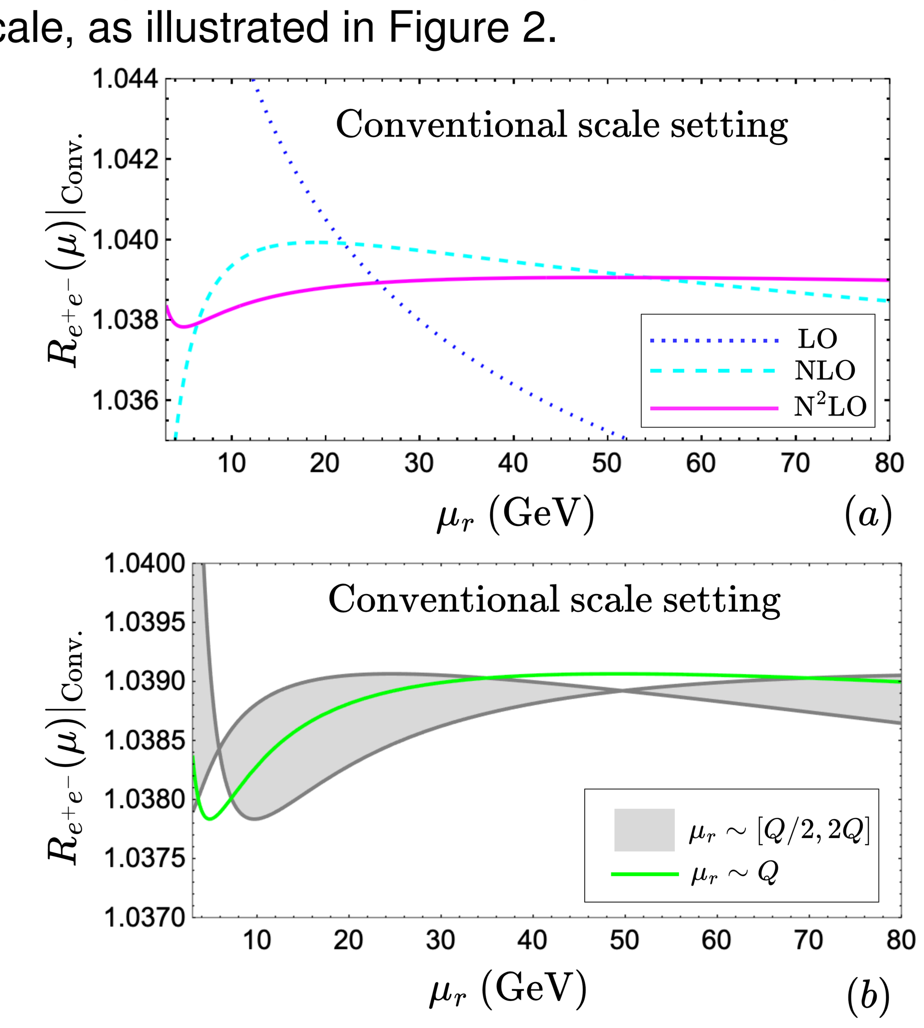

Fig 2. (a) The pQCD prediction for electron-positron annihilation into hadrons up to order N^2LO. (b) Comparison of the observable in the choice of the scale $\mu=Q$ and $\mu\in[Q/2,2Q]$ in CSS.

The Problem in Conv. scale setting

- Scale suppression is only achieved by increasing the order α_s
- The convergence of the series is obtained by eliminating the large Log terms without a physical argument
- There are significant limitations in predicting behavior at higher orders

III. Optimization in scale setting

Different optimizations in scale setting have emerged in the literature, as shown in Fig. 3. The FAC and the PMS, designed to find an optimal renormalization, Whereas BLM, seBLM and PMC are based on the standard RGI and manage to eliminate unnecessary systematic errors for high-precision pQCD predictions.

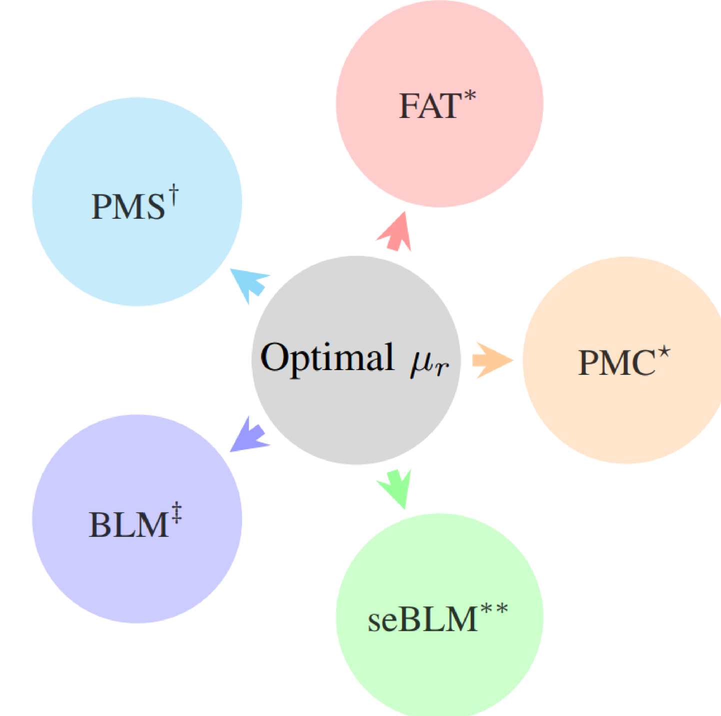

Fig 3. The diagram on the various optimizations of CSS.

IV. The Principle of Maximum Conformality

PMC, underlying the BLM method [1], presents the best alternative to solve the ambiguity problem in the choice of scale. Since it builds a systematic way to remove renormalization scheme and scale ambiguities. In Fig 5. A flow chart is presented to apply the PMC.

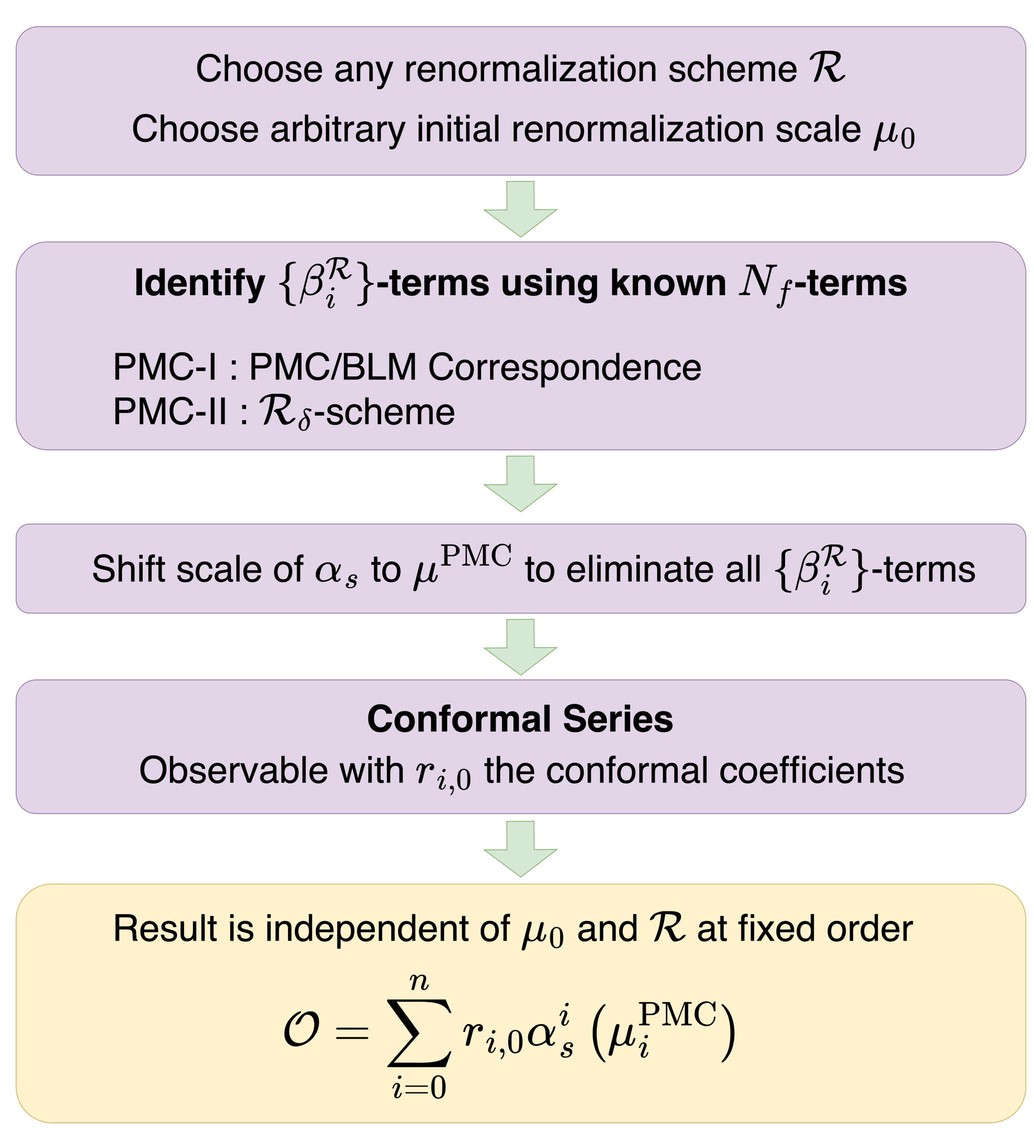

Fig 4. A flow chart which illustrates the PMC procedure. Idea taken from [2].

V. Applications for PMC

A. Relation between Polar and $\overline{\text{MS}}$ Masses of Heavy Quarks

A good prediction of the mass of quarks plays a significant role in the phenomenology of high-energy physics. For example, the b-quark mass is essential for determining the decay of the B meson and is the dominant channel in the decay of the Higgs boson into a pair of quarks, whereas the top quark is used indirectly in the determination of the mass of the Higgs boson. In work [3], it is shown how PMC considerably improves the pole mass predictions for heavy quarks obtaining the following numerical values (given in GeV) for charm, bottom, and top quarks:

	M_c	$\overline{m}_c$	$\lvert\Delta M_c\rvert$
Conv.	2,39129	1,54334	0,847947
PMCs	1,46741	1,57639	0,108979
PMC	2,51128	1,57221	0,939072

	M_b	$\overline{m}_b$	$\lvert\Delta M_b\rvert$
Conv.	5,04326	4,43381	0,609449
PMCs	4,66903	4,51271	0,317034
PMC	4,84294	4,5021	0,340842

	M_t	$\overline{m}_t$	$\lvert\Delta M_t\rvert$
Conv.	173,831	160,593	13,2376
PMCs	172,091	162,86	9,21144
PMC	172,239	162,56	9,67998

Tab 1. Obtained values for the polar, running mass and the difference between them for the heavy quark, using the CSS (Conv.), PMCs, and PMC.

B. New Determination of W Boson Hadronic Decay Width

$$\Gamma^W_{\text{hadrons}}=\frac{\sqrt{2}G_FN_c}{12\pi}M_W^3\sum_{i,j}\left|V_{ij}\right|^2\left(r^W_{\text{NS}}+\delta_{\text{EW}}(\alpha)+\delta_{\text{mix}}(\alpha\alpha_s)\right) \quad (6)$$

Improving the predictions of the hadronic decay of the W-boson directly affects the information of fundamental free parameters of the theory. This decay can be used as an excellent estimator to determine the strong coupling constant, which establishes the scale of the strong interactions, theoretically described by QCD; this impacts by reducing the theoretical uncertainties in the calculations of all high precision pQCD processes, such as the events measured at the LHC. Fig. 5 and 6 show the determination of the hadronic decay width of the W boson in the scale setting PMC [4], in contrast to the results in CSS.

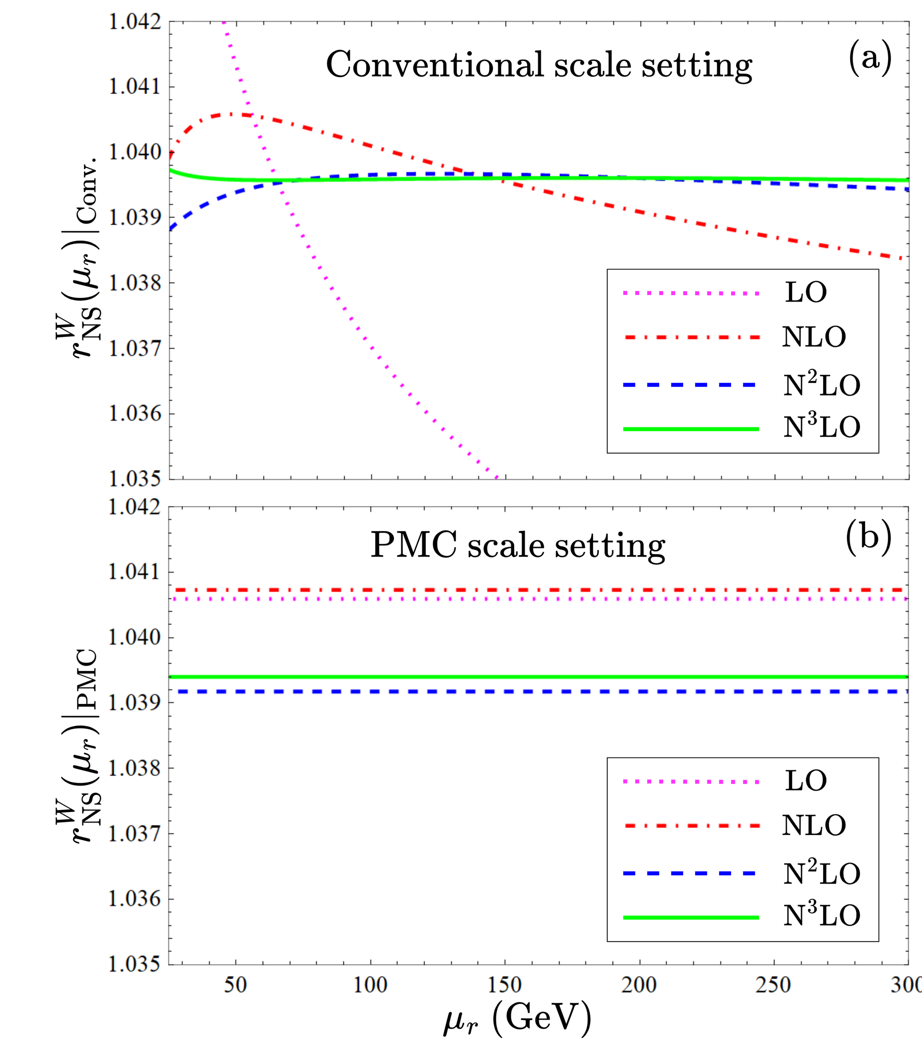

Fig 5. (a) The up to four-loop contributions of the non-singlet ratio r^W_{NS} in the CSS and (b) in the PMC setting scale.

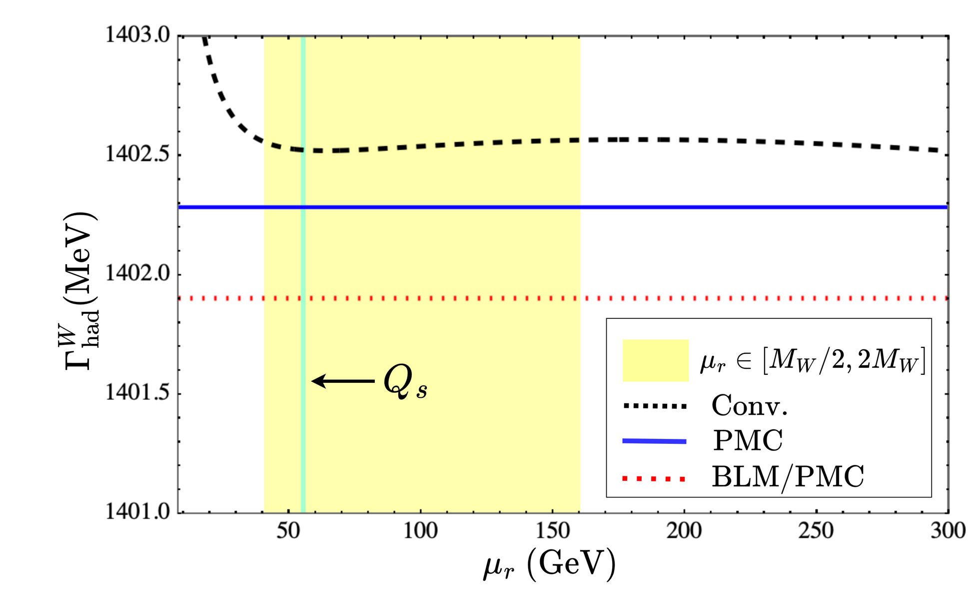

Fig 6. Inclusive hadronic decay of W-boson up a to four-loop level, under CSS (dash curve), PMC (solid curve), and correspondence BLM/PMC (dotted curve). The region highlighted in solid represents the interval of the conventional scale setting, while the vertical line is the value of single-scale PMC.

VI. Summary

- The PMC provides a consistent method for determining the renormalization scale in pQCD.
- The PMC scale-fixed prediction is independent of the choice of renormalization scheme, a key requirement of renormalization group invariance.
- The PMC thus systematically eliminates the scheme and scale uncertainties of pQCD predictions, greatly improving the precision of tests of the Standard Model and the sensitivity of collider experiments to new physics.
- New Determination of W Boson Hadronic Decay.
- An optimal way to determine the mass ratio for heavy quarks.

The PMC eliminates a significant theoretical uncertainty for pQCD predictions at the LHC and other colliders, significantly increasing the precision for testing fundamental theories.

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References

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