Supersymmetry in the adjoint representation of the conformal superalgebra
(aka "Unconventional Supersymmetry")

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8th International conference ON HIGH ENERGY PHYSICS
HEP2023
Valparaíso, Chile
Universidad Técnica Federico Santa María

Punchline 1/2

- We study a general recipe to implement models for gravity, gauge theories and matter using the **adjoint reresentation** of the **superconformal algebra**

$$SU(2,2|N)$$

→ Fermion/boson matching of d.o.f. is not mandatory.

→ Standard gauge kinetic terms are included.

→ Models are highly predictive, very few free parameters in the action.

→ Also included: fermion quartic terms, torsion coupling.

Punchline 2/2

- We constructed a GUT based on

$$SU(2,2|5)_{\rm diag} = [SU(2,2|5) \times SU(2,2|10)]_{\rm diag}$$

That embeds the SU(5) Georgi-Glashow model into the conformal superalgebra.

Model features:

→ All the quarks and leptons of the SM in the 5* + 10

→ Gluons, W and B bosons plus X,Y bosons of the GG model in a 24 of SU(5)

→ Also extra U(1), (5,5*,ynew) + (5*,5,-ynew), SU(5), U(1)_ynew

My collaborators in the topic of susy in the adjoint

- J. Zanelli (CECs)
- P. Pais (U. Austral)
- M. Valenzuela (CECs)
- E. Rodriguez (U. Nac. Colombia)
- P. Salgado (PUCV)
- L. Delage (U. Talca)
- A. Chavez, J. Ortiz (phd students at Universidad de Antofagasta)

Why?

Susy in linear representation → departure from unification

- MSSM ~ 100 free parameters
- SUGRA MSSM ~ 20 free parameters

Where is gravity?

Where are the superpartners?

SUSY in the adjoint representation

All fields in the gauge potential

- Bosons and fermions in the adjoint representation:

$$\mathbb{A} = A^M \mathbb{G}_M + \overline{\mathbb{Q}}\not{e}\psi + \overline{\psi}\not{e}\mathbb{Q}$$

$$A^M \mathbb{G}_M = W^S \mathbb{J}_S + A^I \mathbb{T}_I + A\mathbb{Z}$$

spacetime — internal — susy central

- Spinors matter fields require the introduction of a soldering form → gravity

$$\not{e}\psi = e^a{}_\mu dx^\mu \gamma_a \psi$$

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

Unconventional matter coupling

- Matter in the adjoint representation: $\psi^\alpha \in A_\mu$

- Red. Reps. $\Psi^\alpha_\mu = 1 \otimes 1/2 = 3/2 \oplus 1/2$

(a) Gravitino (SUGRA) $\xi^\alpha_\mu : \gamma^\mu \xi^\alpha_\mu = 0$ $P_{(1/2)} \xi^\alpha_\mu = 0$

(b) USUSY $\psi^\alpha_\mu = \gamma_\mu \psi^\alpha$ $P_{(3/2)} \psi^\alpha_\mu = 0$

Unconventional SUSY: **fields in the adjoint rep**

$$\mathbb{A}_\mu \supset \overline{Q} e^a{}_\mu \gamma_a \psi$$

- We choose a basis of the conformal superalgebra where the Q's carry an R-symmetry rep

  (see Trigiante's lectures on supergravity)

→ **Correct gauge transformations** ↓

- From:

$$\delta \mathbb{A} = DG$$

→

$$\begin{cases} \delta A_{SU(N)} = D\lambda_{SU(N)} \\ \delta\psi = [\lambda_{SU(N)}, \psi] \end{cases}$$

Action inspired by SUGRA a la MacDowell-Mansouri

- The action is written as

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

Townsend '77
MacDowell, Mansouri '77
Castellani 1802.03407
Trigiante 1609.09745

- Obvious resemblance to Yang-Mills

$$S = \int \langle \mathbb{F} * \mathbb{F} \rangle$$

Similarity can be exploited to study field equations and symmetries [PA, Chavez Zanelli 2111.09845 hep-th]

Yang-Mills action

- Wanted:

$$\langle \mathbb{F} \circledast \mathbb{F} \rangle \propto -\frac{1}{2} F * F + d^4 x |e| \overline{\psi} D\!\!\!/ \psi$$

From:

1) Matter in the adjoint rep.

$$\psi^{\alpha} \in A_{\mu}$$

2) Generalized dual operator

$$\circledast = ?$$

Unconventional SUSY

→ Conventional Dirac kinetic term?

$$\overline{\psi}\not{D}\psi$$

$$\langle \mathbb{F} \circledast \mathbb{F} \rangle$$

$\cup$

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q} D(\not{e}\psi)$$

$$\begin{aligned}
\hat{\mathcal{F}}^i \gamma_5 \mathcal{F}_i =& -\overline{\psi}^1 \overleftarrow{D}_\Omega \gamma_5 D_\Omega \chi^1 \\
=& \overline{\psi}^1 (-\overleftarrow{D}^+ + \Omega^-) \gamma_5 (D^+ + \Omega^-) \chi^1 \\
=& -\overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 D^+ \chi^1 - \overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 D^+ \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1 \\
=& d[\overline{\psi}^1 \gamma_5 D^+ \chi^1] + \overline{\psi}^1 \gamma_5 (D^+)^2 \chi^1 + \overline{\psi}(\overleftarrow{D}\not{e} + \not{T}) \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 (-\not{e}D + \not{T})\psi + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1
\end{aligned}$$

(here $\overline{\psi}^1 = \overline{\psi}\not{e}$ and $\chi^1 = \not{e}\psi$)

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	
$[\mathbb{D}, S] = 0$	

Ω_+

$$\Omega_- = \frac{1}{2} f^a \mathbb{J}_a + \frac{1}{2} g^a \mathbb{K}_a$$

- Identification with frames

$$f^a = \rho e^a, \qquad g^a = \sigma e^a,$$

→ **Dirac kinetic term**

Unconventional SUSY

→ Conventional Dirac kinetic term?

$$\overline{\psi} \not{D} \psi$$

$$\langle \mathbb{F} \circledast \mathbb{F} \rangle$$

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q} D(\not{e}\psi)$$

$$\cup$$

$$\hat{\mathcal{F}}^i \gamma_5 \mathcal{F}_i = -\overline{\psi}^1 \overleftarrow{D}_\Omega \gamma_5 D_\Omega \chi^1$$

(here $\overline{\psi}^1 = \overline{\psi}\not{e}$ and $\chi^1 = \not{e}\psi$)

$$= \overline{\psi}^1 (-\overleftarrow{D}^+ + \Omega^-) \gamma_5 (D^+ + \Omega^-) \chi^1$$

$$= -\overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 D^+ \chi^1 - \overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 D^+ \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1$$

$$= d[\overline{\psi}^1 \gamma_5 D^+ \chi^1] + \overline{\psi}^1 \gamma_5 (D^+)^2 \chi^1 + \overline{\psi}(\overleftarrow{D}\not{e} + \not{T}) \gamma_5 \Omega^- \chi^1 + \text{[illegible]} + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1$$

conformal algebra

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	$\Omega_- = \frac{1}{2} f^a \mathbb{J}_a + \frac{1}{2} g^a \mathbb{K}_a$
$[\mathbb{D}, S] = 0$	

(Ω_+ = the even-generator column)

- Identification with frames

$$f^a = \rho e^a , \qquad g^a = \sigma e^a ,$$

→ **Dirac kinetic term**

Unconventional SUSY

→ Conventional Dirac kinetic term?

$$\overline{\psi}\not{D}\psi$$

$$\langle \mathbb{F} \circledast \mathbb{F} \rangle$$
$$\cup$$

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q} D(\not{e}\psi)$$

$$\begin{aligned}
\hat{\mathcal{F}}^i \gamma_5 \mathcal{F}_i &= -\overline{\psi}^1 \overleftarrow{D}_\Omega \gamma_5 D_\Omega \chi^1 \\
&= \overline{\psi}^1 (-\overleftarrow{D}^+ + \Omega^-) \gamma_5 (D^+ + \Omega^-) \chi^1 \\
&= -\overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 D^+ \chi^1 - \overline{\psi}^1 \overleftarrow{D}^+ \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 D^+ \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1 \\
&= d[\overline{\psi}^1 \gamma_5 D^+ \chi^1] + \overline{\psi}^1 \gamma_5 (D^+)^2 \chi^1 + \overline{\psi}(\overleftarrow{D}\not{e} + \not{T}) \gamma_5 \Omega^- \chi^1 + \overline{\psi}^1 \Omega^- \gamma_5 (-\not{e} D + \not{T}) \psi + \overline{\psi}^1 \Omega^- \gamma_5 \Omega^- \chi^1
\end{aligned}$$

$$\left(\text{here } \overline{\psi}^1 = \overline{\psi}\not{e} \text{ and } \chi^1 = \not{e}\psi\right)$$

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Ω_+:

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	$\Omega_- = \frac{1}{2} f^a \mathbb{J}_a + \frac{1}{2} g^a \mathbb{K}_a$
$[\mathbb{D}, S] = 0$	

- Identification with frames

$$f^a = \rho e^a, \qquad g^a = \sigma e^a,$$

→ **Dirac kinetic term**

Unconventional SUSY

→ Conventional Dirac kinetic term? $\overline{\psi}\not{D}\psi$

$\langle \mathbb{F} \circledast \mathbb{F} \rangle$

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q}D(\not{e}\psi)$$

$\cup$

$$\begin{aligned}\hat{\mathcal{F}}^i\gamma_5\mathcal{F}_i =& -\overline{\psi}^1\overleftarrow{D}_\Omega\gamma_5 D_\Omega\chi^1 \\ =& \overline{\psi}^1(-\overleftarrow{D}^+ + \Omega^-)\gamma_5(D^+ + \Omega^-)\chi^1 \\ =& -\overline{\psi}^1\overleftarrow{D}^+\gamma_5 D^+\chi^1 - \overline{\psi}^1\overleftarrow{D}^+\gamma_5\Omega^-\chi^1 + \overline{\psi}^1\Omega^-\gamma_5 D^+\chi^1 + \overline{\psi}^1\Omega^-\gamma_5\Omega^-\chi^1 \\ =& d[\overline{\psi}^1\gamma_5 D^+\chi^1] + \overline{\psi}^1\gamma_5(D^+)^2\chi^1 + \overline{\psi}(\overleftarrow{D}\not{e} + \not{T})\gamma_5\Omega^-\chi^1 + \overline{\psi}^1\Omega^-\gamma_5(-\not{e}D + \not{T})\psi + \overline{\psi}^1\Omega^-\gamma_5\Omega^-\chi^1\end{aligned}$$

(here $\overline{\psi}^1 = \overline{\psi}\not{e}$ and $\chi^1 = \not{e}\psi$)

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	
$[\mathbb{D}, S] = 0$	$\Omega_- = \frac{1}{2}f^a\mathbb{J}_a + \frac{1}{2}g^a\mathbb{K}_a$

(Ω_+: the even generators)

- Identification with frames

$$f^a = \rho e^a, \qquad g^a = \sigma e^a,$$

→ Dirac kinetic term

Unconventional SUSY

→ Conventional Dirac kinetic term? $\overline{\psi}\not{D}\psi$

$\langle \mathbb{F} \circledast \mathbb{F} \rangle$

- Curvature

$$\mathbb{F} = d\mathbb{A} + \mathbb{A}^2 \supset \overline{Q}D(\not{\partial}\psi)$$

$\cup$

$$
\begin{aligned}
\hat{\mathcal{F}}^i\gamma_5\mathcal{F}_i =& -\overline{\psi}^1\overleftarrow{D}_\Omega\gamma_5 D_\Omega\chi^1 \\
=&\overline{\psi}^1(-\overleftarrow{D}^+ + \Omega^-)\gamma_5(D^+ + \Omega^-)\chi^1 \\
=& -\overline{\psi}^1\overleftarrow{D}^+\gamma_5 D^+\chi^1 - \overline{\psi}^1\overleftarrow{D}^+\gamma_5\Omega^-\chi^1 + \overline{\psi}^1\Omega^-\gamma_5 D^+\chi^1 + \overline{\psi}^1\Omega^-\gamma_5\Omega^-\chi^1 \\
=&d[\overline{\psi}^1\gamma_5 D^+\chi^1] + \overline{\psi}^1\gamma_5(D^+)^2\chi^1 + \overline{\psi}(\overleftarrow{D}\not{\partial} + \not{T})\gamma_5\Omega^-\chi^1 + \overline{\psi}^1\Omega^-\gamma_5(-\not{\partial}D + \not{T})\psi + \overline{\psi}^1\Omega^-\gamma_5\Omega^-\chi^1
\end{aligned}
$$

(here $\overline{\psi}^1 = \overline{\psi}\not{\partial}$ and $\chi^1 = \not{\partial}\psi$)

- Grading

$$\Omega = \Omega_+ + \Omega_-$$

Even generators	Odd generators
$[\mathbb{J}_{ab}, S] = 0$	$\{\mathbb{J}_a, S\} = 0$
$[\mathbb{Z}, S] = 0$	$\{\mathbb{K}_a, S\} = 0$
$[\mathbb{T}_I, S] = 0$	
$[\mathbb{D}, S] = 0$	

Ω_+ = { the even generators above }

$$\Omega_- = \frac{1}{2}f^a\mathbb{J}_a + \frac{1}{2}g^a\mathbb{K}_a$$

- Identification with frames

$$f^a = \rho e^a, \qquad g^a = \sigma e^a,$$

→ **Dirac kinetic term**

Identification with frames $f^a = \rho e^a\,, \qquad g^a = \sigma e^a\,,$

Townsend '77

f^a and g^a are nondynamical thanks to the S grading!

$$\begin{cases} \langle \mathbb{E} \circledast \mathbb{O} \rangle = 0 = \langle \mathbb{O} \circledast \mathbb{E} \rangle\,, \\ \langle \mathbb{E}_1 \circledast \mathbb{E}_2 \rangle = \langle \mathbb{E}_2 \circledast \mathbb{E}_1 \rangle\,, \\ \langle \mathbb{O}_1 \circledast \mathbb{O}_2 \rangle = -\langle \mathbb{O}_2 \circledast \mathbb{O}_1 \rangle\,. \end{cases}$$

Very transparent:
- Field equations and integrabillity conditions
- genuine gauge symmetries v/s on-shell symmetries
- Natural definition of self-dual condition

$$\circledast\,(\mathbb{F} - \mathbb{F}^-) = \pm(\mathbb{F} - \mathbb{F}^-)\,.$$

→ Field equations: $D_{\mathbb{A}} \circledast (\mathbb{F} - \mathbb{F}^{-}) = 0\,.$

→ Integrability condition: $[\mathbb{F}, \circledast(\mathbb{F} - \mathbb{F}^{-})] = 0\,.$

→ Symmetry invariance:

$$\delta(-\langle \mathbb{F} \circledast \mathbb{F}\rangle) = -2d\langle D_{\mathbb{A}} G \circledast (\mathbb{F} - \mathbb{F}^{-}) + G D_{\mathbb{A}} \circledast (\mathbb{F} - \mathbb{F}^{-})\rangle + 2\langle G[\mathbb{F}, \circledast(\mathbb{F} - \mathbb{F}^{-})]\rangle\,.$$

→ S-gradding odd generators and supercharges are on-shell sym:

$$\begin{aligned}
[\mathbb{F}, \circledast(\mathbb{F} - \mathbb{F}^{-})] =& (\mathcal{G}^a(\varepsilon_1 *)\mathcal{H} - \varepsilon_s \frac{1}{2}\epsilon^a{}_{bcd}\mathcal{F}^b\mathcal{F}^{cd} + \overline{\mathcal{X}}\gamma^a(-i\varepsilon_\psi\gamma_5)\mathcal{X})\mathbb{J}_a \\
&+ (\mathcal{F}^a(\varepsilon_1 *)\mathcal{H} - \varepsilon_s \frac{1}{2}\epsilon^a{}_{bcd}\mathcal{G}^b\mathcal{F}^{cd} - \overline{\mathcal{X}}\tilde{\gamma}^a(-i\varepsilon_\psi\gamma_5)\mathcal{X})\mathbb{K}_a \\
&+ [(\mathbb{F} - \mathbb{X}), \circledast\mathbb{X}] + [\mathbb{X}, \circledast\mathbb{F}^{+}]\,.
\end{aligned}$$

Grand Unified Theories [Georgi, Glasgow '74]

$(\nu_e, e^-)_L : (1, 2)$

$e_L^+ : (1, 1)$

$(u_\alpha, d_\alpha)_L : (3, 2)$

$u_L^{c\alpha} : (3^*, 1)$

$d_L^{c\alpha} : (3^*, 1)$

- Standard model: 15 left-handed fermions
- Can be accommodated in the SU(5) reps

The fundamental conjugate rep ψ^i $\quad 5^* = (3^*, 1) + (1, 2^*)$

$5^* : (\psi^i)_L = (d^{c1} d^{c2} d^{c3}\ e^- \ -\nu_e)_L$

The antisymmetric $5 \times 5 \psi_{ij} = -\psi_{ji}$ $\quad 10 = (3^*, 1) + (3, 2) + (1, 1).$

$5 : (\psi_i)_R = (d_1\ d_2\ d_3\ e^+ \ -\nu_e^c)_R$

$\varepsilon^{\alpha\beta\gamma}\psi_{\alpha\beta} \sim (3^*, 1)$ $\quad \varepsilon_{rs}\psi^{rs} \sim (1, 1)$

$l^a = (\nu, e)_L$ as a 2 under SU(2) $\quad l^b = \varepsilon^{ab} l_a$

$$10 : (\chi_{ij})_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & u^{c3} & -u^{c2} & u_1 & d_1 \\ -u^{c3} & 0 & u^{c1} & u_2 & d_2 \\ u^{c2} & -u^{c1} & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{bmatrix}_L$$

Conformal superalgebra for SU(2,2|N) GUT

Superconformal algebra for Unified theories
SU(2,2)×SU(N)×U(1) ⊂ SU(2,2|N)

conformal algebra ~ SU(2,2) × SU(N)

$$[\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b \,,$$
$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = -(\eta_{ac}\mathbb{J}_{bd} - \eta_{ad}\mathbb{J}_{bc} - \eta_{bc}\mathbb{J}_{ad} + \eta_{bd}\mathbb{J}_{ac})$$

$$[\mathbb{T}_I, \mathbb{T}_J] = f^{IJK}\mathbb{T}_K$$

$$[\mathbb{K}_a, \mathbb{K}_b] = -\mathbb{J}_{ab} \,.$$
$$[\mathbb{J}_a, \mathbb{K}_b] = s\eta_{ab}\mathbb{D} \,.$$
$$[\mathbb{K}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{K}_c - \eta_{ac}\mathbb{K}_b \,.$$
$$[\mathbb{D}, \mathbb{K}_a] = -s^{-1}\mathbb{J}_a \,.$$
$$[\mathbb{D}, \mathbb{J}_a] = -s\mathbb{K}_a \,.$$

$$W^{mn} = \left[\begin{array}{c|c} \omega^{ab} & e^a \\ \hline e^a & 0 \end{array}\right]$$

Superconformal algebra for Unified theories

$SU(2,2)\times SU(N)\times U(1) \subset SU(2,2|N)$

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{i}{2}\overline{\mathbb{Q}}^j_\alpha(\lambda_I)_j{}^i\,, \quad [\mathbb{T}_I, \mathbb{Q}^\alpha_i] = \tfrac{i}{2}(\lambda_I)_i{}^j\mathbb{Q}^\alpha_j\,,$$

$$[\mathbb{Z}, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{iz}{3}\overline{\mathbb{Q}}^i_\alpha\,, \quad [\mathbb{Z}, \mathbb{Q}^\alpha_i] = \tfrac{iz}{3}\mathbb{Q}^\alpha_i\,,$$

$$\{\mathbb{Q}^\alpha_i, \overline{\mathbb{Q}}^j_\beta\} = \left(\tfrac{1}{2s}(\gamma^a)^\alpha{}_\beta\mathbb{J}_a - \tfrac{1}{2}(\Sigma^{ab})^\alpha{}_\beta\mathbb{J}_{ab} - \tfrac{1}{2}(\tilde\gamma^a)^\alpha{}_\beta\mathbb{K}_a + \tfrac{1}{2}(\gamma^5)^\alpha{}_\beta\mathbb{D}\right)\delta^j_i + \delta^\alpha_\beta\left(-i(\lambda_I)_i{}^j\mathbb{T}_I - \tfrac{i}{4z}\delta^j_i\mathbb{Z}\right)$$

$$[\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b\,,$$

$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = -(\eta_{ac}\mathbb{J}_{bd} - \eta_{ad}\mathbb{J}_{bc} - \eta_{bc}\mathbb{J}_{ad} + \eta_{bd}\mathbb{J}_{ac})$$

$$[\mathbb{T}_I, \mathbb{T}_J] = f^{IJK}\mathbb{T}_K$$

$$[\mathbb{K}_a, \mathbb{K}_b] = -\mathbb{J}_{ab}\,.$$

$$[\mathbb{J}_a, \mathbb{K}_b] = s\eta_{ab}\mathbb{D}\,.$$

$$[\mathbb{K}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{K}_c - \eta_{ac}\mathbb{K}_b\,.$$

$$[\mathbb{D}, \mathbb{K}_a] = -s^{-1}\mathbb{J}_a\,.$$

$$[\mathbb{D}, \mathbb{J}_a] = -s\mathbb{K}_a\,.$$

susy requires to include a central charge $SU(2,2)\times SU(N)\times U(1)$

$$[\mathbb{K}_a, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\tilde\gamma_a)^\beta{}_\alpha\,, \quad [\mathbb{K}_a, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\tilde\gamma_a)^\alpha{}_\beta\mathbb{Q}^\beta_i\,,$$

$$[\mathbb{D}, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\gamma_5)^\beta{}_\alpha\,, \quad [\mathbb{D}, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\gamma_5)^\alpha{}_\beta\mathbb{Q}^\beta_i\,,$$

Superconformal algebra for Unified theories
SU(2,2)×SU(N)×U(1) ⊂ SU(2,2|N)

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{i}{2}\overline{\mathbb{Q}}^j_\alpha (\lambda_I)_j{}^i\,, \quad [\mathbb{T}_I, \mathbb{Q}^\alpha_i] = \tfrac{i}{2}(\lambda_I)_i{}^j \mathbb{Q}^\alpha_j\,,$$

$$[\mathbb{Z}, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{iz}{3}\overline{\mathbb{Q}}^i_\alpha\,, \quad [\mathbb{Z}, \mathbb{Q}^\alpha_i] = \tfrac{iz}{3}\mathbb{Q}^\alpha_i\,,$$

$$\{\mathbb{Q}^\alpha_i, \overline{\mathbb{Q}}^j_\beta\} = \left(\tfrac{1}{2s}(\gamma^a)^\alpha{}_\beta \mathbb{J}_a - \tfrac{1}{2}(\Sigma^{ab})^\alpha{}_\beta \mathbb{J}_{ab} - \tfrac{1}{2}(\tilde{\gamma}^a)^\alpha{}_\beta \mathbb{K}_a + \tfrac{1}{2}(\gamma^5)^\alpha{}_\beta \mathbb{D}\right)\delta^j_i + \delta^\alpha_\beta\left(-i(\lambda_I)_i{}^j \mathbb{T}_I - \tfrac{i}{4z}\delta^j_i \mathbb{Z}\right)$$

Spinors carry a rep. of the bosonic algebra
SU(2,2)×SU(N)×U(1)

$$[\mathbb{J}_a, \mathbb{J}_{bc}] = \eta_{ab}\mathbb{J}_c - \eta_{ac}\mathbb{J}_b\,, \quad [\mathbb{T}_I, \mathbb{T}_J] = f_{IJ}{}^K \mathbb{T}_K$$

$$[\mathbb{J}_{ab}, \mathbb{J}_{cd}] = -(\eta_{ac}\mathbb{J}_{bd} - \eta_{ad}\mathbb{J}_{bc} - \eta_{bc}\mathbb{J}_{ad} + \eta_{bd}\mathbb{J}_{ac})$$

$$[\mathbb{K}_a, \mathbb{K}_b] = -\mathbb{J}_{ab}\,, \quad [\mathbb{D}, \mathbb{J}_a] = -s\mathbb{K}_a\,.$$

$$A^{AB} = \left[\begin{array}{c|c} W^S & \psi \\ \hline \overline{\psi} & A_I \end{array}\right]$$

$$[\mathbb{J}_a, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{s}{2}\overline{\mathbb{Q}}^i_\beta(\gamma_a)^\beta{}_\alpha\,, \quad [\mathbb{J}_a, \mathbb{Q}^\alpha_i] = -\tfrac{s}{2}(\gamma_a)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{J}_{ab}, \overline{\mathbb{Q}}^i_\alpha] = \overline{\mathbb{Q}}^i_\beta(\Sigma_{ab})^\beta{}_\alpha\,, \quad [\mathbb{J}_{ab}, \mathbb{Q}^\alpha_i] = -(\Sigma_{ab})^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{K}_a, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\tilde{\gamma}_a)^\beta{}_\alpha\,, \quad [\mathbb{K}_a, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\tilde{\gamma}_a)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

$$[\mathbb{D}, \overline{\mathbb{Q}}^i_\alpha] = \tfrac{1}{2}\overline{\mathbb{Q}}^i_\beta(\gamma_5)^\beta{}_\alpha\,, \quad [\mathbb{D}, \mathbb{Q}^\alpha_i] = -\tfrac{1}{2}(\gamma_5)^\alpha{}_\beta \mathbb{Q}^\beta_i\,,$$

Superconformal algebra for Unified theories
SU(2,2)×SU(5)×U(1) ⊂ SU(2,2|5)

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^i_\alpha] = -\tfrac{i}{2}\overline{\mathbb{Q}}^j_\alpha(\lambda_I)_j{}^i\,, \quad [\mathbb{T}_I, \mathbb{Q}^\alpha_i] = \tfrac{i}{2}(\lambda_I)_i{}^j\mathbb{Q}^\alpha_j\,,$$

GG model →

$$\mathbf{5^*}: (\psi^i)_L = (d^{c1} d^{c2} d^{c3}\, e^- \; - \nu_e)_L$$

wanted: Q_{ij} →

$$\Psi = \overline{Q}^{ij}\chi_{ij} \subset \mathbb{A}$$

$$[\mathbb{T}_I, \mathbb{Q}^\alpha_{ij}] = i(t_I)_{ij}{}^{kl}\mathbb{Q}^\alpha_{kl}\,,$$

$$[\mathbb{T}_I, \overline{\mathbb{Q}}^{ij}_\alpha] = -i\overline{\mathbb{Q}}^{kl}_\alpha(t_I)_{kl}{}^{ij}\,,$$

GG model →

$$\mathbf{10}: (\chi_{ij})_L = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & u^{c3} & -u^{c2} & u_1 & d_1 \\ -u^{c3} & 0 & u^{c1} & u_2 & d_2 \\ u^{c2} & -u^{c1} & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{bmatrix}_L$$

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New bosons w.r.t. GG model

$$A_X = \{A_I, A_{\tilde{X}}\}$$

$$\nabla'\chi_L^{\text{phys}} = \nabla_{su(5)}\chi_L^{\text{phys}} - ig A^{\tilde{X}} t_{\tilde{X}}\chi_L^{\text{phys}} - ig_{(U(1))}^{(\text{rank } 2)} A\chi_L^{\text{phys}},$$

$SU(2,2)\times SU(10)\times U(1)$

- Group theory decomposition

$$\mathbf{99} = (\mathbf{24},\mathbf{1},0) + (\mathbf{1},\mathbf{24},0) + (\mathbf{1},\mathbf{1},0) + (\mathbf{5},\mathbf{5}^*,-y_{\text{new}}) + (\mathbf{5}^*,\mathbf{5},y_{\text{new}})$$

Charge assignation

5^*

$$(\psi_i)_L = \begin{pmatrix} d_1^c \\ d_1^c \\ d_1^c \\ e^- \\ -\nu_e \end{pmatrix}_L$$

10

$$(\chi_{ij})_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix}_L$$

$$\Psi(x) = \overline{Q}\psi(x)$$

Commutators in the superalgebra!

$$[Q_{elec}, \Psi(x)] = q_{elec}\Psi(x) \qquad [Y, \Psi(x)] = y\Psi(x)$$

GUT model action

$$\mathcal{S} = -\int \left(\langle \xi \mathbb{F} \circledast \mathbb{F} \rangle + \langle \xi' \mathbb{F}' \circledast \mathbb{F}' \rangle \right)$$

$$\circledast \mathbb{F} = (\varepsilon_s S) \left(\frac{1}{2} \mathcal{F}^{ab} \mathbb{J}_{ab} + \mathcal{F}^a \mathbb{J}_a + \mathcal{G}^a \mathbb{K}_a \right) + (\varepsilon_1 *) \mathcal{H} \mathbb{D} + (\varepsilon_2 *) \mathcal{F}^I \mathbb{T}_I + (\varepsilon_3 *) \mathcal{F} \mathbb{Z} + \overline{\mathbb{Q}} (-i \varepsilon_\psi \gamma_5) \mathcal{X} + \overline{\mathcal{X}} (-i \varepsilon_\psi \gamma_5) \mathbb{Q} \,.$$

$$\varepsilon_s = +1 = \varepsilon_1 = \varepsilon_2 = -\varepsilon_3$$

Explicit computation gives:

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} \varepsilon_s (\xi + \xi') \epsilon_{abcd} \mathcal{F}^{ab} \mathcal{F}^{cd} - \varepsilon_1 (\xi + \xi') \mathcal{H} * \mathcal{H} \\ & - \frac{1}{2} \varepsilon_2 \left(\xi \mathcal{F}^I * \mathcal{F}^I + \xi' (n-2) \mathcal{F}'^X * \mathcal{F}'^X \right) \\ & - 4 \varepsilon_3 \left[\xi (4/n - 1) + \xi' (4/d_n - 1) \right] \mathcal{F} * \mathcal{F} \,, \\ & - 2 i \varepsilon_\psi \overline{\mathcal{X}} \gamma_5 \mathcal{X} - \frac{i}{2} \varepsilon_\chi \overline{\mathcal{Y}} \gamma_5 \mathcal{Y} \,. \end{aligned}$$

Dirac terms

$$-2i\varepsilon_\psi \overline{\mathcal{X}}\gamma_5\mathcal{X} - \frac{i}{2}\varepsilon_\chi \overline{\mathcal{Y}}\gamma_5\mathcal{Y}\,.$$

- Action

$$\mathcal{L} \supset \mathscr{L}_{\mathrm{f}} = \mathrm{i}(\bar{\psi}^{\mathrm{c}}_{\mathrm{R}})_a(\not{D}\psi^{\mathrm{c}}_{\mathrm{R}})^a + \mathrm{i}(\bar{\psi}_{\mathrm{L}})_{ac}(\not{D}\psi_{\mathrm{L}})^{ac}$$

$$\not{\nabla}\psi_L^{\mathrm{phys}} = \not{\nabla}_{su(5)}\psi_L^{\mathrm{phys}} - ig^{(\mathrm{rank}\ 1)}_{(U(1))}\not{A}\psi_L^{\mathrm{phys}}\,,$$

$$\not{\nabla}'\chi_L^{\mathrm{phys}} = \not{\nabla}_{su(5)}\chi_L^{\mathrm{phys}} - ig\not{A}^{\tilde{X}}t_{\tilde{X}}\chi_L^{\mathrm{phys}} - ig^{(\mathrm{rank}\ 2)}_{(U(1))}\not{A}\chi_L^{\mathrm{phys}}\,,$$

New w.r.t. the GG model

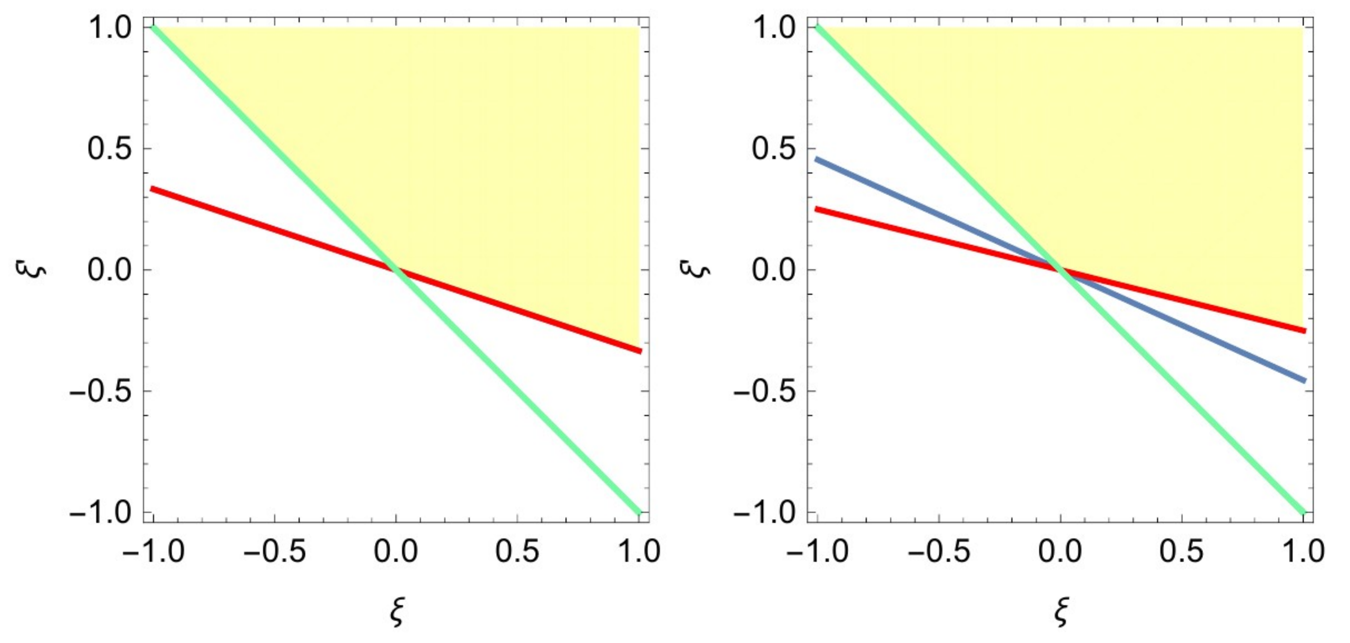
Parameters

- Bosonic part of the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{b}} = & \frac{1}{4}\varepsilon_s(\xi+\xi')\epsilon_{abcd}\mathcal{R}^{ab}\mathcal{R}^{cd} - \varepsilon_1(\xi+\xi')H * H \\
& - \frac{1}{2}\varepsilon_2\left(\xi+\xi'(n-2)\right)F^I * F^I \\
& - 4\varepsilon_3\left[\xi(4/n-1)+\xi'(4/d_n-1)\right]F * F \\
& - \frac{(n-2)}{2}\varepsilon_2\xi'\left[2F^I * F_1^I + F_1^I * F_1^I + F^{\widetilde{X}} * F^{\widetilde{X}}\right],
\end{aligned}
$$

- No ghost conditions

$$
\begin{aligned}
& \xi+\xi' > 0\,, \\
& \xi(4/n-1)+\xi'(4/d_n-1) < 0\,, \\
& \xi+\xi'(n-2) > 0\,.
\end{aligned}
$$



We overcome technical difficulties encountered by Ferrara, Kaku, Townsend, van Nieuwenhuizen in the late 70s

Gauge coupling constants

- Canonical normalization of the fields

$$-aF*F=-\frac{1}{2}F'*F',$$

$$D=d-ig_0\,\rho(T_r)A^r,$$

$$A'=\sqrt{2a}A$$

$$g_{(SU(n))}=g_{(SU(d_n))}=\frac{1}{\sqrt{\xi+\xi'(n-2)}},$$

$$g^{(\text{rank }1)}_{(U(1))}=\frac{4/n-1}{\sqrt{-8(\xi(4/n-1)+\xi'(4/d_n-1))}},$$

$$g^{(\text{rank }2)}_{(U(1))}=\frac{4/d_n-1}{\sqrt{-8(\xi(4/n-1)+\xi'(4/d_n-1))}}.$$

Summary of the model

- Symmetry group

$$SU(2,2|5)_{\text{diag}} = [SU(2,2|5) \times SU(2,2|10)]_{\text{diag}}$$

- All fields in the adjoint rep.

$$\mathbb{A} = \Omega + \overline{\mathbb{Q}}^i \not{e} \psi_i + \overline{\psi}^i \not{e} \mathbb{Q}_i \,,$$

$$\mathbb{A}' = \Omega' + \frac{1}{2} \overline{\mathbb{Q}}^{ij} \not{e} \chi_{ij} + \frac{1}{2} \overline{\chi}^{ij} \not{e} \mathbb{Q}_{ij} \,,$$

$$\Omega = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + f^a\mathbb{J}_a + g^a\mathbb{K}_a + h\mathbb{D} + A^I\mathbb{T}_I + A\mathbb{Z} \,,$$

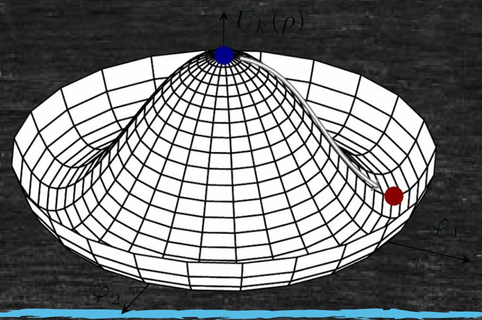
$$\Omega' = \frac{1}{2}\omega'^{ab}\mathbb{J}_{ab} + f'^a\mathbb{J}_a + g'^a\mathbb{K}_a + h'\mathbb{D} + A'^X\mathbb{T}_X + A'\mathbb{Z} \,.$$

- Diagonal symmetry group →

$$\omega'^{ab} = \omega^{ab} \,, \quad f'^a = f^a \,, \quad g'^a = g^a \,, \quad h' = h \,, \quad A' = A \,, \quad A'^X\big|_{X=I} = A^I \,.$$

→ Highly predictive
→ Embedding of SU(5) GG model + new gauge fields
→ Chiral theory from a L-R handed symmetric theory

Outlook



- Pheno. SSB: SU(10) → SU(5) → SU(3) × SU(2) × U(1) → SU(2) × U(1)

$$\mathbf{99} = (\mathbf{24},\mathbf{1},0) + (\mathbf{1},\mathbf{24},0) + (\mathbf{1},\mathbf{1},0) + (\mathbf{5},\mathbf{5}^*,-y_{\text{new}}) + (\mathbf{5}^*,\mathbf{5},y_{\text{new}})$$

- Embedding of other GUT schemes that are phenomenologically more successful, anomaly free? Pati-Salam SO(10)?
- Model with gravitini: full theory and study of the on-shell symmetries (horizontal symmetries)
- USUSY non-renormalization theorems? Nieh-Yang-Weyl symmetry anomaly?(trace anomaly)
- Cosmology USUSY: H0 problem?

$$H^2 = H_0^2(\Omega_m(1+z)^{-3} + \Omega_r(1+z)^{-4} + \Omega_\Lambda) + \left(\frac{\dot{w}_0}{w_0}\right)^2 (1+z)^{-6}$$

References

Thank you

GUT: This work

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