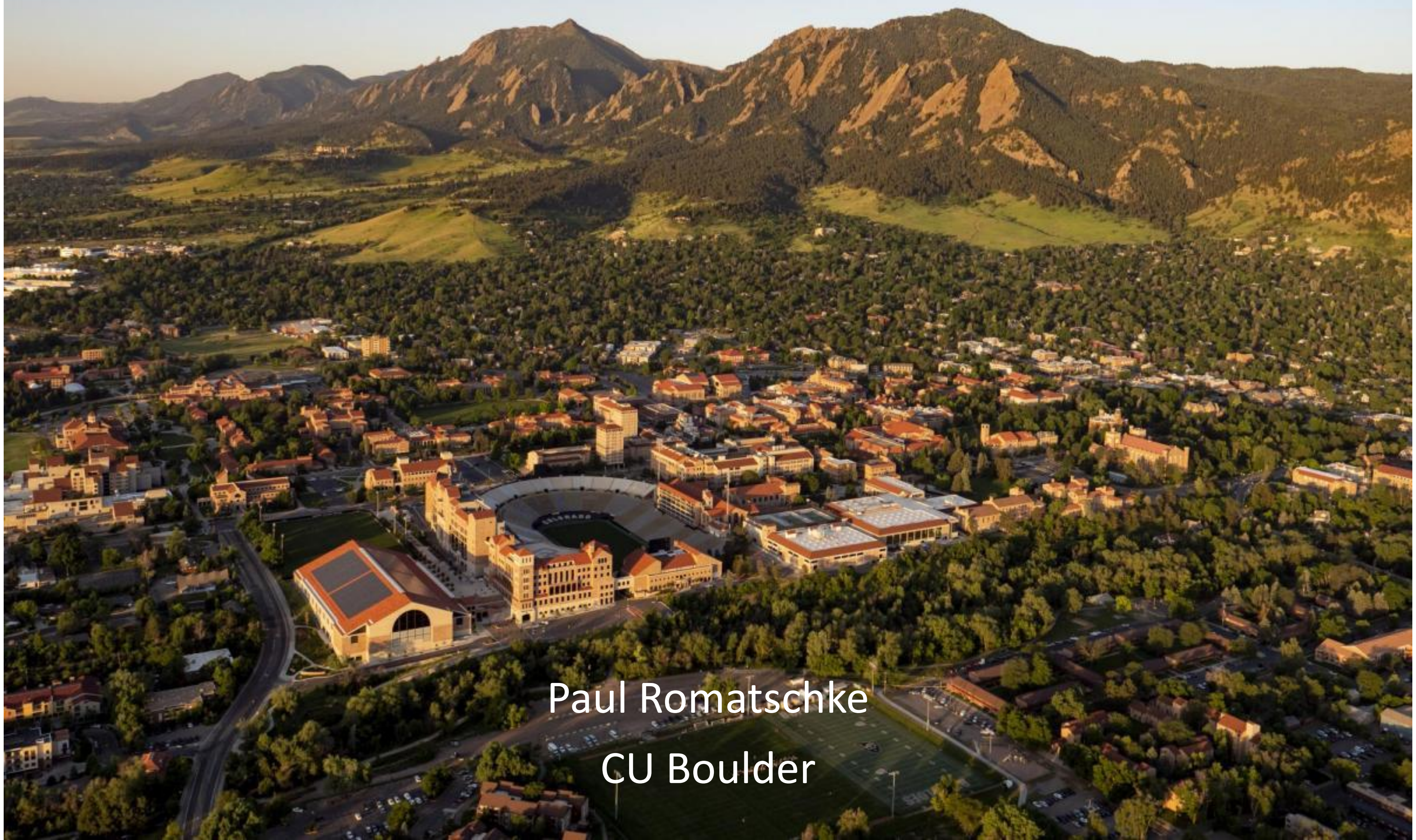
Hydrodynamics

applied to small collision systems



Paul Romatschke
CU Boulder

Three Parts

- Hydro Theory
- Initial Stage Correlations
- Hydro Models and Data

Reminder: what goes into theory calcs

Initial Conditions (e.g. Glauber, AdS/CFT, Glasma)	Model
Pre-Hydrodynamic Evolution	Model
Hydrodynamic Evolution	Theory
Decoupling (Particlization/Cooper-Frye)	Model
Hadronic Evolution	Model
Analysis	Theory

Part I: Hydro Theory

Reminder: Hydro as an Effective Field Theory

- Conservation law for energy-momentum tensor: $\nabla_\mu T^{\mu\nu} = 0$
- Effective description for low wavenumbers/momenta: $T^{\mu\nu}$ must depend on ε, u^μ and source $g_{\mu\nu}$
- Expand in gradients:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \dots$$

Conservation equation $\nabla_\mu T^{\mu\nu} = 0$ gives Euler, Navier-Stokes, BRSSS, etc

Form of equation universal, coefficients (c_s, viscosities, ...) depend on microphysics

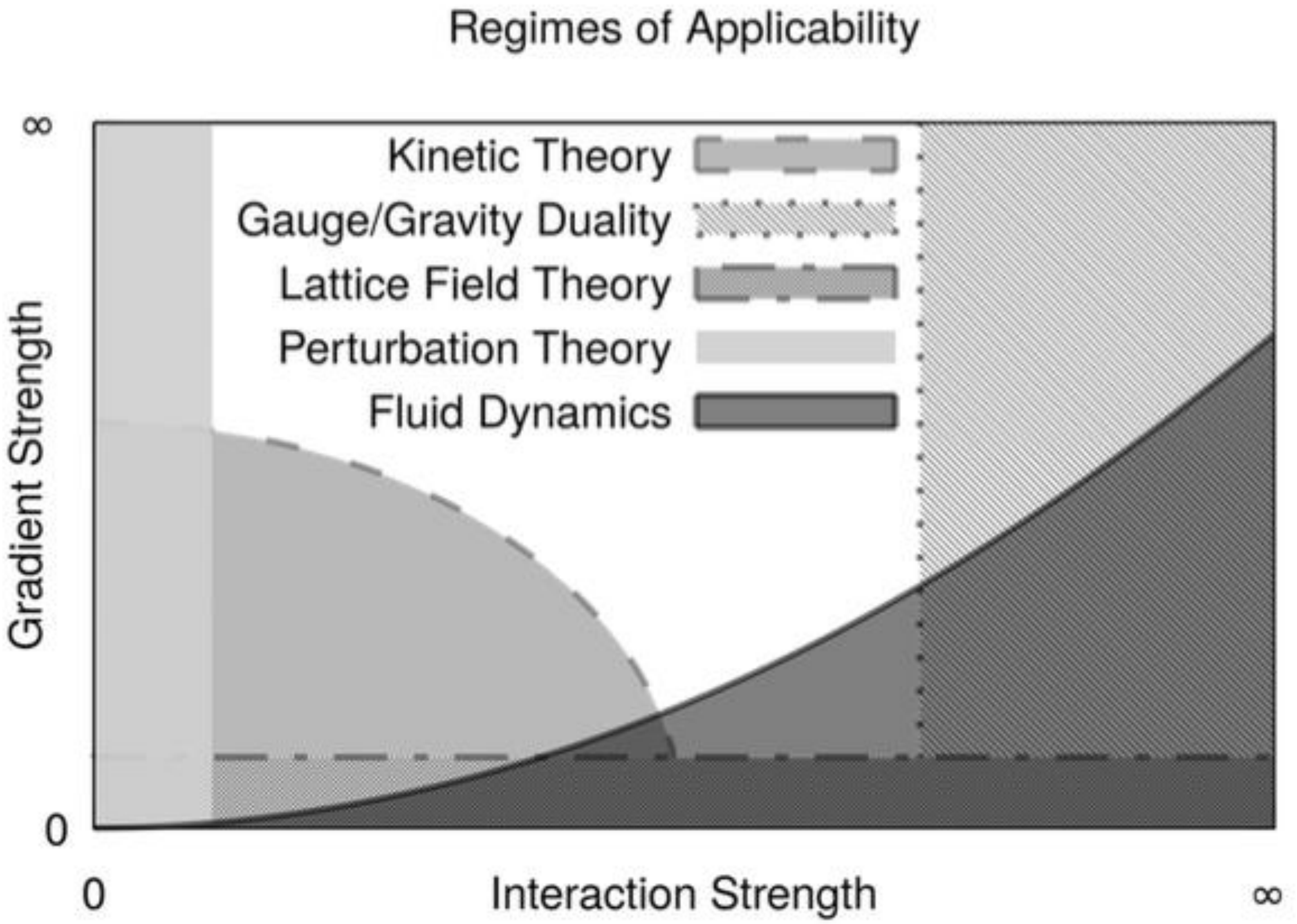
Fluid Dynamics as an Effective Field Theory

Fluid dynamics is ***NOT*** just a particular limit of kinetic theory

Fluid dynamics is ***NOT*** just a particular limit of AdS/CFT

Fluid dynamics is an effective theory which can apply in situations when kinetic theory and/or AdS/CFT do not

Fluid Dynamics as an Effective Field Theory



PR & UR, “Relativistic Fluid Dynamics in and out of Equilibrium”, Cambridge University Press, August 2019; 1712.05815

Off-Equilibrium Fluid Dynamics

EFT Description as a gradient series

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \dots$$

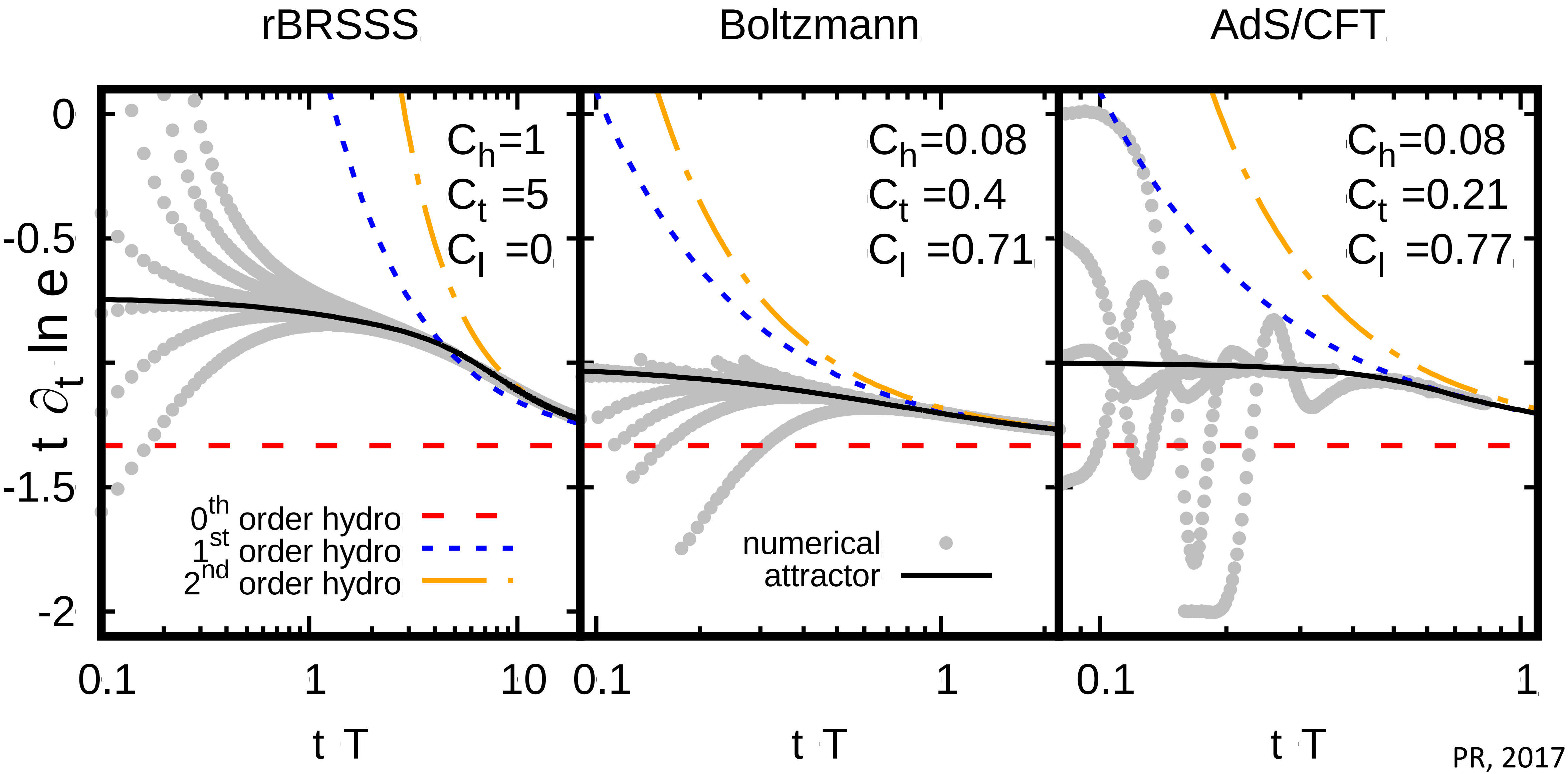
Unfortunately, this series likely is divergent

Fortunately, it seems that the divergent series is Borel-summable

This suggests non-perturbative existence of fluid dynamics for ***LARGE*** gradients: hydrodynamic attractors

Heller & Spalinski, 2015

Borel-summation : Attractor Solutions for large Gradients



PR, 2017

Off-Equilibrium Fluid Dynamics

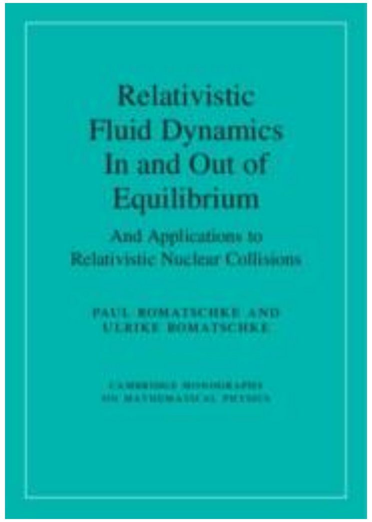
- Textbook criterion for applicability of fluid dynamics

mean free path << system size

OR

mean free path * gradient << 1

- New (attractor) criterion: fluid dynamics applies as long as fluid modes are dominant (longest lived) excitations
- Competition of excitations: hydro modes versus other “non-hydrodynamic” modes
- Will be covered by textbooks (soon)

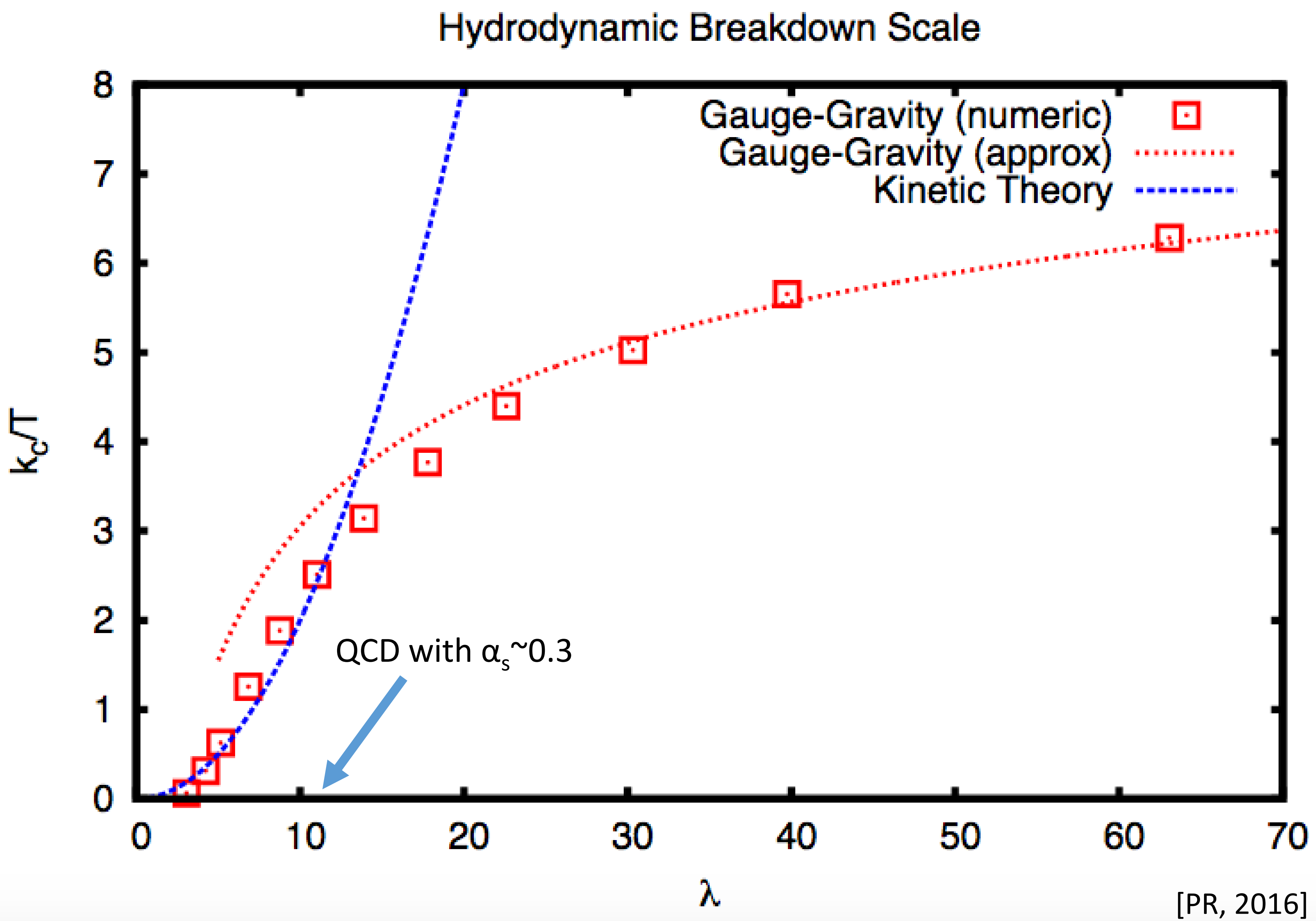


Applicability of Hydrodynamics

Hydro behavior when hydro modes dominate over non-hydro modes:

- Can calculate in kinetic theory (KGB approx)
- Can calculate in AdS/CFT
- [Can NOT calculate in QCD (yet?)]

➔ Get Hydro Breakdown scale k_c as a function of coupling


Hydrodynamic Breakdown Scale
Gauge-Gravity (numeric)
Gauge-Gravity (approx)
Kinetic Theory
k_c/T
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50
60
70
λ
QCD with α_s~0.3

[PR, 2016]

Applicability of Hydrodynamics

- I don't know what k_c is for QCD, but k_c~3.5 T seems OK?
- Hydro breaks down for momenta k>3.5 T
- Hydro breaks down for system sizes L<1/k~0.29/T
- Hydro breaks down for times t<1/k~0.29/T

But T changes as a function of t;

Educated Guess: $T(t)=(t0/t)^{1/3}$

Applicability of Hydrodynamics

Educated Guess: $T(t)=(t0/t)^{1/3}$

Combine with hydro start time

$t>0.29/T$

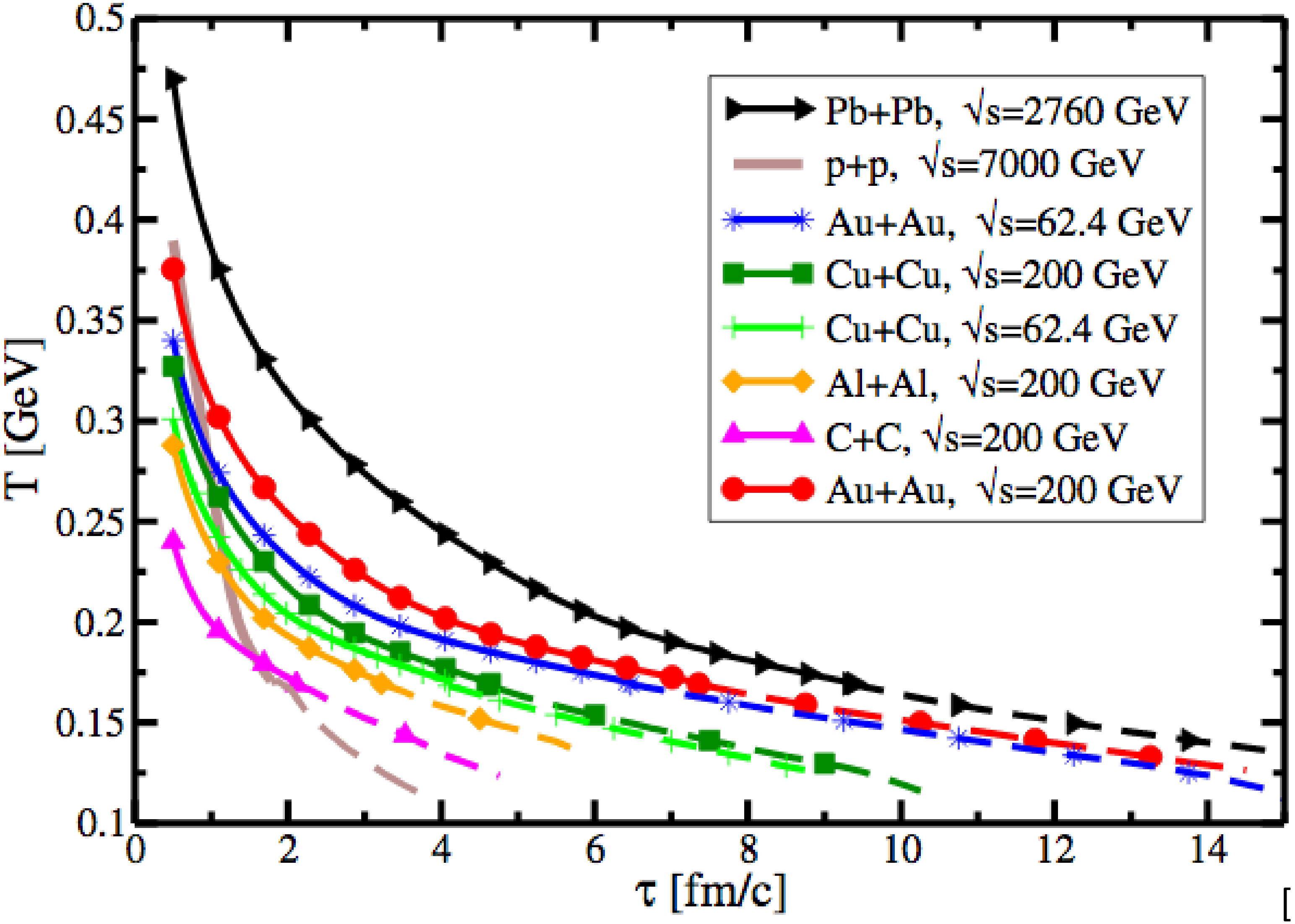
leads to self-consistency condition for hydro start time:

$t>0.29\ (t/t0)^{1/3}$ or

$$\mathbf{t>0.29^{1.5}\ t0^{-0.5}}$$

Need one input number to fix t0

Temperature evolution for heavy-ion collisions



For PbPb:
T(t=1 fm)~0.4 GeV

so t0~0.32 GeV^2

[Habich et al., 1409.0040]

Applicability of Hydrodynamics: PbPb

- t0~0.32 GeV^2
- Hydro breaks down for times t_S<0.05 fm (and $T(t_S)$~1.06 GeV)
- Hydro breaks down for momenta k>3.5x1.06 GeV ~3.7 GeV
- Hydro breaks down for system sizes L<1/k<0.05 fm

Extremely early times!
(Hydro starts early, not late)

Tide Tables

6 EASTPORT.—HIGH WATER, 1867.

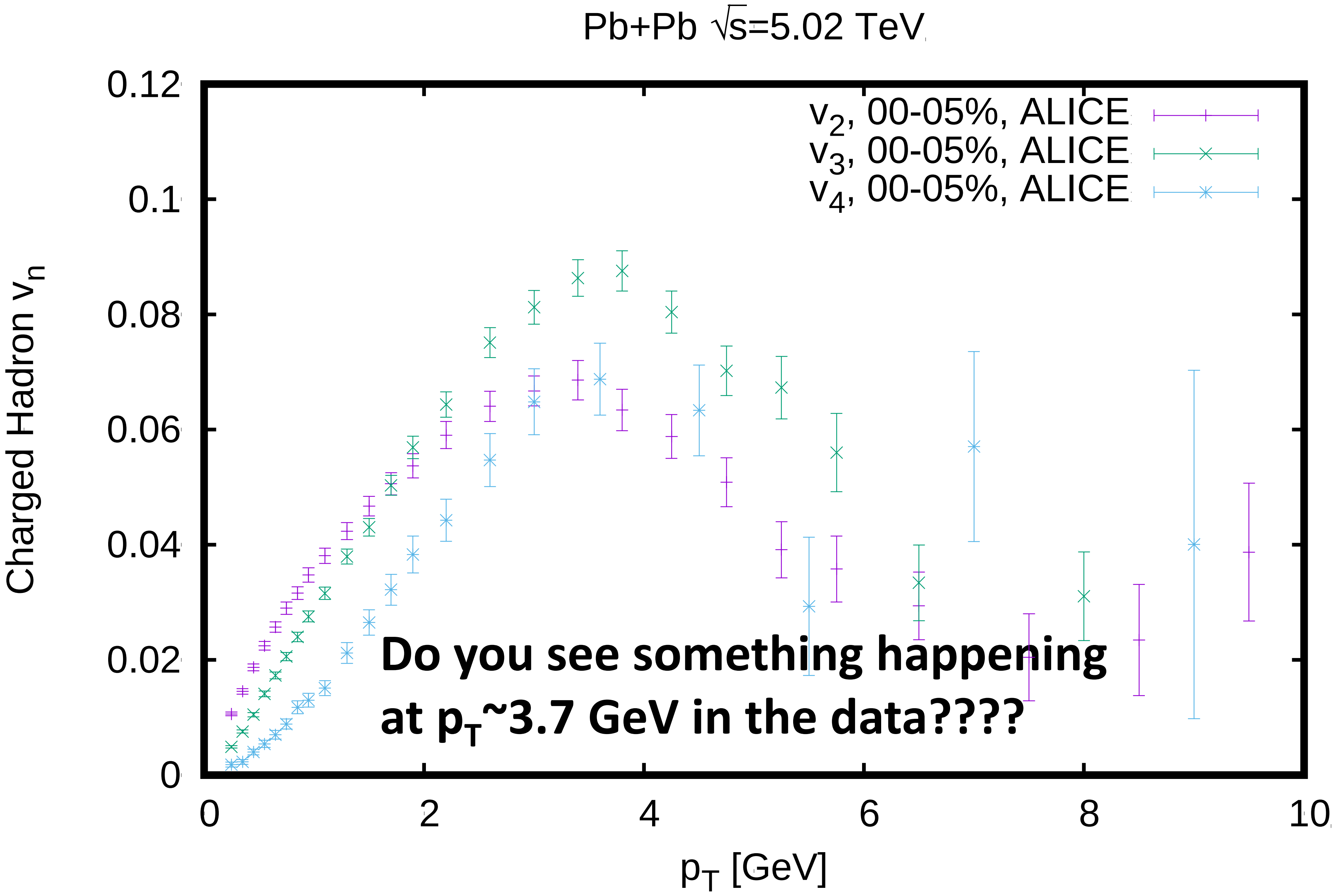
Day of the month.	JANUARY.				FEBRUARY.			
	A. M.		P. M.		A. M.		P. M.	
	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.
	h. m.	*Feet.*	*h. m.*	*Feet.*	*h. m.*	*Feet.*	*h. m.*	*Feet.*
1	. .	. .	8 09	17·4	8 50	17·8	9 14	18·1
2	8 34	17·6	8 58	17·9	9 39	18·3	10 01	18·6
3	9 21	18·2	9 43	18·4	10 23	18·9	10 43	19·2
4	10 06	18·7	10 27	19·0	11 04	19·4	11 24	19·4
5	10 48	19·2	11 08	19·4	11 43	19·4	. .	. .
6	11 28	19·4	11 47	19·4	0 04	19·4	0 24	19·3
7	. .	. .	0 08	19·4	0 44	19·2	1 05	19·1
8	0 28	19·3	0 50	19·2	1 26	18·9	1 47	18·8
9	1 09	19·1	1 29	18·9	2 08	18·5	2 31	18·3

Ion Collision Tide Tables - Incomplete

Collision System	Pb+Pb				Au+Au			
	2.76 TeV		5.02 TeV		130 GeV		200 GeV	
	Time.	Length	Time.	Length	Time.	Length	Time.	Length
	fm/c	fm/c	fm/c	fm/c	fm/c	fm/c	fm/c	fm/c
			0.05				0.09	

Applicability of Hydrodynamics: PbPb

- t0~0.32 GeV^2
- Hydro breaks down for times t_S<0.05 fm (and $T(t_S)$~1.06 GeV)
- **Hydro breaks down for momenta k>3.5x1.06 GeV ~3.7 GeV**
- Hydro breaks down for system sizes L<1/k<0.05 fm


Pb+Pb √s=5.02 TeV
0.12
0.1
0.08
0.06
0.04
0.02
0
0
2
4
6
8
10
Charged Hadron v_n
p_T [GeV]
v_2, 00-05%, ALICE
v_3, 00-05%, ALICE
v_4, 00-05%, ALICE
Do you see something happening
at p_T~3.7 GeV in the data????

[PR, 2016]

Hydro Prediction

Turn-over point in v_2,v_3,v_4 etc should decrease in momentum as collision energy is lowered for heavy-ion collisions

Prediction:

For AuAu @ $s^{0.5}$=200 GeV, turn-over point should occur at around

p_T~2.7 GeV

Hydro EFT take-home messages

- Hydro applies OUT OF EQUILIBRIUM (and we know why)
- Low momentum modes ALWAYS behave hydrodynamically
- For momenta larger than k_c~3.5 T, hydro breaks down (we can see this in experimental heavy-ion data)
- For system sizes/time-scales smaller than 1/(3.5 T) hydro breaks down
- For HICs, self-consistent estimate leads to

$$t_{hydro}>0.05 \text{ fm}, L_{hydro}>0.05 \text{ fm}$$

(Consistent with my 2016 result L>0.15 fm @ T=0.2 GeV after time evolution and expansion)

Discussion on Part I: Hydro Theory

Part II: Initial Stage Correlations

Simulation of small collision systems: inputs

Hydro models

- Initial Geometry
- Pre-Hydrodynamic Flow
- Hydrodynamic Evolution
- Hadronization/Hadron Gas Dynamics
- Analysis

Initial State Models

- Initial Geometry
- Pre-Hydrodynamic Flow
- Analysis

Simulation of small collision systems: inputs

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These steps are essentially the same

Simulation of small collision systems: inputs

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Initial State Models

- Initial Geometry
- Pre-Hydrodynamic Flow
- Analysis

These steps are essentially the same...

...but initial state models ***assume*** hydro/HRG stages are absent!

Q: Is that assumption OK?

Initial Stages

Can't calculate pre-hydro stage for QCD

CAN calculate pre-hydro evolution
ASSUMING EITHER

Weak coupling (Classical YM):

- Conformal, $g=0$, $N=3$
- "IP-Glasma"
- Gives rise to energy deposition/pre-hydro flow

Strong Coupling (AdS/CFT)

- Conformal, $\lambda \to \infty$, $N \to \infty$
- "AdS/CFT"
- Gives rise to energy deposition/pre-hydro flow

Initial Stages: weak coupling (IP-Glasma)

- Recall: working in classical limit g->0
- In this limit approximately

$$\langle T^{\tau\tau}\rangle_{\rm cf} \propto g^2 T_{A_1}(\mathbf{x}_\perp) T_{A_2}(\mathbf{x}_\perp + \mathbf{b}_\perp) + \mathcal{O}(\tau^2)\,.$$

and

$$\langle T^{\tau\perp}\rangle_{\rm cf} = -\frac{\tau}{2}\frac{\partial}{\partial \mathbf{x}_\perp}\langle T^{\tau\tau}\rangle_{\rm cf} + \mathcal{O}(\tau^3)$$

"IP-Jazma"
cf. Jamie's talk

for early times (1712.05815, cf. Chen et al, 1507.03524, earlier work by Lappi)

Just like Glauber binary collision scaling. There is initial stage flow (long range correlation).

Initial Stages: AdS/CFT

- Recall: working in the large ‘t Hooft coupling, large N limit
- Can show analytically that

$$T^{\tau\tau} \propto T_{A_1}(\mathbf{x}_\perp) T_{A_2}(\mathbf{x}_\perp + \mathbf{b}_\perp)\tau^2 + \mathcal{O}(\tau^4)\,,$$
$$T^{\tau\perp} = -\frac{\tau}{2}\frac{\partial}{\partial \mathbf{x}_\perp} T^{\tau\tau} + \mathcal{O}(\tau^5)\,,$$

for early times (1712.05815, cf. PR & Hogg 1301.2635)

Just like Glauber binary collision scaling. There is initial stage flow (long range correlation).

Initial Stages

- There are initial stage velocity correlations for weak coupling
- There are initial stage velocity correlations for strong coupling
- Correlations are “universal”

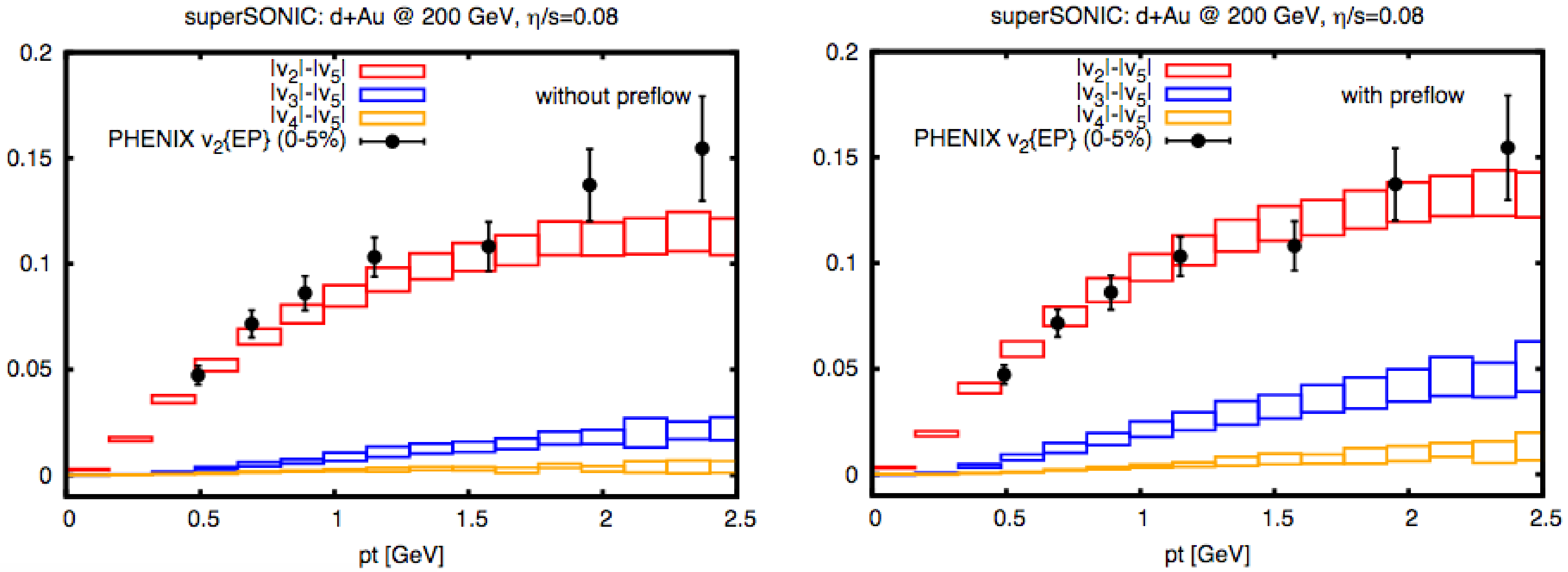
cf. Vredevoogd & Pratt 2008

Long Range Correlations from the Initial Stage are real

Initial Stage Long Range Correlations are *included* in some hydro models (e.g. SONIC)

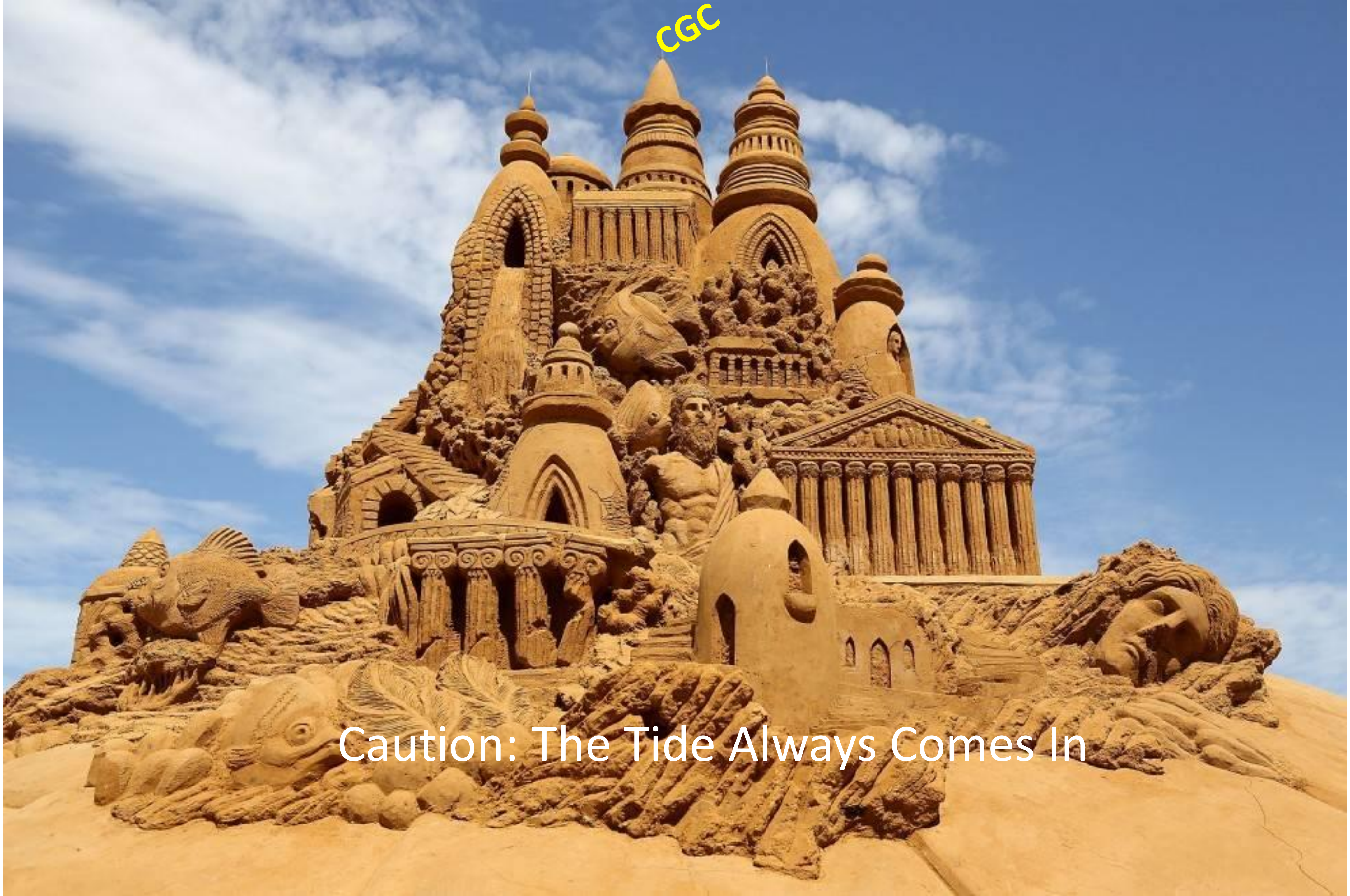
Do Initial Stage Correlations matter in practice?

How relevant are Initial Stage Correlations?



There is some effect. It is minor for v_2 in d+Au.

1502.04745


CGC
Caution: The Tide Always Comes In

Initial Stages: Take Home Message

- Initial stage correlations are real
- Initial stage correlations are included in hydro models (SONIC)
- Small effect for some observables (v_2 in d+Au)
- Larger effects for other observables/collision systems:

Light-Heavy Ion Collisions: A window into pre-equilibrium QCD dynamics?

P. Romatschke[1]

[1] *University of Colorado at Boulder*

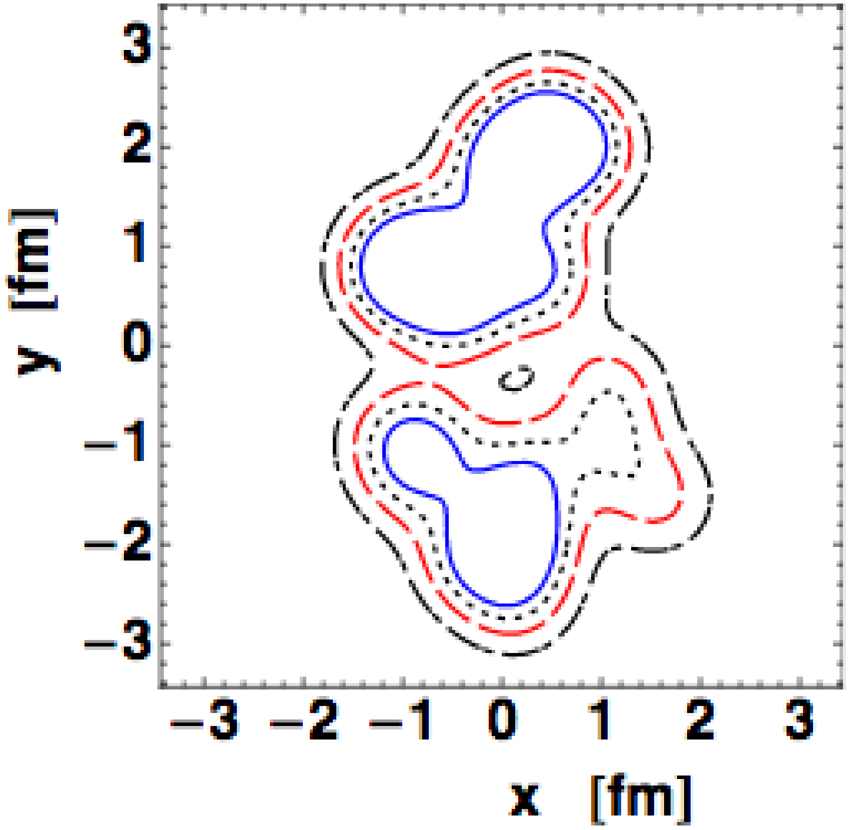
(Dated: September 28, 2018)

Relativistic collisions of light on heavy ions (p+Au at $\sqrt{s}$=7.7 GeV, p+Au , d+Au , ^{3}He+Au at $\sqrt{s}$ =62.4 GeV and 200 GeV and p+Pb , ^{3}He+Pb at $\sqrt{s}$ = 5.02 TeV) are simulated using "super-SONIC", a model that includes pre-equilibrium flow, viscous hydrodynamics and a hadronic cascade

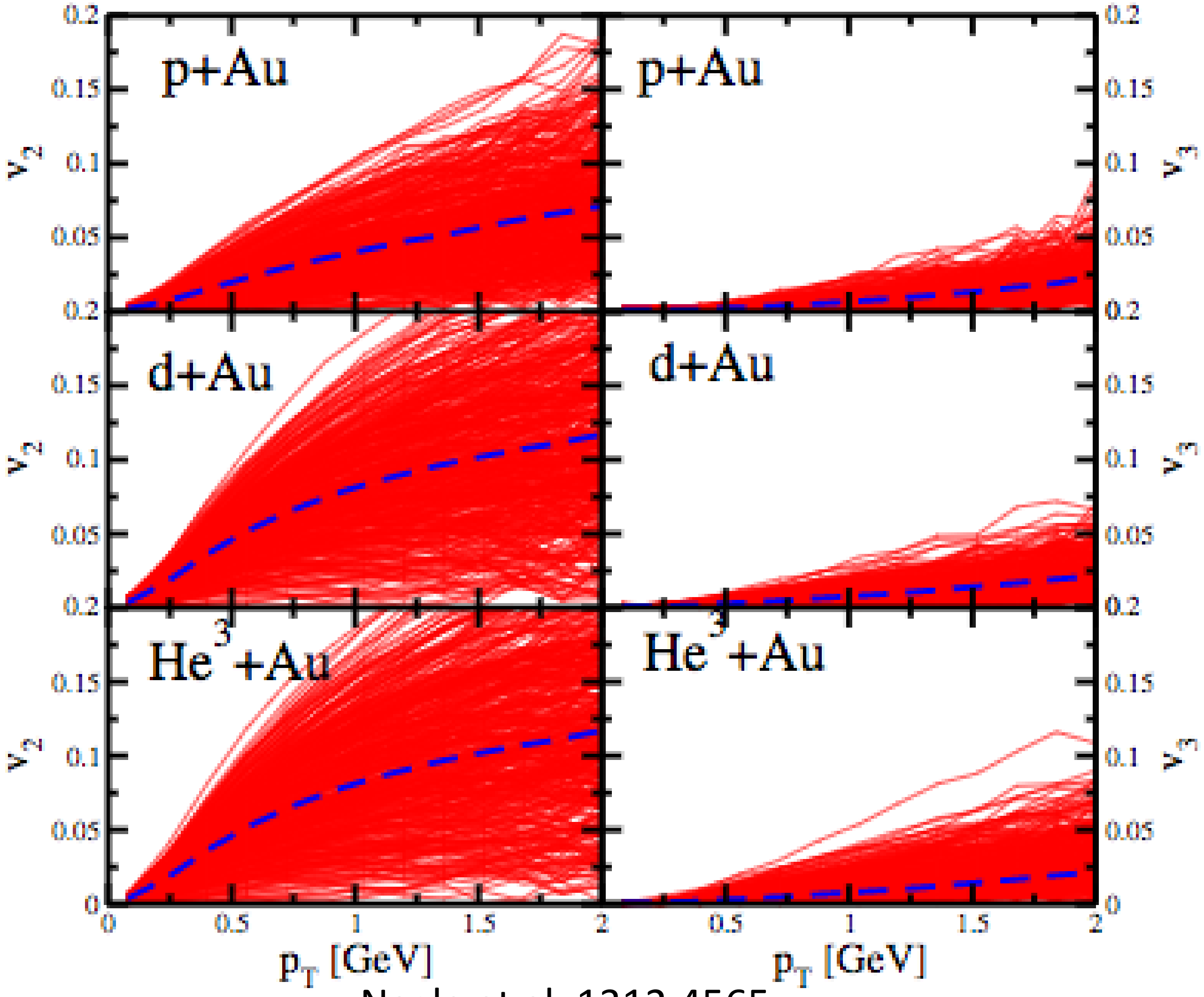
Discussion on Part II:
Initial Stage Correlations

Part III: Hydro Models and Data

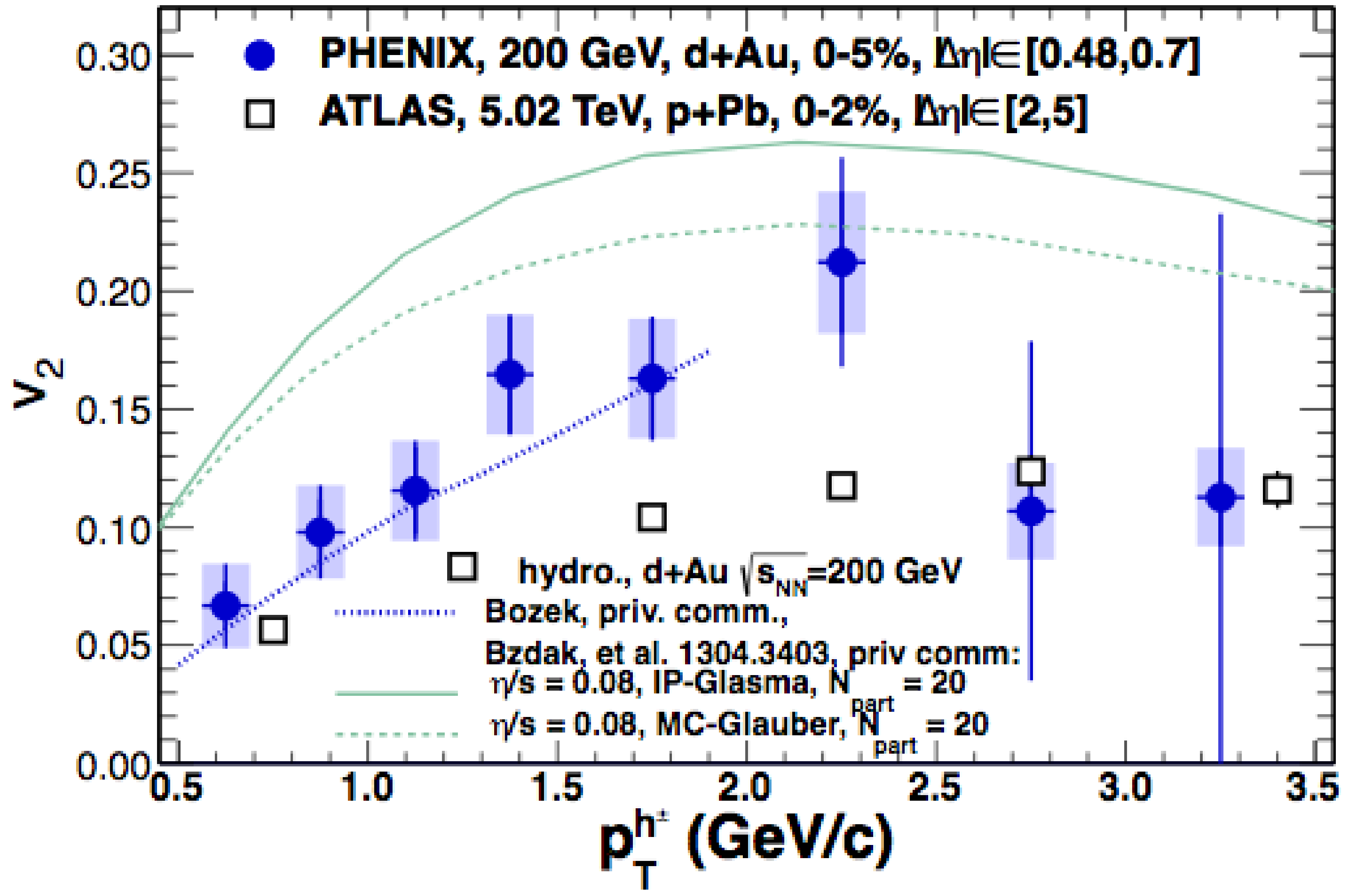
Hydro for Small Systems: Predictions



P. Bozek 1112.0915

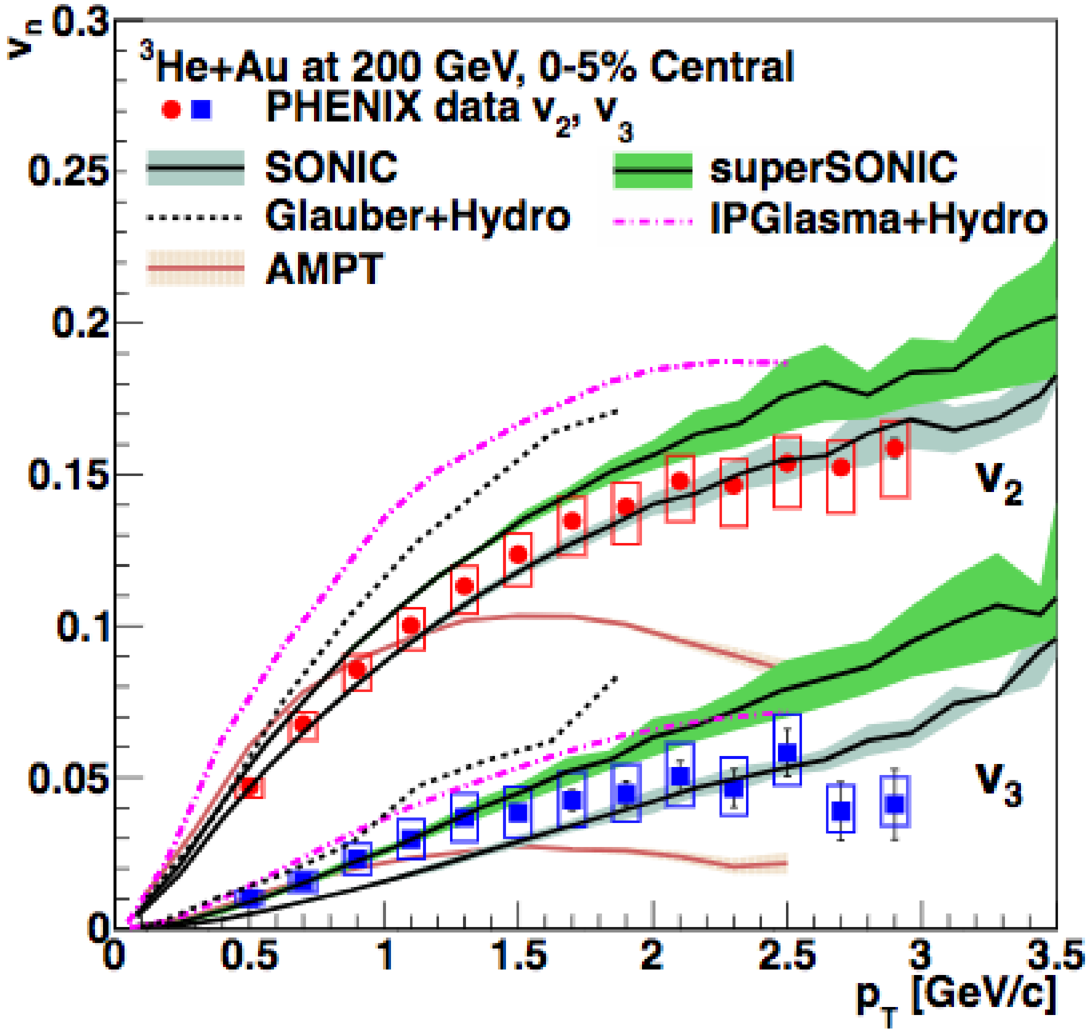


Nagle et al. 1312.4565



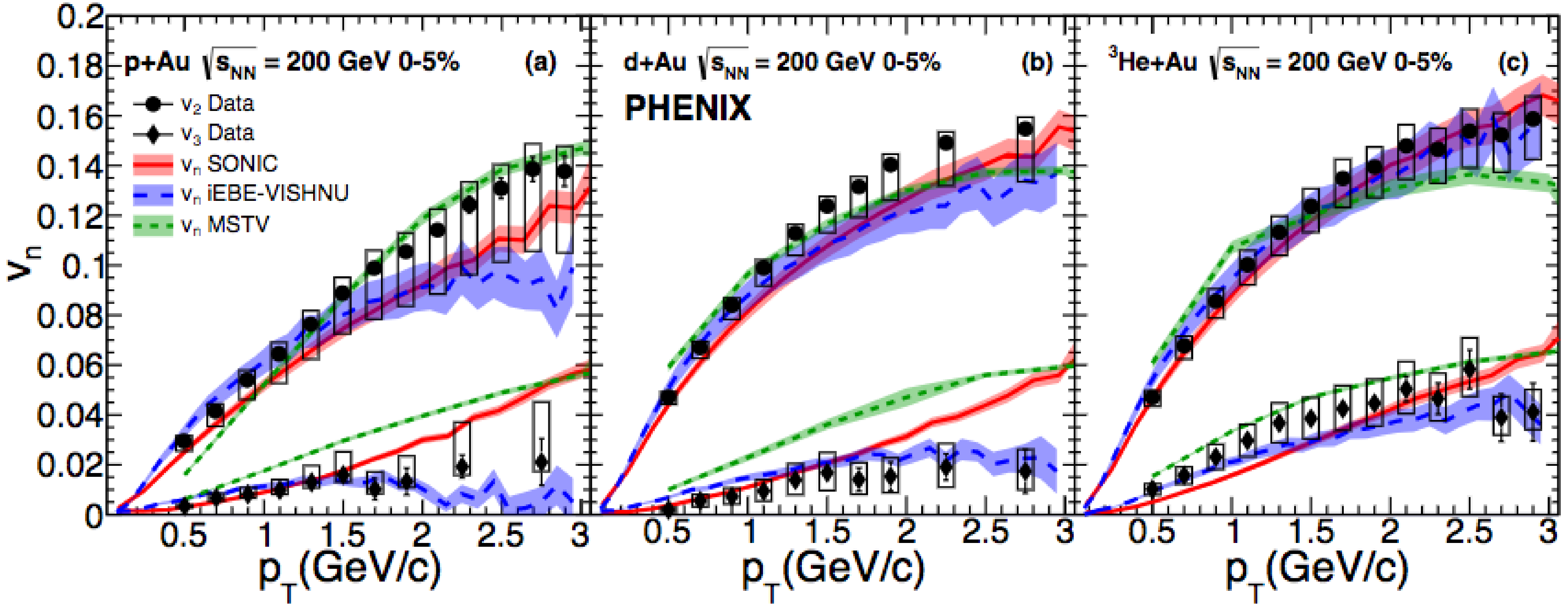
PHENIX, 1303.1794

Hydro for Small Systems: Measurements



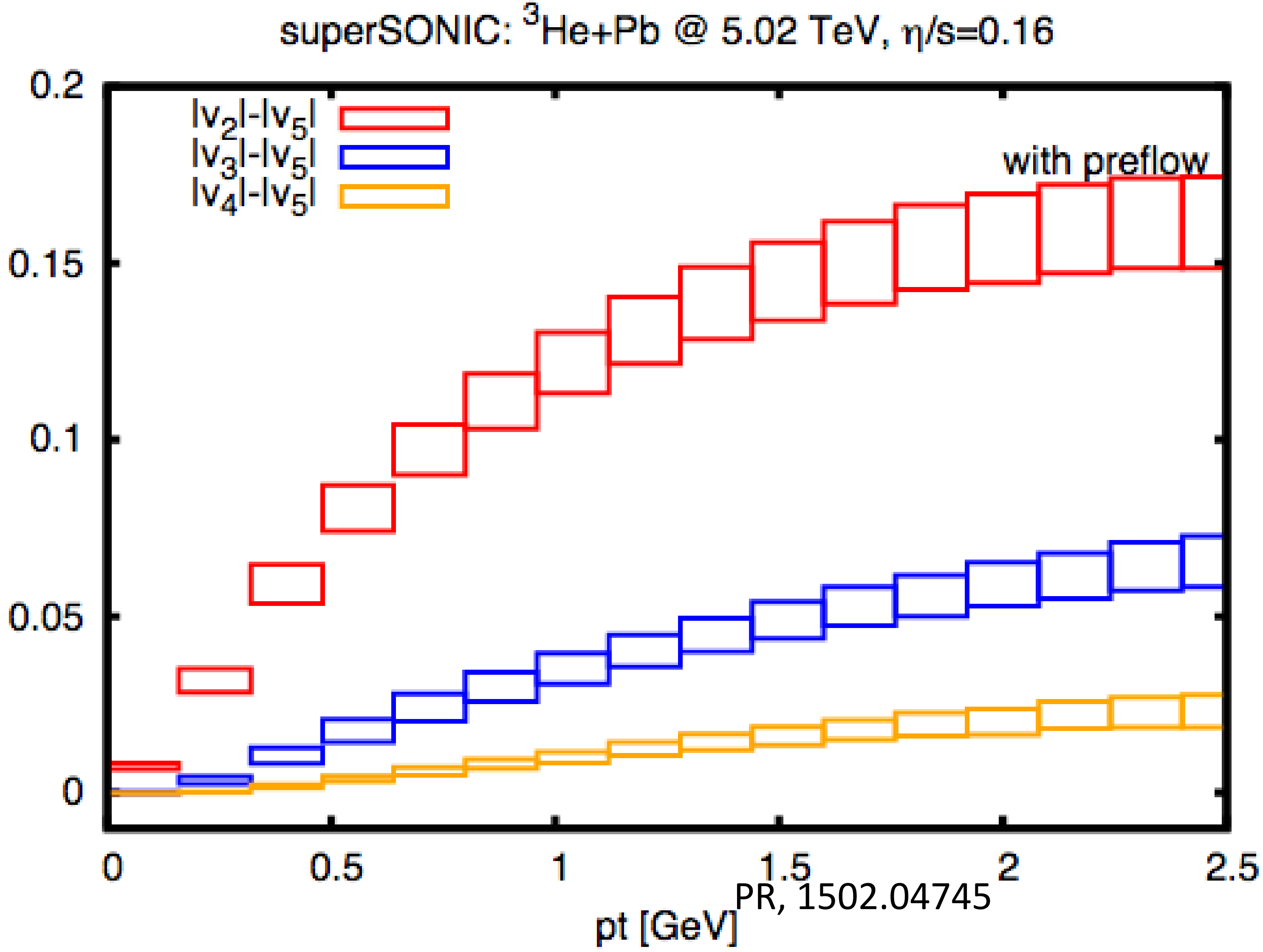
PHENIX, 1507.06273

2017: RHIC results for p+Au, d+Au and ^{3}He+Au



PHENIX, 1805.02973

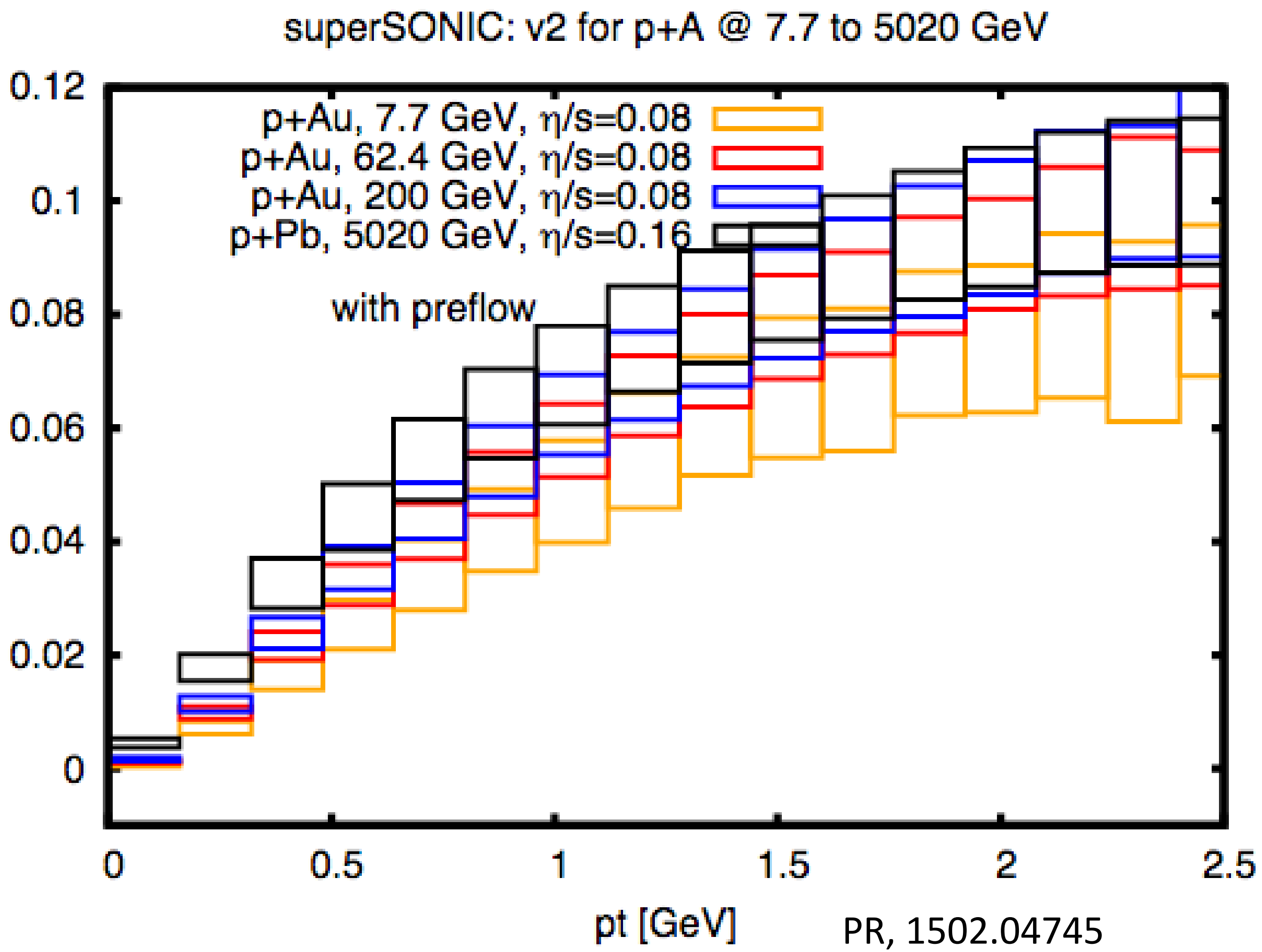
Hydro for Small Systems: Further Predictions



PR, 1502.04745

- Non-vanishing v3 in d+Au @ 200 GeV
- Flow in ^3He+Pb @ 5 TeV

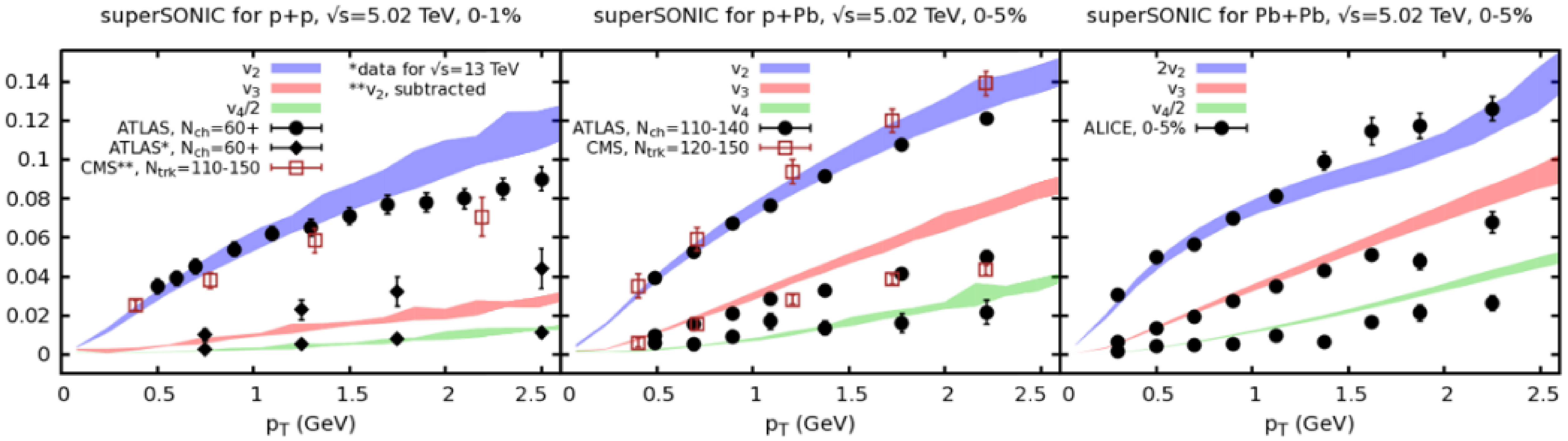
Hydro for Small Systems: Further Predictions



PR, 1502.04745

- Non-vanishing v3 in d+Au @ 200 GeV
- Flow in ^{3}He+Pb @ 5 TeV
- Energy-Scan of p+A v2,v3,v4
- much more (e.g. HBT Radii)

One Fluid To Rule Them All



Initial Stage Flow included in Hydro Simulations!

Simultaneous Description of all collision systems without tuning

Hydro Models and Data Take-Home Message

- Initial Stage Flow should not be neglected
- Hydro evolution should be neglected even less
- The IS + fluid paradigm is able to describe all available experimental data for all collision systems* with minimal #s
- The IS + fluid paradigm has made successful predictions for experiment

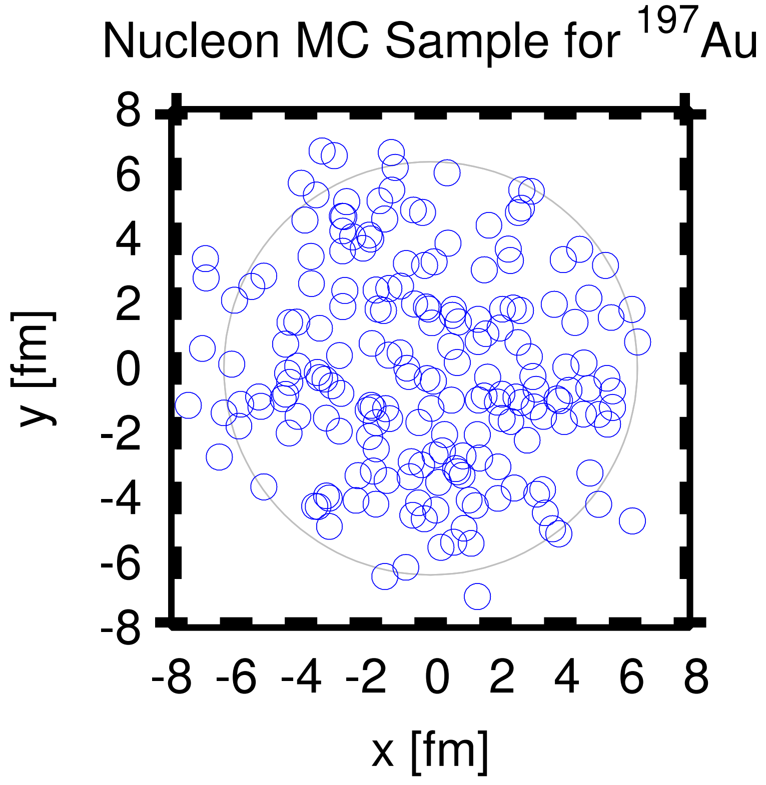
IMHO it is time to fully embrace the IS+fluid paradigm

* For momenta k<kc

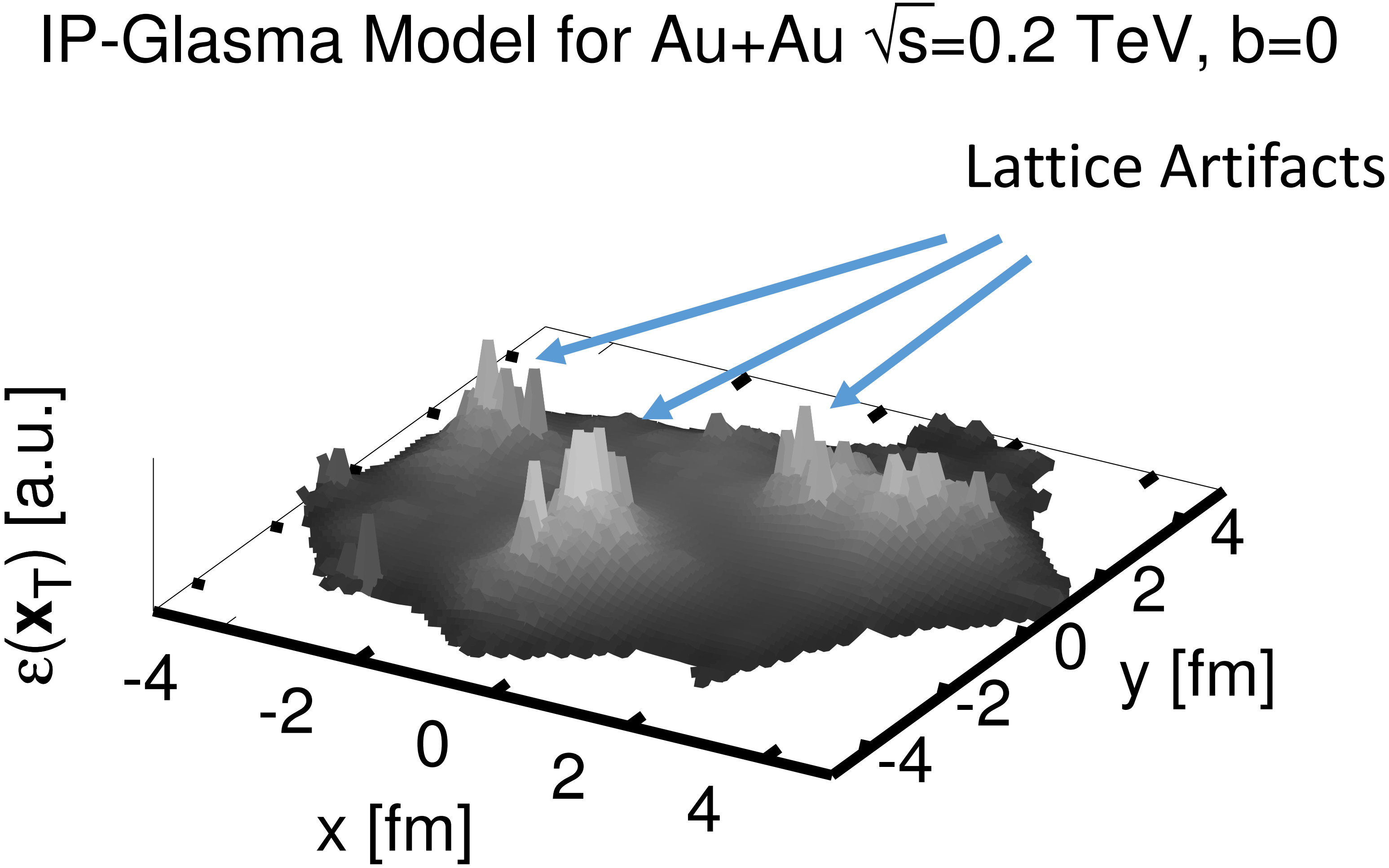
Backup

Simulation of small collision systems: inputs (1)

First things first: initial state



IP-Glasma



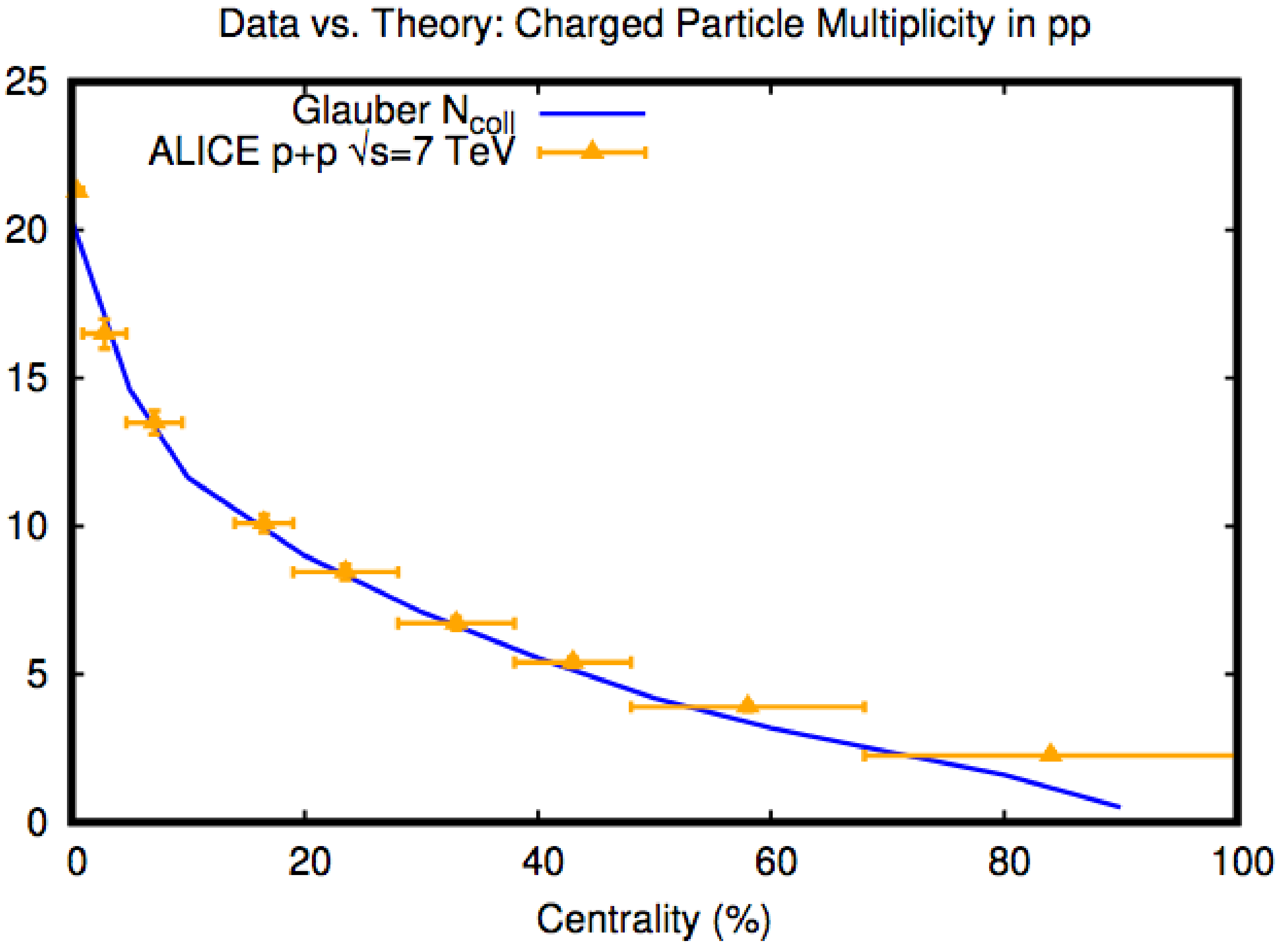
1712.05815

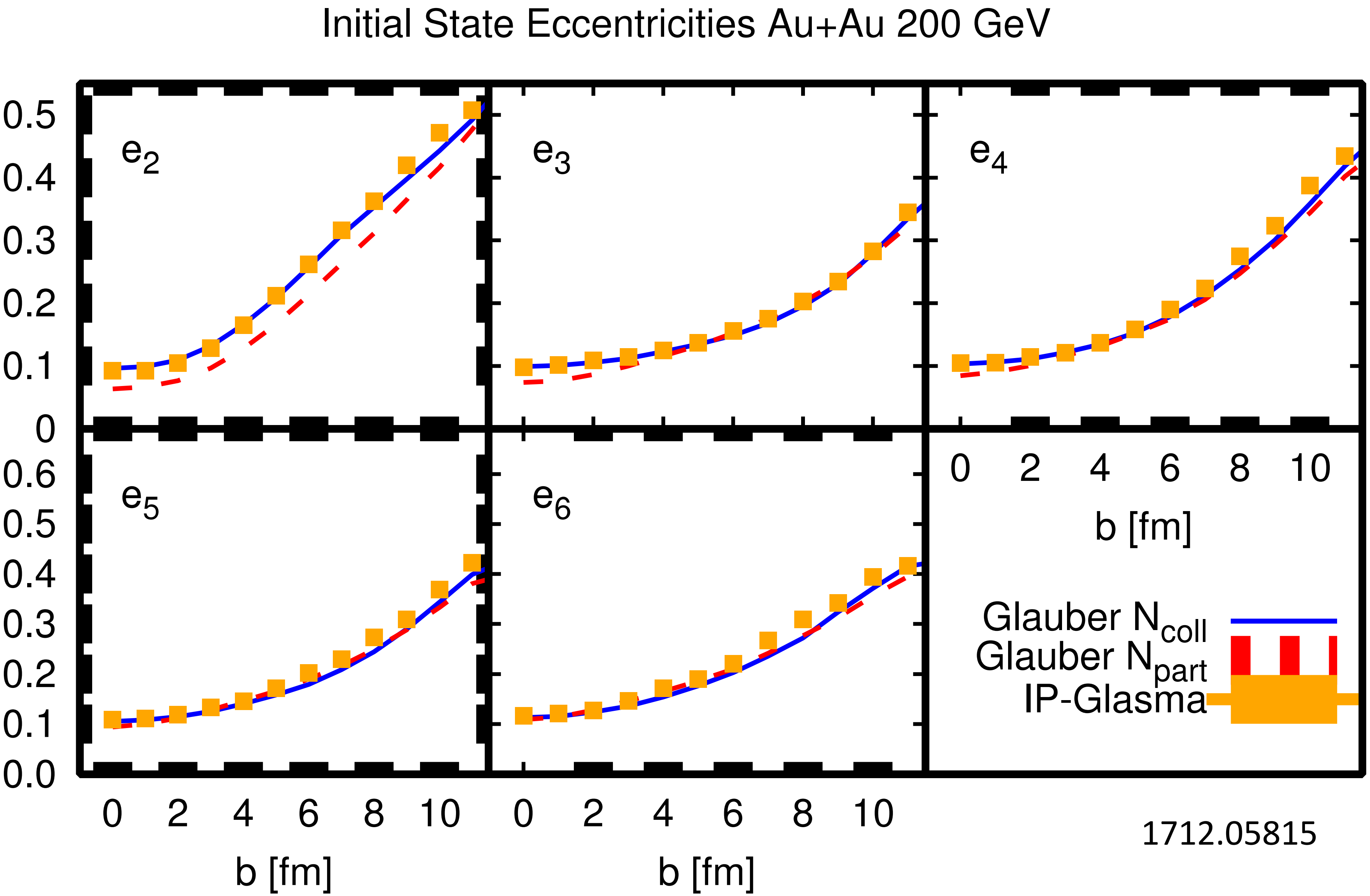
Hydro for small systems

- Small systems: p+Au, ^{3}He+Au, p+p
- System sizes L~0.7 fm to a few fm
- Hydro applicability for pp simulation:

t_{init}=0.25 fm, T~0.5 GeV:

Check: t_{hydro}>0.08 fm; L_{hydro}>0.08 fm



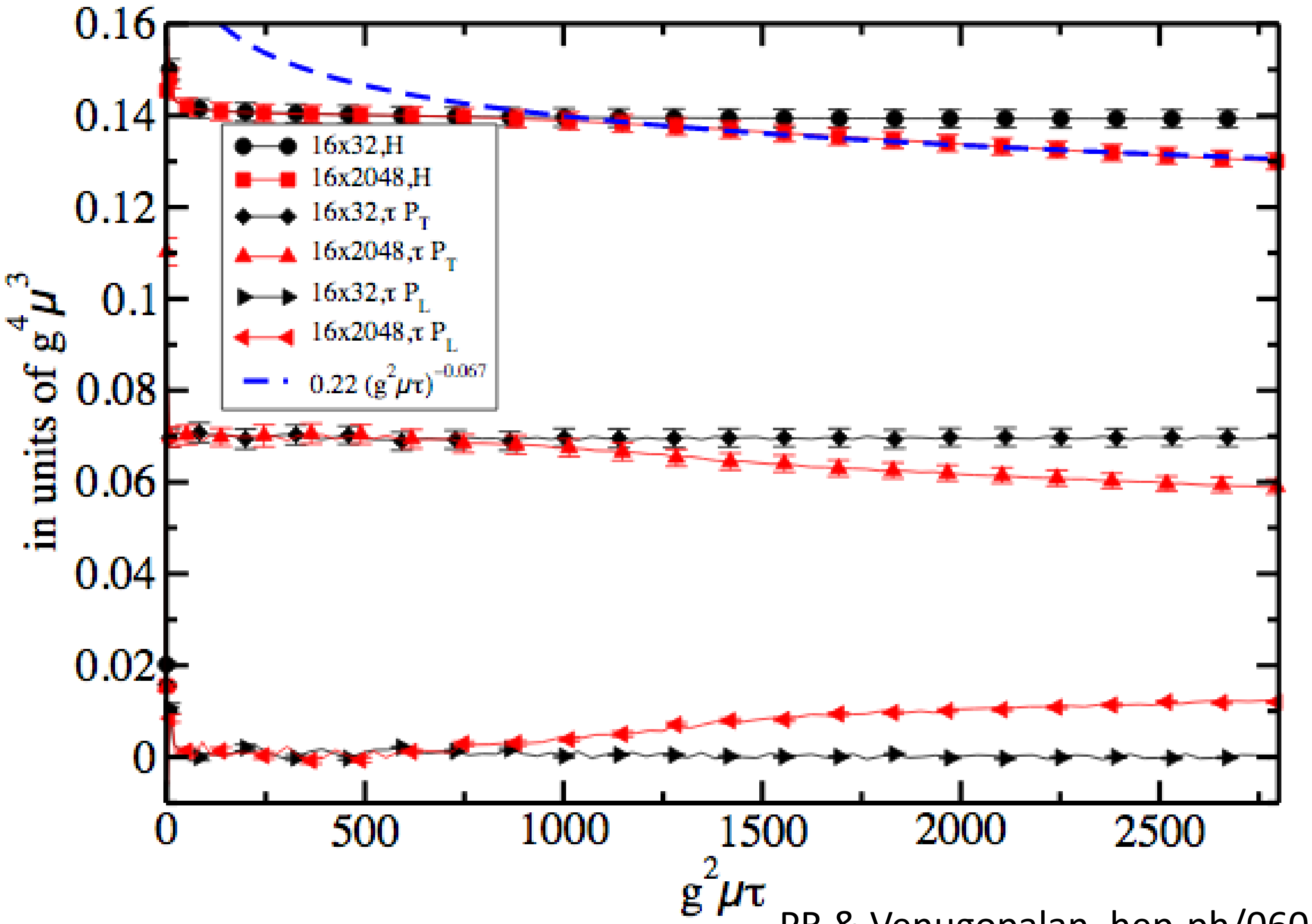


Noticed already for e_2 Lappi & Venugopalan in nucl-th/0609021

Cf. IP-Jazma by Nagle & Zajc, 1808.01276

IP-Glasma

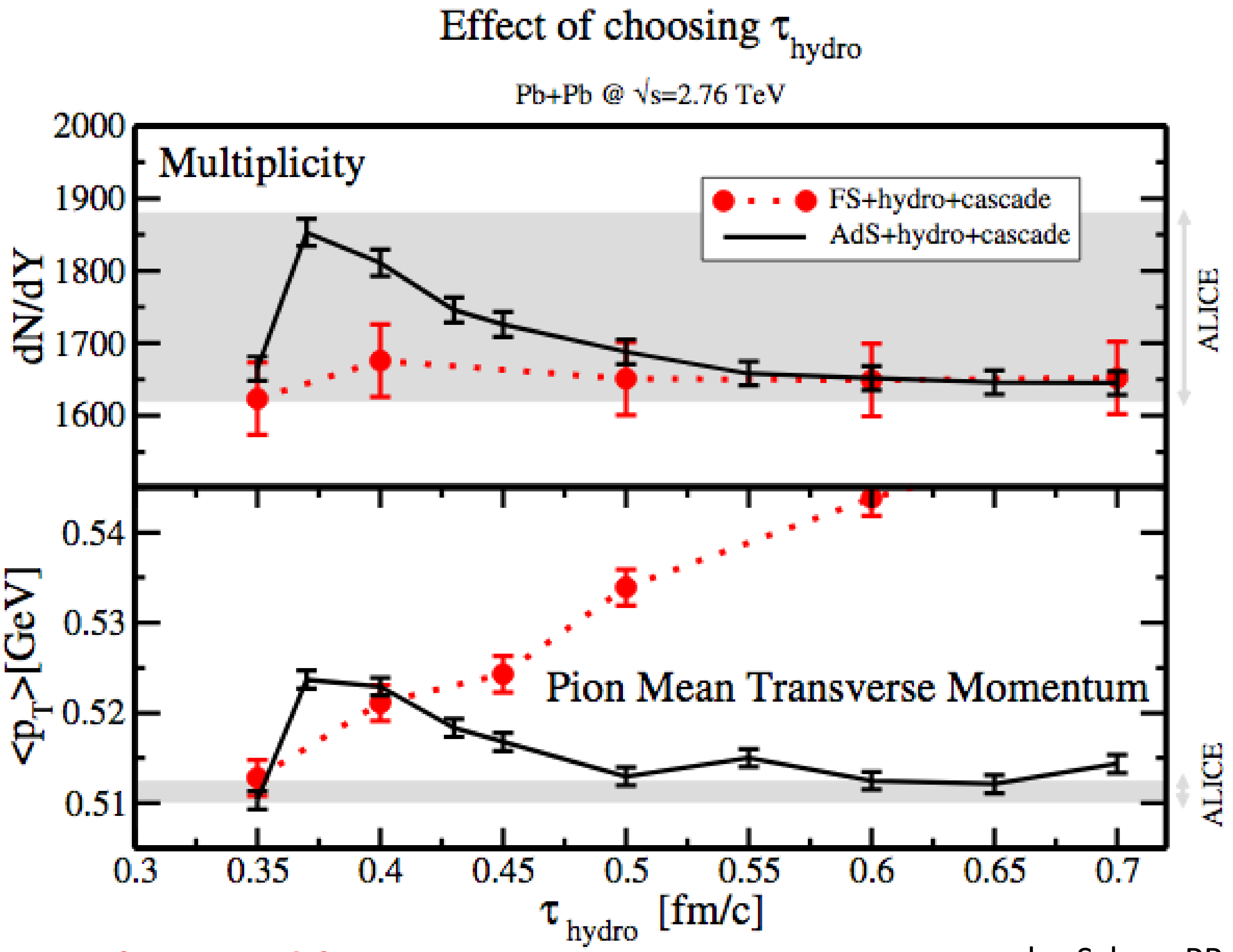
- Has energy-deposition and pre-hydro flow
- Is missing important physics: hard collisions
- ~free-streaming at late times



PR & Venugopalan, hep-ph/0605045

Only use classical YM at very early times

AdS/CFT



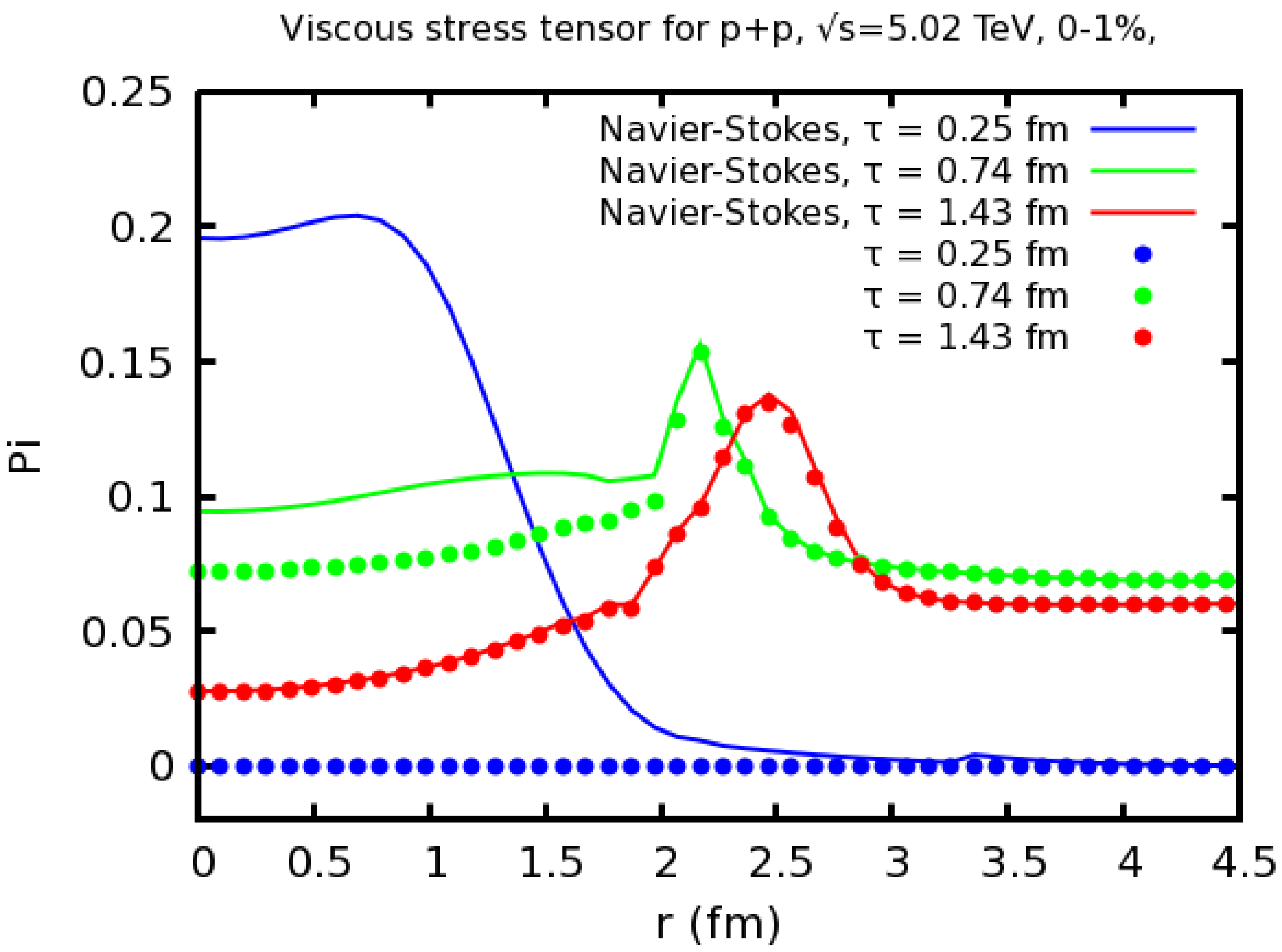
Can use AdS/CFT at early *and* late times

van der Schee, PR & Pratt, 1307.2539

Hydro Model: SONIC

- Uses pre-hydro flow (“initial state flow”)
- Hydro evolution
- Hadronization/Hadron Cascade
- Analysis

Does the Initial Stage Info survive?



R. Weller, unpublished

Once hydro switches on, initial state info is lost quickly