# Particle production in CGC

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# How do we calculate particle production in CGC?

Basics: eikonal scattering of the projectile gluons on the target fields.

Projectile color charge density $\rho^a(x)$ produces soft gluons via the Weiszacker-Williams field.
Gluons "decohere" due to eikonal interaction with the target and are scattered into final state.

$$a^{\dagger a}(x) \rightarrow U^{ab}(x_\perp) a^{\dagger b}(x)$$

$U^{ab}(x_\perp)$ - eikonal scattering matrix.
Work within the dilute-dense CGC approach: "p-A at mid rapidity".
No hadronization is accounted for: final state gluons = final state hadrons.
Final state interactions not considered until recently - perturbative classical field dynamics after collision may be taken into account.

# The CGC hadron wave function.

High energy factorisation: the fast partons are dressed by the soft gluon cloud.
Fast partons: color charge density in the transverse plane $\rho^a(x_\perp)$.
Soft gluons: the Weiszacker-Williams cloud.
CGC wave function of the projectile (approximate!):

$$\Psi[A] = e^{i\int_{x_\perp} b_i[\rho]A_i(x_\perp)}|0\rangle_{soft}|v\rangle$$

Solution of classical Yang-Mills equation:

$$\partial_i b_i^a(x_\perp) = g\rho^a(x_\perp); \qquad b_i^a(k) = g\frac{ik_i}{k^2}\rho^a(k)$$

$\rho$ has to be averaged over with some weight functional. Need to know $|v\rangle$.
Don't know - therefore model e.g. simplest Gaussian:
McLerran-Venugopalan model.

# Single inclusive gluon production

The master formula:

$$\frac{dN}{d^2p d\eta} = \langle \int_{z,\bar{z}} e^{ik(z-\bar{z})} \int_{x_1,x_2,\bar{x}_1,\bar{x}_2} \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)$$

$$\mathrm{Tr}\ \tilde{\rho}(x_1)[U^\dagger(x_1)-U^\dagger(z)][U(\bar{x}_1)-U(\bar{z})]\ \tilde{\rho}(\bar{x}_1)\rangle_{P,T}$$

$$f_i(x-y) = \frac{(x-y)_i}{(x-y)^2}, \qquad \tilde{\rho} \equiv -iT^a\rho^a$$

Have to average over the projectile (distribution of $\rho$) and target (distribution of $S$)

# Double inclusive production.

The master formula:

$$\frac{dN}{d^2p d^2k d\eta d\xi} = \langle\, \sigma(k)\, \sigma(p) + O(\rho^2) + O(\rho^3)\,\rangle_{P,T}$$

with

$$\sigma(k) = \int_{z,\bar{z}} e^{ik(z-\bar{z})} \int_{x_1,x_2,\bar{x}_1,\bar{x}_2} \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)$$
$$\mathrm{Tr}\tilde{\rho}(x_1)[U^\dagger(x_1)-U^\dagger(z)][U(\bar{x}_1)-U(\bar{z})]\ \tilde{\rho}(\bar{x}_1)$$

$O(\rho^2)$ - production of both gluons from a single "Pomeron". Contributes to back-to-back correlations. Subleading at high density - not frequently discussed.

$O(\rho^3)$ - production from "Odderon". Also subleading at large $\rho$.

We focus on $\langle\sigma\sigma\rangle$ term - the main source of correlations in CGC calculations.

# Nature of correlations

$$\sigma(k) = \int_{z,\bar{z}} e^{ik(z-\bar{z})} \int_{x_1,x_2,\bar{x}_1,\bar{x}_2} \vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)$$

$$\mathrm{Tr}\tilde{\rho}(x_1)[U^\dagger(x_1)-U^\dagger(z)][U(\bar{x}_1)-U(\bar{z})]\ \tilde{\rho}(\bar{x}_1)$$

"Accidental symmetry" $\sigma(k)=\sigma(-k)$.

Thus double inclusive production is symmetric under $(k,p)\to(k,-p)$.

This means absence of odd harmonics $v_3$, $v_5$ etc. configuration by configuration in $\rho$ and $U$!

"Accidental" - only the property of $\sigma\sigma$ term.

Is violated by the single Pomeron term (but leads to wrong sign $(v_3)^2$).

Is also violated by various corrections - more later.

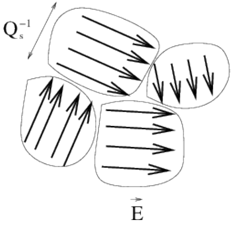
**Correlations are there.**

Correlations come in two varieties: Quantum and Classical

# Anatomy of correlations: Classical correlations 1

**Local anisotropy**

$Q_s$ - color correlation length in the target: "color field domains".




Purely classical correlation: gluons that hit the same domain scatter in the same direction (if they have the same charge.)

# Anatomy of correlations: Classical correlations 2

**Density variation**

If the density profile is not constant, a dipole scatters differently depending on its orientation. Scattering is more efficient for dipole oriented along the density gradient rather than perpendicular to it.

More particles are produced with momentum parallel to density gradient.

**Both mechanisms are purely classical:** in order to produce correlated particles, the incoming gluons have to sit close to each other in the transverse plain so that they feel the same local structure of the target.

But different momentum scales involved: $Q_s > \frac{dQ_s}{db}/Q_s$.

Color domains more relevant for higher $p_\perp$ while density variation affects more lower $p_\perp$.

# Anatomy of correlations: Quantum correlations 1

Bose enhancement in the incoming wave function a.k.a. "Glasma graphs"

The CGC soft gluon state is "classical" - coherent state.

But when averaged over the valence color charges, the density matrix is not classical!

$$\hat{\rho} = \mathcal{N} \int D[\rho]\; W[\rho] e^{i\int_q b^i_b(q)[a^i_b(-q)+a^{\dagger i}_b(q)]} |0\rangle\langle 0|\; e^{-i\int_p b^j_c(p)[a^j_c(-p)+a^{\dagger j}_c(p)]}$$

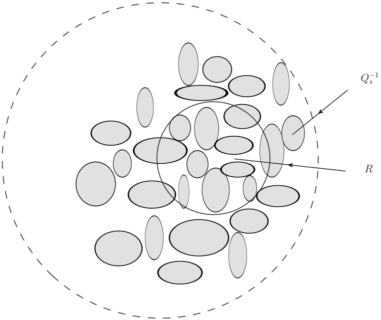
Explicit calculation with McLerran-Venugopalan model

$$W[\rho] = e^{-\int_k \frac{1}{2\mu^2}\rho_a(k)\rho_a(-k)}$$

yields Bose enhancement effect: enhanced probability to find two gluons with the same transverse momentum.

More gluons come in with $k_\perp = p_\perp \rightarrow$ more gluons are produced with $k_\perp \approx p_\perp$.

Gluonic Hanbury-Brown - Twiss effect




Scattering randomizes color phases in the projectile on transverse scale $Q_s^{-1}$ - the projectile after scattering turns into a bunch of sources of incoherent emission. Typical HBT situation.

# How to separate classical from quantum?

Classical are suppressed by $1/Q_s^2 S_{proj}$.

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2 d\eta_2} = \alpha_s^2 (4\pi)^2 \int_{z_1 \bar{z}_1 z_2 \bar{z}_2; x_1 x_2 y_1 y_2} e^{ik_1\cdot(z_1-\bar{z}_1)+ik_2\cdot(z_2-\bar{z}_2)}$$
$$\times f^i(x_1 - z_1) f^i(\bar{z}_1 - y_1) f^j(x_2 - z_2) f^j(\bar{z}_2 - y_2)$$
$$\times \Big\langle \rho^{a_1}(x_1)\big[U(z_1) - U(x_1)\big]\big[U^\dagger(\bar{z}_1) - U^\dagger(y_1)\big]^{a_1 b_1} \rho^{b_1}(y_1)$$
$$\times \rho^{a_2}(x_2)\big[U(z_2) - U(x_2)\big]\big[U^\dagger(\bar{z}_2) - U^\dagger(y_2)\big]^{a_2 b_2} \rho^{b_2}(y_2)\Big\rangle_{P,T}.$$

with the Weizsacker-Williams field

$$f^i(x-y) = -\frac{1}{2\pi}\frac{(x-y)_i}{(x-y)^2}$$

Points close together - classical; points far away - quantum.

# Separating quantum I

Concentrate on target averages.

E.g.:

$$\int_{z,\bar{z}} F(z,\bar{z})\langle [U(z_1)U^\dagger(\bar{z}_1)]^{ab}[U(z_2)U^\dagger(\bar{z}_2)]^{cd}\rangle_T$$

Roughly: The further apart the points are, the larger the contribution due to integration (no area suppression.)

But has to be color invariant!

Color neutralization in the target on distance scales $r \sim 1/Q_s$.

So points should be pairwise close to each other - otherwise the average vanishes.

Any product of $U$'s factorizes *a la* Wick with basic "contraction"

$$\left\langle U^{ab}(x)U^{cd}(y)\right\rangle_T = \delta^{ac}\delta^{bd}\frac{1}{N_c^2-1}d(x,y),$$

# Separating quantum - II

So, for example

$$\frac{1}{N_c^2-1}\langle \mathrm{Tr}[U(x)U^\dagger(y)U(z)U^\dagger(v)]\rangle_T = d(x,y)d(z,v) + d(x,v)d(z,y)$$
$$+\frac{1}{N_c^2-1}d(x,z)d(y,v),$$
$$\frac{1}{(N_c^2-1)^2}\langle \mathrm{Tr}[U(x)U^\dagger(y)][\mathrm{Tr}[U(z)U^\dagger(v)\rangle_T = d(x,y)d(z,v)$$
$$+\frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right].$$

with $d(x,y) = \frac{1}{N_c^2-1}\mathrm{Tr}[U(x)u^\dagger(y)]$

Not large $N_c$ limit - rather dense target limit!

All points close together - factorization does not hold. This is where "classical" contributions come from.

# BE vs HBT

$$\langle Q(z_1,\bar z_1,z_2,\bar z_2)\rangle_T = d(z_1,\bar z_1)d(z_2.\bar z_2)+d(z_1,\bar z_2)d(z_2,\bar z_1) + \frac{1}{N_c^2-1}d(z_1,z_2)d(\bar z_1,\bar z_2),$$

First term: two gluons scatter independently. But arise with larger probability from the wave function. Bose enhancement.
Width of the correlation $\Delta k \sim Q_s^T$

Second term: correlates directly momenta of produced gluons. Hanbury Brown - Twiss effect.
Width of the correlation $\Delta k \sim 1/R$ - size of the projectile.

# Some are more equal than others?

**Quantum and classical are different.**

Quantum correlations are leading in $1/SQ_s^2$, where $S$ - the area of the projectile.

So Quantum is leading? Yes, but...

Narrow in momentum.

Width of BE correlation: $\Delta k \sim Q_s$

Width of the HBT correlation $\Delta k \sim \Lambda_{QCD}$

Both sharply peaked around $|k| = |p|$. If the momentum bin is too wide - quantum correlations are suppressed.

**Classical are present also if $|k| \ll |p|$.**

Classical 1: local anisotropy - scales inverse proportionally to the number of domains, and thus $1/Q_s^2$. Should decrease as the energy grows.

Low momenta sample more local density variations - no obvious $1/Q_s$ scaling.

# Challenges

$v_2^2$ is straightforward to get.

What is not straightforward to get/understand?

$v_3$ $v_4$ $v_2^n$

# Challenges: $v_3$

Does $v_3$ vanish in CGC?

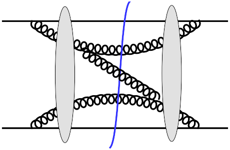
No. The symmetry $(k, p) \to (k, -p)$ is accidental and does not survive corrections to the leading CGC approximation.

**a.** Single Pomeron contribution is not symmetric. But it does not help, since it produces mostly back-to-back correlations, and thus the wrong sign $v_3^2$.

**b.** Corrections to the CGC wave function:

$$e^{i\int_{x_\perp} b_i[\rho]A_i(x_\perp)}|0\rangle \to e^{i\int_{x_\perp} b_i(x_\perp)A_i(x_\perp)}e^{-\int_{x_\perp,y_\perp} A_i(x_\perp)\Lambda_{ij}(x_\perp,y_\perp)A_j(y_\perp}|0\rangle$$

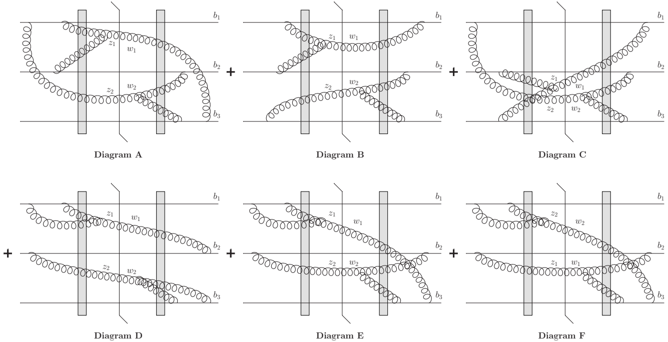
No accidental symmetry. Produces positive $v_3^2$ in a certain kinematical range. Diagrammatically:



Order $\alpha_s$ relative to $(v_2)^2$.

# Challenges: $v_3$

**c.** Classical evolution in the final state.
Diagrammatically:




Order $(\alpha_s\rho)^2$ relative to $(v_2)^2$. Leading in the classical regime of strong fields $\rho \sim 1/g^2$.

# Challenges: $v_4$

This is not necessarily a real problem, but can teach us something.

Quantum interference correlations have fixed width in momentum space $\Delta k \sim Q_s$ at $\Delta\phi = 0$ and $\Delta\phi = \pi$.

For $|k| \approx |p| \gg Q_s \sim$ sum of two delta functions. This leads to $v_4 \approx v_2$ - very different from experiment.

But classical contributions are different and dominate when the bin width is larger than $Q_s$.

Numerically CGC calculations (Mace's talk?) give reasonable values for $p_\perp$ integrated $v_4$.

# Challenges: $v_2^4$

This is still a puzzle.
Is it physics or not?
What is $c_2^4$?

$$V_n = \int_0^{2\pi} \bar{d}\phi \int dkk \left. \frac{dN}{d^2kdy} \right|_{\rho,U} \exp\left(ni\phi\right) \; ;$$

$$-(v_2^4)^2 = c_2^4 = \langle \frac{V_2 V_2^* V_2 V_2^*}{V_0 V_0^* V_0 V_0^*} \rangle - 2 \left[ \langle \frac{V_2 V_2^*}{V_0 V_0^*} \rangle \right]^2$$

$$\neq \frac{\langle V_2 V_2^* V_2 V_2^* \rangle}{\langle V_0 V_0^* \rangle \langle V_0 V_0^* \rangle} - 2 \left[ \frac{\langle V_2 V_2^* \rangle}{\langle V_0 V_0^* \rangle} \right]^2$$

In hydro the difference is small.

But if two particle correlations are most important (no real flow) - the difference between the two is of the same order as each one of them.
Is the sign the same? We don't know.

# Challenges: $v_2^4$

If this is physics: negative "connected" four particle correlation.

Recent paper by Blok and Wiedemann: emission of $m$ gluons from $N$ sources with $m > N$ leads to such negative correlation.

If this is what we are looking for? The jury is still out...

# What are we missing?

The most obvious thing: hadronization corrections - going beyond local parton-hadron duality.

Final state interactions. Taken into account perturbatively.

The real issue is the real structure of the proton. If we try to explain correlations based on the initial state, we better know the projectile wave function pretty well. And we don't.

We use MV model for the projectile. Iffy. It was derived for a large nucleus, but we use it for proton. Rare dense configurations in proton can be quite different than typical configurations in a large nucleus.

Some modifications are included: i.e. distribution of $Q_s$ is tantamount to modifying the gaussian distribution of color charge densities. But is it enough?

# What are we missing?

JIMWLK evolved MV - perhaps better, hopefully more universal. But we still have to model the initial conditions. Also is JIMWLK precise enough for studying correlations in the wave function?

Is transverse shape of the projectile important? If so, JIMWLK may be no good - it generates long transverse tails.

Does not give us a tool to study density - flow correlations in the wave function. Should be especially important for rare configurations which initiate high multiplicity events. Density matrix/Wigner functional approach?

Perhaps things are related: good understanding of the projectile wave function is the same as accounting for final state interactions. After all exact eigenstates evolve after collision without interaction - it is only in the partonic/classical approximation "particles" interact in the final state.