Comments on Current-Current Correlators

- Structure Functions from Euclidean Hadronic Tensor
- Compton Amplitude with OPE
- Quasi-PDF
- Inclusive Semi-leptonic B Decay Shape Function

c QCD Collaboration

U. Maryland, Apr. 8, 2018

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering
  In Minkowski space

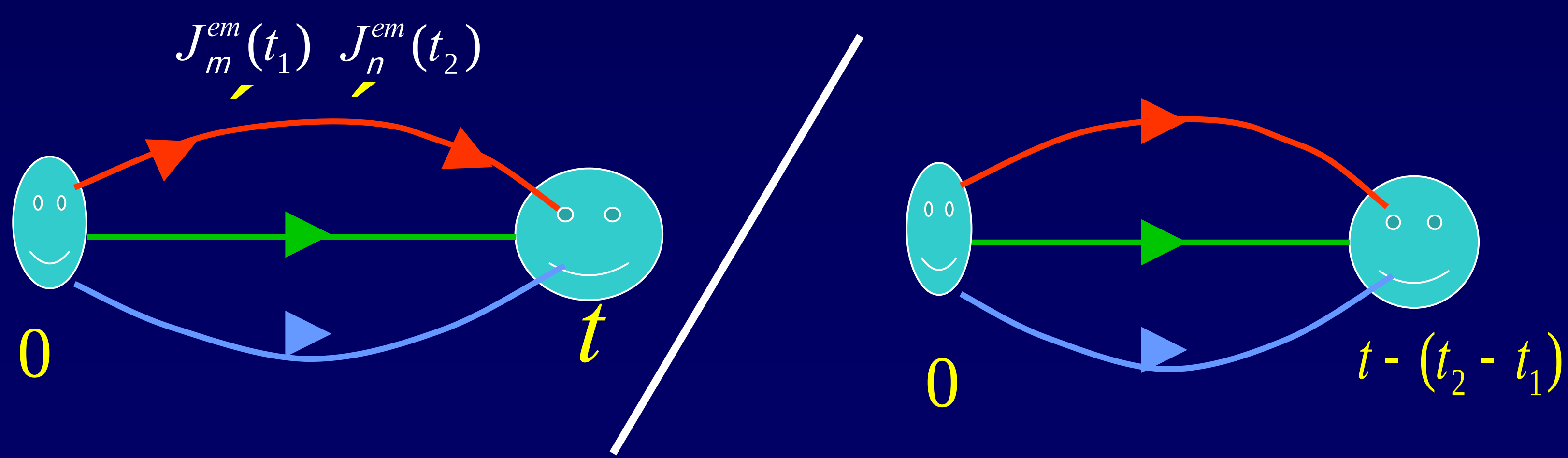
$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu} = \left\langle N(\vec{p}) \right| \int \frac{d^4x}{4\pi}\, e^{iq\cdot x} J_\mu(x) J_\nu(0) \left| N(\vec{p}) \right\rangle_{\text{spin avg}}$$

$$= \frac{1}{2}\sum_n \int \prod_{i=1}^{n} \left[\frac{d^3p_i}{(2\pi)^3 2E_{pi}}\right] (2\pi)^3 \delta^4(p_n - p - q) < N(\vec{p}) | J_\mu | n >< n | J_\nu | N(\vec{p}) >_{\text{spin avg}}$$

- Euclidean path-integral

KFL and S.J. Dong, PRL 72, 1790 (1994)
KFL, PRD 62, 074501 (2000)



$W_{\mu\nu}$ in Euclidean Space

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau = t_2 - t_1) = \frac{\frac{E_P}{M_N}\mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t) \sum_{\vec{x}} \frac{1}{4\pi}\ e^{-i\vec{q}\cdot\vec{x}} J_\mu(\vec{x},t_2) J_\nu(0,t_1) \chi_N^\dagger(\vec{p},0) >}{\mathrm{Tr} < \Gamma_e \chi_N(\vec{p},t) \chi_N^\dagger(\vec{p},0) >}$$

$$\xrightarrow[t-t_2 >> 1/\Delta E_P,\ t_1 >> 1/\Delta E_P]{}$$

$$= \frac{1}{4\pi} \sum_n \left(\frac{2m_N}{2E_n}\right)\ \delta_{\vec{p}_n - \vec{p} - \vec{q}}\ \ < N(\vec{p}) \,|\, J_\mu \,|\, n >< n \,|\, J_\nu \,|\, N(\vec{p}) >_{\text{spin avg}}\ \ e^{-(E_n - E_P)\tau}$$

$$= \ \ < N(\vec{p}) \,|\, \sum_{\vec{x}} \frac{e^{-i\vec{q}\cdot\vec{x}}}{4\pi} J_\mu(\vec{x},\tau) J_\nu(0,0) \,|\, N(\vec{p}) >_{\text{spin avg}}$$

Inverse Laplace transform

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{i} \int_{c-i\infty}^{c+i\infty} d\tau\ e^{\nu\tau}\ \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$$

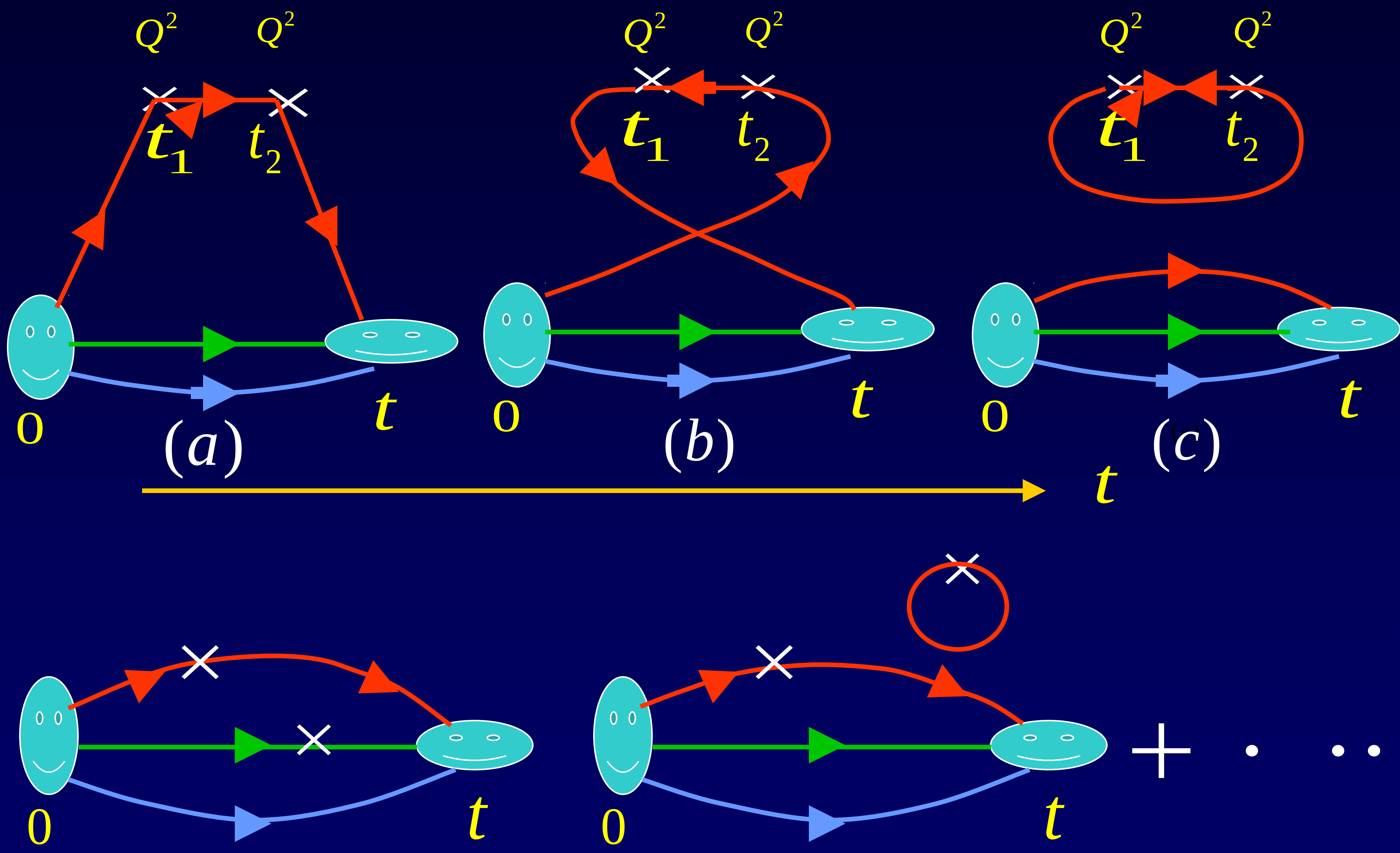
Numerical Challenge of an Inverse Problem

Corresponding Laplace transform $$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau) = \int_0^\infty d\nu\, e^{-\nu\tau}\, \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\nu)$$

Entails an inverse problem which maps the hadonic tensor in Euclidean time to the spectral function in Minkowski space.

- Improved Maximum Entropy method
- Backus-Gilbert method
- Fitting with a prescribed functional form for the spectral distribution.
- Compton Amplitude approach

$q = q_V + q_{CS}$ $\qquad \bar{q}_{CS}$ $\qquad q_{DS} = (\neq ?)\ \bar{q}_{DS}$



Cat’s ears diagrams are suppressed by $O(1/Q^2)$.

- $W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2,\nu)(p_\mu - \frac{p\cdot q}{q^2} q_\mu)(p_\nu - \frac{p\cdot q}{q^2} q_\nu)$

- Bjorken limits

$$\nu W_2(q^2,\nu) \longrightarrow F_2(x,Q^2) = x\sum_i e_i^2 (q_i(x,Q^2) + \bar{q}_i\left(x,Q^2\right)); \quad x = \frac{Q^2}{2p\cdot q}$$

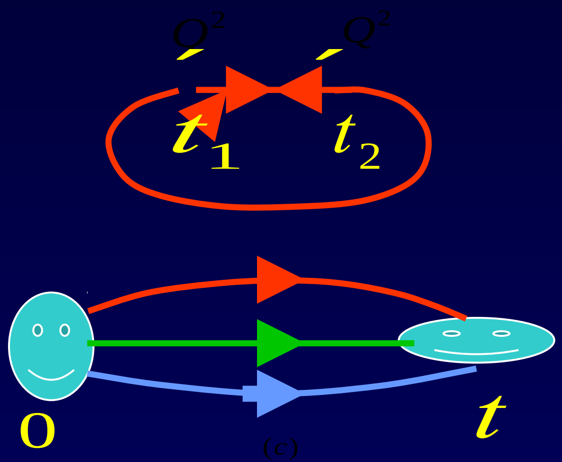
- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

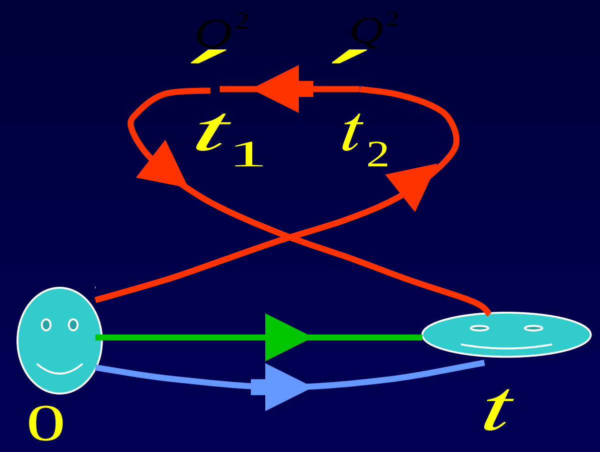
Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_P(x) - \bar{d}_P(x)); \quad S_G(0,1;Q^2) = \frac{1}{3} \text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\ \text{GeV}^2) = 0.240 \pm 0.016\ (5\sigma \text{ from GSR})$



two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$)



one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right]\ (1 + O(\alpha_s^2))$$

Properties of this separation

- No renormalization (finite normalization with local current)
- Gauge invariant
- Topologically distinct as far as the quark lines are concerned
- $W_1(x, Q^{2)}$ and $W_2(x, Q^2)$ are frame independent.
- Small x behavior of CS and DS are different.

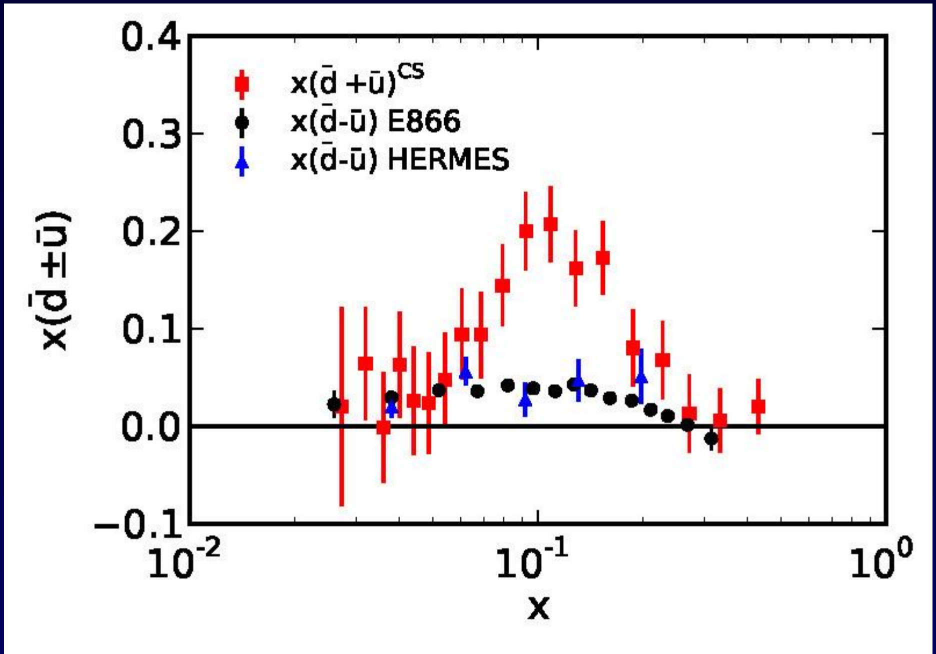
$$q_V,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0}\ x^{-a_R}(x^{-1/2})$$

$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0}\ x^{-1}$$

- Short distance expansion (Taylor expansion) ⟹ OPE

How to Extract Connected Sea Partons ?

K.F. Liu, W.C. Chang, H.Y. Cheng, J.C. Peng, PRL 109, 252002 (2012)

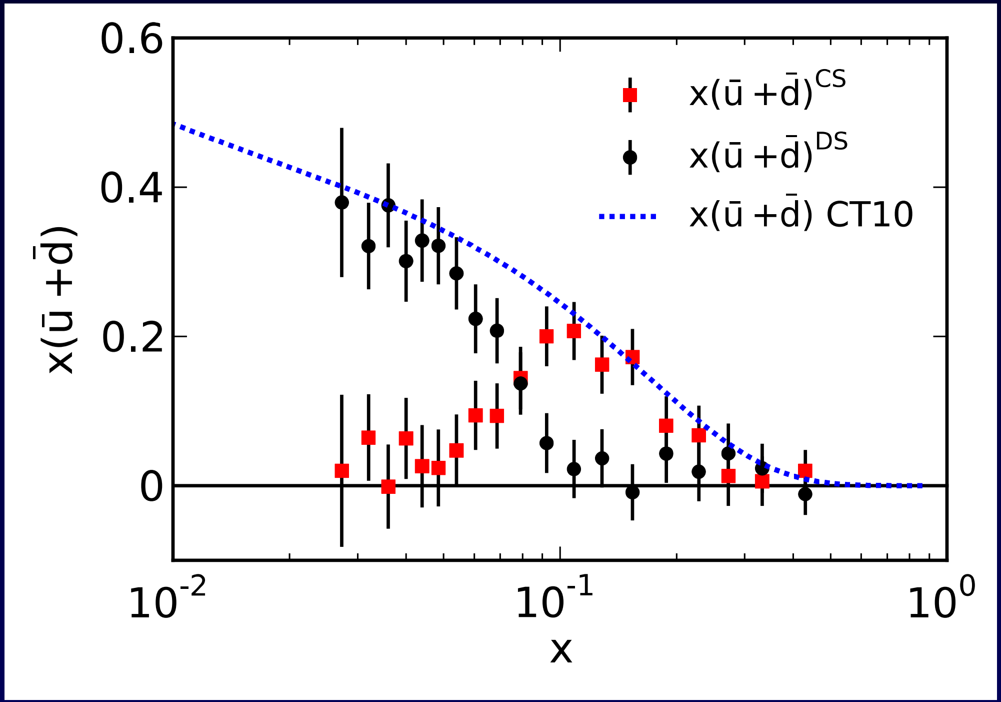


Q^2=2.5 GeV^2

$$x(\bar{d}+\bar{u})_{CS}(x) = x(\bar{d}+\bar{u})(x) - \frac{1}{R}x(s+\bar{s})(x);$$

CT10 lattice expt

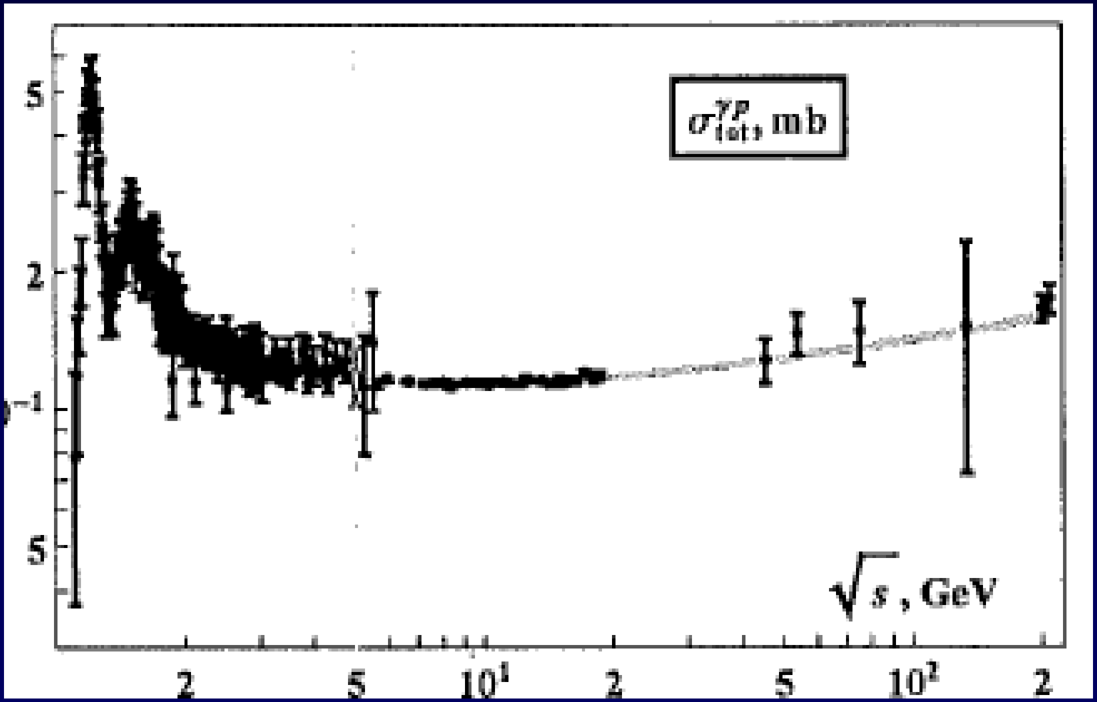
$$R = \frac{\langle x \rangle_s}{\langle x \rangle_u (DI)} (\text{lattice}): \ 0.857$$



$$q_V,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0} \ x^{-a_R}(x^{-1/2})$$

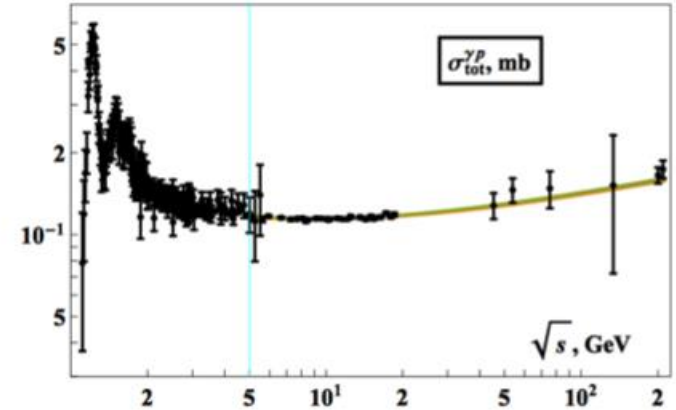
$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0} \ x^{-1}$$

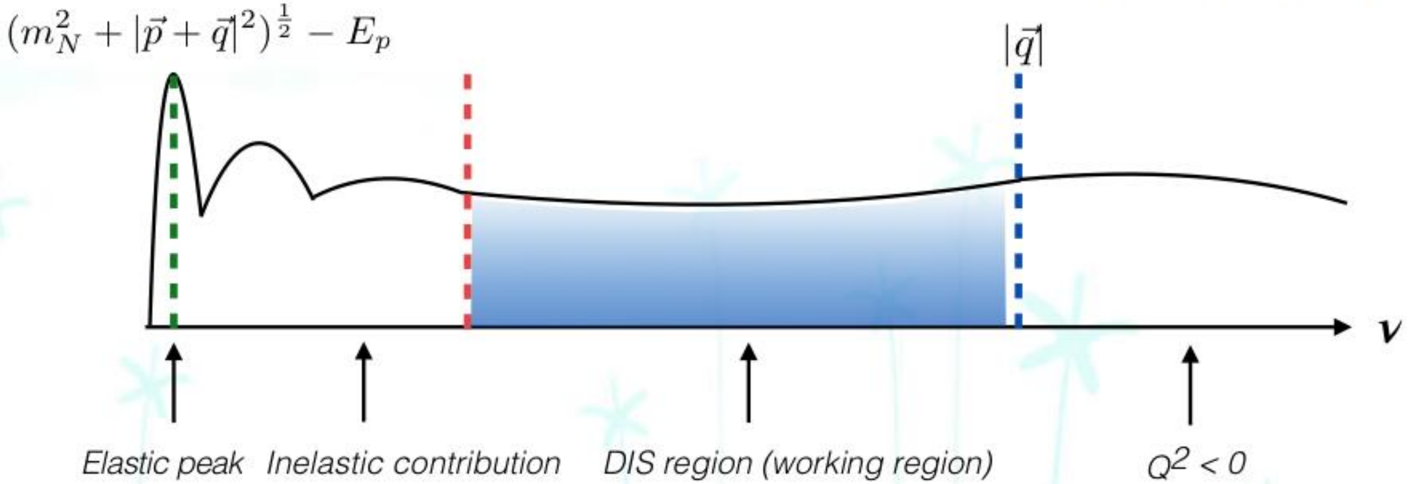
Photo-proton Inclusive X-section



Constraint on kinetics

More structures arise for none-zero p and q





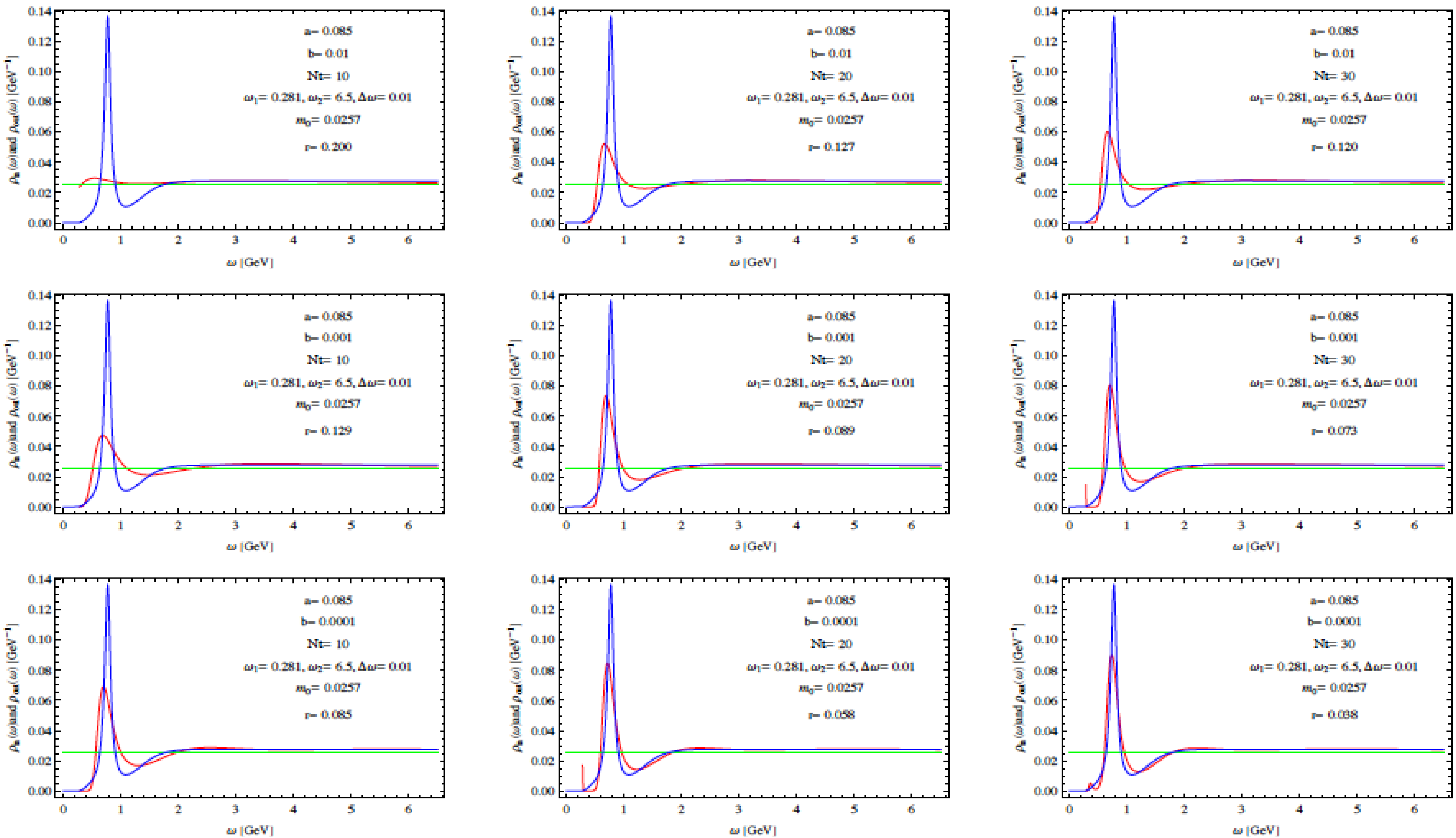
$$Q^2 = |\vec{q}|^2 - \nu^2 > 0 \rightarrow \nu < |\vec{q}|$$

$$\nu > (E_{n=0} - E_P) + \Delta E$$ away from the elastic peak

to enlarge the working region

- **increase momentum transfer**
- **move the position of elastic peak far away from** $|\vec{q}|$ $\quad \vec{q} \sim -\vec{p}$
- **increase energy resolution**

Reconstruction of spectral function



$e^{+}e^{-} \rightarrow r$

Compton Amplitude Approach

- A. J. Chambers et al. (QCDSF), PRL 118, 24001 (2017)
- OPEw/oOPE
  - In Minkowski space, the Compton amplitude is

$$T_{\mu\nu}(p,q) = \rho_{\lambda\lambda'} \int d^4x\, e^{iq\cdot x} \left\langle p,\lambda' \,|\, T(J_\mu(x) J_\nu(0)) \,|\, p,\lambda \right\rangle$$

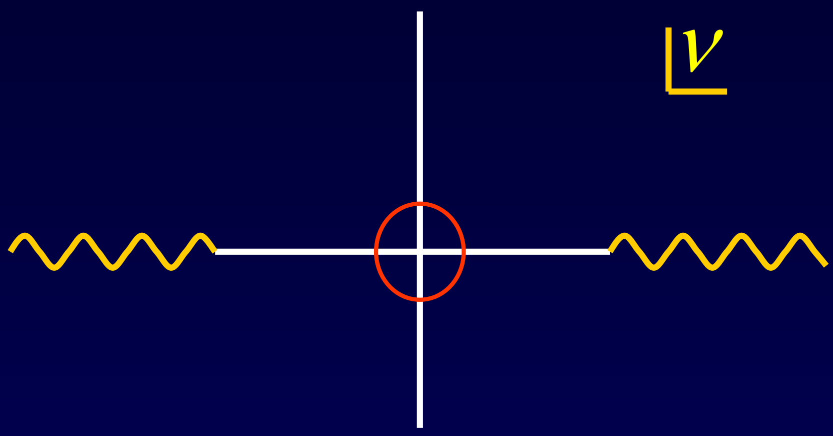
  - and $$W_{\mu\nu}(p,q) = \frac{1}{\pi}\, \mathrm{Im}\, T_{\mu\nu}(p,q)$$

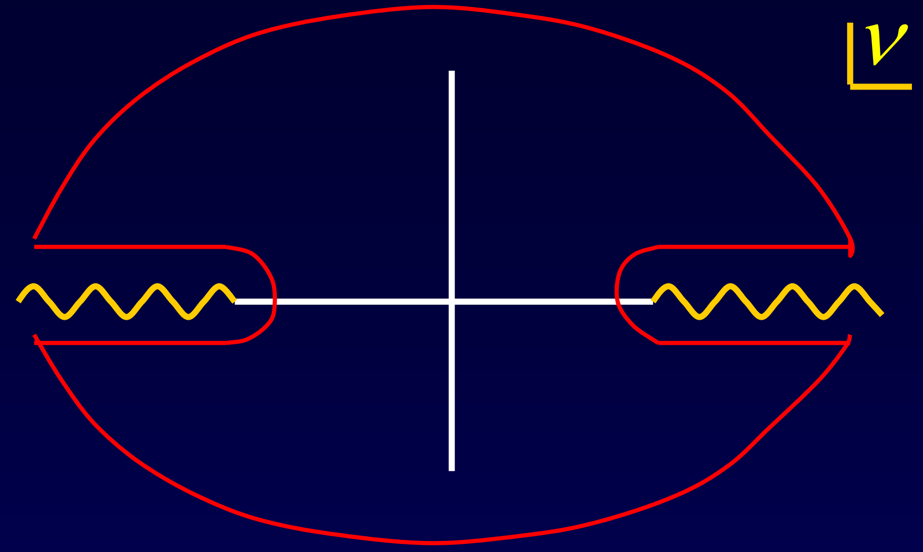
  - In the Euclidean space

$$T_{\mu\nu} \sim \int dn \frac{J_\mu^{pn} J_\nu^{np} \delta^3(\vec{p}_n - \vec{p} - \vec{q})}{\nu - (E_n - E_p)} \lim_{\tau\to\infty} [e^{[\nu-(E_n-E_p)]\tau} - 1]$$

$$+\mu \leftrightarrow \nu,\ q \to -q \qquad \nu > (E_n - E_p)$$

Moments of PDF





$$I_n = \oint \frac{d\nu}{2\pi i}\frac{1}{\nu^{n-1}} T_2(Q^2,\nu),$$

$$= \sum_f 8\, e_f^2 \left(\frac{2M_N}{Q^2}\right)^{n-1} A_f^n$$

$$I_n = 2\int_{Q^2}^{\infty} \frac{d\nu\, 2M_N}{2\pi i}\frac{2i}{\nu^{n-1}} W_2(Q^2,\nu),$$

$$= 8\left(\frac{2M_N}{Q^2}\right)^{n-1}\int_0^1 dx\, x^{n-2}\,\frac{2M_N \nu W_2(Q^2,\nu)}{4}$$

- $A_f^{n=even}(CI) \equiv M_f^n(CI) = \int_0^1 dx\, x^{n-1}(q_V(x) + q_{CS}(x) + \bar{q}_{CS}(x))_f$
- $A_f^{n=odd}(CI) \equiv M_f^n(CI) = \int_0^1 dx\, x^{n-1} q_V(x)_f$
- $A_f^{n=even}(DI) \equiv M_f^n(DI) = \int_0^1 dx\, x^{n-1}(q_{DS}(x) + \bar{q}_{DS}(x))_f$

OPE

- After OPE and relating the matrix elements to the moments of the structure function, it is shown (R. Devenish and A. Cooper-Sarkar) that

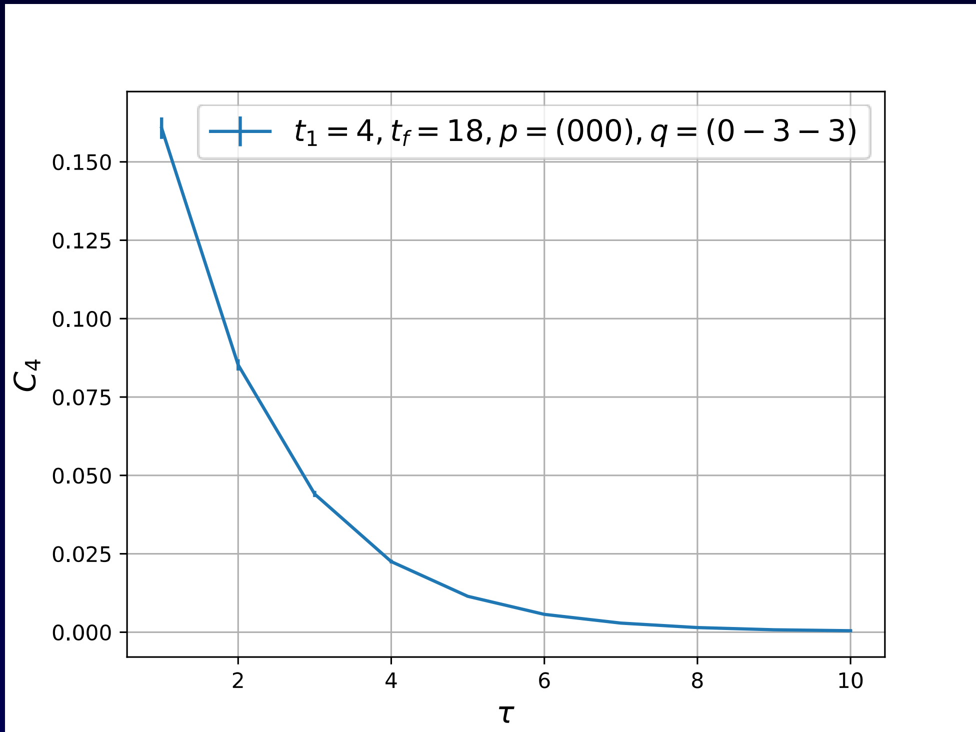
$$T_{33}(p,q) = \sum_{n=2,4,\cdots} 4\omega^n \int_0^1 dx x^{n-1} F_1(x,q^2),$$

$$= 4\omega \int_0^1 dx \frac{\omega x}{1-(\omega x)^2} F_1(x,q^2),$$

where,

$\omega = 2p\cdot q/(-q^2) < 1$ for judiciously chosen $(\vec{p},\vec{q})$ and $\nu=0$

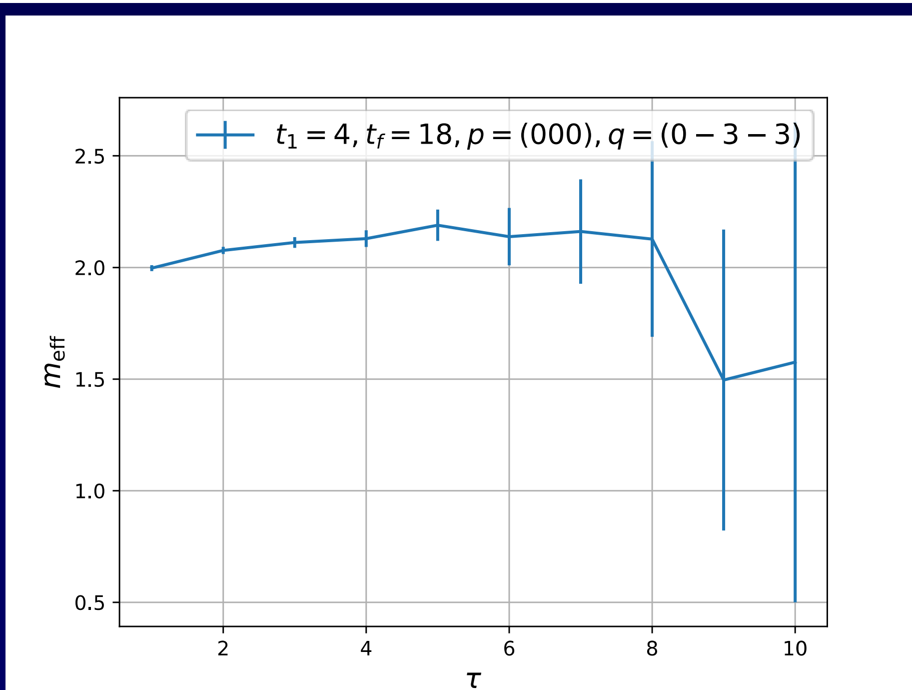
- Note, this is carried out in Minkowski space.

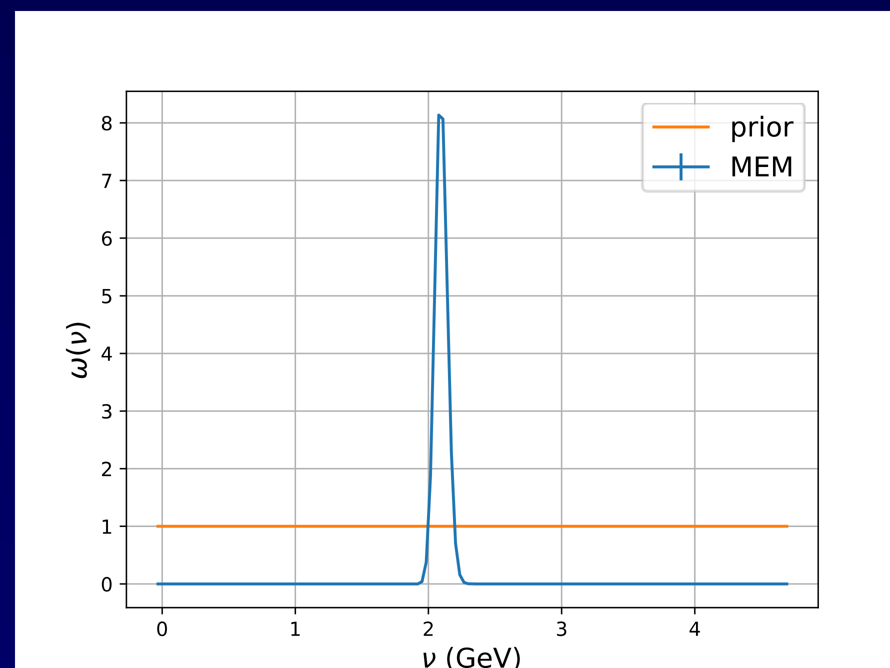


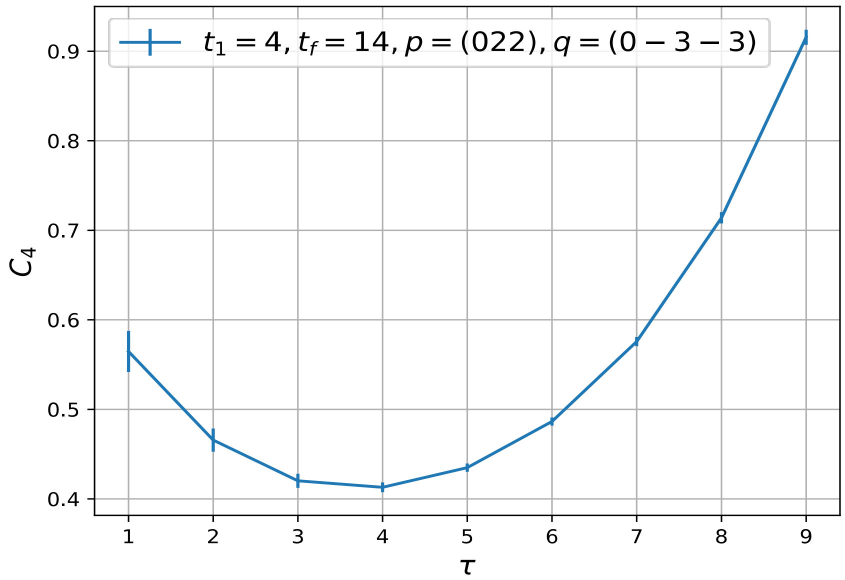
Jian Liang

N_f =2+1
32^3 x 64 DWF lattice
a = 0.06 fm,
m_π=370 MeV
Clover valence

The elastic peak is ~ 100%

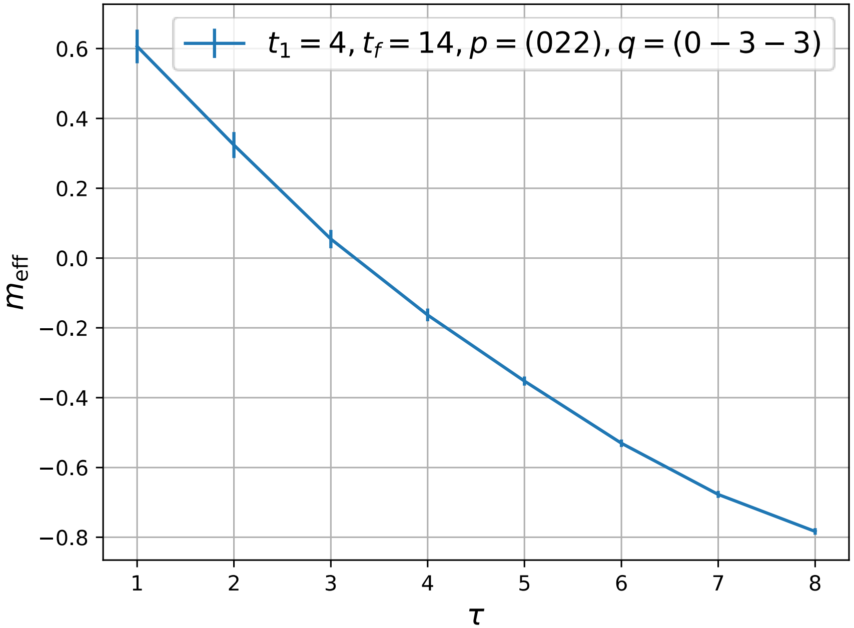


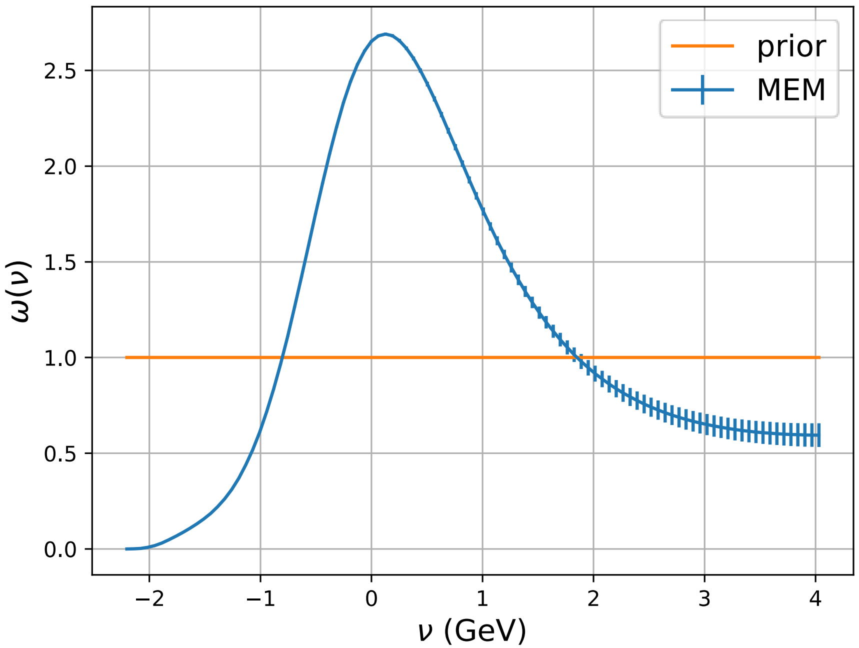




N_f =2+1
32^3 x 64 DWF lattice
a = 0.06 fm, m_π=370 MeV
Clover valence

The elastic peak is ~ 70%





Large Momentum Approach

X. Ji, PRL, 110, 262002 (2013)

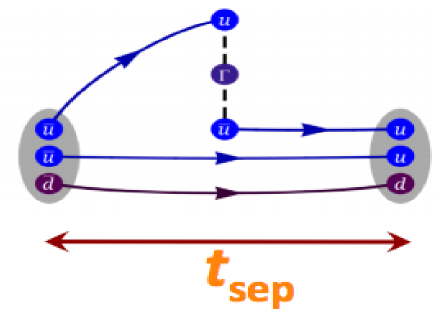
§ Take the large-P_z limit:

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

$x = k^z/P^z$

Lattice z coordinate

Nucleon momentum $P^\mu = \{P^0,0,0,P^z\}$



Product of lattice gauge links

- At $P^z \longrightarrow \infty$ limit, twist-2 parton distribution is recovered
- For finite P^z, corrections are needed

Xiangdong Ji, this Thursday; HWL et al in progress

Quasi-PDF

- Comment by G.C. Rossi and M. Testa (1706.04428)
  - Mixing of lower dimension operators in the trace term in OPE leads to $1/(P_z a)^2$ divergences which need to be subtracted.
- Renormalization of the non-local operator

$$O_R = Z_{\bar{\psi}}^{-1} Z_{\psi}^{-1} e^{\delta m\, z/a}\, \bar{\psi}(z)\Gamma L(z,0)\psi(0), \qquad \delta m = 2\pi\alpha_s/3$$

  - T. Ishikawa, Y.Q. Ma, J.W. Qiu, S. Yoshida, PRD, 1609.02018; X.Ji, Z.H. Zhang, and Y. Zhao, PRL, 1706.08962; J. Green, K. Jansen, and F. Steffens, 1707.07152.

- Fourier transform of the unrenormalized operator which contains the self-energy of the Wilson line

$$\tilde{q}(x,P_z)_{unren} = \int dz\, \mathrm{e}^{ixP_z z} e^{-\delta m z/a} f(z,P_z) = \frac{1}{P_z}\tilde{f}(x + i\frac{\delta m}{P_z a}, P_z)$$

$$= \frac{1}{P_z}[\tilde{f}(x,P_z) + \sum_{n=1}(-1)^n (\frac{\delta m}{P_z a})^{2n} \tilde{f}^{(2n)}(x,P_z)]$$

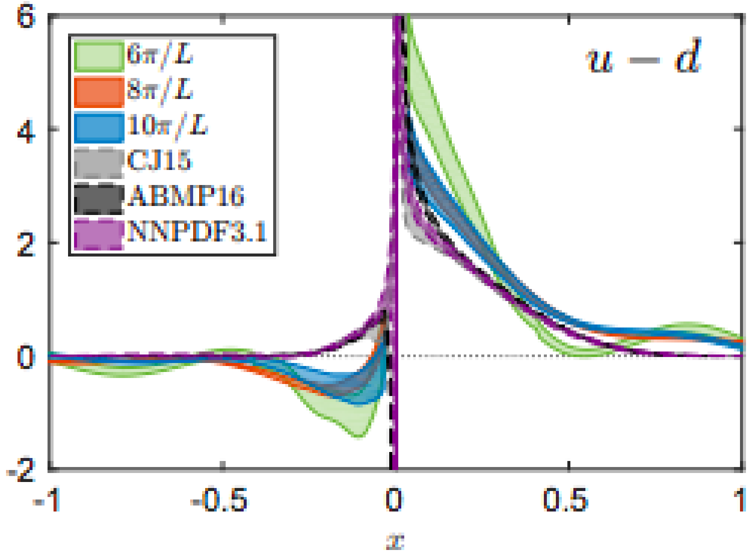
Quasi-PDF

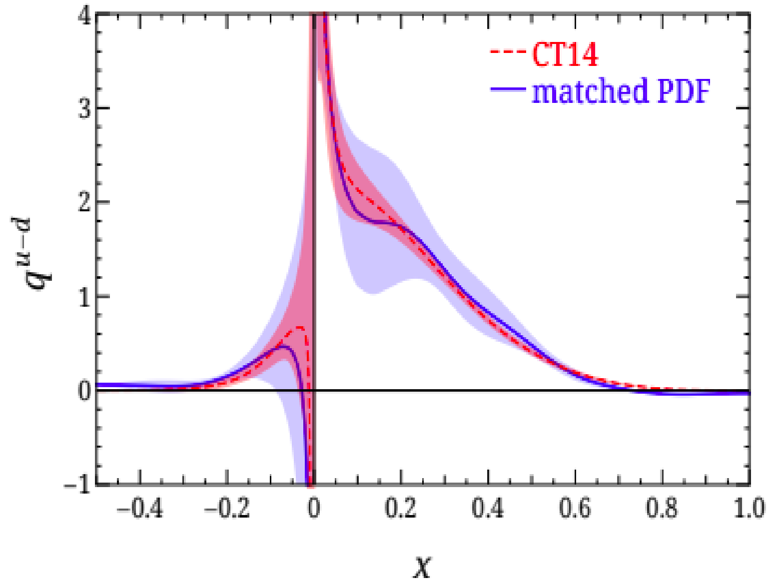
- Renormalization of quasi-distribution (LaMET)

$$\tilde{q}(x,\mu^2,P_z) = \int_0^1 \frac{dy}{y} Z(\frac{x}{y},\frac{\mu}{P_z})\ q(y,\mu^2) + O(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2})$$

  - Unlike global fitting with limited x data at each Q^2, the inversion is not a problem here.
  - The connected insertion has no mixing from the glue and disconnected insertions (M. Deka et al., 1312.4816)

$$\begin{pmatrix} <x>_q^R(CI) \\ <x>_q^R(DI) \\ <x>_G^R \end{pmatrix} = \begin{pmatrix} Z_{qq}(CI) & 0 & 0 \\ Z_{qq}(DI) & Z_{qq}(CI+DI) & Z_{qG} \\ Z_{Gq} & Z_{Gq} & Z_{GG} \end{pmatrix} \begin{pmatrix} <x>_q^L(CI) \\ <x>_q^L(DI) \\ <x>_G^L \end{pmatrix}$$





C. Alexandrou, et al., 1803.02685

J.W. Chen et al. (LP3), 1803.04393

- Useful and meaningful to separately look at

$u(x)$, $\bar{u}_{CS}(x)=-u(-x)$, $d(x)$, $\bar{d}_{CS}(x)=-d(-x)$ (N.B. $x > 0$)

Inclusive B meson semileptonic decay

- Shape function including $B \to D(D^*)l\nu$
  - U. Aglietti et al., hep-ph/9804416
  - S. Hashimoto, 1703.01881

$$W_{\mu\nu}(\vec{q},\vec{p},\nu) = \frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu} = \left\langle B \middle| \int \frac{d^4x}{4\pi} e^{iq\cdot x} J_\mu(x) J_\nu(0) \middle| B \right\rangle$$

Summary

- Quasi-PDF has made a great progress in just a few years.
- Hadronic tensor is being studied with various inversion algorithms. It can address the elastic scattering as well as deep inelastic scattering structure functions.
- It would be useful to compare different approaches to PDF which are subjected to different systematics as a cross check for validity and reliability of lattice methods.