Nucleon's axial charge

Lattice PDF Workshop
Univ. of Maryland
6-8 April, 2018

BERKELEY LAB
André Walker-Loud

A percent-level determination of the nucleon axial coupling from QCD
arXiv:1704.01114: (updated data set)

Lattice QCD Team

Glasgow: Chris Bouchard
INT: Chris Monahan
JLab: Balint Jóo
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LLNL: Pavlos Vranas
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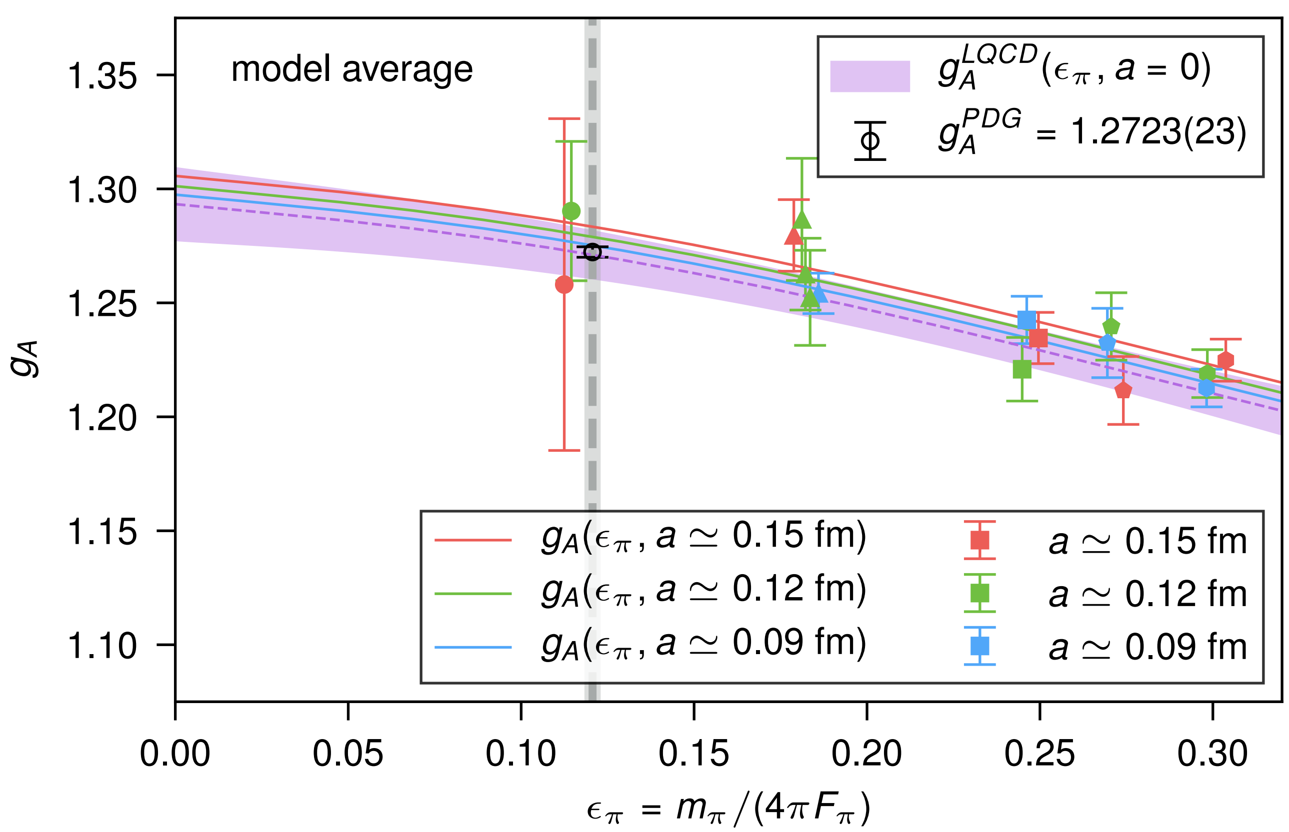
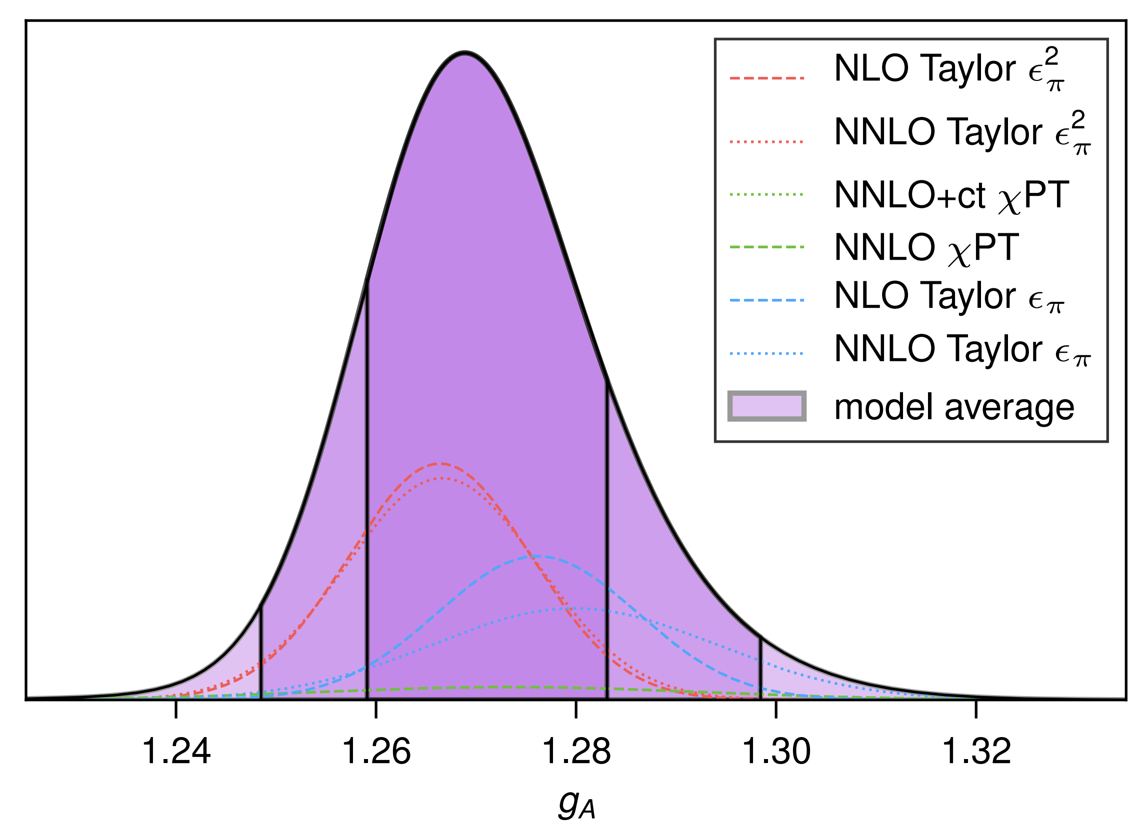
UNC: Amy Nicholson
William and Mary: Kostas Orginos

plus a few others

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$
$$= 1.2711(126)$$
$$g_A^{\rm UCNA} = 1.2772(020)$$

experiment factor of 6 more precise





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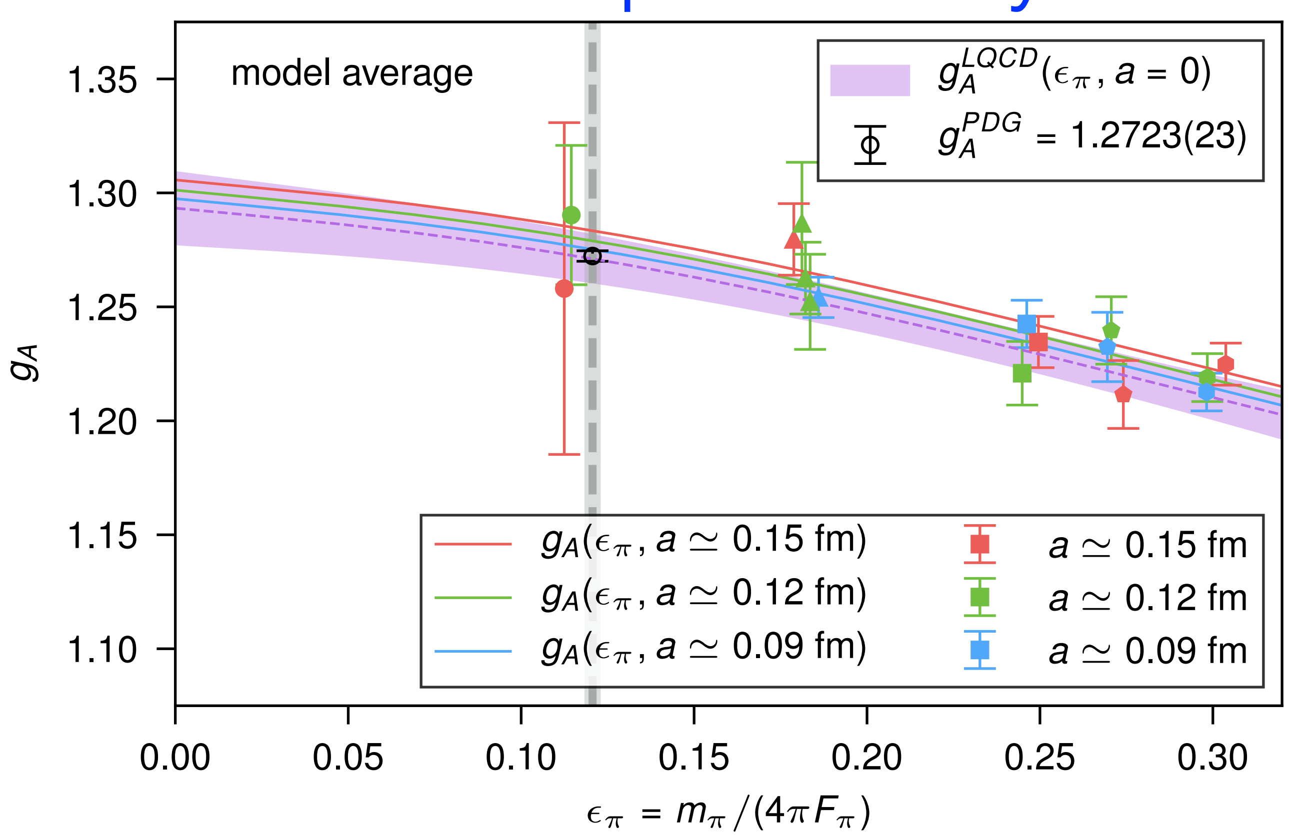
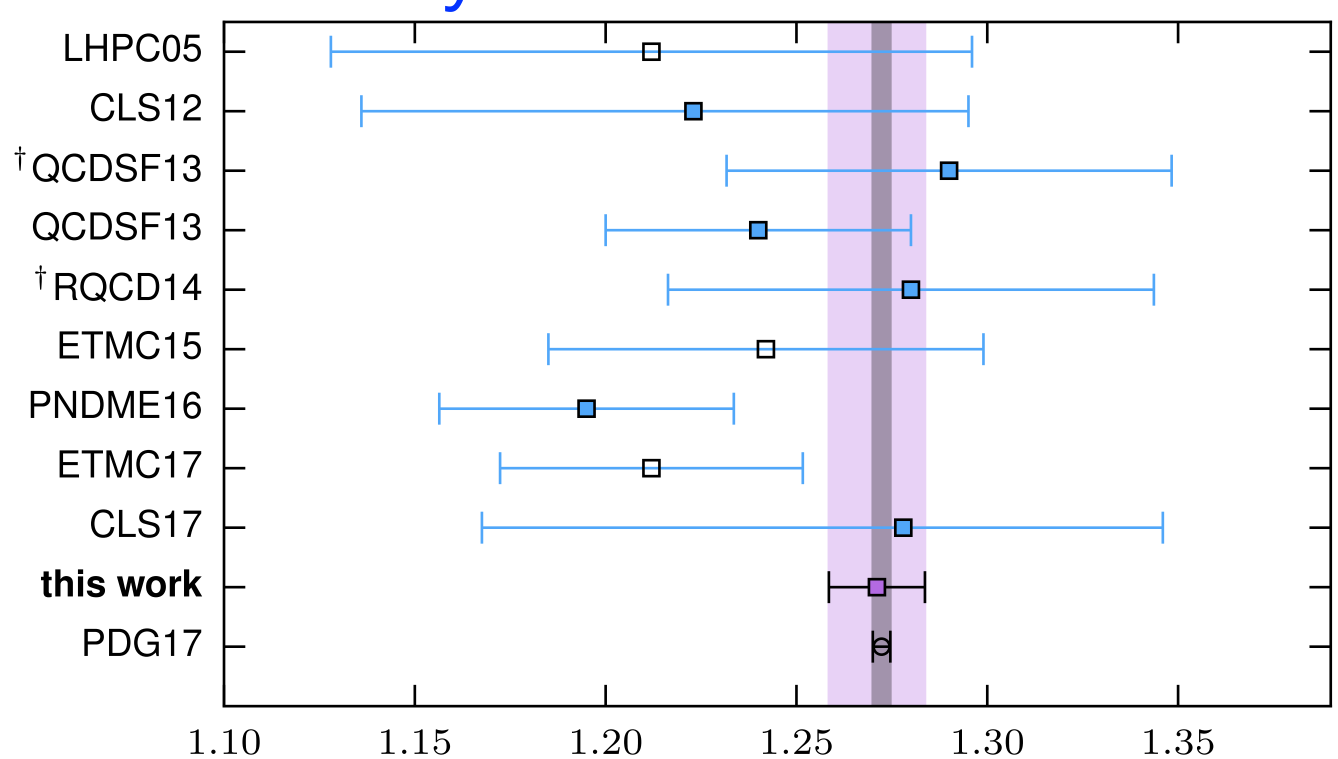
$$= 1.2711(126)$$

previously estimated 2% by 2020

LQCD results





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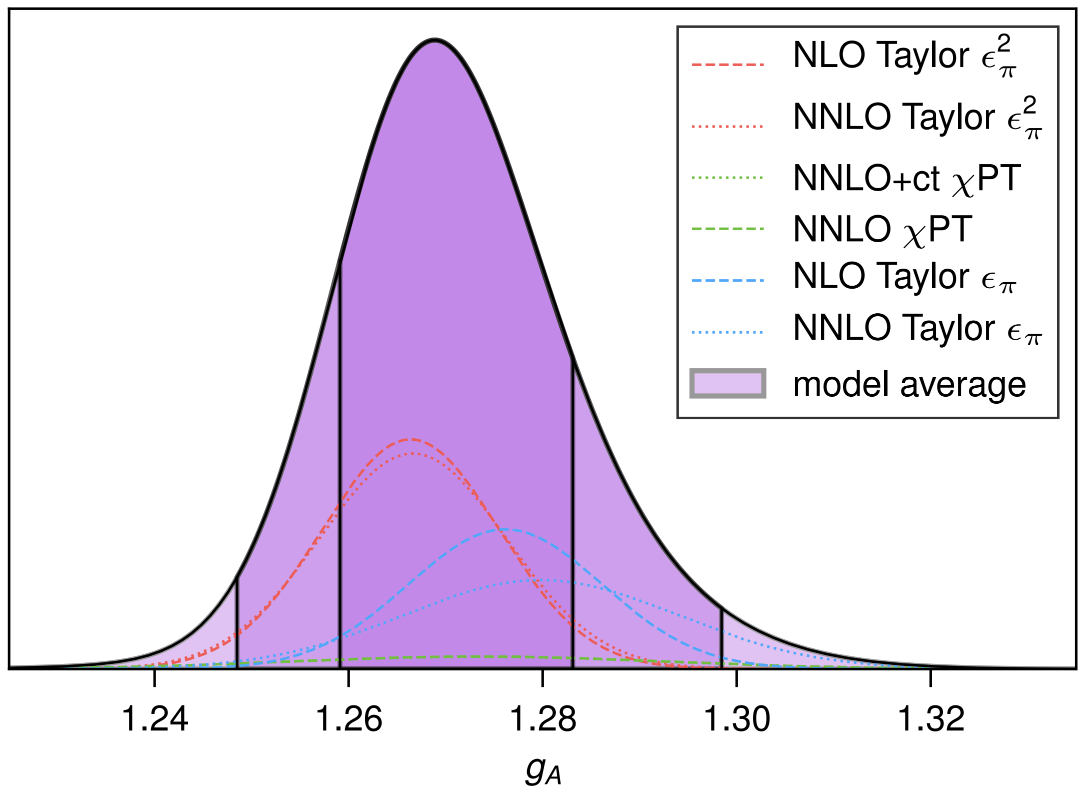
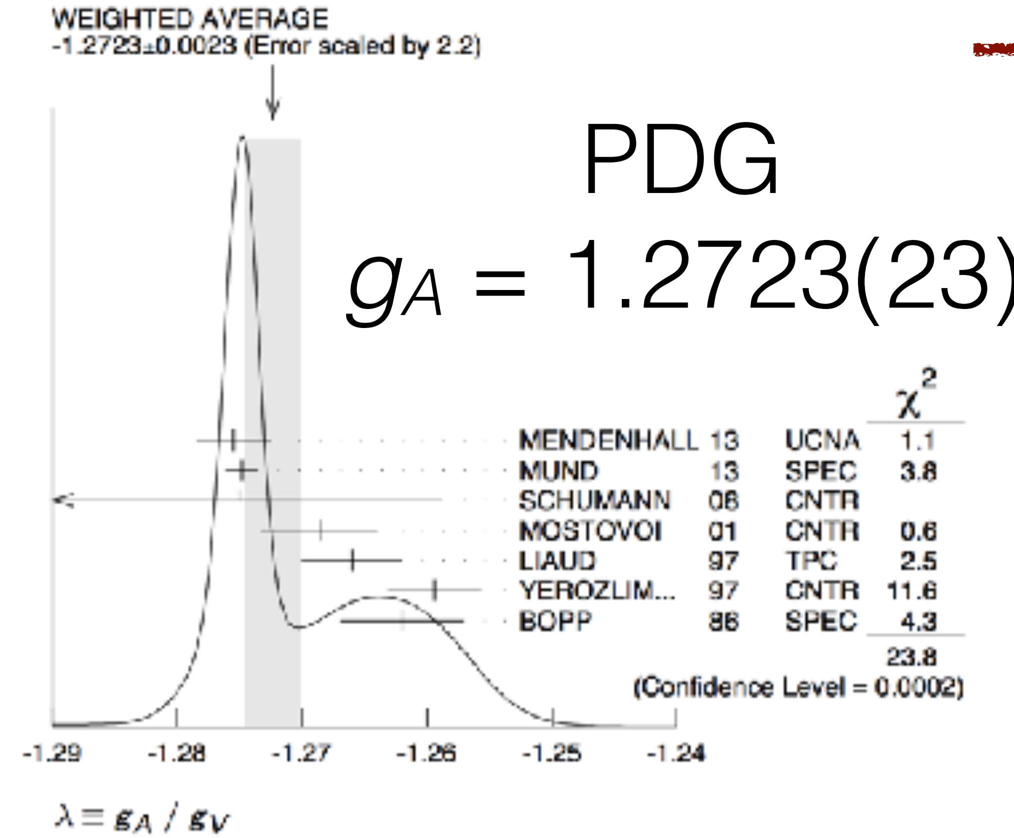




$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- The success of this result was enabled through a few features of the calculation:
  - an improved strategy that can exploit **exponentially more precise data** at early time and has **demonstrable control of excited state** contributions
  - an action with **improved stochastic behavior**, a very **mild continuum** extrapolation, **highly suppressed chiral symmetry breaking**
  - access to a set of ensembles (**MILC**) that allowed for control over all standard lattice systematics (physical pion mass, continuum and infinite volume limits)
  - ***ludicrously fast*** GPU code (Quda)

An Improved Computational Strategy

arXiv.org > hep-lat > arXiv:1612.06963 ***Phys. Rev.* D96 (2017)**

High Energy Physics – Lattice

On the Feynman–Hellmann Theorem in Quantum Field Theory and the Calculation of Matrix Elements

Chris Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

(Submitted on 21 Dec 2016 (v1), last revised 5 Jul 2017 (this version, v2))

- Take the Feynman-Hellmann (FH) Theorem as a starting point: $\partial_\lambda E_n = \langle n|H_\lambda|n\rangle$
- conceptually very simple and straightforward
- applying the FH Theorem to the effective mass directly leads to the method we use
  - relates matrix elements to functional derivatives of the partition function
  - reduces the dependence on two time variables (operator insertion time, τ, and source/sink separation time, t) to a single variable, t
  - allows for demonstrable control of excited state systematics, reduced sensitivity to correlated fluctuations & the extraction of the signal early in Euclidean time (exponentially smaller relative noise)

An Improved Computational Strategy

arXiv:1612.06963

Consider a two point correlation function in the presence of some source

$$C_\lambda(t) = \langle\lambda|\hat{O}(t)\hat{O}^\dagger(0)|\lambda\rangle$$
$$= \frac{1}{\mathcal{Z}_\lambda}\int D\Phi e^{-S-S_\lambda}O(t)O^\dagger(0)$$

$$|\lambda\rangle \equiv \lambda\text{-vacuum}$$

$$|\Omega\rangle \equiv \lim_{\lambda\to 0}|\lambda\rangle$$

$$\mathcal{Z}_\lambda \equiv \mathcal{Z}[\lambda] = \int D\Phi e^{-S}e^{-S_\lambda}$$

$$S = \text{action for sourceless theory}$$

$$S_\lambda = \lambda\int d^4x j(x)$$

$$j(x) = \text{ some bi-linear current density}$$

$$\text{e.g. } \lambda j(x) = \bar{q}(x)m_q q(x)$$

An Improved Computational Strategy

arXiv:1612.06963

We can differentiate the correlation function with respect to **λ** (this can be built from a sum of functional derivatives over all spacetime)

$$-\frac{\partial C_\lambda}{\partial \lambda} = \frac{\partial_\lambda \mathcal{Z}_\lambda}{\mathcal{Z}_\lambda} C_\lambda(t) + \frac{1}{\mathcal{Z}_\lambda} \int D\Phi e^{-S-S_\lambda} \int d^4x' j(x')\ \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

The first term is proportional to a vacuum matrix element and the second contains the matrix element we are interested in. We are really interested in the linear-response

$$-\frac{\partial C_\lambda(t)}{\partial \lambda}\bigg|_{\lambda=0} = -C_\lambda(t) \int d^4x' \langle \Omega | j(x') | \Omega \rangle + \int dt' \langle \Omega | T\{\mathcal{O}(t) J(t') \mathcal{O}^\dagger(0)\} | \Omega \rangle$$

$$J(t) = \int d^3x\ j(t, \mathbf{x})$$

An Improved Computational Strategy

arXiv:1612.06963

The FHT relates matrix elements to the spectrum. Can we find something similar in QFT?

Let us try the first obvious thing, take a derivative of the effective mass:

$$m^{eff}(t,\tau) = \frac{1}{\tau}\ln\left(\frac{C(t)}{C(t+\tau)}\right) \underset{t\to\infty}{\longrightarrow} \frac{1}{\tau}\ln(e^{E_0\tau})$$

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

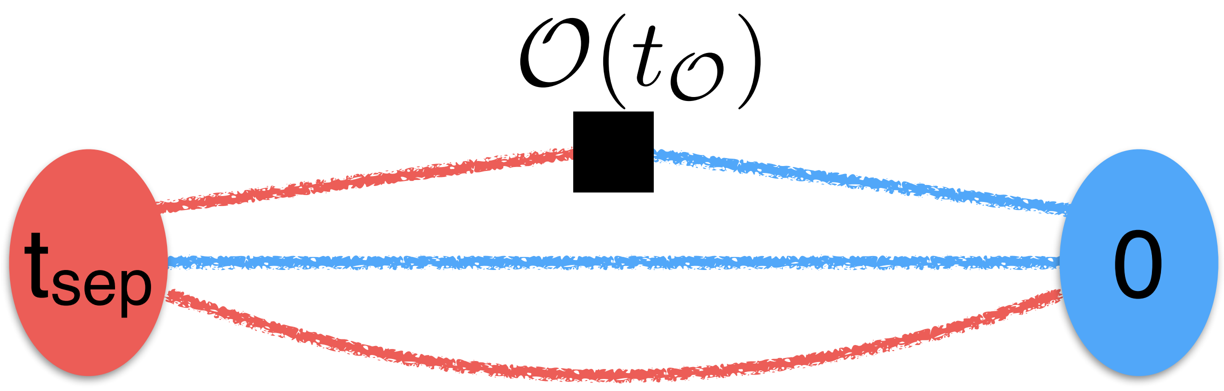
NOTE: even for currents with non-vanishing vacuum matrix elements, this contribution exactly cancels in this quantity

$$-\left.\frac{\partial C_\lambda(t)}{\partial\lambda}\right|_{\lambda=0} = -\,C_\lambda(t)\int d^4x'\langle\Omega|j(x')|\Omega\rangle$$
$$+\int dt'\langle\Omega|T\{\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)\}|\Omega\rangle$$

An Improved Computational Strategy

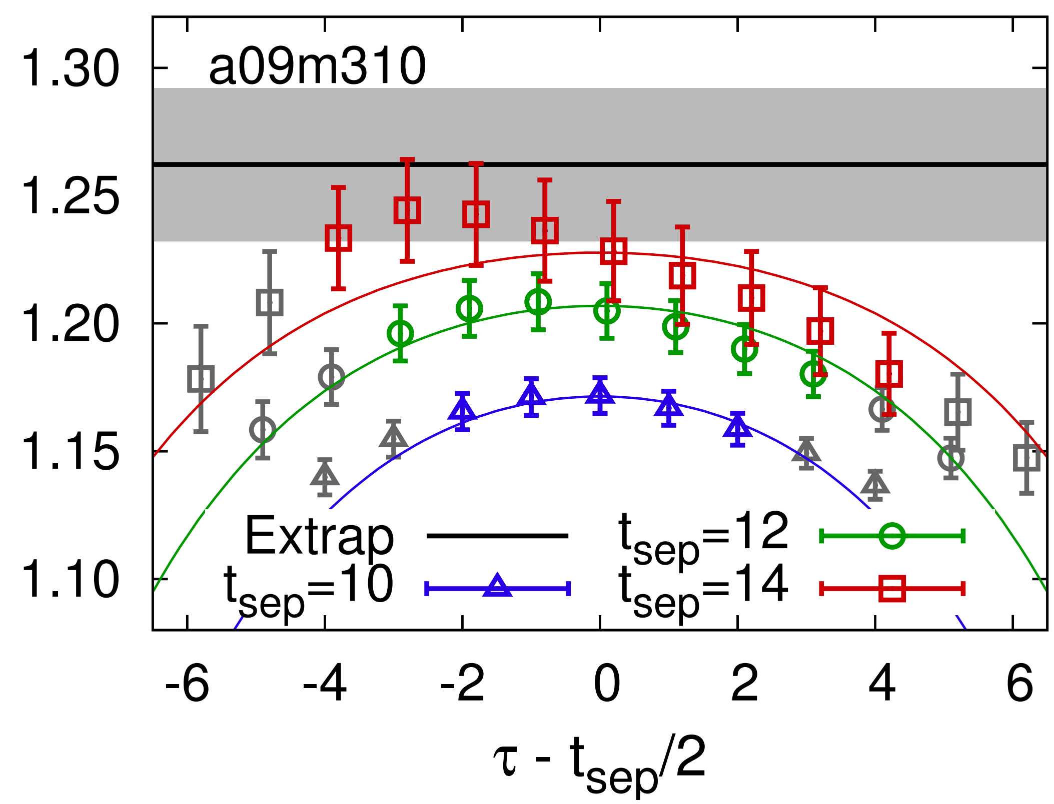
arXiv:1612.06963

standard method



fixed source-sink separation time

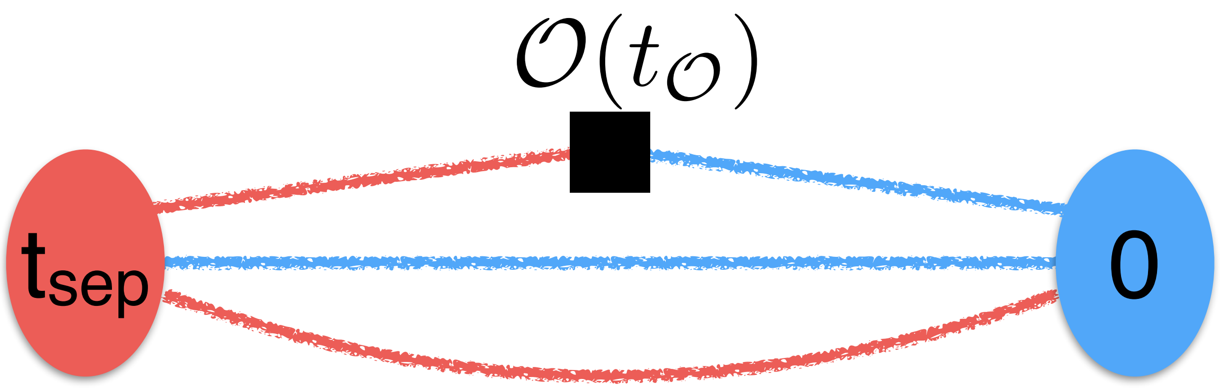
PNDME arXiv:1606.07049



An Improved Computational Strategy

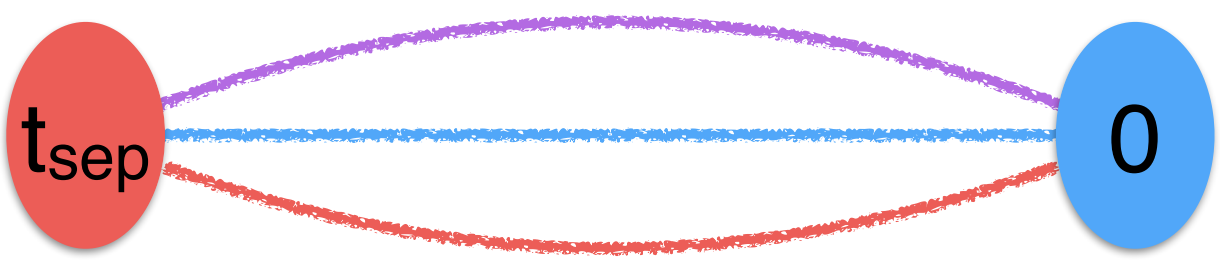
arXiv:1612.06963

standard method



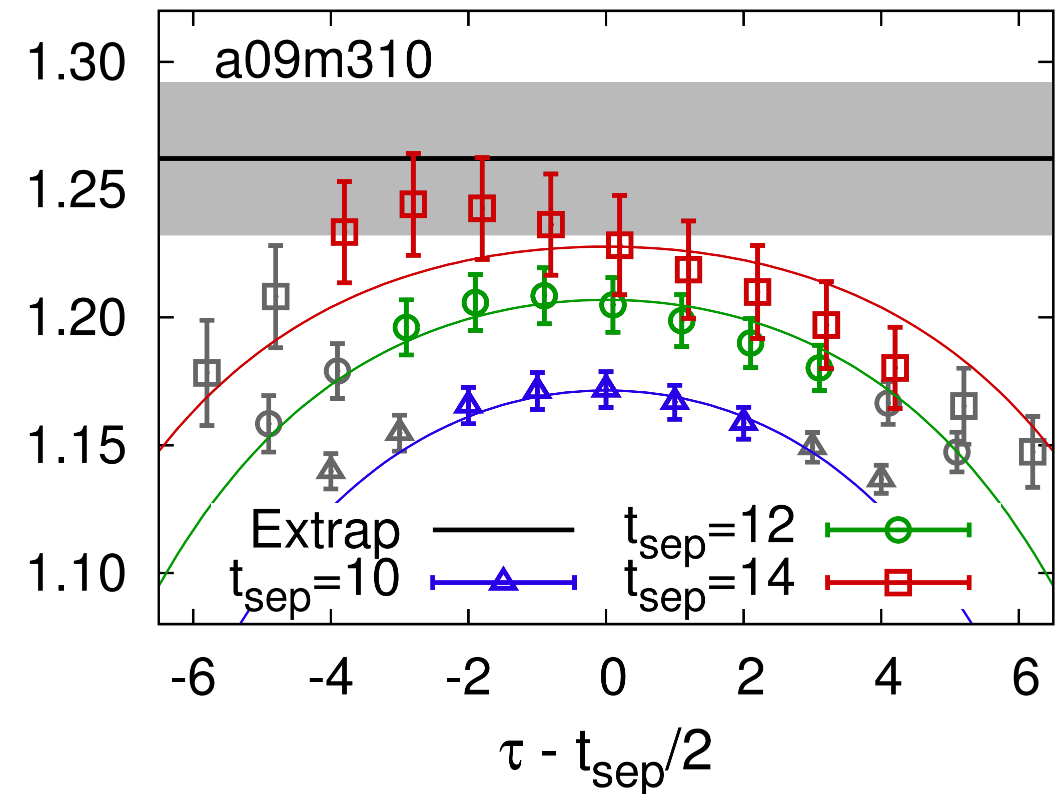
fixed source-sink separation time

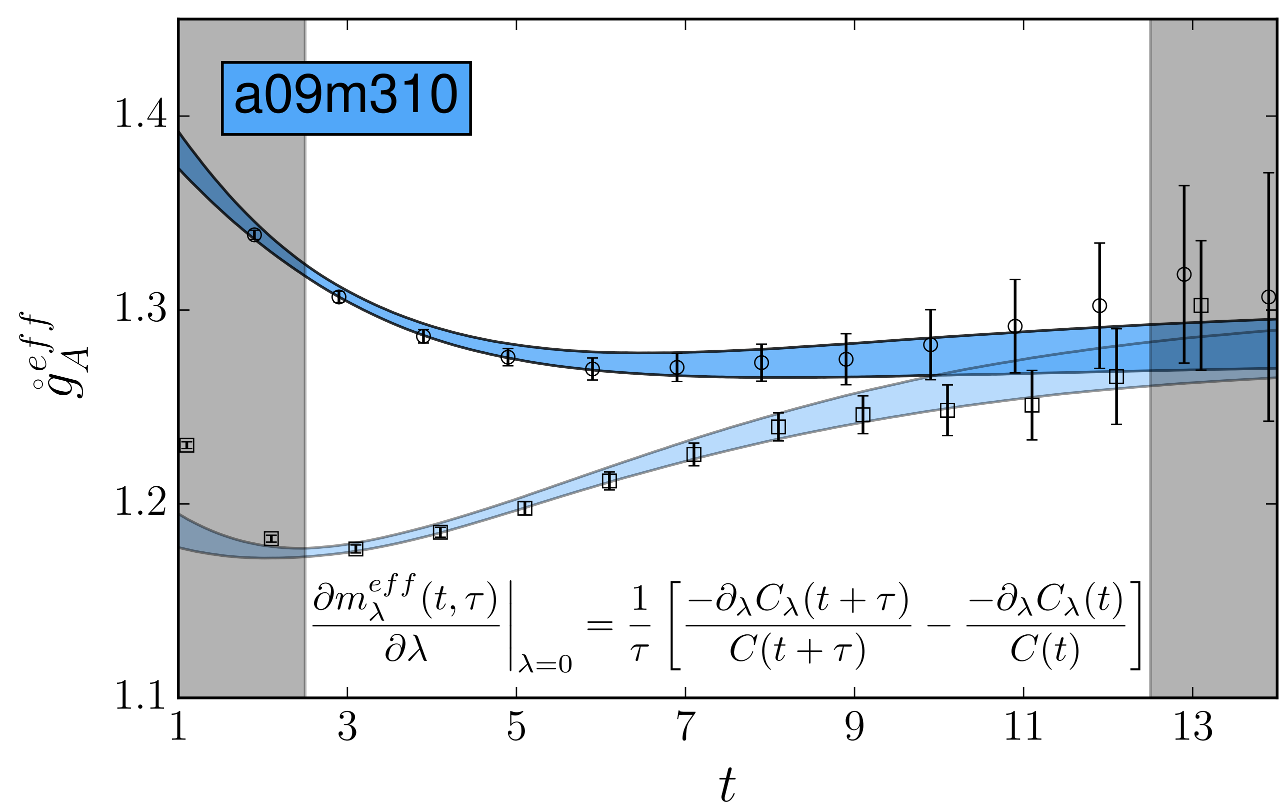
our new method



numerical cost = single fixed src/snk separation

PNDME arXiv:1606.07049





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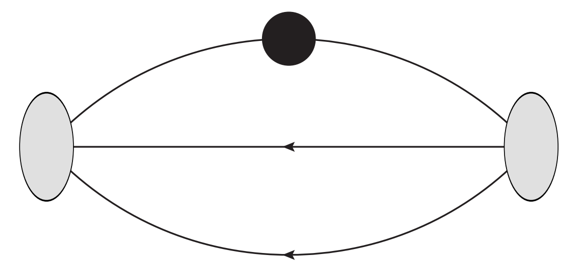
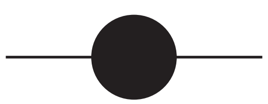
Numerical Implementation:



the “Feynman-Hellman” propagator is given by



$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

$S(z,x)$ standard quark propagator off some source at x, to all z

$\Gamma(z)$ some bi-linear operator (can be constant)
e.g., γ_4 for the vector current

$\Gamma(z)S(z,x)$ treat like a source to invert off of

NOTE: this is the same equation as appears in de Divitiis, Petronzio, Tantalo, PLB718 (2012) can be traced back to Maiani, Martinelli, Paciello and Taglienti Nucl. Phys. B293 (1987)

Similar ideas in literature:
Chambers et. al. Phys.Rev. D90 [arXiv:1405.3019]
Chambers et. al. Phys.Rev. D92 [arXiv:1508.06856]
Savage et. al. Phys.Rev.Lett. 119 [arXiv:1610.04545]

Already used for new processes!
(related to the topic of this workshop)
Orginos, Radyushkin, Karpie, Zafeiropoulos
Phys.Rev. D96 [arXiv:1706.05373]

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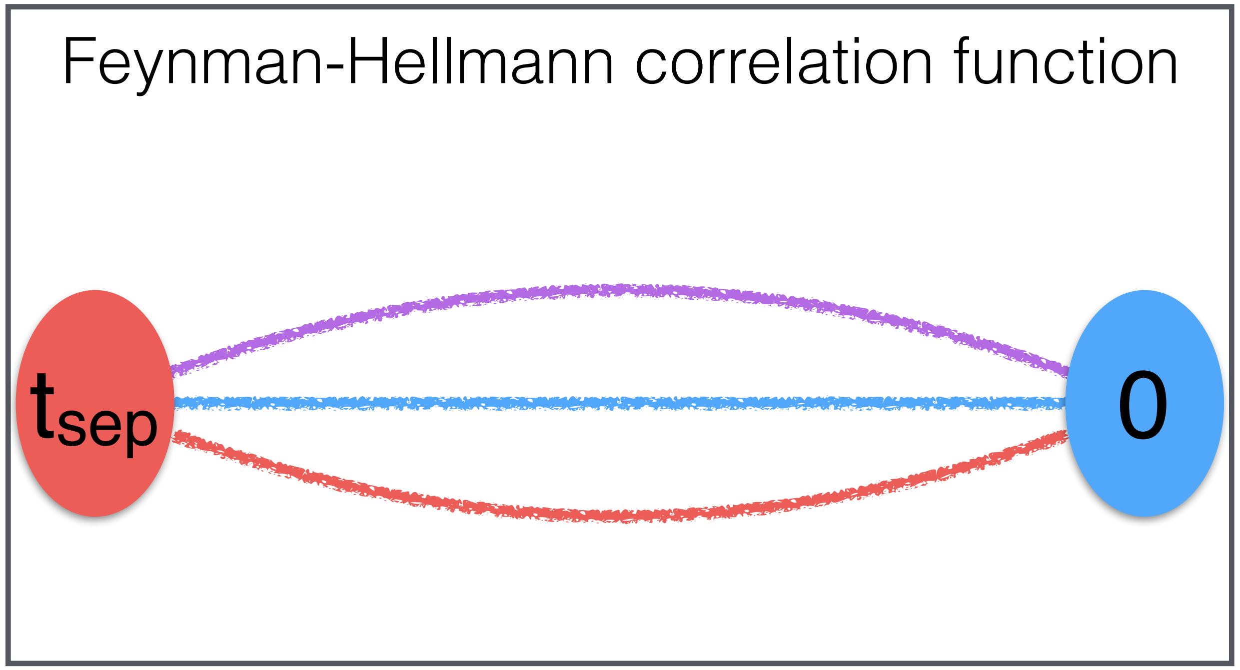

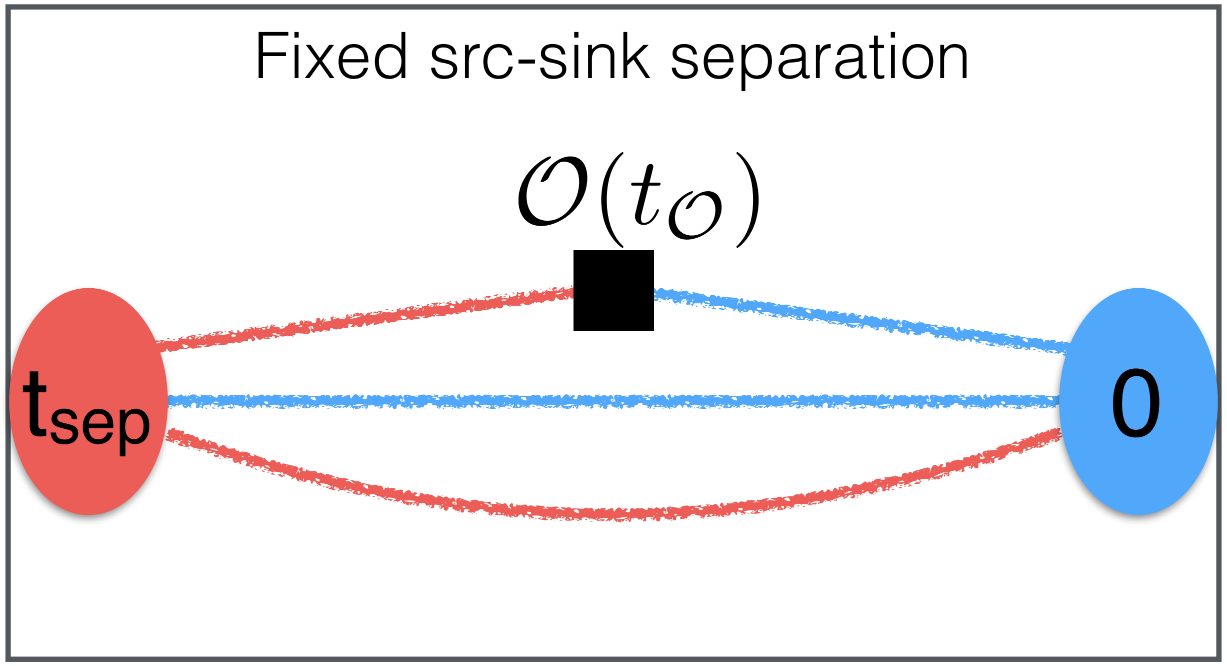

- Our Feynman-Hellmann method is similar to the Summation Method, in which several calculations with fixed src-sink separation times are performed, and the current is summed between the src and sink
- We sacrifice the flexibility to do any current insertion to perform the sum over all current insertion times at the cost of a single fixed src-sink calculation, providing access to short and long time separations
- The short time separation has exponentially better signal-to-noise, allowing for a more precise determination (order of magnitude)

An Improved Computational Strategy

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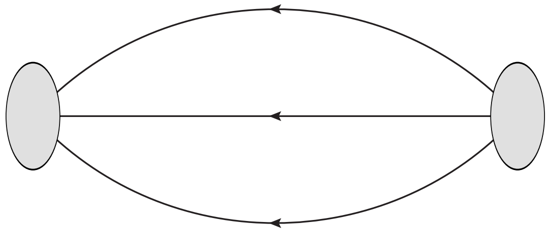
Proton

$$\langle \Omega | N_\gamma(y) \bar{N}_{\gamma'}(x) | \Omega \rangle$$



creation/annihilation operators

$$\bar{N}_{\gamma'} = \epsilon_{i'j'k'} P_{\gamma'\rho'} \; \bar{u}^{i'}_{\rho'} (\bar{u}^{j'}_{\alpha'} \Gamma^{\dagger, src}_{\alpha'\beta'} \bar{d}^{k'}_{\beta'})$$

$$N_\gamma = \epsilon_{ijk} P_{\gamma\rho} \; u^i_\rho (u^j_\alpha \Gamma^{snk}_{\alpha\beta} d^k_\beta)$$

quark propagators

$$U(y,x)^{ii'}_{\alpha\alpha'} = u^i_\alpha(y) \bar{u}^{i'}_{\alpha'}(x) \,,$$

$$D(y,x)^{ii'}_{\alpha\alpha'} = d^i_\alpha(y) \bar{d}^{i'}_{\alpha'}(x) \,,$$

$\langle \Omega | N_\gamma(y) \bar{N}_{\gamma'}(x) | \Omega \rangle$ =

$$C_{\gamma\gamma'} = \epsilon_{ijk} \epsilon_{i'j'k'} P_{\gamma'\rho'} \Gamma^{src}_{\alpha'\beta'} \left[P_{\gamma\rho} \Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha} \Gamma^{snk}_{\rho\beta} \right] U^{ii'}_{\rho\rho'} U^{jj'}_{\alpha\alpha'} D^{kk'}_{\beta\beta'}$$

An Improved Computational Strategy

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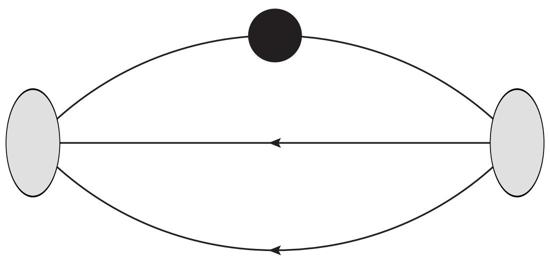
Proton - with FH propagator $\langle \Omega | N_\gamma(y) \bar{N}_{\gamma'}(x) | \Omega \rangle$



down-quark

$$C^{\Gamma d}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} P_{\gamma'\rho'} \Gamma^{src}_{\alpha'\beta'} \left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta} \right] U^{ii'}_{\rho\rho'} U^{jj'}_{\alpha\alpha'} \underline{D^{\Gamma,kk'}_{\beta\beta'}}$$

up-quark

$$C^{\Gamma u}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} P_{\gamma'\rho'} \Gamma^{src}_{\alpha'\beta'} \left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta} \right] \Big[\underline{U^{\Gamma,ii'}_{\rho\rho'}} U^{jj'}_{\alpha\alpha'} D^{kk'}_{\beta\beta'} + U^{ii'}_{\rho\rho'} \underline{U^{\Gamma,jj'}_{\alpha\alpha'}} D^{kk'}_{\beta\beta'} \Big]$$

up←down

$$C^{u\leftarrow d}_{\gamma\gamma'} = \epsilon_{ijk}\epsilon_{i'j'k'} \left[P_{\gamma\rho}\Gamma^{snk}_{\alpha\beta} + P_{\gamma\alpha}\Gamma^{snk}_{\rho\beta} \right] \left[P_{\gamma'\rho'}\Gamma^{src}_{\alpha'\beta'} + P_{\gamma'\beta'}\Gamma^{src}_{\alpha'\rho'} \right] U^{ii'}_{\alpha\alpha'} D^{jj'}_{\beta\beta'} \underline{(U \leftarrow D)}^{kk'}_{\rho\rho'}$$

NOTE: this method does NOT require any actual background field. Instead, we have analytically determined the linear-response correlation function

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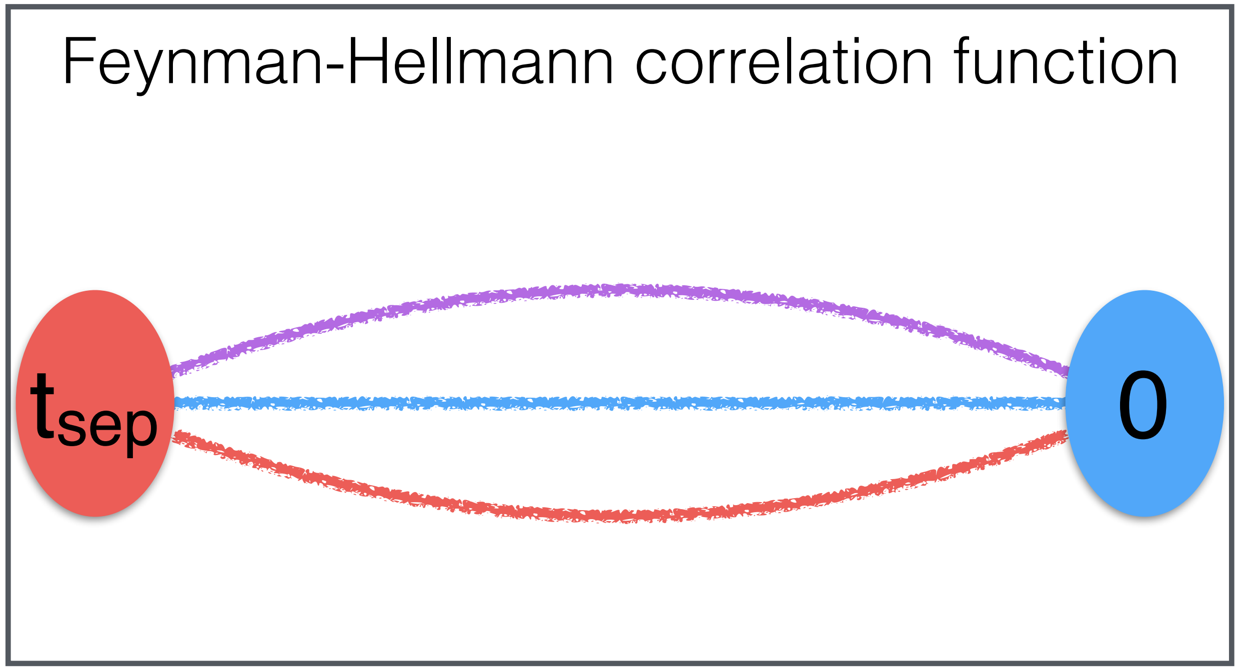
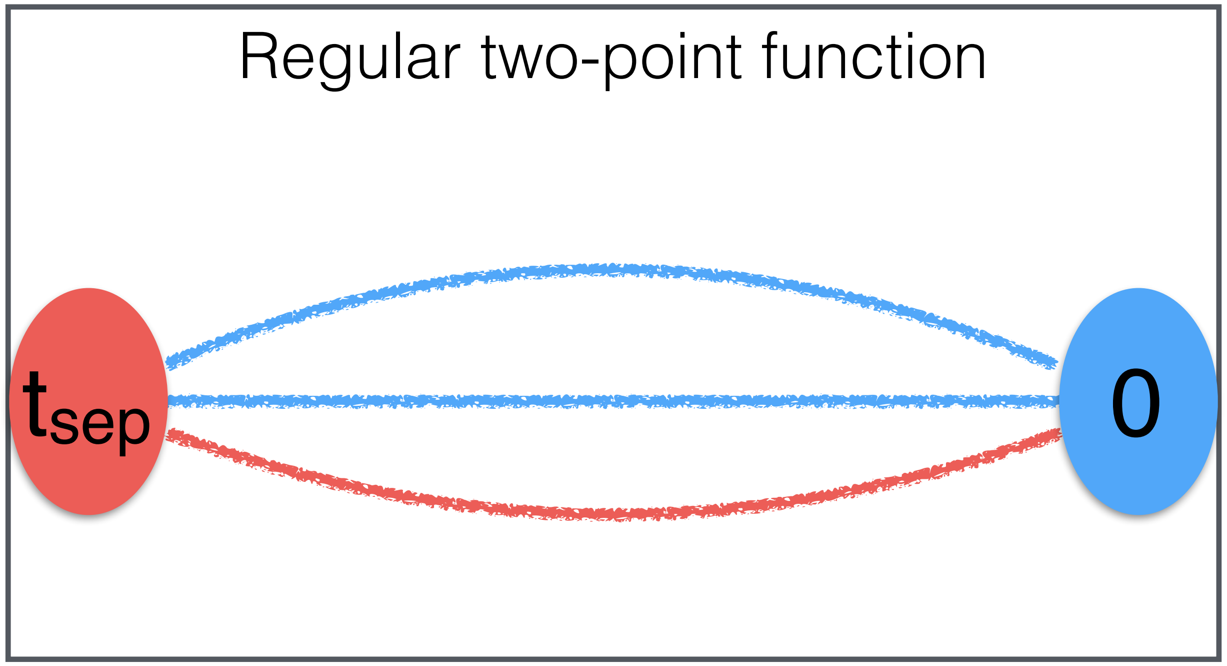
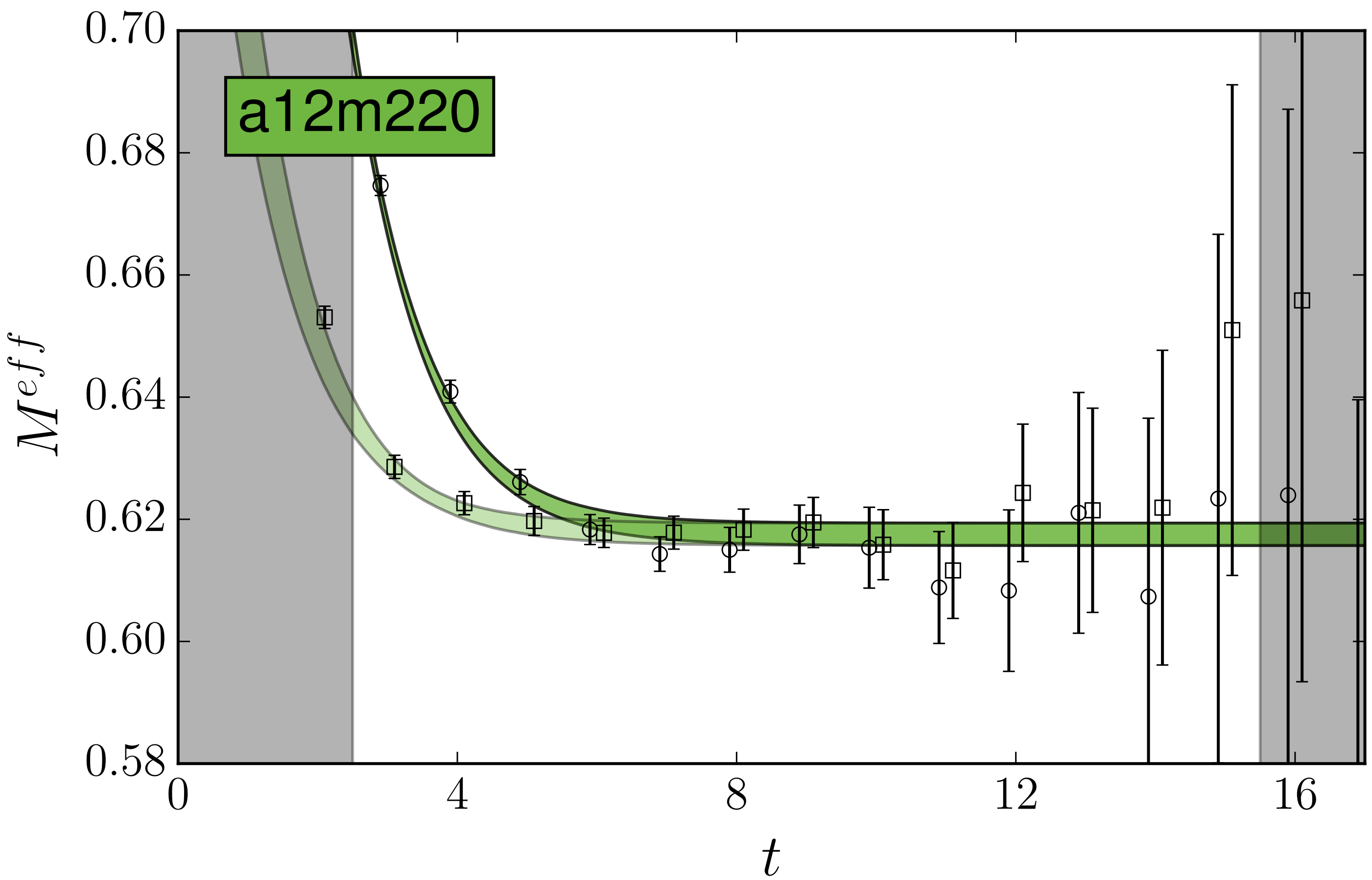




$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

What is spectral decomposition of Feynman-Hellmann correlation function?



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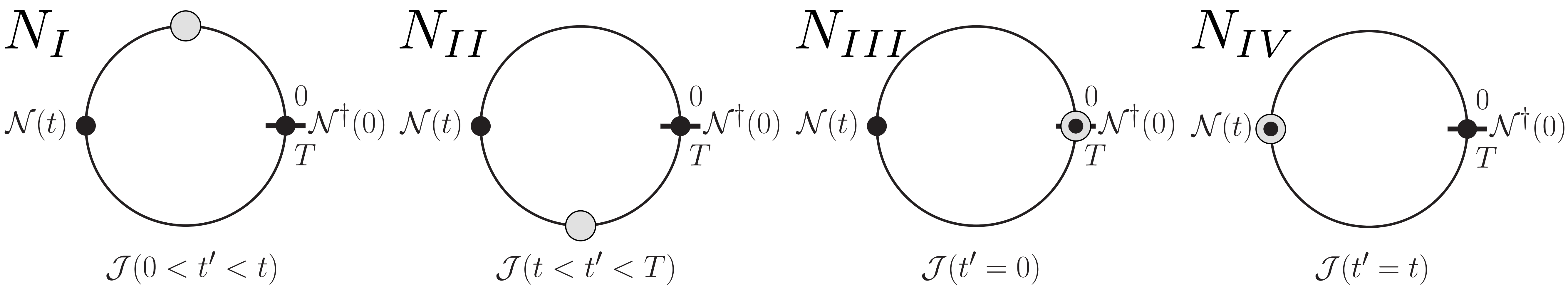

$$N(t) = \sum_n \left[(t-1) z_n g_{nn} z_n^\dagger + d_n\right] e^{-E_n t} + \sum_{n \neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\Delta_{nm}/2} - e^{-E_m t} e^{\Delta_{mn}/2}}{e^{\Delta_{mn}/2} - e^{\Delta_{nm}/2}}$$

$$z_n \equiv \frac{Z_n}{\sqrt{2E_n}} \qquad g_{nm} \equiv \frac{J_{nm}}{\sqrt{4E_n E_m}} \qquad d_n \equiv Z_n Z_{J:n}^\dagger + \text{h.c.} + Z_n Z_n^\dagger J_{\Omega\Omega} + \sum_j \frac{Z_n Z_{nj}^\dagger J_j^\dagger + \text{h.c.}}{2E_j(e^{E_j} - 1)}$$

$$\Delta_{nm} \equiv E_n - E_m$$

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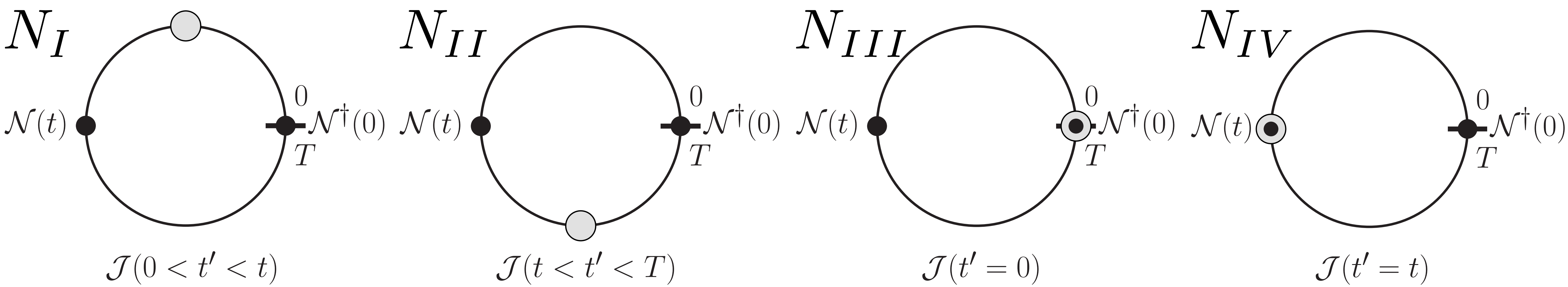

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matrix elements of interest

$$g_{00} = g_A \qquad \mathcal{J} = \bar{u}\gamma_3\gamma_5 d$$

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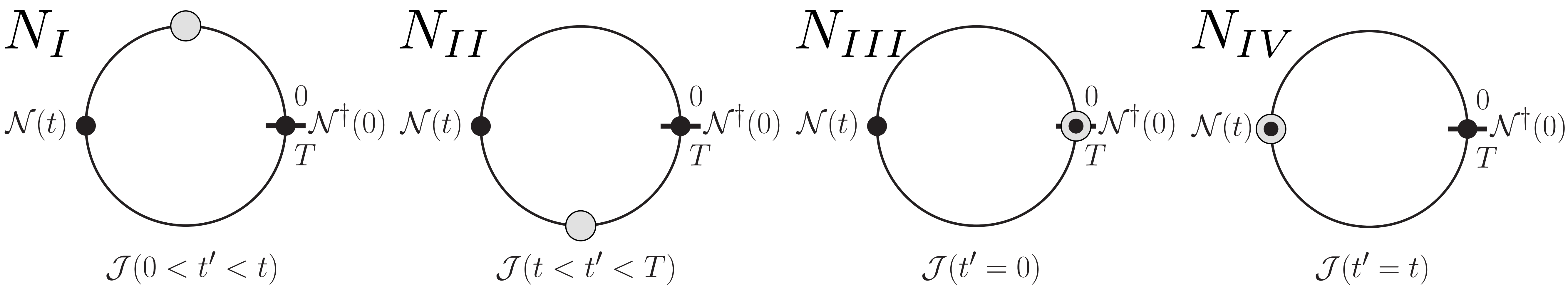

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transition matrix elements

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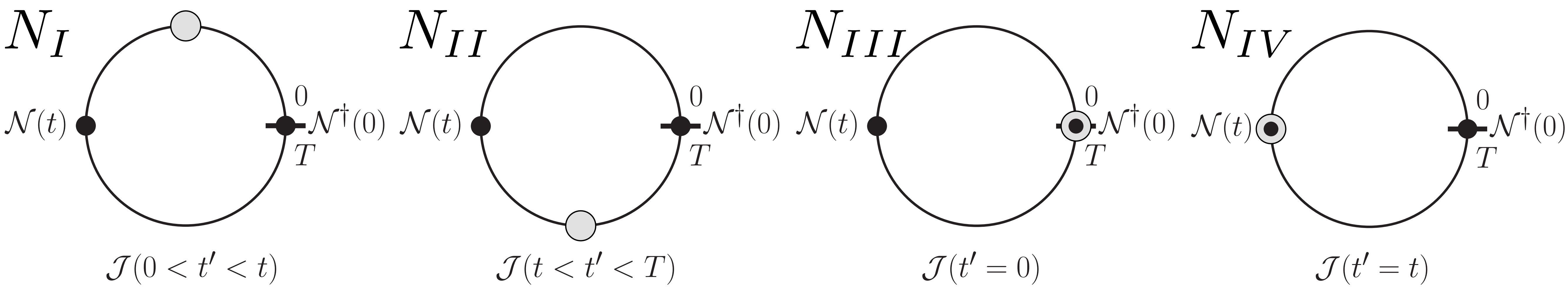

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undesired systematic contamination, II, III, IV

contact terms

undesired time orderings

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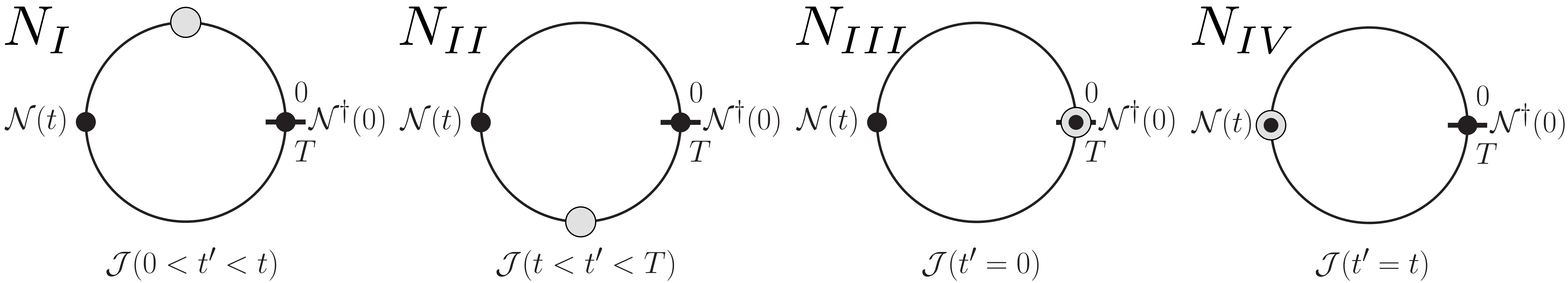

$$N(t) = \sum_n \left[(t-1) z_n g_{nn} z_n^\dagger + d_n \right] e^{-E_n t} + \sum_{n \neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\Delta_{nm}/2} - e^{-E_m t} e^{\Delta_{mn}/2}}{e^{\Delta_{mn}/2} - e^{\Delta_{nm}/2}}$$

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NOTE: unique time-dependence (t-1) of matrix elements of interest. This allows us to cleanly isolate them in numerical analysis

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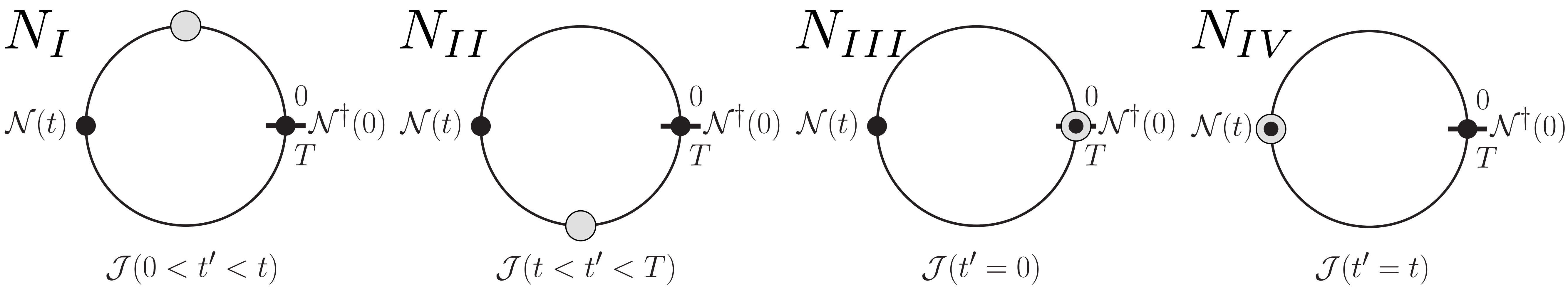

$$N(t) = \sum_n \left[(t-1) z_n g_{nn} z_n^\dagger + d_n \right] e^{-E_n t} + \sum_{n \neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\Delta_{nm}/2} - e^{-E_m t} e^{\Delta_{mn}/2}}{e^{\Delta_{mn}/2} - e^{\Delta_{nm}/2}}$$

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not immediately obvious, but at t=1, all terms cancel except contact + wrong time-ordering terms

$$N_J(1) = \sum_n d_n^J e^{-E_n}$$

which allows us to estimate these contributions in a controlled way

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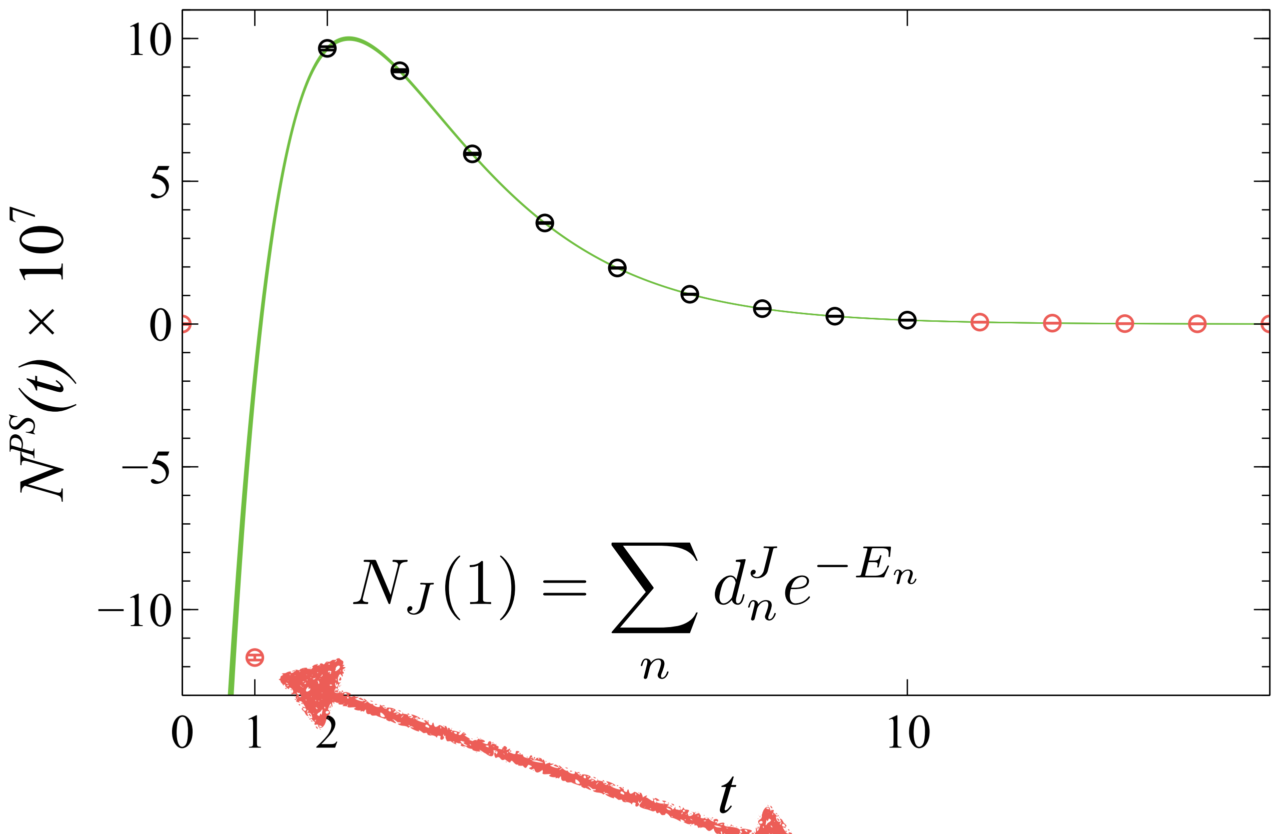
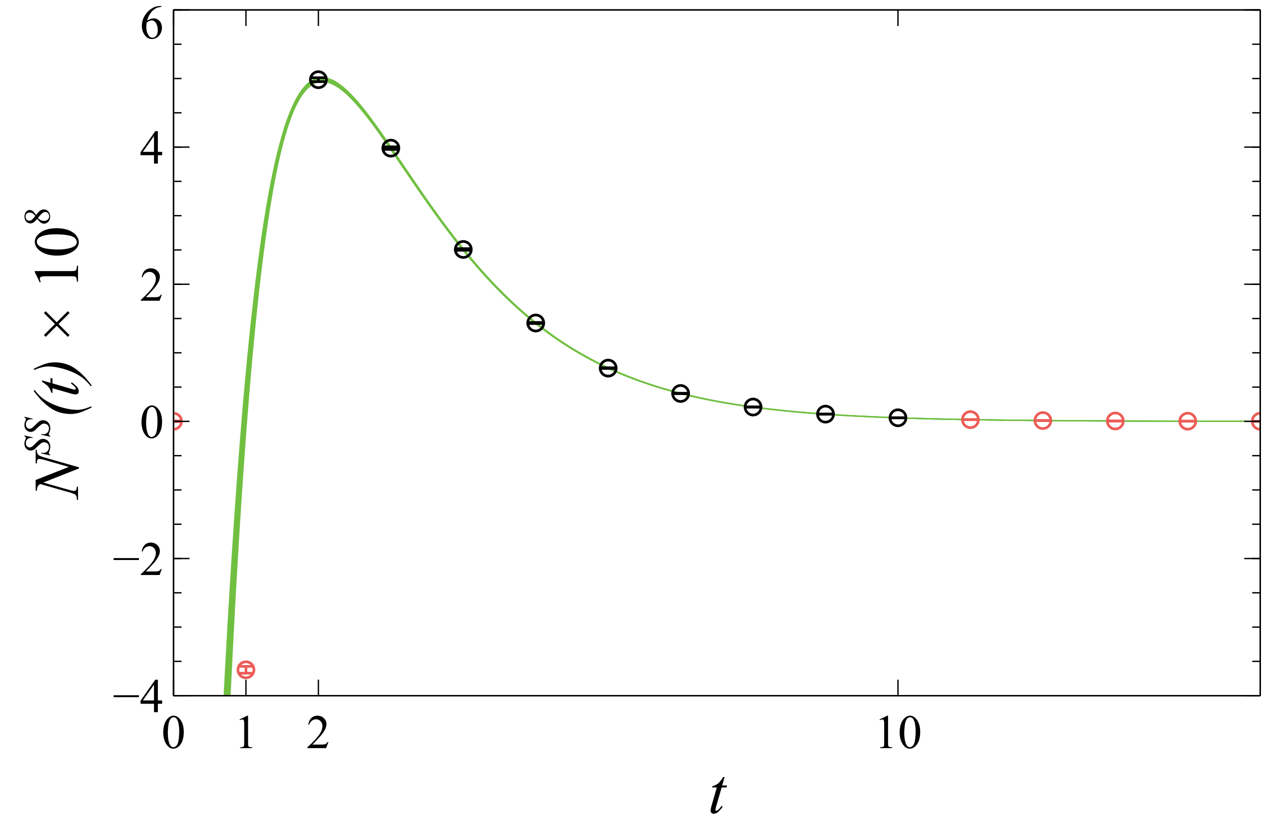
simultaneous fit to two-point and Numerator correlation functions



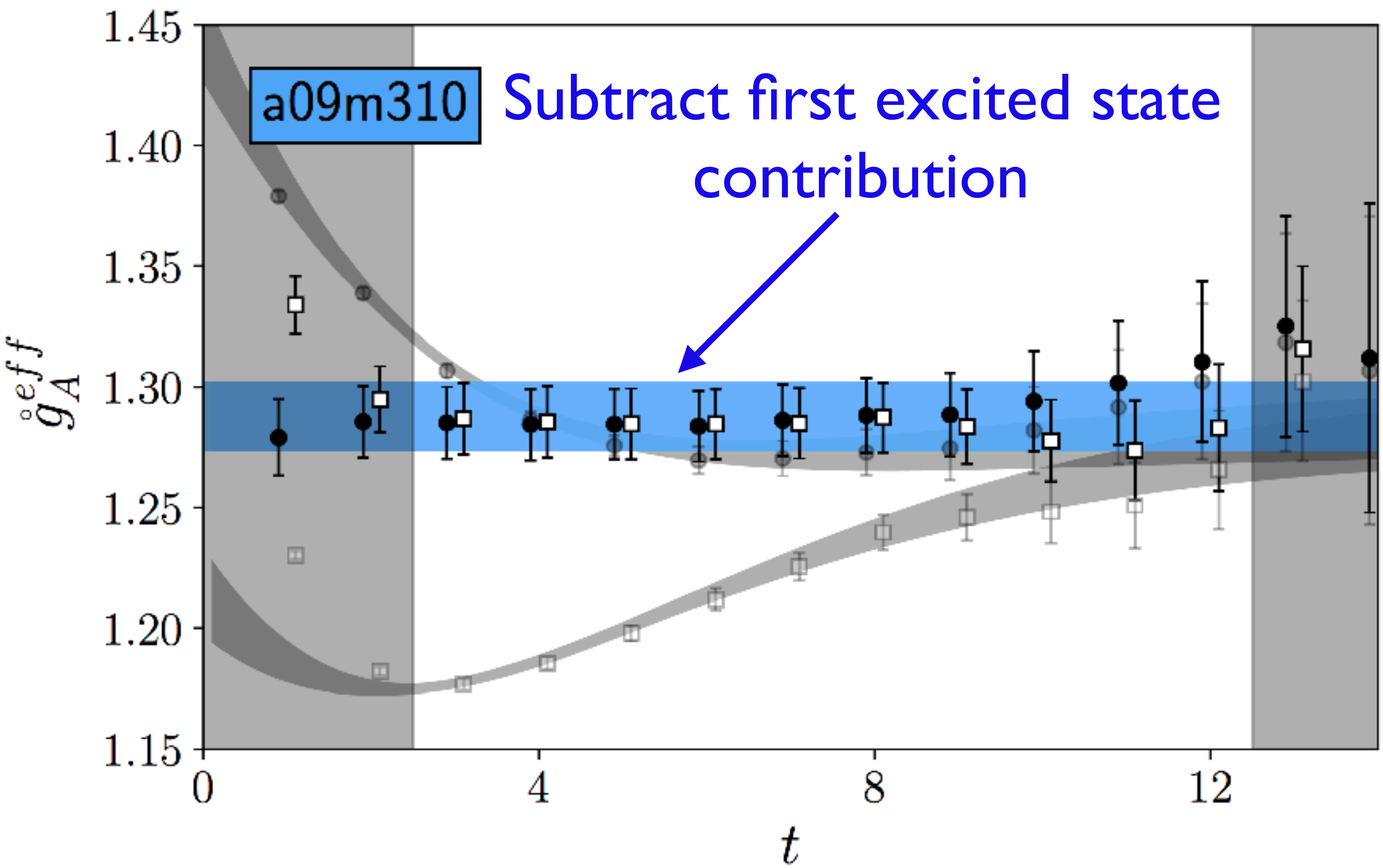


$$N(t) = \sum_n \left[(t-1) z_n g_{nn} z_n^\dagger + d_n \right] e^{-E_n t}$$

$$+ \sum_{n \neq m} z_n g_{nm} z_m^\dagger \frac{e^{-E_n t} e^{\frac{\Delta_{nm}}{2}} - e^{-E_m t} e^{\frac{\Delta_{mn}}{2}}}{e^{\frac{\Delta_{mn}}{2}} - e^{\frac{\Delta_{nm}}{2}}}$$

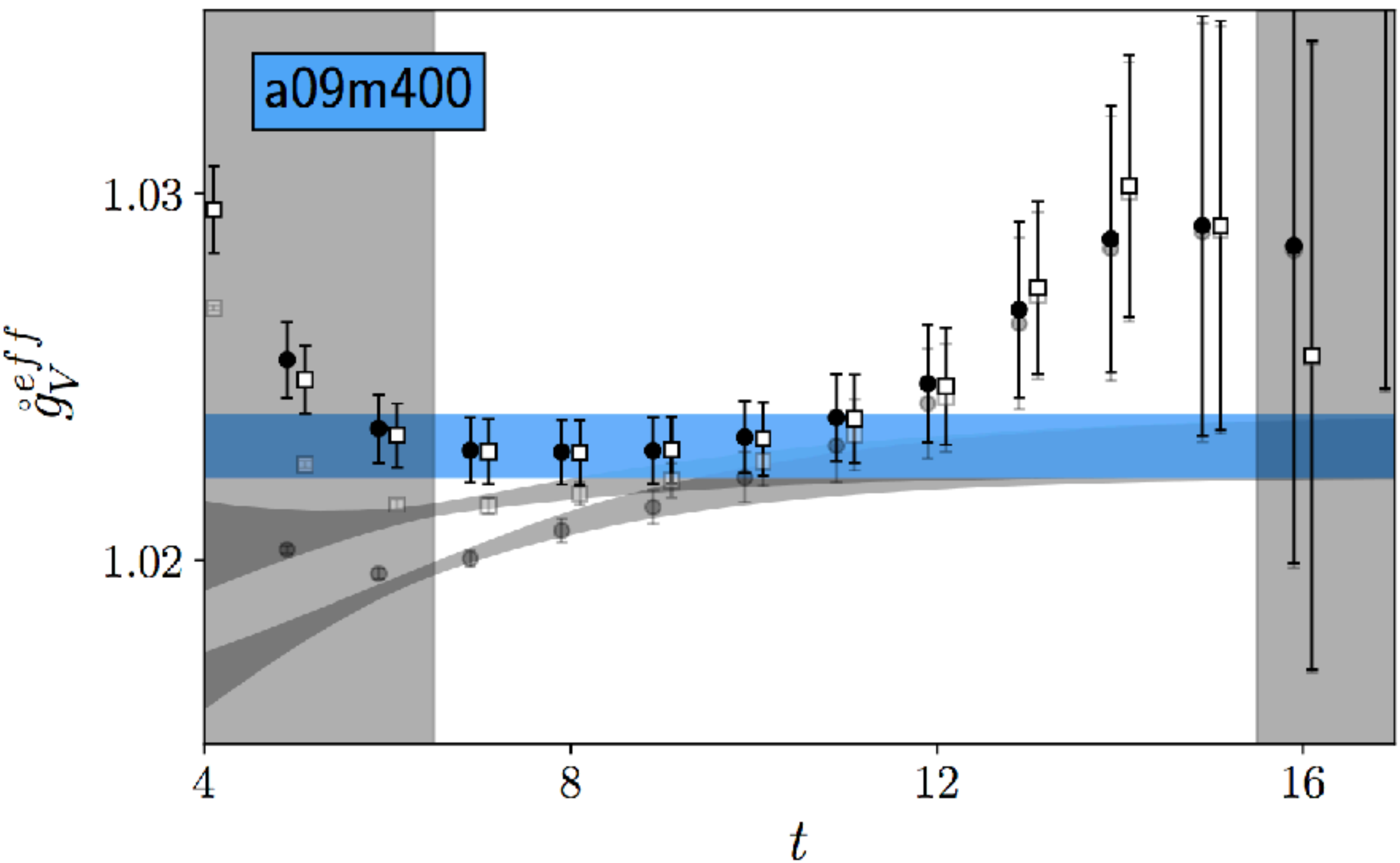
- Fit to numerator data yields consistent results
- Numerator suffers from more excited state contamination than Feynman-Hellmann correlator

Comparison with a Standard Method



Slides adapted from A. Nicholson

Comparison with a Standard Method



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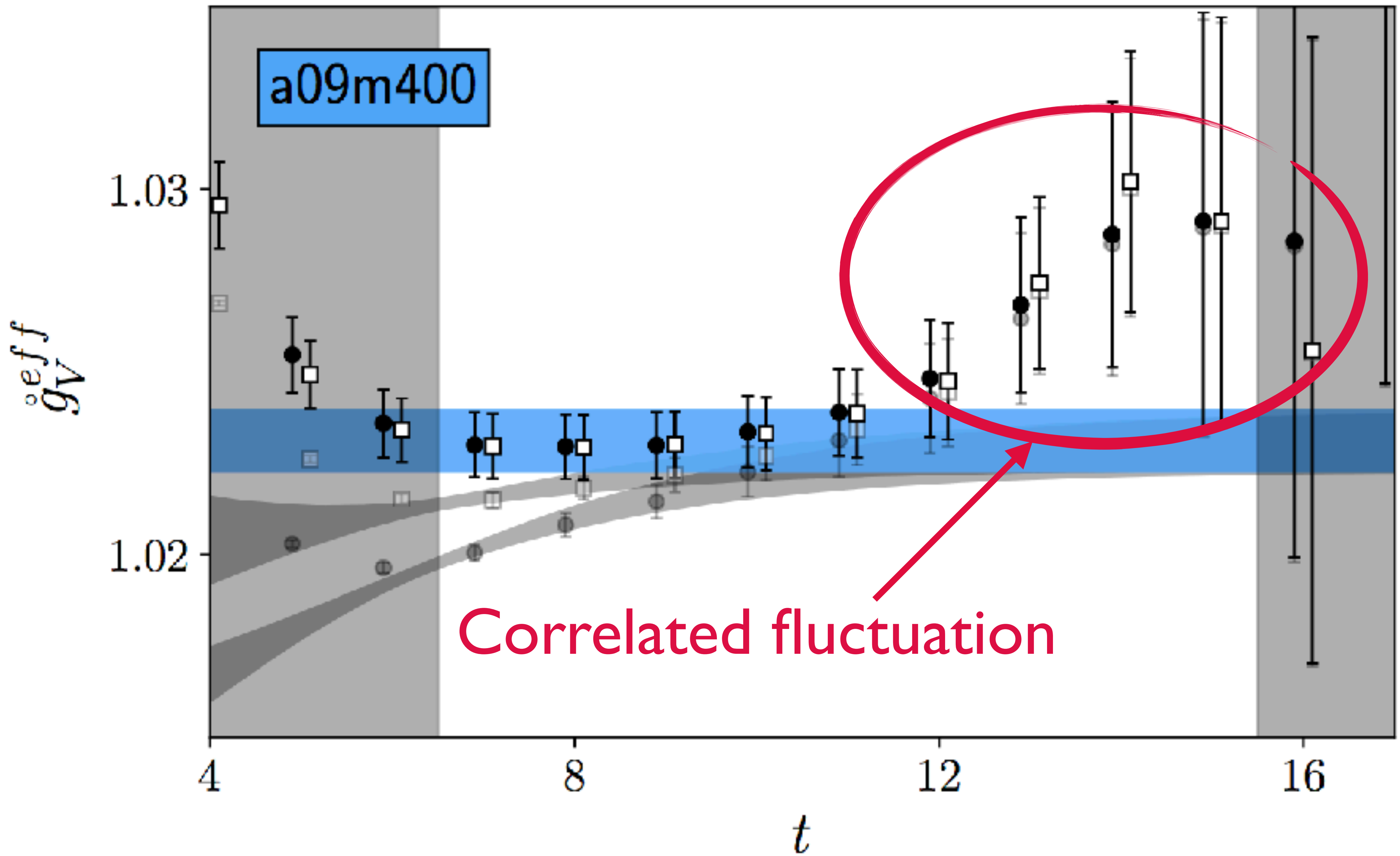

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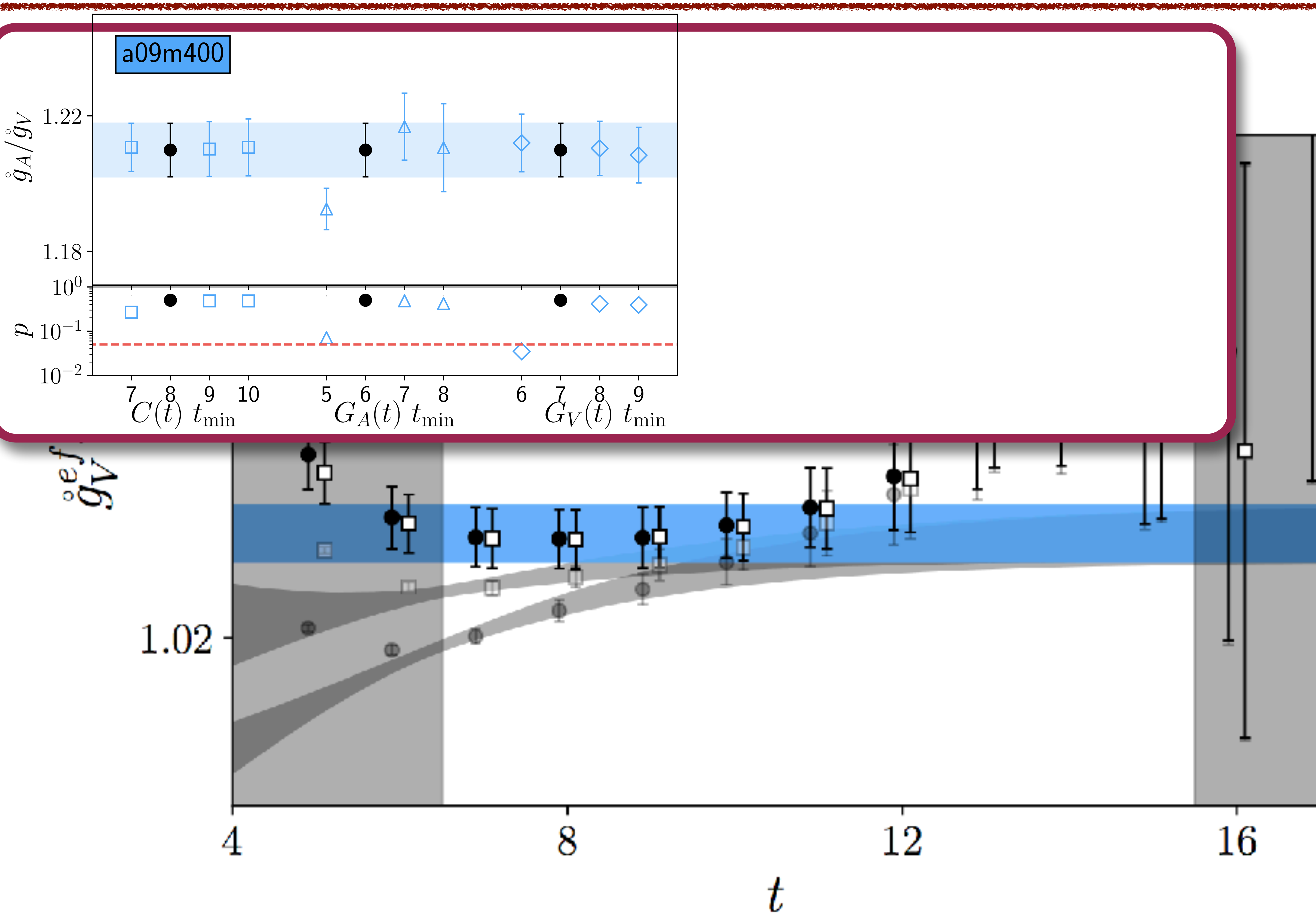

Slide adapted from A. Nicholson

Our Recent Lattice QCD Calculation

arXiv.org > hep-lat > arXiv:1704.01114 **UPDATE of arXiv**

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High Energy Physics – Lattice

An accurate calculation of the nucleon axial charge with lattice QCD

Evan Berkowitz, David Brantley, Chris Bouchard, Chia Cheng Chang, M. A. Clark, Nicholas Garron, Balint Joo, Thorsten Kurth, Chris Monahan, Henry Monge-Camacho, Amy Nicholson, Kostas Orginos, Enrico Rinaldi, Pavlos Vranas, Andre Walker-Loud

(Submitted on 4 Apr 2017)

HISQ ensembles

$a[fm] : m_\pi[MeV]$	310	220	135
0.15	$16^3 \times 48, m_\pi L \sim 3.78$	$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12		$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$	$32^3 \times 64, m_\pi L \sim 4.29$	
0.12		$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$		

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	HISQ gauge configuration parameters							valence parameters							
	abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{val.}$	σ_{smr}	N_{smr}
*	a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
*	a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
	a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
	a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
	a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
*	a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
*	a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
	a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
	a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
*	a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
*	a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
*	a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
	a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
*	a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

* New calculation

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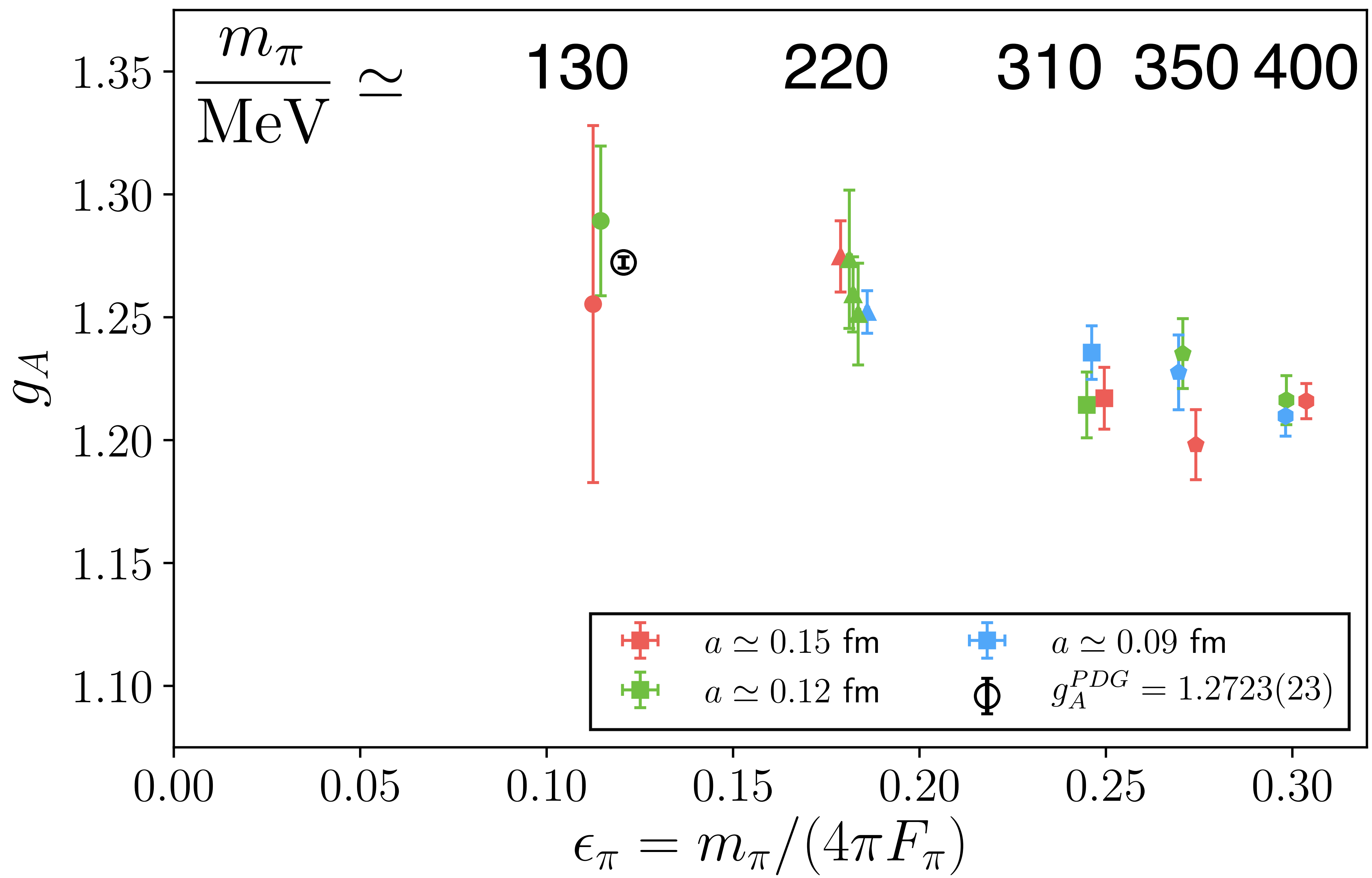
* New calculation

additional HISQ ensembles generated @ LLNL



Slide adapted from A. Nicholson

Extrapolations

Dimensionless parameters:

lattice spacing, volume, pion mass

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad m_\pi L \qquad \epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

- ChiPT: EFT expanding around m_π = 0
  - best hope for model-independent extrapolation
  - not guaranteed to converge around m_π = 135 MeV
- Mild m_π,a dependence
  - Taylor expansion works well for extrapolation/interpolation

Slide adapted from A. Nicholson

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lattice spacing, volume, pion mass

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$$\delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

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$$F_1(x) = \sum_{\mathbf{n}\neq 0}\left[K_0(x|\mathbf{n}|) - \frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}\right]$$

$$\delta_L = \frac{8}{3}\varepsilon_\pi^2\left[g_0^3 F_1(m_\pi L) + g_0 F_3(m_\pi L)\right] + f_3\varepsilon_\pi^3 F_1(m_\pi L)$$

$$F_3(x) = -\frac{3}{2}\sum_{\mathbf{n}\neq 0}\frac{K_1(x|\mathbf{n}|)}{x|\mathbf{n}|}$$

Beane and Savage
Phys.Rev.D70 [hep-ph/0404131]

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Beane and Savage
Phys.Rev.D70 [hep-ph/0404131]

$$g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0 c_3\epsilon_\pi^3$$

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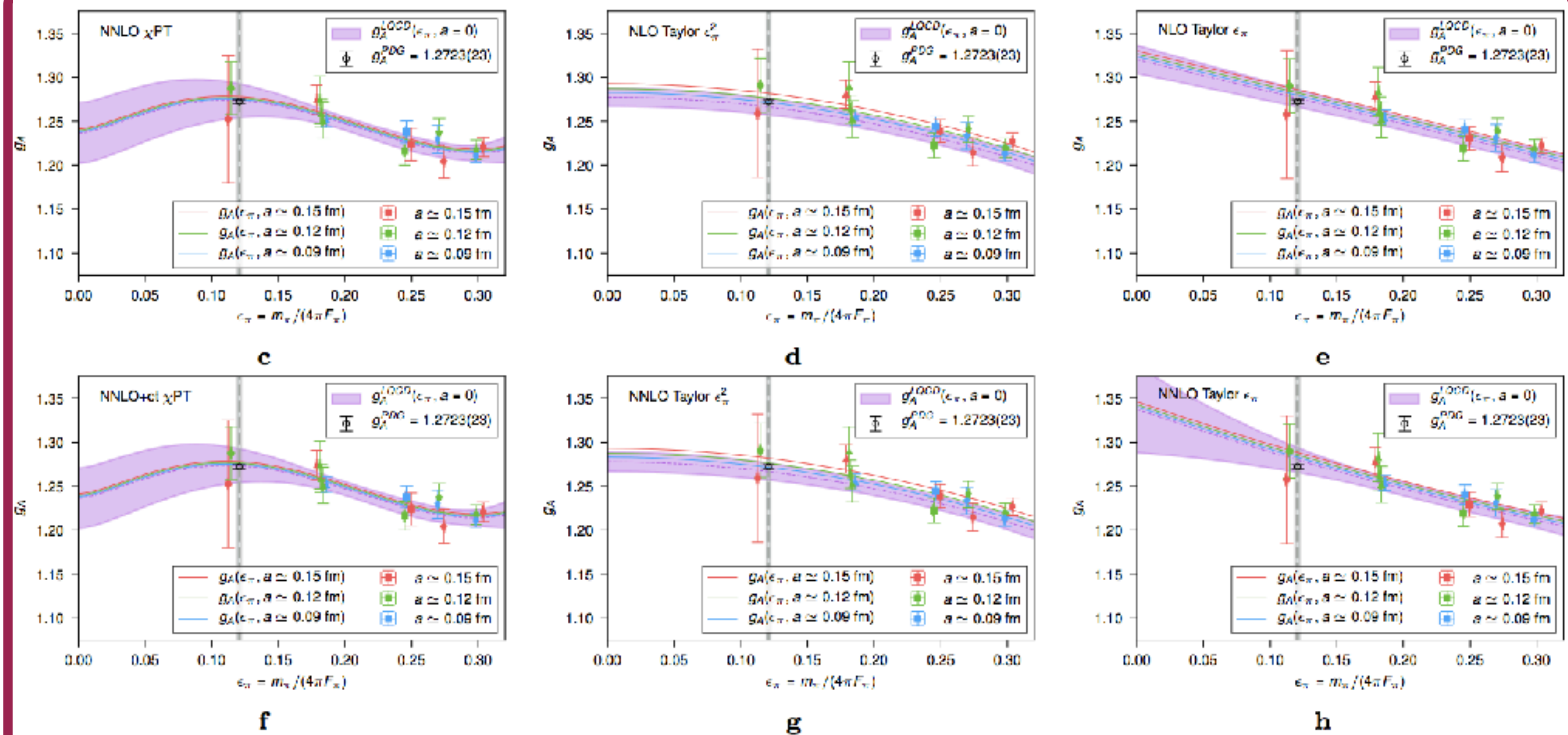
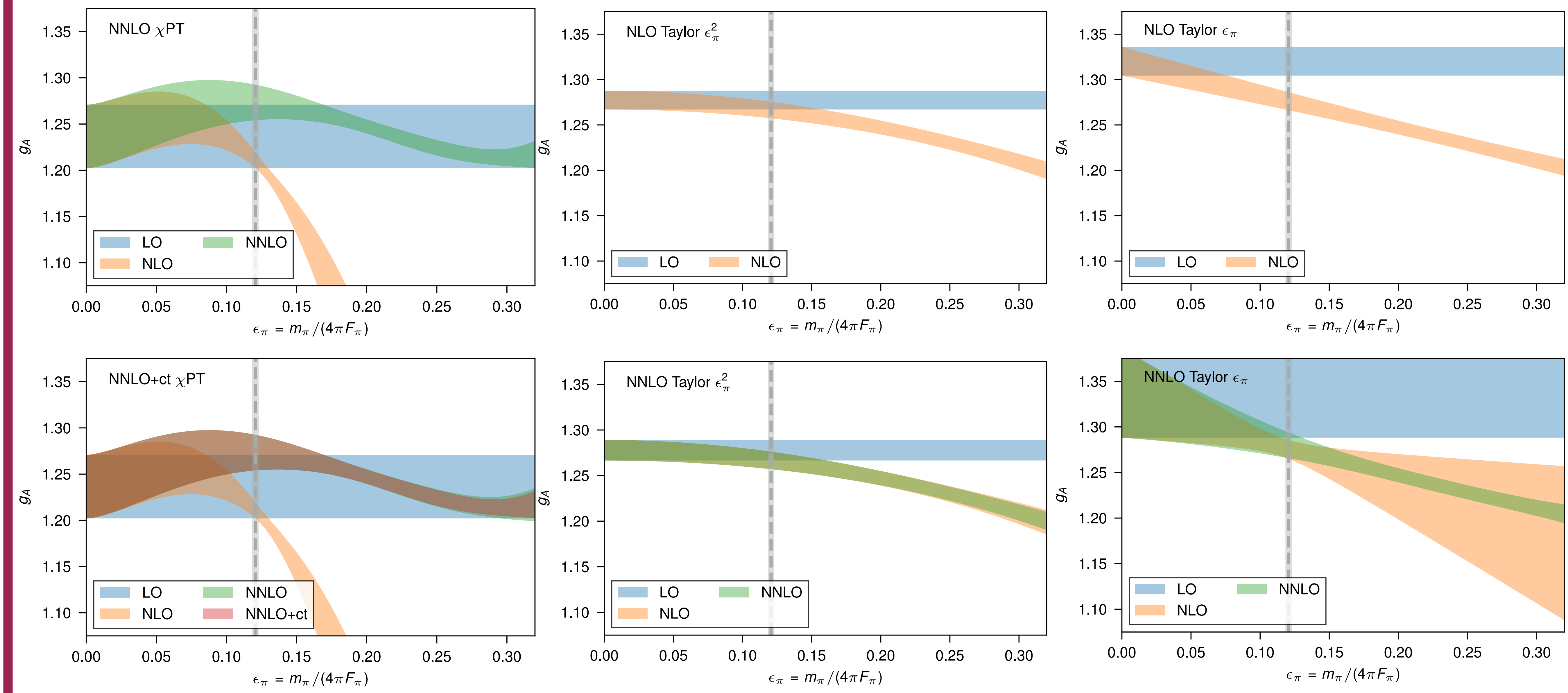
Extrapolations



Slide adapted from A. Nicholson

Convergence



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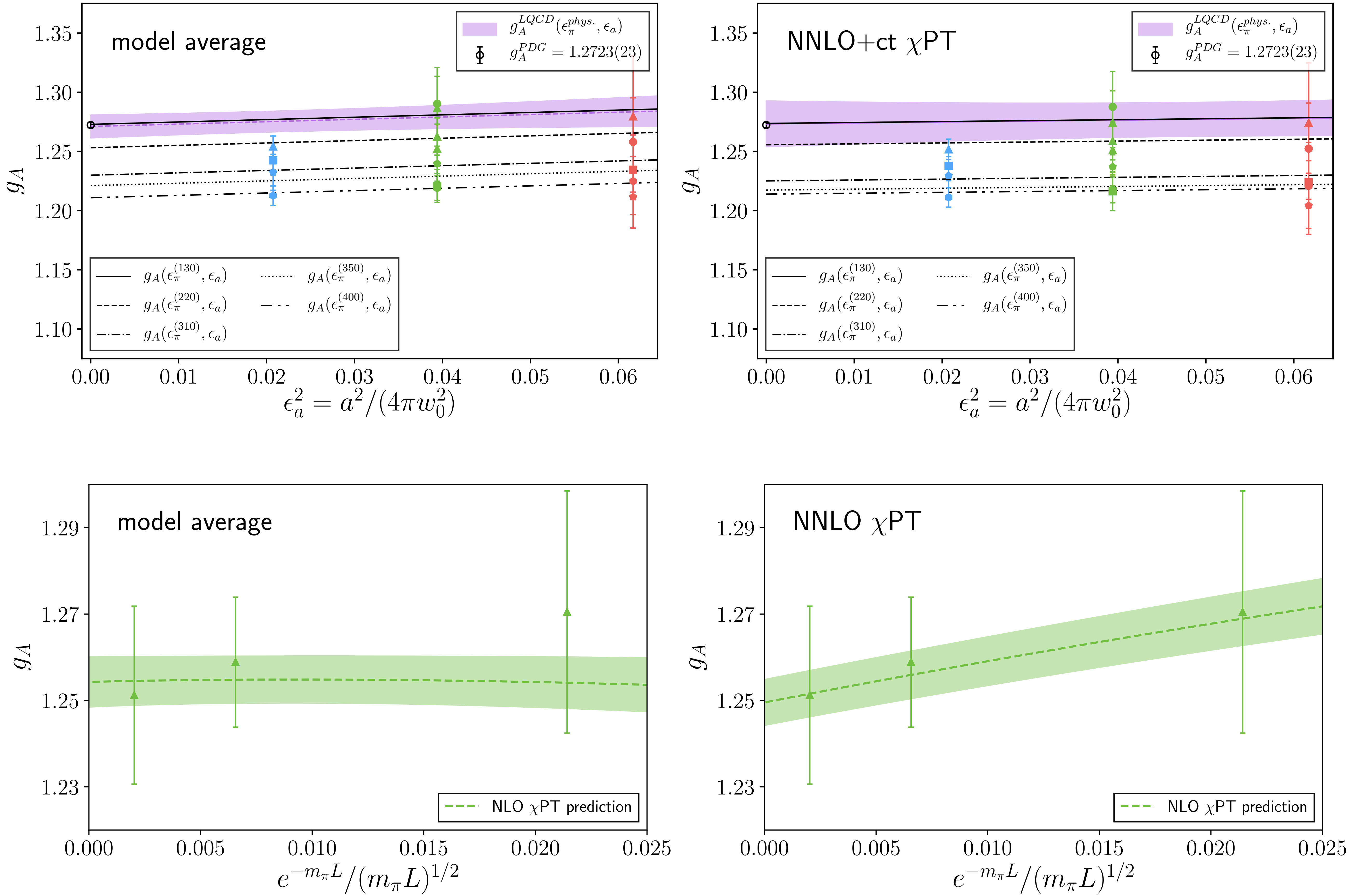
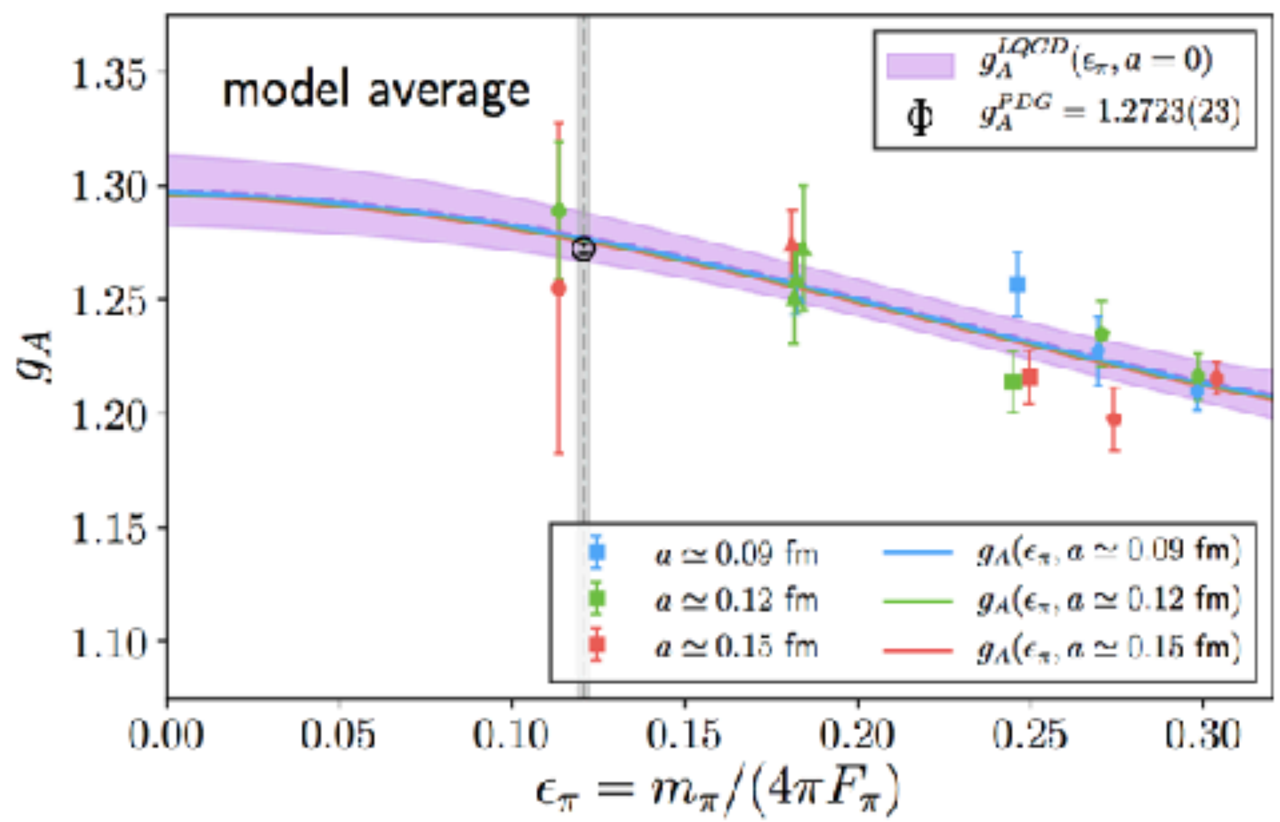
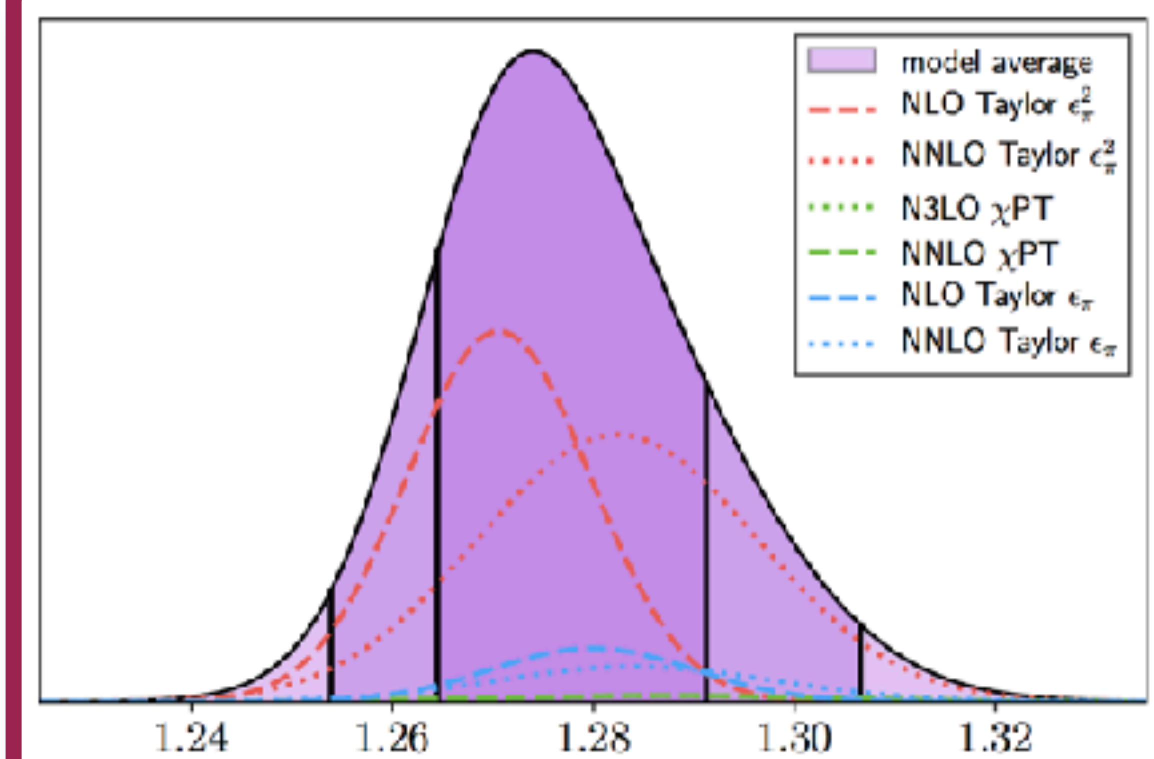
Continuum and FV Extrapolation



Model Average





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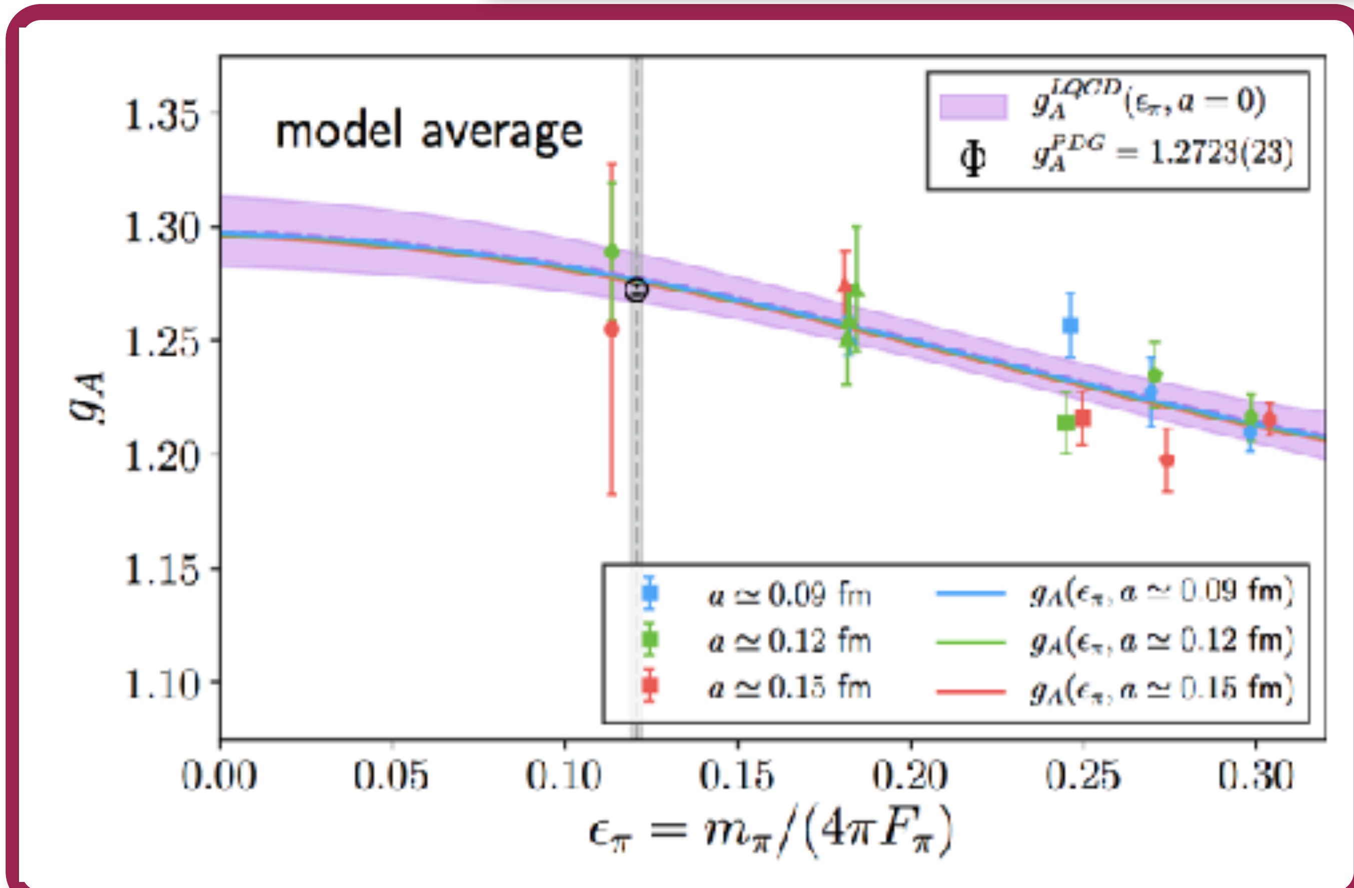

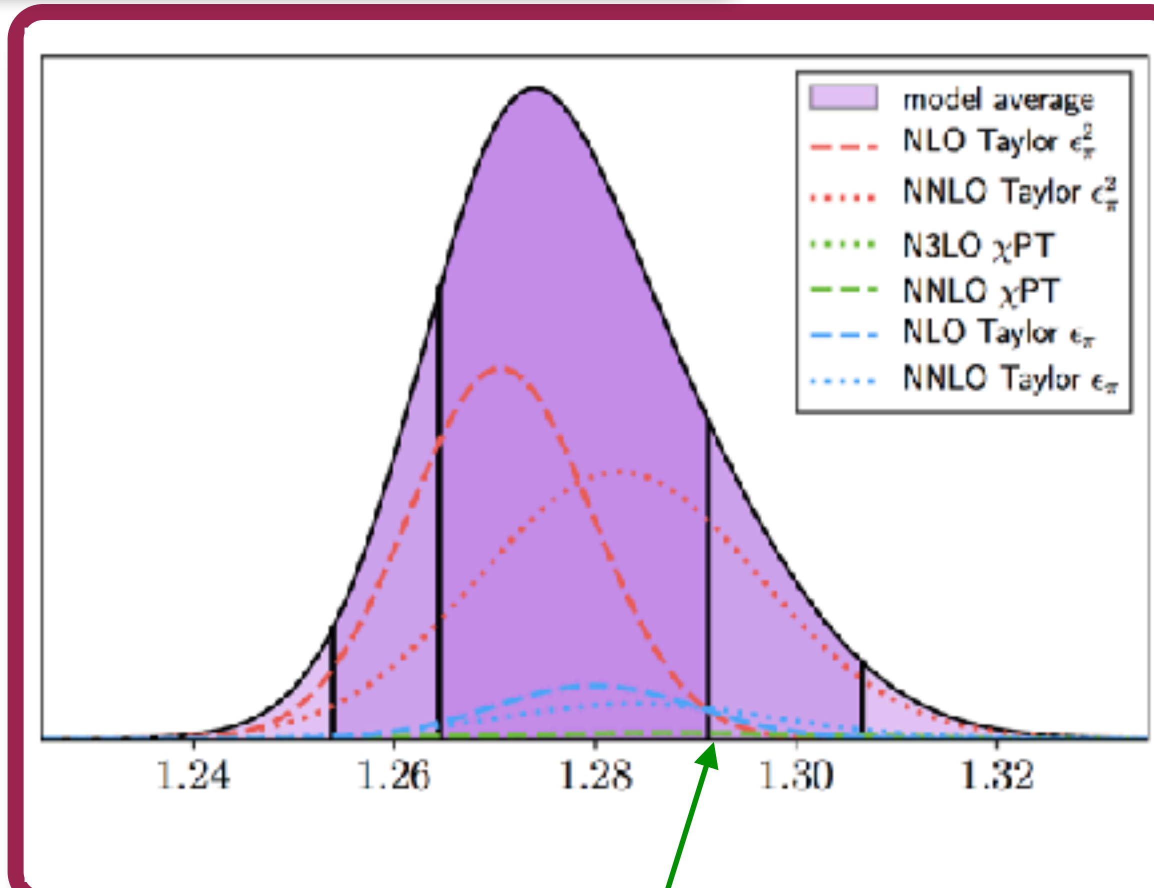

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Slide adapted from A. Nicholson

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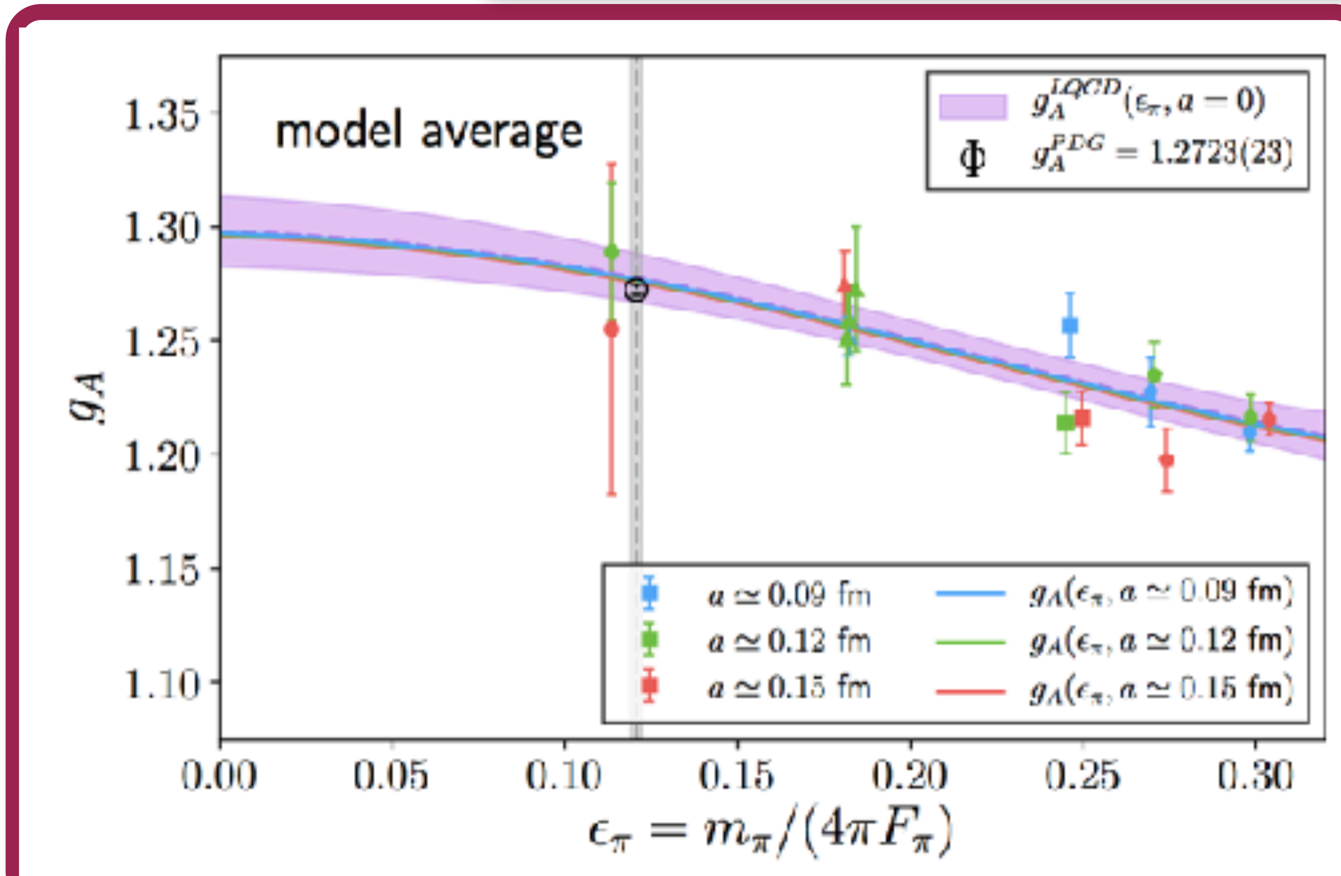

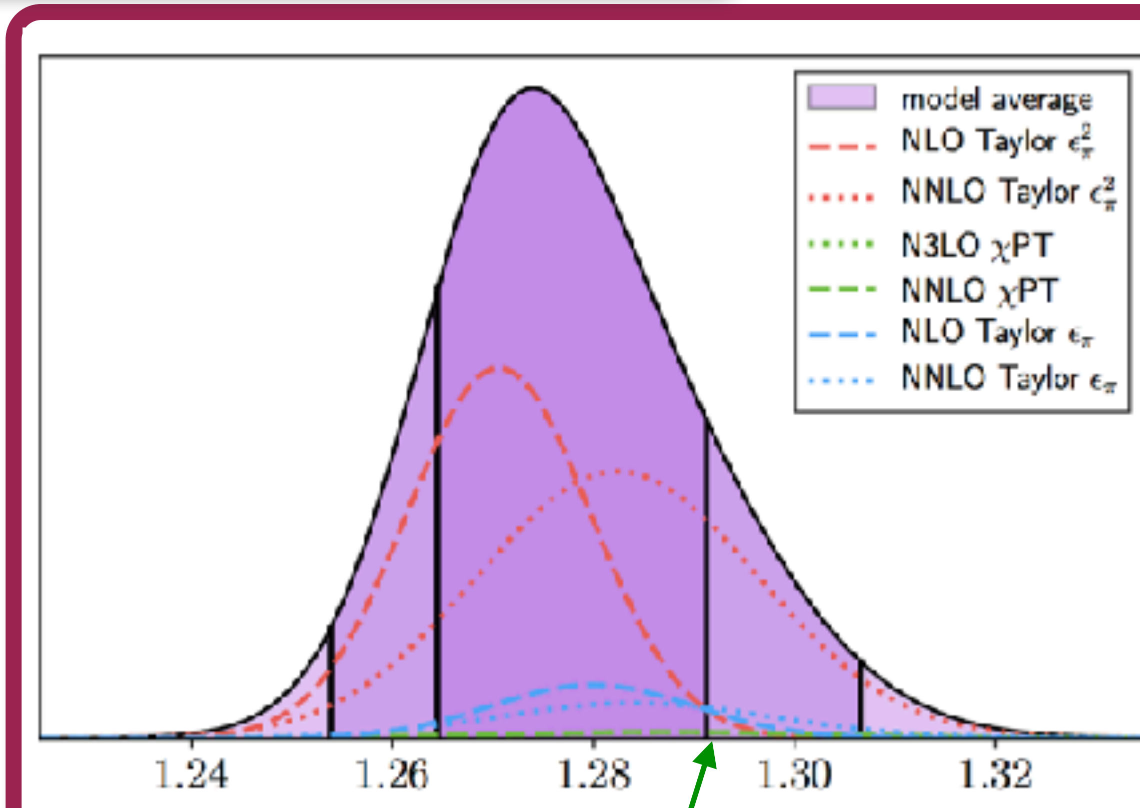

Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

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Slide adapted from A. Nicholson

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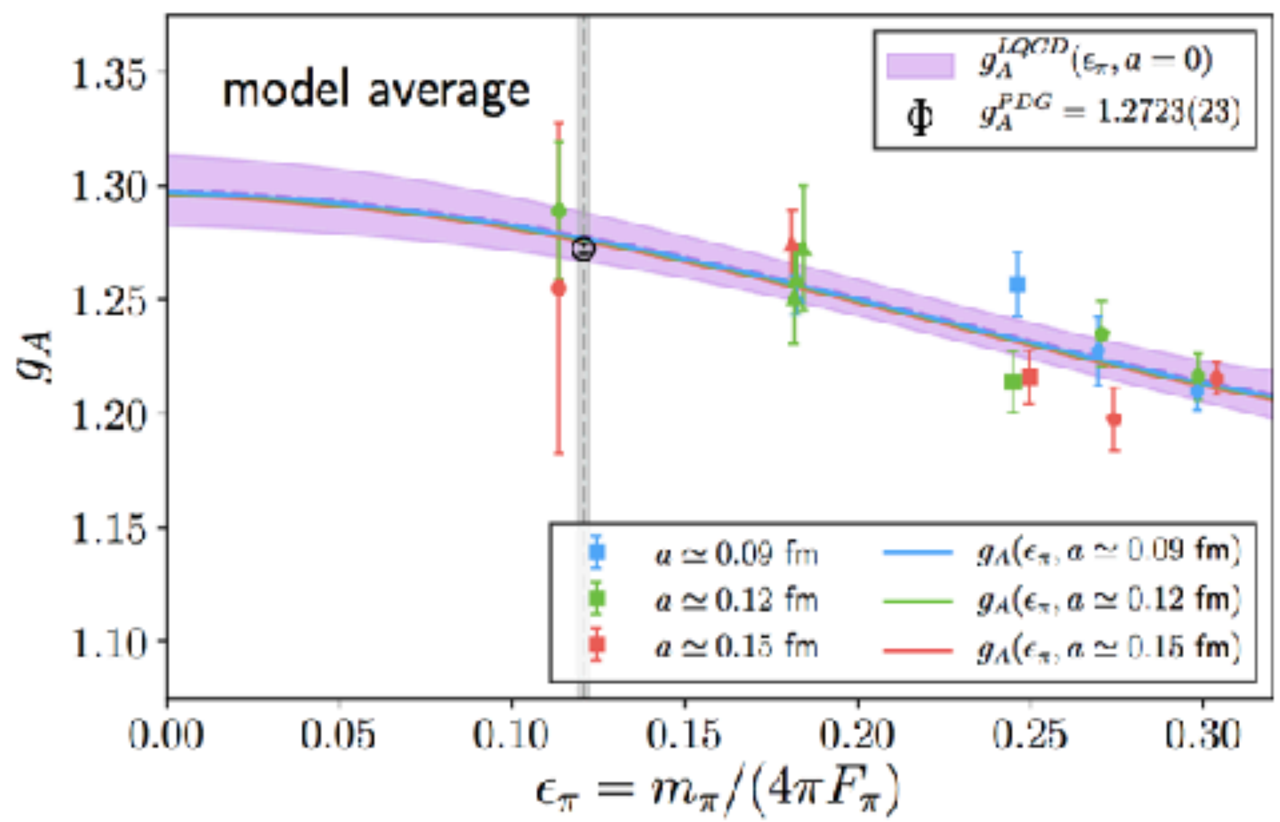

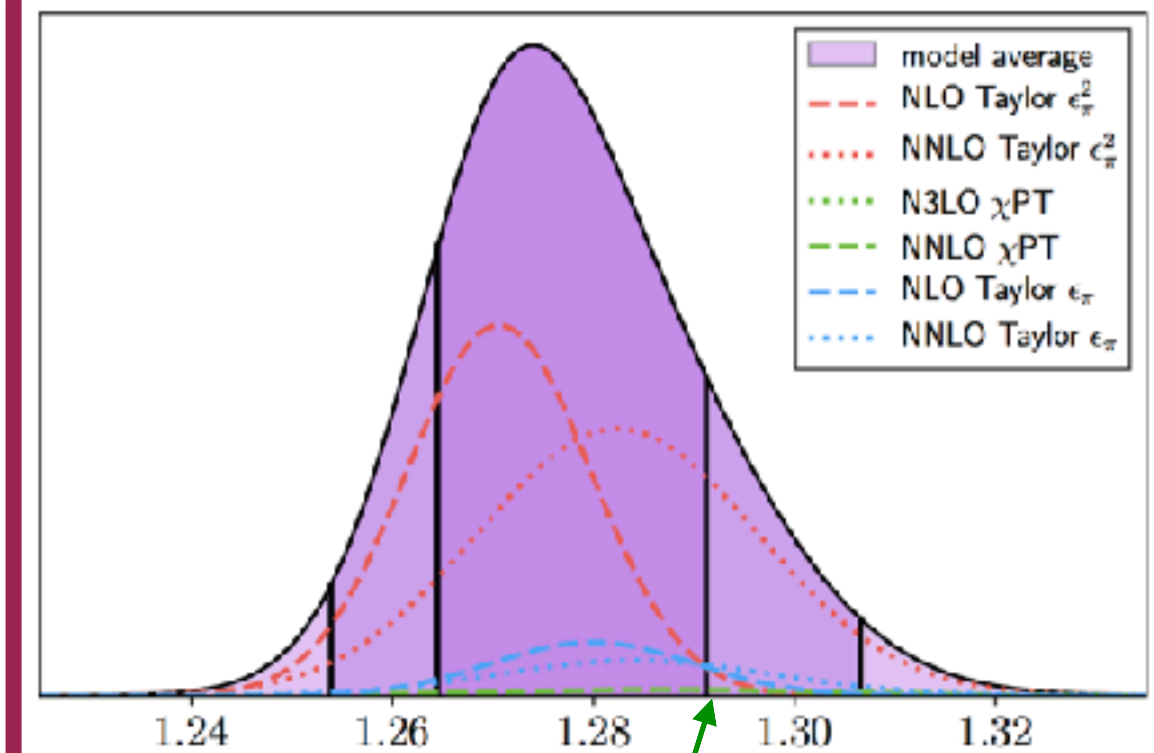

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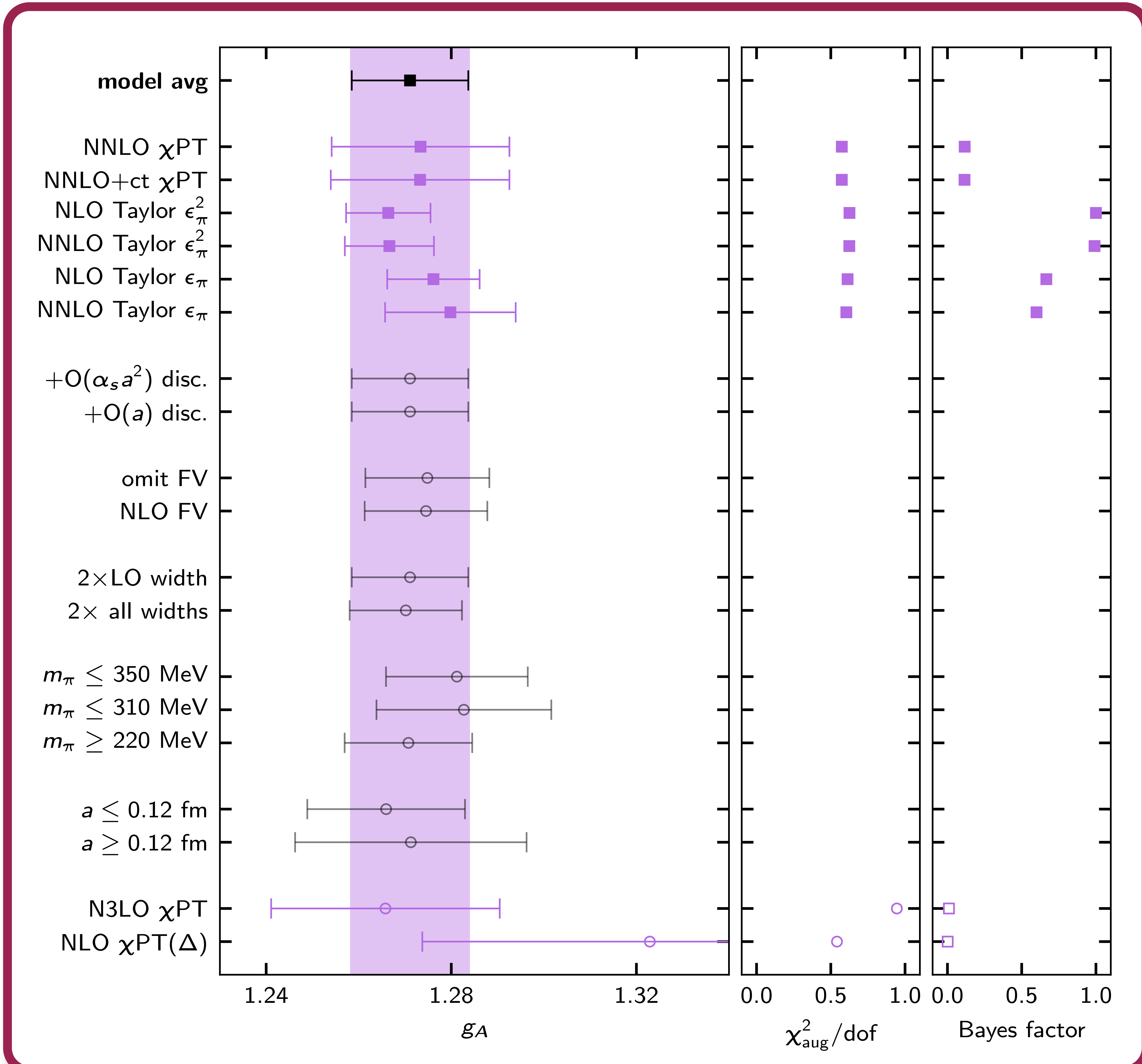
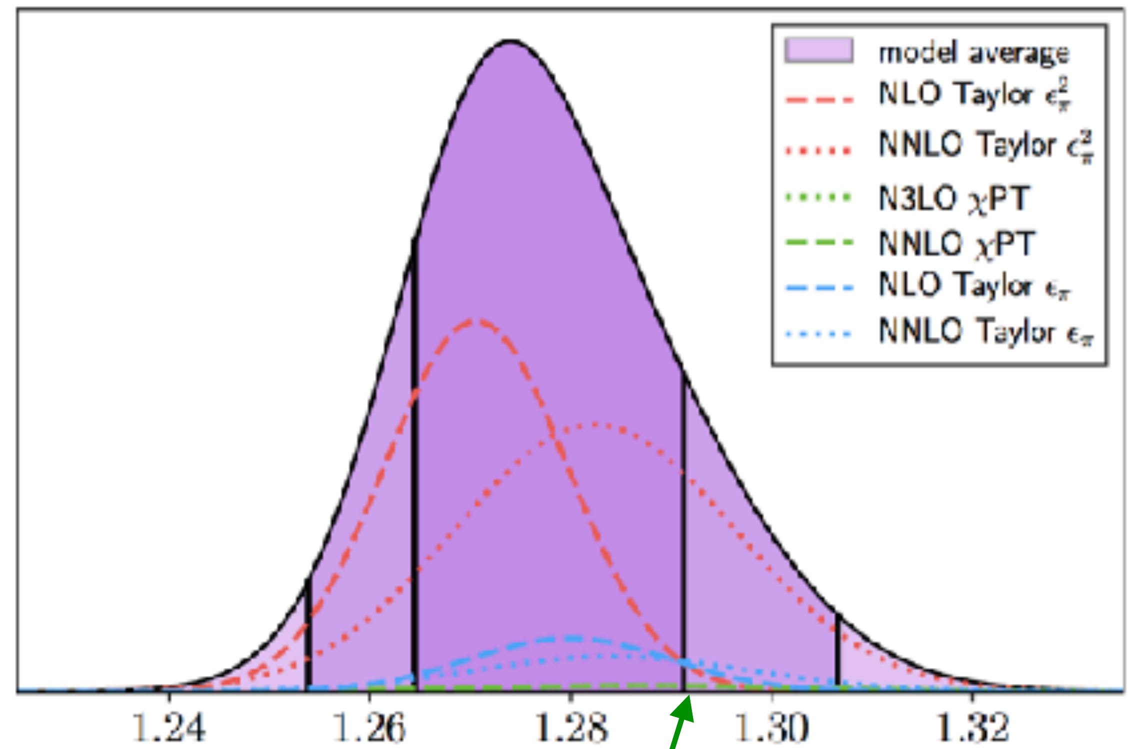
Slide adapted from A. Nicholson



Slide adapted from A. Nicholson

Final result: $g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
model selection	0.43%
total	0.99%



Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

NNLO χPT : Eq. (S8) $+ \delta_a + \delta_L$

NNLO+ct χPT : Eq. (S8) $+ c_4\epsilon_\pi^4 + \delta_a + \delta_L$

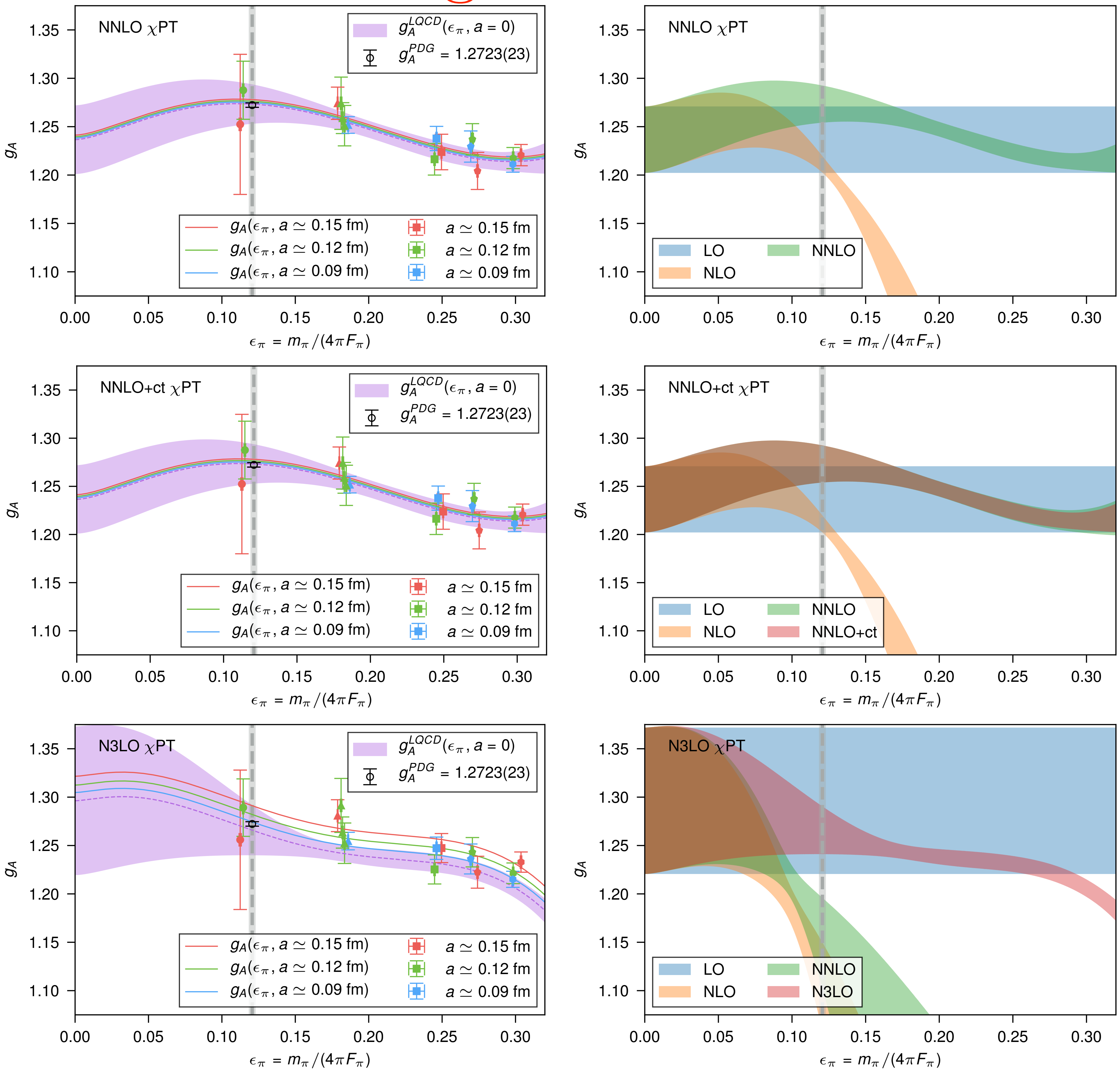
NLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

NNLO Taylor ϵ_π^2 : $c_0 + c_2\epsilon_\pi^2 + c_4\epsilon_\pi^4 + \delta_a + \delta_L$

NLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + \delta_a + \delta_L$

NNLO Taylor ϵ_π : $c_0 + c_1\epsilon_\pi + c_2\epsilon_\pi^2 + \delta_a + \delta_L$

convergence of the chiral expansion…



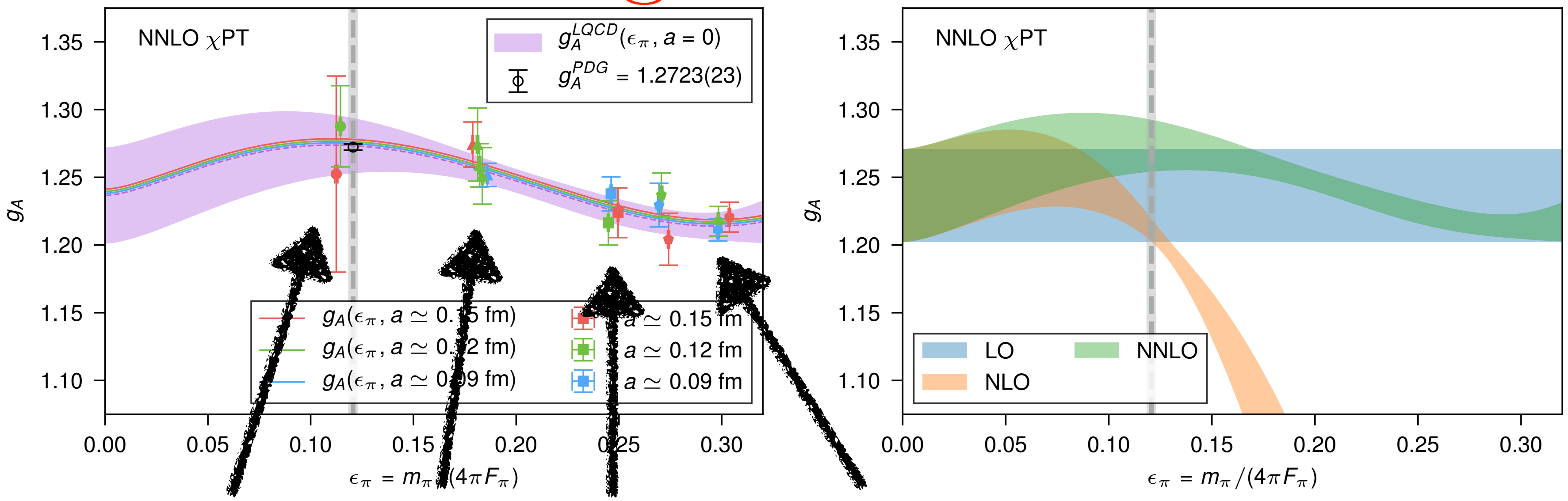
$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4\epsilon_\pi^4$$

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

convergence of the chiral expansion…



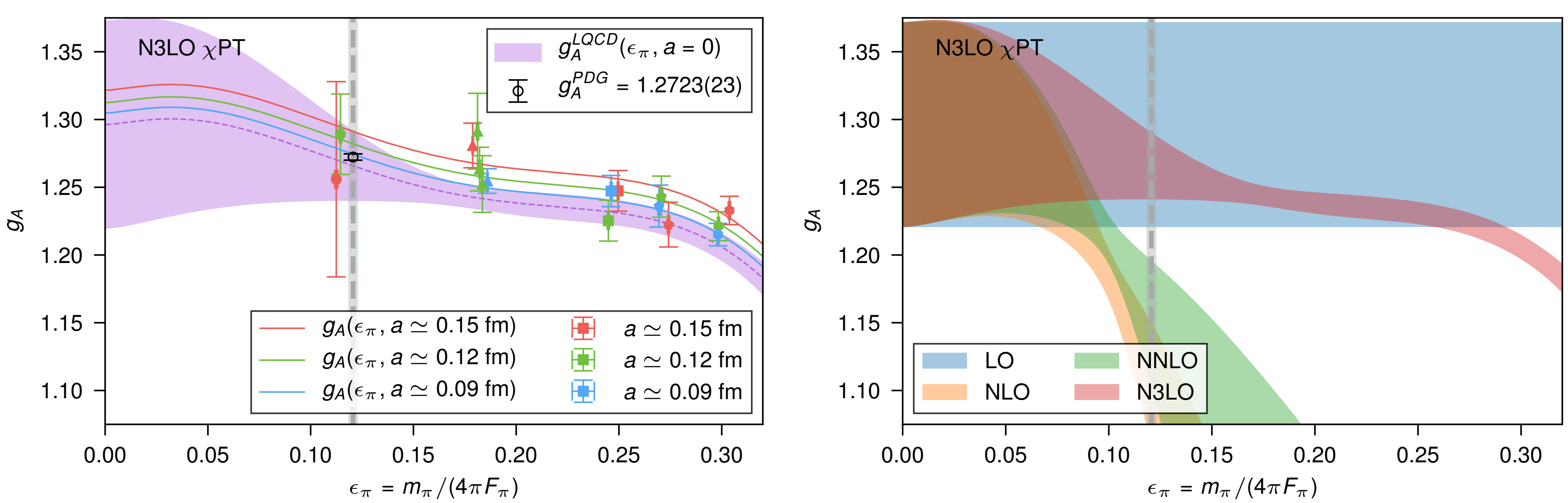
$m_\pi \sim 130$ MeV 220 310 400

$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

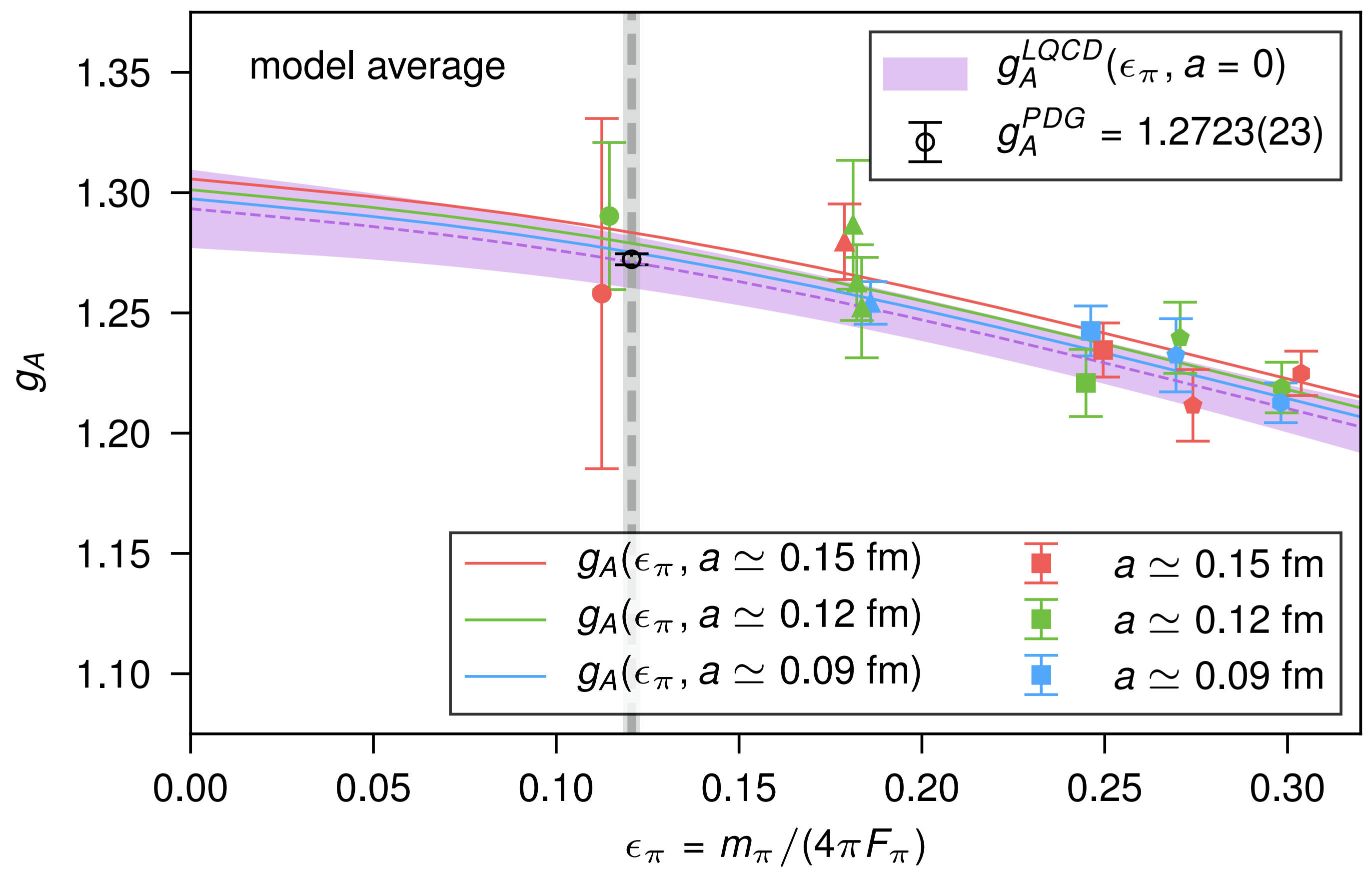
can we trust extrapolation of quantities with chiraly-enhanced behavior?

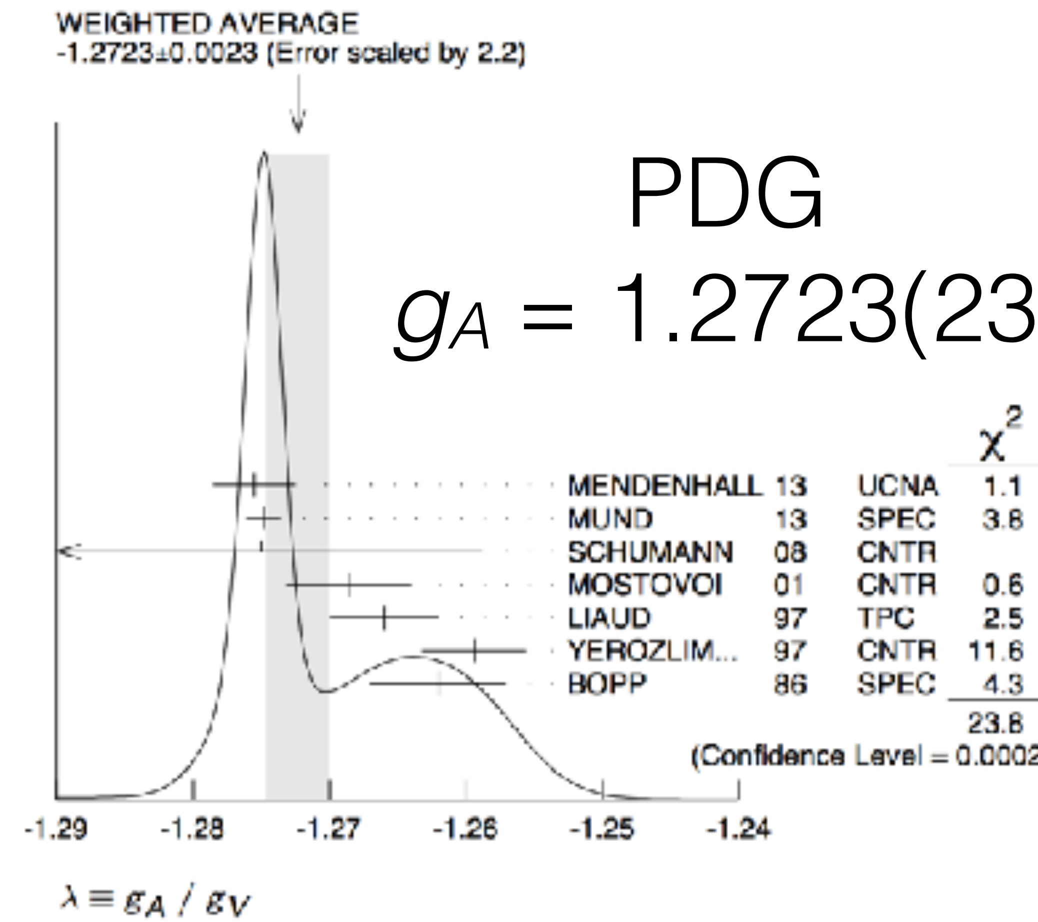
if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?



$$g_A = g_0 - \epsilon_\pi^2(g_0 + 2g_0^3)\ln(\epsilon_\pi^2) + c_2\epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4\left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) + \left(\frac{2}{3}g_0 + \frac{37}{12}g_0^3 + 4g_0^5\right)\ln^2(\epsilon_\pi^2)\right]$$

Our Recent Lattice QCD Calculation arXiv:1704.01114





PDG

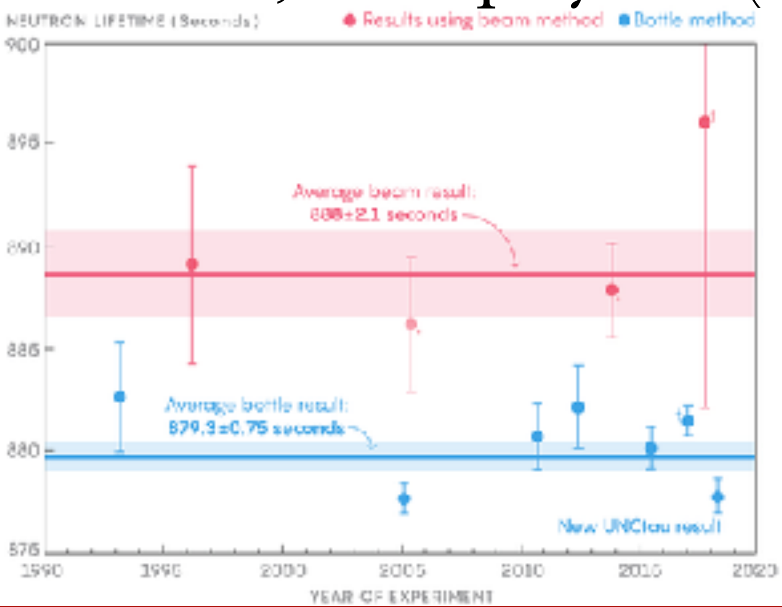
$g_A = 1.2723(23)$

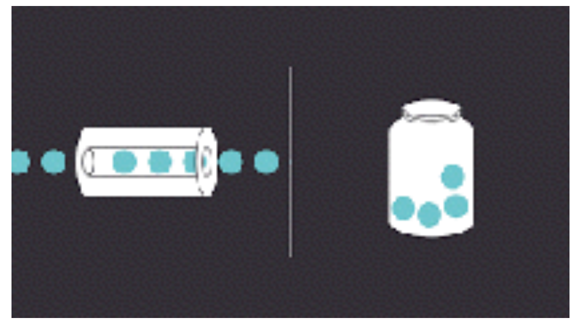
- All of our results (raw correlators, analysis results, LQCD code, etc.) will be made available with the publication
- Our result is statistics limited - paving the way for a determination of g_A, approaching the experimental precision

$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

Neutron Lifetime…

- there is a 4-sigma discrepancy: beam $\tau_n^{\text{beam}} = 888.0(2.0)s$ and bottle $\tau_n^{\text{bottle}} = 879.4(0.6)s$ measurements of the neutron lifetime, new physics (dark matter) or unknown systematic?





Czarnecki, Marciano, Sirlin
arXiv:1802.01804

$$\tau_n = \frac{5172.0(1.0)\,s}{1+3g_A^2}$$

arXiv.org > hep-ph > arXiv:1802.01804

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High Energy Physics – Phenomenology

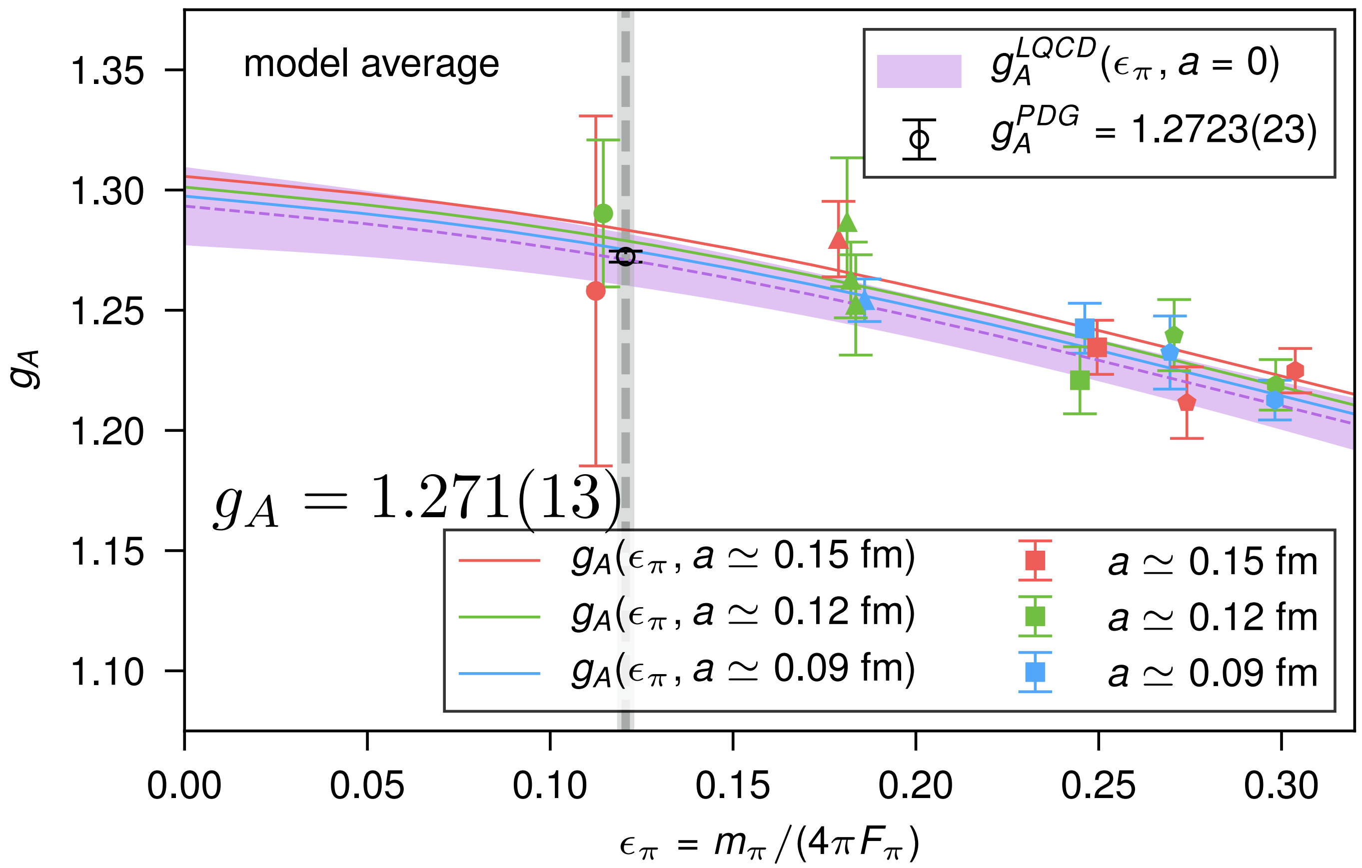
The Neutron Lifetime and Axial Coupling Connection

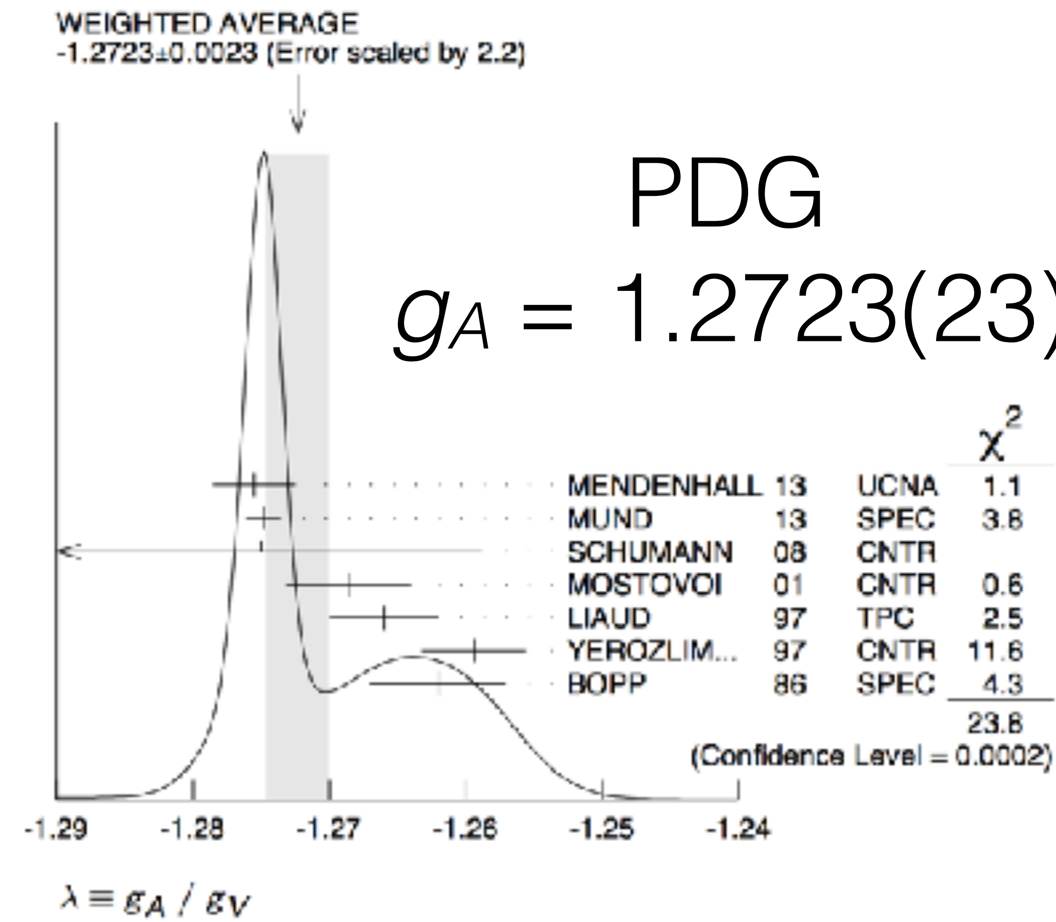
Andrzej Czarnecki, William J. Marciano, Alberto Sirlin

(Submitted on 6 Feb 2018 (v1), last revised 22 Feb 2018 (this version, v2))

Experimental studies of neutron decay, $n \to pe\bar{\nu}$, exhibit two anomalies. The first is a 8.6(2.1)s, roughly 4σ difference between the average beam measured neutron lifetime, $\tau_n^{\text{beam}} = 888.0(2.0)$s, and the more precise average trapped ultra cold neutron determination, $\tau_n^{\text{trap}} = 879.4(6)$s. The second is a 5σ difference between the pre2002 average axial coupling, g_A, as measured in neutron decay asymmetries $g_A^{\text{pre2002}} = 1.2637(21)$, and the more recent, post2002, average $g_A^{\text{post2002}} = 1.2755(11)$, where, following the UCNA collaboration division, experiments are classified by the date of their most recent result. In this study, we correlate those τ_n and g_A values using a (slightly) updated relation $\tau_n(1+3g_A^2) = 5172.0(1.1)$s. Consistency with that relation and better precision suggest $\tau_n^{\text{favored}} = 879.4(6)$s and $g_A^{\text{favored}} = 1.2755(11)$ as preferred values for those parameters. Comparisons of g_A^{favored} with recent lattice QCD and muonic hydrogen capture results are made. A general constraint on exotic neutron decay branching ratios is discussed and applied to a recently proposed solution to the neutron lifetime puzzle.

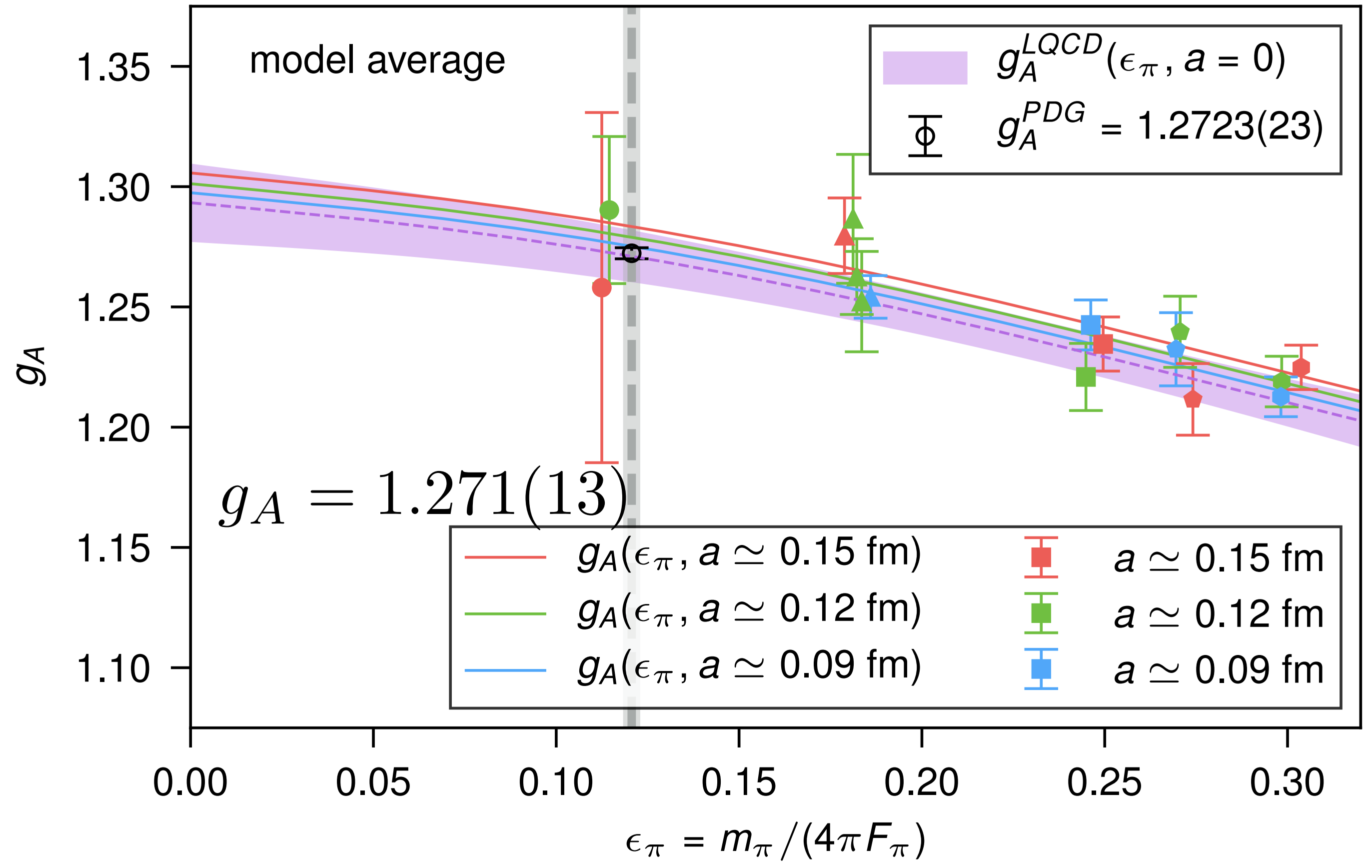
Our Recent Lattice QCD Calculation arXiv:1704.01114

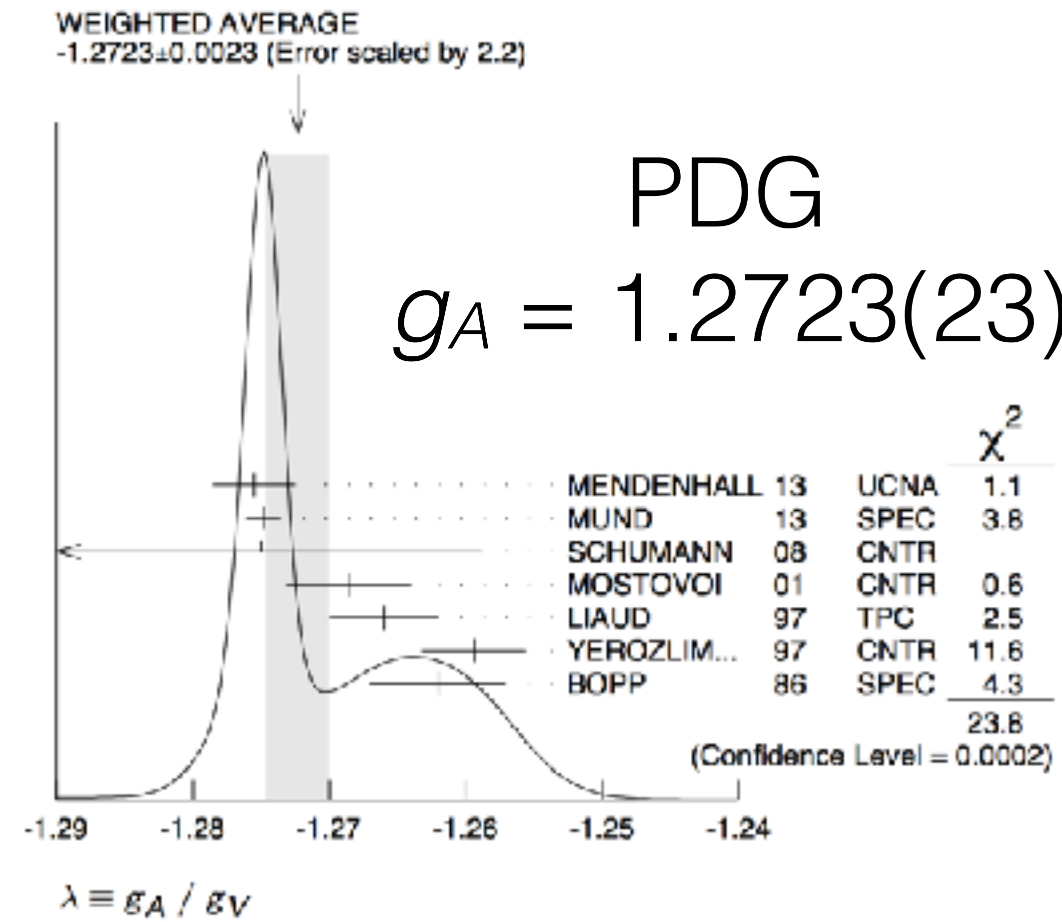




- The success of this result was enabled through a few features of the calculation:
  - an improved strategy that can exploit **exponentially more precise data** at early time and has **demonstrable control of excited state** contributions
  - an action with **improved stochastic behavior**, a very **mild continuum** extrapolation, **highly suppressed chiral symmetry breaking**
  - access to a set of ensembles (**MILC**) that allowed for control over all standard lattice systematics (physical pion mass, continuum and infinite volume limits)
  - ***ludicrously fast*** GPU code (NVIDIA)

Our Recent Lattice QCD Calculation arXiv:1704.01114





- The method is readily extended to
  - flavor changing currents
  - non-zero momentum transfer
  - multiple current insertions
  - multi-nucleon systems

Thank You

The axial coupling of the nucleon from QCD

Jülich	Evan Berkowitz
LBL/UCB	David Brantley, Chia Cheng (Jason) Chang, Thorsten Kurth, Henry Monge Camacho, AWL
Glasgow	Chris Bouchard
NVIDIA	Kate Clark
Liverpool/Plymouth	Nicolas Garron
JLab	Balint Jóo
Rutgers	Chris Monahan
Univ. of North Carolina	Amy Nicholson
William and Mary	Kostas Orginos
RIKEN/BNL	Enrico Rinaldi
LLNL	Pavlos Vranas

red = postdoc

blue = grad student

Some Lattice QCD Details

arXiv.org > hep-lat > arXiv:1701.07559 ***Phys. Rev.* D96 (2017)**

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High Energy Physics – Lattice

Möbius domain-wall fermions on gradient-flowed dynamical HISQ ensembles

Evan Berkowitz, Chris Bouchard, Chia Cheng Chang, M. A. Clark, Balint Joo, Thorsten Kurth, Christopher Monahan, Amy Nicholson, Kostas Orginos, Enrico Rinaldi, Pavlos Vranas, Andre Walker-Loud

(Submitted on 26 Jan 2017 (v1), last revised 21 Sep 2017 (this version, v3))

HISQ ensembles

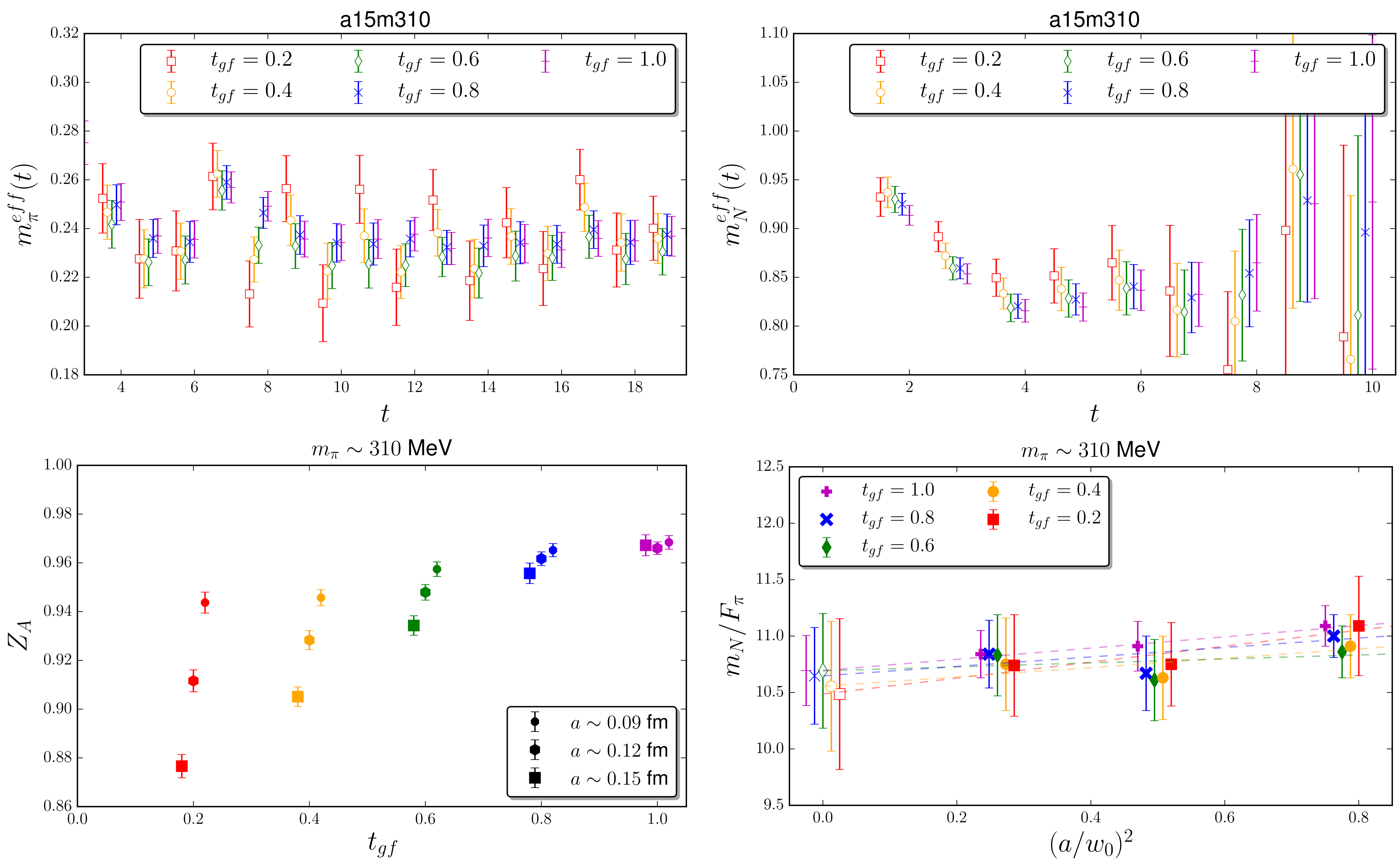
$a[fm] : m_\pi[MeV]$	310	220	135
0.15	$16^3 \times 48, m_\pi L \sim 3.78$	$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12		$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$	$32^3 \times 64, m_\pi L \sim 4.29$	$48^3 \times 64, m_\pi L \sim 3.91$
0.12		$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$	$48^3 \times 96, m_\pi L \sim 4.73$	

For the experts:

- Möbius DWF on HISQ: chiral symmetry in valence sector
- Gradient flow method for smearing configs
  - $m_{res} < 0.1\ m_l$ for moderate L_5
- Leading discretization errors:
  - HISQ $O(\alpha_S a^2)$, MDWF $O(a\ m_{res})$, $O(a^2)$

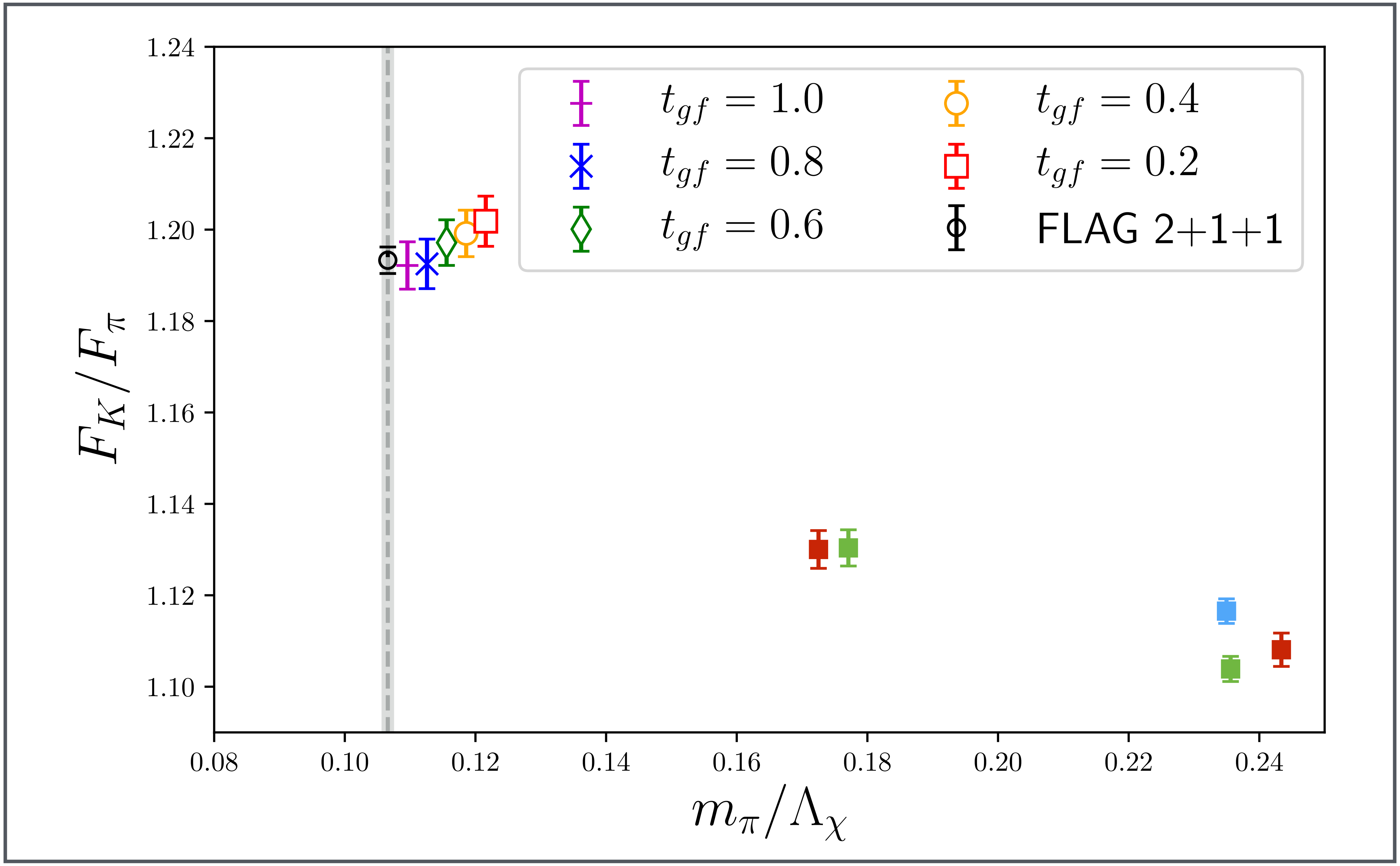
Some Lattice QCD Details

Möbius Domain-Wall Fermions on the dynamical Nf=2+1+1 HISQ Configurations from MILC (freely available, multiple lattice spacings, pion masses, etc., control of continuum, infinite volume, physical pion mass extrapolations)



Some Lattice QCD Details

Möbius Domain-Wall Fermions on the dynamical Nf=2+1+1 HISQ Configurations from MILC (freely available, multiple lattice spacings, pion masses, etc., control of continuum, infinite volume, physical pion mass extrapolations)



Some Lattice QCD Details