Quasi-PDF with non-perturbative renormalization

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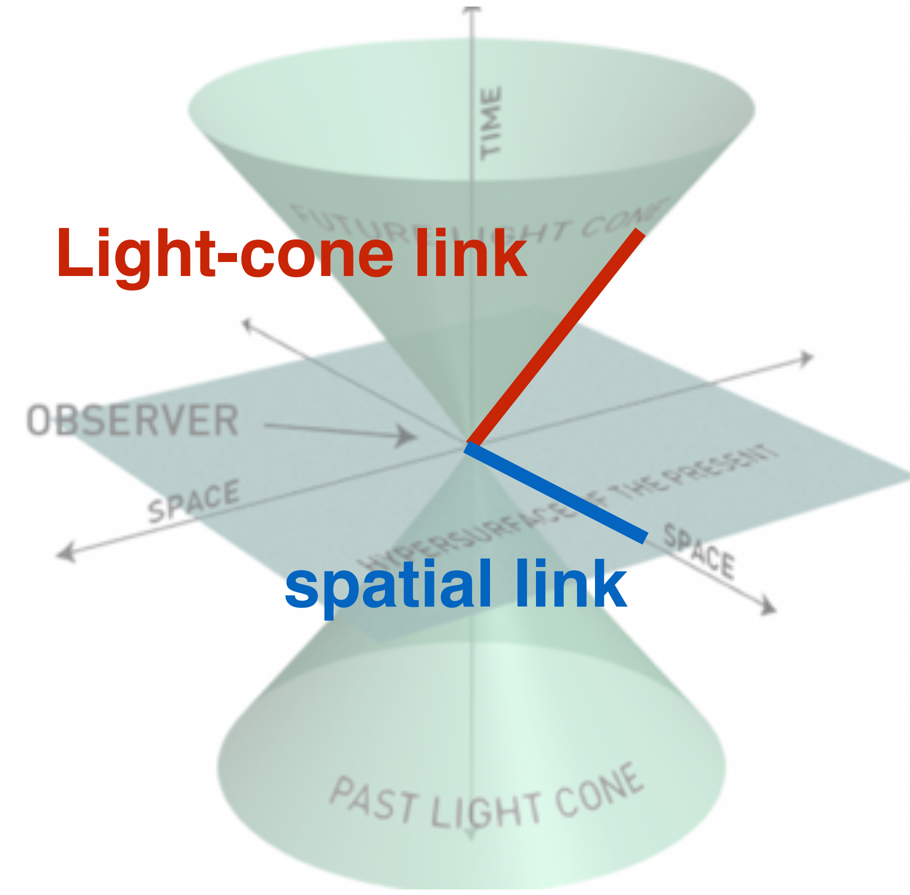
Definition of the quasi-PDF

The original quark PDF defined in the light front frame is,

$$\boxed{\begin{aligned} q(x,\mu^2) &= \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \\ &\times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle \end{aligned}}$$


Light-cone link

spatial link

The quasi-PDF is defined by

$$\boxed{\tilde{q}(x,P_z,\tilde{\mu})_\Gamma = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_z z}\langle P|O_\Gamma(z)|P\rangle}$$

$$O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0) \qquad U(z,0) = P\exp\left(-ig\int_0^z dz' A^z(z')\right)$$

$$\Gamma = \gamma_z \text{ or } \gamma_t$$

← *Modified definition*

↑ *Original definition*

X.D. Ji, PRL 110 (2013) 262002
X. Xiong, X. Ji, J.-H. Zhang, and Y. Zhao, PRD 90 (2014) 014051

From the bare quasi-PDF

to the real PDF

Y-Q. Ma, J-W. Qiu, 1404.6860
C. Alexandrou et. al., Phys. Rev. D92 014502
J.-W. Chen, X. Ji, J. Zhang, Nucl.Phys. B915 (2017) 1
LP³, 1803.04393

$$q(x,\mu) \; = \int_{-\infty}^{\infty} \frac{dy}{|y|} \left\{ \delta(1-\frac{x}{y}) - \frac{\alpha_s C_F}{2\pi} \left[f_1\left(\frac{x}{y}, \frac{yP_z}{\mu}\right) - \frac{yP_z}{p_z^R} f_2(1+\frac{yP_z}{p_z^R}(\frac{x}{y}-1), \frac{\mu_R^2}{p_z^{R2}}) \right] \right\}$$

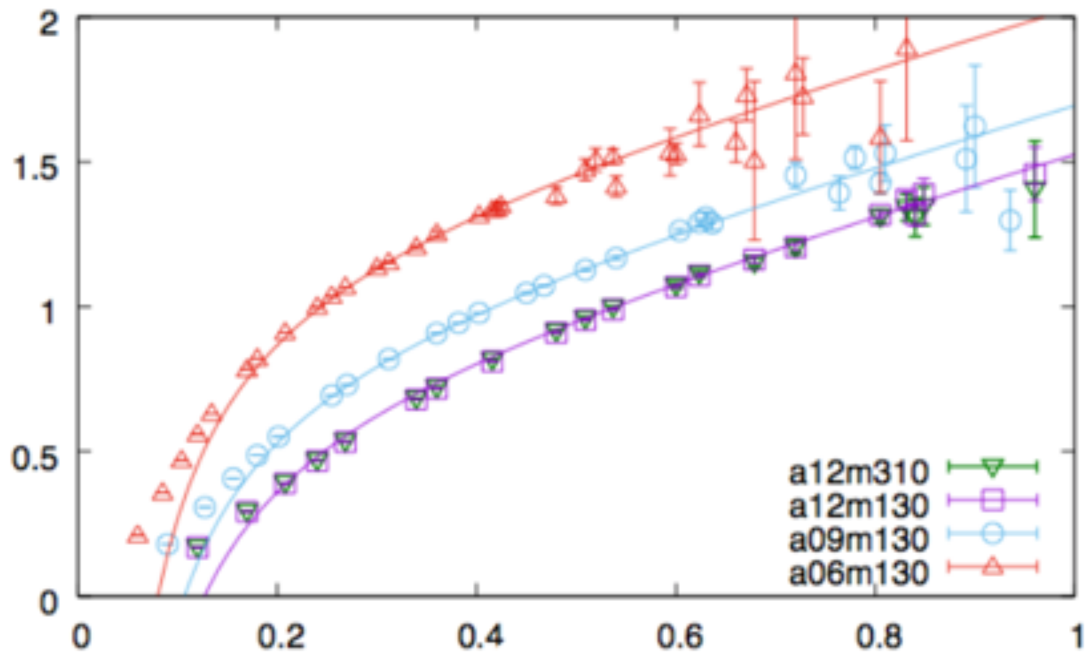
$$\int_{-\infty}^{\infty} e^{iyP_z z} \langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R_{p^2=\mu_R^2, p_z=p_z^R}$$

$$+\mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}, \alpha_s^2\right).$$

The linear divergence under the lattice regularization can break down the power counting!

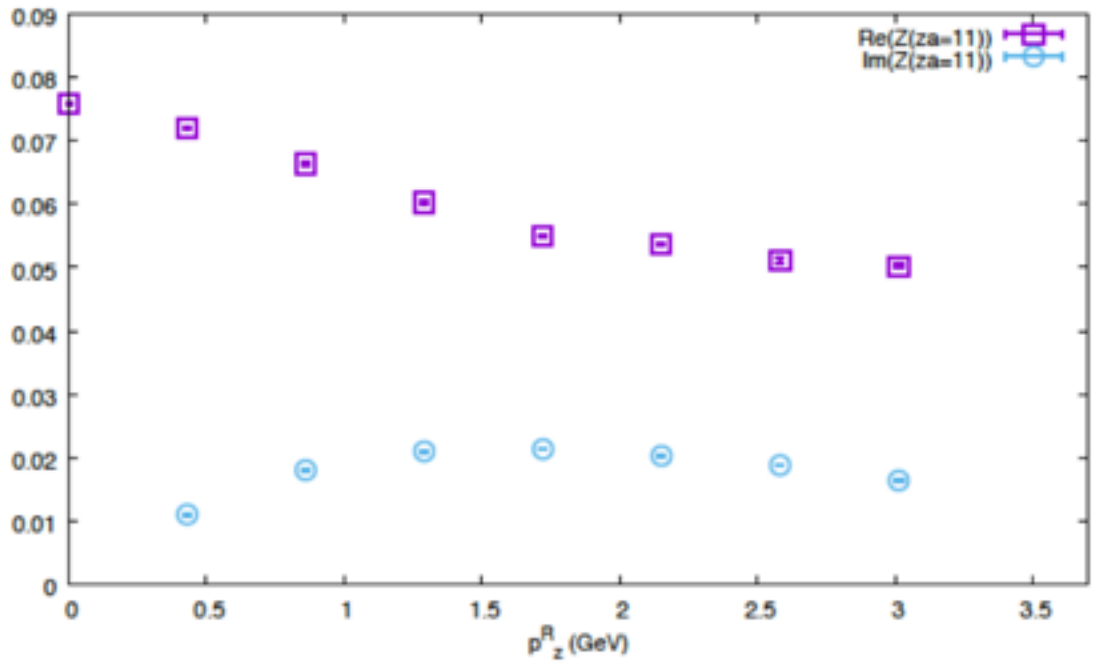
$$\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R = (1+\frac{\alpha_s}{4\pi}(\frac{C}{a}+\mathrm{Log}(\mu_R^2 a^2)+...)+\mathcal{O}\left(\alpha_s^2\right))\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle_{bare}$$

Outline



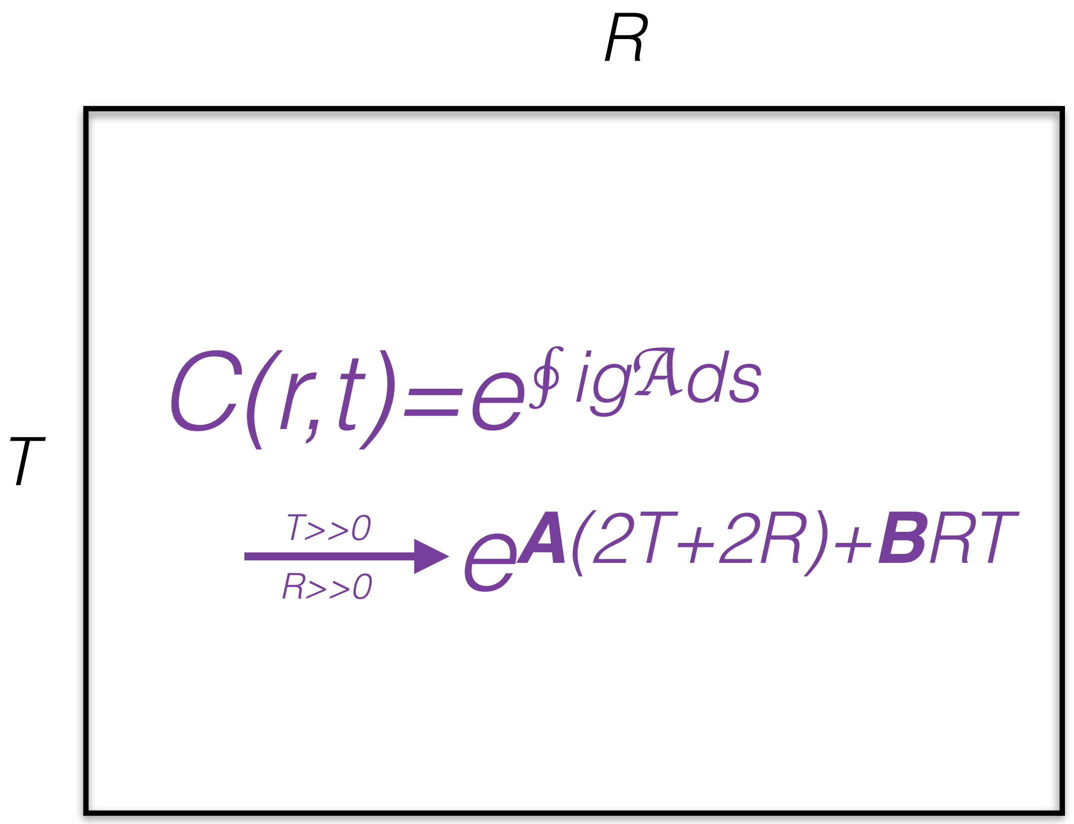
- **Linear divergence (LD) in the wilson loop and quasi-PDF operator**

- The external momentum dependence of RI/MOM



Linear divergence (LD)

in the wilson loop



$$C(r,t)=e^{\oint ig\mathcal{A}ds}$$

$$\xrightarrow[R>>0]{T>>0} e^{\mathbf{A}(2T+2R)+\mathbf{B}RT}$$

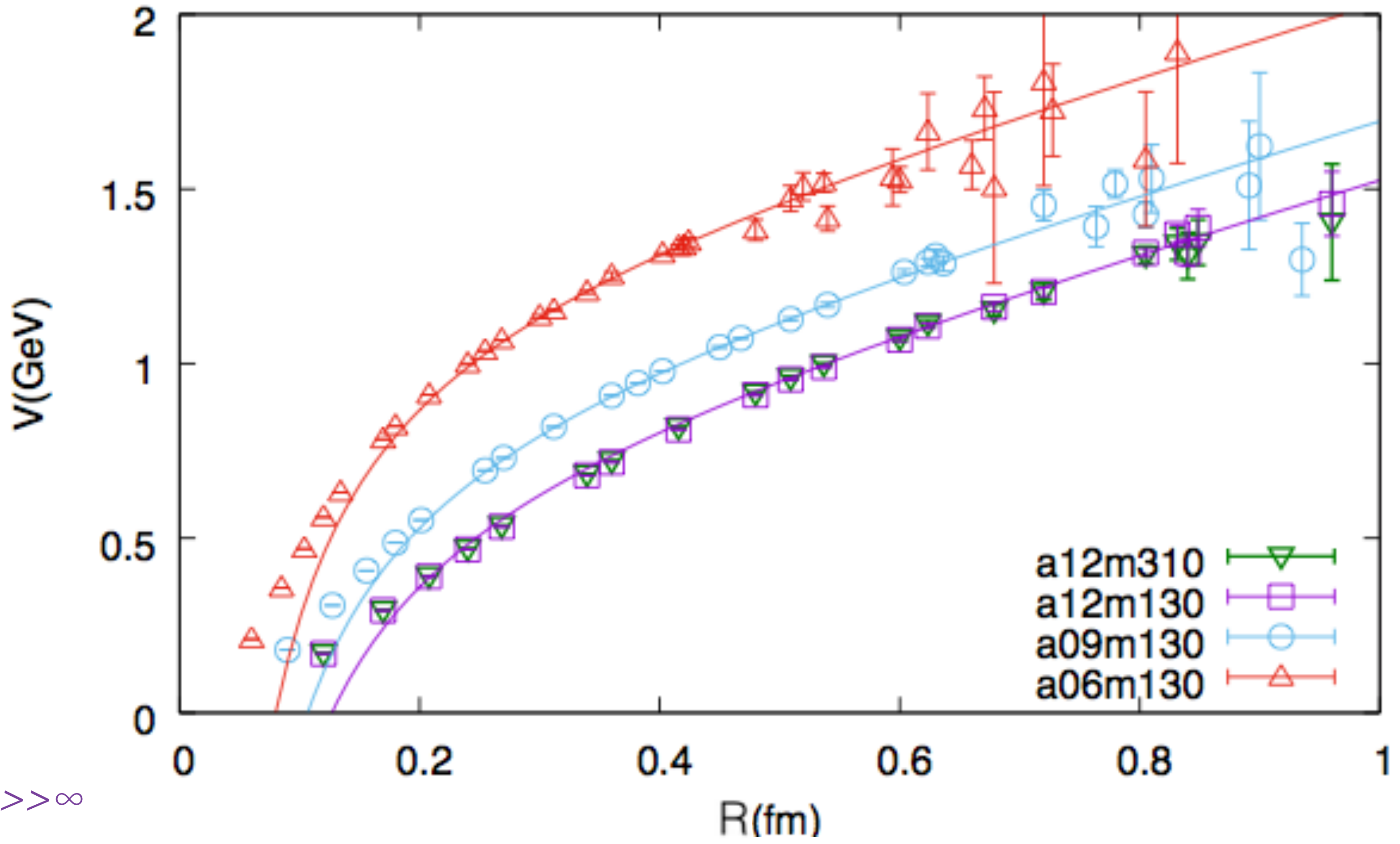
The statical potential is defined by,

$$V(R)=Log[\langle C(R,T)\rangle/\langle C(R,T+1)\rangle]\big|_{T\to\infty,\ R>>\infty}$$

$$=\alpha/R+2\mathbf{A}+\mathbf{B}R,$$

with

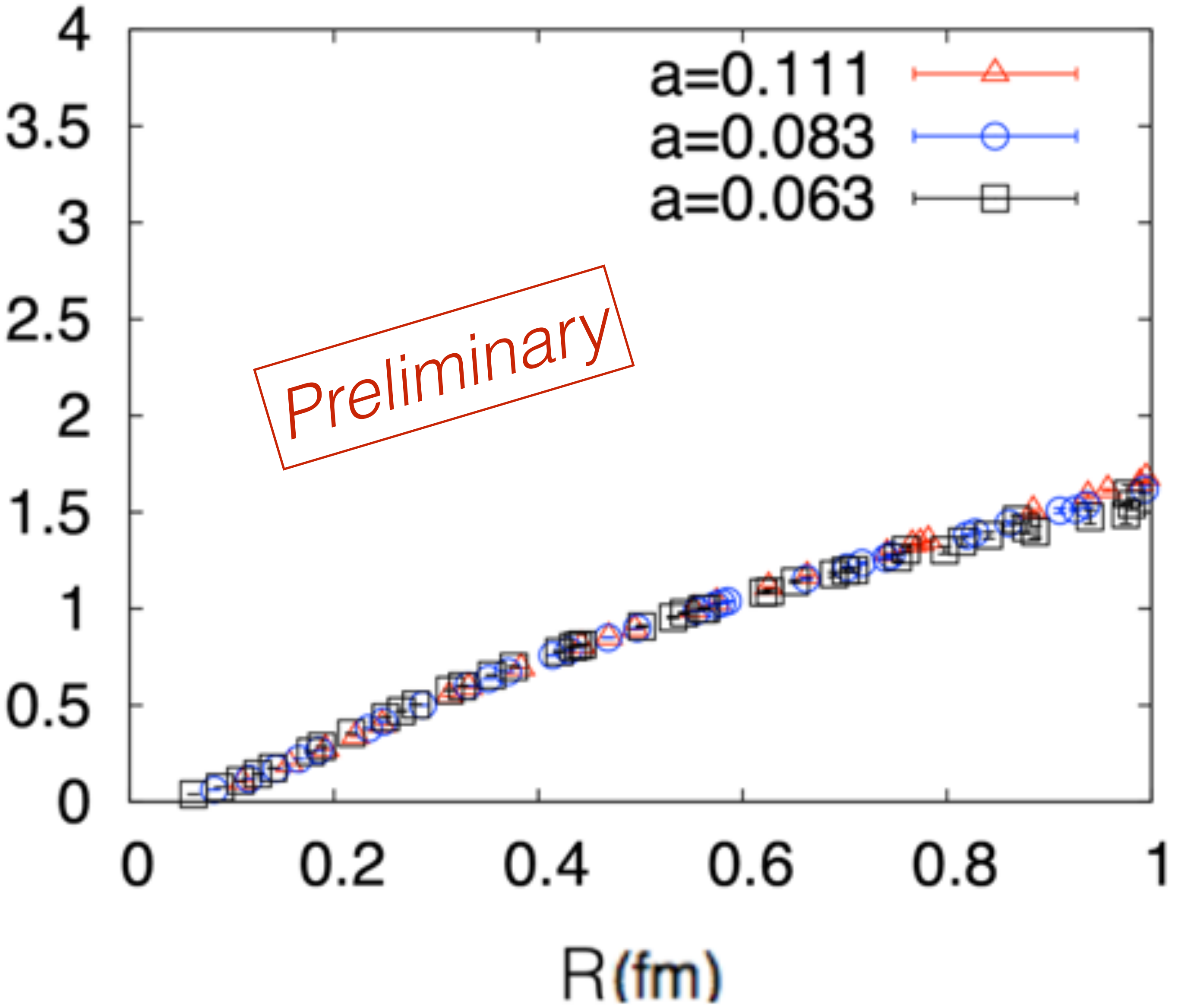
$$\mathbf{A}\sim \Delta m/\mathbf{a} + \mathbf{A}_0$$



*We can see **V(R) shifts vertically** when the lattice spacing **a** becomes **smaller**, and a joint fit get **Δm=0.154(2)**.*

Gradient flow?

- *With the same t (Sqrt[1/t] ~ 3.3 GeV) the statical potential with different lattice spacings are **almost the same**.*
- *The effective δm can be smaller or even negative with even lower Sqrt[1/t].*
- *But the matching in the continuum will be highly non-trivial.*



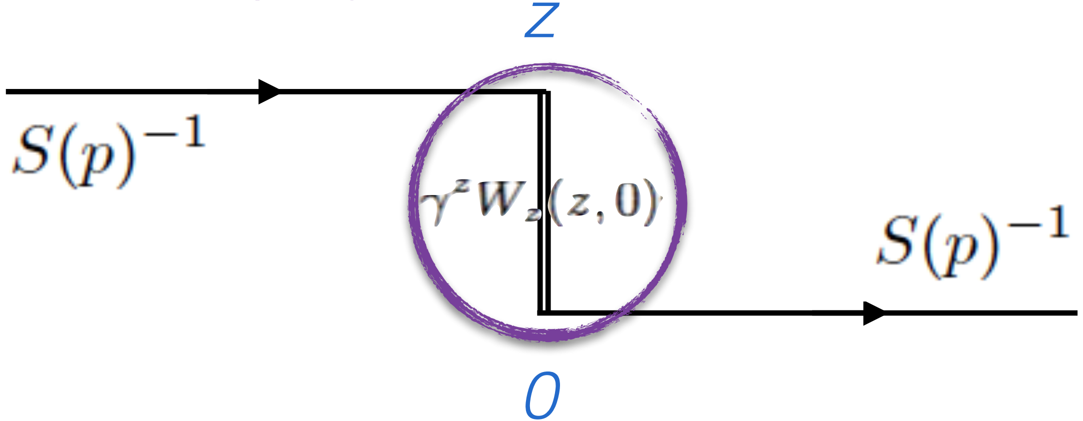
Non-perturbative renormalization (NPR)

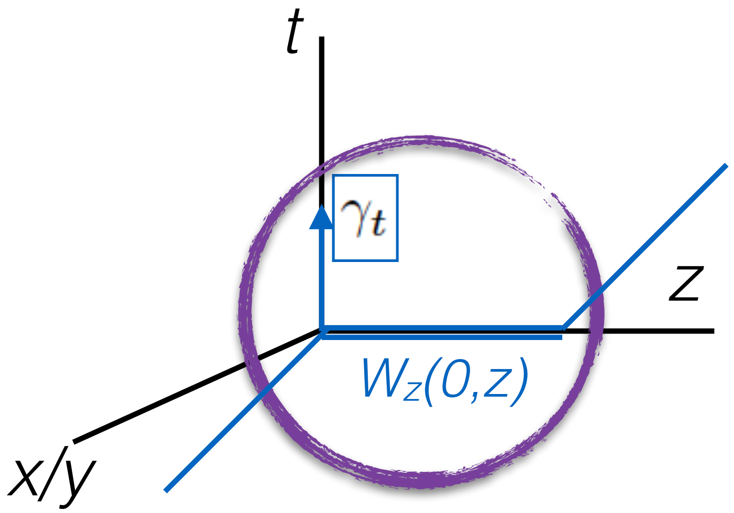
*The non-perturbative renormalized quasi-PDF matrix element **$\tilde{h}^R$** in the RI/MOM scheme is defined by*

$$\tilde{h}^R(z, P_z, p_z^R, \mu_R) \;=\; \tilde{Z}^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{a\to 0}$$

where $\tilde{h}(z, P_z, a^{-1}) \;=\; \frac{1}{2P^0}\langle P|O_{\gamma_t}(z)|P\rangle$ *is the lattice bare quasi-PDF matrix elements.*

To get Z, we project the dressed vertex function,





$$p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$

by p-slash:

→ ***the renormalization constan***

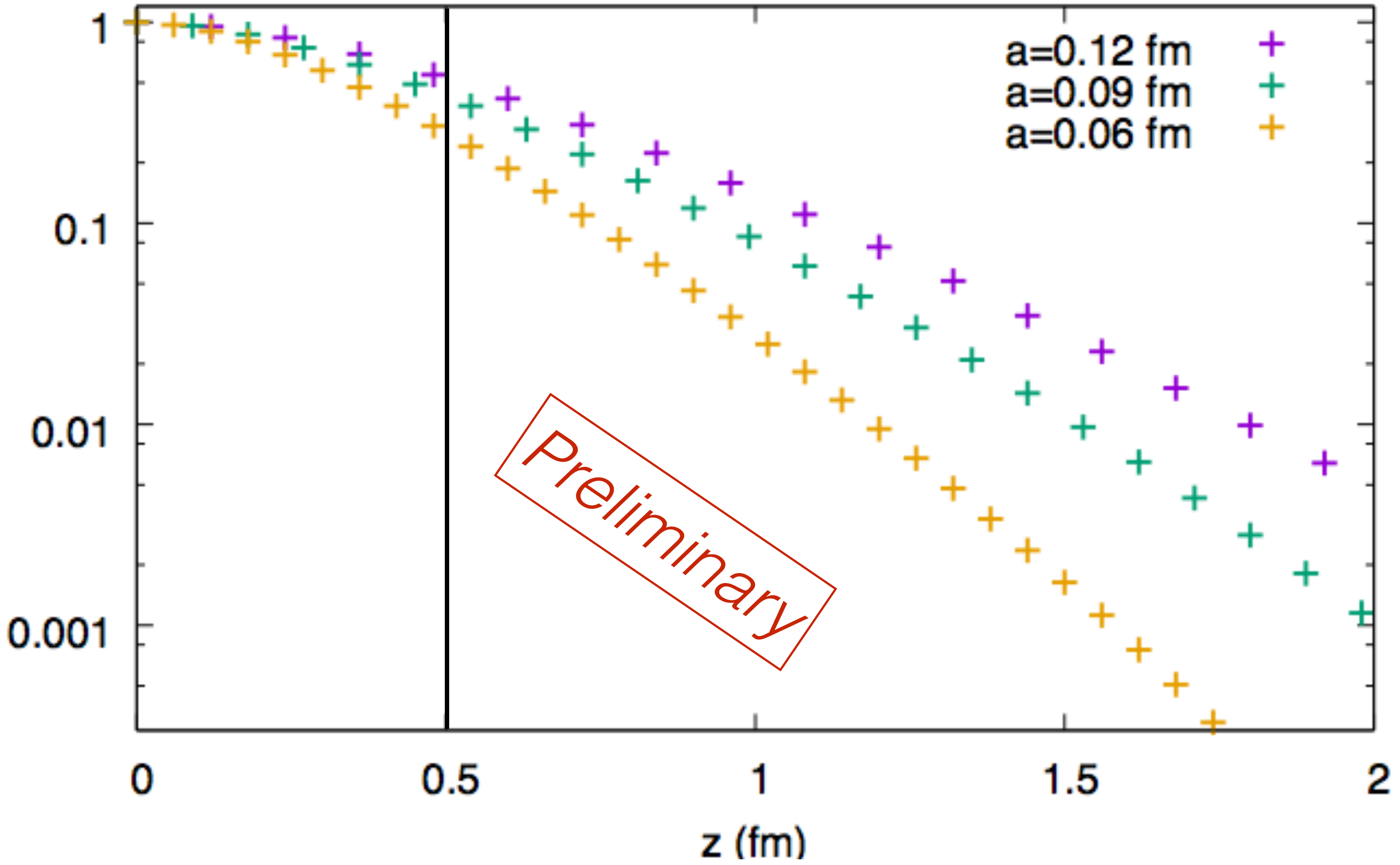
$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 \;=\; -\mu_R^2 \\ p_z \;=\; p_z^R}}$$

Linear divergence (LD)

based on the NPR

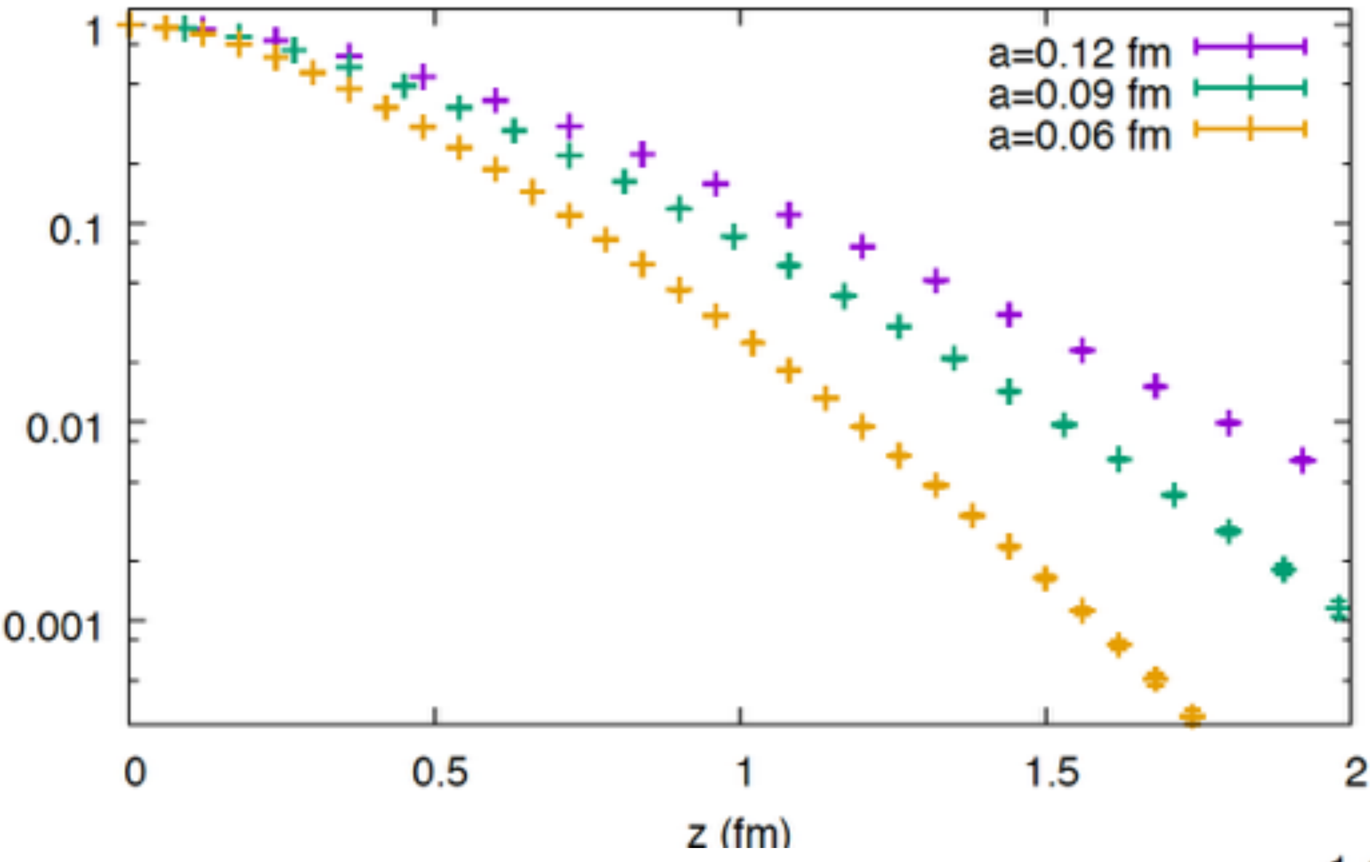
$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

*is purely **real** when p_z^R is **zero**.*



- *$h_R = h_{bare}/Z$;*
- With a=0.06 fm and z~0.5 fm, Z~0.3;
- The linear divergence should be **resumed** to describe the Z from non-perturbative renormalization (NPR).

Linear divergence (LD)



based on the NPR

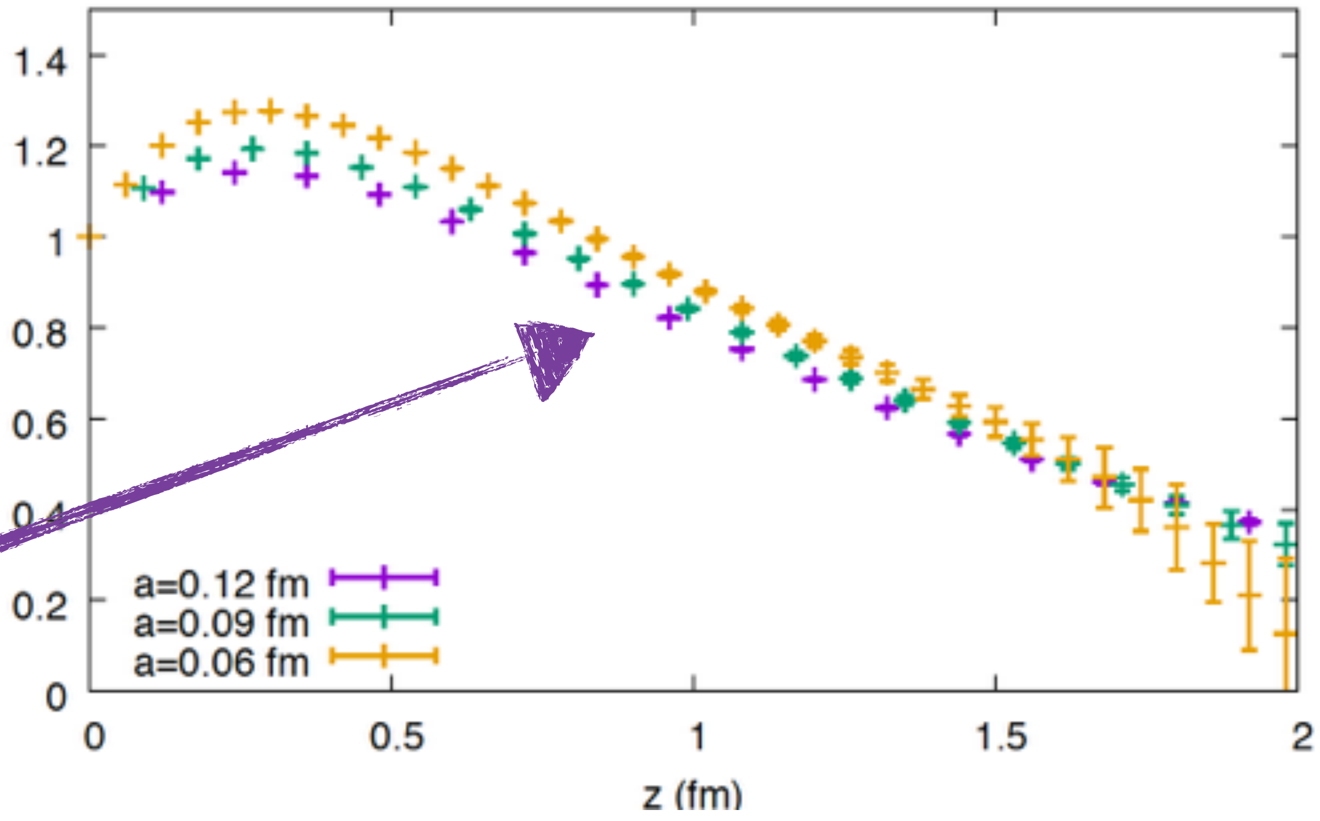
Fit in the range z=(0.45, 1.5) fm, we can get the linear divergence terms as

$$e^{(-0.136(2)z+0.011(2)z^2-0.083(4)z^3)/a}$$

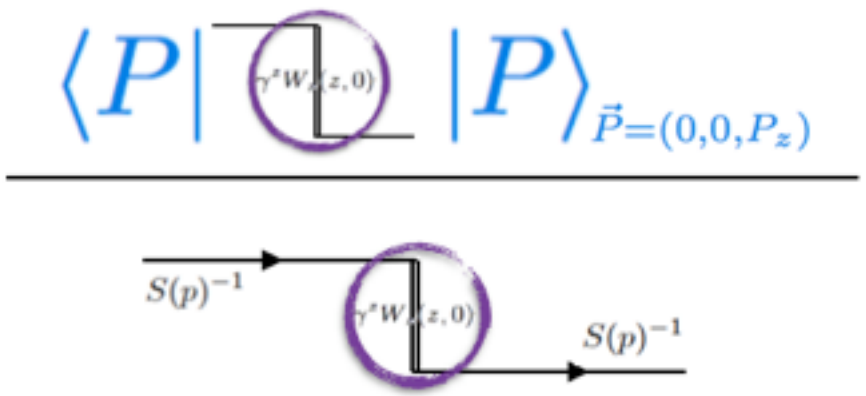
with χ^2/d.o.f.<1.

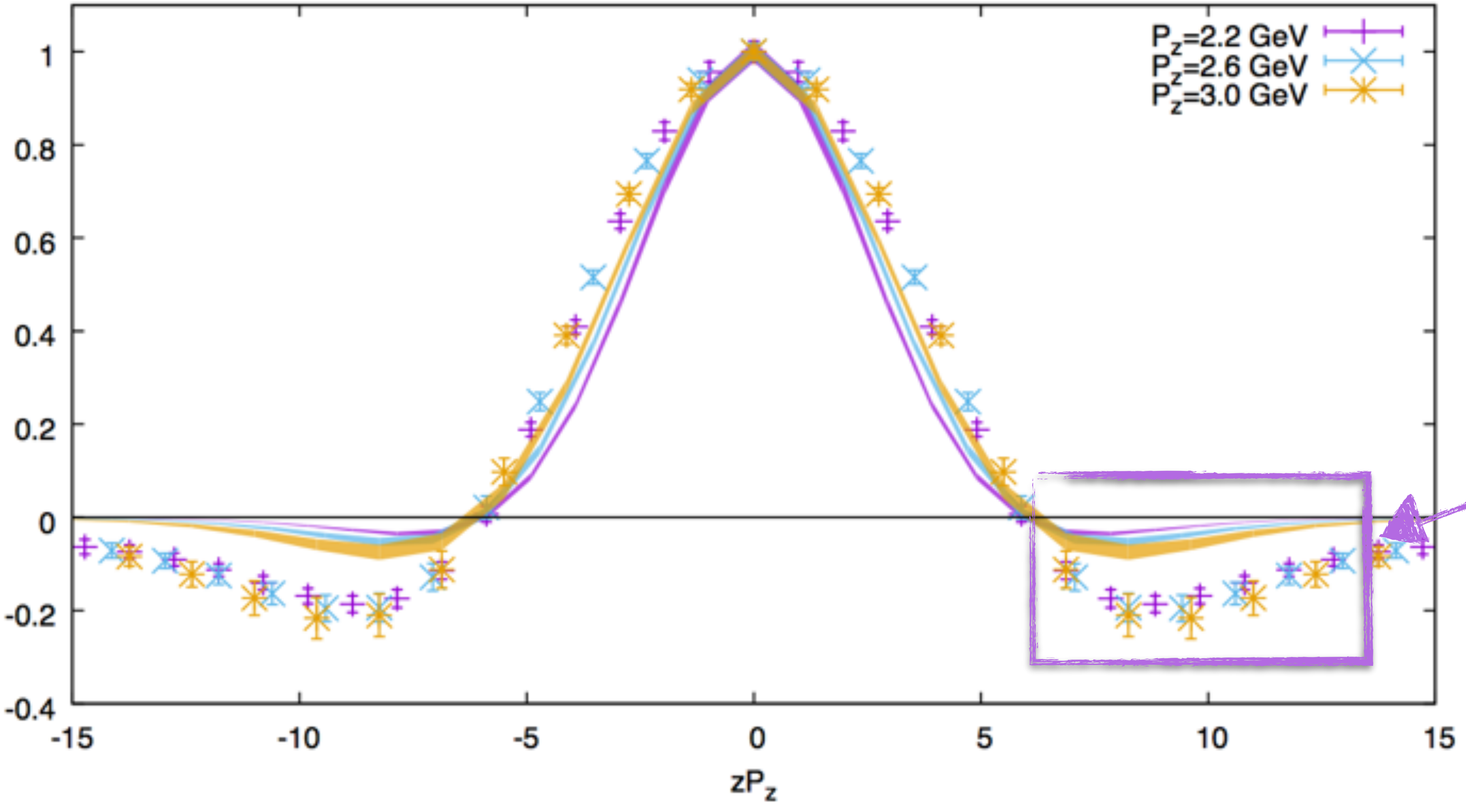
Preliminary

- We got **Δm=0.154(2)** from the **wilson loop**;
- The small z limit of the linear divergence **from the quasi-PDF operator** is **0.136(2)**.
- The renormalization constants are $\mathcal{O}(1)$ after the LDs are removed.



The renormalization effect





- **Curves** show the bare results.
- **Data points** shows the renormalized ones with p_z^R =0.
- The renormalization enlarged the MEs much at the long tails of z.

Another way

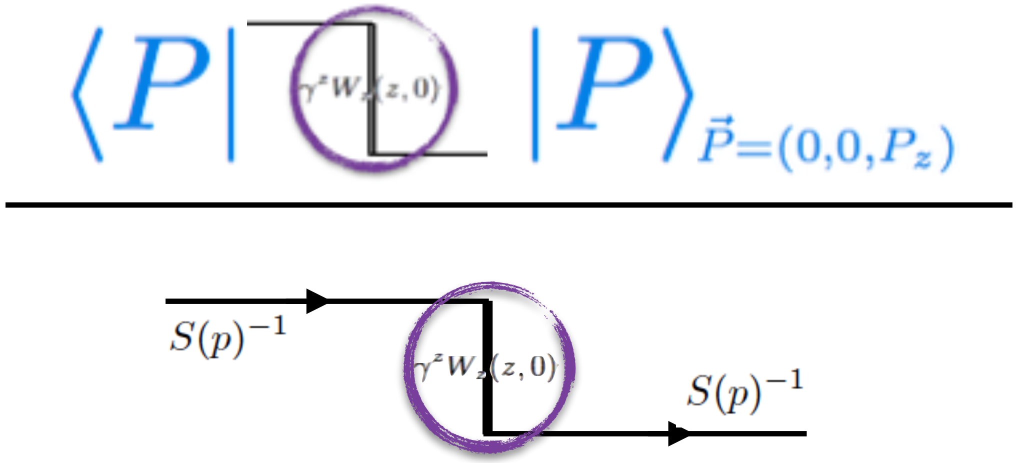
to remove the linear divergence

- No matter the hadron is in the moving frame or rest frame, the operator used by $\tilde{h}(z, P_z)$ and $\tilde{h}(z,0)$ are the same.
- So by taking the ratio $\tilde{h}(z, P_z)/\tilde{h}(z,0)$, the linear divergence will be canceled.

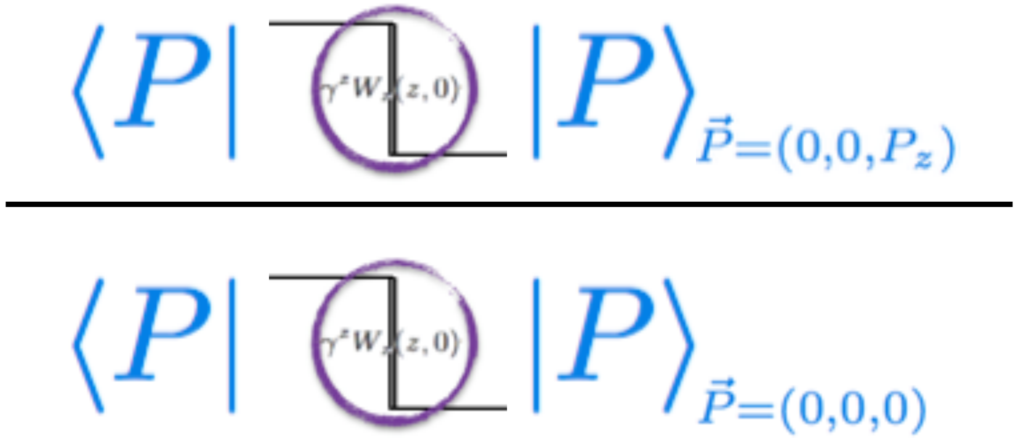
A.V. Radyushkin, 1705.01488

- It is similar to the idea of RI/MOM renormalization, while using the hadron matrix element instead of the quark one to canceled the linear divergence.
- The matching needed to connect $\tilde{h}(z, P_z)/\tilde{h}(z,0)$ to PDF will be different from the quasi-PDF case, but it can be done.

T. Izubuchi, et.al , 1801.03917

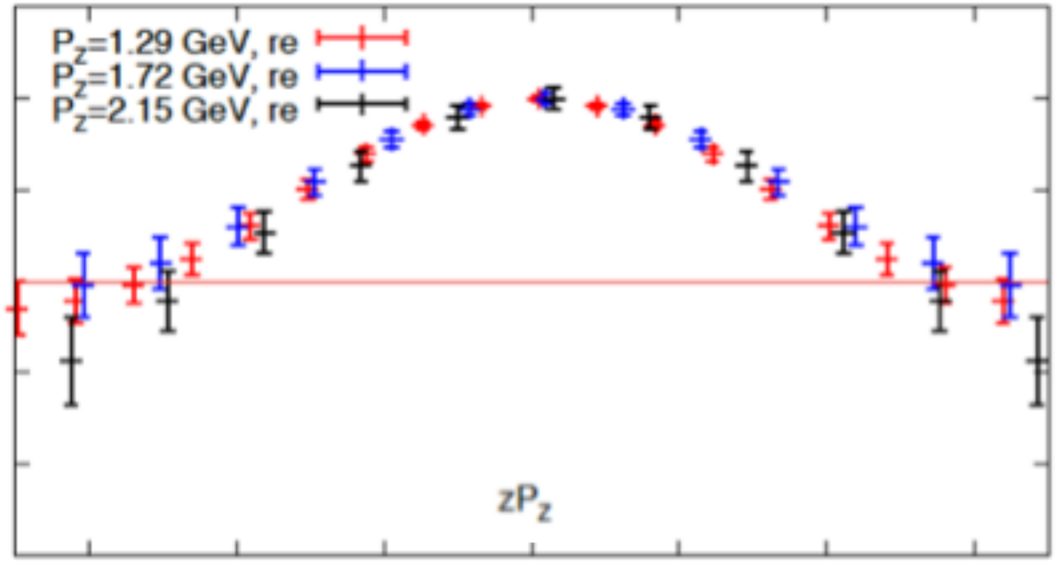


RI/MOM

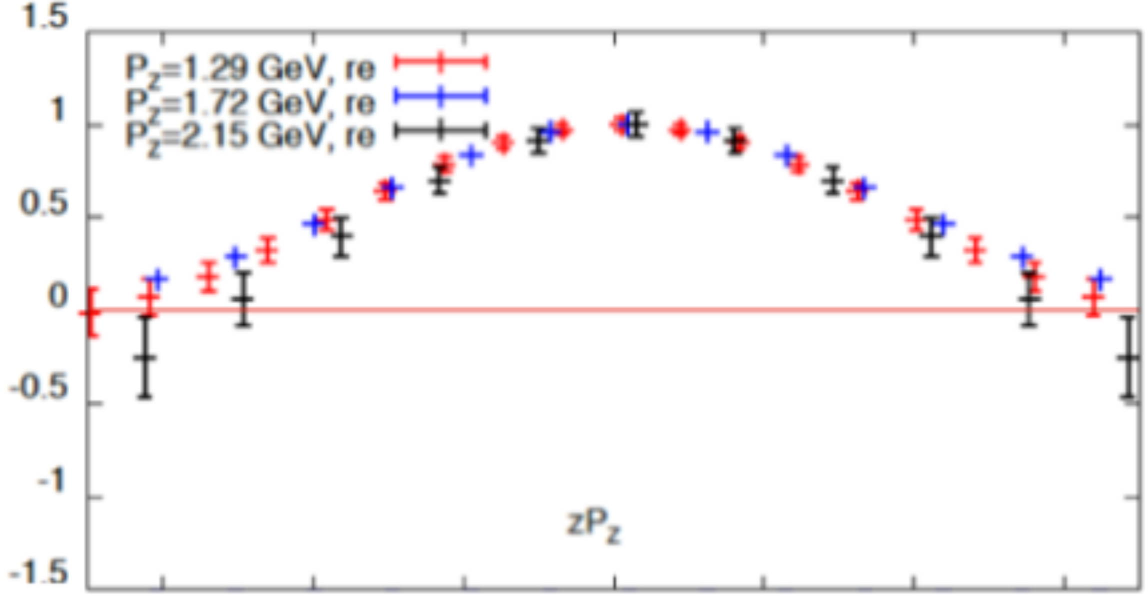


Ratio method

Another way



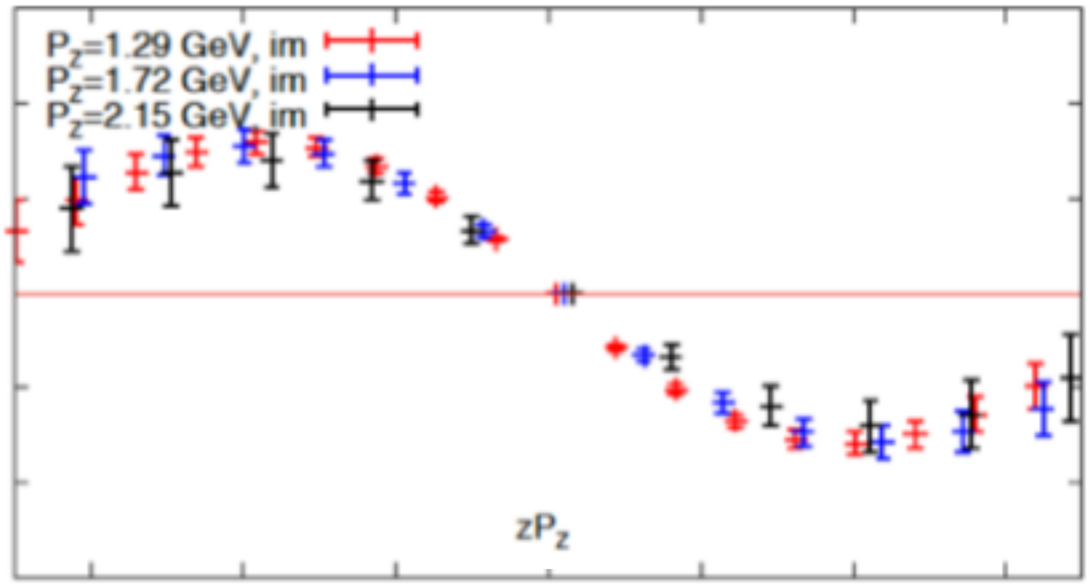
to remove the linear divergence

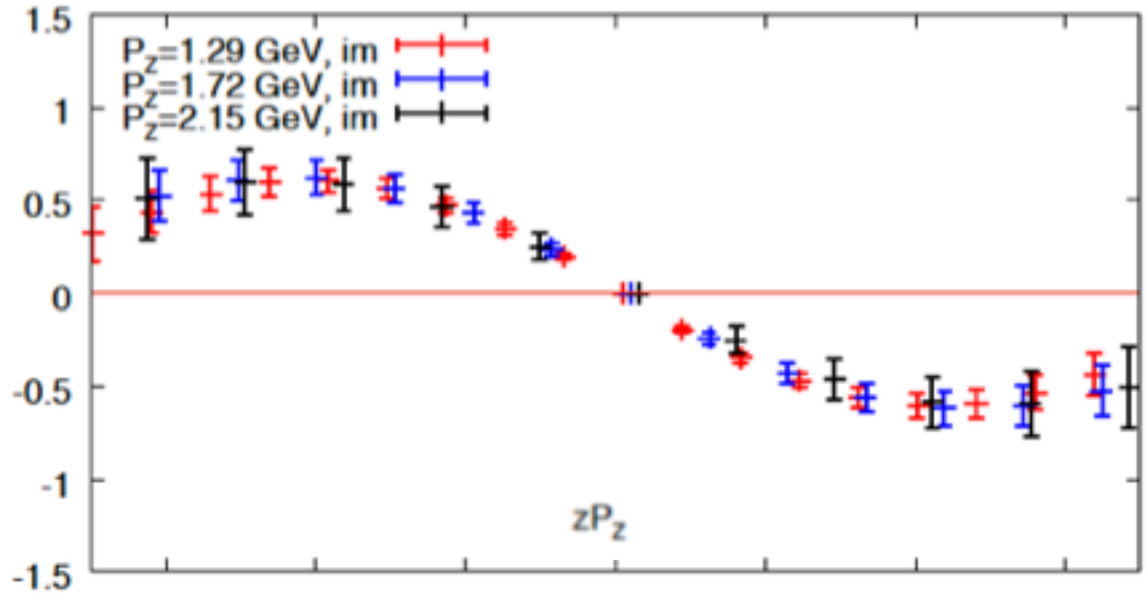


RI/MOM renormalized $\tilde{h}^R$ with $p_z^R = 0$

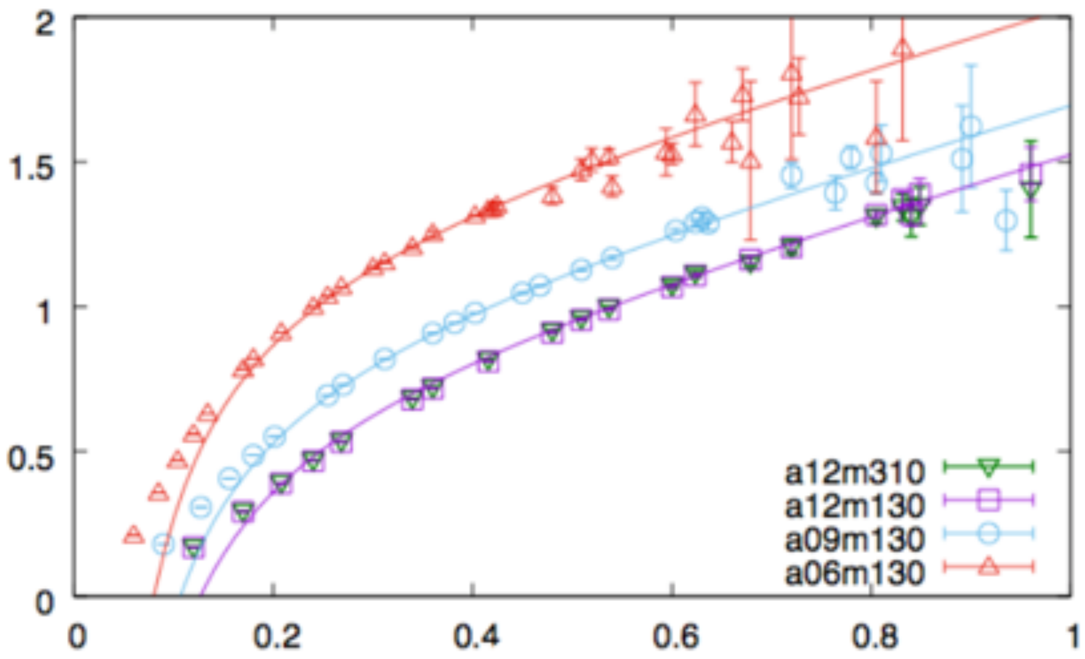
The results are very close to each other.

$\tilde{h}(z, P_z)/\tilde{h}(z,0)$



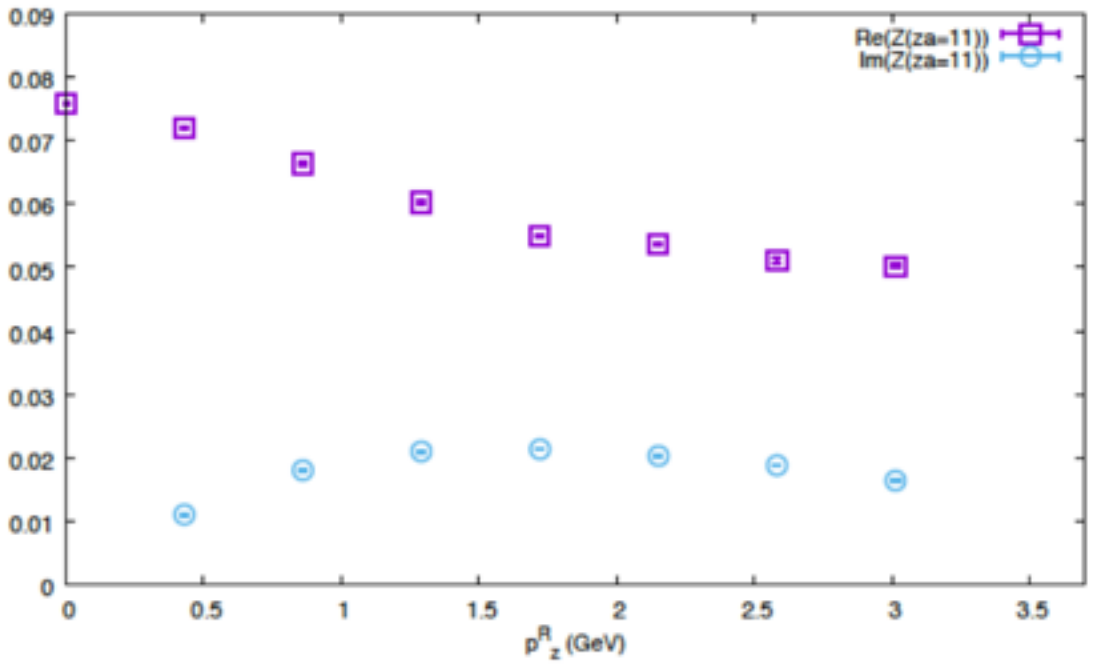


Outline



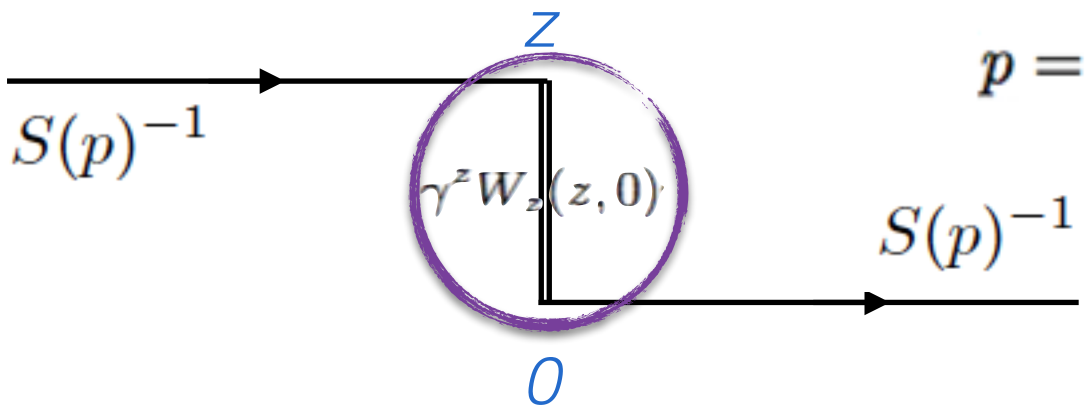
- Linear divergence (LD) in the wilson loop and quasi-PDF operator

- **The external momentum dependence of RI/MOM**



External momentum dependence

To get Z, we project the dressed vertex function,



$$p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$

by p-slash:
→ the renormalization constant

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \left. \frac{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]} \right|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

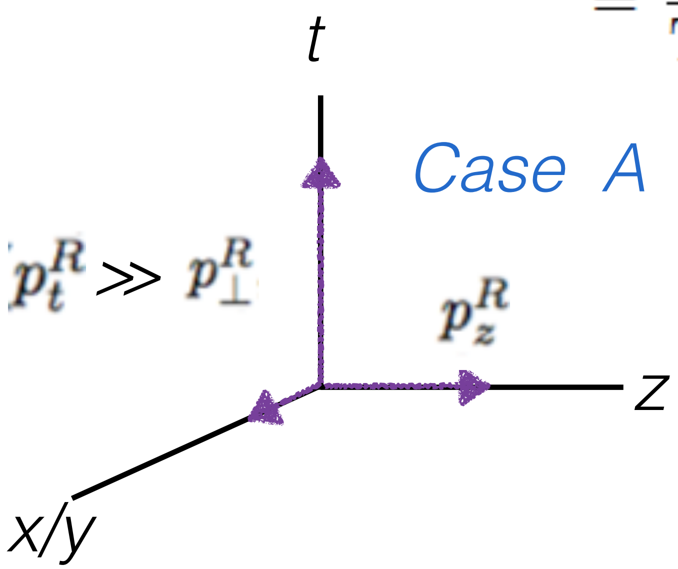
- z direction is special since the wilson link is along z; so **p_z^R dependence** starts from **1-loop level.**
- Even though Z is independent to p_t^R at 1-loop level, **p_t^R dependence** would still exist at **2-loop level** since the gamma matrix is along t;
- At 1-loop level, **μ_R dependences** will be in the **$\log(a^2p^2)$** term, but also in the finite pieces through a ratio **$(\mu_R/p_t^R)^2$**;

p_t^R dependence

of the RI/MOM constants

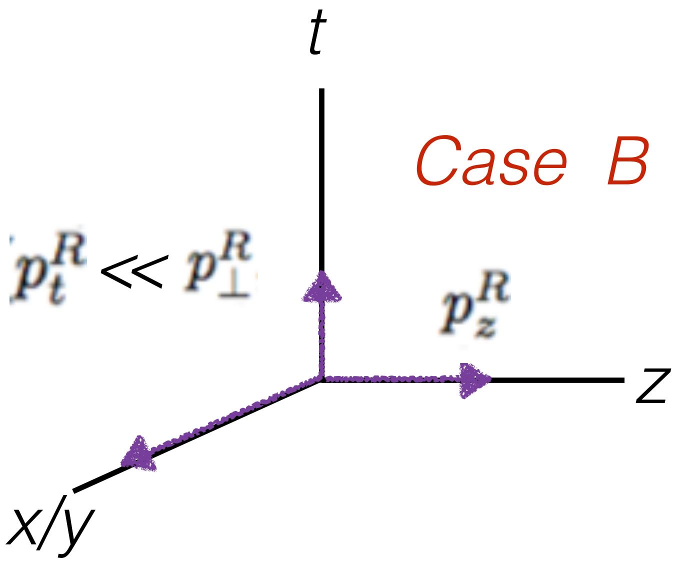
$$\tilde{Z}(z,p_z^R,a^{-1},\mu_R) = \frac{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}$$

$p=(p_t^R,p_\perp^R,p_z^R)$ with $p^2=\mu_R^2$



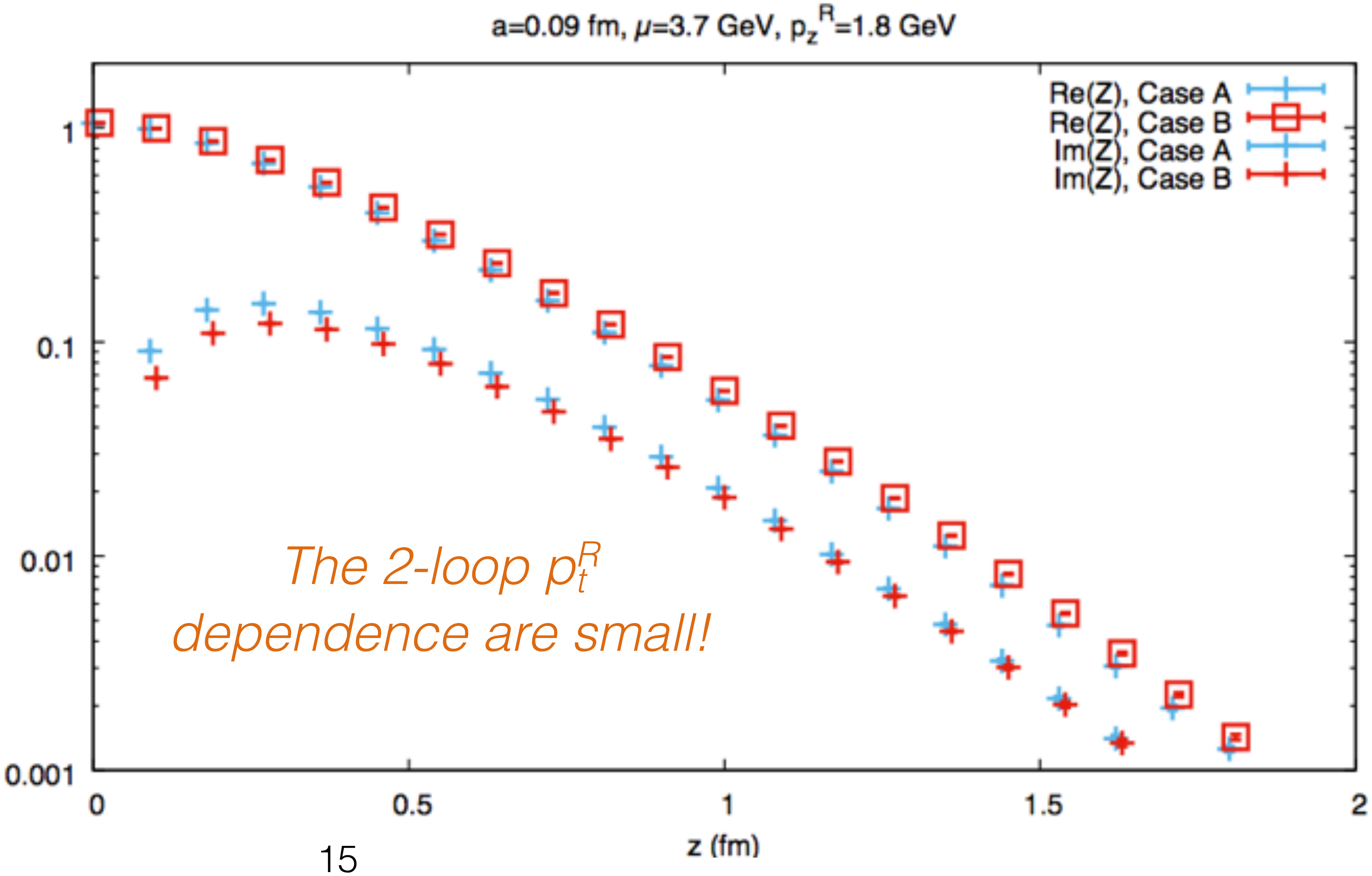
Case A

$p_t^R \gg p_\perp^R$



Case B

$p_t^R \ll p_\perp^R$



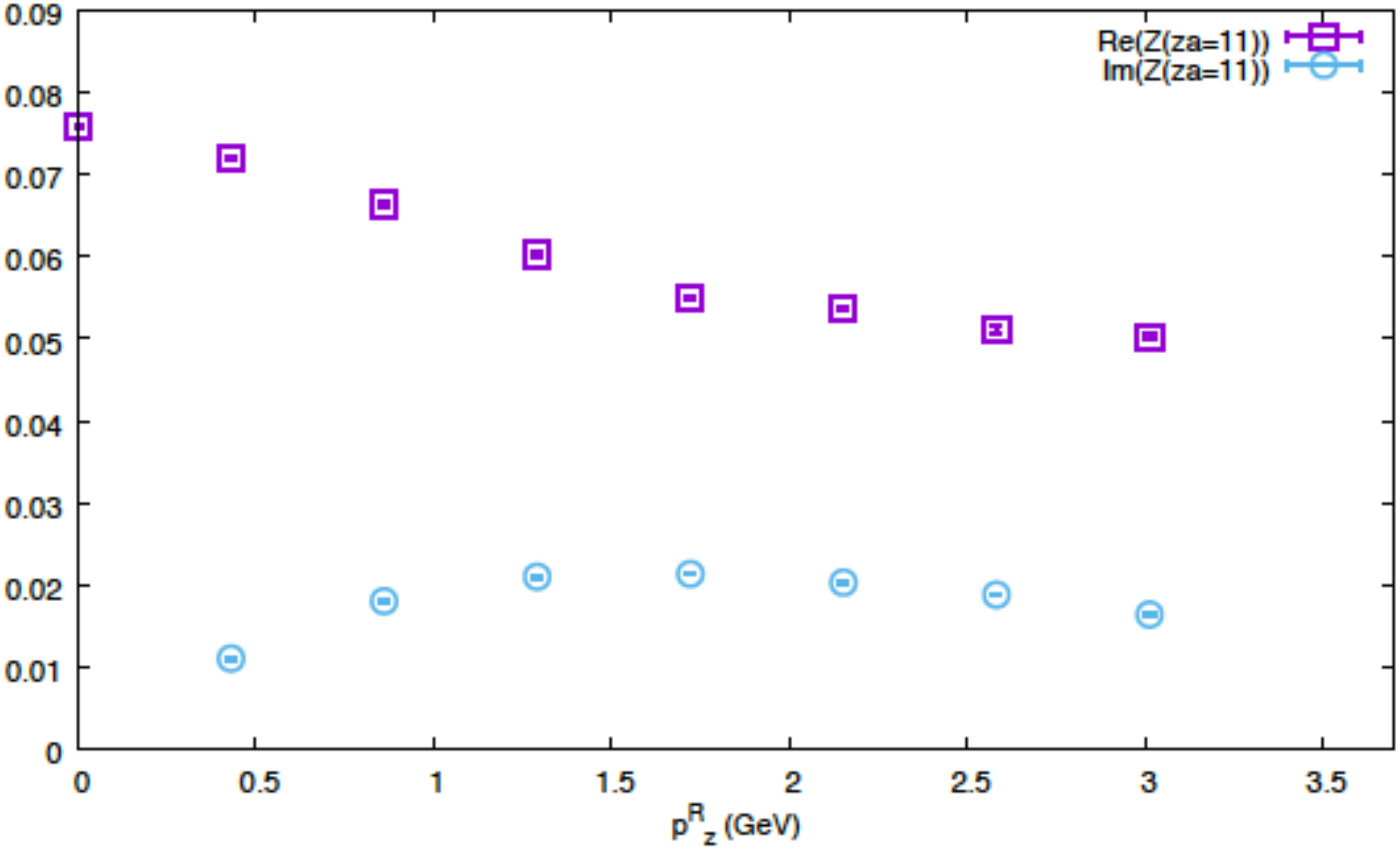
The 2-loop p_t^R dependence are small!

p_z^R dependence

of the RI/MOM constants

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p} \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

$$p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$



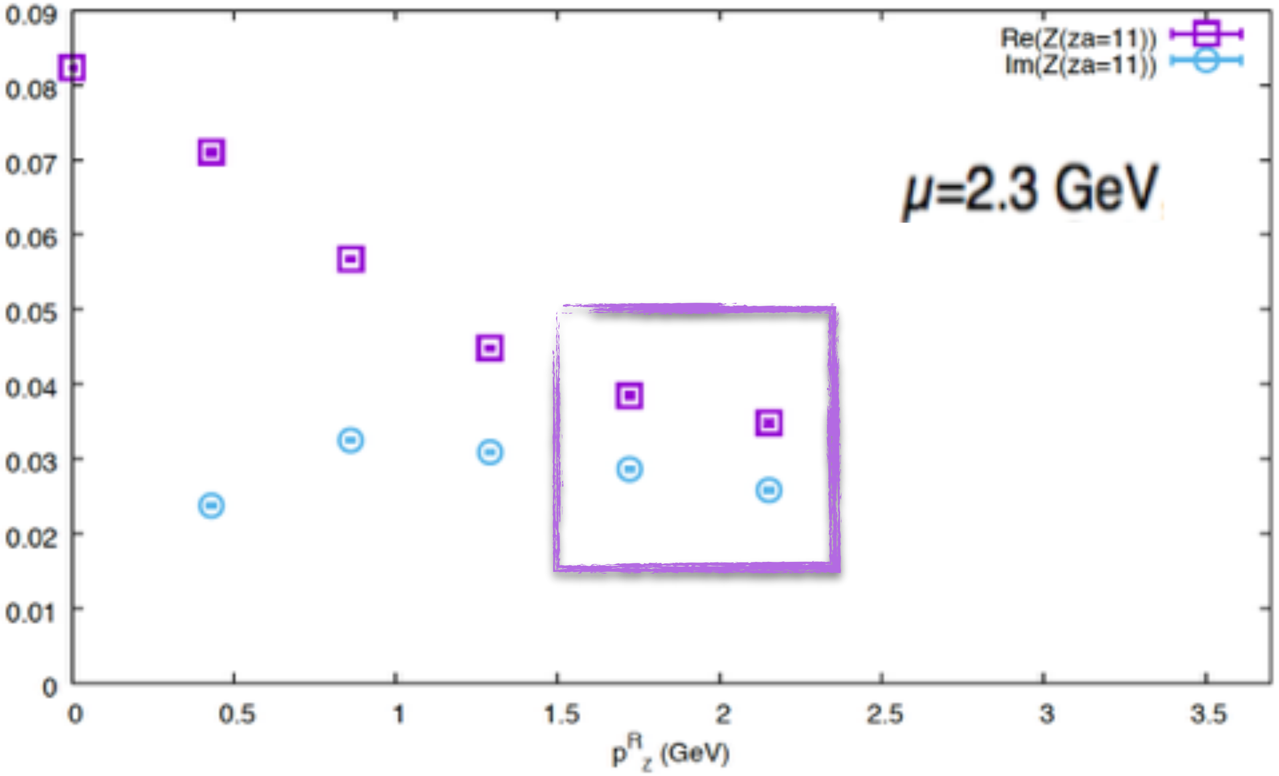
- ***Consider the z~1fm case as an example***
- *μ~3.7 GeV. p_z^R ~ 0.0, 0.4, 0.9, 1.3, 1.7, 2.2, 2.6. 3.0 GeV*
- *Strong p_z^R dependence at small p_z^R, while somehow flat at large p_z^R.*

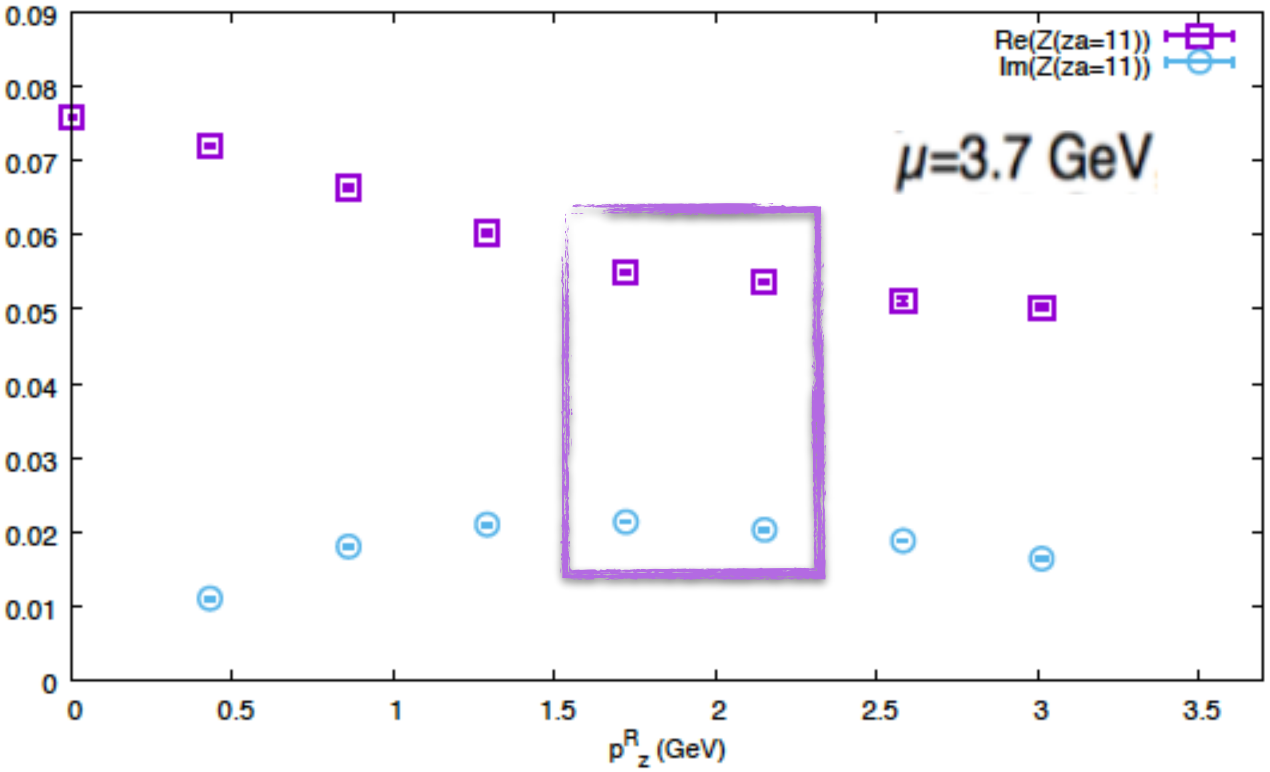
μ_R dependence

of the RI/MOM constants

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

$$p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$





- *The μ dependence with p_z^R =0 is a few percents (purely in the log term);*
- *But the μ dependence are much stronger when p_z^R is larger.*

z

The cancellation

from the matching

$$\tilde{Z}(z,p_z^R,a^{-1},\mu_R) = \left.\frac{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}} \quad p=(p_t^R,p_\perp^R,p_z^R) \text{ with } p^2=\mu_R^2$$

$$\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R = \frac{\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle_{bare}}{\tilde{Z}(z,p_z^R,a^{-1},\mu_R)}$$

$$\begin{aligned} q(x,\mu) = & \int_{-\infty}^{\infty}\frac{dy}{|y|}\left\{\delta(1-\frac{x}{y}) - \frac{\alpha_s C_F}{2\pi}\left[f_1\left(\frac{x}{y},\frac{yP_z}{\mu}\right) - \frac{yP_z}{p_z^R}f_2(1+\frac{yP_z}{p_z^R}(\frac{x}{y}-1),\frac{\mu_R^2}{p_z^{R2}})\right]\right\} \\ & \int_{-\infty}^{\infty} e^{iyP_z z}\langle P|\bar{\psi}(z)\gamma_t U_z(z,0)\psi(0)|P\rangle^R_{p^2=\mu_R^2,p_z=p_z^R} \\ & +\mathcal{O}\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2},\alpha_s^2\right). \end{aligned}$$

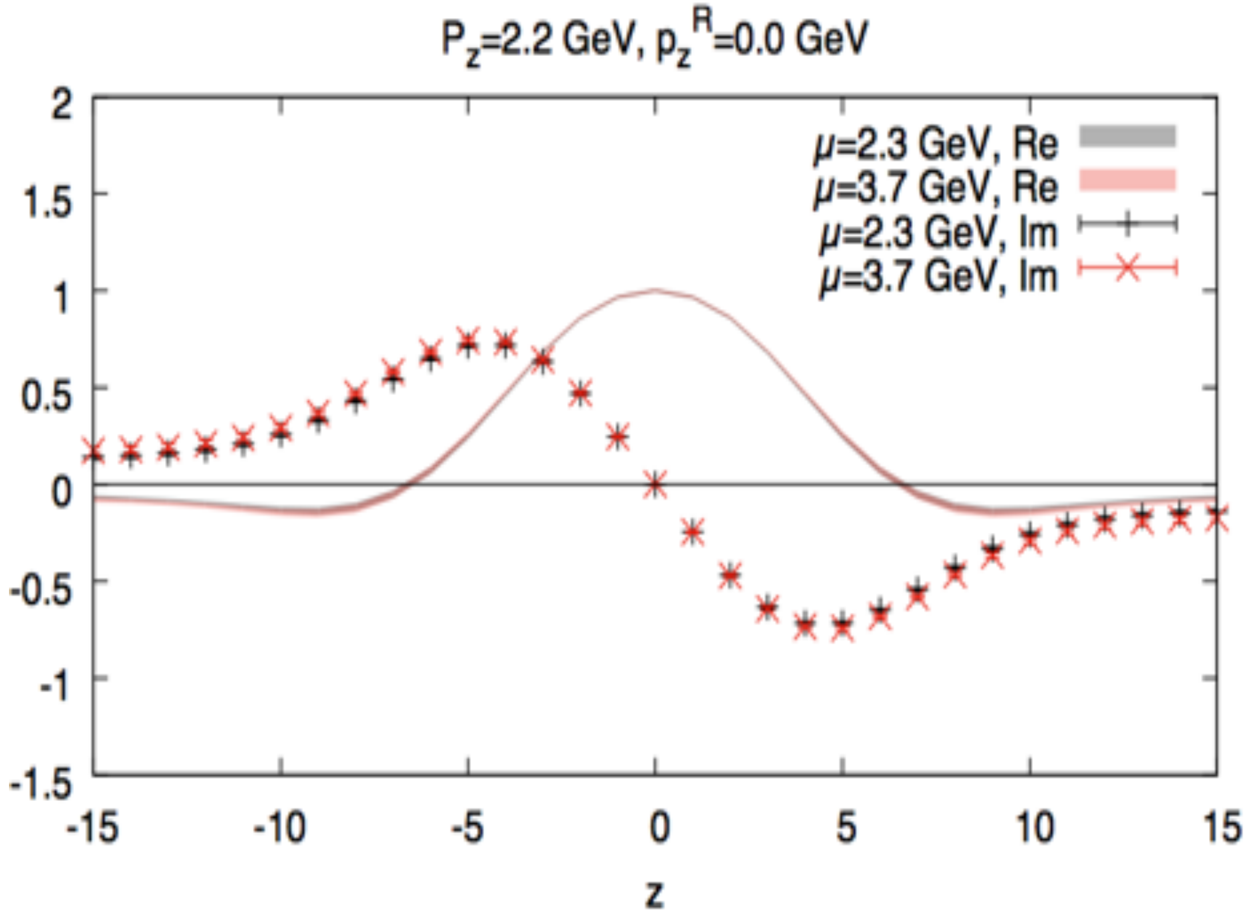
- *The μ_R and p_z^R dependence should be cancelled with the matching in the continuum;*
- *But 1-loop matching may not be good enough to reach the goal.*

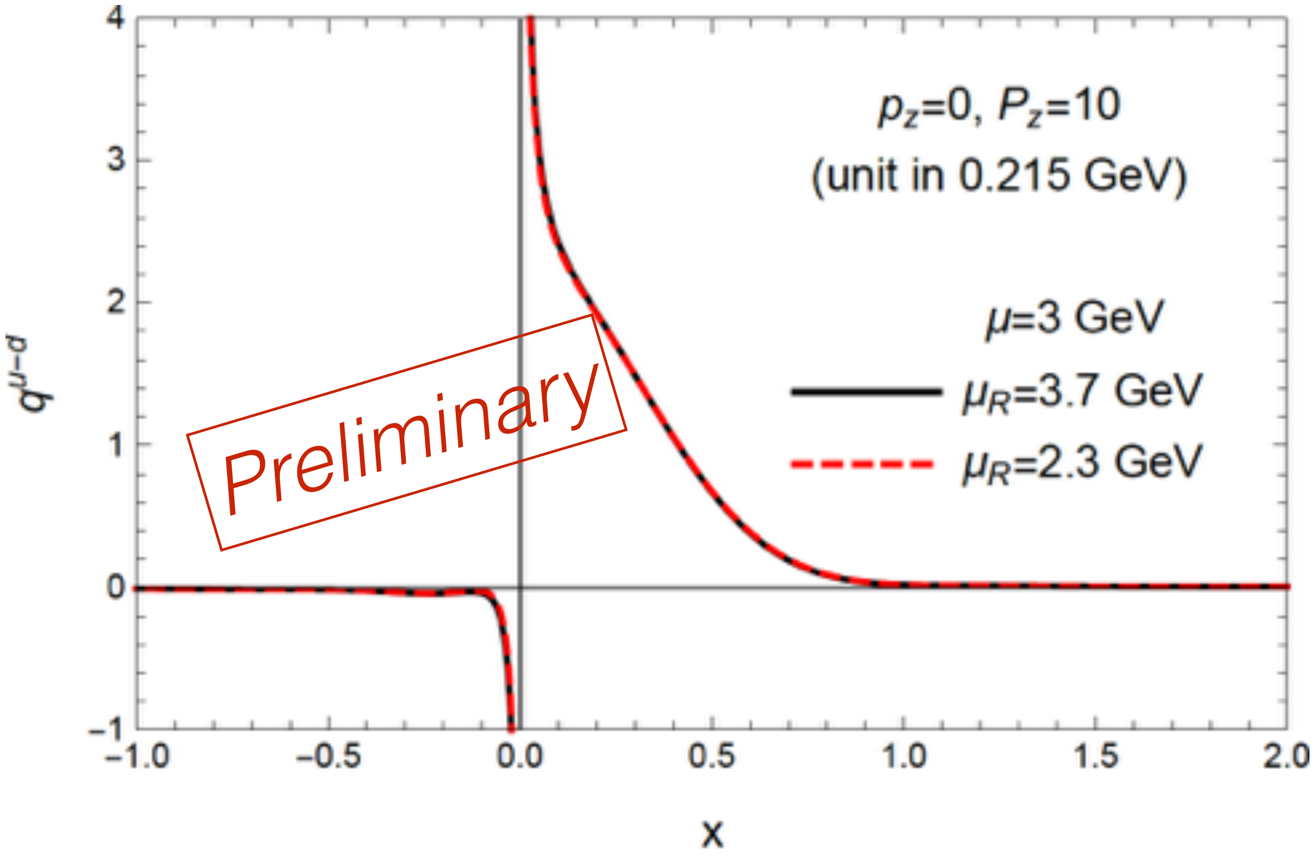
z

The result after matching

p_z^R =0 case

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not{p}\sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}} \qquad p = (p_t^R, p_\perp^R, p_z^R) \text{ with } p^2 = \mu_R^2$$





- *Some small μ_R dependence at large z;*
- *And the final result with different μ_R are also almost the same.*

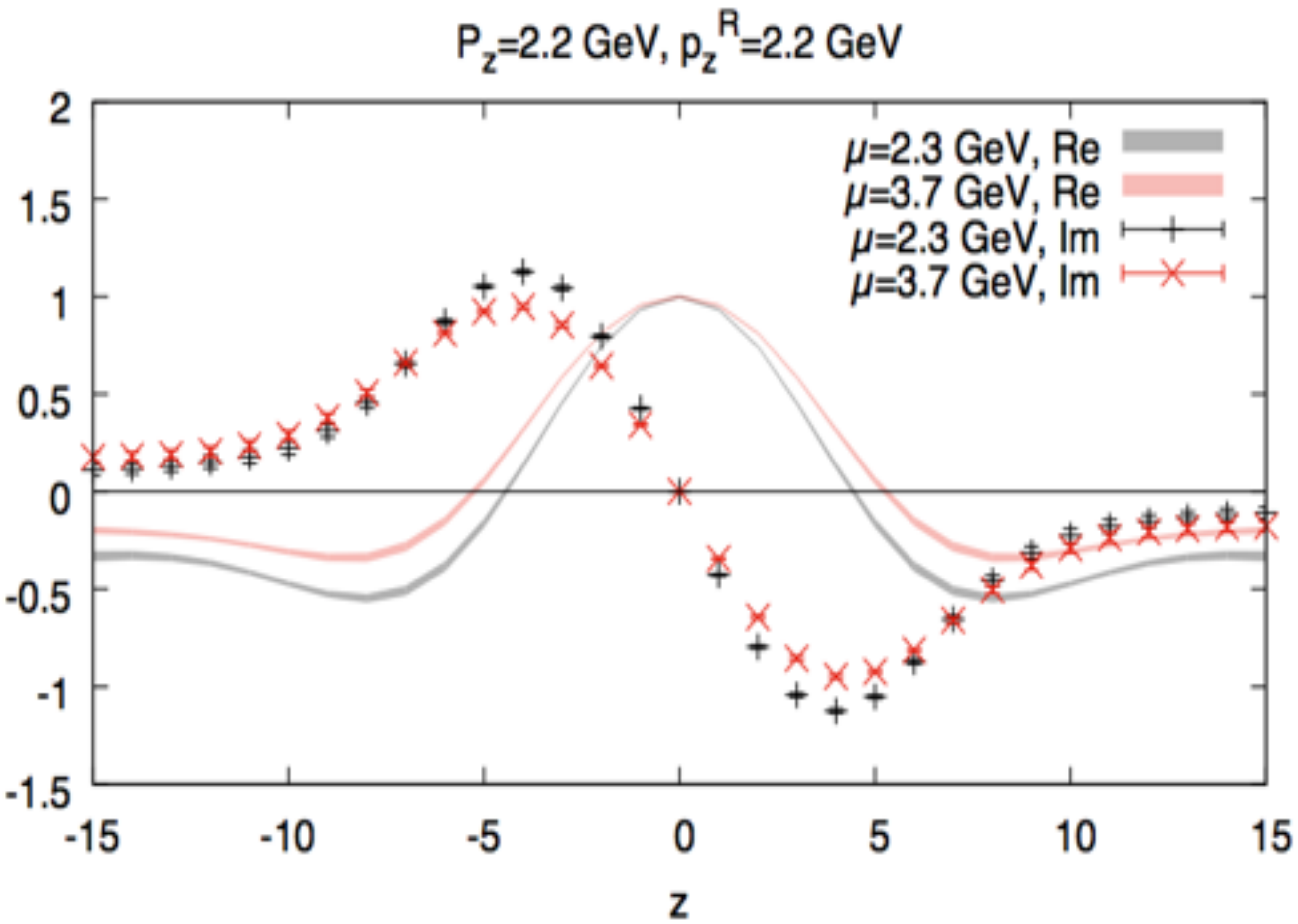
z

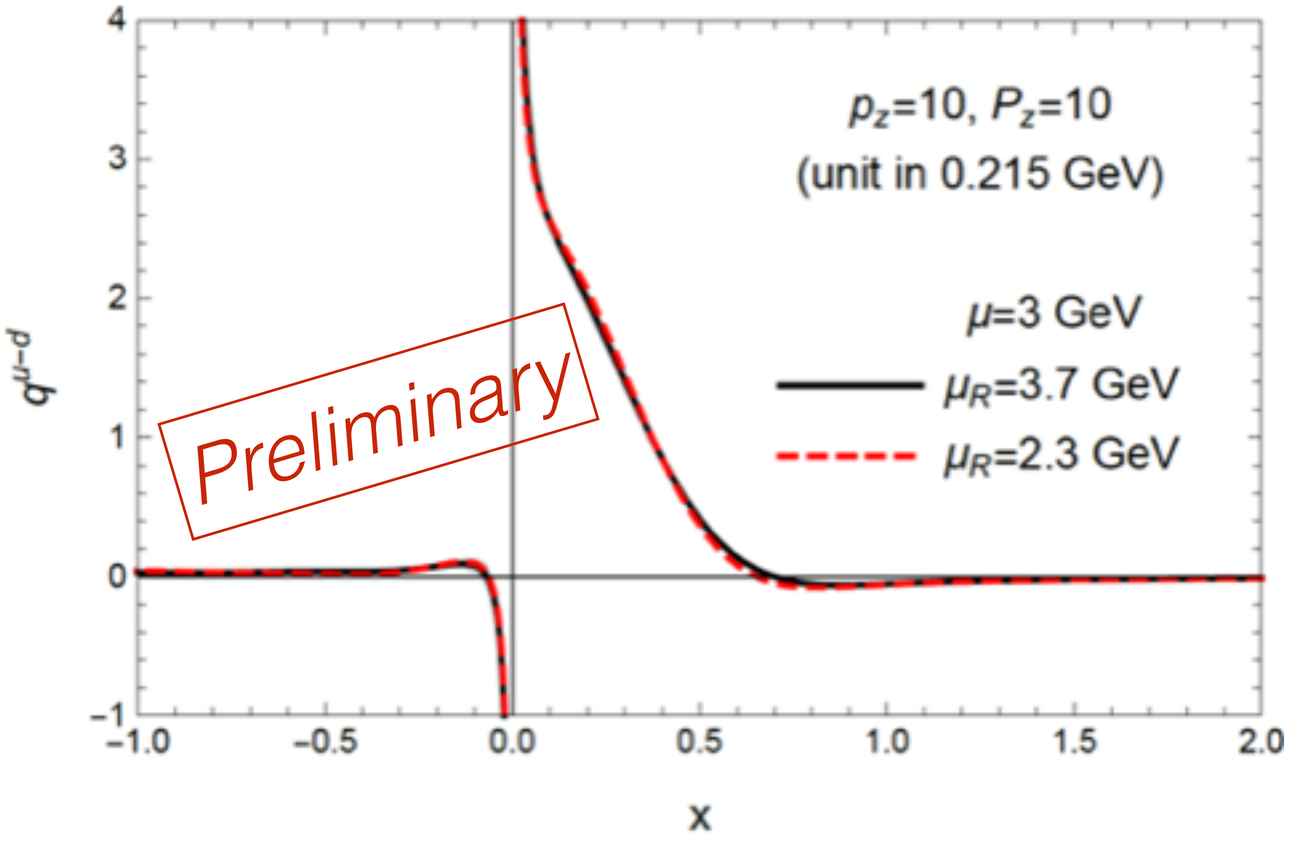
The result after matching

p_z^R =2.2 GeV case

$$\tilde{Z}(z, p_z^R, a^{-1}, \mu_R) = \frac{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle]}{\mathrm{Tr}[\not p \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle_{tree}]}\Bigg|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}}$$

$p = (p_t^R, p_\perp^R, p_z^R)$ with $p^2 = \mu_R^2$



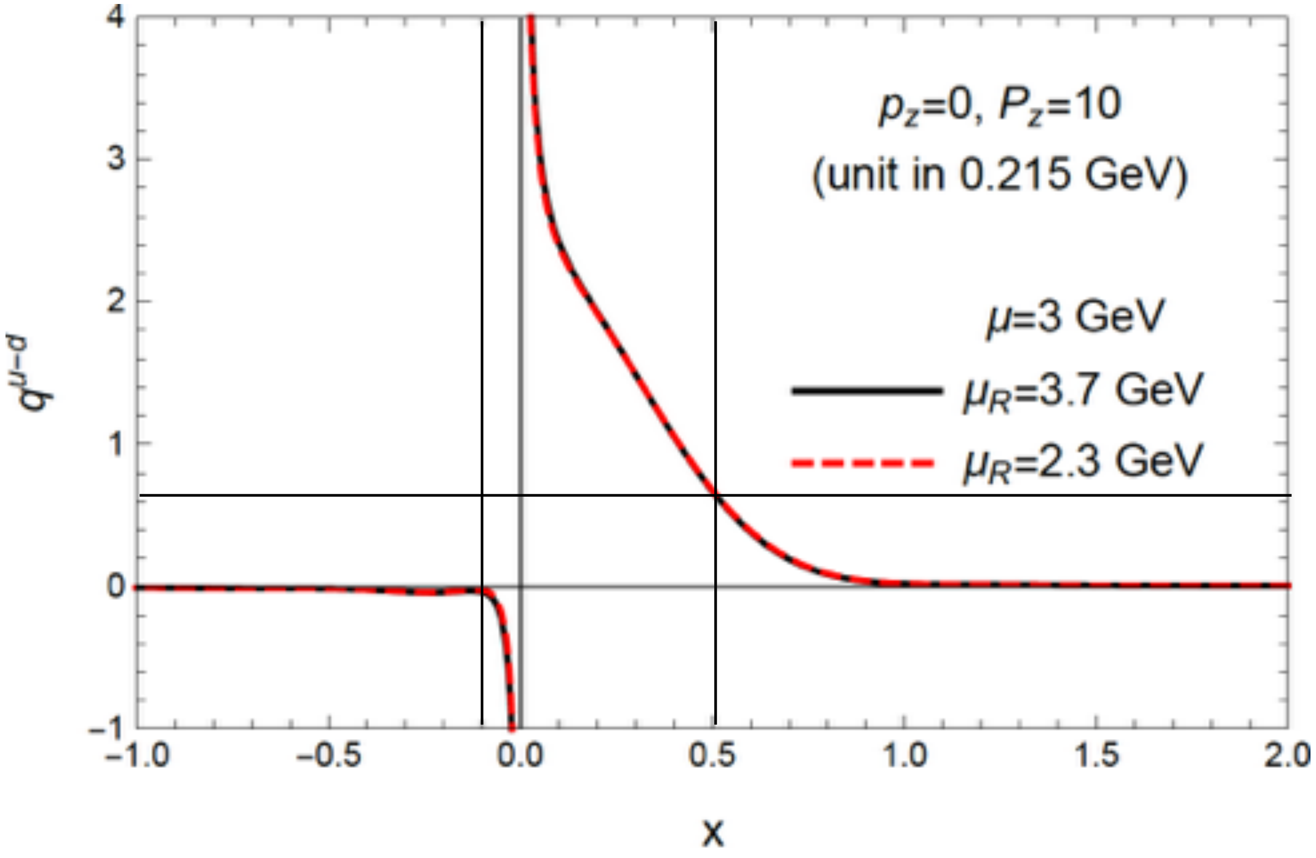


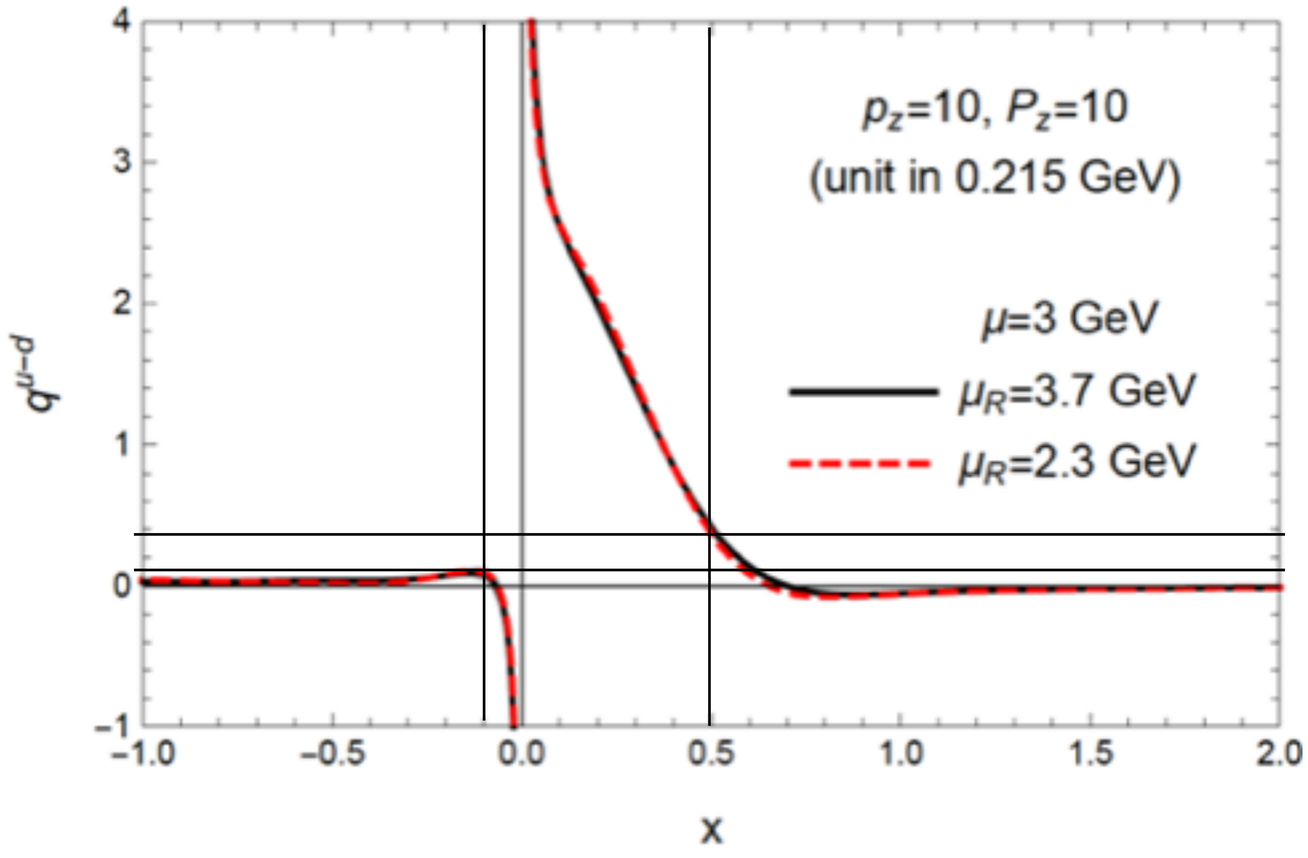
- *Obvious μ_R dependence when p_z^R is large;*
- *But the final result are still almost independent to μ_R!*

z

The result after matching

Residual p_z^R dependence

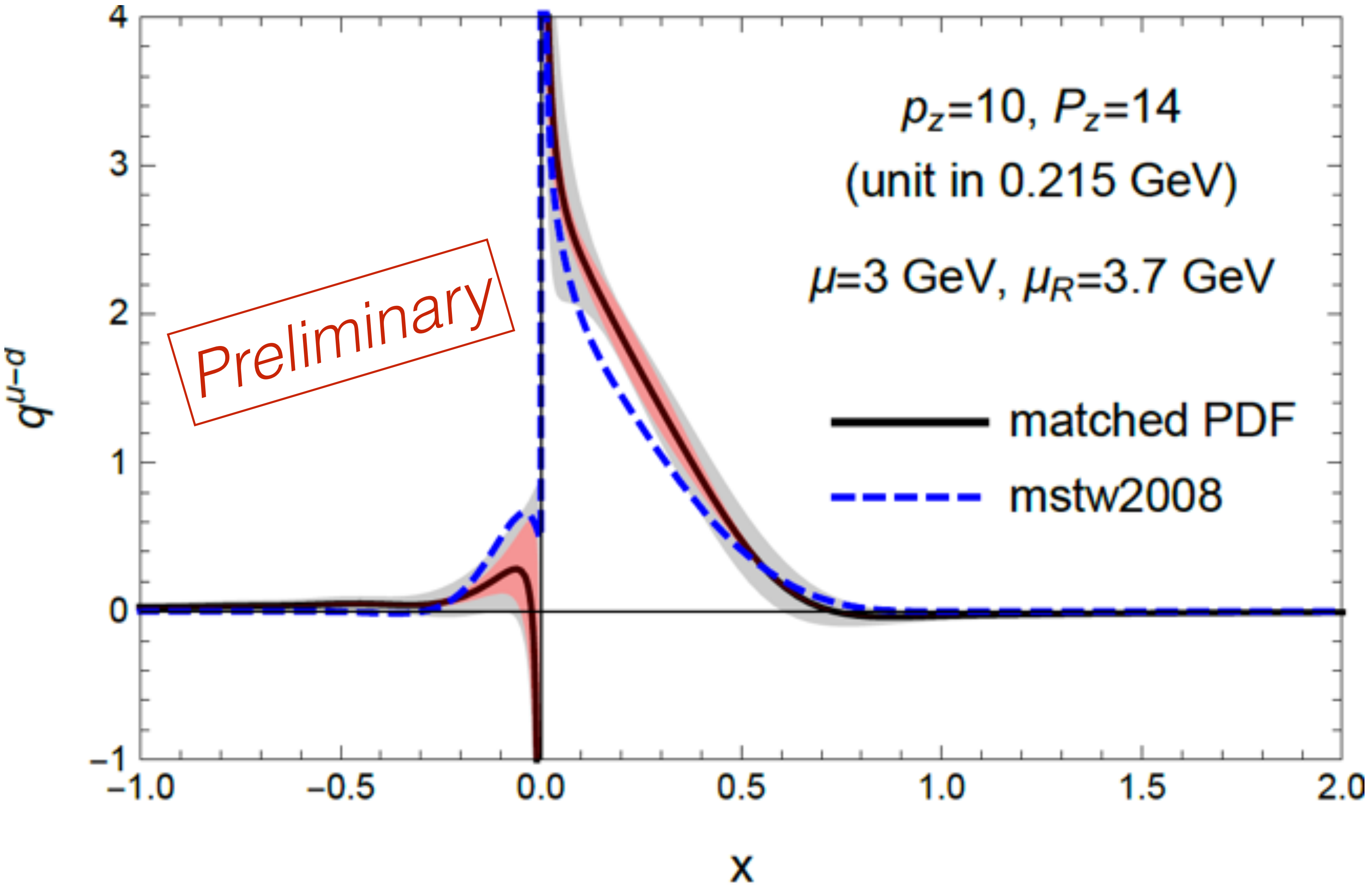




- *Large p_z^R makes the positive x to be lower and the negative x to be higher.*

The final prediction

with systematic uncertainties



- *The red error bar shows the statistical errors.*
- *The gray error bar shows the total uncertainties including the systematic uncertainties:*

1. *Higer-twist corrections;*
2. *Truncation errors;*
3. ***Dependence on the RI/MOM scheme parameters p_z^R** and μ_R;*
4. *Matching-inverse-matching mismatch effects.*

Summary

- The central problem of the bare quasi-PDF matrix element is the linear divergence from the wilson links;

- The linear divergence can be removed by applying the Wilson link counter term from Wilson loop, using the RI/MOM scheme, or forming a ratio of the matrix elements with different hadron momenta.

- Most of the RI/MOM parameter dependence can be cancelled, while the cancellation on p_z^R will need the perturbative calculation at higher loop level.