

Pion distribution amplitudes from Euclidean correlation functions

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with

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- DAs: definition and moments of DAs
- Lattice calculation of the 2nd moment of the pion DA¹
- “Direct” lattice calculation of the pion DA²
- Preliminary X-space results²
- Outlook

1) RQCD: GB, VM Braun, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, 1705.10236 + in preparation

2) RQCD: GB, VM Braun, B Gläsel, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, P Wein, J-H Zhang, 1709.04325 + in preparation

Not covered:

ρ DAs [RQCD: VM Braun et al, 1612.02955]

octet baryon DAs [RQCD: GB et al, 1512.02050]

Attention: Many results shown are still preliminary.

What are distribution amplitudes?

Wavefunction of a hadron (here pion) near the infinite momentum frame, written as a superposition of different Fock states:

$$|\pi\rangle = c_1|\bar{q}q\rangle + c_2|\bar{q}gq\rangle + c_3|\bar{q}q\bar{q}q\rangle + \dots$$

Light front wavefunction (Distribution amplitude, DA) describes the distribution of the longitudinal momentum among the partons.

Momentum fractions $0 \leq u_i \leq 1$, $\sum_{i \in \{q, \bar{q}, g\}} u_i = 1$.

At leading twist (twist 2) only the valence quarks contribute:

$$u = u_q = 1 - u_{\bar{q}}, \quad \xi = u_q - u_{\bar{q}} = 2u - 1 \in [-1, 1].$$

In hard processes higher Fock states are power suppressed.

PDFs are (within the parton model) single particle probability densities and can **directly be extracted from fits to DIS and SIDIS data**.

DAs are wavefunctions and only appear within convolutions in hard exclusive processes. Due to other hadronic uncertainties (and experimental techniques), it is much **harder to extract these reliably from experimental data**.

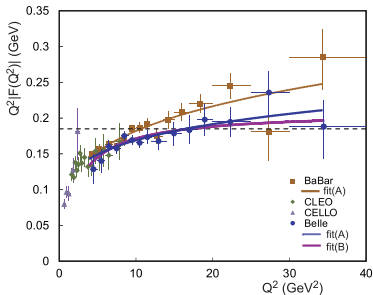
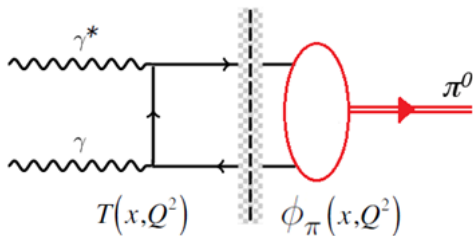
Distribution amplitudes II

DAs are needed for the theoretical description of hard exclusive processes.

Example: collinear factorization of the $\gamma\gamma^* \rightarrow \pi^0$ photoproduction

formfactor ($Q \gtrsim \mu \gg \Lambda$)

[Belle, 1205.3249]



$$F_{\pi\gamma}(Q^2) = \frac{2F_\pi}{3} \int_0^1 du \underbrace{T_{q\bar{q}\gamma}(u, \mu_F, Q^2)}_{\text{hard matching function}} \cdot \underbrace{\phi_\pi(u, \mu_F)}_{\text{soft factor (DA)}} + \underbrace{\text{higher twist}}_{F_\pi \cdot \mathcal{O}(1/Q^2)}$$

μ_F is the factorization scale and we renormalize the hard coefficient function T at the scale $\mu_R^2 = Q^2$. $F_\pi \approx 90 \text{ MeV}$

Definition of DAs

Non-local light front matrix element at a separation n ($n^2 = 0$):

$$\begin{aligned} & \langle 0 | \bar{d} \left(\frac{n}{2} \right) \not{n} \gamma_5 \left[\frac{n}{2}, -\frac{n}{2} \right] u \left(-\frac{n}{2} \right) | \pi^+(p) \rangle \\ & = i F_\pi n \cdot p \int_0^1 du \exp \left\{ -i \underbrace{[-u + (1-u)]}_{=-\xi} \frac{(n \cdot p)}{2} \right\} \phi_\pi(u, \mu) \end{aligned}$$

$[n/2, -n/2]$ above denotes a gauge covariant connection.

The DA is not accessible in Euclidean spacetime but moments of DAs are:

$$\langle \xi^n \rangle = \int_0^1 du (2u - 1)^n \phi_\pi(u, \mu), \quad \langle \xi^0 \rangle = 1, \quad \langle \xi^1 \rangle = 0.$$

$\langle \xi^{0,2} \rangle$ can be extracted from local matrix elements $\langle 0 | O_{\mu\nu\rho}^\pm | \pi^+(p) \rangle$ with

$$O_{\mu\nu\rho}^\pm = \bar{d} \left\{ \left[\overleftarrow{D}_{(\mu} \overleftarrow{D}_{\nu} \pm 2 \overleftarrow{D}_{(\mu} \overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu} \overrightarrow{D}_{\nu)} \right] \gamma_\rho \right\} \gamma_5 u,$$

where (\dots) gives a traceless, symmetrized expression.

Gegenbauer expansion:

$$\phi_\pi(u, \mu) = 6u(1-u) \left[1 + \sum_{n \in \mathbb{N}} a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1) \right]$$

Collinear conformal symmetry: $C_n^{3/2}(\xi)$ in $SL(2, \mathbb{R})$ analogous to $Y_{\ell m}(\theta, \phi)$ in $SO(3)$.
 $\langle \xi^{2n} \rangle$ and a_{2n}^π are related by simple algebraic expressions:

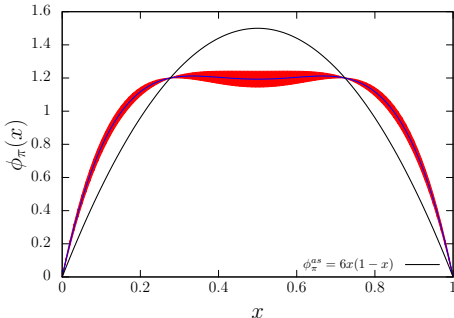
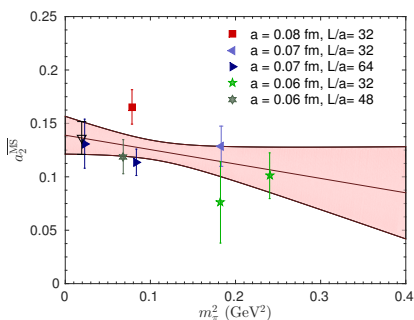
$$a_{2n}^\pi(\mu) = \frac{7}{12} (5\langle \xi^2 \rangle - 1) = \frac{7}{12} (5\langle \xi^2 \rangle - \langle \xi^0 \rangle)$$

$a_{2n}^\pi(\mu) \rightarrow 0$ as $\mu \rightarrow \infty$: At large scales the lower moments will dominate.

Note that the counting is different: 2nd DA-moment \sim 3rd PDF-moment.

Previous results ($\mu = 2 \text{ GeV}$)

$N_f = 2$, $M_\pi = 150 - 490 \text{ MeV}$, $LM_\pi = 3.4 - 6.7$.



$$a_2^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.136(15)(15)(?)$$

[RQCD: VM Braun et al, 1503.03656],

$$a_2^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.233(30)(60) (N_f = 2 + 1)$$

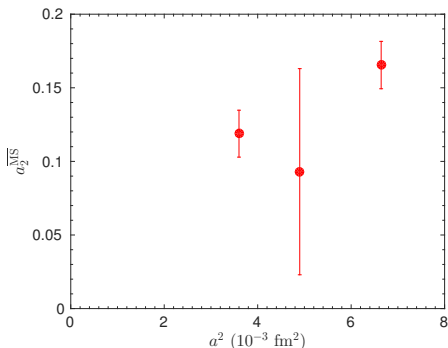
[RBC/UKQCD: R Arthur et al, 1011.5906]

$$a_2^{\overline{\text{MS}}}(2 \text{ GeV}) = 0.211(114)$$

[QCDSF/UKQCD: VM Braun et al, hep-lat/0606012]

Challenge: statistical errors

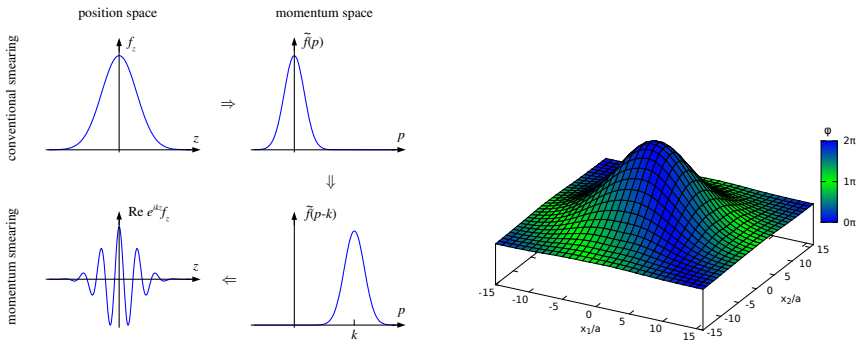
- Continuum extrapolation could not be carried out:



- ⚡⚡⚡ Second moment of pion DA requires at least two non-vanishing momentum components, e.g., $\vec{p} = (1, 1, 0)2\pi/L$
- ⚡⚡⚡ Employing two derivatives considerably deteriorates the signal-to-noise ratio.

Momentum smearing

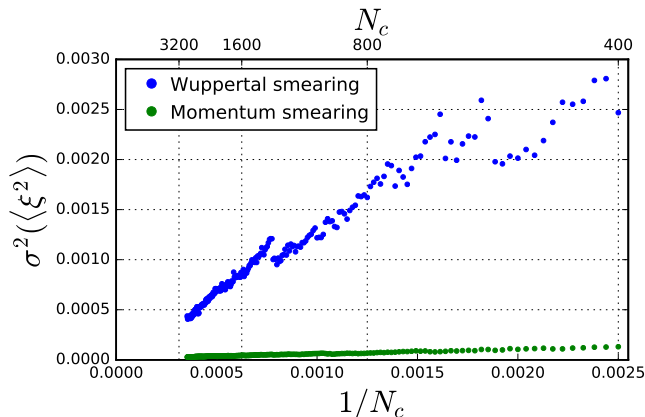
Intuition: interpolating wavefunction used to create $|\pi^+(p)\rangle$ should acquire a phase for $\vec{p} \neq \vec{0} \Rightarrow$ **momentum smearing**.



[RQCD: GB, B Lang, B Musch, A Schäfer, 1602.05525]

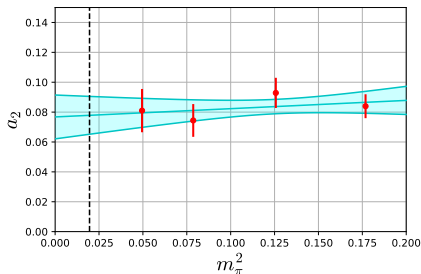
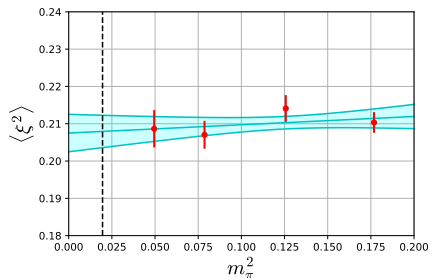
Squared error as a function of the statistics

- $N_f = 2 + 1$ CLS ensemble "H105" for $\vec{n}_{\vec{p}} = (110), (101), (011)$
- Each momentum requires 2 inversions for the momentum smearing (6 inversions).
- Additional momenta only require additional Fourier sums for the Wuppertal smearing (1 inversion).



Chiral Extrapolation

Note that in one loop ChPT \nexists chiral logs in this DA moment.



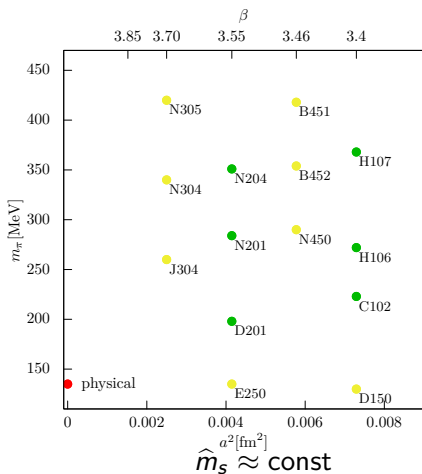
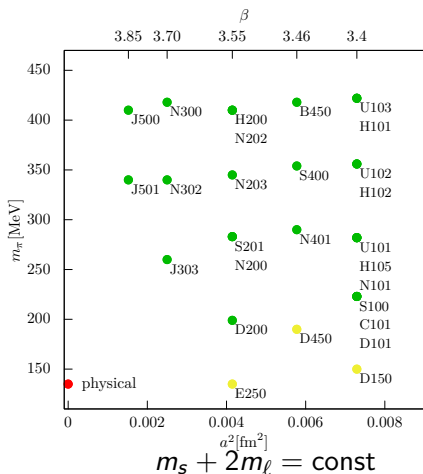
$N_f = 2 + 1$, where $m_s + 2m_{ud} = \text{Tr } M = \text{const}$

[RQCD: GB, VM Braun, M Göckeler, M Gruber, F Hutzler, P Korcyl, B Lang, A Schäfer, 1705.10236]

NLO matching between RI'/SMOM and $\overline{\text{MS}}$. NNLO matching to $\overline{\text{MS}}$?

Above was at $a \approx 0.086$ fm. In preparation: the continuum limit.

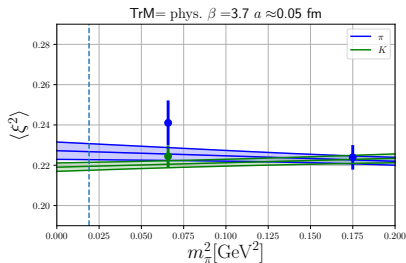
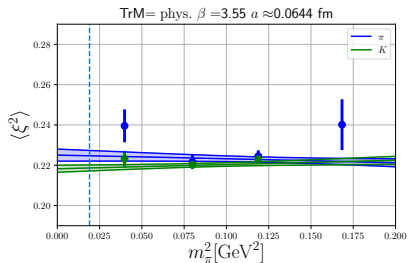
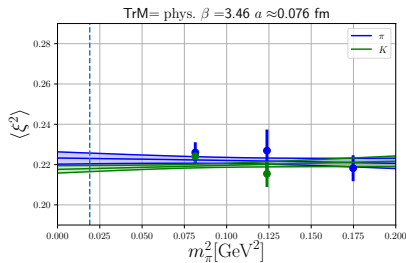
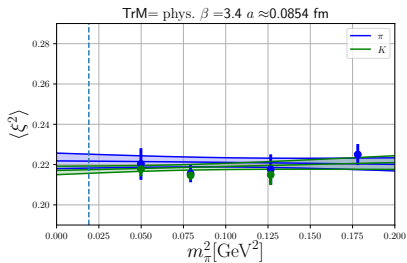
CLS ensemble overview



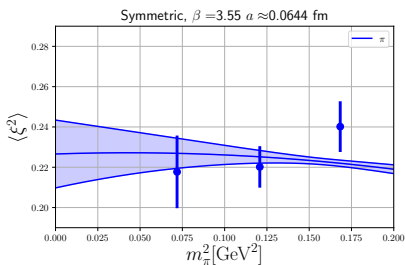
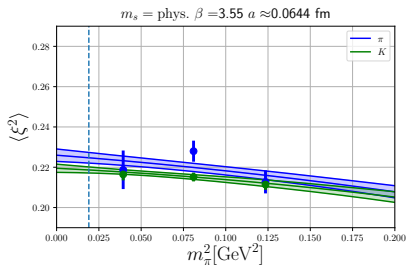
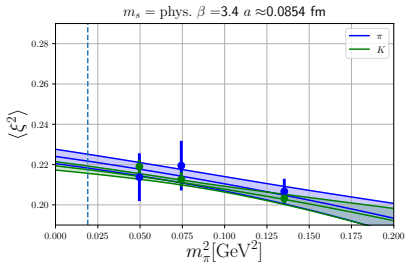
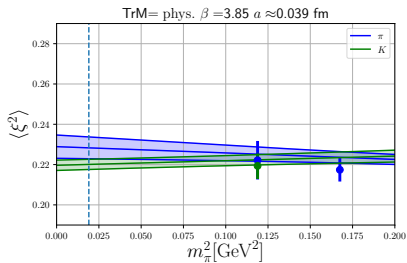
E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
 S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

☐ additional ensembles with $m_s = m_\ell$.

Chiral extrapolation: $\text{Tr } M = \text{const}$ (preliminary)



More chiral extrapolations (preliminary)

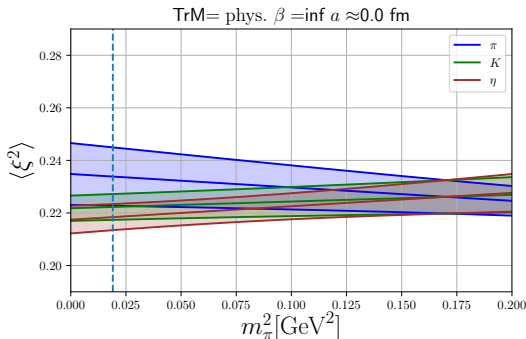


Continuum limit $\text{Tr } M = \text{const}$ (preliminary)

SU(3) NLO ChPT and linear lattice spacing effects (7 parameters):

$$\xi_X^2(M_\pi, M_K, a) = \left[\xi_0^2 + \bar{A}\bar{M}^2 + A_X\delta M^2 \right] \cdot \left[1 + a \left(c_0 + \bar{c}\bar{M}^2 + c_X\delta M^2 \right) \right],$$

$$X \in \{\pi, K, \eta_8\}, A_\pi = -2A_K, \delta M^2 = M_K^2 - M_\pi^2, \bar{M}^2 = \frac{2M_K^2 + M_\pi^2}{3}.$$



This corresponds to $a_{2,\pi}^{\overline{\text{MS}}}(2 \text{ GeV}) \approx 0.105(35)$. **PRELIMINARY!**
More statistics and simulation points are in preparation.

“Direct” determination of the DA

Large momentum effective theory (LaMET) [X Ji, 1305.1539]:
compute “quasi-distribution”, in analogy to “quasi-PDF”.

$$\tilde{\phi}_\pi(u, a^{-1}, p_z) = \frac{i}{F_\pi} \int_{-\infty}^{\infty} \frac{dZ}{2\pi} e^{-i(u-1)p_z Z} \langle \pi(p) | \bar{\psi}(0) \gamma_z \gamma_5 [0, z] \psi(z) | 0 \rangle,$$

with quark fields separated along the spatial z direction ($(z^\mu) = (0, 0, 0, Z)$).
Then match to pion DA (like [X Ji, 1506.00248] for PDFs):

$$\tilde{\phi}_\pi(u, a^{-1}, p_z) = \int_0^1 dv Z_\phi(u, v, a^{-1}, \mu, p_z) \phi_\pi(v, \mu) + \mathcal{O}\left(\frac{\Lambda^2}{p_z^2}, \frac{M_\pi^2}{p_z^2}\right).$$

Practical problems: Z_ϕ has many arguments and is power divergent.

However: [K Orginos, A Radyushkin, J Karpie, S Safeiropoulos, 1706.05373]

Perturbative matching to $\overline{\text{MS}}$ at large Z ? Contribution suppressed for large p_z !

Discard large Z data and integrate over $d(p_z Z)/p_z$ to minimize corrections?

Does \exists enough data in relevant $p_z Z$ region to carry out Fourier transform?

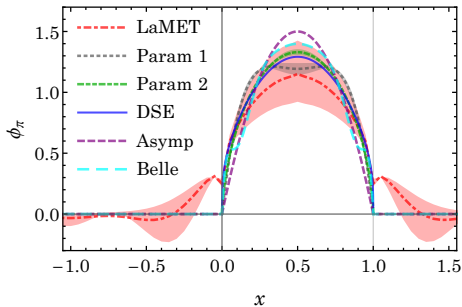
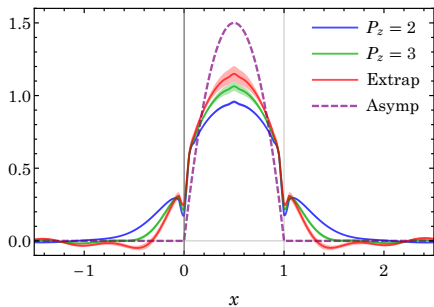
Window and statistics: $\pi/a \gg p_z \gg m_N$ for nucleon PDF.

The pion DA is an interesting test case.

First numerical study of the pion “quasi-DA”

$N_f \approx 2 + 1 + 1$, $M_\pi = 310 \text{ MeV}$, $a = 0.12 \text{ fm}$, $LM_\pi = 4.5$

[J-H Zhang, J-W Chen, X Ji, L Jin, H-W Lin, 1702.00008]



$$x = u$$

What do we do differently?

We will **directly** compute the DA in X -space. The method is related to “quasi-PDFs”, however, there are **essential** differences:

- We **do not** employ a Schwinger line $[z/2, -z/2]$, simplifying renormalization. Also we through away $2/|z| < 1$ GeV data.
- Most importantly: we **do not** transform the DA to the longitudinal momentum fraction space.

We compute an object ($z = (0, \vec{z}) \Rightarrow z^2 = -\vec{z}^2 < 0, |p \cdot z| = |-\vec{p} \cdot \vec{z}|$)

$$\sqrt{2} T(p \cdot z, z^2) = \langle 0 | \bar{d}(z/2) \Gamma_A \overbrace{q(z/2) \bar{q}(-z/2)} \Gamma_B u(-z/2) | \pi^+(p) \rangle$$

q is an auxiliary field. We use $J_{\Gamma_A} J_{\Gamma_B} = SP - PS, VA + AV, VV + AA$. $T(z^2)$ can then be factorized just like

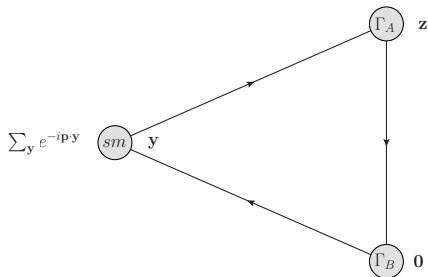
$$F_{\pi\gamma}(Q^2) = \frac{2F_\pi}{3} \int_0^1 du H_{q\bar{q}\gamma}(u, \mu, Q^2) \phi_\pi(u, \mu) + \text{higher twist.}$$

into a hard, perturbative function and the DA, as long as $2/|z| \gg \Lambda$. We set $\mu_R = 2/|z|$, which “plays” the role of $|Q|$. ($|z| = \sqrt{-z^2}$)

$T(p \cdot z, z^2)$

Following [VM Braun, D Müller, 0709.1348], we use a light quark propagator for $q(z/2)\bar{q}(-z/2)$. Advantage: Renormalization factor of $\bar{q}\Gamma u$ known. Other suggestions:

- “Static” propagator: Large Energy Effective Theory (LEET) [MJ Dugan, B Grinstein, PLB255(91)583], Large Momentum Effective Theory (LaMET) [X Ji, 1305.1539],
- Scalar propagator: [U Aglietti et al, hep-ph/9806277]; [A Abada et al, hep-ph/0105221],
- Heavy quark propagator: [W Detmold, CJD Lin, hep-lat/0507007].



Tree level result ($\Gamma_A, \Gamma_B = \gamma_5, \mathbb{1}$):

$$T(p \cdot z, z^2) = F_\pi \frac{p \cdot z}{2\pi^2 z^4} \Phi_\pi(p \cdot z),$$

with the X -space (Ioffe time) DA

$$\Phi_\pi(p \cdot z) = \int_0^1 du e^{i(u-1/2)p \cdot z} \phi_\pi(u).$$

The DA in momentum and in X -space

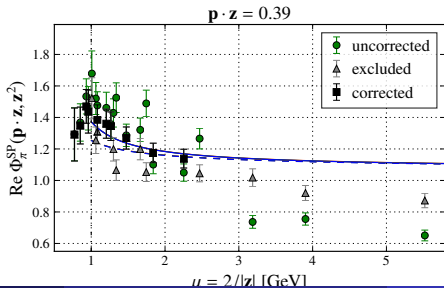
In practice we compute for large t (example: $\Gamma_A \Gamma_B = \frac{1}{2}[\gamma_5, \mathbb{1}]$)

$$\frac{T(\mathbf{p} \cdot \mathbf{z}, z^2)}{F_\pi} = \frac{Z_S Z_P}{Z_A} \frac{\langle 0 | [\bar{d} \mathbb{1} q](0, \vec{z}/2) [\bar{q} \gamma_5 u](0, -\vec{z}/2) O_\pi^\dagger(-t, \vec{p}) | 0 \rangle}{\langle 0 | [\bar{d} \gamma_0 \gamma_5 u](0, \vec{0}) O_\pi^\dagger(-t, \vec{p}) | 0 \rangle} E(\vec{p}),$$

where the lattice currents are renormalized to the $\overline{\text{MS}}$ scheme via $Z_S(\mu_R a, g^2)$, $Z_P(\mu_R a, g^2)$ and $Z_A(g^2)$ and $\mu_R = \mu_F = 2/|z|$.

We tree-level correct for \vec{z} -dependent lattice artefacts:

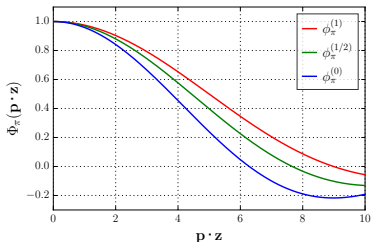
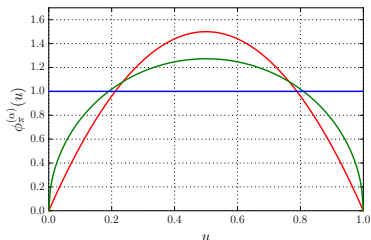
$$T(\mathbf{p} \cdot \mathbf{z}, z^2) \mapsto T(\mathbf{p} \cdot \mathbf{z}, z^2) \frac{\text{tr} [\not{z} G_{\text{cont}}^{\text{tree}}(z)]}{\text{tr} [\not{z} G_{\text{latt}}^{\text{tree}}(z, a)]} \quad (\text{for the chiral even part})$$



Three illustrative models of the DA (taken at a scale $\mu_0 = 1$ GeV):

$$\phi_\pi^{(1)}(u) = 6u(1-u), \quad \phi_\pi^{(1/2)}(u) = \frac{8}{\pi} \sqrt{u(1-u)}, \quad \phi_\pi^{(0)}(u) = 1$$

$$\phi_\pi^{(\alpha)} = \frac{\Gamma[2(\alpha+1)]}{[\Gamma(\alpha+1)]^2} [u(1-u)]^\alpha$$



$|z| < 2/\text{GeV} \approx 0.4$ fm is **necessary** for factorization, $\overline{\text{MS}}$ scheme matching!

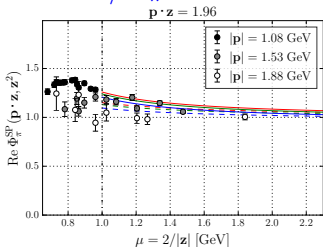
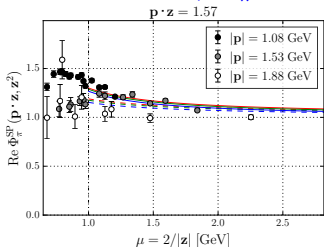
Large $|\vec{p} \cdot \vec{z}|/2$ values are **desirable** as this is conjugate to $\xi = 2u - 1$.

$|z|$ small \Rightarrow large $|\vec{p}|$. No other reason why $|\vec{p}|$ has to be large!

Results: factorization scale dependence of the X -space DA

$N_f = 2$ NP improved Wilson-clover quarks (old QCDSF ensemble)

$a^{-1} \approx 2.76$ GeV, $M_\pi \approx 290$ MeV, $L = 32a \approx 3.4/M_\pi$.



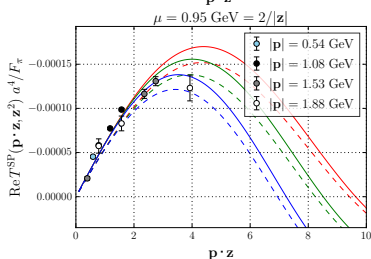
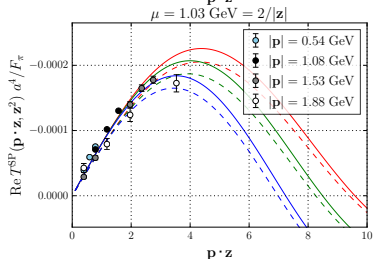
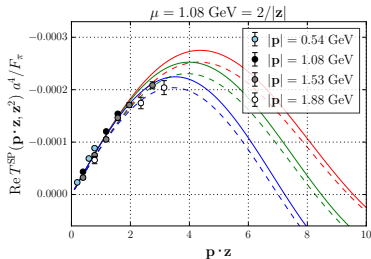
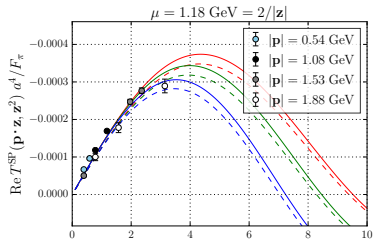
Large $|\vec{p}|a$, $a/|z| \rightarrow$ bigger lattice corrections. QCD factorization:

$$\frac{T(p \cdot z, z^2)}{F_\pi} = \frac{p \cdot z}{2\pi^2 z^4} \int_0^1 du e^{i(u-1/2)p \cdot z} H(u, \mu, z^2) \phi_\pi(u, \mu) + T^{\text{HT}},$$

We have computed $H(u, \mu, z^2)$ to order α_s . $T^{\text{HT}} = \mathcal{O}(M_\pi^2 z^2, \Lambda^2 z^2)/z^4$ can be estimated from LCSR. [VA Novikov et al, NPB237(84)525; VM Braun, A Khodjamirian, M Maul, hep-ph/9907495]

Results II: $|\vec{p} \cdot \vec{z}|$ dependence for SP-PS ($\mu = \mu_R = 2/|z|$)

Solid curves include higher twist and mass correction estimates.

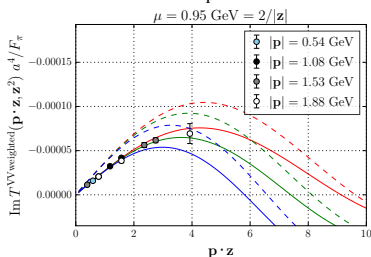
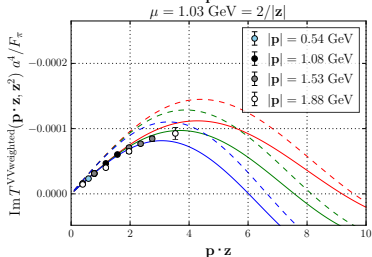
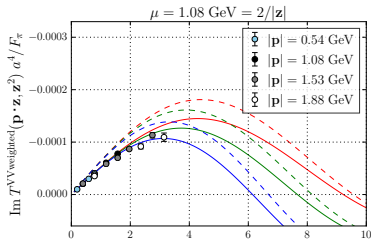
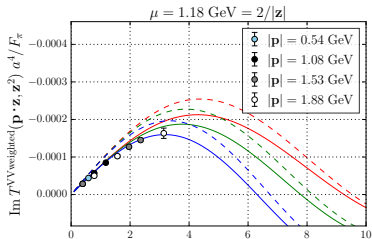


The future: smaller a , larger $|\vec{p}|$.

Results III: $|\vec{p} \cdot \vec{z}|$ dependence for VV+AA

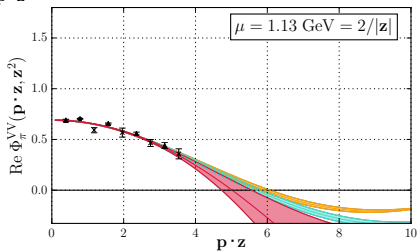
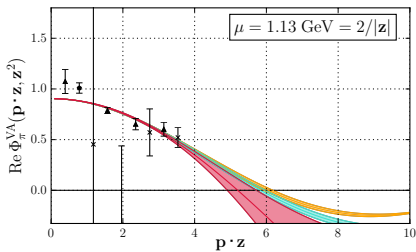
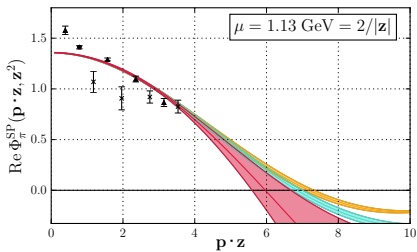
VV+AA current:

Higher twist effects (and data!) go in the different direction.



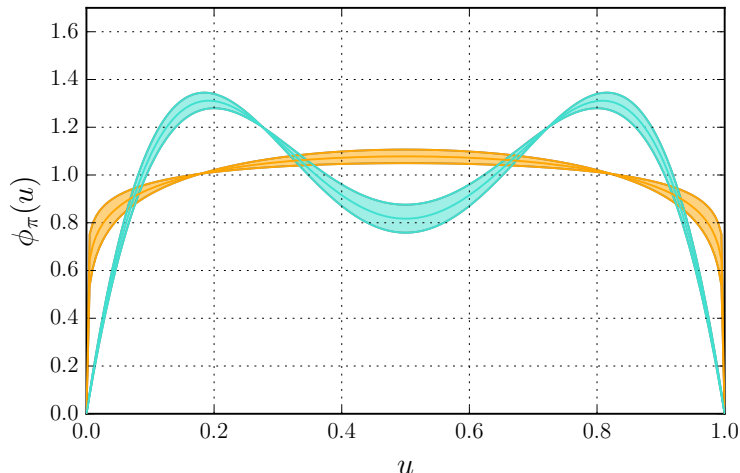
Results IV: $|\vec{p} \cdot \vec{z}|$ dependence of X-space DA ($\mu = \mu_R = 2/|z| \approx 1.17$ GeV, preliminary)

Error bands are different fits.



Results V (preliminary)

Two DAs at $\mu = 2 \text{ GeV}$, both of which describe the data well.
The shapes are very different but a_2 is the same.



Also different to discriminate experimentally!!!

- The future of Gegenbauer/Mellin moments:
 - Continuum limit.
 - So far NP matching to RI'-MOM/SMOM schemes and from there perturbatively to $\overline{\text{MS}}$ at NLO. We explore NNLO matching.
 - Also other meson and baryon DAs.
- “Direct” method:
 - We presented a proof of concept.
 - For $2/|z| \gtrsim 1 \text{ GeV}$ we need $|\vec{p}| \gtrsim 3 \text{ GeV}$ to reach “loffe times” $|p \cdot z|$ large enough to discriminate between DA parametrizations \Rightarrow small lattice spacings are necessary.
 - Different current combinations depend differently on higher twist contributions: constrain higher twist DAs?
 - Ideal: smaller $|z|$ to suppress higher twist effects \Rightarrow even higher $|\vec{p}|$.
 - Kaon X -space DA.
 - Already at the limit of the state-of-the art for the pion DA: what about nucleon PDFs?
 - Good news: matching to the $\overline{\text{MS}}$ scheme is easier for currents without derivatives. The matching function requires “only” a continuum calculation \Rightarrow NNLO calculation planned.