

Understanding the structure of the proton through large-scale simulations



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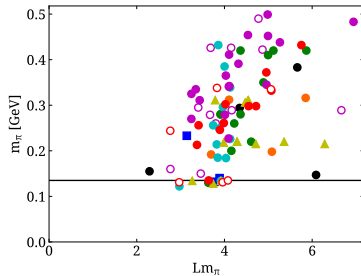
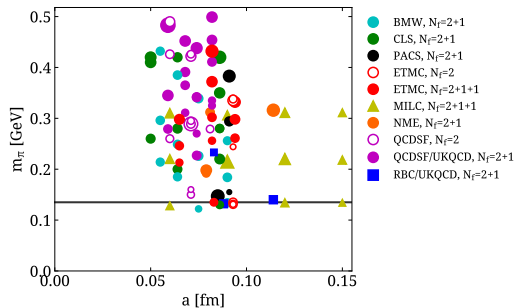
Lattice PDF Workshop, 6-8 April 2018



Outline

- 1 Current status of simulations
- 2 Spin content of the nucleon
- 3 The quark content of the nucleon (σ -terms)
- 4 Nucleon form factors
- 5 Conclusions

Status of simulations



Questions we would like to address

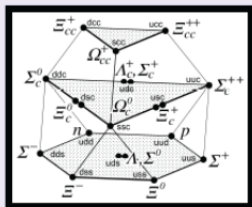
With simulations at the physical value of the pion mass there is a number of interesting questions we want to address:

In this talk I will address three topics:

- The nucleon spin decomposition of the nucleon
- The nucleon scalar content or σ -terms as a probe of new physics
- Nucleon form factors

Low-lying spectrum

20'-plet of spin-1/2 baryons

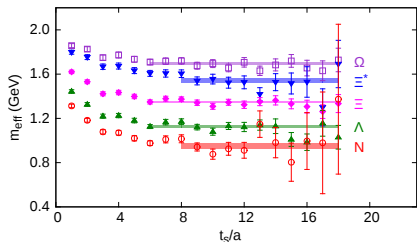
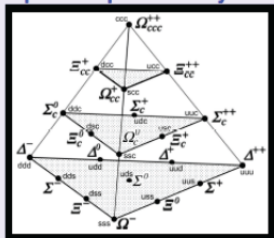


⇐ Two charm quarks ⇒

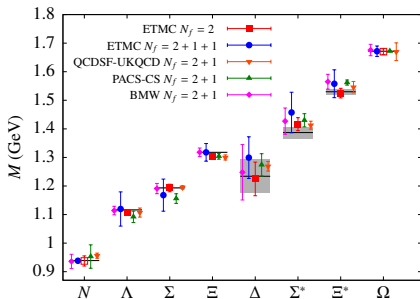
⇐ One charm quarks ⇒

⇐ No charm quarks ⇒

20-plet of spin-3/2 baryons

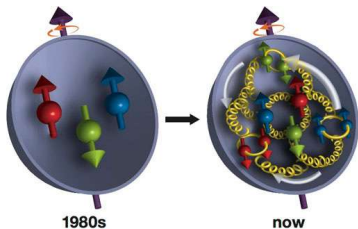


Using $N_f = 2$ simulations at a physical value of the pion mass



Proton spin puzzle

European Muon Collaboration (EMC) experiment at CERN: Deep Inelastic Scattering (DIS) of high energy polarized muons on polarized protons , J. Ashman *et al.* (EMC) Phys. Lett. B206 (1988) 364 and Nucl. Phys. B328 (1989) 1.



Naive quark model: Only valence quarks $\frac{1}{2} = \frac{1}{2}(\Delta u_v + \Delta d_v)$ where $\Delta u_v = \frac{4}{3}$ and $\Delta d_v = -\frac{1}{3}$

EMC result: $\frac{1}{2} \sum_q \Delta \Sigma_q \sim \frac{1}{4}$ → Spin puzzle

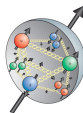
What carries the proton spin?

Gluons and sea quarks are important → ΔG and Δq_{sea}

But also orbital angular momentum of quarks and gluons.

Spin of the nucleon

$$J^3 \text{ decomposition: } \frac{1}{2} = \underbrace{\sum_q \frac{1}{2} \Delta \Sigma^q}_{\text{quark spin}} + \underbrace{\sum_q L^q + J^g}_{\text{dark spin}}$$



$$\Delta \Sigma_q \equiv \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} + \dots$$

Total quark angular momentum $J^q = \frac{1}{2} \Delta \Sigma^q + L^q$ and total gluon angular momentum J^g .

The total quark angular momenta J^q can be extracted from generalized form factors at zero momentum transfer $Q^2 = 0$ (unpolarized and helicity PFDs):

$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ while } \Delta \Sigma^q = \tilde{A}_{10}^q(0).$$

Need to compute nucleon matrix elements of local operators.

Matrix elements for quark spin

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)

Consider one-particle states p' and $p \rightarrow$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_{\Gamma}(x, \xi, q^2) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i x \lambda} \langle p' | \bar{\psi}(-\lambda n/2) \Gamma \mathcal{P} e^{ig \int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(n\alpha)} \psi(\lambda n/2) | p \rangle$$

where $q = p' - p$, $\bar{P} = (p' + p)/2$, n is a light-cone vector with $\bar{P} \cdot n = 1$

Expansion of the light cone operator leads to a tower of local operators $\mathcal{O}^{\mu_1 \mu_2 \dots \mu_n}$

\rightarrow Entails computing nucleon matrix elements of quark bilinears: $\langle N(p', s') | \mathcal{O}_\Gamma^{\mu_1 \dots \mu_n} | N(p, s) \rangle$

- **Unpolarized:**

$$\mathcal{O}_V^{\mu_1 \mu_2 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} \overleftrightarrow{D}^{\mu_2} \dots \overleftrightarrow{D}^{\mu_n\}} \psi(x)$$

$$n = 0 : \rightarrow \langle 1 \rangle_q = g_V^q, \quad n = 1 : \rightarrow J^q = \frac{1}{2} \left[A_{20}^q(0) + B_{20}^q(0) \right] \text{ and}$$

$$\langle x \rangle_q = A_{20}^q(0)$$

- **Helicity:**

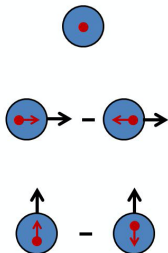
$$\mathcal{O}_A^{\mu_1 \mu_2 \dots \mu_n} = \bar{\psi}(x) \gamma^{\{\mu_1} \overleftrightarrow{D}^{\mu_2} \dots \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \psi(x)$$

$$n = 0 : \rightarrow \langle 1 \rangle_{\Delta q} = \Delta \Sigma^q = g_A^q, \quad n = 1 : \rightarrow \langle x \rangle_{\Delta q} = \tilde{A}_{20}^q(0)$$

- **Transversity:**

$$\mathcal{O}_T^{\nu \mu_1 \mu_2 \dots \mu_n} = \bar{\psi}(x) \sigma^{\{\nu, \mu_1} \overleftrightarrow{D}^{\mu_2} \dots \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2} \psi(x)$$

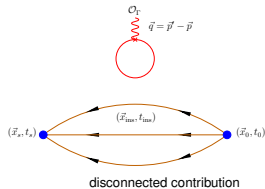
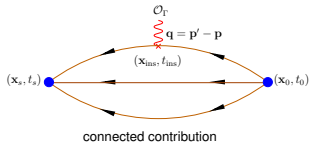
$$n = 0 : \rightarrow \langle 1 \rangle_{\delta q} = g_T^q, \quad n = 1 : \rightarrow \langle x \rangle_{\delta q} = \tilde{\tilde{A}}_{20}^q(0)$$



Evaluation of matrix elements

Three-point functions:

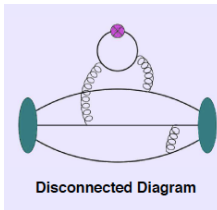
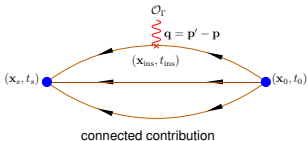
$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_S, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_S, t_s) O_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



Evaluation of matrix elements

Three-point functions:

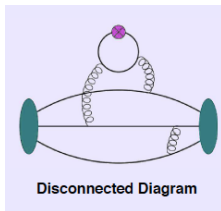
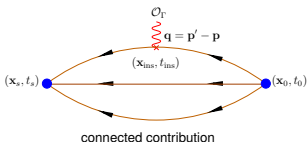
$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\text{ins}}) = \sum_{\vec{x}_S, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_S, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_S, t_{\text{ins}}) = \sum_{\vec{x}_S, \vec{x}_{\text{ins}}} e^{i\vec{x}_{\text{ins}} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_S, t_S) O_\Gamma^{\mu\nu}(\vec{x}_{\text{ins}}, t_{\text{ins}}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



- Plateau method:

$$R(t_S, t_{\text{ins}}, t_0) \xrightarrow[\substack{(t_S - t_{\text{ins}})\Delta \gg 1 \\ (t_{\text{ins}} - t_0)\Delta \gg 1}]{\mathcal{M}} [1 + \dots e^{-\Delta(\mathbf{p})(t_{\text{ins}} - t_0)} + \dots e^{-\Delta(\mathbf{p}')(t_S - t_{\text{ins}})}]$$

- Summation method: Summing over t_{ins} :

$$\sum_{t_{\text{ins}}=t_0}^{t_S} R(t_S, t_{\text{ins}}, t_0) = \text{Const.} + \mathcal{M}[(t_S - t_0) + \mathcal{O}(e^{-\Delta(\mathbf{p})(t_S - t_0)}) + \mathcal{O}(e^{-\Delta(\mathbf{p}')(t_S - t_0)})].$$

Excited state contributions are suppressed by exponentials decaying with $t_S - t_0$, rather than $t_S - t_{\text{ins}}$ and/or $t_{\text{ins}} - t_0$

However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

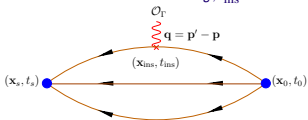
- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

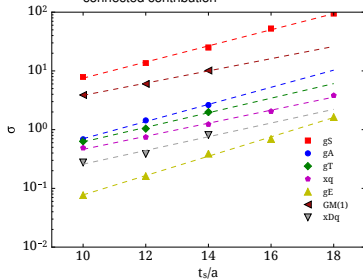
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{ins}) = \sum_{\vec{x}_S, \vec{x}_{ins}} e^{i\vec{x}_{ins} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_S, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{ins}, t_{ins}) \bar{J}_\beta(\vec{x}_0, t_0) \rangle$$



connected contribution

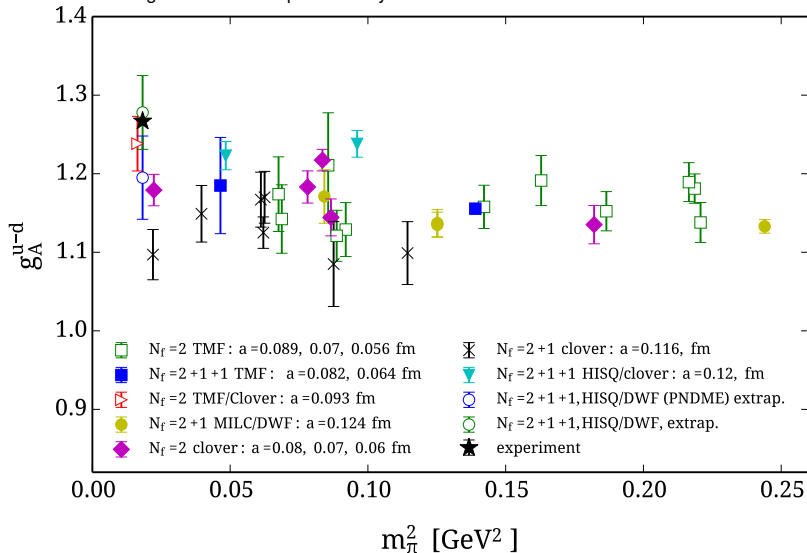


- \mathcal{M} the desired matrix element
- t_s, t_{ins}, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

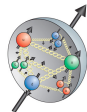
To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

Nucleon axial charge g_A

Nucleon axial charge well known experimentally \rightarrow benchmark for lattice QCD

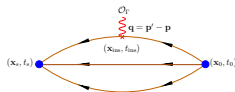


Quark intrinsic spin



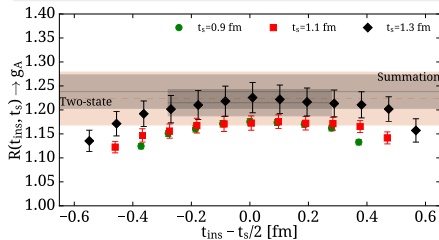
$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2} \Delta\Sigma^q + L^q \right)}_{J^q} + J^g$$

$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ and } \Delta\Sigma^q = g_A^q$$

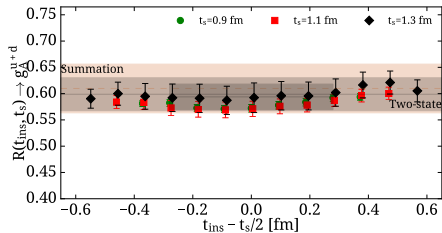


Need isoscalar g_A , which has disconnected contributions

- $N_f = 2$ twisted mass fermions with a clover term at a **physical value of the pion mass**, $48^3 \times 96$ and $a = 0.093(1)$ fm, using 9264 measurements for $t_s/a = 10, 12, 14, \sim 47,600$ $t/a = 16$ and $\sim 70,000$ for $t/a = 18$
- Intrinsic quark spin: $\Delta\Sigma^q = g_A^q$



Isovector



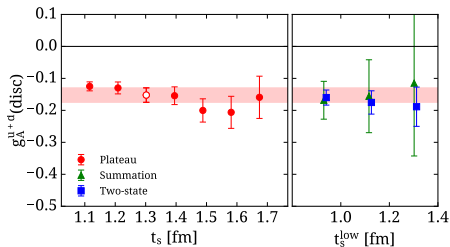
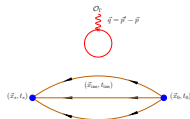
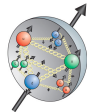
Connected isoscalar

Quark intrinsic spin

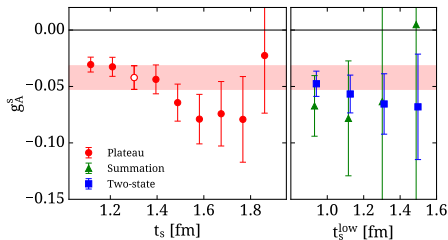
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$$J^q = \frac{1}{2} (A_{20}^q(0) + B_{20}^q(0)) \text{ and } \Delta \Sigma^q = g_A^q$$

Need isoscalar g_A , which has disconnected contributions



Isoscalar disconnected



Strange

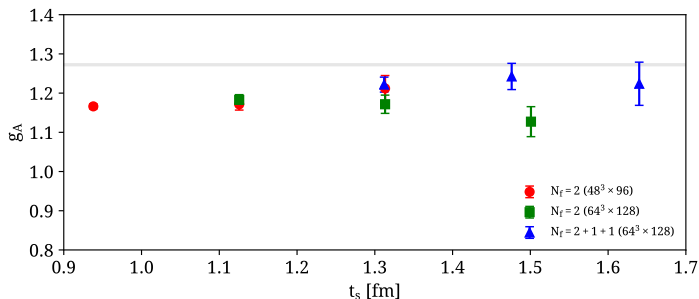
We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ (disconnected only) with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Volume and unquenching effects

Investigation of volume and quenching effects using:

- $N_f = 2$ twisted mass plus clover, $64^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV, with ~ 5000 statistics
- $N_f = 2 + 1 + 1$ twisted mass plus clover $64^3 \times 96$, $a = 0.081(1)$ fm, $m_\pi = 135$ MeV, with ~ 9000 measurements

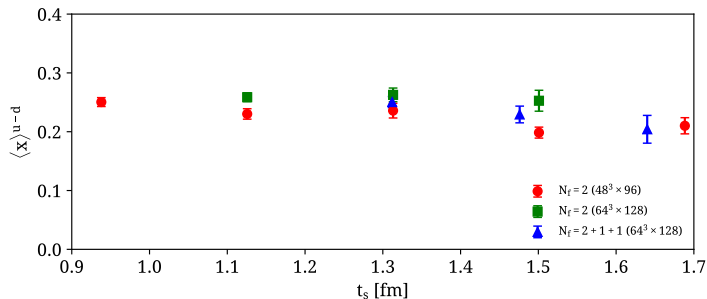


Preliminary results for the nucleon axial charge

Volume and unquenching effects

Investigation of volume and quenching effects using:

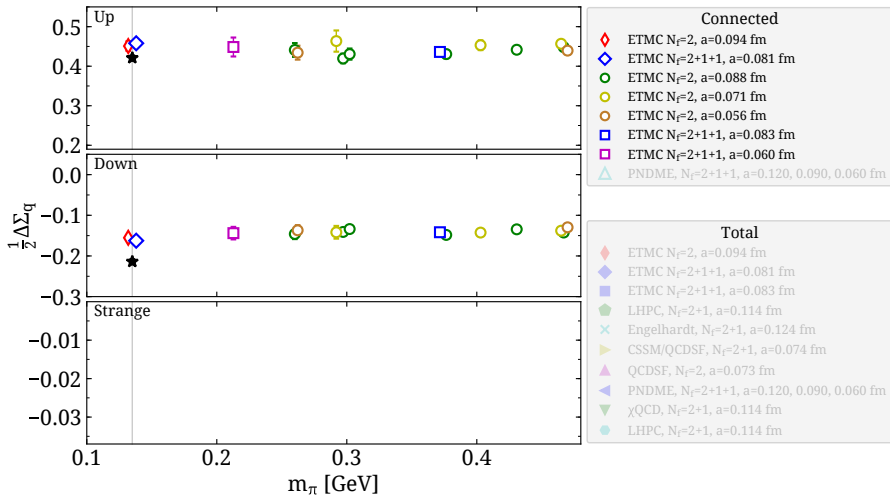
- $N_f = 2$ twisted mass plus clover, $64^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV, with ~ 5000 statistics
- $N_f = 2 + 1 + 1$ twisted mass plus clover $64^3 \times 96$, $a = 0.081(1)$ fm, $m_\pi = 135$ MeV, with ~ 9000 measurements



Preliminary results for $\langle x \rangle_{u-d}$

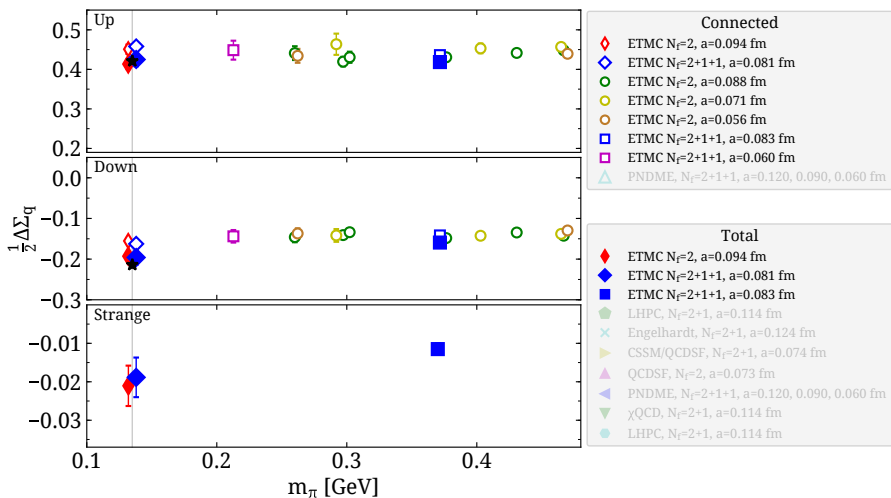
Quark intrinsic spin

- Volume smaller than statistical errors; cut-off effects negligible at heavier than physical pion masses



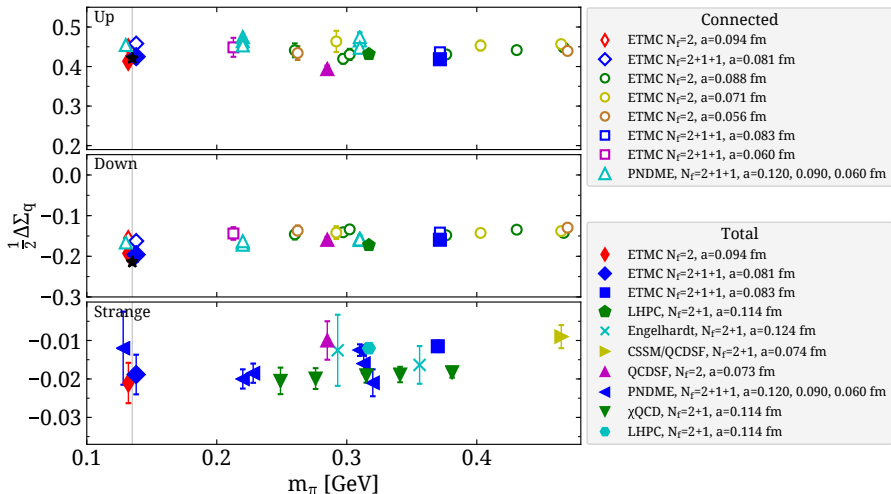
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- Volume smaller than statistical errors; cut-off effects negligible at heavier than physical pion masses
- Disconnected contributions non-zero. Our result agrees with recent analysis by COMPASS that found $0.13 < \frac{1}{2} \Delta \Sigma < 0.18$ C. Adolph et al., Phys. Lett. B753, 18 (2016), 1503.08935



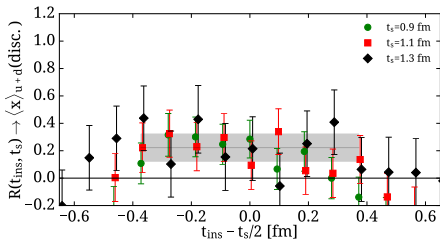
Quark intrinsic spin

- Volume smaller than statistical errors; cut-off effects negligible at heavier than physical pion masses
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- Good agreement with other lattice QCD results



Momentum fraction $\langle x \rangle_{u-d}$

- $N_f = 2$ twisted mass fermions with a clover term at a **physical value of the pion mass, $48^3 \times 96$** and $a = 0.093(1)$ fm
- Intrinsic quark spin: momentum fraction

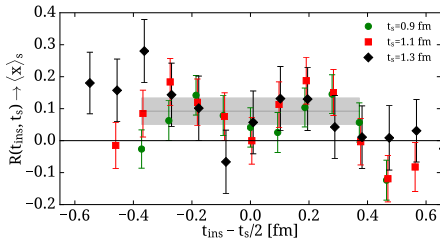


Results for the disconnected isoscalar

At the physical point we find in the \overline{MS} at 2 GeV from the plateau method ($\mathcal{O}(860,000)$ statistics):

- $\langle x \rangle_{u-d} = 0.194(9)(10)$
- $\langle x \rangle_{u+d+s} = 0.80(12)_{\text{stat}}(10)_{\text{syst}}$

$\langle x \rangle_{u+d+s}$ is perturbatively renormalized to one-loop due to its mixing with the gluon operator.



Results for the strange

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Gluon content of the nucleon

- Gluons carry a significant amount of momentum and spin in the nucleon

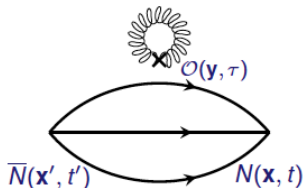
- ▶ Compute gluon momentum fraction : $\langle x \rangle_g = A_{20}^g$
- ▶ Compute gluon spin: $J^g = \frac{1}{2}(A_{20}^g + B_{20}^g)$

- Nucleon matrix of the gluon operator: $O_{\mu\nu} = -G_{\mu\rho}G_{\nu\rho}$
→ gluon momentum fraction extracted from
 $\langle N(0) | O_{44} - \frac{1}{3} O_{ij} | N(0) \rangle = m_N \langle x \rangle_g$

- Disconnected correlation function, known to be very noisy

⇒ we employ several steps of **stout smearing** in order to remove fluctuations in the gauge field

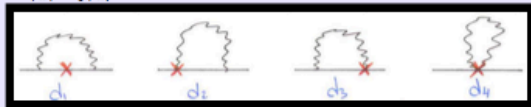
- Results are computed on the $N_f = 2$ ensemble at the physical point, $m_\pi = 131$ MeV, $a = 0.093$ fm, $V = 48^3 \times 96$, A. Abdel-Rehim *et al.* (ETMC):1507.04936
- The methodology was tested for $N_f = 2 + 1 + 1$ twisted mass at $m_\pi = 373$ MeV, C. Alexandrou, V. Drach, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 (2014) 289



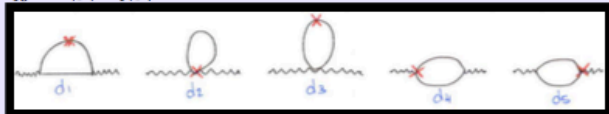
Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \Rightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

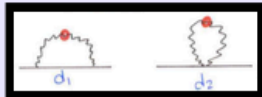
$$\times Z_{qq} : \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$$



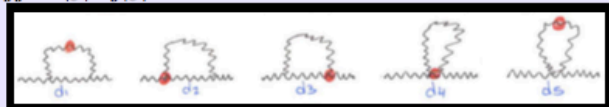
$$\times Z_{gg} : \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$$



$$\bullet Z_{gq} : \Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$$



$$\bullet Z_{gg} : \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$$



Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \implies$ Perturbation theory - M. Constantinou and H. Panagopoulos

$$\times Z_{qq} : \Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$$

$$Z_{gg} = 1 + \frac{g^2}{16\pi^2} \left(1.0574 N_f + \frac{-13.5627}{N_c} - \frac{2N_f}{3} \log(a^2 \bar{\mu}^2) \right)$$

$$\times Z_{gq} : \Lambda_{gq} = \langle g | \mathcal{O}_q | g \rangle$$

$$Z_{gq} = 0 + \frac{g^2 C_f}{16\pi^2} \left(0.8114 + 0.4434 c_{SW} - 0.2074 c_{SW}^2 + \frac{4}{3} \log(a^2 \bar{\mu}^2) \right)$$

$$\bullet Z_{qq} : \Lambda_{qq} = \langle q | \mathcal{O}_g | q \rangle$$

$$Z_{qq} = 1 + \frac{g^2}{16\pi^2} \left(-1.8557 + 2.9582 c_{SW} + 0.3984 c_{SW}^2 - \frac{8}{3} \log(a^2 \bar{\mu}^2) \right)$$

$$\bullet Z_{gg} : \Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$$

$$Z_{gg} = 0 + \frac{g^2 N_f}{16\pi^2} \left(0.2164 + 0.4511 c_{SW} + 1.4917 c_{SW}^2 - \frac{4}{3} \log(a^2 \bar{\mu}^2) \right)$$

Results for the gluon content

- 2094 gauge configurations with 100 different source positions each \rightarrow more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, [M. Constantinou and H. Panagopoulos](#)

- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}}$

$\langle x \rangle_g^R = Z_{gg} \langle x \rangle_g + Z_{gq} \langle x \rangle_{u+d+s} = 0.267(12)_{\text{stat}}(10)_{\text{syst}}$. The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.

- Momentum sum is satisfied:

$$\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d} |_{\text{conn.}} + \langle x \rangle_{u+d+s} |_{\text{disconn.}} + \langle x \rangle_g = 1.07(12)_{\text{stat}}(10)_{\text{syst}}$$

Nucleon spin

$$\text{Spin sum: } \frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2} \Delta \Sigma^q + L^q \right)}_{J^q} + J^g$$

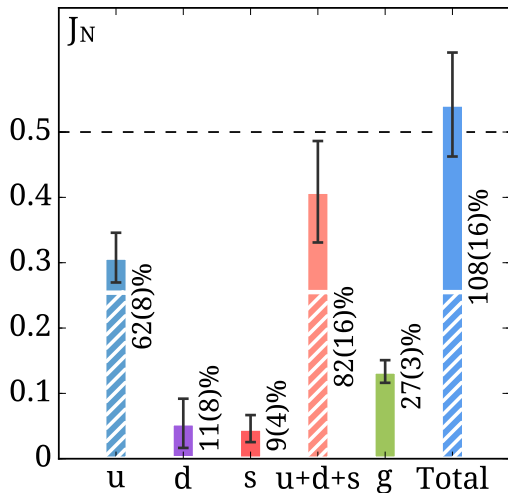
$$\begin{aligned} \frac{1}{2} \Delta \Sigma^u &= 0.415(13)(2), & \frac{1}{2} \Delta \Sigma^d &= -0.193(8)(3), & \frac{1}{2} \Delta \Sigma^s &= -0.021(5)(1) \\ J^u &= 0.308(30)(24), & J^d &= 0.054(29)(24), & J^s &= 0.046(21) \\ L^u &= -0.107(32)(24), & L^d &= 0.247(30)(24), & L^s &= 0.067(21)(1) \end{aligned} \quad (1)$$

We find that $B_{20}^q(0) \sim 0 \rightarrow$ taking $B_{20}(0)^g \sim 0$ we can directly check the nucleon spin sum:

$$J_N = (0.308)_u + (0.054)_d + (0.046)_s + (0.133)_g = 0.54(6)(5)$$

The proton spin puzzle

1987: the European Muon Collaboration showed that only a fraction of the proton spin is carried by the quarks \Rightarrow ETMC has now provided the solution



Recent results from lattice QCD at the physical point

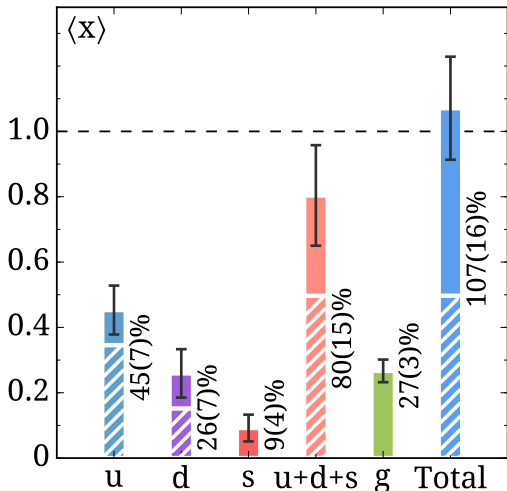
C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973



The proton momentum sum

⇒ Momentum sum also satisfied

$$\sum_q \langle x \rangle_q + \langle x \rangle_g = 0.497(12)(5)|_{\text{conn.}} + 0.307(121)(95)|_{\text{disc.}} + 0.267(12)(10)|_{\text{gluon}} = 1.07(12)(10)$$

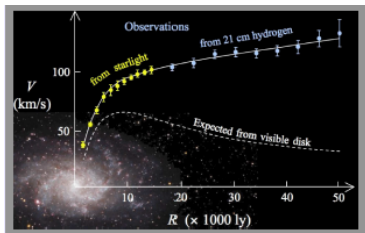


Recent results from lattice QCD at the physical point

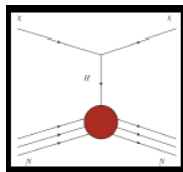
C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973



The quark content of the nucleon



Rotational curve of M33 galaxy



- $\sigma_f \equiv m_f \langle N | \bar{q}_f q_f | N \rangle$: measures the explicit breaking of chiral symmetry
 Largest uncertainty in interpreting experiments for direct dark matter searches - Higgs-nucleon coupling depends on σ ,
 e.g. spin-independent cross-section can vary an order of magnitude if $\sigma_{\pi N}$ changes from 35 MeV to 60 MeV, J. Ellis, K. Olive, C. Savage, arXiv:0801.3656

In lattice QCD:

- ▶ Feynman-Hellmann theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$

Similarly $\sigma_s = m_s \frac{\partial m_N}{\partial m_s}$, S. Dürr *et al.* (BMW_C) Phys.Rev.Lett. 116 (2016) 172001

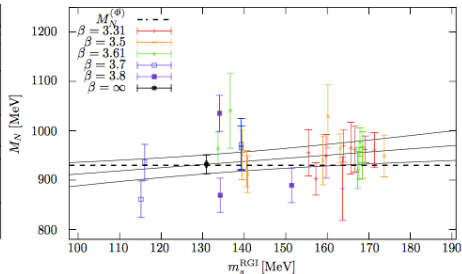
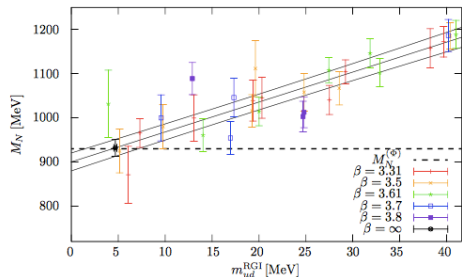
- ▶ Direct computation of the scalar matrix element

G. Bali, *et al.* (RQCD) Phys.Rev. D93 (2016) 094504, arXiv:1603.00827; Yi-Bo Yang *et al.* (χ QCD) Phys.Rev. D94 (2016) no.5, 054503;

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001;

The quark content of the nucleon via Feynman-Hellmann

BMW Collaboration: 47 lattice ensembles with $N_f = 2 + 1$ clover fermions, 5 lattice spacings down to 0.054 fm, lattice sizes up to 6 fm and pion masses down to 120 MeV.

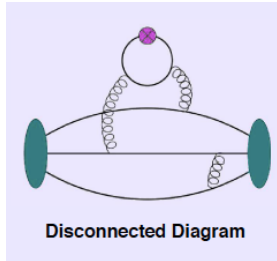


$$\sigma_{\pi N} = 38(3)(3) \text{ MeV}$$

$$\sigma_S = 105(41)(37) \text{ MeV}$$

The quark content of the nucleon via direct determination

Need disconnected contributions



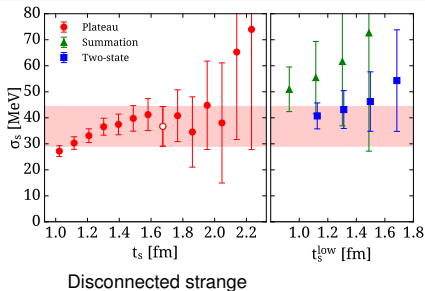
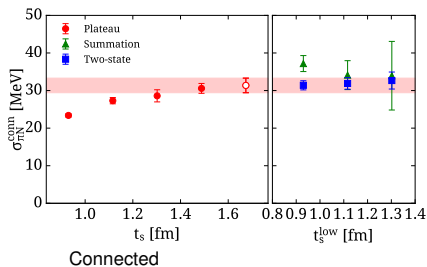
- RQCD: $N_f = 2$ clover fermions with a range of pion masses down to $m_\pi = 150$ MeV and $a = 0.06 - 0.08$ fm G. Bali, *et al.*, Phys.Rev. D93 (2016) 094504, arXiv:1603.00827
- χ QCD: Valence overlap fermions on $N_f = 2 + 1$ flavor domain-wall fermion (DWF) configurations, 3 ensembles of $m_\pi = 330$ MeV, $m_\pi = 300$ MeV and $m_\pi = 139$ MeV Yi-Bo Yang *et al.*, Phys.Rev. D94 (2016) no.5, 054503; M/ Gong *et al.*, Phys. Rev. D 88 (2013) 014503 arXiv:1304.1194
- ETM Collaboration: $N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV, A. Abdel-Rehim *et al.*, arXiv:1601.3656, PRL116 (2016) 252001

The quark content of the nucleon from ETMC

$N_f = 2$ twisted mass plus clover, $48^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV

- Connected: $t/a = 10, 12, 14$ 9264 statistics, $t/a = 16 \sim 47,600$ statistics and $t/a = 18 \sim 70,000$ statistics
- Disconnected: $\sim 213,700$ statistics

A. Abdel-Rehim *et al.* arXiv:1601.3656, PRL116 (2016) 252001

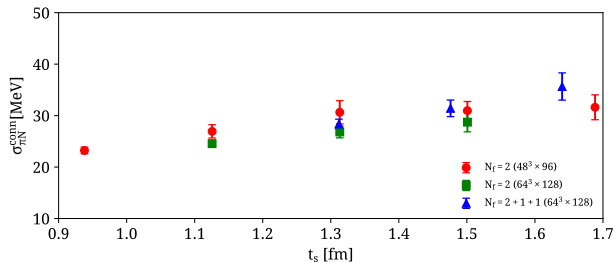


Our results are: $\sigma_{\pi N} = 36(2)$ MeV $\sigma_s = 37(8)$ MeV $\sigma_c = 83(17)$ MeV

Volume and unquenching effects

Investigation of volume and quenching effects using:

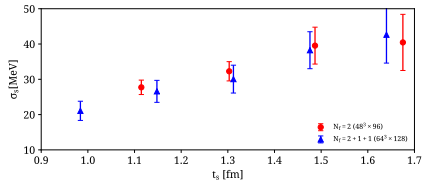
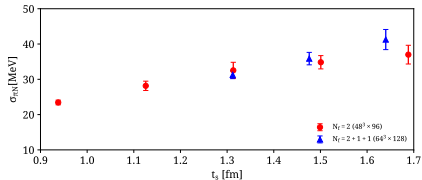
- $N_f = 2$ twisted mass plus clover, $64^3 \times 96$, $a = 0.093(1)$ fm, $m_\pi = 131$ MeV
- $N_f = 2 + 1 + 1$ twisted mass plus clover $64^3 \times 96$, $a = 0.081(1)$ fm, $m_\pi = 135$ MeV



Volume and unquenching effects

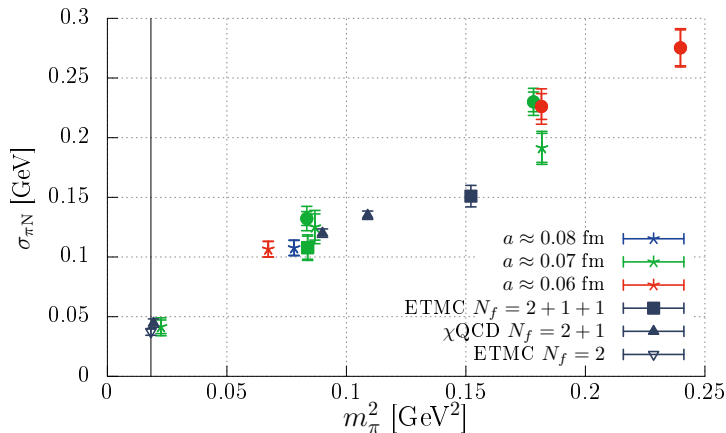
Investigation of volume and quenching effects using:

- $N_f = 2 + 1 + 1$ twisted mass plus clover $64^3 \times 96$, $a = 0.081(1)$ fm, $m_\pi = 135$ MeV



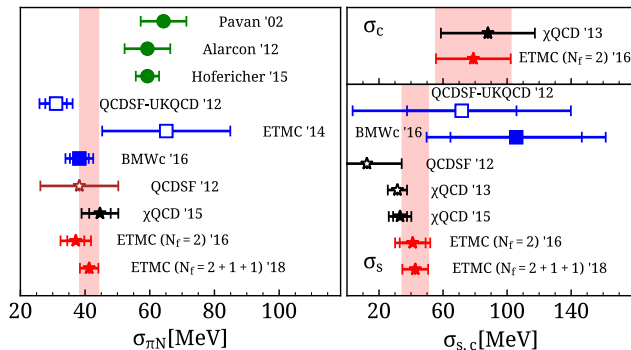
The quark content of the nucleon

Comparison of results



The quark content of the nucleon

Comparison of results



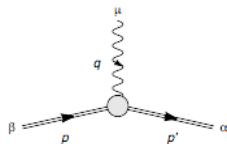
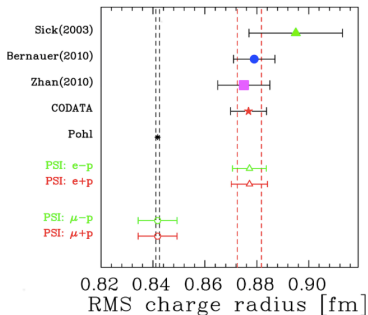
- Recent results from lattice QCD at the physical point and from phenomenology
- Filled symbols for lattice QCD results include simulations with pion mass close to its physical value, *A. Abdel-Rehim et al. arXiv:1601.3656, PRL116 (2016) 252001*
- New preliminary results using $N_f = 2 + 1 + 1$ twisted mass corroborate our $N_f = 2$ value
- With current statistics no unquenching effects seem for σ_s and σ_c .

Electromagnetic form factors

$$\langle N(p', s') | j^\mu(0) | N(p, s) \rangle = \bar{u}_N(p', s') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u_N(p, s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2} F_2(q^2)$$

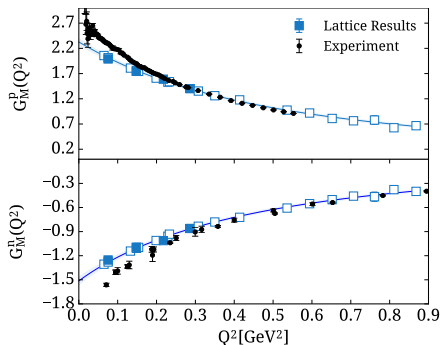
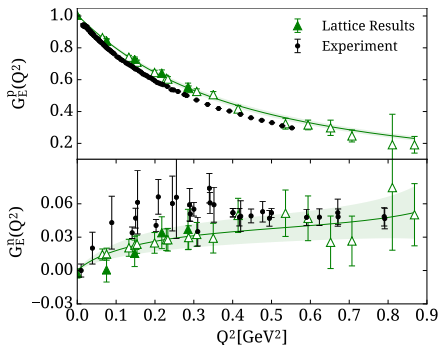
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



E. J. Downie, EPJ Conf. 113 (2016) 05021

- Proton radius extracted from muonic hydrogen is 7.9 σ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate and a reanalysis of electron scattering data may give agreement with muonic measurement

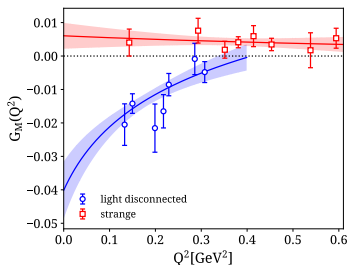
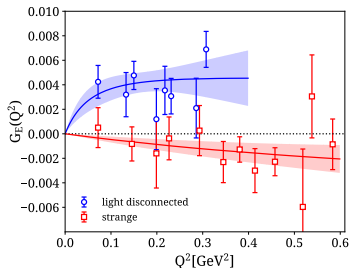
Recent results on the electric and magnetic form factors



- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$

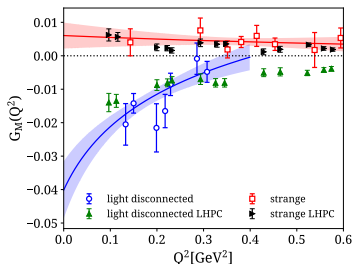
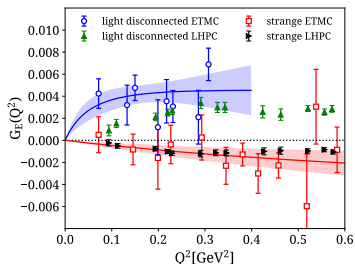
C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou and A. Vaquero Aviles-Casco. Phys. Rev. D96 (2017) 034503, arXiv:1706.00469

Recent results on the electric and magnetic form factors



- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$
- Disconnected uses about 200,000 statistics

Recent results on the electric and magnetic form factors

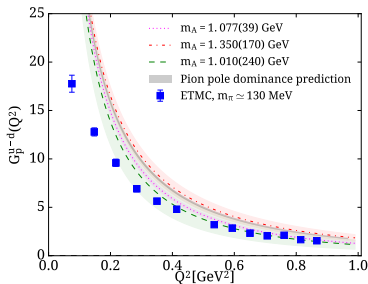
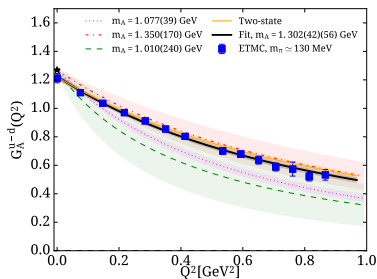


- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$
- Disconnected uses about 200,000 statistics
- LHPC using $N_f = 2 + 1$ clover fermions with $m_\pi \sim 320$ MeV

Recent results on the axial form factors

$$\langle N(p', s') | A_\mu | N(p, s) \rangle = i \sqrt{\frac{m_N^2}{E_N(\vec{p}') E_N(\vec{p})}} \bar{u}_N(p', s') \left(\gamma_\mu G_A(Q^2) - i \frac{Q_\mu}{2m_N} G_P(Q^2) \right) \gamma_5 u_N(p, s)$$

Isovector



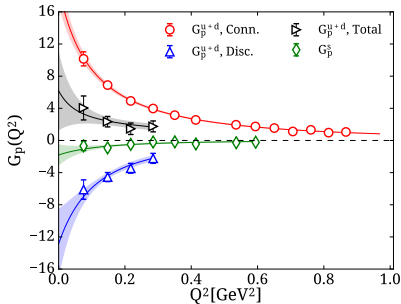
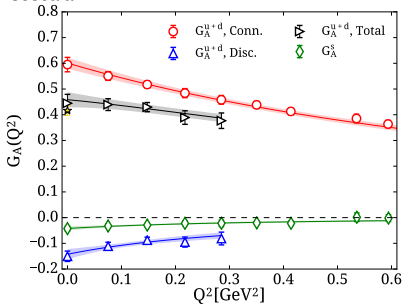
- ETMC using $N_f = 2$ twisted mass fermions (TMF), $a = 0.093$ fm, $48^3 \times 96$
- Connected contributions: G_E with $t_s = 1.7$ fm and 66,000 statistics, G_M with $t_s = 1.3$ fm and 9,300 statistics

C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou and A. Vaquero Aviles-Casco. Phys. Rev. D, arXiv:1705.03399 [hep-lat]

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Isoscalar



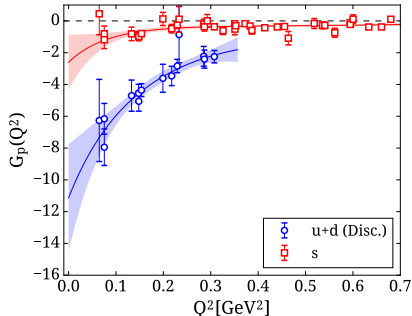
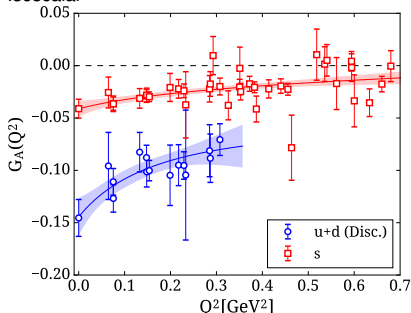
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- Disconnected uses about 200,000 statistics

C. A. *et al.* (ETMC), Phys. Rev. D, arXiv:1705.03399 [hep-lat]

Conclusions

Simulations at the physical point → that's where we always wanted to be!

Future Perspectives

- Computation of g_A , $\langle X \rangle_{u-d}$, etc, at the physical point allows direct comparison with experiment
 - can provide reliable predictions for g_S , g_T , tensor moment, σ -terms, etc
 - can address long-standing puzzles like the spin decomposition of the nucleon

On-going studies within ETMC

- Assess volume effects using $N_f = 2$ and lattice size $64^3 \times 128$ at same pion mass
- Analyze a new ensemble of $N_f = 2 + 1 + 1$ twisted clover improved configurations with $a \sim 0.08$ fm and lattice size $64^3 \times 128$ and $m_\pi \sim 135$ MeV
 - ▶ Preliminary results for nucleon charges and σ -terms
 - ▶ Other quantities under study include the nucleon form factors, the proton and gluonic observables

European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)



Cyprus (Univ. of Cyprus, Cyprus Inst.),
France (Orsay, Grenoble), **Germany**
(Berlin/Zeuthen, Bonn, Frankfurt, Ham-
burg, Münster), **Italy** (Rome I, II, III, Trento),
Netherlands (Groningen), **Poland** (Poznan),
Spain (Valencia), **Switzerland** (Bern), **UK**
(Liverpool)

Collaborators:

A. Abdel-Rehim, S. Bacchio, K. Cichy, M.
Constantinou, J. Finkenrath, K. Hadjiyan-
nakou, K.Jansen, Ch. Kallidonis, G. Kout-
sou, K. Ottnad, M. Petschlies, F. Steffens, A.
Vaquero, C. Wiese



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