Pion light-cone distribution amplitude from lattice QCD with valence heavy quark

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Collaborators

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Outline

- Motivation and general strategy.
- Lattice OPE and the pion light-cone wavefunction.
- Exploratory numerical result.
- Outlook.

Motivation and general strategy

W.Detmold and CJDL, Phys.Rev.D73, 014501 (2006) [hep-lat/0507007]

Parton distribution from lattice QCD

The "traditional" approach

- Hadronic tensor

$$W_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S|\, [J^\mu(x), J^\nu(0)]\, |p,S\rangle$$

optical theorem

Imaginary part → challenging in Euclidean QCD

$$T_S^{\mu\nu}(p,q) = \int d^4x\, \mathrm{e}^{iq\cdot x} \langle p,S| T\, [J^\mu(x) J^\nu(0)]\, |p,S\rangle$$

- Light-cone OPE

$$T[J^\mu(x)J^\nu(0)] = \sum_{i,n} \mathcal{C}_i\left(x^2,\mu^2\right)\, x_{\mu_1}\dots x_{\mu_n}\, \mathcal{O}_i^{\mu\nu\mu_1\dots\mu_n}(\mu)$$

local operators, issue of operator mixing

Power divergences arising from Lorentz symmetry breaking

leading moments in practice

Introducing the valence heavy quark

- Valence → not in the action.

- The "heavy quark" is relativistic.

  → propagating in both space and time

- The current for computing the even moments of the PDF

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

↓ Compton tensor

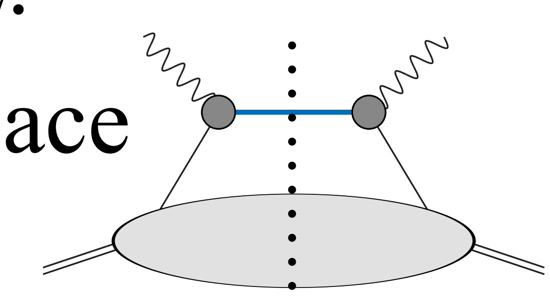
$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_{S} \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_{S}\int d^4x\ \mathrm{e}^{iq\cdot x}\langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

Strategy for extracting the moments

$$T^{\mu\nu}_{\Psi,\psi}(p,q) \equiv \sum_S \langle p,S|t^{\mu\nu}_{\Psi,\psi}(q)|p,S\rangle = \sum_S \int d^4x\; \mathrm{e}^{iq\cdot x} \langle p,S|T\left[J^{\mu}_{\Psi,\psi}(x)J^{\nu}_{\Psi,\psi}(0)\right]|p,S\rangle$$

$$J^{\mu}_{\Psi,\psi}(x) = \overline{\Psi}(x)\gamma^{\mu}\psi(x) + \overline{\psi}(x)\gamma^{\mu}\Psi(x)$$

- Simple renormalisation for quark bilinears.
- Work with the hierarchy of scales $\Lambda_{\mathrm{QCD}} << \sqrt{q^2} \leq m_{\Psi} << \frac{1}{a}$
  - → Heavy scales for short-distance OPE.
  - → Avoid branch point in Minkowski space at $(q+p)^2 \sim (m_N + m_\Psi)^2$



- Extrapolate $T^{\mu\nu}_{\Psi,\psi}(p,q)$ to the continuum limit first.
  - → Then match to the short-distance OPE results.
  - → Extract the moments without power divergence.

Lattice OPE
and pion light-cone distribution amplitude

W.Detmold, I.Kanamori, CJDL, S.Mondal, Y.Zhao, work in progress

Pion light-cone wavefunction

Important input for flavour physics

$$\langle 0|\bar{u}(z/2)\gamma_5\gamma_\mu d(-z/2)|\pi^+(p)\rangle = -ip_\mu f_\pi \int_0^1 d\xi\ \mathrm{e}^{i\left(\bar{\xi}p\frac{z}{2}-\xi p\frac{z}{2}\right)}\phi_\pi(\xi)$$

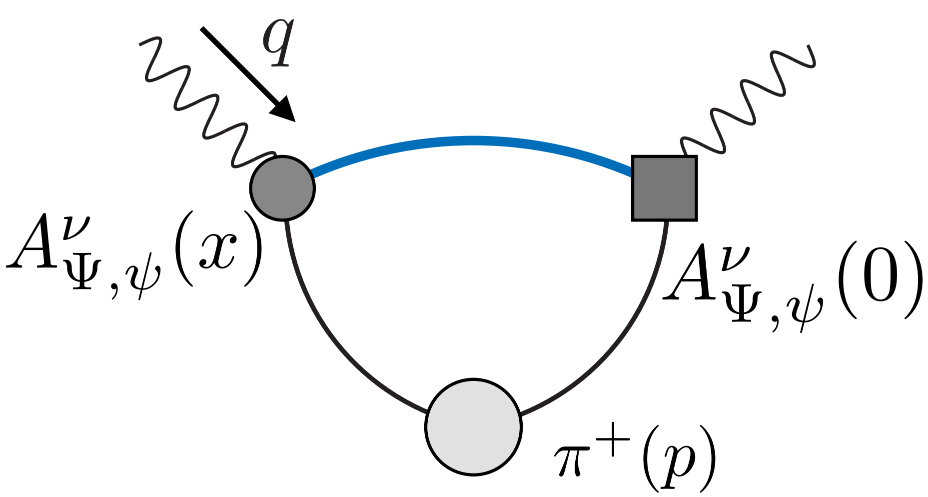
$$a_n = \int_0^1 d\xi\ \xi^n \phi_\pi(\xi)$$

OPE

$$\langle 0|O_\psi^{\mu_1\ldots\mu_n}|\pi^+(p)\rangle = f_\pi a_{n-1}[p^{\mu_1}\ldots p^{\mu_n} - \mathrm{traces}]$$

$$O_\psi^{\mu_1\ldots\mu_n} = \overline{\psi}\gamma_5\gamma^{\{\mu_1}(iD^{\mu_2})\ldots(iD^{\mu_n\}})\psi - \mathrm{traces}$$

Lattice OPE, AA amplitude



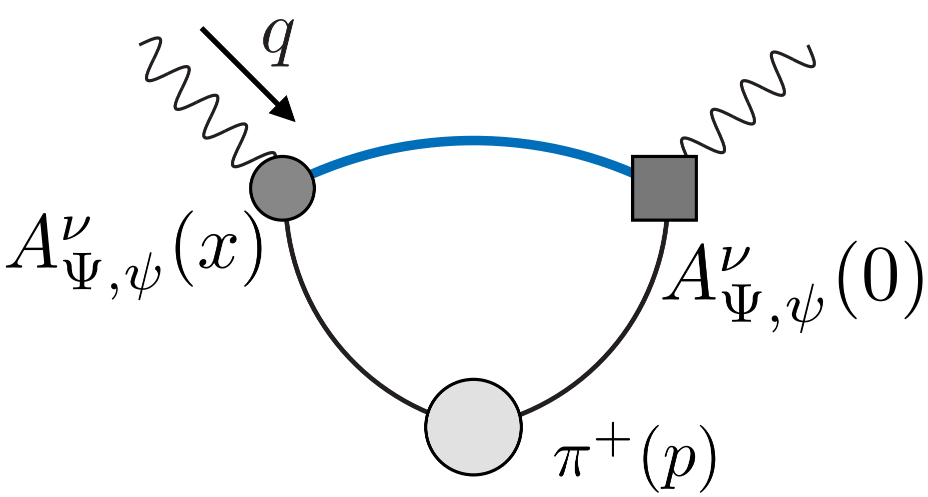
$$+ \begin{pmatrix} \mu \leftrightarrow \nu \\ x \leftrightarrow 0 \end{pmatrix}$$

$$A^{\mu}_{\Psi,\psi} = \overline{\Psi}\gamma^{\mu}\gamma_5\psi + \overline{\psi}\gamma^{\mu}\gamma_5\Psi$$

$$U^{\mu\nu}_{A}(q,p) = \int d^4x \; e^{iqx} \; \langle 0|T[A^{\mu}_{\Psi,\psi}(x)A^{\nu}_{\Psi,\psi}(0)]|\pi^+(p)\rangle$$

μ and ν anti-symmetrised

Lattice OPE, AA amplitude result



$$+ \begin{pmatrix} \mu \leftrightarrow \nu \\ x \leftrightarrow 0 \end{pmatrix}$$

$$U_A^{[\mu\nu]}(q,p) = \left(\frac{2\epsilon^{\mu\nu\rho\sigma} \; p^{[\rho} q^{\sigma]} \; f_\pi}{\tilde{Q}^2} \right) \sum_{n=0,\text{even}}^{\infty} \frac{\zeta^n \; \mathcal{C}_n^2(\eta) \; C_n(q, m_\Psi, \mu)}{n+1} a_n(\mu)$$

$$\mu = 1 \quad \nu = 2$$

$$\zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}, \qquad \eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}$$

$$C_n = 1$$

$$U_A^{[12]}(q,p) = \left[\frac{2 \; \left(p^3 q^4 - p^4 q^3\right) \; f_\pi}{\tilde{Q}^2} \right] \sum_{n=0,\text{even}}^{\infty} F_n \left(q^2, p \cdot q, m_\Psi\right) a_n$$

Exploratory numerical result

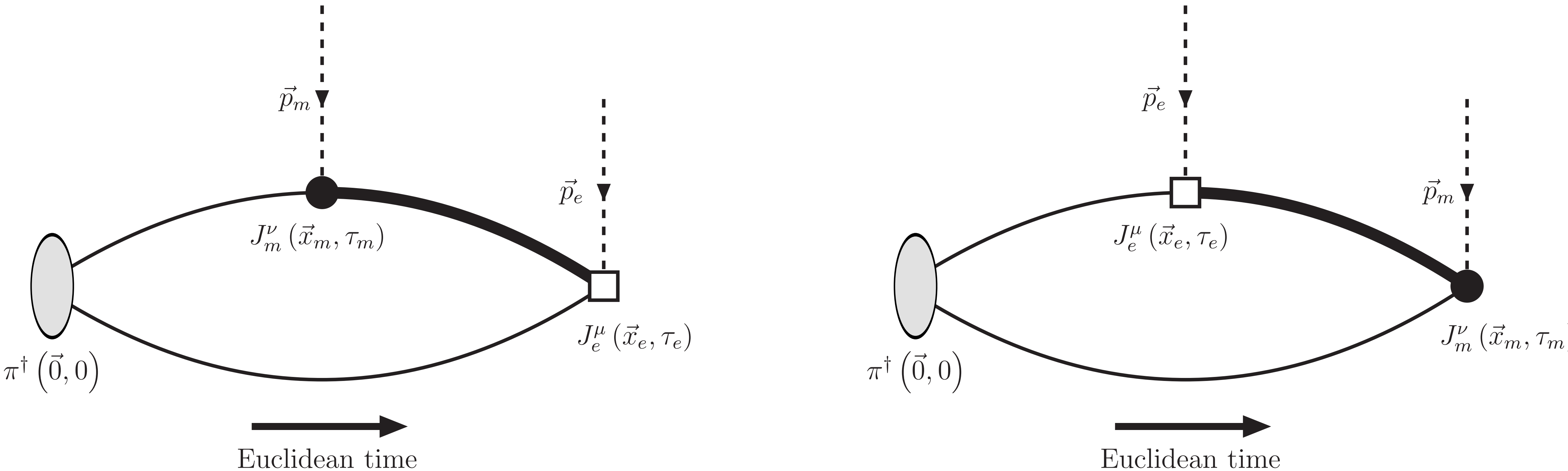
W.Detmold, I.Kanamori, CJDL, S.Mondal, Y.Zhao, work in progress

Simulation details

Testing our approach with fine quenched lattices

- In this talk: exploratory study with ~350 configs:
  - → $a^{-1} \sim 4$ GeV, $L^3 \times T = 48^3 \times 96$.
  - → NP-improved clover valence fermion.
  - → $m_\Psi \sim 2$ GeV, $M_\pi \sim 450$ MeV.
  - → Four locations for the (static) Gaussian pion source.
- Available (> 200 each) configs at $L = 32, 48, 64, 96$.
  - → Cut-off scale as high as 8 GeV.

The correlators



$$C_3^{\mu\nu}(\tau_e, \tau_m; \vec{p}_e, \vec{p}_m) = \int \mathrm{d}^3 x_e \int \mathrm{d}^3 x_m \; \mathrm{e}^{i\vec{p}_e\cdot\vec{x}_e} \mathrm{e}^{i\vec{p}_m\cdot\vec{x}_m} \left\langle 0 \middle| \mathrm{T}\left[J_e^\mu(\vec{x}_e, \tau_e) \, J_m^\nu(\vec{x}_m, \tau_m) \, \mathcal{O}_\pi^\dagger(\vec{0}, 0) \right] \middle| 0 \right\rangle$$

$$C_\pi(\tau_\pi; \vec{p}) = \int \mathrm{d}^3 x \; \mathrm{e}^{i\vec{p}\cdot\vec{x}} \left\langle 0 \middle| \mathcal{O}_\pi(\vec{x}, \tau_\pi) \mathcal{O}_\pi^\dagger(\vec{0}, 0) \middle| 0 \right\rangle \xrightarrow{\tau_\pi \to \infty} \frac{\left| \left\langle \pi(\vec{p}) \middle| \mathcal{O}_\pi^\dagger(\vec{0}, 0) \middle| 0 \right\rangle \right|^2}{2 V_3 E_\pi} \times \mathrm{e}^{-E_\pi \tau_\pi}$$

Analysis strategy

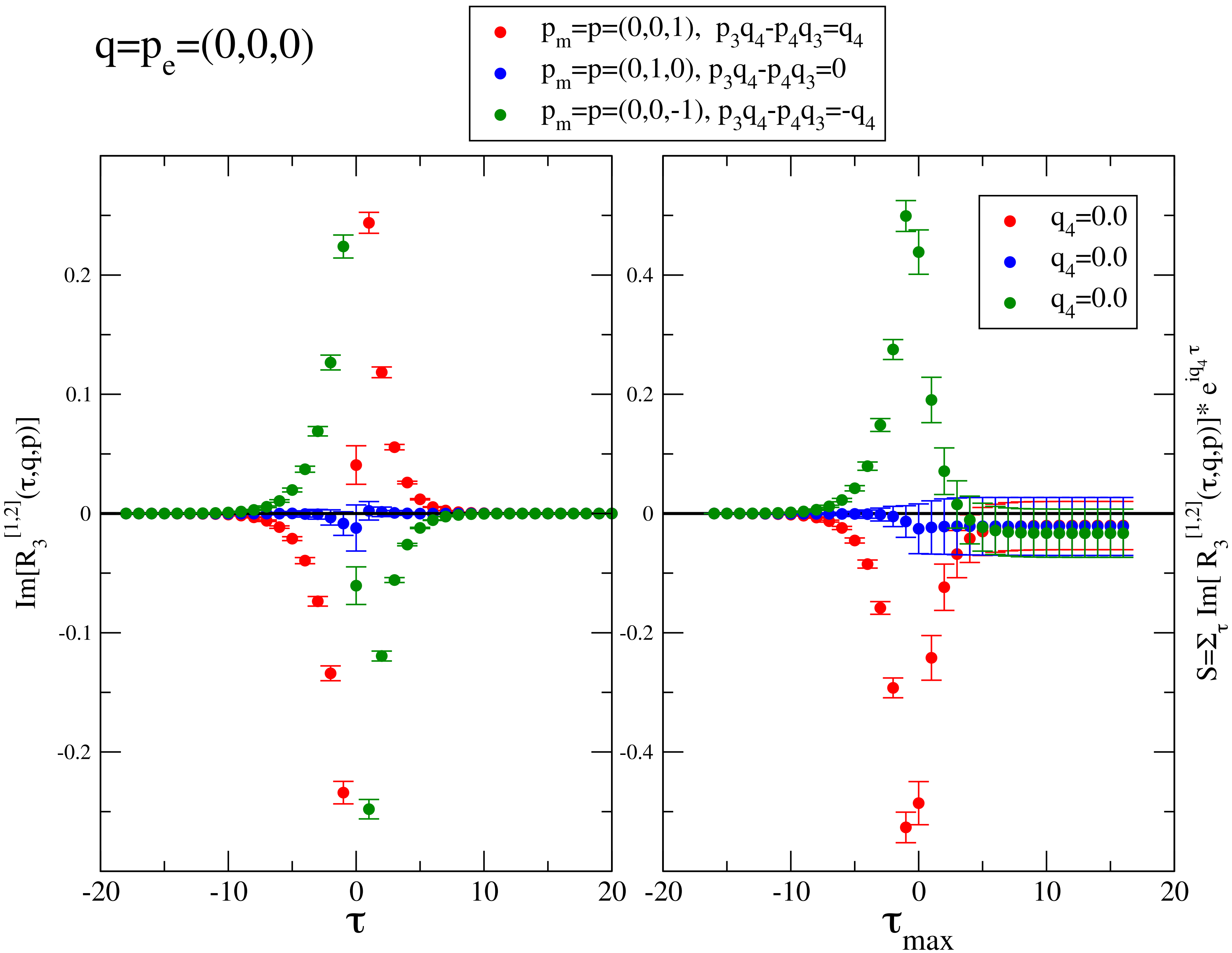
$$R_3^{\mu\nu}(\tau, \vec{q}, \vec{p}) \equiv \int \mathrm{d}^3x \; \mathrm{e}^{i\vec{q}\cdot\vec{x}} \left\langle 0 \middle| \mathrm{T}\left[J_e^{\mu}(\vec{x}, \tau)\, J_m^{\nu}(\vec{0}, 0) \right] \middle| \pi(\vec{p}) \right\rangle$$

$$U^{\mu\nu}(p, q) = i \int \mathrm{d}\tau \; \mathrm{e}^{iq_4\tau} \; R_3^{\mu\nu}(\tau, \vec{q}, \vec{p})$$

real

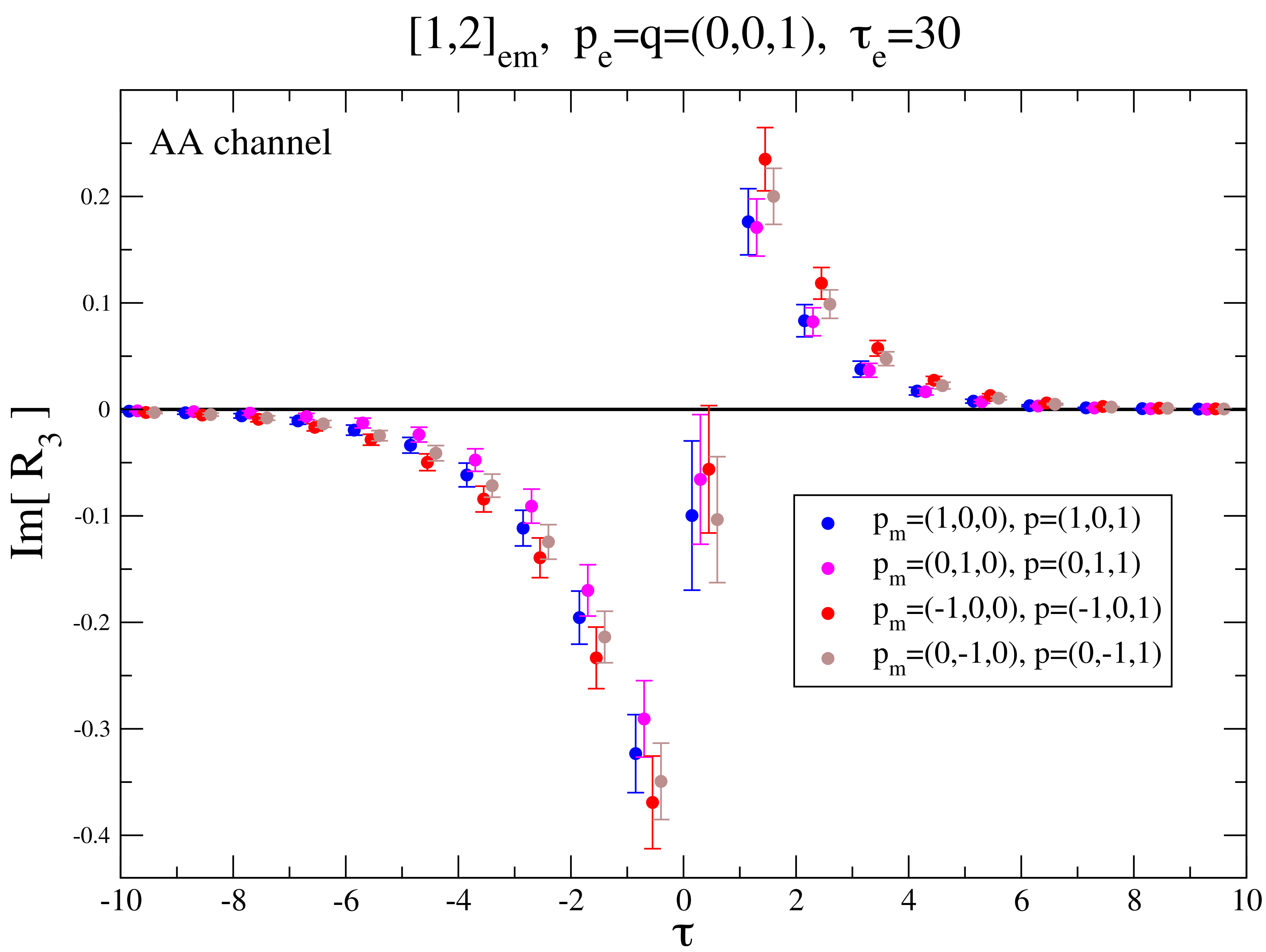
pure imaginary

Exploratory numerical results



$$U_A^{[12]}(q,p) = \left[\frac{2 \left(p^3 q^4 - p^4 q^3\right) f_\pi}{\tilde{Q}^2} \right] \sum_{n=0,\text{even}}^{\infty} F_n\left(q^2, p \cdot q, m_\Psi\right) a_n$$

Exploratory numerical results



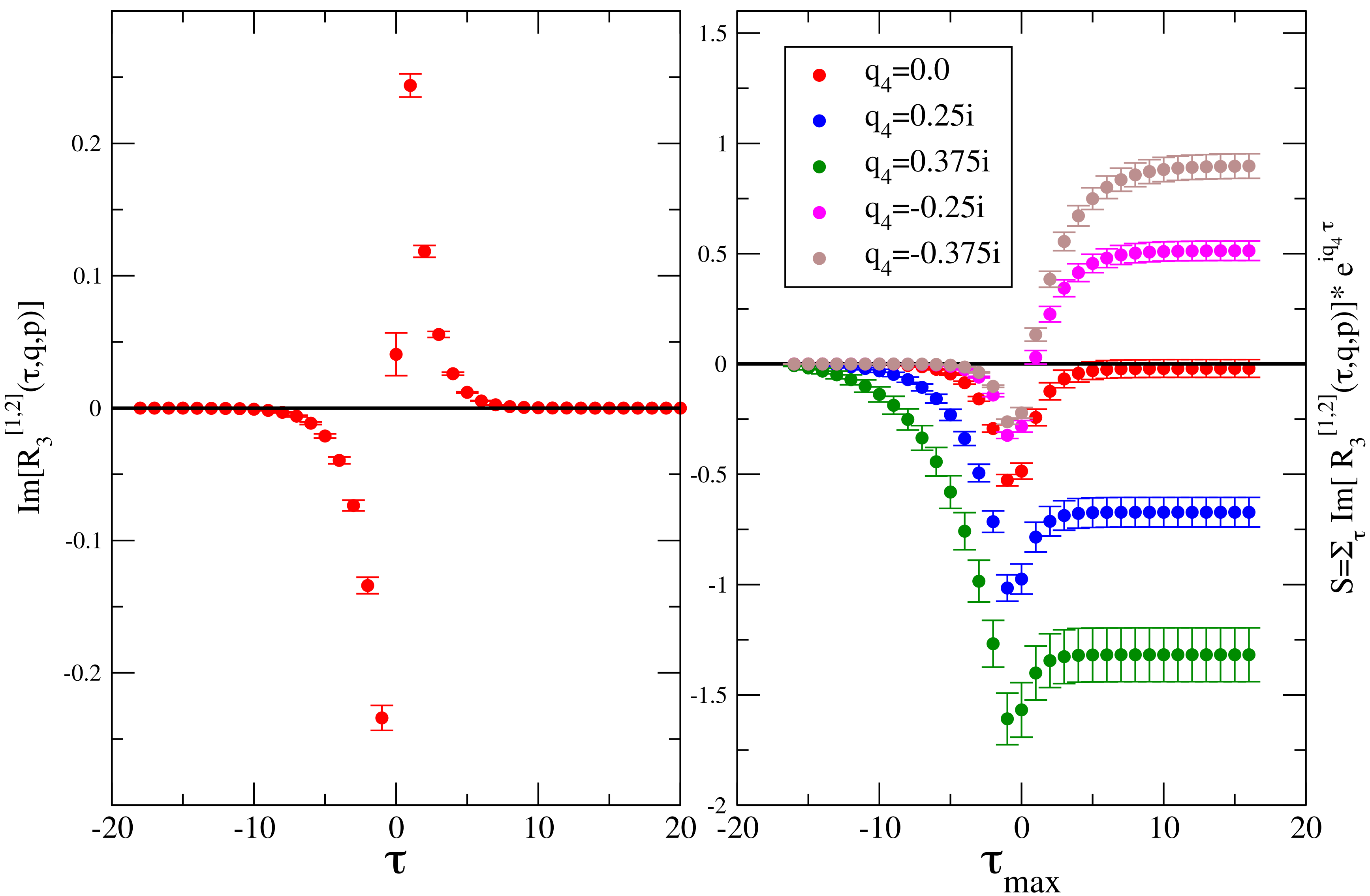
All have same $p^3q^4 - p^4q^3$, $p \cdot q$ and q^2

$$U_A^{[12]}(q,p) = \left[\frac{2\ \left(p^3q^4 - p^4q^3\right)\ f_\pi}{\tilde{Q}^2}\right] \sum_{n=0,\text{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

Exploratory numerical results

q=p$_e$=(0,0,0), p=p$_m$=(0,0,1)

Changing $p \cdot q$ and q^2



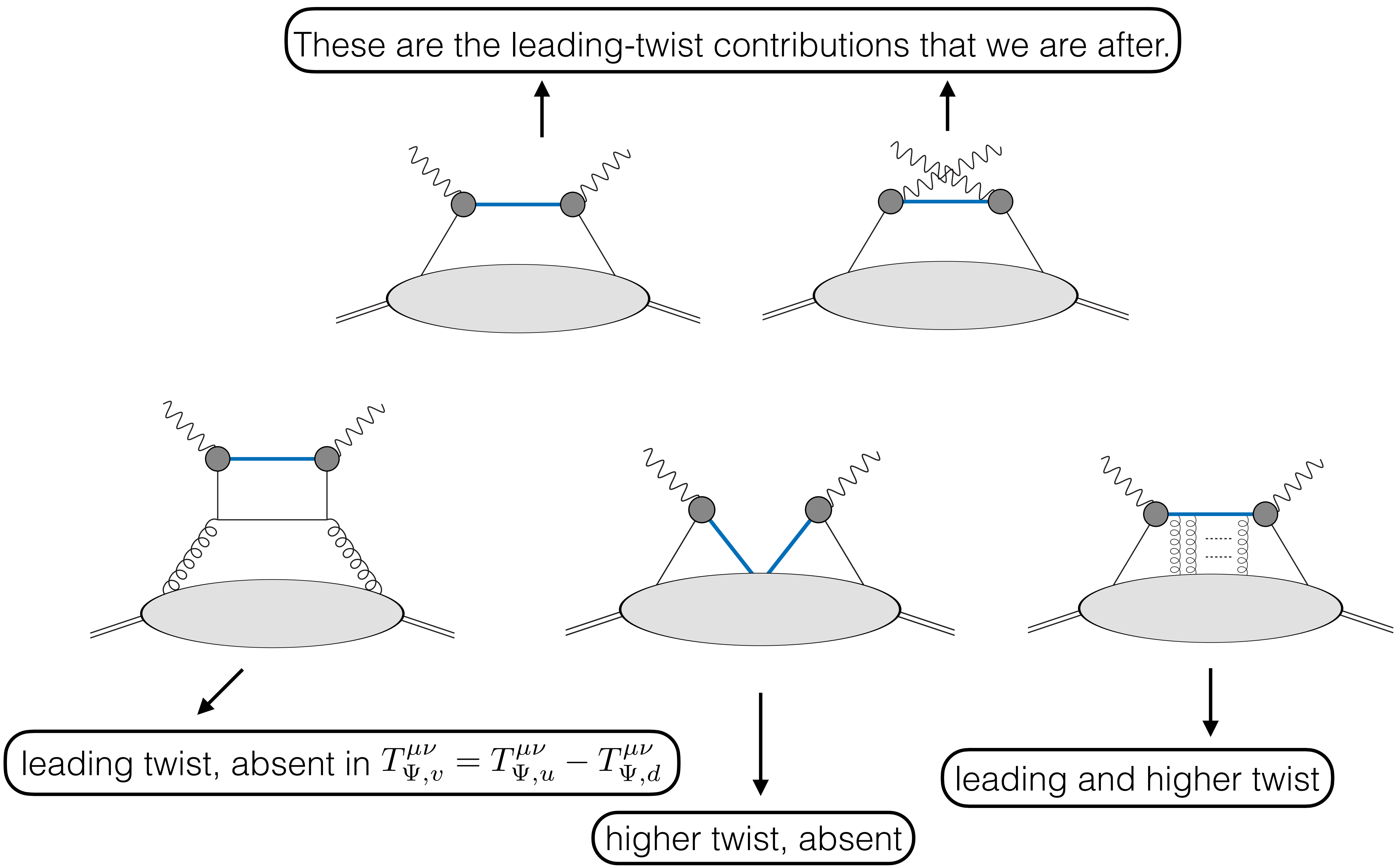
$$U_A^{[12]}(q,p) = \left[\frac{2 \ \left(p^3 q^4 - p^4 q^3\right) \ f_\pi}{\tilde{Q}^2}\right] \sum_{n=0,\text{even}}^{\infty} F_n\left(q^2, p\cdot q, m_\Psi\right) a_n$$

Outlook

- Our method has the potential to give viable information relevant to partonic physics.
- Thorough understanding of issues in numerical implementation, including the continuum limit.
- Attempt the extraction of the moments soon.
- Investigate the relation to other methods.
- Many opportunities ahead.

Backup slides

Short-distance OPE & valence heavy quark



Analysis details

$$R^{\mu\nu}_{3,\pi\to m\to e}(\tau_{3,m},\vec{q}_e,\vec{p}) = \theta(\tau_e-\tau_m)\times\frac{C_3^{\mu\nu}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m)}{C_\pi(\tau_m;\vec{p})}\times\left\langle\pi(\vec{p})\left|\mathcal{O}^\dagger_\pi(\vec{0},0)\right|0\right\rangle$$

$$= \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}_e\cdot\vec{x}}\left\langle 0\left|J^\mu_e(\vec{x},\tau_{3,m})\,J^\nu_m(\vec{0},0)\right|\pi(\vec{p})\right\rangle_{\tau_{3,m}=\tau_e-\tau_m\geq 0;\ \vec{p}=\vec{p}_e+\vec{p}_m}$$

$$R^{\mu\nu}_{3,\pi\to e\to m}(\tau_{3,e},\vec{q}_m,\vec{p}) = \theta(\tau_m-\tau_e)\times\frac{C_3^{\mu\nu}(\tau_e,\tau_m;\vec{p}_e,\vec{p}_m)}{C_\pi(\tau_e;\vec{p})}\times\left\langle\pi(\vec{p})\left|\mathcal{O}^\dagger_\pi(\vec{0},0)\right|0\right\rangle$$

$$= \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}_m\cdot\vec{x}}\left\langle 0\left|J^\nu_m(\vec{x},\tau_{3,e})\,J^\mu_e(\vec{0},0)\right|\pi(\vec{p})\right\rangle_{\tau_{3,e}=\tau_m-\tau_e\geq 0;\ \vec{p}=\vec{p}_e+\vec{p}_m}$$

$$R^{\mu\nu}_3(\tau,\vec{q},\vec{p}) \equiv \int \mathrm{d}^3x\ \mathrm{e}^{i\vec{q}\cdot\vec{x}}\left\langle 0\left|\mathrm{T}\left[J^\mu_e(\vec{x},\tau)\,J^\nu_m(\vec{0},0)\right]\right|\pi(\vec{p})\right\rangle$$

$$= \theta(\tau)\times R^{\mu\nu}_{3,\pi\to m\to e}(\tau,\vec{q},\vec{p}) + \theta(-\tau)\times \mathrm{e}^{-E_\pi(\vec{p})\tau}\times R^{\mu\nu}_{3,\pi\to e\to m}(-\tau,\vec{q},\vec{p})$$

$$U^{\mu\nu}(p,q) = i\int \mathrm{d}\tau\ R^{\mu\nu}_3(\tau,\vec{q},\vec{p})$$