Lattice Calculation on Quasi-PDF at Physical Pion Mass

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Lattice PDF Workshop

Lattice Parton Physics Project (LP3)

Author list of arXiv:1803.04393

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LaMET & Quasi-PDF

- Light-Cone PDF

$$q(x,\mu) \equiv \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ U(\xi^-,0)\psi(0)|P\rangle$$

- Quasi-PDF

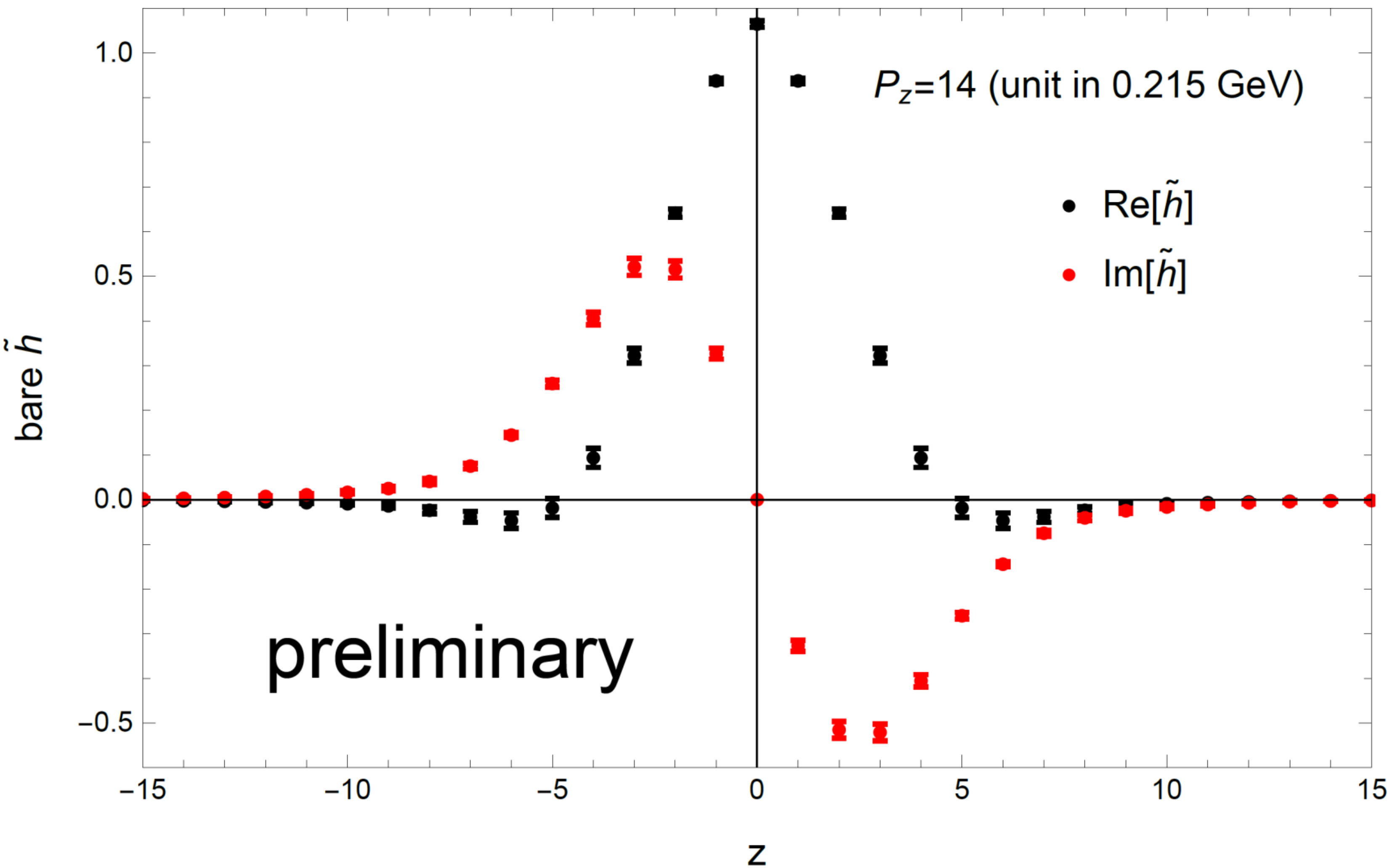
$$\tilde{q}(x,P_z,\Gamma,\tilde{\mu}) \equiv \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{ixP_z z} \langle P|\bar{\psi}(z)\Gamma U(z,0)\psi(0)|P\rangle$$

- Choose $\Gamma = \gamma_t$ to avoid mixing with scalar operator ($\Gamma = 1$) at $\mathcal{O}(a^0)$ order.
- unpolarized $u - d$ PDF

Lattice Calculation

- Lattice space $\mathrm{a} = 0.09$ fm
- Box size $64^3 \times 96$ ($L = 5.8\ \mathrm{fm}$)
- $m_\pi = 135$ MeV
- The nucleon momentum $P_z = \{10,12,14\}\frac{2\pi}{L}$ ($\frac{2\pi}{L} = 0.215$ GeV)
- $N_f = 2 + 1 + 1$, the gauge links are hypercubic (HYP)-smeared, include excited states, etc.

Bare Matrix Element



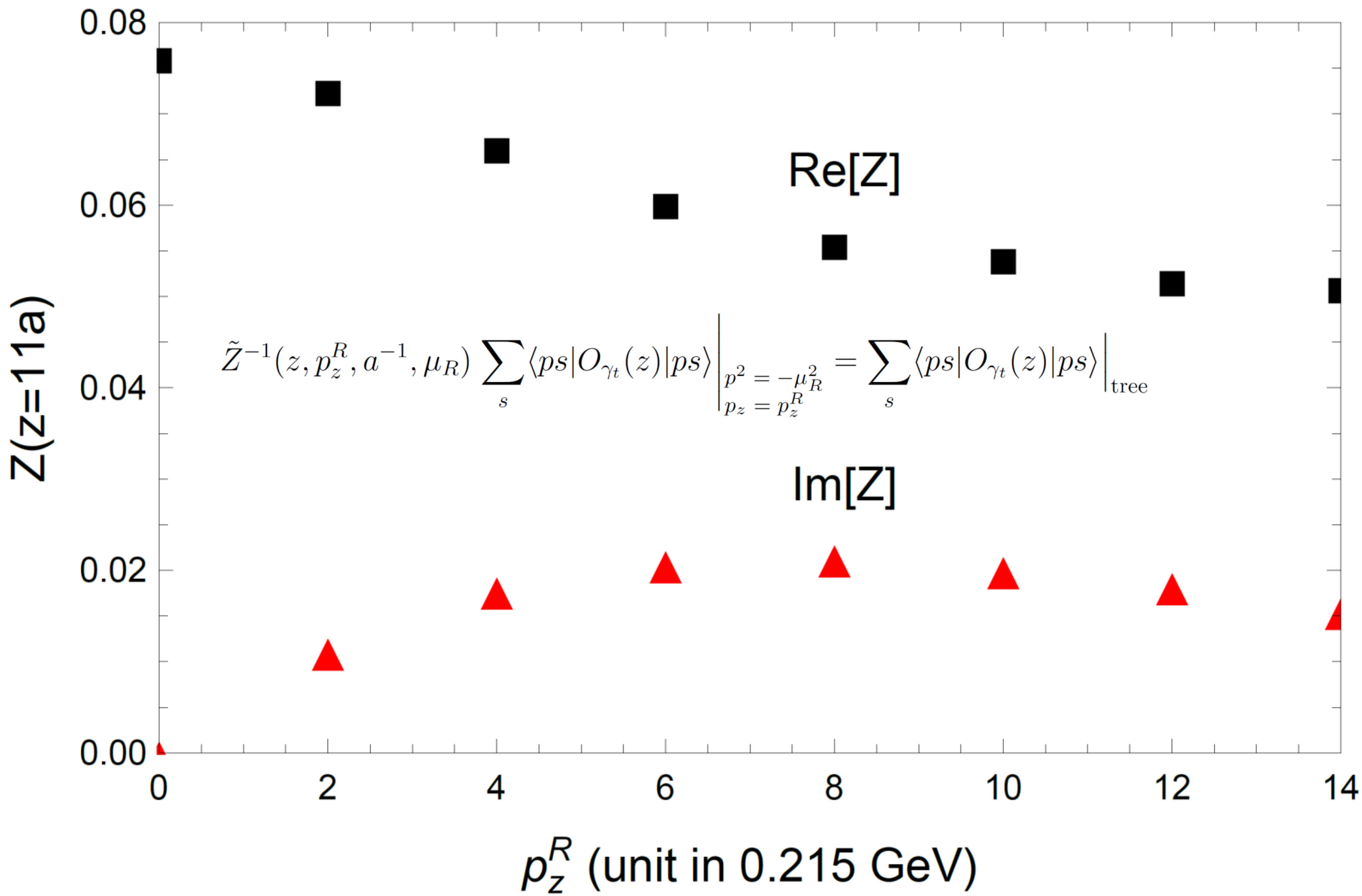
Renormalization

- Multiplicative
- Linear Divergence—Wilson line self energy
- RI/MOM scheme

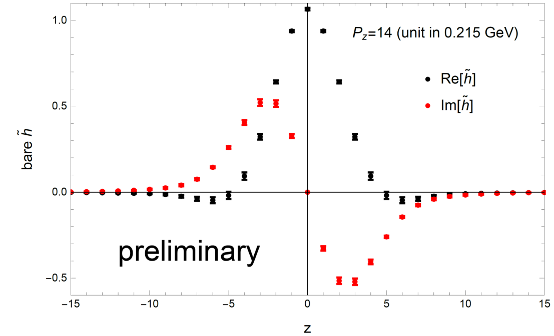
$$\tilde{Z}^{-1}(z, p_z^R, a^{-1}, \mu_R) \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle \Big|_{\substack{p^2 = -\mu_R^2 \\ p_z = p_z^R}} = \sum_s \langle ps|O_{\gamma_t}(z)|ps\rangle \Big|_{\text{tree}}$$

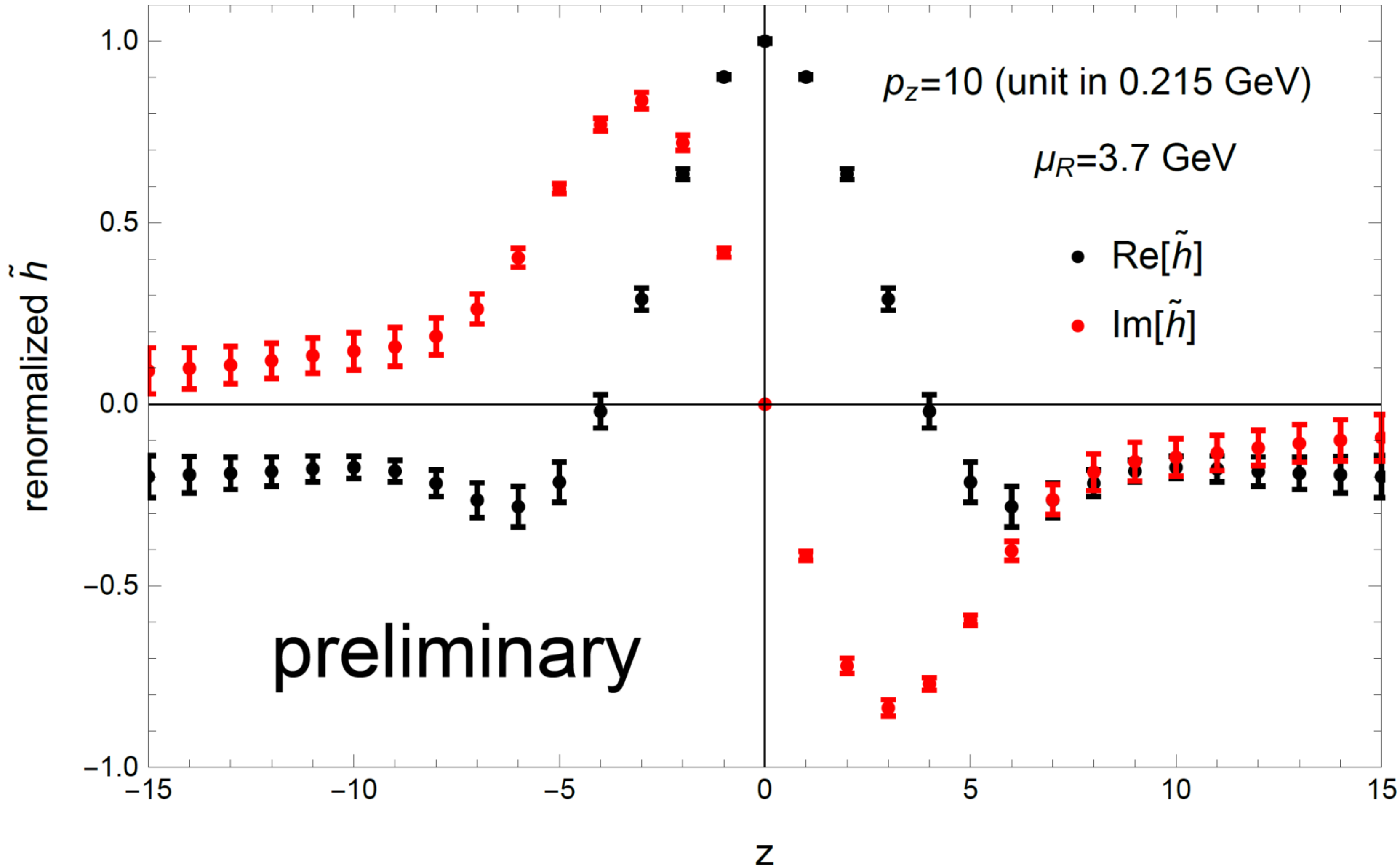
- Non-perturbative/perturbative

Renormalization Constant

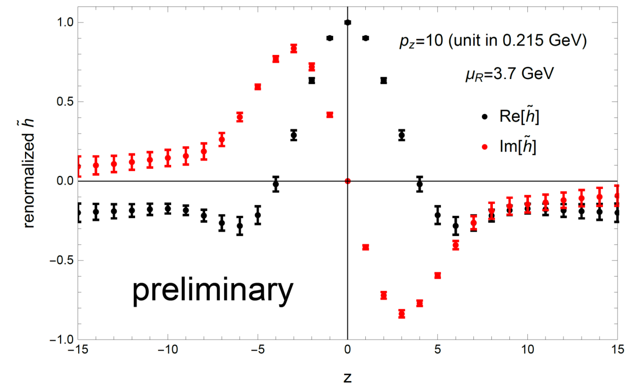


Renormalized ME





Fourier Transformation



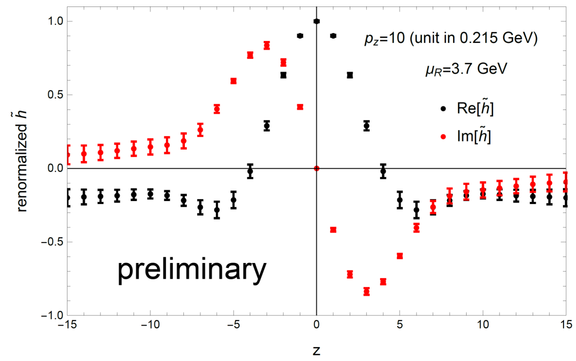
$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = P_z \int \frac{dz}{2\pi}\, e^{ixP_z z}\tilde{h}_R(z, P_z, p_z^R, \mu_R)$$

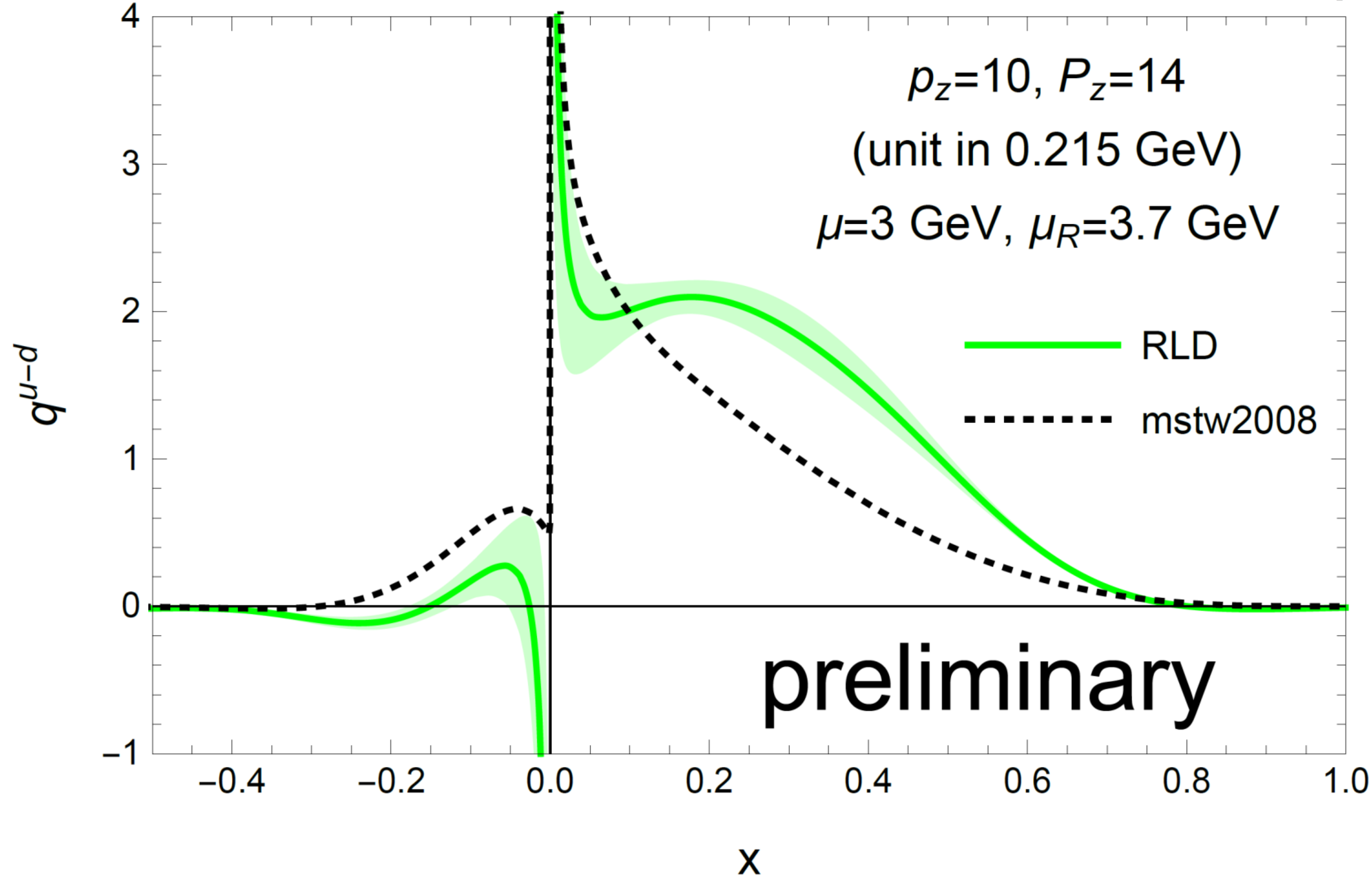
- Derivative method

$$\tilde{q}(x) = \int_{-z_{\max}}^{+z_{\max}} dz \frac{e^{ixP_z z}}{ix} \partial_z \tilde{h}_R(z)$$

- Equivalent to set $\tilde{h}_R(z) = \tilde{h}_R(z_{max})$ if $|z| \geq z_{max}$.
- $\partial_z \tilde{h}_R(z)$ is consistent with zero for $|z| \geq 15a$ and we take $z_{max} = 15a$.

Renorm. Lattice Data



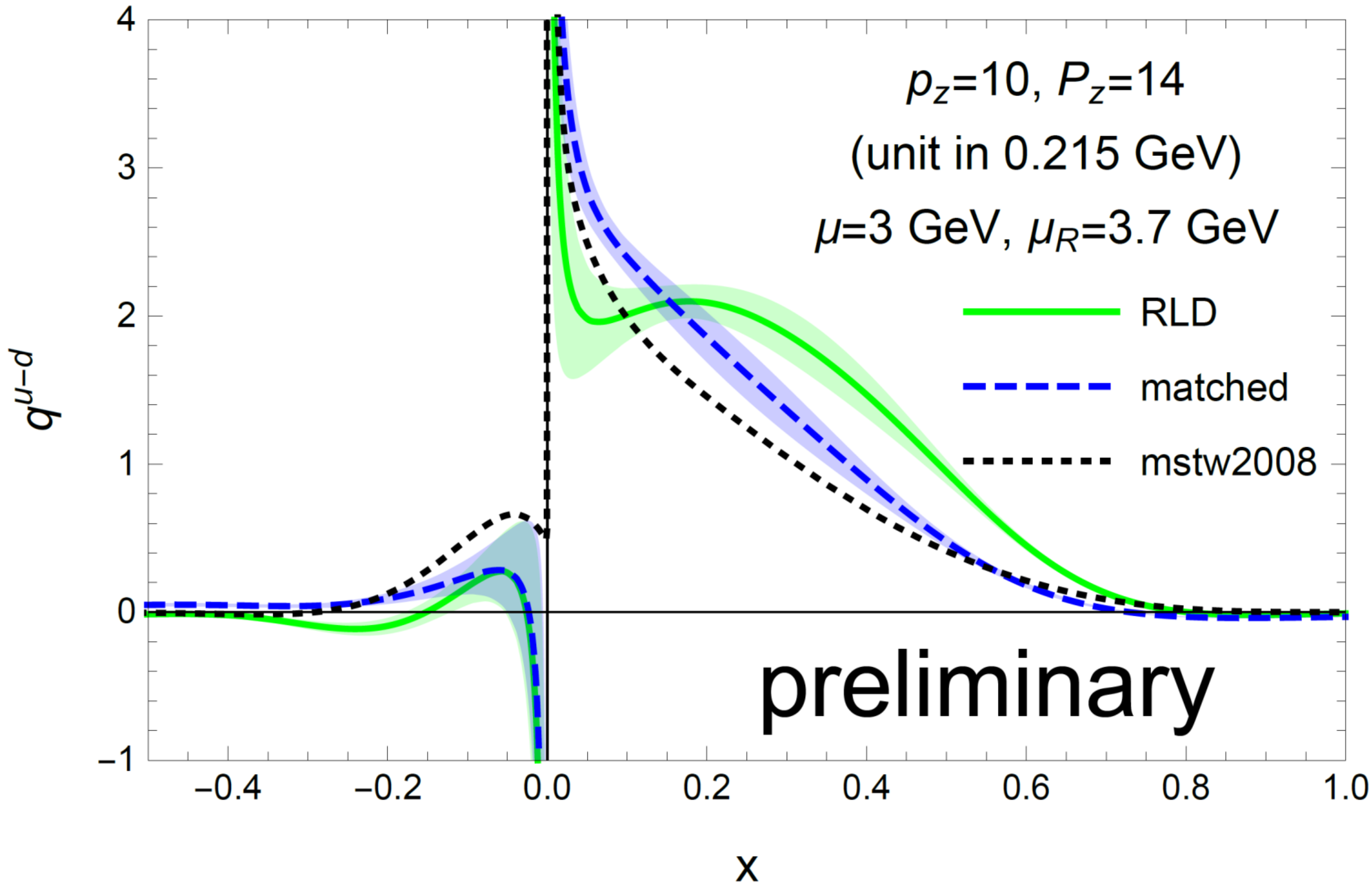


Matching

- Light-cone and quasi-PDF have the same IR, but differ in UV physics.
- The difference in UV can be controlled by perturbation theory.
- Factorization formula:

$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

Matched PDF



p_z=10, P_z=14
(unit in 0.215 GeV)
μ=3 GeV, μ_R=3.7 GeV
RLD
matched
mstw2008
preliminary
q^{u-d}
x
4
3
2
1
0
−1
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0

Error Analysis

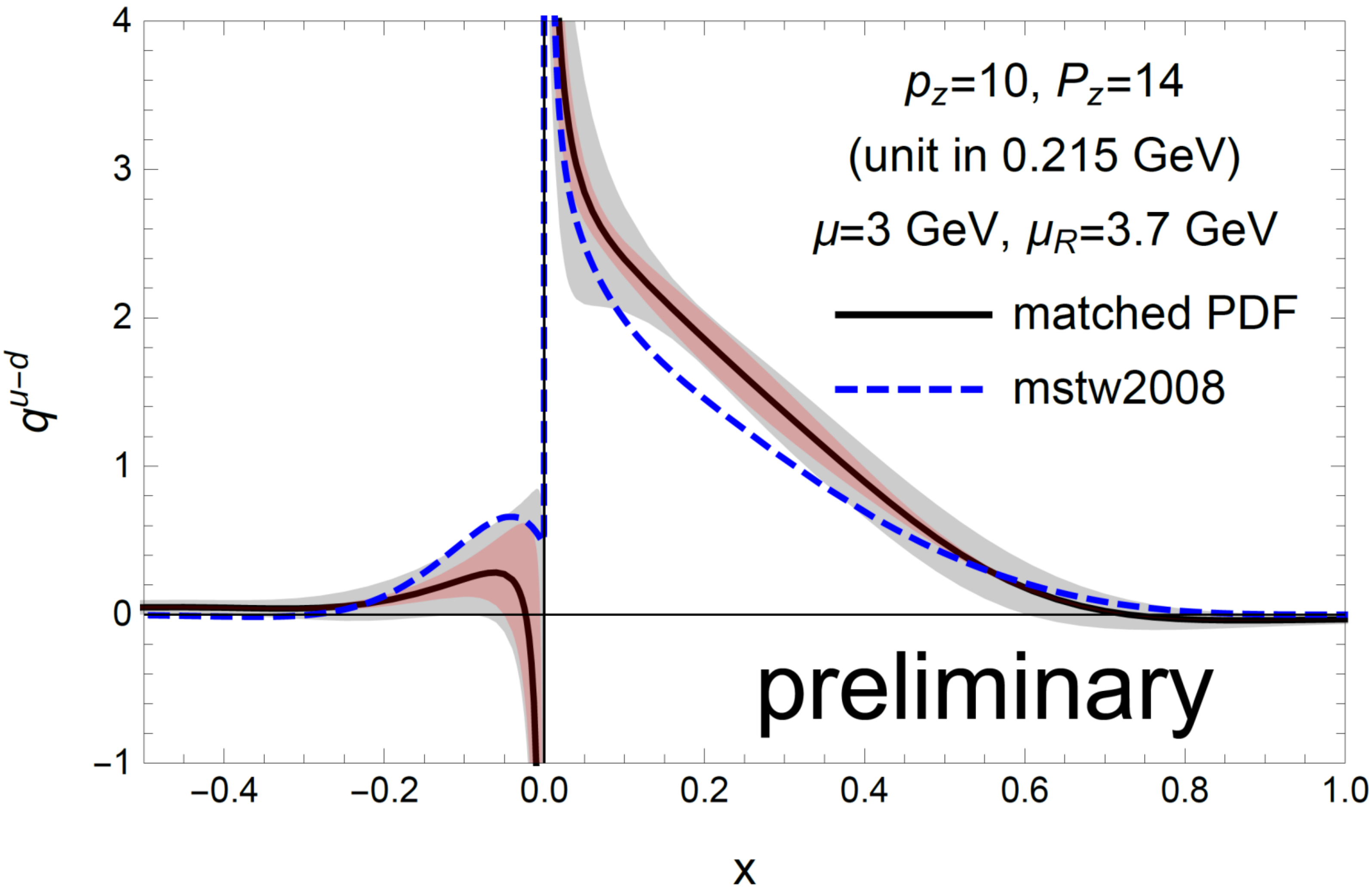
Already have

- Statistial Error
- Excited State Contamination

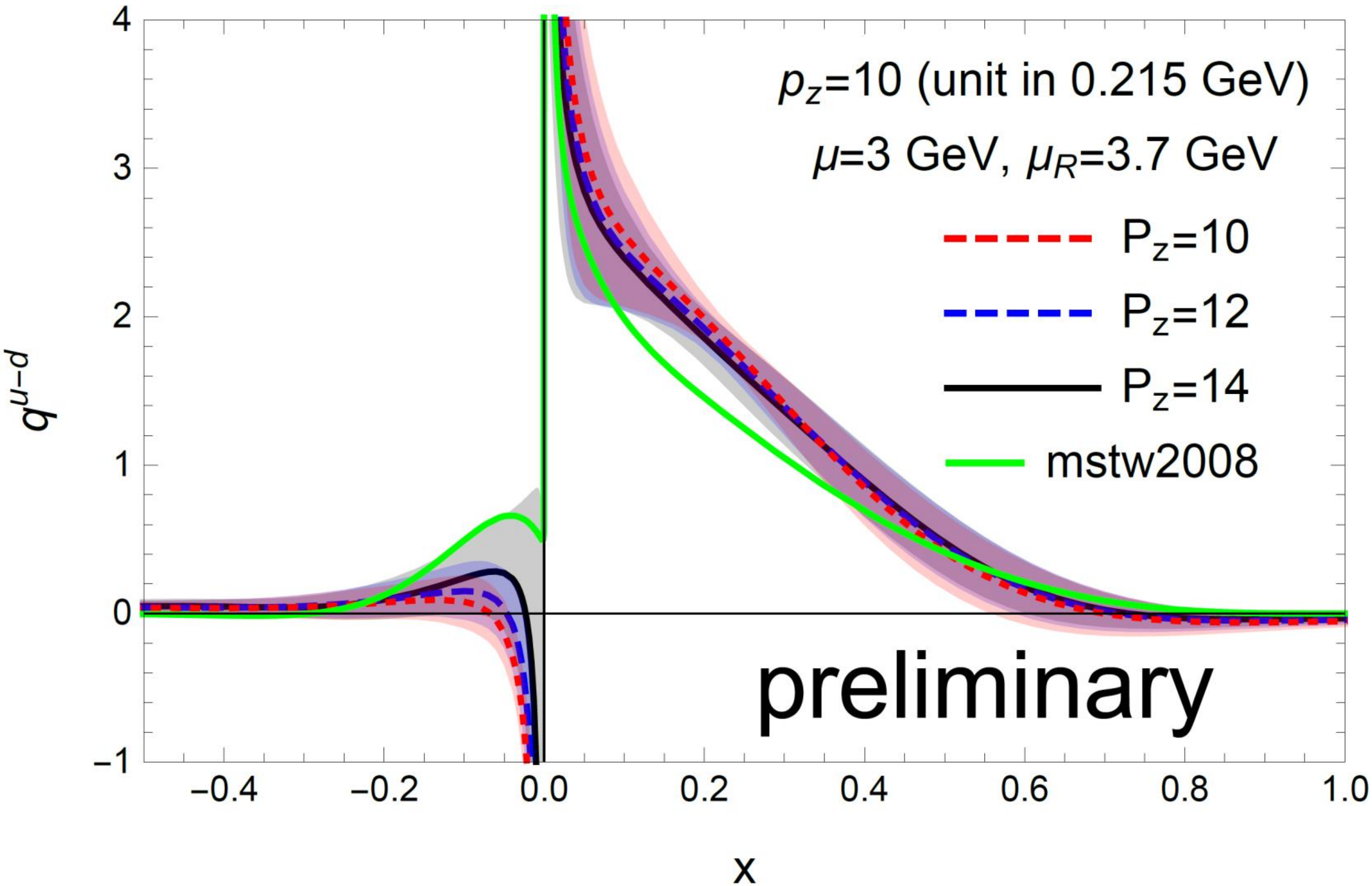
To be included

- One-loop matching error $\mathcal{O}(\alpha_s^2)$
- μ_R dependence
- p_z^R dependence

Matched PDF with Systematics



Different Nucleon Momentum



Future Work

- Finer lattice spacing
- Higher nucleon momentum
- Higher order loop matching kernel
- Other physical quantities