Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point

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Outline of the talk

1. Introduction
   - Basics
   - Lattice techniques
   - Excited states
   - Computational costs
   - Procedure
2. Results
   - Bare ME
   - Renormalized ME
   - Matching
   - Final result
   - Comparison of physical and non-physical m_π
3. Conclusions and prospects

Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, "Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point", arXiv: 1803.02685 [hep-lat]
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (Frontiers Article)
- M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", Phys. Rev. D96 (2017) 054506
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "Updated Lattice Results for Parton Distributions", Phys. Rev. D96 (2017) 014513
- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "A Lattice Calculation of Parton Distributions", Phys. Rev. D92 (2015) 014502

Introduction

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider expriments.
- PDFs have non-perturbative nature $\Rightarrow$ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian – problem for the lattice!

$$q(x) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!
- Accessible on the lattice – moments of the distributions, but ...

Moments of PDFs on the lattice

- Moments of PDFs are defined via matrix elements of local operators:

$$\int dx\, x^{n-1} q(x) = \langle N | \mathcal{O}^{\{\mu_1 \ldots \mu_n\}} | N \rangle,$$

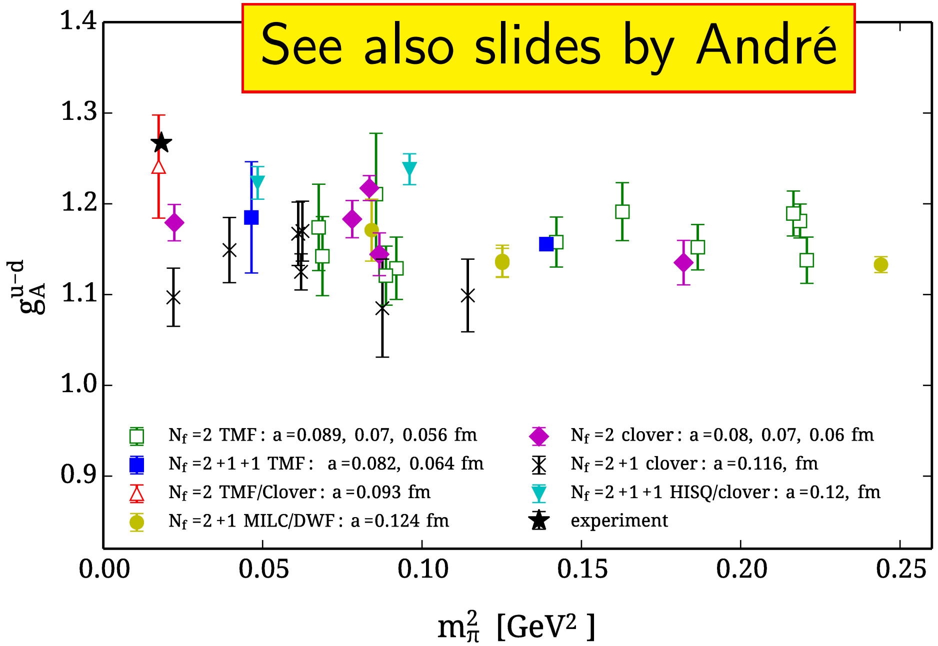
with:
$$\mathcal{O}^{\{\mu_1 \ldots \mu_n\}} = \bar{\psi} \left(\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \ldots i \overleftrightarrow{D}^{\mu_n\}} \right) \frac{\tau^a}{2} \psi.$$

- Example – isovector quark momentum fraction ($q(x) = u(x) - d(x)$):

$$\langle x \rangle_{u-d} = \int dx\, x \, \left(q(x) + \bar{q}(x) \right).$$

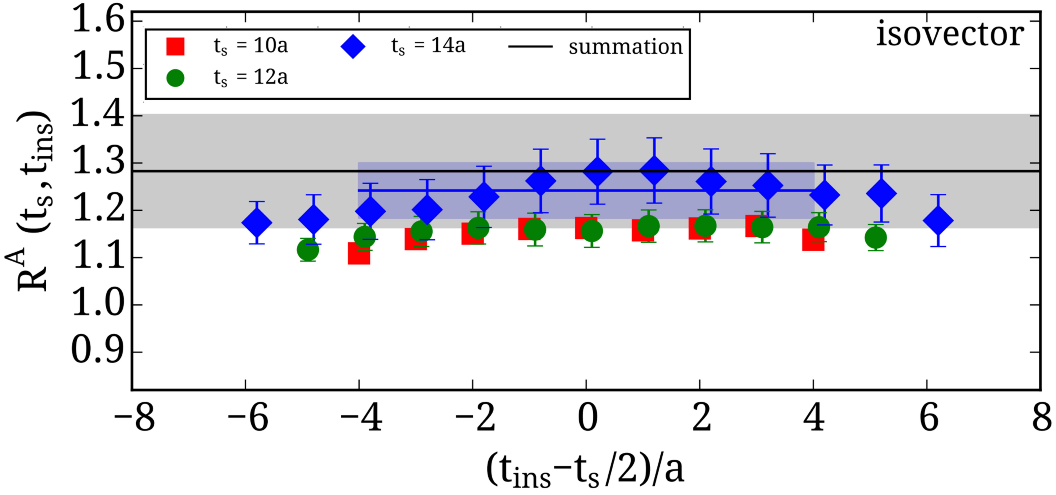
- However, higher moments are difficult for technical reasons:
  - ⋆ higher derivatives noisy,
  - ⋆ operator mixing.

See also slides by André

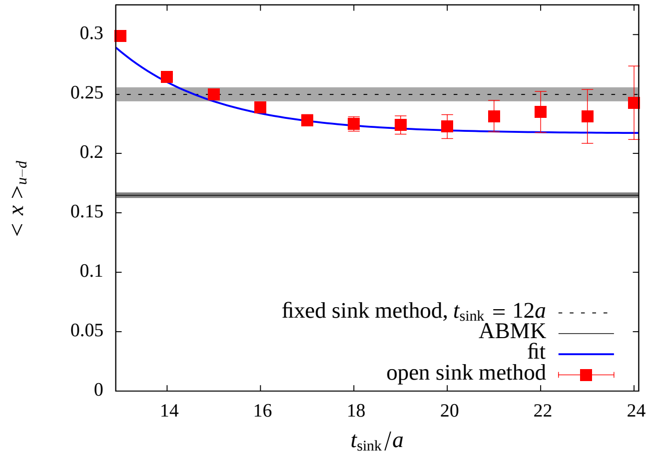


ETMC, C. Alexandrou et al., 1509.04936

There is, however, an important lesson to be learned from moments calculations:

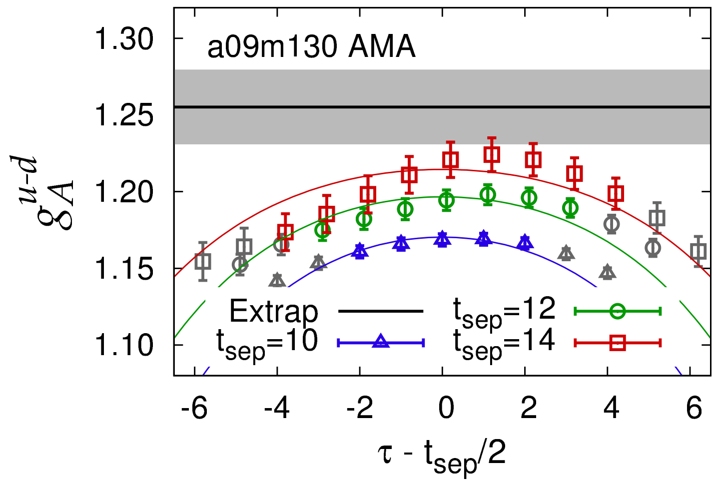


ETMC, C. Alexandrou et al.
Phys. Rev. D93, 039904 (2016)



ETMC, S. Dinter et al.
Phys. Lett. B704, 89 (2011)

- source-sink separation T_s has to be at least 1 fm!
- simultaneous fits to different T_s make sense **only** if one can get the safe=large T_s with similar precision as the lower ones
- else, the simultaneous fit is certainly dominated by the lower T_s



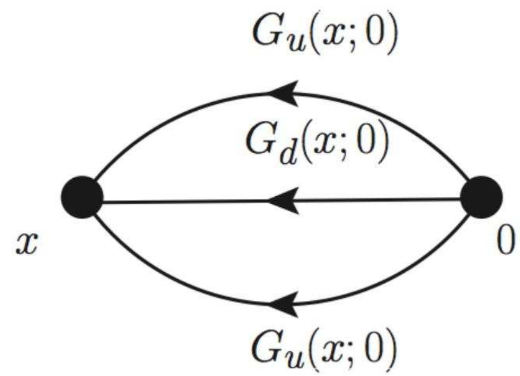
PNDME, T. Bhattacharya et al.
Phys. Rev. D94, 054508 (2016)

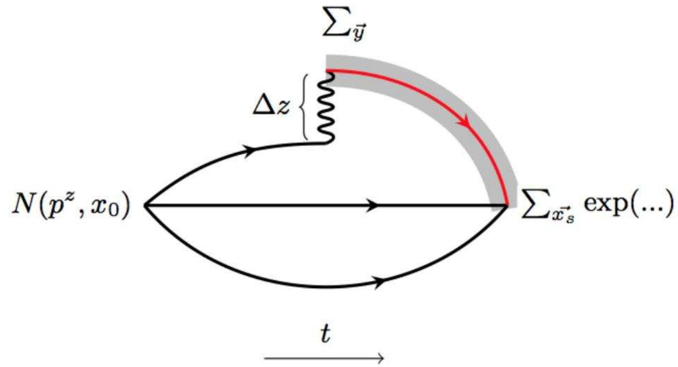
See also slides by André

- Quasi-PDF approach:
  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002
- Compute a **quasi distribution** $\tilde{q}$, which is purely spatial and uses nucleons with finite momentum:

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{ixP_3z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$

- z – distance in any *spatial* direction z,
- P_3 – momentum boost in this direction.
- e.g. $\Gamma=\gamma_0,\ \gamma_3$ – unpolarized, $\Gamma=\gamma_5\gamma_3$ – helicity
- Theoretically very appealing and intuitive!
- Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2},\frac{m_N^2}{P_3^2}\right)$.
- The highly non-trivial aspect:
  how to relate $\tilde{q}(x,\mu^2,P_3)$ to the light-front PDF $q(x,\mu^2)$ (infinite momentum frame)
  $\Longrightarrow$ **LaMET**





Beautiful idea and solid theoretical framework!

BUT: lattice realization far from trivial!

- Signal for the relevant nucleon 2-pt and 3-pt function depends on:
  - ⋆ nucleon momentum P_3 – exponentially decaying with P_3!
  - ⋆ source-sink separation T_s – exponentially decaying with T_s!
  - ⋆ quark mass – worsens for smaller masses.
- Many systematics to control!

HENCE: Choice of the pairs (T_s, P_3) is **crucial!**

Lattice setup

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

β=2.10,	$c_{\rm SW}$=1.57751,	a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

Wilson twisted mass fermions

The Wilson twisted mass fermion action for the 2 light (u, d quarks) is given in the so-called twisted basis by: [R. Frezzotti, P. Grassi, G.C. Rossi, S. Sint, P. Weisz, 2000-2004]

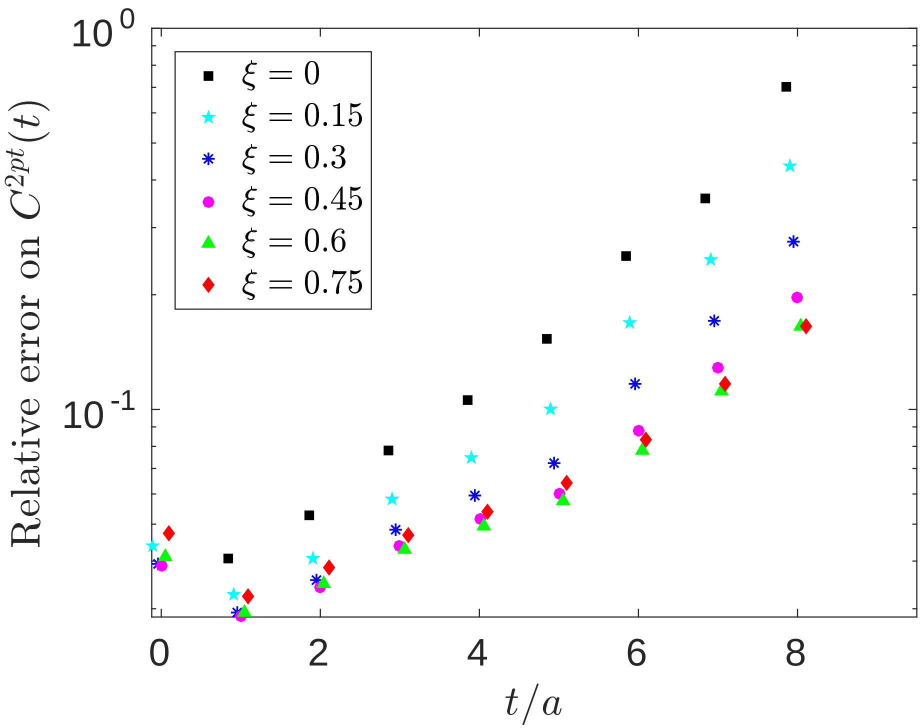
$$S_l[\psi, \bar{\psi}, U] = a^4 \sum_x \bar{\chi}_l(x) \big(D_W + m_{0,l} + i\mu_l \gamma_5 \tau_3 \big) \chi_l(x),$$

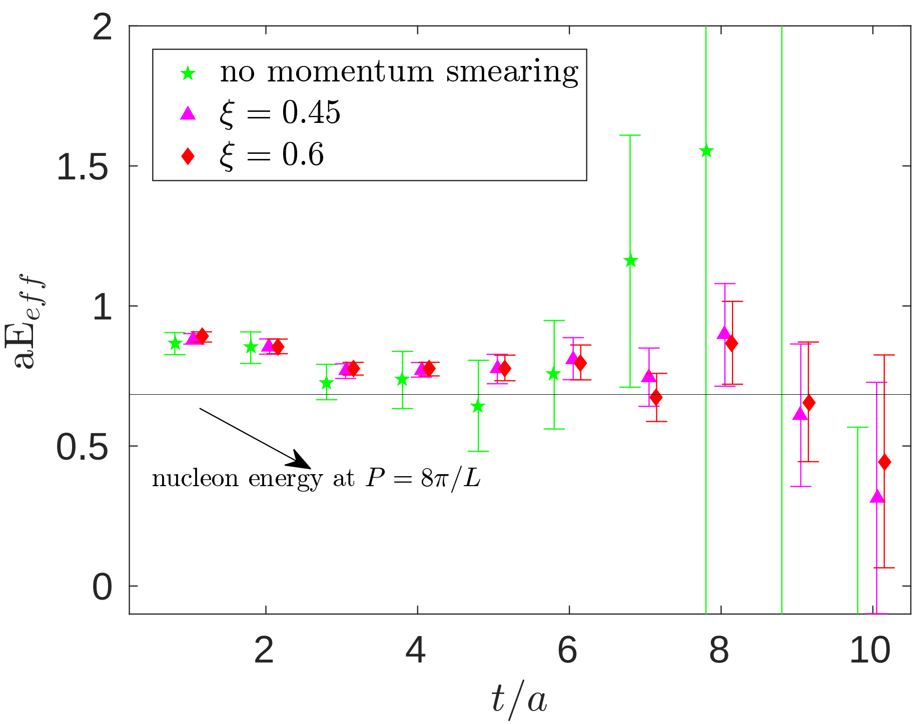
- D_W – Wilson-Dirac operator,
- $m_{0,l}$ and μ_l – bare untwisted and twisted light quark masses,
- $\chi_l = (\chi_u, \chi_d)$ – 2-component vector in flavor space; chiral rotation of standard one: $\psi = e^{i\gamma_5 \tau_3 \omega/2} \chi$
- Maximal twist: $\omega = \pi/2$ by tuning the PCAC mass to zero $\Rightarrow$ automatic $\mathcal{O}(a)$-improvement.

Momentum smearing

$$S_{\text{mom}}\psi(x) = \frac{1}{1+6\alpha}\left(\psi(x) + \alpha\sum_{j=\pm1}^{\pm3} U_j(x)e^{i\xi\hat{j}}\psi(x+\hat{j})\right)$$

G. Bali et al., Phys. Rev. D93, 094515 (2016)





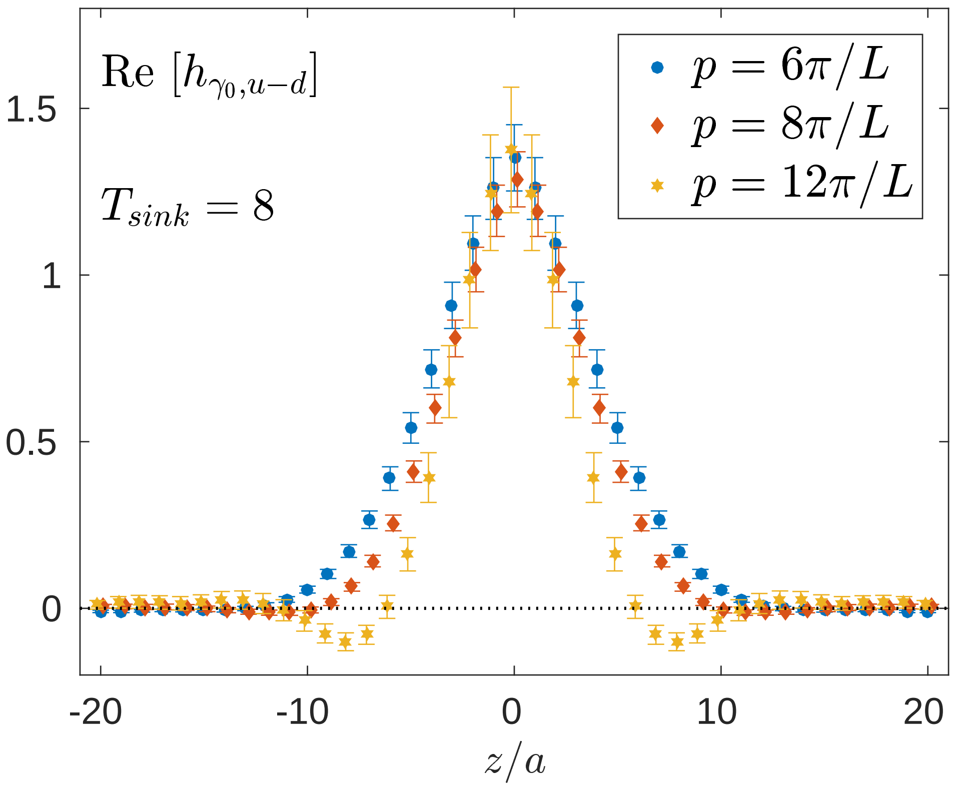
50 iterations of (Gaussian) momentum smearing, $\alpha = 4$

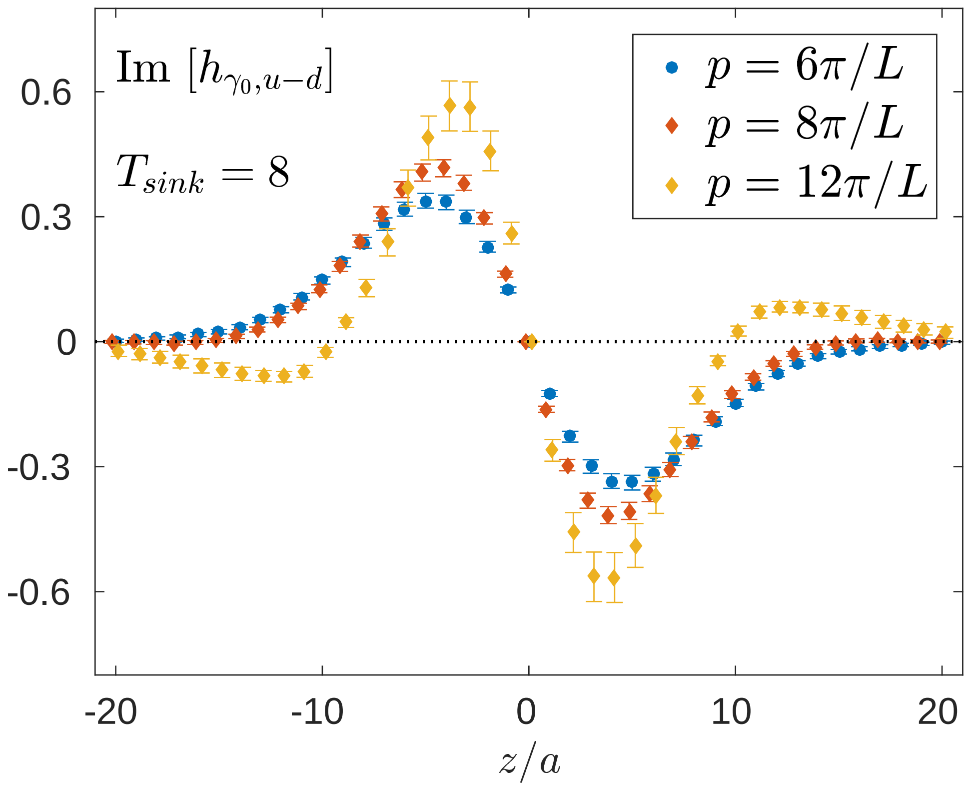
For each gauge field configuration, we use:

- 6 directions of Wilson line: $\pm x, \pm y, \pm z$
- 16 source positions:
  - ⋆ 1 high precision (HP) inversion
  - ⋆ 16 low precision (LP) inversions
- Bias from the LP inversions corrected using the Covariant Approximation Averaging technique (CAA)
  E. Shintani et al., Phys. Rev. D91, 114511 (2015)

Crucial thing: choice of nucleon momenta
Needs careful choice of source-sink separation

Source-sink separation $T_s = 8a \approx 0.75$ fm

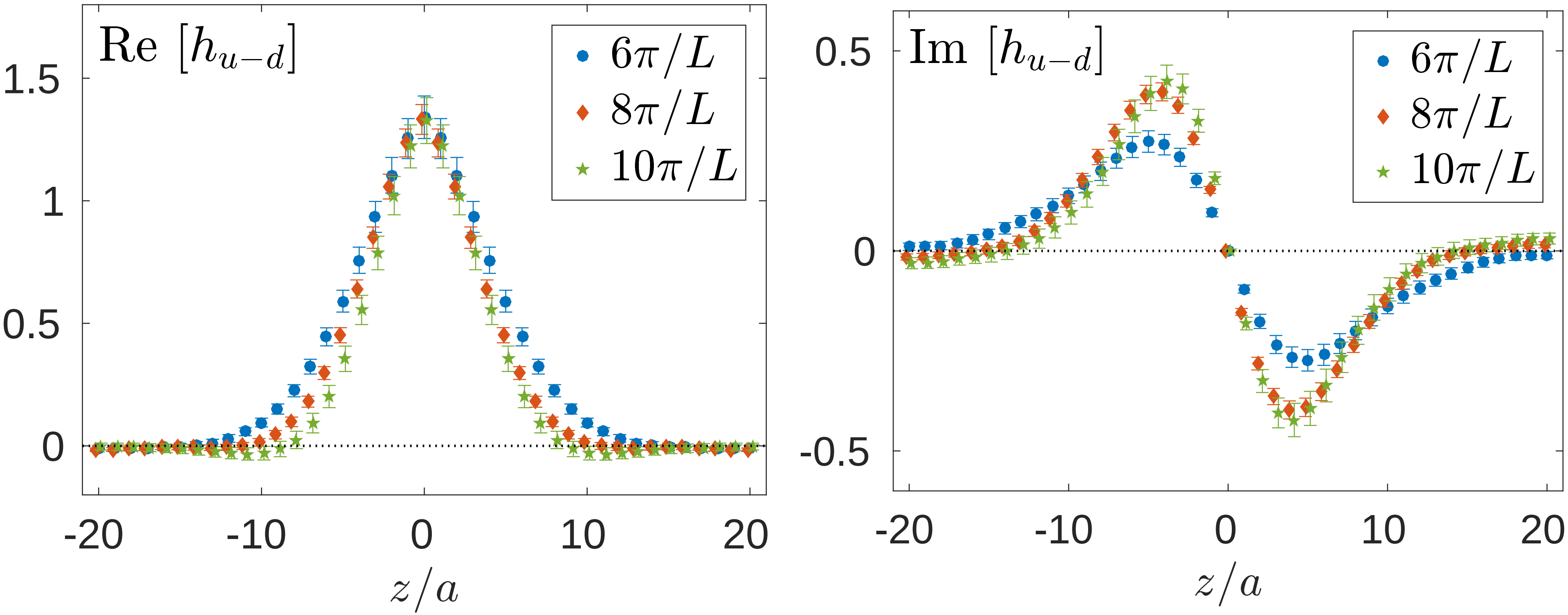




Huge excited states effects at large momenta!

See also slides by Jeremy

Source-sink separation $T_s = 12a \approx 1.13$ fm

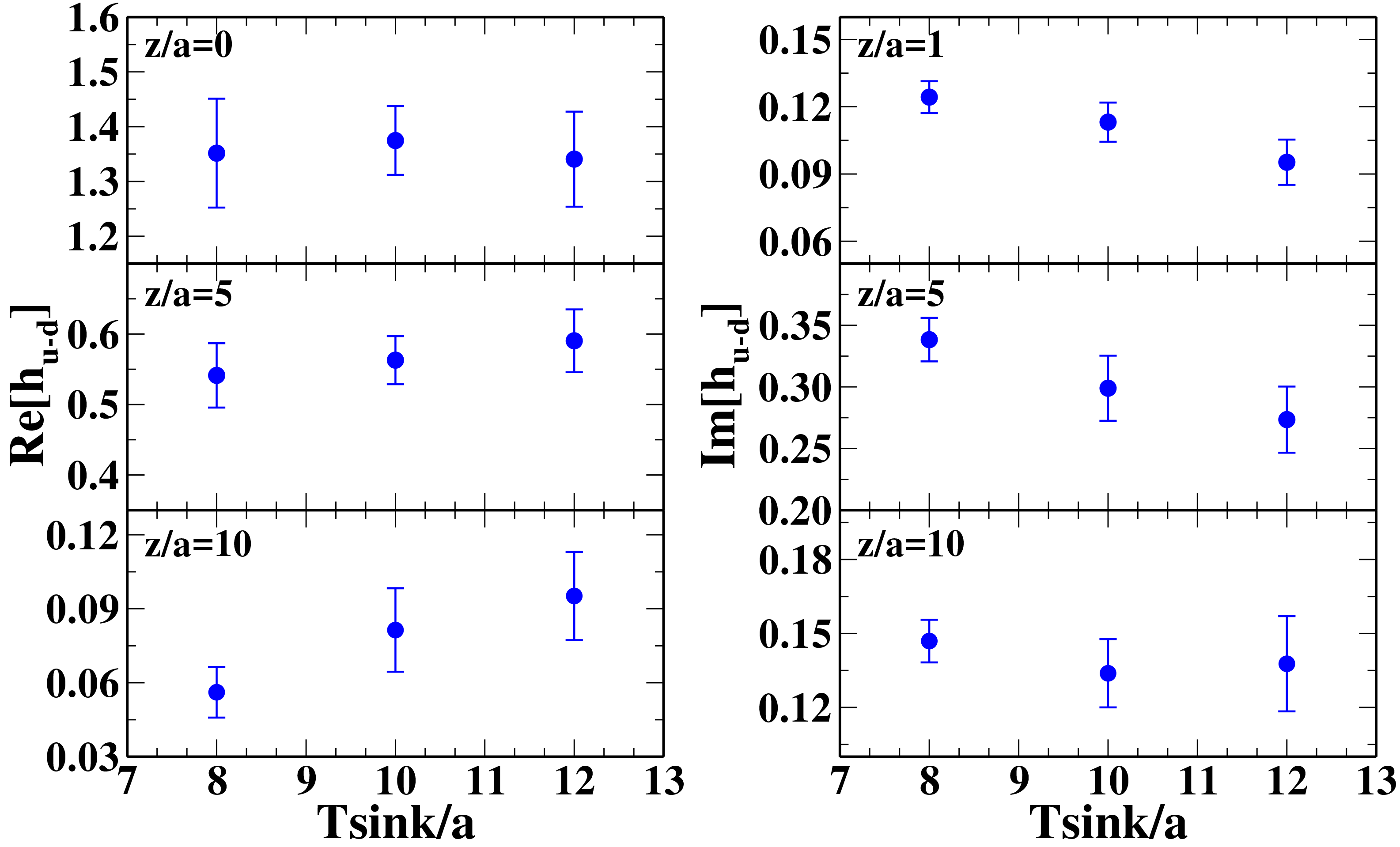


Excited states effects seem to be under control!

See also slides by Jeremy

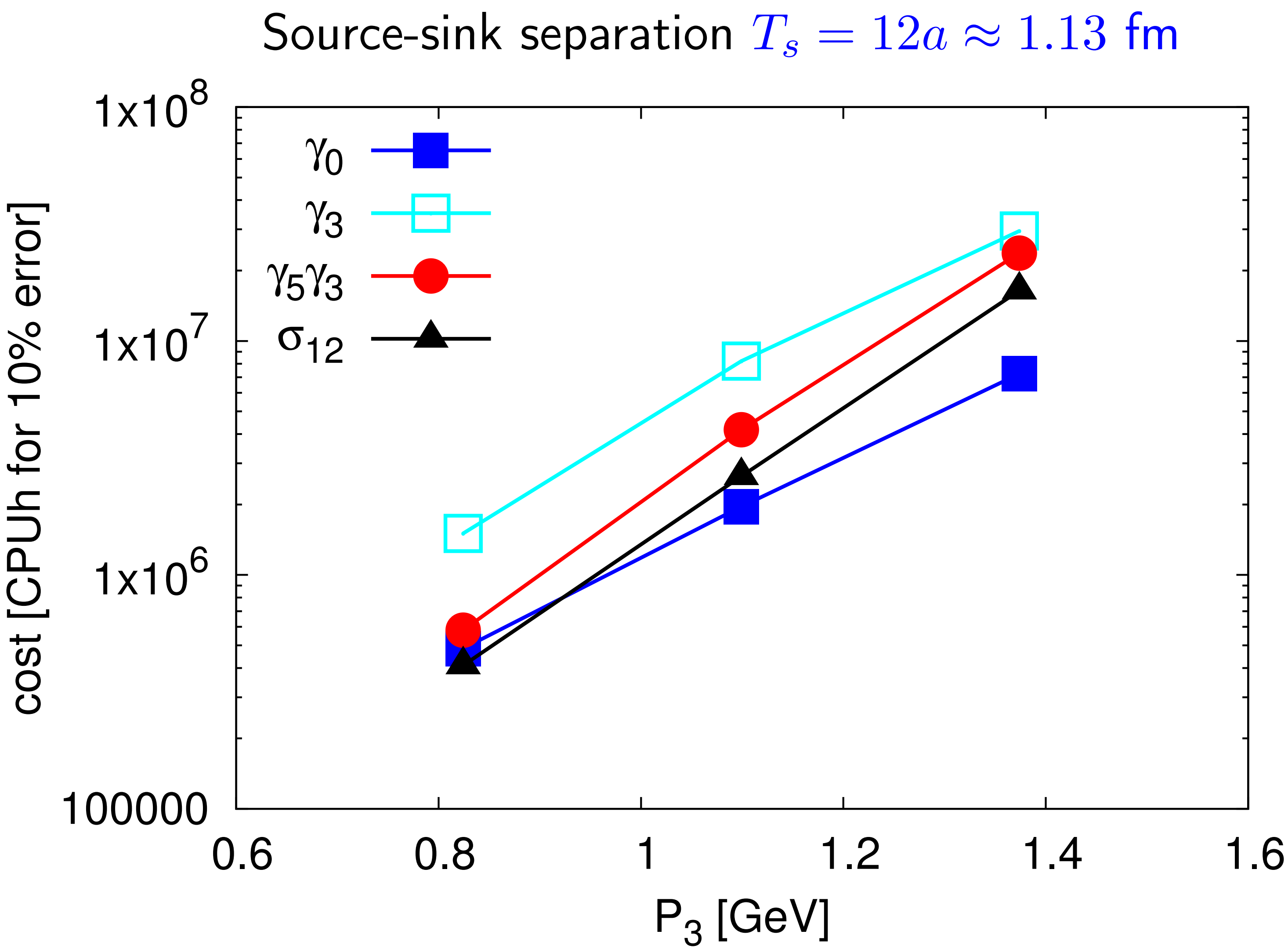
Source-sink separation

Nucleon momentum $\frac{6\pi}{48} \approx 0.83$ GeV



Certain regions of z are very much affected by excited states!

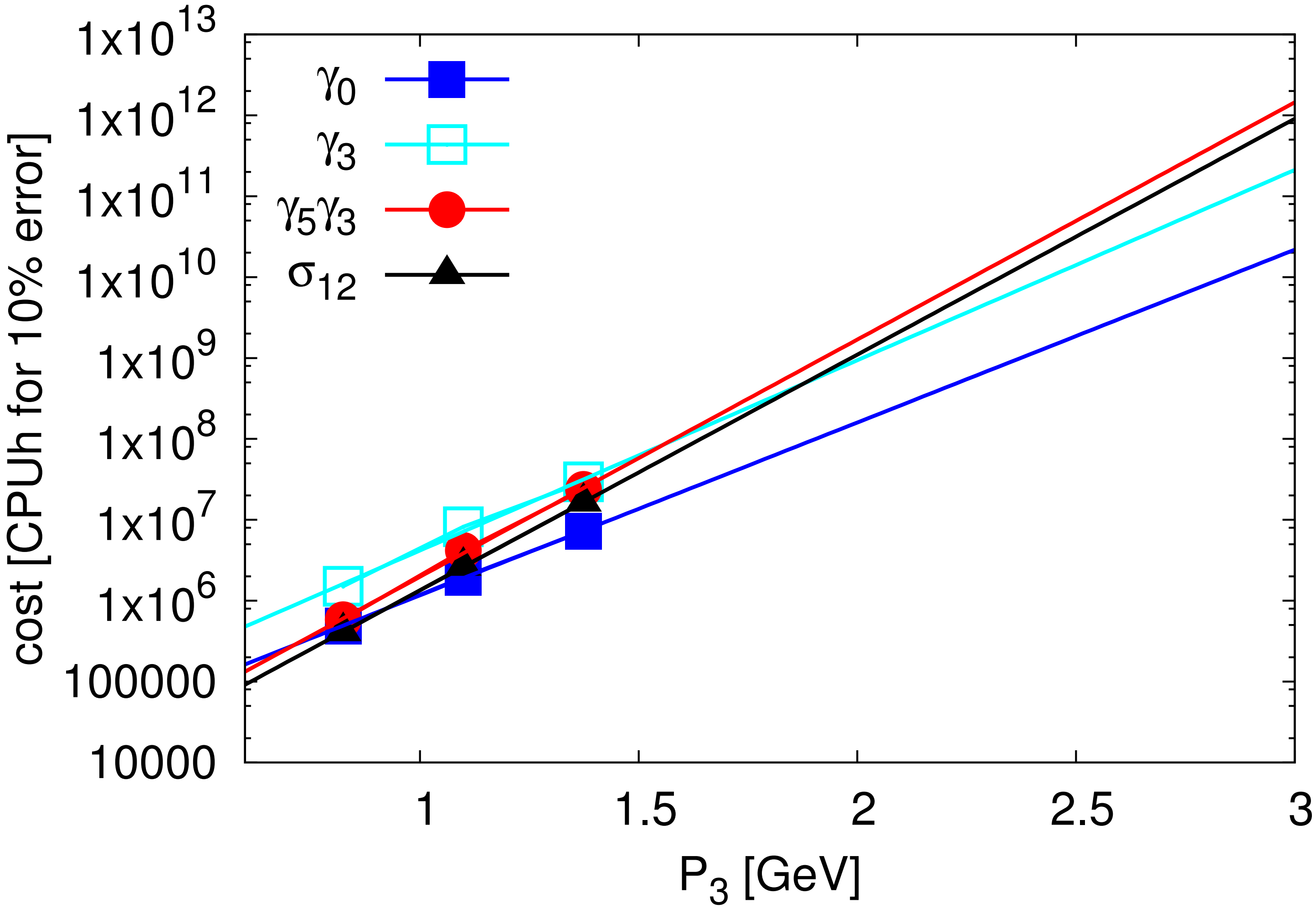
Cost of the computation



Reaching 1.5 GeV @ $T_s \approx 1.1$ fm needs already $\mathcal{O}(20)$ million CPUh

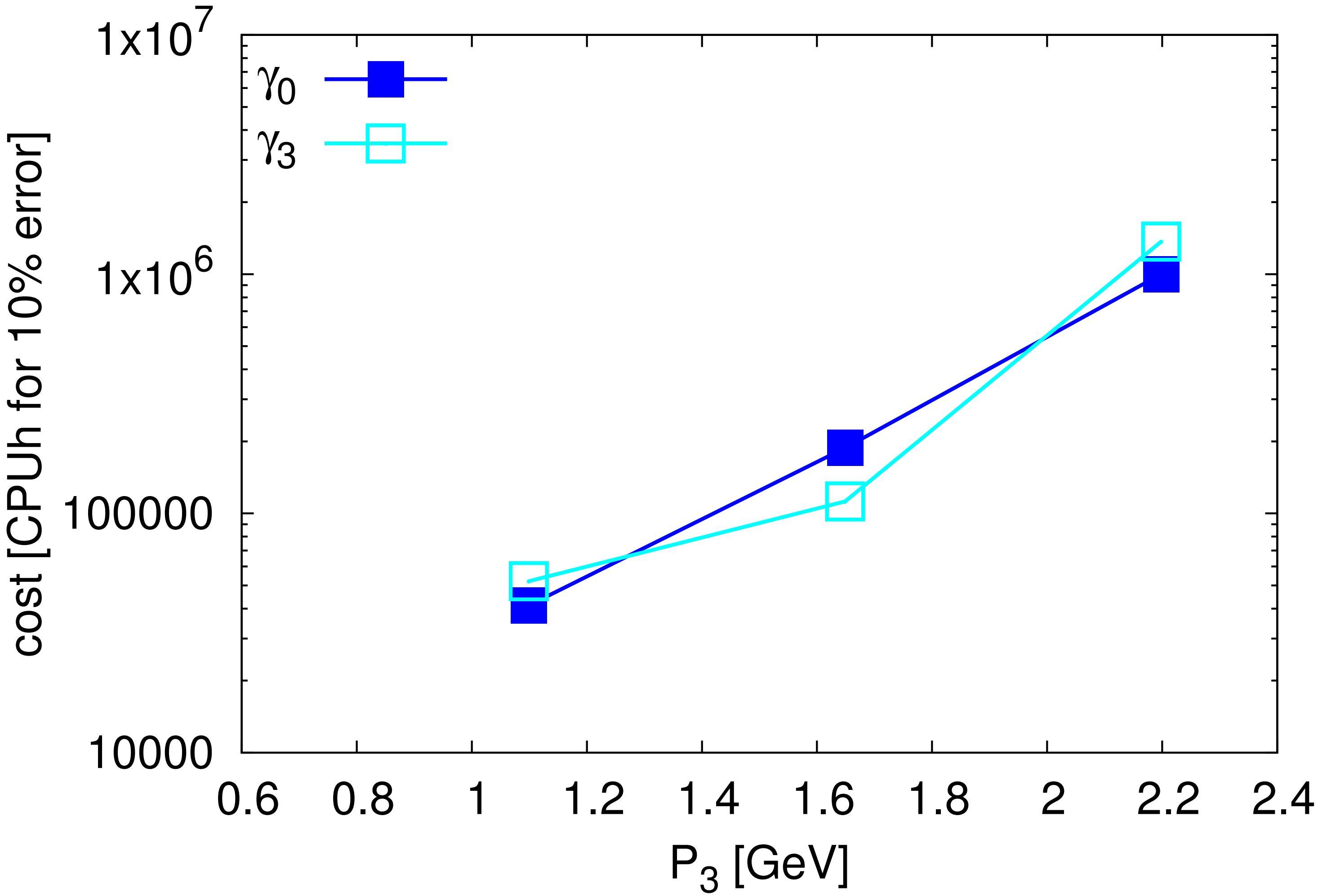
Cost extrapolation to 3 GeV

Source-sink separation $T_s = 12a \approx 1.13$ fm



Going to 3 GeV @ $T_s \approx 1.1$ fm needs already TENS/HUNDREDS THOUSAND million CPUh

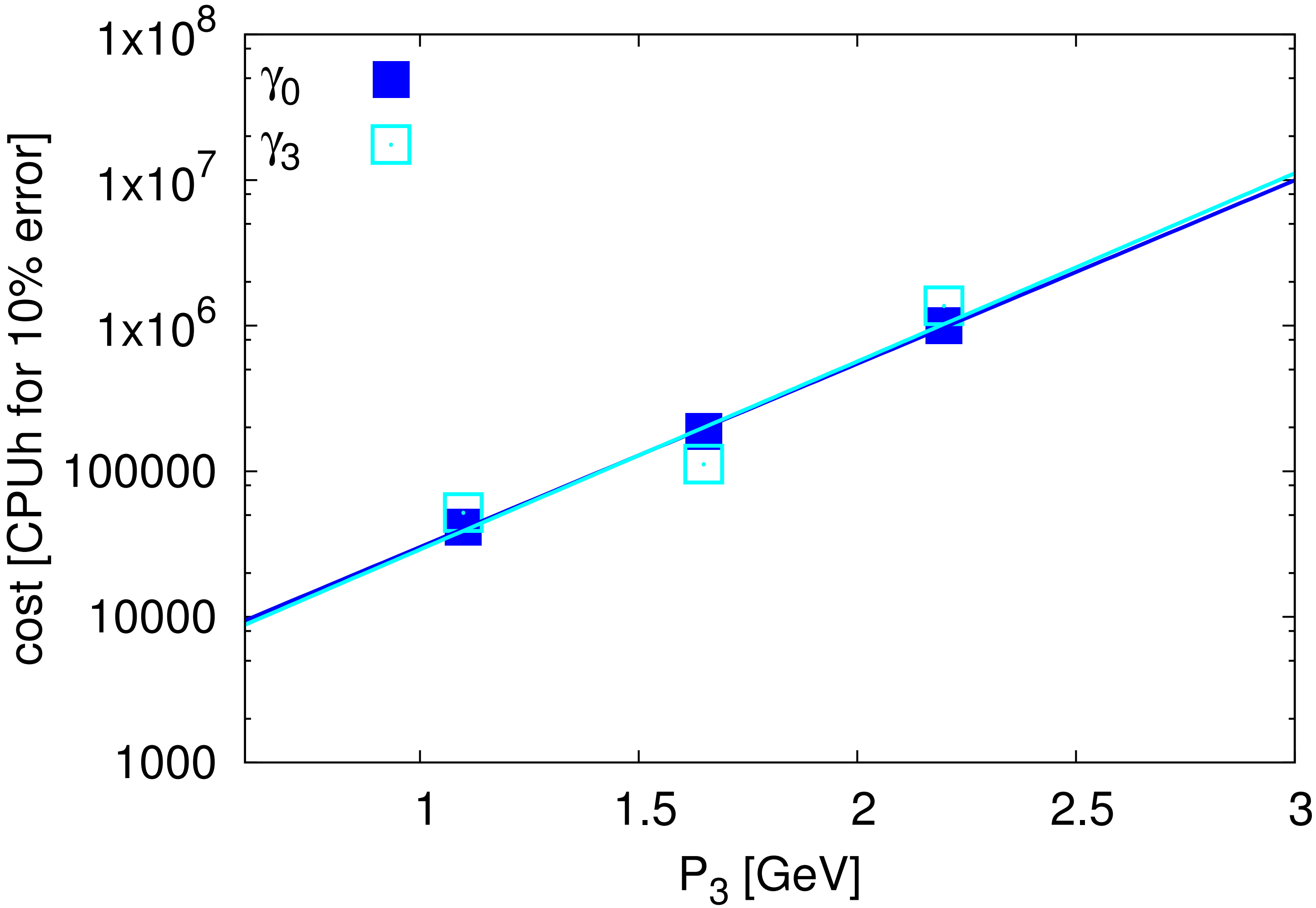
Source-sink separation $T_s = 8a \approx 0.75$ fm



Reaching 2.2 GeV @ $T_s \approx 0.75$ fm pretty cheap – $\mathcal{O}(1)$ million CPUh

Cost extrapolation to 3 GeV – lower T_s

Source-sink separation $T_s = 8a \approx 0.75$ fm



Going to 3 GeV @ $T_s \approx 0.75$ fm feasible – $\mathcal{O}(10)$ million CPUh.

BUT: definitely too large excited states contamination

Conclusion from this

- Elimination of excited states must not be compromised – reaching really large momenta **extremely difficult** if one takes excited states seriously.
- Note that the log-linear extrapolation of the cost is likely to underestimate this cost.
- Momentum smearing technique is **extremely useful**, but it does not kill the exponential signal-to-noise problem.
- It moves it to somewhat higher momenta:
  - ⋆ without it, momentum 0.8-0.9 GeV at $T_s \approx 1.1$ fm becomes the borderline (tens of million CPUh),
  - ⋆ with it, the same cost makes 1.4-1.5 GeV reachable.
- Key aspect for the future: how to tackle the signal-to-noise problem at safe source-sink separations.

Our choice of nucleon momenta

- In our work, we decided to learn from:
  - ⋆ the many-year effort to compute moments on the lattice,
  - ⋆ tests of 3 source-sink separations: $T_s \approx 0.75, 0.94, 1.13$ fm.
- $T_s \approx 1.1$ fm seems to be the lowest justifiable choice, i.e. it should be safe from excited states at the $\sim 10\%$ level.
- With $T_s \approx 0.75$ fm, excited states totally uncontrolled (20%, 30%, 50% ???) – may affect different x-ranges in a different way.
- Simultaneous fit of different T_s? Makes sense only if similar statistical precision of all $T_s \Longrightarrow$ impossible here without investing resources beyond the current computing capabilities.
- Hence, given these capabilities, we take:
  - ⋆ $aP_3 = 6\pi/48 \Rightarrow P_3 \approx 0.83$ GeV
  - ⋆ $aP_3 = 8\pi/48 \Rightarrow P_3 \approx 1.11$ GeV
  - ⋆ $aP_3 = 10\pi/48 \Rightarrow P_3 \approx 1.38$ GeV

  all at $T_s \approx 1.13$ fm

Statistics

$P_3 = \frac{6\pi}{L}$			$P_3 = \frac{8\pi}{L}$			$P_3 = \frac{10\pi}{L}$		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_0	50	4800	γ_0	425	38250	γ_0	655	58950
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	655	58950

Summary of the procedure

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Results

Step 1

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
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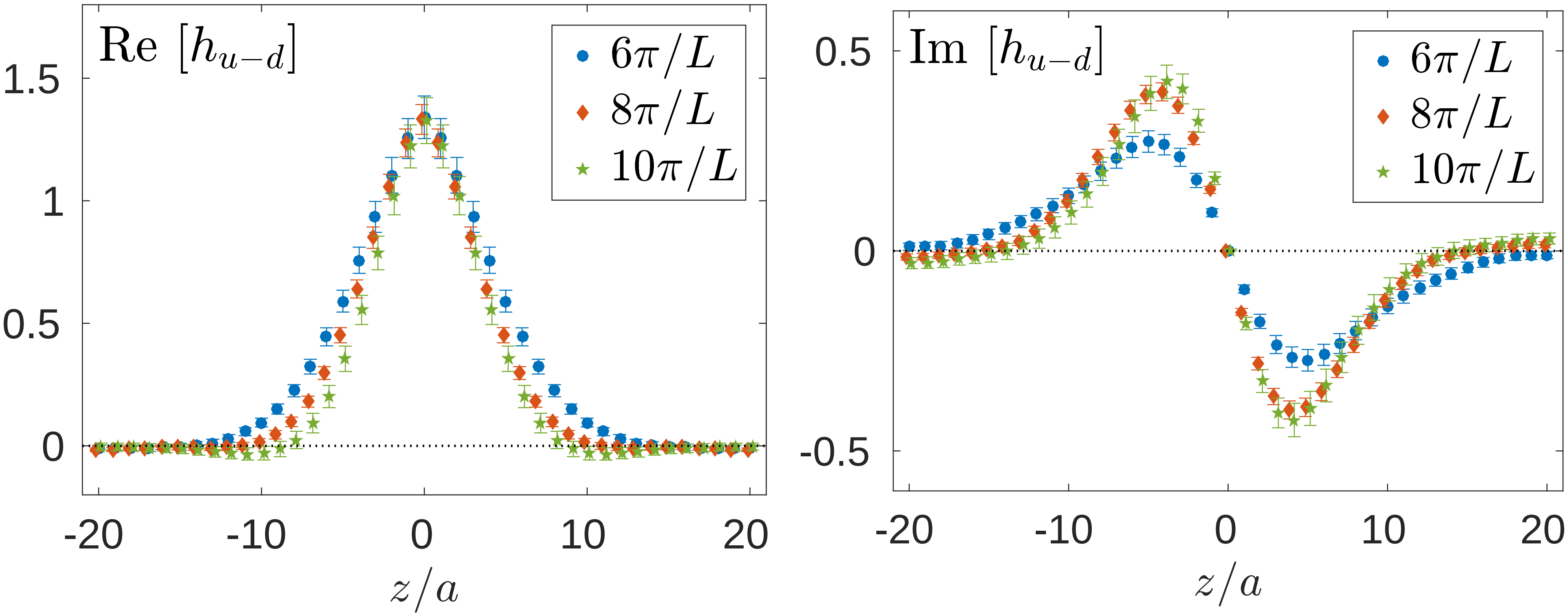
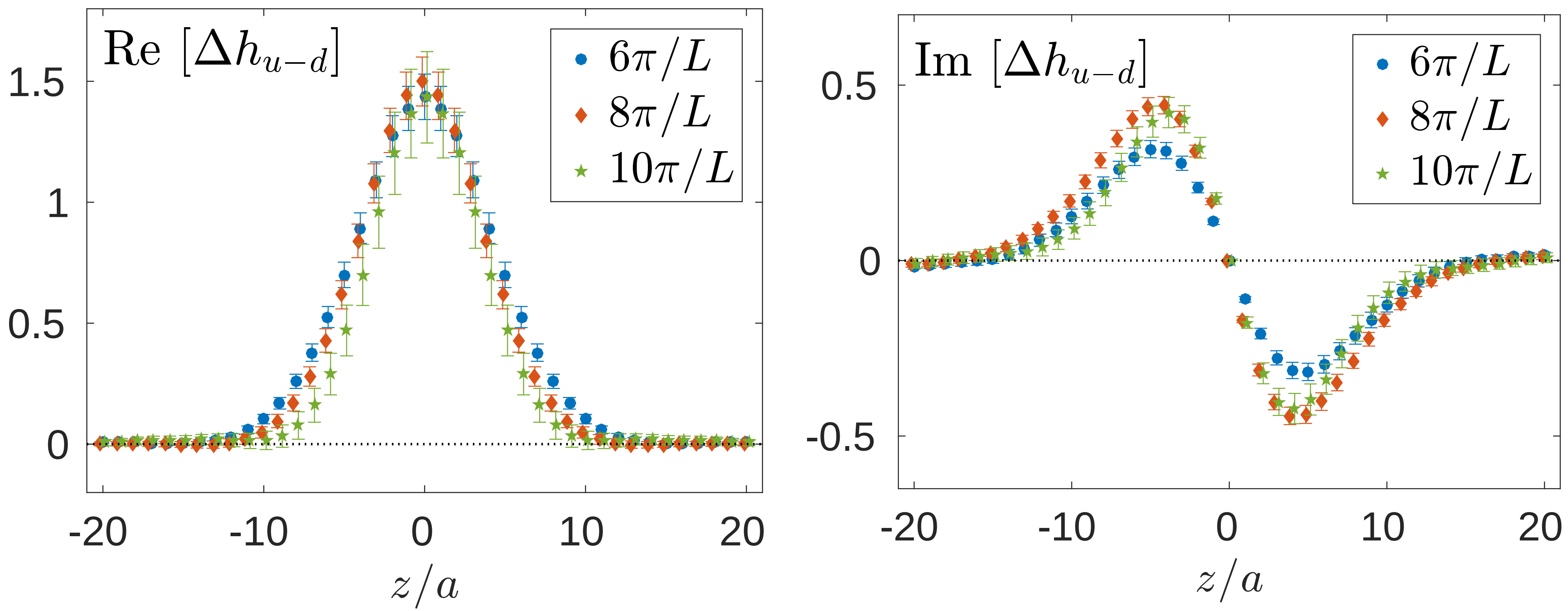
Bare matrix elements for unpolarized PDFs



Bare matrix elements for helicity PDFs



Steps 2-4

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV. See slides by Martha
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Renormalization

Bare matrix elements $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$ contain divergences that need to be removed:

- standard logarithmic divergence with respect to the regulator, $\log(a\mu)$,
- power divergence related to the Wilson line; resums into a multiplicative exponential factor, $\exp\left(-\delta m|z|/a + c|z|\right)$
  δm – strength of the divergence, operator independent,
  c – arbitrary scale (to be fixed by the renormalization prescription).

See slides by Martha

Proposed renormalization programme described in:
C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", Nucl. Phys. B923 (2017) 394-415 (Frontiers Article)

Important insights also from the lattice perturbative paper:
M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", Phys. Rev. D96 (2017) 054506
$\rightarrow$ discovered mixing between the vector and scalar matrix elements (unpolarized PDF). **This perturbative analysis is very important guidance to non-perturbative renormalization!**

Non-perturbative renormalization scheme: RI'-MOM.
G. Martinelli et al., Nucl. Phys. B445 (1995) 81

See slides by Martha

RI'-MOM renormalization conditions (for cases without mixing):
for the operator:

$$Z_q^{-1}\, Z_{\mathcal{O}}(z)\, \frac{1}{12} \mathrm{Tr} \left[\mathcal{V}(p,z) \left(\mathcal{V}^{\mathrm{Born}}(p,z) \right)^{-1} \right] \Big|_{p^2=\bar{\mu}_0^2} = 1\,,$$

for the quark field:

See slides by Martha

$$Z_q = \frac{1}{12} \mathrm{Tr} \left[(S(p))^{-1}\, S^{\mathrm{Born}}(p) \right] \Big|_{p^2=\bar{\mu}_0^2}\,.$$

- momentum p in the vertex function is set to the RI′ renormalization scale $\bar{\mu}_0$
- $\mathcal{V}(p,z)$ – amputated vertex function of the operator,
- $\mathcal{V}^{\mathrm{Born}}$ – its tree-level value, $\mathcal{V}^{\mathrm{Born}}(p,z) = i\gamma_3\gamma_5\, e^{ipz}$ for helicity,
- $S(p)$ – fermion propagator ($S^{\mathrm{Born}}(p)$ at tree-level).

This prescription handles all divergences that are present and applies the necessary finite renormalization related to the lattice regularization.

Stout smearing

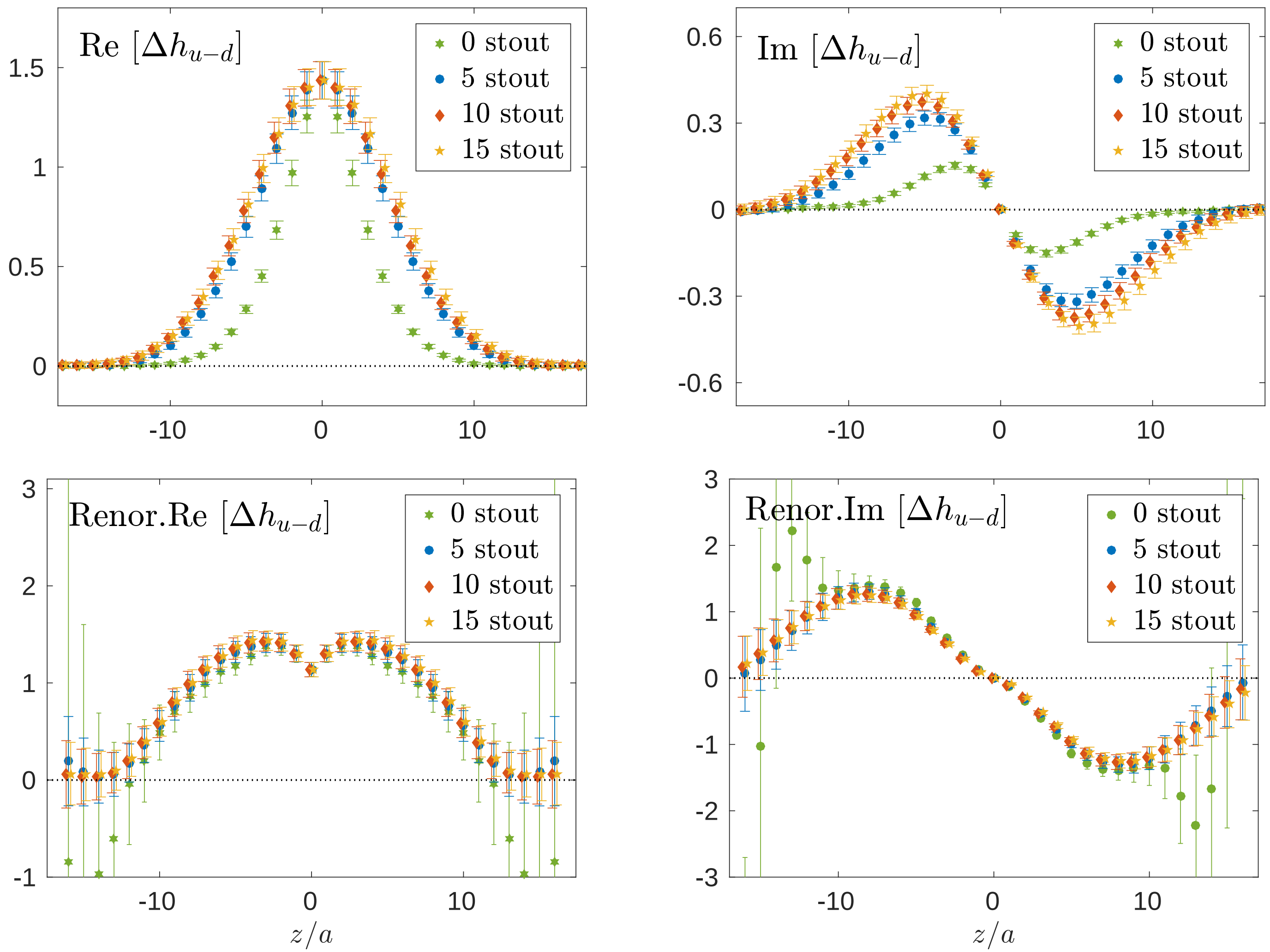
- The power divergence related to the Wilson line makes the values of Z-factors very large at large lengths.
- Hence, we mildly smoothen the divergence by applying stout smearing **only to the Wilson line**.
- Note: we do not apply it to the Dirac operator – potentially dangerous procedure.
- We test:
  - ⋆ 5 stout smearing steps
  - ⋆ 10 stout smearing steps
  - ⋆ 15 stout smearing steps

See also slides by Martha

- This influences both bare matrix elements and the values of Z-factors.
- **But:** renormalized matrix elements should be **independent** of the number of stout steps!

Renormalized matrix elements for helicity PDFs

Nucleon momentum $\frac{6\pi}{48}$



Important self-consistency check for the renormalization procedure!

See also slides by Martha

Steps 5-6

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
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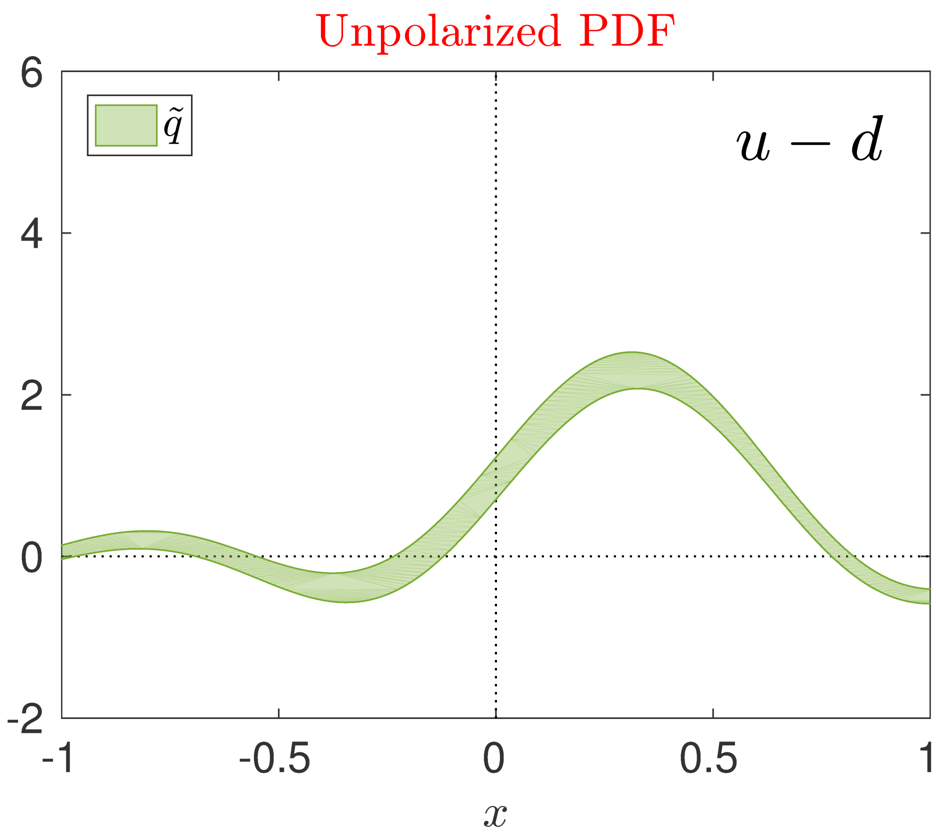
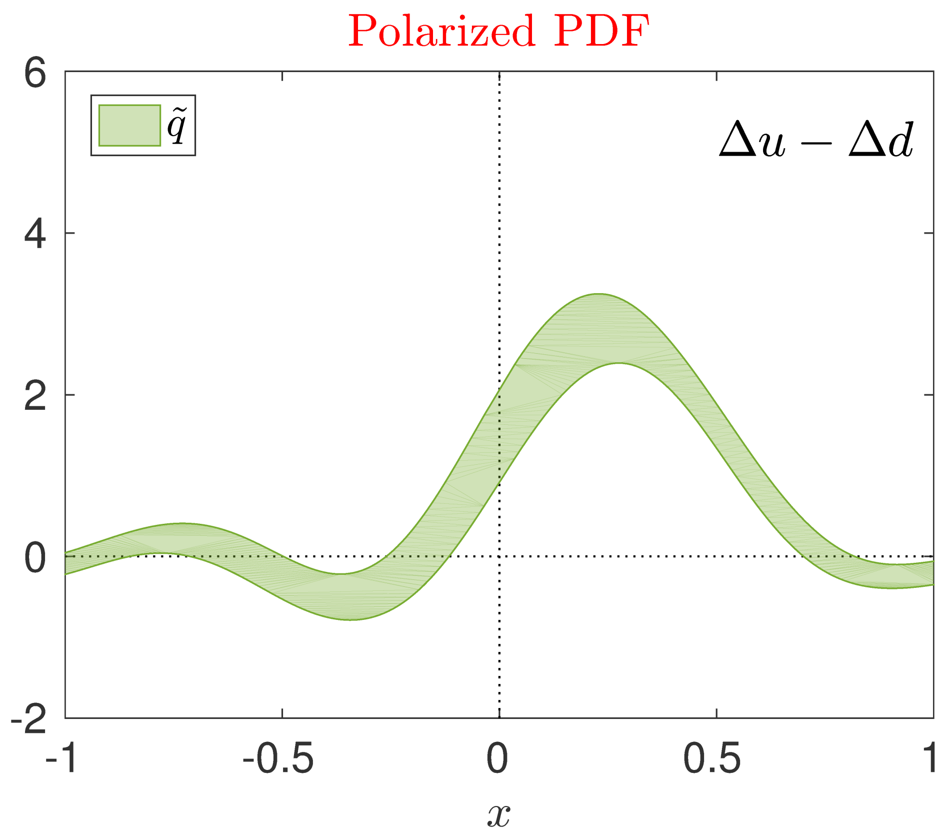
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.

   See slides by Fernanda

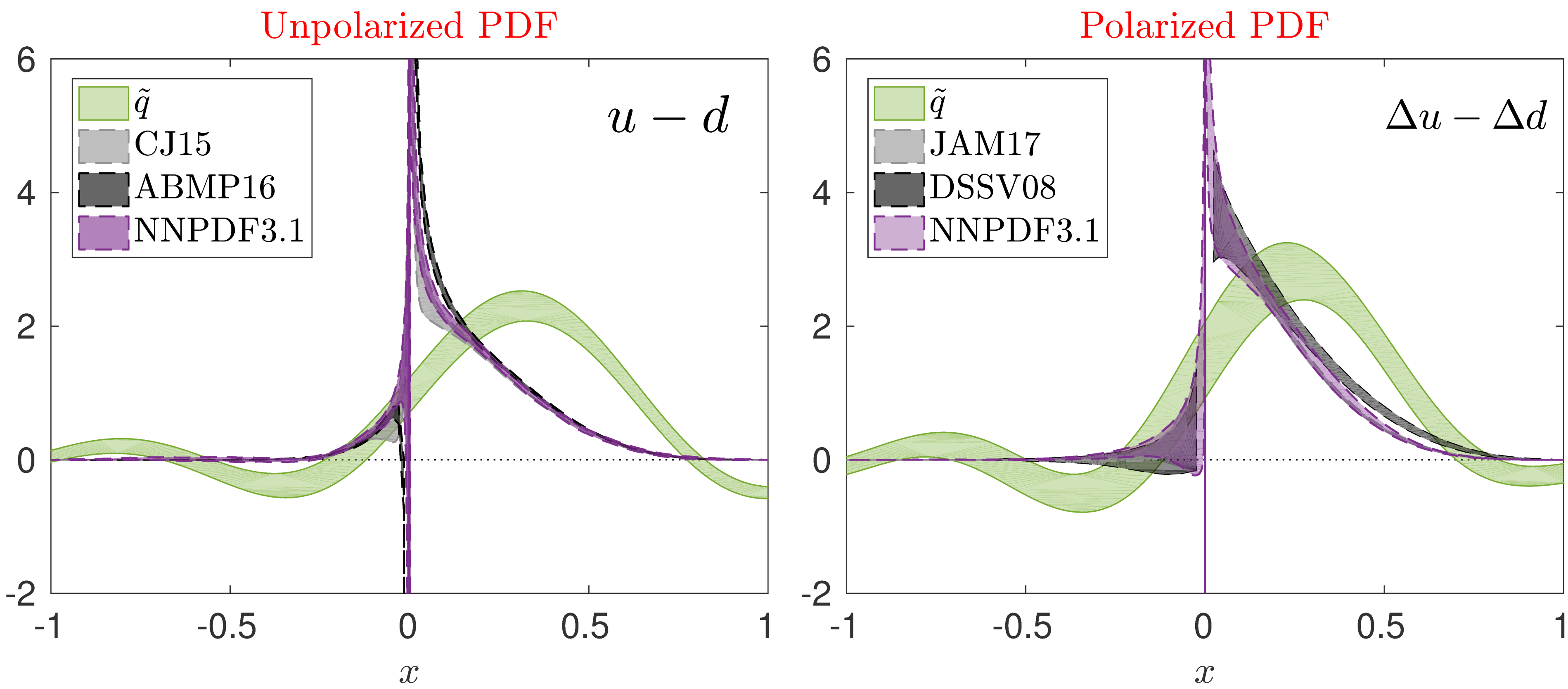
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Nucleon momentum $\frac{10\pi}{48}$





Nucleon momentum $\frac{10\pi}{48}$



The matching formula can be expressed as:

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C – matching kernel:

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi > 1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln\frac{x^2P_3^2}{\xi^2\mu^2}(4\xi(1-\xi)) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0<\xi<1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln\frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi < 0, \end{cases}$$

[T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., arXiv:1803.02685 [hep-lat]]

$\iota{=}0$ for γ_0 and $\iota{=}1$ for $\gamma_3/\gamma_5\gamma_3$.

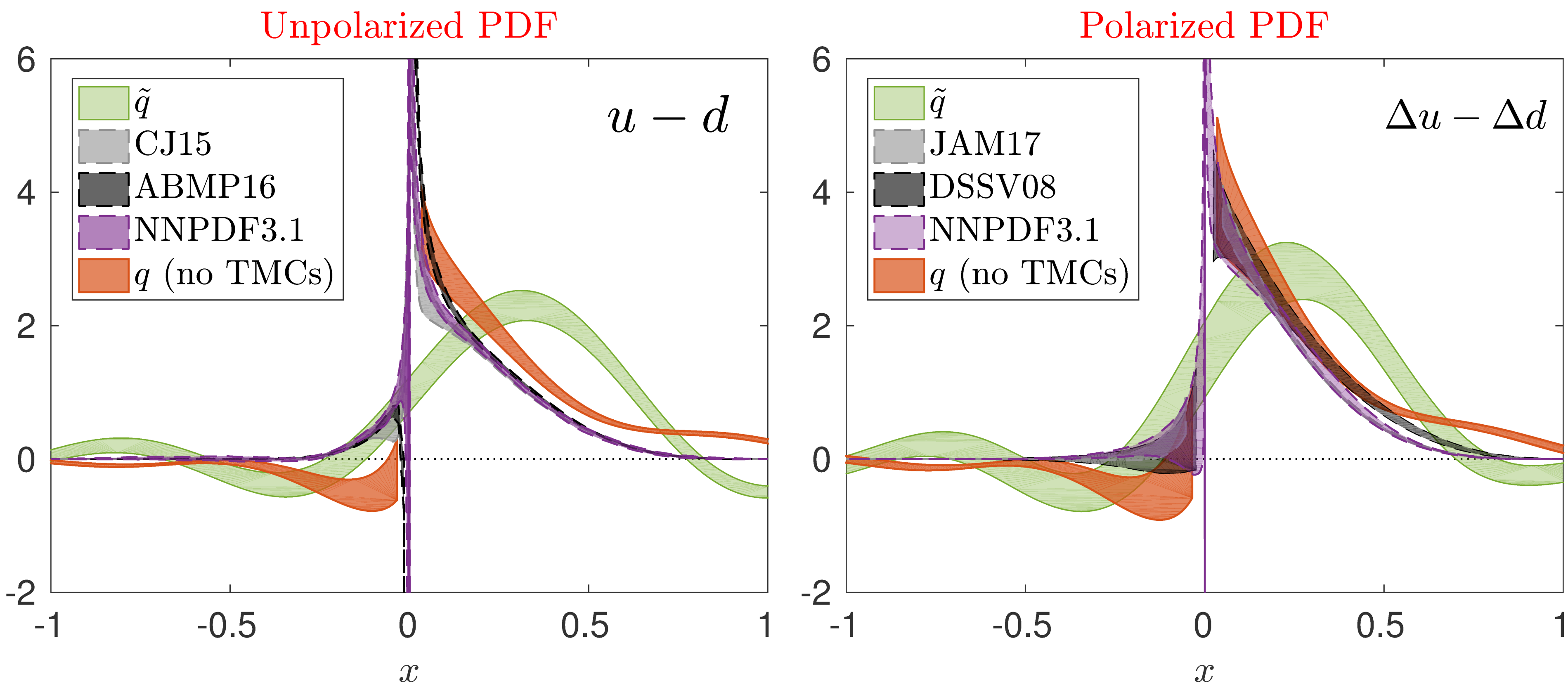
Plus prescription at $\xi{=}1$:

See slides by Fernanda

$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|} C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}(x) \int d\xi\, C\left(\xi, \frac{\mu}{xP_3}\right).$$

Matched PDF

Nucleon momentum $\frac{10\pi}{48}$



Step 7

The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

1. Compute bare matrix elements: $\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$
2. Compute vertex functions and the resulting renormalization functions in the intermediate RI'-MOM scheme $Z^{\mathrm{RI}'}(z,\mu)$.
3. Convert the renormalization functions to the $\overline{\mathrm{MS}}$ scheme and evolve to $\bar{\mu}=2$ GeV.
4. Apply the renormalization functions to the bare matrix elements, obtaining renormalized matrix elements in the $\overline{\mathrm{MS}}$ scheme.
5. Calculate the Fourier transform, obtaining quasi-PDFs:
$$\tilde{q}(x,\mu^2,P_3)=\int\frac{dz}{4\pi}\,e^{ixP_3z}\langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle.$$
6. Relate quasi-PDFs to light-cone PDFs via a matching procedure.
7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

Target mass corrections

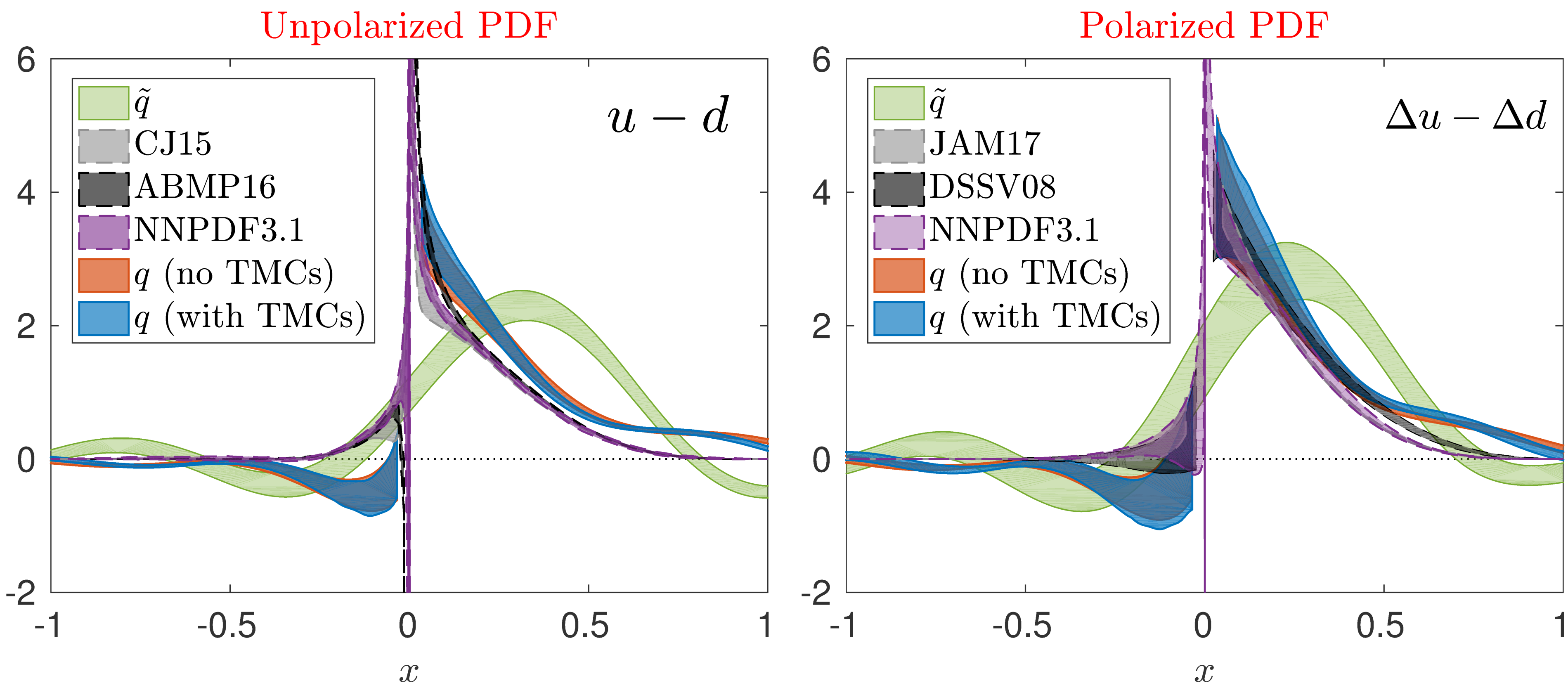
In the infinite momentum frame, nucleon mass does not matter, i.e. $m_N/P_3 = 0$.

Here, we work with nucleon boosted to finite momentum P_3 and we need to correct for $m_N/P_3 \neq 0$.

We use formulae derived in:
[J.W. Chen et al., Nucl.Phys. B911 (2016) 246-273, arXiv:1603.06664 [hep-ph]]

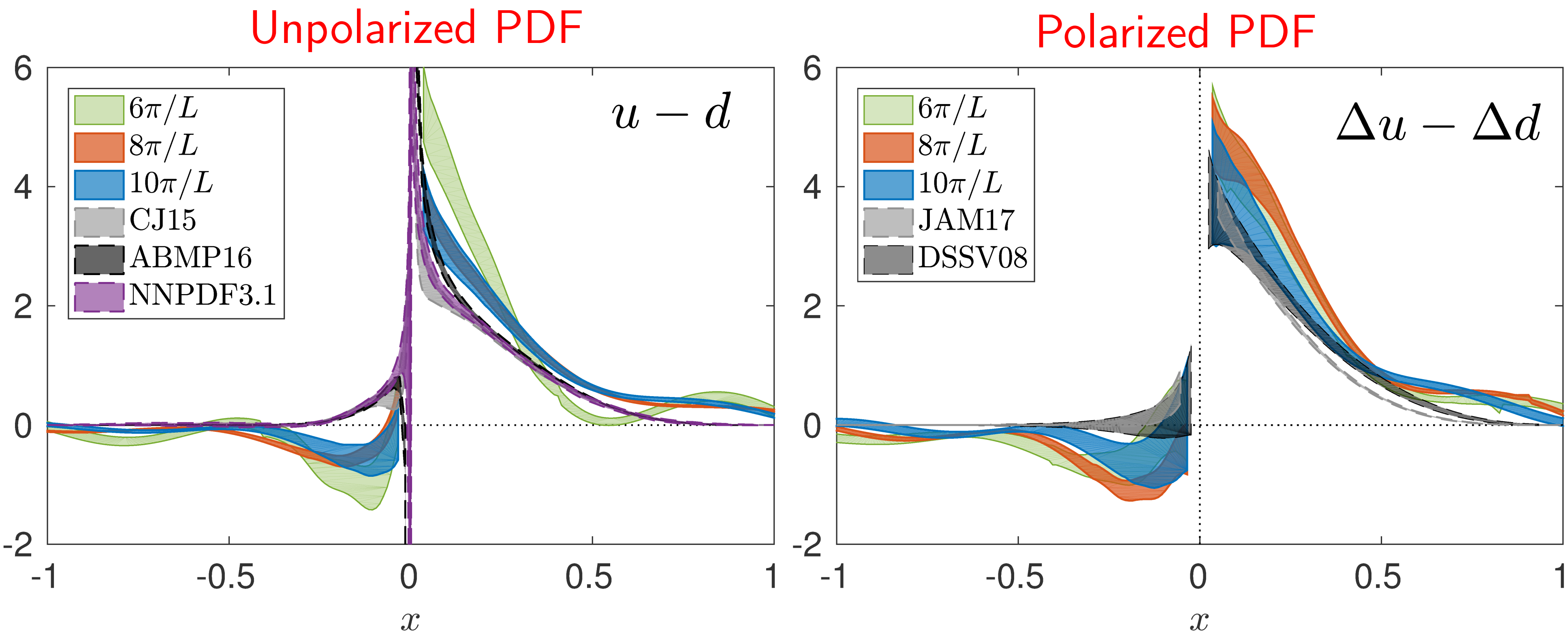
Important feature: particle number is conserved in nucleon mass corrections.

Nucleon momentum $\frac{10\pi}{48}$

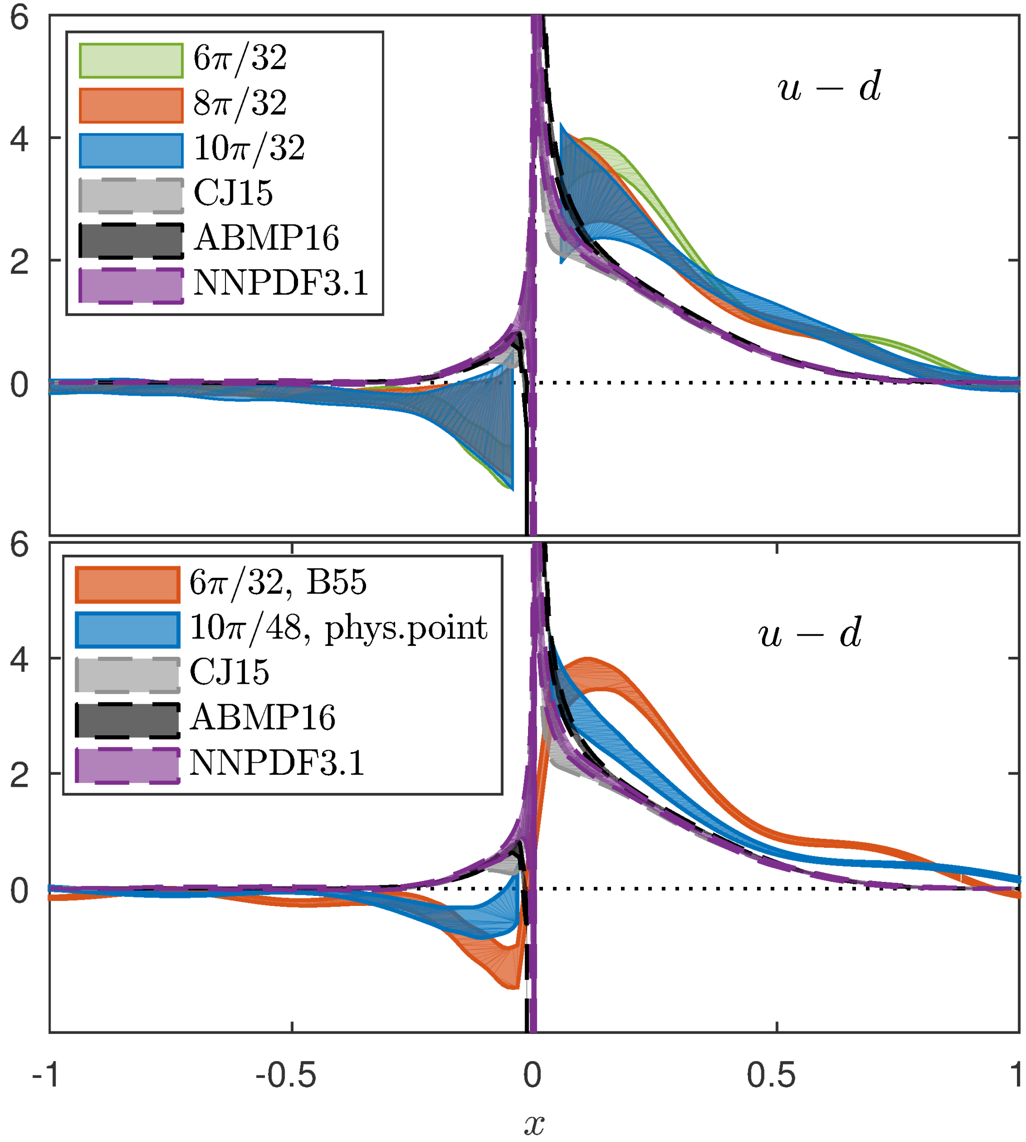


Momentum dependence of final PDF

Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$



Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF



Systematics

Different systematic effects still need to be addressed:

- pion mass ✓
- cut-off effects ✓✗
- finite volume effects ✓✗
- contamination by excited states ✓✗
- higher-twist effects ✓✗
- truncation of conversion, evolution and matching ✗
- lattice artifacts in renormalization functions ✓✗
- ...

Biggest challenge:
Reach large momenta at large source-sink separations

Conclusions and prospects

- **We have shown a computation of the full Bjorken-x dependence of PDFs from first principles at a physical pion mass.**
- Very encouraging results and already agreement with pheno for a range of x values.
- But: still a long way to go to control all systematics.
- We need to be slow and careful, go one step at a time.
- There will always be room for improvement of precision and given the importance of the subject, a better precision will always be desired.
- In the future: also other kinds of structure functions: GPDs, TMDs, gluon PDFs etc.

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Thank you for your attention!

Standard vs. derivative Fourier transform

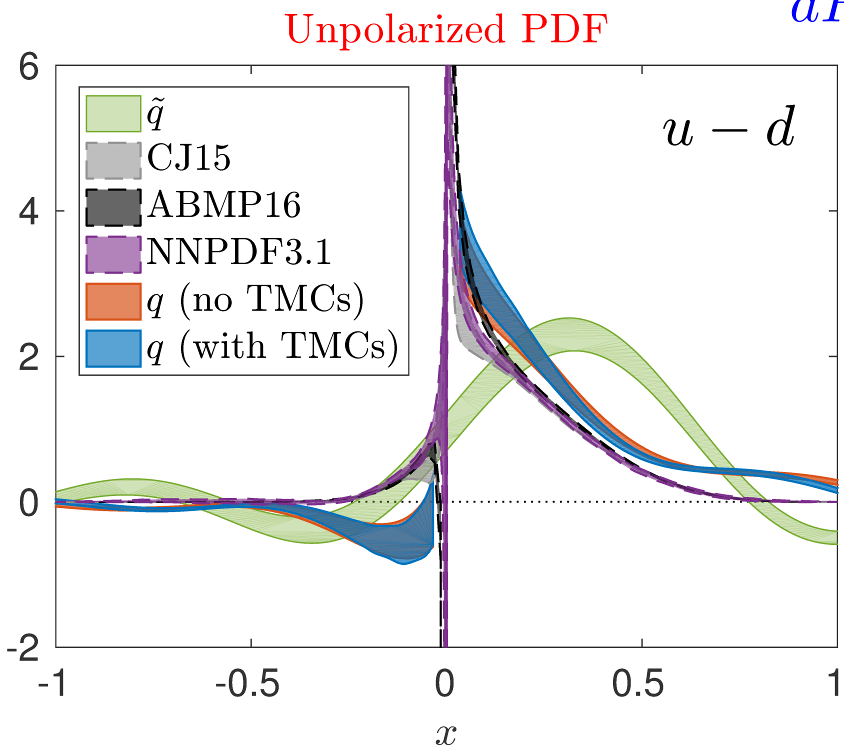
Standard Fourier transform defining qPDFs: $\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$

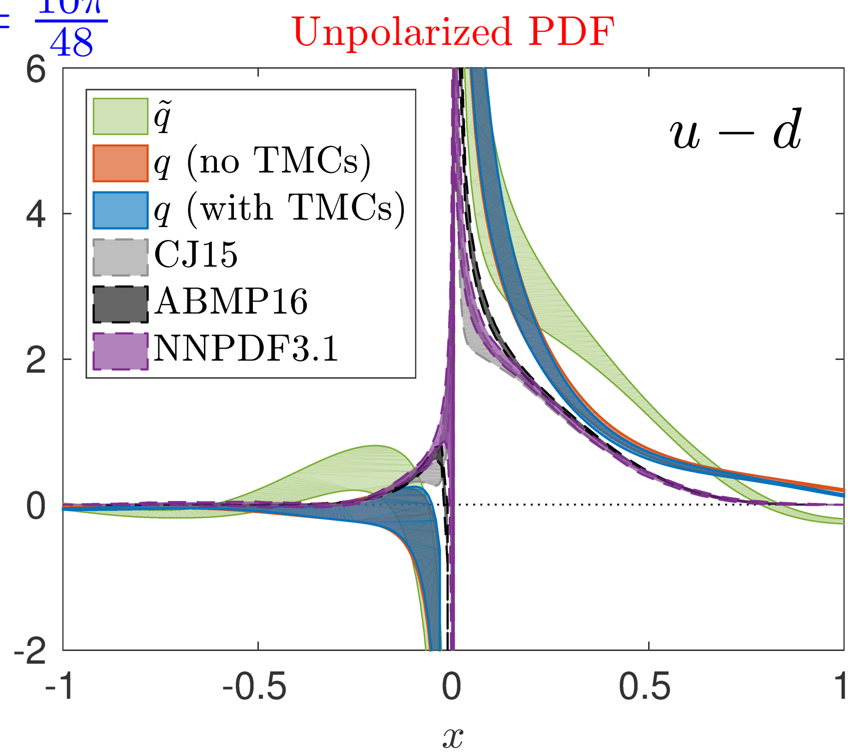
can be rewritten using integration by parts as: [H.W. Lin et al., arXiv:1708.05301]

$$\tilde{q}(x) = h(z) \frac{e^{ixzP_3}}{2\pi i x}\Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP_3}}{ix} h'(z).$$

Truncation: $h(|z| \geq z_{\max}) = 0$ is equivalent to neglecting the surface term.

$$aP_3 = \frac{10\pi}{48}$$





Oscillations ✓ Small-x ✗