Auxiliary field approach for nonperturbative renormalization

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Outline

1. Quasi-PDF operator
2. Auxiliary field approach
3. Non-perturbative renormalization
4. Renormalized quasi-PDF data

Partly based on JG, F. Steffens, and K. Jansen, 1707.07152

Operator for quark quasi-PDFs

We compute nucleon matrix elements of the operator

$$O_\Gamma(\boldsymbol{x}, \xi, \boldsymbol{n}) \equiv \bar{\psi}(\boldsymbol{x} + \xi\boldsymbol{n})\Gamma W(\boldsymbol{x} + \xi\boldsymbol{n}, \boldsymbol{x})\psi(\boldsymbol{x}),$$

where $\boldsymbol{n}^2 = 1$ is a unit vector, ξ is the separation, and W is a straight Wilson line.

On the lattice we can restrict $\boldsymbol{n}$ to point along an axis, and simply form W from a product of gauge links:

$$\text{e.g. for } \xi > 0,\ W(\xi\hat{\boldsymbol{z}}, \boldsymbol{0}) = U_z^\dagger\Big((\xi - a)\hat{\boldsymbol{z}}\Big)U_z^\dagger\Big((\xi - 2a)\hat{\boldsymbol{z}}\Big)\cdots U_z^\dagger\Big(a\hat{\boldsymbol{z}}\Big)U_z^\dagger\Big(\boldsymbol{0}\Big).$$

This is a nonlocal operator. How can we understand its renormalization?

Whole operator approach

Simplest approach: ignore that $\mathcal{O}_\Gamma$ is nonlocal. Impose renormalization conditions independently for each ξ to obtain $Z(\xi)$.

1. Perturbative study: one loop in lattice and continuum.

   M. Constantinou and H. Panagopoulos, Phys. Rev. D **96**, 054506 (2017)

   - Renormalization in lattice perturbation theory. Chiral symmetry breaking allows mixing between $\mathcal{O}_\Gamma$ and $\mathcal{O}_{\{\not{n},\Gamma\}}$.
   - Definition of RI-MOM type scheme and perturbative matching to $\overline{\text{MS}}$.

2. Nonperturbative studies: used RI-MOM type schemes.

   C. Alexandrou *et al.* (ETMC), Nucl. Phys. B **923**, 394 (2017)

   J.-W. Chen *et al.* (LP^3), Phys. Rev. D **97**, 014505 (2018)

Issues with this approach:

- Imposing condition at each ξ means an infinite number of conditions.
- Perturbation theory may be unreliable for matching at large ξ.

Auxiliary field approach

(Loosely following H. Dorn, Fortsch. Phys. **34**, 11 (1986))
The Wilson line satisfies the equation of motion

$$\left[\frac{d}{d\xi} + ig\boldsymbol{n}\cdot A(\boldsymbol{x}+\xi\boldsymbol{n})\right] W(\boldsymbol{x}+\xi\boldsymbol{n},\boldsymbol{x}) = \delta(\xi).$$

Introduce a scalar, color triplet field $\zeta_{\boldsymbol{n}}(\xi)$ that is defined on the line $\boldsymbol{x}+\xi\boldsymbol{n}$. (We omit the subscript $\boldsymbol{n}$ most of the time.) Give it the action

$$S_\zeta = \int d\xi\, \bar{\zeta} \left[\frac{d}{d\xi} + ig\boldsymbol{n}\cdot A + m\right]\zeta.$$

Then its propagator for fixed gauge background is

$$\left\langle \zeta(\xi)\bar{\zeta}(0)\right\rangle_\zeta = \theta(\xi) W(\boldsymbol{x}+\xi\boldsymbol{n},\boldsymbol{x}) e^{-m\xi}.$$

We want zero mass but there is no symmetry that forbids it. Unless we use dimensional regularization, a power-divergent counterterm is needed.

Auxiliary field approach: quark operator

Introduce the spinor-valued color singlet ζ-quark bilinear

$$\phi \equiv \bar{\zeta}\psi.$$

Then the extended operator for quasi-PDFs is given (for $m = 0$ and $\xi > 0$) by

$$\mathcal{O}_\Gamma(\boldsymbol{x}, \xi, \boldsymbol{n}) = \left\langle \bar{\phi}(\boldsymbol{x} + \xi \boldsymbol{n}) \Gamma \phi(\boldsymbol{x}) \right\rangle_\zeta .$$

For $\xi < 0$, we can use the relation

$$\mathcal{O}_\Gamma(\boldsymbol{x}, \xi, \boldsymbol{n}) = \mathcal{O}_\Gamma(\boldsymbol{x}, -\xi, -\boldsymbol{n}).$$

Thus, any QCD correlator involving $\mathcal{O}_\Gamma$ can be rewritten as a correlator in QCD+ζ involving the *local operators* ϕ and $\bar{\phi}$. To renormalize this, we need:

1. Z_ϕ to renormalize the local operators.
2. The mass counterterm.

Auxiliary field on the lattice

Discretize S_ζ, restricting $\boldsymbol{n}$ to be $\boldsymbol{n} = \pm\hat{\boldsymbol{\mu}}$:

$$S_\zeta = a \sum_\xi \frac{1}{1 + am_0} \bar{\zeta}(\boldsymbol{x} + \xi\boldsymbol{n})[\nabla_n + m_0]\zeta(\boldsymbol{x} + \xi\boldsymbol{n}),$$

where

$$\nabla_n = \begin{cases} \boldsymbol{n} \cdot \nabla^* = \nabla^*_\mu, & \text{if } \boldsymbol{n} = \hat{\boldsymbol{\mu}} \\ \boldsymbol{n} \cdot \nabla = -\nabla_\mu, & \text{if } \boldsymbol{n} = -\hat{\boldsymbol{\mu}} \end{cases}.$$

For $\boldsymbol{n} = +\hat{\boldsymbol{\mu}}$, this yields the bare propagator on fixed gauge field:

$$\left\langle \zeta(\boldsymbol{x} + \xi\hat{\boldsymbol{\mu}})\bar{\zeta}(\boldsymbol{x}) \right\rangle_\zeta = \theta(\xi) e^{-m\xi} U^\dagger_\mu\left(\boldsymbol{x} + (\xi - a)\hat{\boldsymbol{\mu}}\right) U^\dagger_\mu\left(\boldsymbol{x} + (\xi - 2a)\hat{\boldsymbol{\mu}}\right) \cdots U^\dagger_\mu(\boldsymbol{x}),$$

where $m = a^{-1}\log(1 + am_0)$.
(We could use smeared links U in defining the covariant derivative.)

Auxiliary field on the lattice: renormalization and mixing

In our approach, mixing appears between ϕ and $\not{n}\phi$ when chiral symmetry is broken. The ζ-quark bilinear $\phi = \bar{\zeta}\psi$ renormalizes as

$$\phi_R = Z_\phi\left(\phi + r_{\text{mix}}\not{n}\phi\right), \qquad \bar{\phi}_R = Z_\phi\left(\bar{\phi} + r_{\text{mix}}\bar{\phi}\not{n}\right).$$

We can use $P_\pm \equiv \frac{1}{2}(1 \pm \not{n})$ to define operators that don't mix:

$$\phi^\pm \equiv P_\pm\phi \implies \phi_R^\pm = Z_\phi^\pm\phi^\pm, \quad \text{where } Z_\phi^\pm = Z_\phi(1 \pm r_{\text{mix}}).$$

The renormalized extended quark bilinear has the form (for all $\xi \neq 0$)

$$\mathcal{O}_\Gamma^R(\boldsymbol{x},\xi,\boldsymbol{n}) = {Z_\phi}^2 e^{-m|\xi|}\bar{\psi}(\boldsymbol{x}+\xi\boldsymbol{n})\Gamma' W(\boldsymbol{x}+\xi\boldsymbol{n},\boldsymbol{x})\psi(\boldsymbol{x}),$$
$$\text{where } \Gamma' = \Gamma + \operatorname{sgn}(\xi) r_{\text{mix}}\{\not{n},\Gamma\} + {r_{\text{mix}}}^2\not{n}\Gamma\not{n}.$$

Three parameters needed: m, Z_ϕ, r_{mix}.

Relation to static quark theory

The Lagrangian for a static quark on the lattice is

$$\mathcal{L}(\boldsymbol{x}) = \frac{1}{1 + am_0} \bar{Q}(\boldsymbol{x}) \left[\nabla_t^* + m_0\right] Q(\boldsymbol{x}),$$

where Q is a color triplet spinor satisfying $\frac{1}{2}(1+\gamma_t)Q = Q$.
Other than the spin degres of freedom (which don't couple in the action) this is the same as for ζ with $\boldsymbol{n} = \hat{\boldsymbol{t}}$. The propagators are also related:

$$\left\langle Q(\boldsymbol{x})\bar{Q}(\boldsymbol{y})\right\rangle_Q = \left\langle \zeta(\boldsymbol{x})\bar{\zeta}(\boldsymbol{y})\right\rangle_\zeta P_+.$$

With broken chiral symmetry, there are two renormalization factors for static-light bilinears:

$$Z_V^{\text{stat}} \text{ for } \bar{\psi}\gamma_t Q \quad \text{and} \quad Z_A^{\text{stat}} \text{ for } \bar{\psi}\gamma_t\gamma_5 Q.$$

Inserting P_+, we identify $Z_V^{\text{stat}} = Z_\phi^+$ and $Z_A^{\text{stat}} = Z_\phi^-$.

Further implications

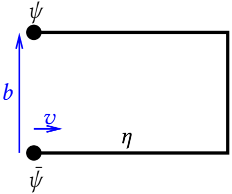
1. Lattice artifacts are $O(a)$.
   Even with chiral symmetry, the static-light currents need improvement at $O(a)$: e.g.

$$A_0^{\mathrm{stat},I} = \bar{\psi}\gamma_0\gamma_5 Q + ac_A^{\mathrm{stat}}\bar{\psi}\gamma_j\gamma_5\frac{1}{2}\left(\overleftarrow{\nabla}_j + \overleftarrow{\nabla}^*_j\right)Q.$$

2. No mixing of $\mathcal{O}_\Gamma$ with gluon operator, even for flavor singlet.
   - The local bilinear $\phi = \bar{\zeta}\psi$ is in the flavor fundamental irrep.
     The corresponding gluon operator is flavor singlet.
   - Mixing between quark and gluon PDFs must occur in:
     a. the matching from quasi-PDF to PDF,
     b. the dependence of quasi-PDFs on p_z.

Generalization: staple-shaped gauge link

Transverse momentum-dependent (TMD) PDFs are studied on the lattice using operators with staple-shaped gauge links:



$$\mathcal{O}^{\mathrm{TMD}} = \bar\psi(\mathbf{0})\Gamma W(\mathbf{0},\eta\boldsymbol{v})W(\eta\boldsymbol{v},\eta\boldsymbol{v}+\boldsymbol{b})W(\eta\boldsymbol{v}+\boldsymbol{b},\boldsymbol{b})\psi(\boldsymbol{b}).$$

We introduce the auxiliary fields $\zeta_{\boldsymbol{v}}$, $\zeta_{-\boldsymbol{v}}$, and $\zeta_{-\hat{\boldsymbol{b}}}$. Using

1. the ζ-quark bilinear $\phi_{\boldsymbol{n}} = \bar\zeta_{\boldsymbol{n}}\psi$,
2. the ζ-ζ "corner" bilinear $C_{\boldsymbol{n}',\boldsymbol{n}} = \bar\zeta_{\boldsymbol{n}'}\zeta_{\boldsymbol{n}}$,

we obtain

$$\mathcal{O}^{\mathrm{TMD}} = \left\langle \bar\phi_{-\boldsymbol{v}}(\mathbf{0})\Gamma C_{-\boldsymbol{v},-\hat{\boldsymbol{b}}}(\eta\boldsymbol{v})C_{-\hat{\boldsymbol{b}},\boldsymbol{v}}(\eta\boldsymbol{v}+\boldsymbol{b})\phi_{\boldsymbol{v}}(\boldsymbol{b})\right\rangle_{\zeta}.$$

The corner operators also must be renormalized with a factor Z_C.
In this case mixing will occur between TMD operators with Γ and $[\not{\boldsymbol{v}},\Gamma]$.

Nonperturbative approach

In Landau gauge, compute the position-space ζ propagator

$$S_\zeta(\xi) \equiv \left\langle \zeta(\boldsymbol{x} + \xi\boldsymbol{n})\bar{\zeta}(\boldsymbol{x}) \right\rangle_{\text{QCD}+\zeta} = \langle W(\boldsymbol{x} + \xi\boldsymbol{n}, \boldsymbol{x}) \rangle_{\text{QCD}},$$

the momentum-space quark propagator

$$S_q(\boldsymbol{p}) \equiv \sum_x e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \left\langle \psi(\boldsymbol{x})\bar{\psi}(\boldsymbol{0}) \right\rangle,$$

and the mixed-space Green's function for $\phi^\pm$:

$$G^\pm(\xi, \boldsymbol{p}) \equiv \sum_x e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \left\langle \zeta(\xi\boldsymbol{n})\phi^\pm(\boldsymbol{0})\bar{\psi}(\boldsymbol{x}) \right\rangle_{\text{QCD}+\zeta}.$$

These renormalize as

$$S_\zeta^R(\xi) = e^{-m\xi} Z_\zeta S_\zeta(\xi), \quad S_q^R(\boldsymbol{p}) = Z_q S_q(\boldsymbol{p}),$$
$$G_R^\pm(\xi, \boldsymbol{p}) = e^{-m\xi} \sqrt{Z_\zeta Z_q} Z_\phi^\pm G^\pm(\xi, \boldsymbol{p}).$$

Power divergence

Take the effective energy of the ζ propagator:

$$E_{\text{eff}}(\xi) \equiv -\frac{d}{d\xi} \log \operatorname{Tr} S_\zeta(\xi).$$

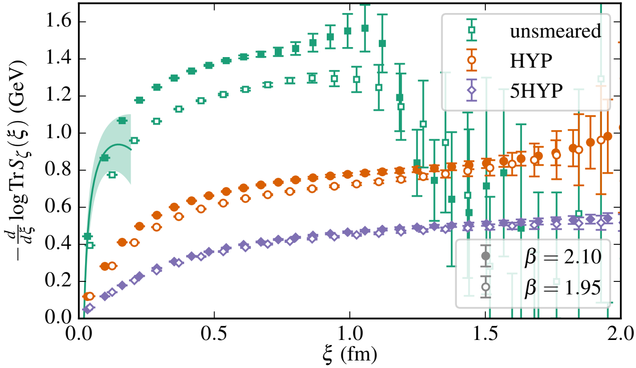
This renormalizes as $E^R_{\text{eff}}(\xi) = m + E_{\text{eff}}(\xi)$. We can impose the condition $E_{\text{eff}}(\xi_0) = 0$ for some ξ_0.

Convert to $\overline{\text{MS}}$ at small ξ using perturbation theory: static quark propagator known to three-loop order.

K. Melnikov and T. van Ritbergen, Nucl. Phys. B **591**, 515 (2000)

K. G. Chetyrkin and A. G. Grozin, Nucl. Phys. B **666**, 289 (2003)

Effective energy



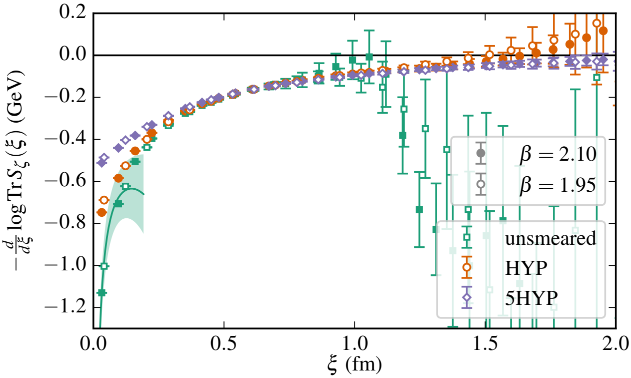
Bare E_{eff} computed on $N_f = 4$ twisted mass ensembles.

Two lattice spacings: 0.064 fm ($\beta = 2.10$), 0.082 fm ($\beta = 1.95$).

Three different link discretizations used.

Match thin links on fine lattice with perturbation theory at small ξ, then match thin with smeared links at larger ξ.

Effective energy



Renormalized E_{eff} computed on $N_f = 4$ twisted mass ensembles.

Two lattice spacings: 0.064 fm ($\beta = 2.10$), 0.082 fm ($\beta = 1.95$).

Three different link discretizations used.

Match thin links on fine lattice with perturbation theory at small ξ, then match thin with smeared links at larger ξ.

RI-xMOM scheme

We use momentum space for quark fields and position space for ζ.

For Z_ζ, we use the condition

$$\left[\tfrac{1}{3}\operatorname{Tr} S^R_\zeta(\xi_0)\right]^2 = \tfrac{1}{3}\operatorname{Tr} S^R_\zeta(2\xi_0)$$

For $\phi^\pm$, "amputate" the Green's function:

$$\Lambda^\pm(\xi, \boldsymbol{p}) \equiv S^{-1}_\zeta(\xi) G^\pm(\xi, \boldsymbol{p}) S^{-1}_q(\boldsymbol{p}).$$

Both of these serve to eliminate the dependence on m.

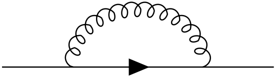
Then to determine $Z^\pm_\phi$ we impose the condition

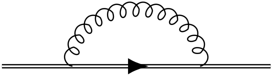
$$\frac{1}{6}\Re \operatorname{Tr} \Lambda^\pm_R(\boldsymbol{p}_0, \xi_0) = 1$$

at some scale $\mu^2 = \boldsymbol{p}_0^2$. This is a two-parameter family of schemes, which depends on the dimensionless parameters $y \equiv |\boldsymbol{p}_0|\xi_0$ and $z \equiv (\boldsymbol{n} \cdot \boldsymbol{p}_0)/|\boldsymbol{p}_0|$.

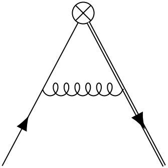
Conversion to $\overline{\text{MS}}$

Using dimensional regularization, we computed at one-loop order the propagators:

S_q: 

S_ζ: 

and the vertex function, restricting to $\boldsymbol{p} \propto \boldsymbol{n}$ ($z = 1$).

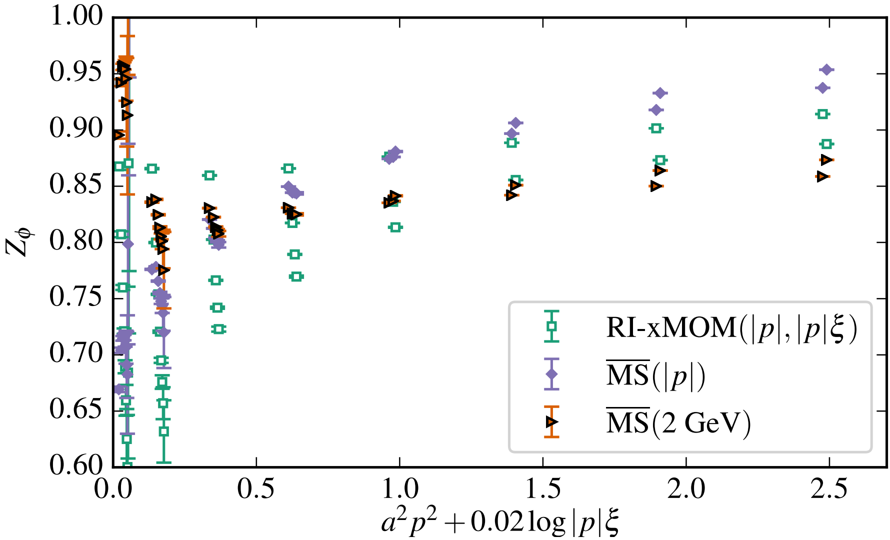


This gave the conversion factor:

$$C_\phi \equiv \frac{Z_\phi^{\overline{\text{MS}}}(\mu)}{Z_\phi^{\text{RI-xMOM}}(\mu, y, z = 1)} = 1 + \alpha_s(\mu) C_F f(y) + O(\alpha_s^2).$$

Anomalous dimension for ϕ taken from static-light bilinear.

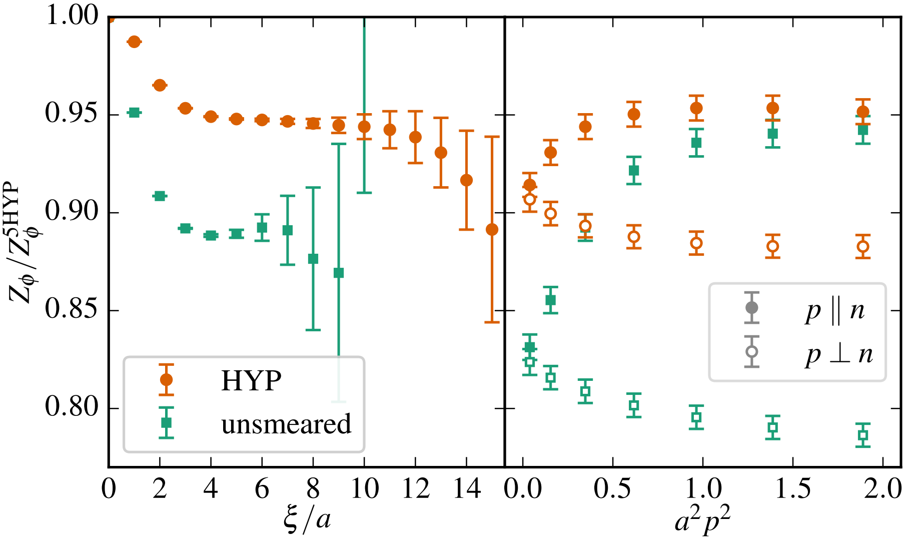
Renormalization of bilinear



Unsmeared links on fine lattice spacing.

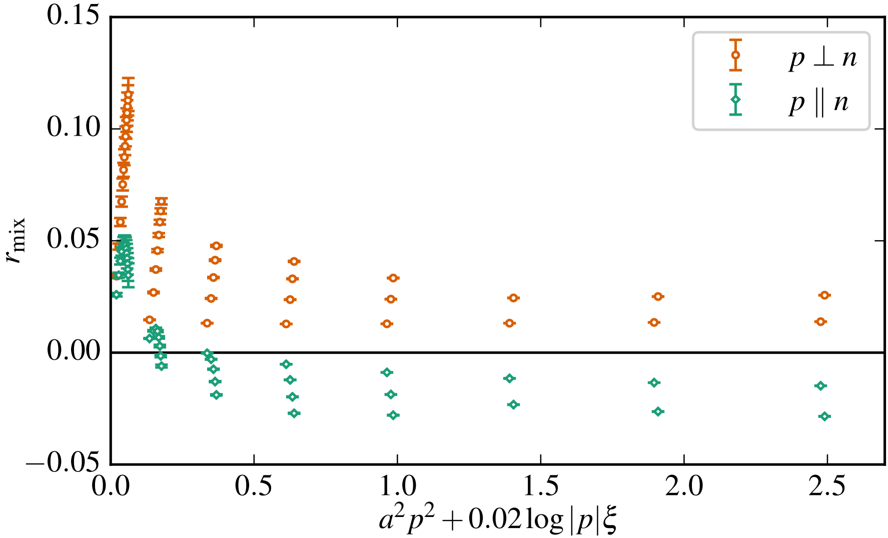
4d volume sources used to get good signal inexpensively.

Ratio between different link smearings



Nonperturbative matching at long distance and low momentum.

Mixing parameter?



Likely affected by $O(a)$ lattice artifacts. Avoid mixing for now.

Helicity quasi-PDF

The quark helicity matrix element $\Delta h(\xi, p_z)$ is given by

$$\langle \boldsymbol{p} | \mathcal{O}_{\not{n}\gamma_5}(\xi, \boldsymbol{n}) | \boldsymbol{p} \rangle = \Delta h(\xi, \boldsymbol{n} \cdot \boldsymbol{p}) \bar{u}(\boldsymbol{p}) \not{n} \gamma_5 u(\boldsymbol{p}),$$

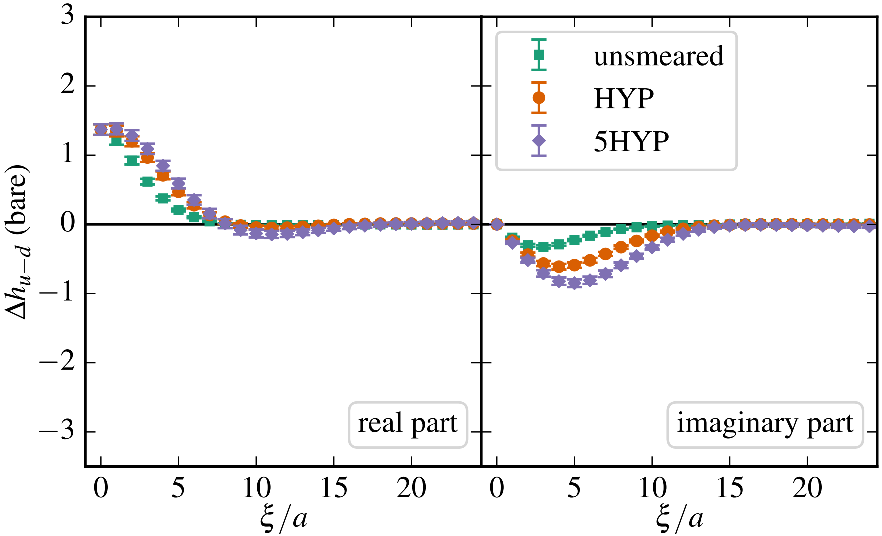
and the quark helicity quasi-PDF $\Delta \tilde{q}(x, p_z)$ by

$$\Delta \tilde{q}(x, p_z) = p_z \int \frac{d\xi}{2\pi} e^{-i\xi p_z x} \Delta h(\xi, p_z).$$

Unaffected by mixing because $\{\not{n}, \not{n}\gamma_5\} = 0$.
Term with $r_{\text{mix}}^2 (\lesssim 0.01)$ can be neglected.

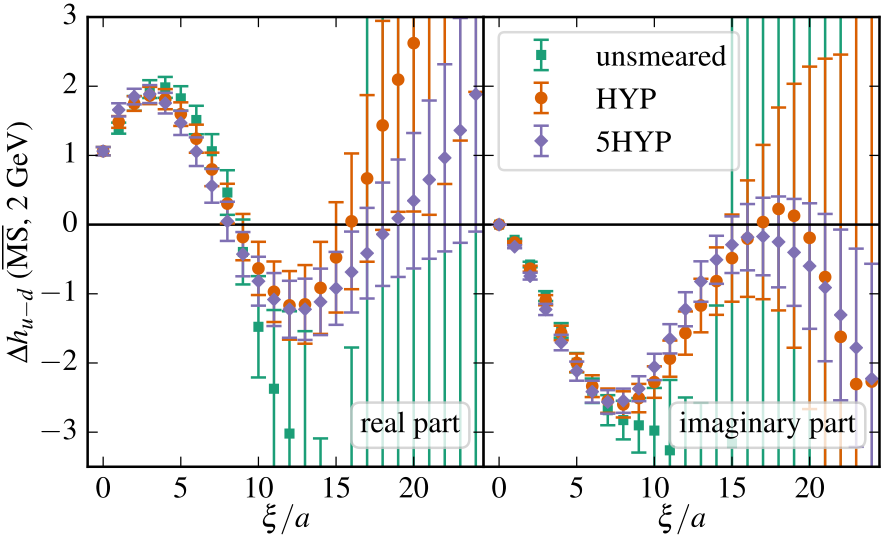
Calculations performed on $N_f = 2 + 1 + 1$ twisted mass ensembles with $m_\pi \approx 370$ MeV, using $p_z \approx 1.85$ GeV.

Helicity matrix element: bare



Fine lattice spacing.

Helicity matrix element: renormalized



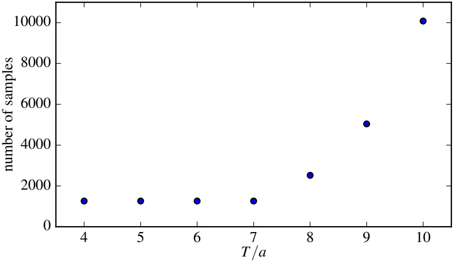
Fine lattice spacing.

Toward the continuum limit

We want to show that the power divergence $m \sim a^{-1}$ is under control and that the $a \to 0$ limit can be taken.

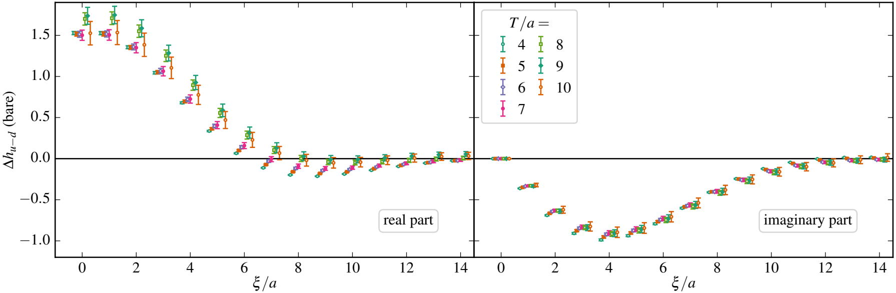
First steps to take:

- Revisit existing calculations to reduce excited-state effects.
- Add a third lattice spacing, $a = 0.0934$ fm.



Increase statistics exponentially with t to reduce exponential growth of noise.

Excited-states study

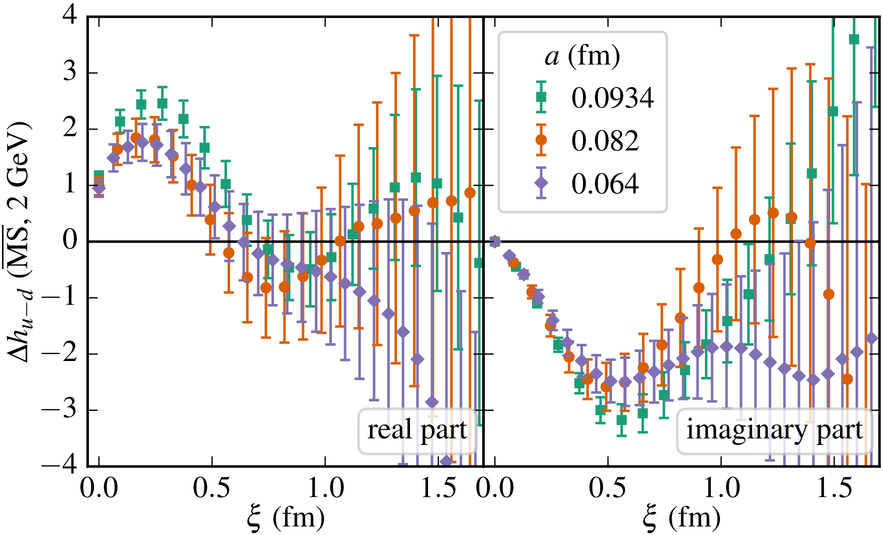


Study on coarsest lattice spacing.

Excited states cause real part to dip below zero.

For continuum-limit study, take $T > 0.9$ fm.

Helicity matrix element: discretization effects



Taking the continuum limit of Δh

For fixed ξ and p_z we could take the continuum limit of $\Delta h(\xi, p_z)$.

But this is not continuous (even divergent) as $|\xi| \to 0$.
Needs a trickier double extrapolation $|\xi| \to 0$ and $a \to 0$.

Instead, try computing $\Delta\tilde{q}$ for each lattice spacing and then take $a \to 0$.

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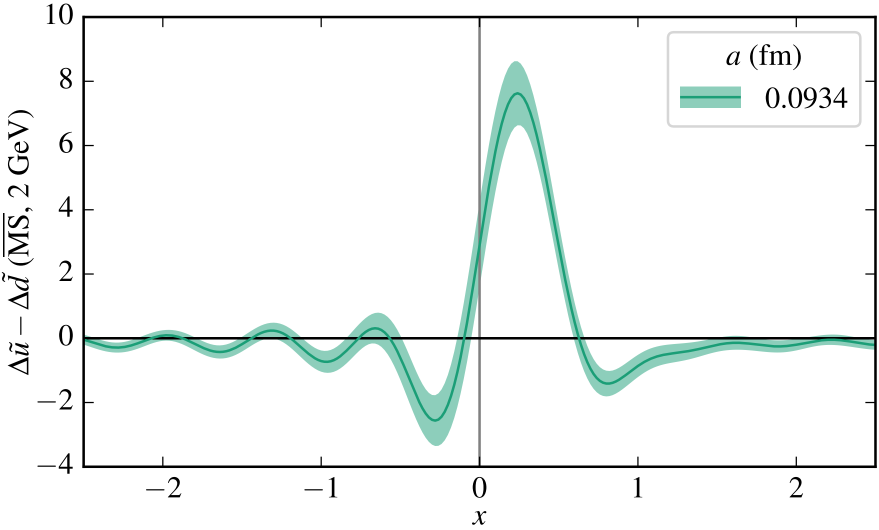
Additional problem: noise becomes very large at large $|\xi|$.
Proposed solutions:

- Match to a model for the large-$|\xi|$ behavior.
- Use $h'(\xi)$ rather than $h(\xi)$. LP^3, 1708.05301
- Use a Gaussian to suppress large-$|\xi|$ region. LP^3, 1711.07858

Here we will simply use a hard cutoff. This leads to some unphysical oscillations in $\tilde{q}$.

Helicity quasi-PDF: discretization effects



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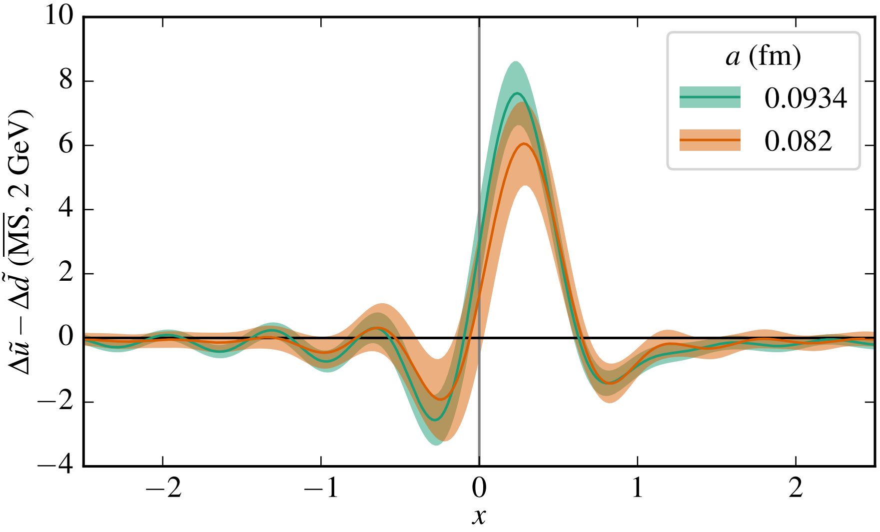
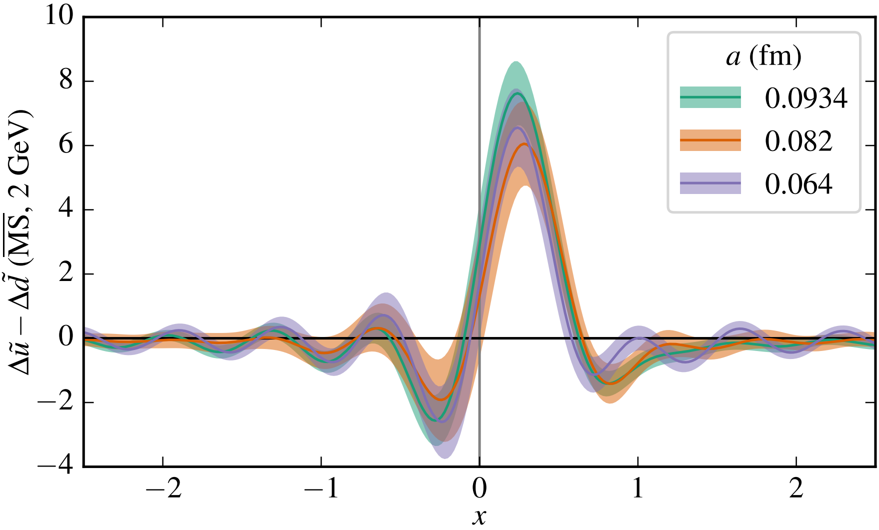
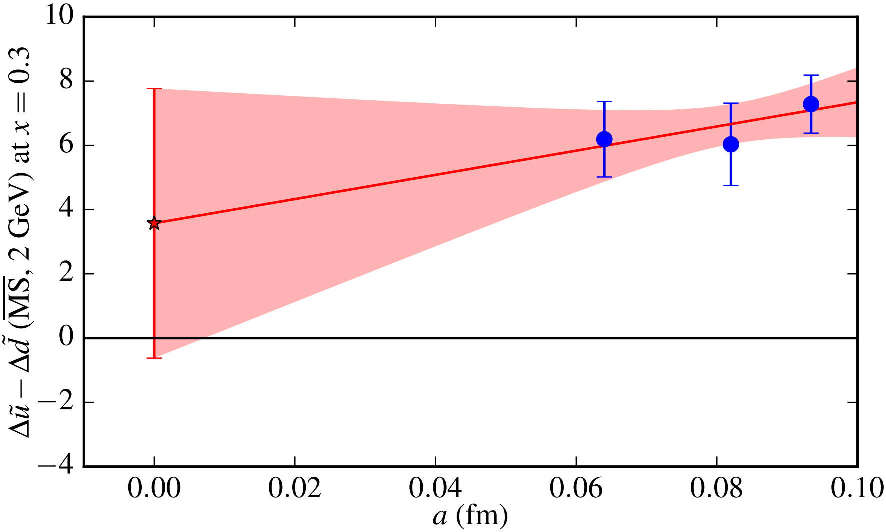


Helicity quasi-PDF: discretization effects



No significant discretization effects seen.

Continuum limit?



$O(a)$ improvement would help a lot.

Outlook

We are now able to renormalize the nonlocal operator for quasi-PDFs.
Work still to be done:

- $O(a)$ improvement and $a \to 0$ limit.
- Comparison with whole operator approach.
- Control over large-$|\xi|$ region.

It is important to demonstrate control over systematics for quasi-PDFs, even at heavy m_π and small p_z.

Once we have quasi-PDF:

1. Perturbatively match to PDF. Currently available at one-loop order.
2. Take the $p_z \to \infty$ limit.

Results appearing at physical m_π:

C. Alexandrou *et al.* (ETMC), 1803.02685

J.-W. Chen *et al.* (LP^3), 1803.04393