Perturbative and non-perturbative renormalization for quasi-PDFs

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in collaboration with:

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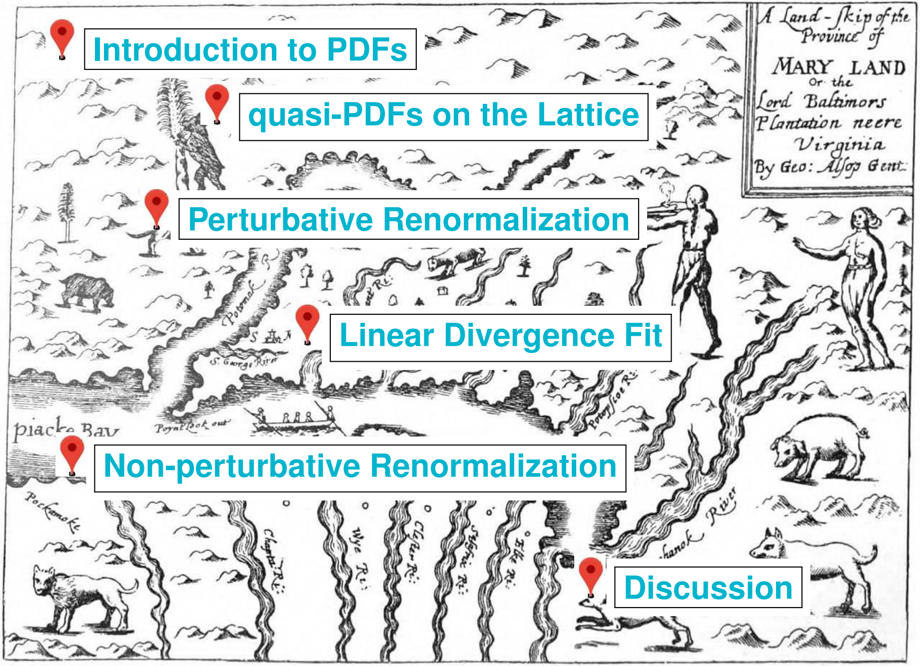
Lattice PDF Workshop
Maryland, USA
April 6, 2018

ROADMAP OF TALK

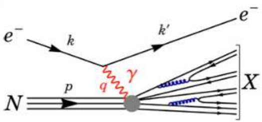


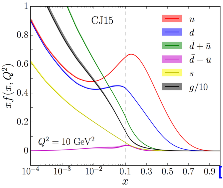
- Introduction to PDFs
- quasi-PDFs on the Lattice
- Perturbative Renormalization
- Linear Divergence Fit
- Non-perturbative Renormalization
- Discussion

A

Introduction to PDFs

Probing Nucleon Structure





CJ15 PDFs

[A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- ★ **powerful tool to describe the structure of a nucleon**
- ★ **necessary for the analysis of Deep inelastic scattering (DIS) data**
- ★ **Parametrization of off-forward matrix of a bilocal quark operator (light-like)**

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

$q = p' - p$, $\bar P = (p' + p)/2$, n: **light-cone vector (**$\bar P.n = 1$**),** $\xi = -n \cdot \Delta/2$

PDFs on the Lattice

★ **first principle calculations of PDFs are necessary**

★ **On the lattice: long history of moments of PDFs**

$$\boxed{f^n = \int_{-1}^{1} dx\, x^n f(x)}$$

★ **rely on OPE to reconstruct the PDFs (difficult task):**

- **signal-to-noise is bad for higher moments**
- **n $>$ 3: operator mixing (unavoidable!)**

★ **Alternative investigation of PDFs ?**

Types:

- **Unpolarized** (vector current)
- **Polarized** (axial current)
- **Transversity** (tensor current)

PDFs on the Lattice

Novel direct approach: [X.Ji, arXiv:1305.1539]

- ★ compute **quasi-PDF** on the lattice
- ★ contact with physical PDFs in two steps:
  1. Renormalization of quasi-PDFs in Lattice Regularization
  2. Matching procedure (LaMET)

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★ **contact with physical PDFs in two steps:**

1. **Renormalization of quasi-PDFs in Lattice Regularization**
2. **Matching procedure (LaMET)**

Exploratory studies are maturing:

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],
[Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575],
[J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018],
[C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],
[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],
[C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962],
[K. Orginos et al., arXiv:1706.05373], [T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152],
[Y-Q Ma et al., arXiv:1709.03018], [J. Karpie et al., arXiv:1710.08288, [J-W Chen et al., arXiv:1711.07858],
[C.Alexandrou et al., arXiv:1710.06408], [T. Izubuchi et al., arXiv:1801.03917], [C.Alexandrou et al., arXiv:1803.02685],
[J-W Chen et al, arXiv:1803.04393], · · ·

B

quasi-PDFs

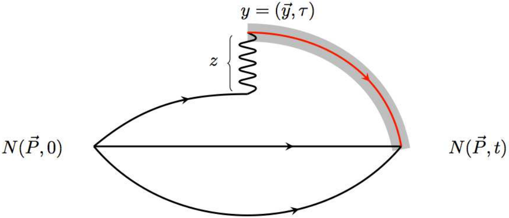
on the Lattice

Access of PDFs on a Euclidean Lattice

★ **quasi-PDF purely spatial for nucleons with finite momentum**

$$\boxed{\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z}\,\langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}}$$

- $\mathcal{A}(z,0)$: **Wilson line from $0 \to z$**
- z: **distance in any spatial direction** **(momentum boost in z direction)**



★ **At finite but feasibly large momenta on the lattice:**

a large momentum EFT can relate Euclidean $\tilde{q}$ to PDFs through a factorization theorem

★ **use of Perturbation Theory for the matching**

C

History of Renormalization

Renormalization History

Prior 2017:

- ★ **Results available only for bare matrix elements (ME)**
- ★ **Comparison with phenomenology: only qualitative**

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- ★ Undertook a perturbative calculation that revealed important features of Wilson line operators (mixing!)

  M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]

- ★ Renormalization pattern led to development of non-perturbative prescription

  C. Alexandrou, et al., Nucl. Phys. B 923 (2017) 394 (Frontier Article), [arXiv:1706.00265]

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2017:

- ★ Results available for renormalized matrix elements (ME)
- ★ Comparison with phenomenology becomes a real possibility

C

Perturbative Renormalization

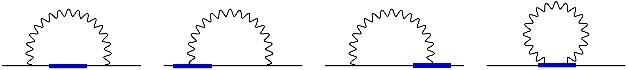
Perturbative Calculation

★ **Operators**

$$\mathcal{O}^{\mu}_{\Gamma} \equiv \overline{\psi}(x)\,\Gamma\,\mathcal{P}\,e^{i\,g\,\int_0^z A(\zeta)d\zeta}\,\psi(x+z\hat{\mu})$$

Scalar, Pseudoscalar, Vector, Axial, Tensor

★ **Feynman Diagrams**



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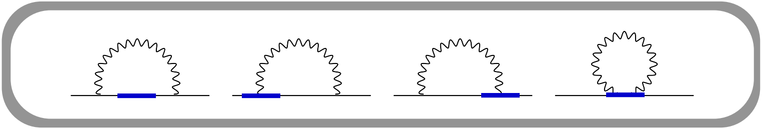

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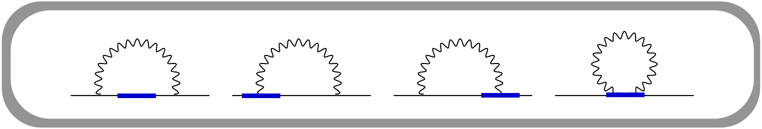

★ **Main components of calculation**

A Dimensional Regularization (DR)

- **Compute conversion factor between $\overline{MS}$ and other schemes**

Perturbative Calculation

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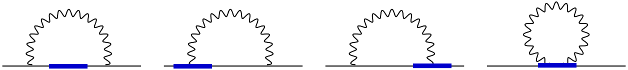


★ **Main components of calculation**

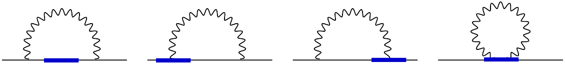
A **Dimensional Regularization (DR)**

- **Compute conversion factor between $\overline{MS}$ and other schemes**

B **Lattice Regularization (LR)**

- **Extract full pert. Z-factors using Green's functions in both DR and LR**

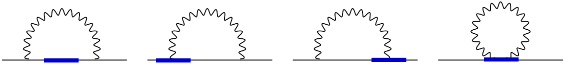
A. Dimensional Regularization



Features of DR Calculation

- ★ **No linear divergence**
- ★ **Z-factors in $\overline{\mathrm{MS}}$: real function**
- ★ **Conversion factor: a complex function**

A. Dimensional Regularization



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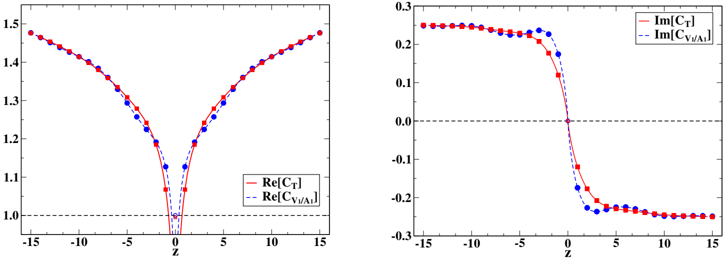
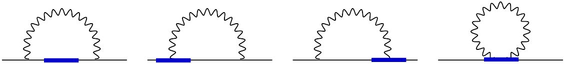


Parameters chosen based on B55 ETMC ensemble, Nucleon momentum: $P_3\!=\!4$

- ★ **Necessary ingredient for non-perturbative renormalization**

B. Lattice Regularization



- ★ Linear divergence **from tadpole diagram:** $\propto |z|/a$
- ★ **To all orders in pert. theory:** $e^{-c\,\frac{|z|}{a}}$ [Dotsenko et al., NPB169 (1980) 527]
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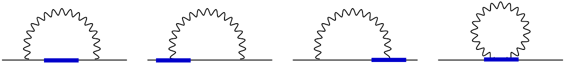

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$$\boxed{\langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{DR,\overline{\rm MS}}_{\rm amp}-\langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle^{LR}_{\rm amp}=\frac{g^2\,C_f}{16\,\pi^2}\,e^{i\,q_\mu\,z}\;\times\mathcal{F}}$$

$$\mathcal{F}=\Gamma\Big(c_1+c_2\,\beta+c_3\,\frac{|z|}{a}+\log\left(a^2\bar{\mu}^2\right)(4-\beta)\Big)+(\Gamma\cdot\gamma_\mu+\gamma_\mu\cdot\Gamma)\,\Big(c_4+c_5\,c_{\rm SW}\Big)$$

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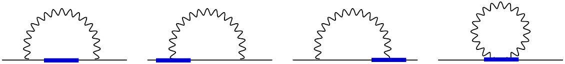

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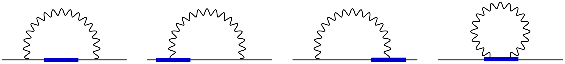

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linear divergence

mixing term

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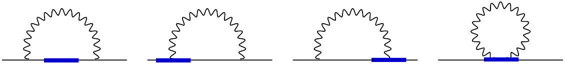

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linear divergence

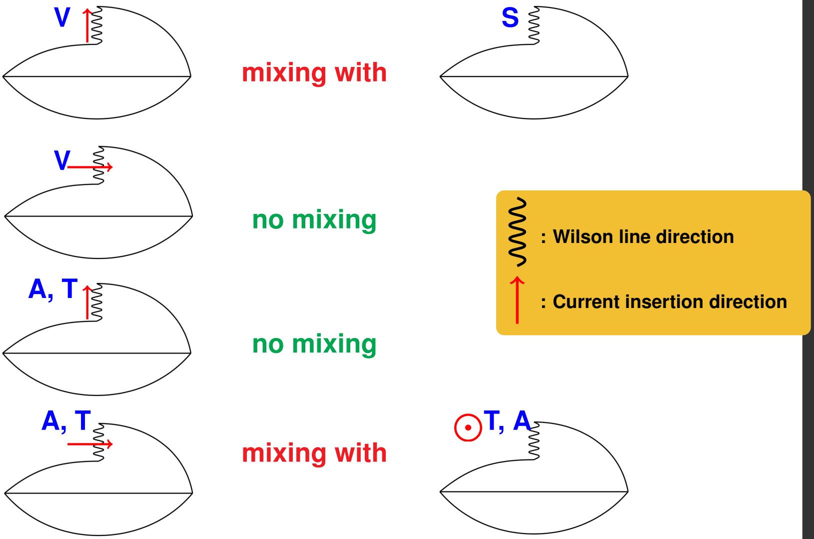
mixing term

c_i **depend on gluonic action**

mixing vanishes if $c_{\rm SW}=-c_4/c_5$

Mixing pattern

Depends on the relation between the current & Wilson line direction



mixing with

no mixing

no mixing

mixing with

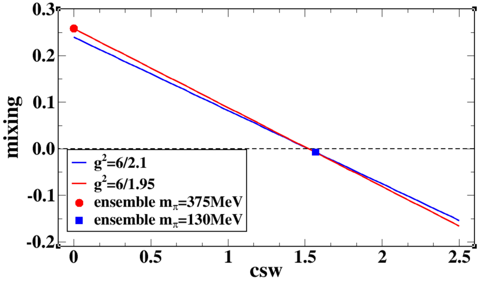
〰 : Wilson line direction

↑ : Current insertion direction

Mixing pattern

⇒ **Perturbative results can guide simulations on quasi-PDFs**

⇒ **If mixing present, it must be taken into account (e.g. γ_3)**



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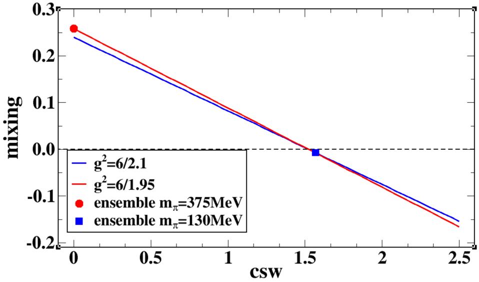

★ **Mixing may be suppressed because:**

- **mixing vanishes at some value of $c_{\rm SW}$ ($=-c_4/c_5$)**
- **numerical application show no mixing for $c_{\rm SW}\sim 1$ (for Wilson-like actions)**

D

Linear Divergence

Linear Divergence

Absence of mixing:

presence of c_0:

$$\mathcal{R}=\frac{q(P_3,z)}{q(P_3',z')}=e^{\left(-\frac{c}{a}+c_0\right)(|z|-|z'|)}\left(\frac{P_3}{P_3'}\right)^{-6\frac{g^2C_f}{16\pi^2}},\ zP_3=z'P_3'$$

[R. Sommer, arXiv[1501.03060]]

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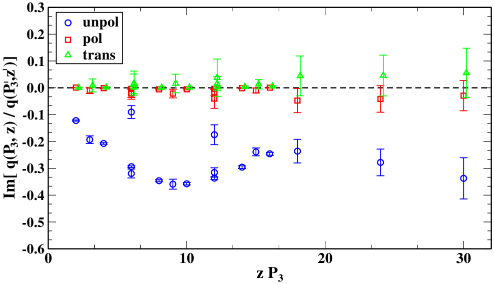
open symbols:

Twisted Mass

$m_\pi=375$MeV



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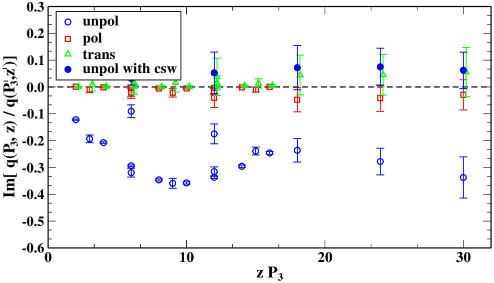


filled symbols:

Twisted Mass

& $c_{\rm SW}$

m_π=130MeV

- ★ Mixing must be treated for the unpolarized case
- ★ Presence of $c_{\rm sw}$ suppresses mixing

E

Non-Perturbative Renormalization

Non-perturbative Renormalization

- ★ **RI$'$ scheme**
- ★ **Use 1-loop conversion factor to convert to the $\overline{\mathrm{MS}}$ at 2 GeV**
- ★ **Vertex function has the same divergence as the nucleon matrix element**

No mixing

helicity, transversity, unpolarized (γ_0)

$$Z_{\mathcal{O}}(z) = \frac{Z_q}{\mathcal{V}_{\mathcal{O}}(z)}$$

$$\mathcal{V}_{\mathcal{O}} = \frac{\mathrm{Tr}}{12}\left[\mathcal{V}(p)\left(\mathcal{V}^{\mathrm{Born}}(p)\right)^{-1}\right]\Big|_{p=\bar{\mu}}$$

- ★ Z_q**: fermion field renormalization**
- ★ $Z_{\mathcal{O}}$ **includes the linear divergence**

Mixing

Unpolarized (γ_3)

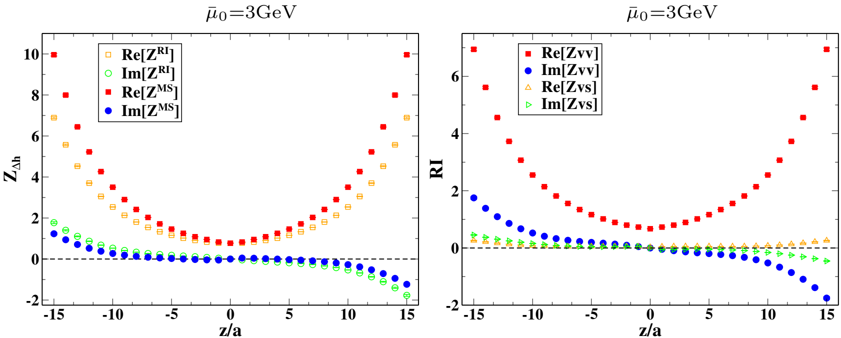
$$\begin{pmatrix}\mathcal{O}_V^R(P_3,z)\\ \mathcal{O}_S^R(P_3,z)\end{pmatrix} = \hat{Z}(z)\cdot\begin{pmatrix}\mathcal{O}_V(P_3,z)\\ \mathcal{O}_S(P_3,z)\end{pmatrix}$$

$$Z_q^{-1}\,\hat{Z}(z)\,\hat{\mathcal{V}}(p,z)\Big|_{p=\bar{\mu}} = \hat{1}$$

$$\begin{aligned} h_V^R(P_3,z) &= Z_{VV}(z)\ h_V(P_3,z)\\ &+ Z_{VS}(z)\ h_S(P_3,z)\end{aligned}$$

Numerical Results

- ★ **Twisted Mass fermions,** m_π=375MeV, $32^3 \times 64$, **HYP smearing**
- ★ **Conversion** & **Evolution to** $\overline{\mathrm{MS}}$**(2GeV)** **(Perturbatively)**



Plot from [C. Alexandrou at al., arXiv:1706.00265]

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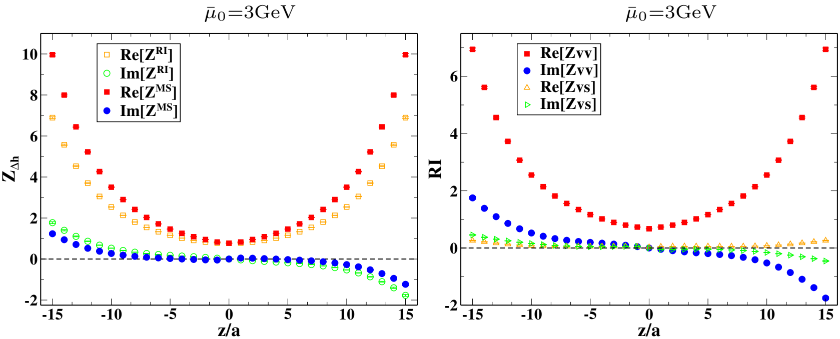

Plot from [C. Alexandrou at al., arXiv:1706.00265]

- ★ **Z-factors are complex functions**
- ★ $Im[Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}] < Im[Z_{\mathcal{O}}^{RI'}]$ **(expected from pert. theory)**

Systematic uncertainties

Ultimate goal: Reliability in final estimates

Systematic uncertainties

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↓

Systematic uncertainties need to be addresses

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Ultimate goal: Reliability in final estimates

Systematic uncertainties need to be addresses

- **Upper bounds estimated in** [C. Alexandrou et al., arXiv:1706.00265]
- **Both the ME and Z-factors are complex functions, in absence of mixing, e.g. unpolarized with γ_0 ($h \equiv h_{u-d}$):**

$$\begin{aligned} h^{ren} = Z_h\, h \quad &= \quad Re[Z_h]\, Re[h] - Im[Z_h]\, Im[h] \\ &+ \quad I\ \left(Re[Z_h]\, Im[h] + Im[Z_h]\, Re[h]\right) \end{aligned}$$

Systematic uncertainties

Ultimate goal: Reliability in final estimates

Systematic uncertainties need to be addresses

- ★ **Upper bounds estimated in** **[C. Alexandrou et al., arXiv:1706.00265]**
- ★ **Both the ME and Z-factors are complex functions, in absence of mixing, e.g. unpolarized with γ_0 ($h \equiv h_{u-d}$):**

$$\begin{aligned} h^{ren} = Z_h\, h \quad &= \quad Re[Z_h]\, Re[h] - Im[Z_h]\, Im[h] \\ &+ \quad I\ \left(Re[Z_h]\, Im[h] + Im[Z_h]\, Re[h]\right) \end{aligned}$$

- ★ **Uncertainties in Z-factors may have important implications on the final estimates for PDFs**

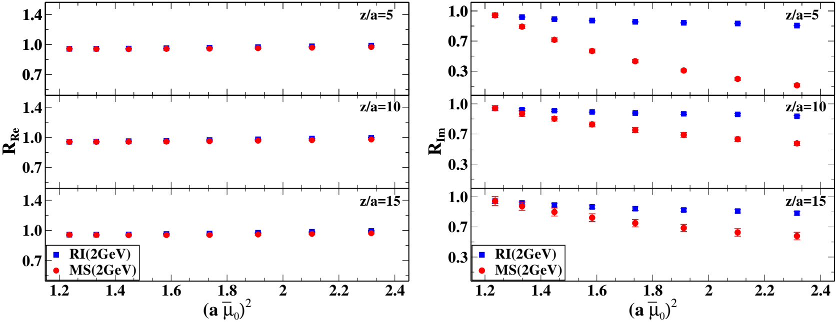
Systematic uncertainties

Truncation effects in C:

$$R^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re\,(Im)}}(z,\bar\mu_0,\bar\mu_0';\bar\mu)\equiv\frac{Z^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re\,(Im)}}(z,\bar\mu_0;\bar\mu)}{Z^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re\,(Im)}}(z,\bar\mu_0';\bar\mu)},$$

($\bar\mu_0'$=2.67GeV)

Evolution to 2 GeV in RI' and $\overline{\mathrm{MS}}$ schemes:
slope in R reveals truncation effect in conversion factor



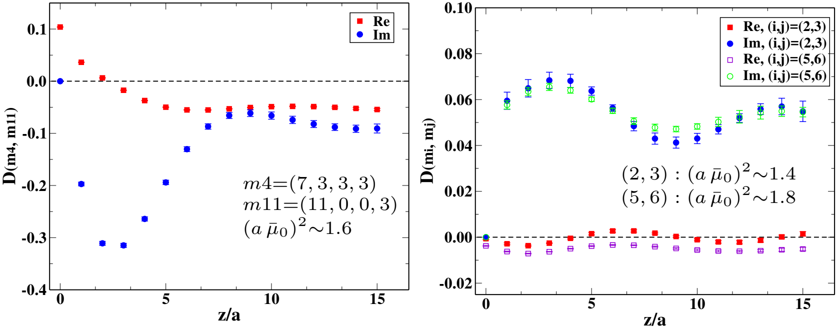
- ★ **Effect in Real part:** $\sim 2\%$
- ★ **Effect in Imaginary part:** $\sim 100\%$ ($Im[Z^{\overline{\mathrm{MS}}}]$=0 in Dim. Regul.)

Systematic uncertainties

Lattice artifacts:

$$D_{\mathrm{Re\,(Im)}}(\bar{\mu}_0,\bar{\mu}'_0) \equiv \frac{Z^{\mathrm{RI}'}_{\mathrm{Re\,(Im)}}(\bar{\mu}_0) - Z^{\mathrm{RI}'}_{\mathrm{Re\,(Im)}}(\bar{\mu}'_0)}{Z^{\mathrm{RI}'}_{\mathrm{Re\,(Im)}}(\bar{\mu}_0)}$$

Comparison of Z-factors on same $(a\,\bar{\mu}_0)^2$, different components:



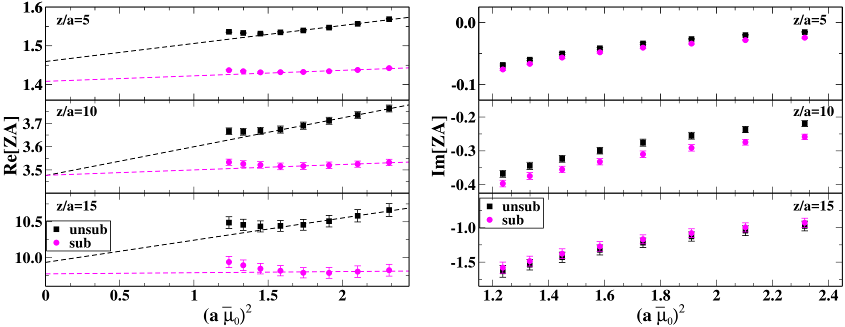
- ★ **Effect in Real part:** $\sim 2-5\%$
- ★ **Effect in Imaginary part:** $\sim 10\%$

Refining Renormalization

★ Improvement Technique:

- **Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2\, a^\infty)$**
- **Subtraction of lattice artifacts from non-perturbative estimated**

★ Application to the quasi-PDFs: PRELIMINARY



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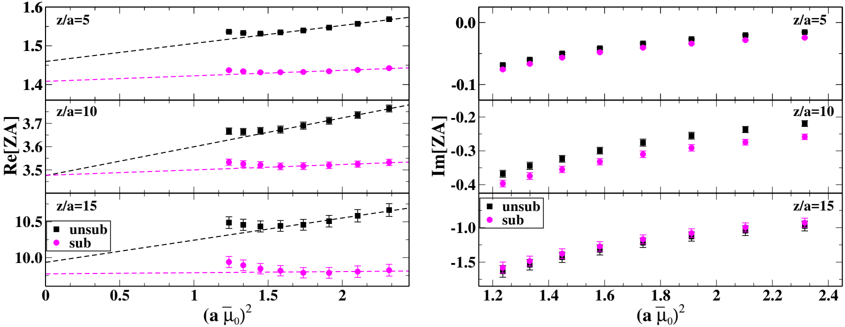

★ **Real part significantly improved**

★ **Mild change in imaginary part (expected to change with smearing)**

- Behavior might be a consequence of absence of smearing in pert. calculation

Renormalization of ensemble @ physical point

- ★ **Application of stout smearing as in ME (0, 5, 10, 15, 20 steps)**
- ★ **Calculation for a range of RI' scales:** $a\,\bar{\mu}_0 = \frac{2\pi}{48}\left(\frac{n_t}{2}+\frac{\pi}{2}, n_x, n_y, n_z\right)$

(n_t, n_x, n_y, n_z)	$(a\bar{\mu}_0)^2$	$\bar{\mu}$ (GeV)	$\hat{P}$
(8,3,3,3)	0.772	3.465	0.280
(9,3,3,3)	0.849	3.811	0.306
(10,3,3,3)	0.935	4.195	0.337
(6,4,4,4)	1.003	4.503	0.256
(7,4,4,4)	1.063	4.772	0.251
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(10,4,4,4)	1.295	5.810	0.268
(11,4,4,4)	1.389	6.233	0.283
(5,5,5,5)	1.415	6.348	0.283
(7,5,5,5)	1.526	6.848	0.261
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(11,5,5,5)	1.852	8.308	0.254
(6,6,6,6)	2.032	9.116	0.285
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fit:

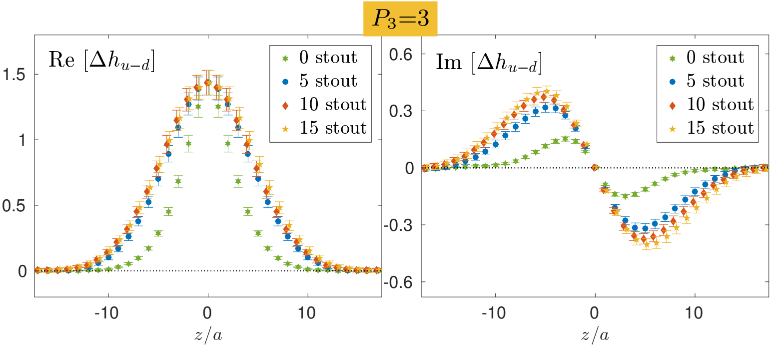
$$Z^{\overline{\rm MS}} = Z_0^{\overline{\rm MS}} + (a\bar{\mu}_0)^2\, Z_1^{\overline{\rm MS}}$$

final estimates:

$(a\bar{\mu}_0)^2 \epsilon [1 - 2.6]$

Renormalization of ensemble @ physical point

- Smearing improves the signal-to-noise ratio
- Smearing important when renormalization not available (suppresses linear divergence)
- Stout smearing used (up to 20 steps)



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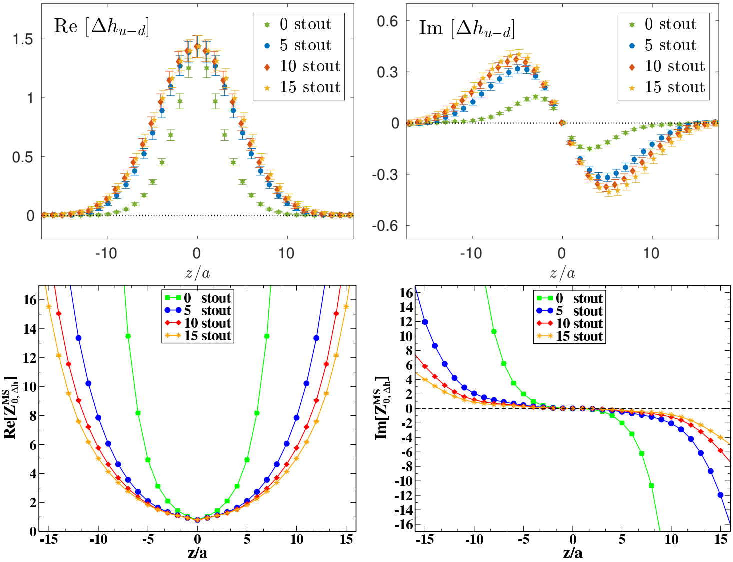
$P_3\!=\!3$



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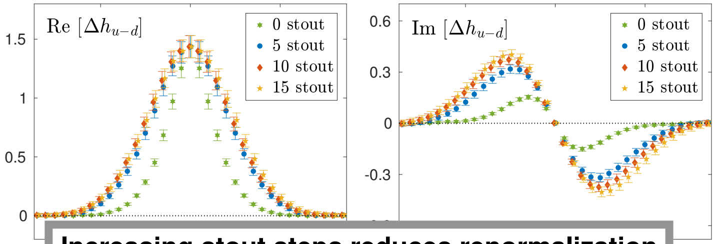
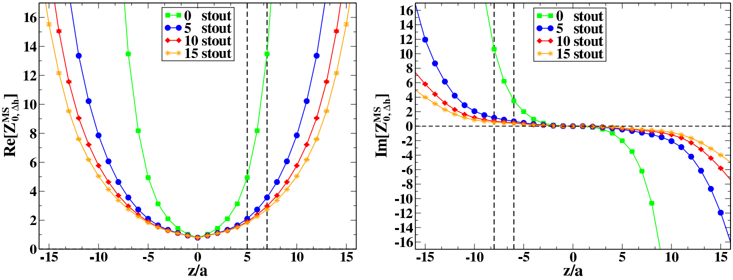


Increasing stout steps reduces renormalization

$P_3=3$



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★ **Renormalized ME must be independent of stouts steps**

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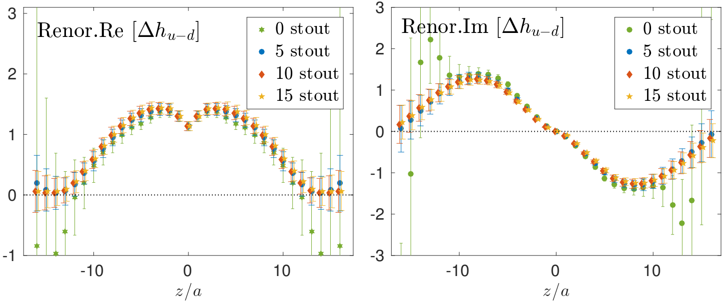

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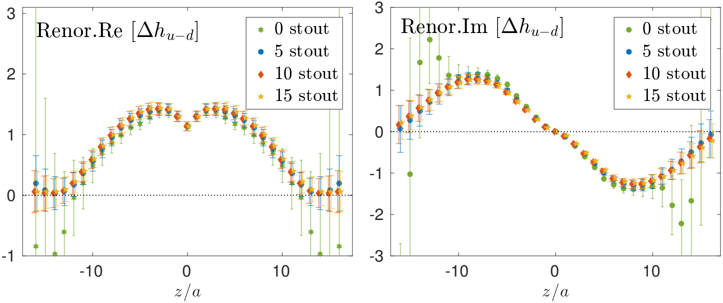

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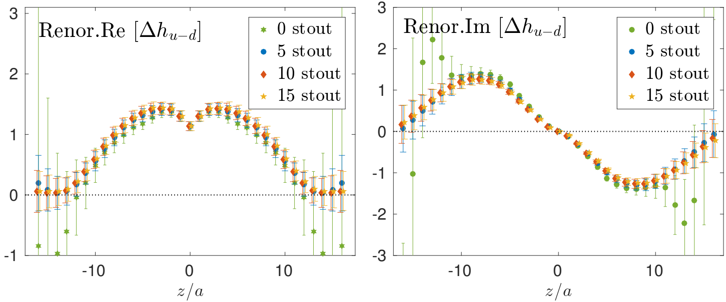

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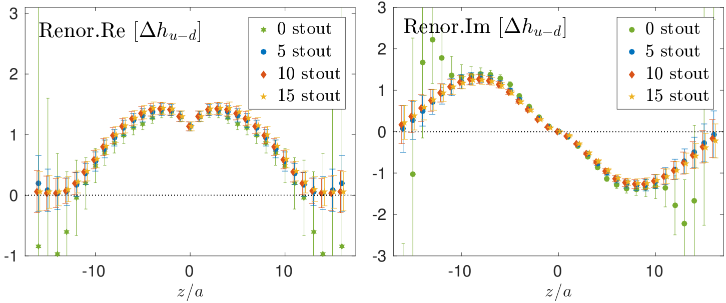

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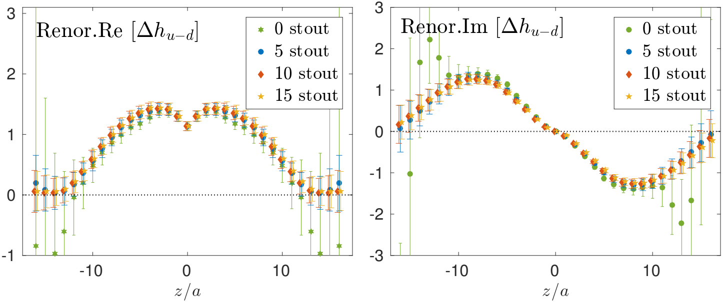

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Beyond the scope of this talk:

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- ★ Matching to Physical PDFs
- ★ Target mass corrections

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See talks by:

- ★ Fernanda Steffens, Friday @ 16:30pm
- ★ Krzysztof Cichy, Saturday @ 09:40am

E

DISCUSSION

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  Talk by K. Cichy, Saturday @ 09:40am

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  → **pert. calculation: renorm. pattern & conversion to $\overline{\mathrm{MS}}$**
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THANK YOU

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