Renormalization in large momentum effective theory

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Ji and JHZ, arXiv: 1505.07699, PRD 15'
Chen, Ji and JHZ, arXiv: 1609.08102, NPB 17'
Ji, JHZ and Zhao, arXiv: 1706.08962, PRL 18'

Lattice PDF Workshop, April 6, 2018, UMD, College Park

Large momentum effective theory

- Allows to compute light-cone or parton physics from Euclidean correlations [Ji, PRL 13', Sci. China Phys. Mech. Astron., 14']
  - Parton physics corresponds to taking $P_h \to \infty$ prior to any other scale, including the UV cutoff Λ
    - Leads to light-cone correlations
  - If $\Lambda \to \infty$ is taken prior to $P_h \to \infty$
    - *"Parton physics"* will depend on P_h
    - Not light-cone correlations, but may be calculable on the lattice
  - **The two limits differ only in UV region, they can be connected to each other by perturvatively calculable functions (asymptotic freedom of QCD)**

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  - **Similar to an effective theory setup**

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Effective theory

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    - Not light-cone correlations, but may be calculable on the lattice

Full theory

  - **Similar to an effective theory setup**
  - **Momentum of external state plays the role of perturbative d.o.f.**

Other proposals

- **Computing correlations at spacelike separations**

  - Current-current correlation functions
  - [Liu and Dong, PRL 94’]
  - [Detmold and Lin, PRD 06’]
  - [Braun and Müller, EPJC 08’]
  - [Davoudi and Savage, PRD 12’]
  - [Chambers et al., PRL 17’]

  - Lattice cross sections
  - [Ma and Qiu, 14’ & PRL 17’]

  - Ioffe-time /pseudo-distribution
  - [Radyushkin, PRD 17’]

An example: unpolarized quark PDF

- Light-cone PDF

$$q(x,\mu^2)=\int\frac{d\xi^-}{4\pi}e^{-ix\xi^-P^+}\langle P|\overline{\psi}(\xi^-)\gamma^+\exp\left(-ig\int_0^{\xi^-}d\eta^-A^+(\eta^-)\right)\psi(0)|P\rangle$$

- The quasi-PDF can be constructed as [Ji, PRL 13′]

$$\tilde{q}(x,\Lambda,P^z)=\int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{izk^z}\langle P|\overline{\psi}(0,0_\perp,z)\gamma^z\exp\left(-ig\int_0^z dz'A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - z is a spatial direction
  - It approaches $q(x)$ in the limit $P_z\to\infty$
  - An alternative choice is to replace $\gamma^z\to\gamma^0$

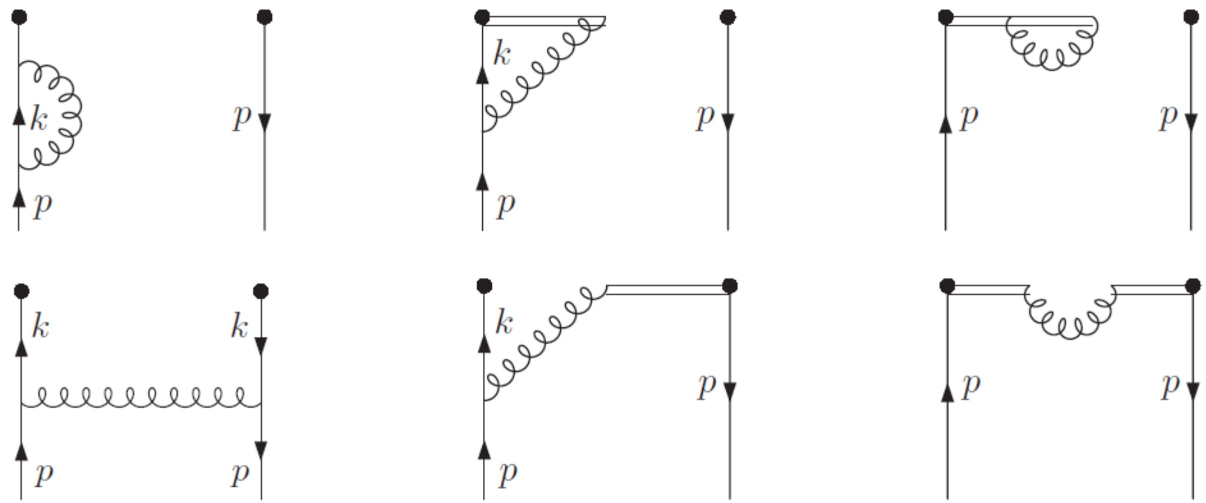
- Factorization (for bare quantities) [Ji, PRL 13′, Xiong, Ji, JHZ and Zhao, PRD 13′]

$$\tilde{q}(x,\Lambda,P^z)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\Lambda}{P^z},\frac{\mu}{P^z}\right)q(y,\mu)+\mathcal{O}\left(\Lambda_{\rm QCD}^2/(P^z)^2,M^2/(P^z)^2\right)$$

- Further refined in [Chen, Ji and JHZ, NPB 17′, Stewart and Zhao, PRD 17′, Izubuchi, Ji, Jin, Stewart and Zhao, 18′; see also Ma and Qiu, PRL 18′]

Renormalization of quasi-PDF

- Renormalization of PDF has been well studied [See e.g. Collins' book]
- Quasi-PDF renormalization
  - DR [Ji and JHZ, PRD 15']



  - Self-energy diagrams contain usual UV divergences
    - Renormalization similar to heavy-light current renormalization
  - Vertex diagrams
    - Momentum fraction x extends between $[-\infty, +\infty]$, and is left unintegrated
    - Power of UV divergences reduced, UV convergent@1-loop

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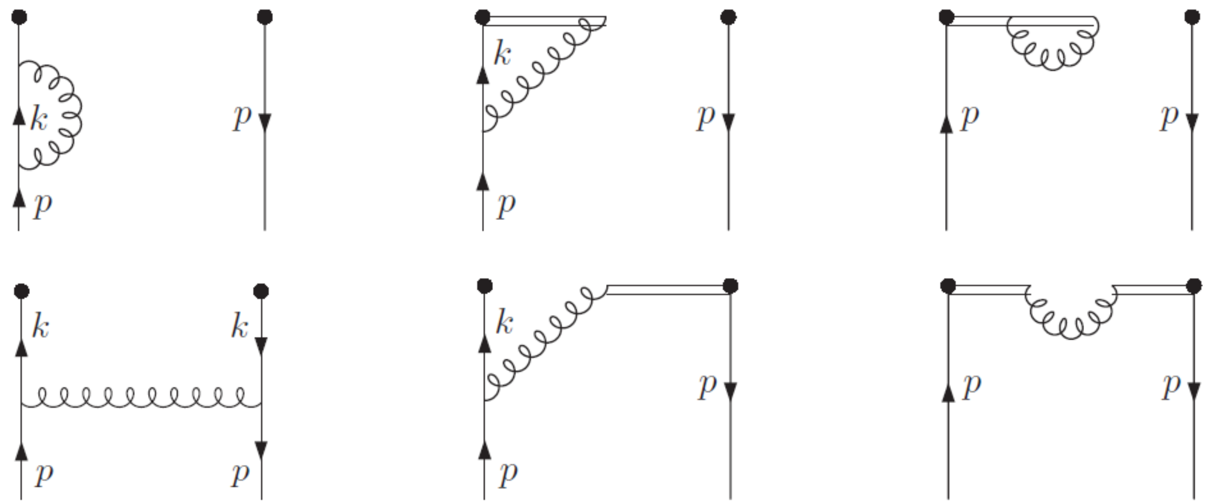

  - Renormalization@1-loop

$$\tilde{q}_R(x, p^z) = \int \frac{dy}{|y|}[\tilde{Z}_F Z(\frac{x}{y})][\tilde{Z}_F^{-1}\tilde{q}(y, p^z)]$$

with

$$Z(\eta) = \delta(\eta - 1)(1 + Z^{(1)}) = \delta(\eta - 1) - \frac{3\alpha_S C_F S_\epsilon}{4\pi}\frac{1}{\epsilon}\delta(\eta - 1)$$

Renormalization of quasi-PDF

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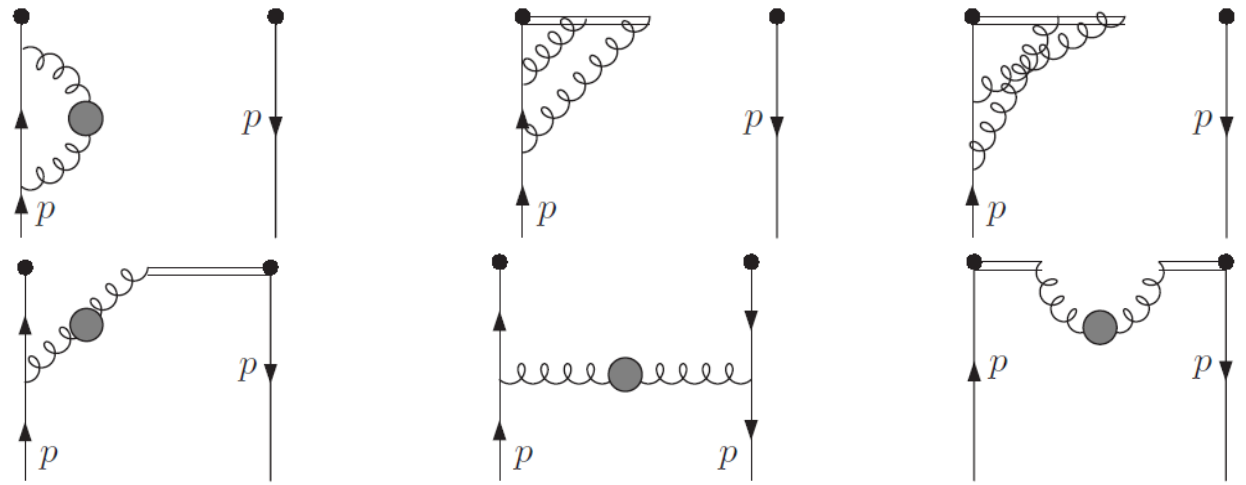
  - Example diagrams@2-loop



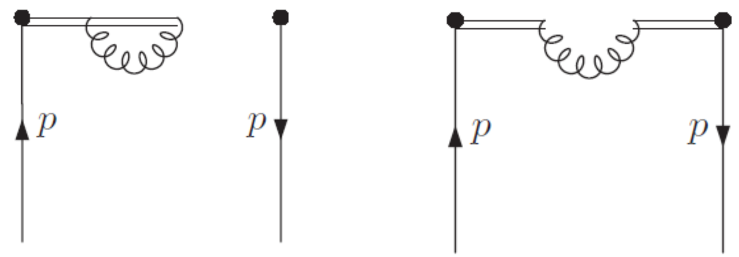
  - No overall UV divergence, only subdivergence in vertex diagrams
    - Can be removed by counterterms from interaction
  - Renormalization of self-energy

$$Z^{(2)}(\eta) = [(\frac{\alpha_S}{4\pi})^2 S_\epsilon^2(\frac{a}{\epsilon^2} + \frac{b}{\epsilon}) + (Z^{(1)})^2]\delta(\eta - 1)$$

  - Multiplicative renormalization up to two-loop

Renormalization of power divergence

- Power divergence is hidden in DR, it comes from Wilson line self energy [Ishikawa, Ma, Qiu and Yoshida, 16′, Chen, Ji and JHZ, 16′; Monahan and Orginos, JHEP 17′]
  - @1-loop, a linear divergence is associated with



  - It is well-known that such a linear divergence can be removed by a mass renormalization [Polyakov, NPB 80′, Dotsenko and Vergeles, NPB 80′, Dorn, Fortsch. Phys. 86′ (auxiliary *z*-field formalism)]
  - In a sense, the auxiliary field can be understood as a Wilson line extending between $[z, \infty]$

$$Z(z) = L(z, \infty) \qquad\qquad [\partial_z - igA_z(z)]\, Z(z) = 0$$

  - Analogous to a heavy quark field

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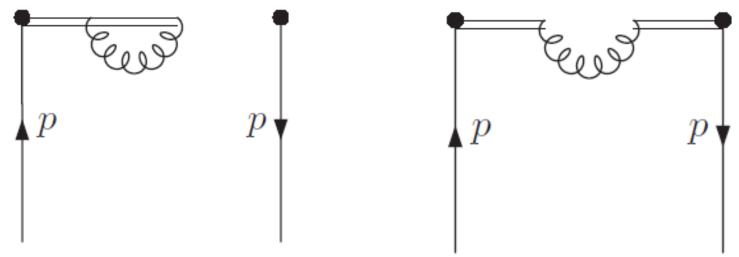

  - Non-local Wilson line can be interpreted as a two-point function of z-field

$$L(z,0) = Z(z)Z^{\dagger}(0)$$

  - Renormalizes analogously to a heavy quark two-point function [Dotsenko and Vergeles, NPB 80', Dorn, Fortsch. Phys. 86']

$$L^{\mathrm{ren}}(z,0) = \mathcal{Z}_Z^{-1} e^{-\delta m|z|} L(z,0)$$

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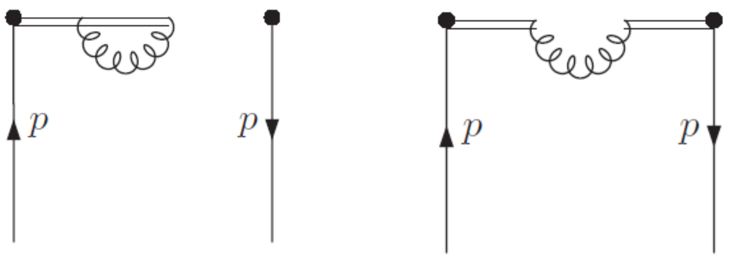


  - The Wilson line self energy diagram gives ($\bar{x}$=1-x)

$$\lim_{\epsilon\to 0}\int dk_z \frac{\alpha_s C_F \Lambda}{2\pi}\frac{[\delta(k_z-\bar{x}p_z)-\delta(\bar{x}p_z)]p_z}{k_z^2+\epsilon^2}$$

  - Mass counterterm contributes [Chen, Ji and JHZ, 16']

$$-\int \frac{dz}{2\pi}p_z\, e^{i(x-1)p_z z}|z|\,\delta m = -\lim_{\epsilon\to 0}\int \frac{dz}{2\pi}p_z\, e^{-i\bar{x}p_z z}\frac{1-e^{-\epsilon|z|}}{\epsilon}\delta m$$
$$= -\lim_{\epsilon\to 0}\int \frac{dk_z}{\pi}p_z\frac{\delta(\bar{x}p_z)-\delta(k_z-\bar{x}p_z)}{k_z^2+\epsilon^2}\delta m.$$

  - $\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda)$ is gauge-independent

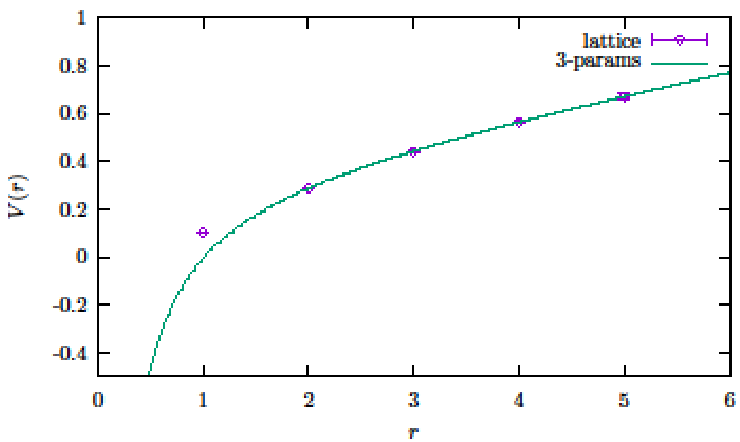
Non-perturbative determination of δm

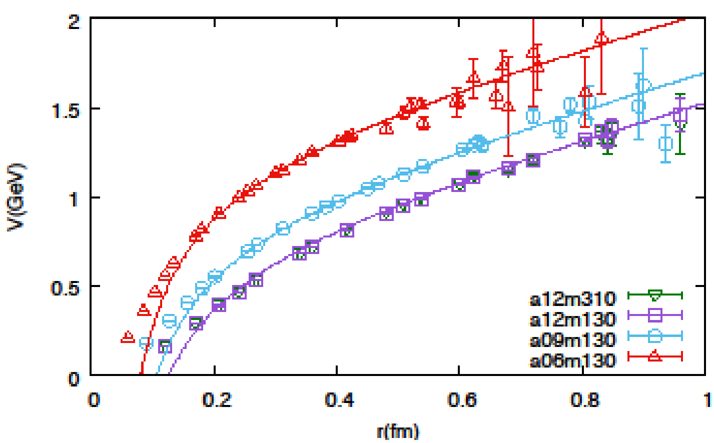
- Can be done following [Musch, Hägler, Negele and Schäfer, PRD 11']
  - Static heavy quark-antiquark potential can be obtained from asymptotic behavior of a rectangular Wilson loop

$$W(R,T) = c(R)e^{-V(R)T} + \text{higher excitations},$$

  - Choose a Wilson loop long in t-direction such that higher excitations are sufficiently suppressed
  - Fit the quark potential [JHZ, Chen, Ji, Jin and Lin, PRD 17', LP3, 17']

$$V(r) = -\frac{1}{a}\lim_{t\to\infty} \ln \frac{\langle \mathrm{Tr}[W(t,r)]\rangle}{\langle \mathrm{Tr}[W(t-a,r)]\rangle} \quad \text{to} \quad V(r) = \frac{c_1}{r} + c_2 + c_3 r$$





  - $\delta m = -c_2/2 \approx -253 \pm 3$ MeV

Implementation of mass renormalization

- We can define an improved quasi-PDF without power divergence [Chen, Ji and JHZ, NPB 17']

$$\tilde{q}_{\rm imp}(x,\Lambda,p^z)=\int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{izk^z-\delta m|z|}\langle p|\overline{\psi}(0,0_\perp,z)\gamma^z L(z,0)\psi(0)|p\rangle$$

- Similarly for the pion quasi-DA [JHZ, Chen, Ji, Jin and Lin, PRD 17']

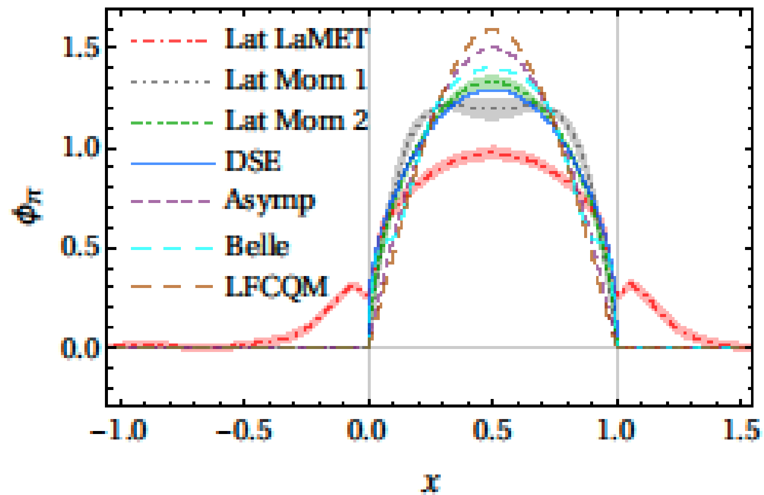
$$\tilde{\phi}_{\rm imp}(x,P_z)=\frac{i}{f_\pi}\int\frac{dz}{2\pi}e^{-i(x-1)P_z z-\delta m|z|}\langle\pi(P)|\bar{\psi}(0)\gamma^z\gamma_5\Gamma(0,z)\psi(z)|0\rangle$$

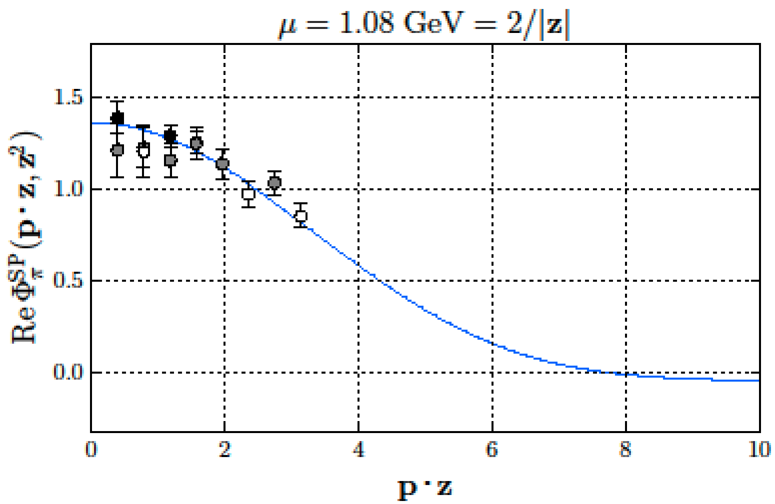
- Apart from the exponential mass renormalization to remove power divergence, there supposed to be other renormalization factors at the endpoint, which are local

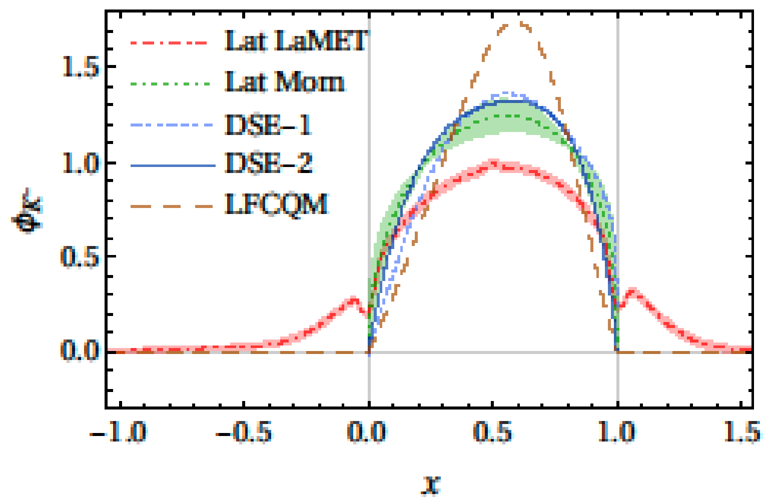
- Normalization condition roughly means an implementation of renormalization

Results with mass renormalization

- Meson distribution amplitudes [LP3, 17′]

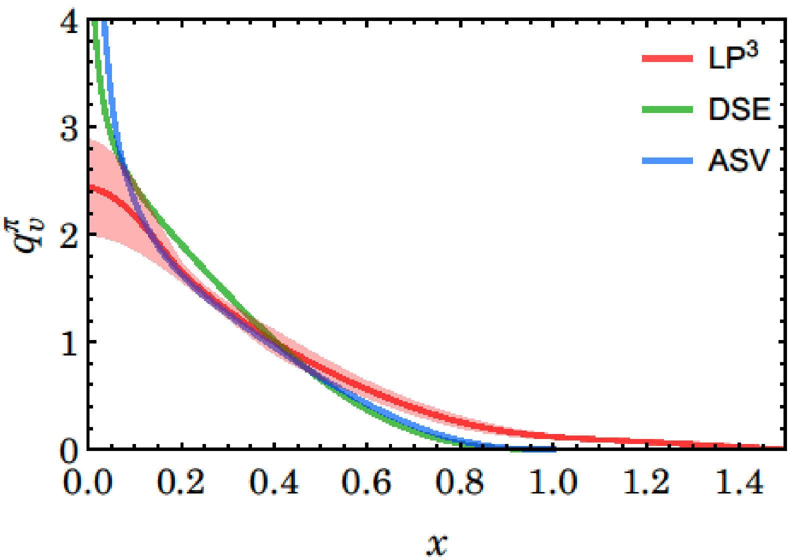


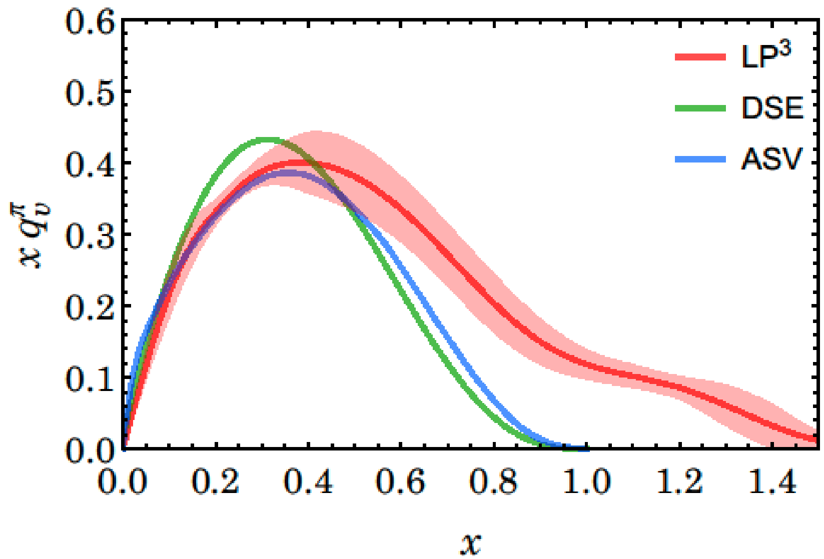




Results with mass renormalization

- Meson PDF [LP3, 18′]





- To be improved with larger momentum, physical pion mass, higher-order matching ……

Complete renormalization of quasi-PDF

- For the non-local quark bilinear operator

$$O(x,y) = \overline{\psi}(x)\Gamma L(x,y)\psi(y)$$

- We are motivated to introduce the following auxiliary heavy quark Lagrangian [Ji, JHZ and Zhao, PRL 18']

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{Q}(x) in \cdot D Q(x)$$

  - For a real heavy quark, n is timelike, and Q is a dynamical field
  - For an auxiliary heavy quark, n is spacelike, no dynamical evolution

  - After integrating out the heavy quark field, we have

$$\int \mathcal{D}\overline{Q}\mathcal{D}Q\, Q(x)\overline{Q}(y) e^{i\int d^4x\mathcal{L}} = S_Q(x,y) e^{i\int d^4x \mathcal{L}_{\rm QCD}}$$

up to a constant that can be absorbed into the overall normalization

Complete renormalization of quasi-PDF

- $S_Q(x, y)$ satisfies

$$n \cdot D\, S_Q(x, y) = \delta^{(4)}(x - y),$$

  with the solution

$$S_Q(x, y) = \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x, y)$$
$$= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x^z, y^z)$$

  - δ-function ensures that the time and transverse components are equal, and thereby generates a spacelike Wilson line

- We can do the replacement (restrict to $x^z > y^z$ for the moment)

$$O(x, y) = \overline{\psi}(x)\Gamma Q(x)\overline{Q}(y)\psi(y)$$

  - The non-local operator can be replaced by a product of two heavy-light currents

Complete renormalization of quasi-PDF

- Renormalization in HQET [Maiani et al., NPB 92′]
  - The HQET Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{Q}(x) i n \cdot D Q(x)$$

  - Takes infinite heavy quark mass limit, and does not contain any mass term
  - In a cutoff regularization like lattice regularization, heavy quark self-energy does generate a linear divergence, which has to be absorbed into an effective mass counterterm

$$\delta \mathcal{L}_m = -\delta m \overline{Q} Q$$

with $\delta m \sim 1/a$

Complete renormalization of quasi-PDF

- Renormalization in HQET [Maiani et al., NPB 92′]
  - Infinitely heavy quark behaves like a static color source, its energy will have a Coulomb-like form $1/r$, and diverges linearly if the source is a pointlike particle
  - This physical picture is lost for an auxiliary heavy quark, but the linear divergence can be removed in the same way
  - The mass counterterm shall not be understood as a physical mass, but as a parameter with mass dimension
  - The total heavy quark Lagrangian now becomes

$$\mathcal{L}_Q = \bar{Q}(in \cdot D - \delta m)Q$$

  - Integrating over the auxiliary heavy quark field, our non-local operator renormalization becomes [Ji, JHZ and Zhao, PRL 18′]

$$O_R = Z_{\bar{j}}^{-1} Z_j^{-1} e^{\delta \bar{m}|z_2 - z_1|} \overline{\psi}(z_2) \Gamma L(z_2, z_1) \psi(z_1)$$

  - See also [Ishikawa, Ma, Qiu and Yoshida, PRD 17′, Green, Jansen and Steffens, 17′]

Practical implementation

- In practice, the overall renormalization factor can be determined as a whole
  - Nonperturbative renormalization in RI/MOM scheme
  - For a local operator

$$O_\Gamma = \bar{\psi}\Gamma\psi$$

  - The renormalization factor is defined as

$$Z_O \langle p|O_\Gamma|p\rangle_{p^2=\mu^2} = \langle p|O_\Gamma|p\rangle_{\text{free}}$$

  - Generalization to non-local operator [Alexandrou et al., NPB 17', Stewart and Zhao, PRD 18']

$$\tilde{Z}^{\text{OM}}(z, p^z, \Lambda, \mu_R)^{-1} \sum_s \langle ps|\bar{\psi}(z)\gamma^z W(z,0)\psi(0)|ps\rangle\Big|_{p^2=-\mu_R^2}$$
$$= \sum_s \langle ps|\bar{\psi}(z)\gamma^z W(z,0)\psi(0)|ps\rangle\Big|_{\text{tree}}$$

  - Subtraction also for UV finite contributions, renormalization factor is in general complex
  - Renormalization factor in previous slide corresponds to minimal subtraction and is real

Practical implementation

- In practice, the overall renormalization factor can be determined as a whole
  - Multiplicative renormalization factor is independent of external momentum

$$O_R = Z_{\bar{j}}^{-1} Z_{j}^{-1} e^{\delta \bar{m}|z_2 - z_1|} \overline{\psi}(z_2) \Gamma L(z_2, z_1) \psi(z_1)$$

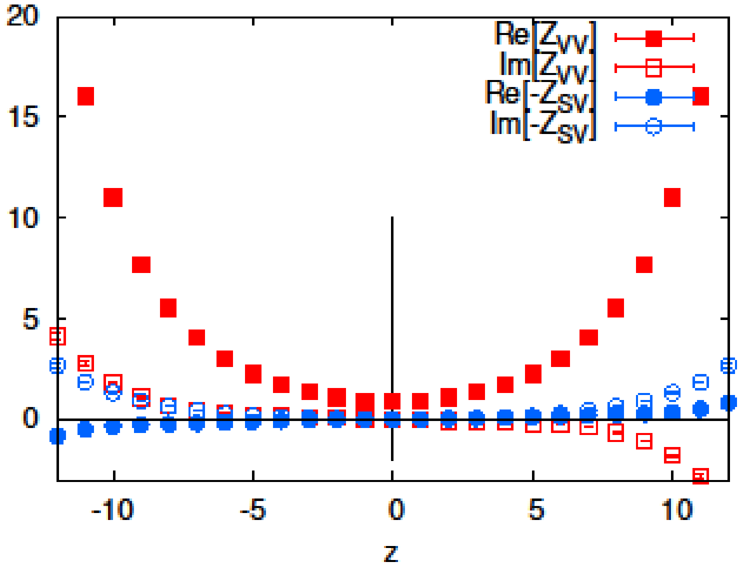
  - By taking the ratio of coordinate space matrix elements for the quasi-PDF at two different momenta, the renormalization factor is completely canceled out [Radyushkin, PRD 17', Orginos et al., PRD 17']

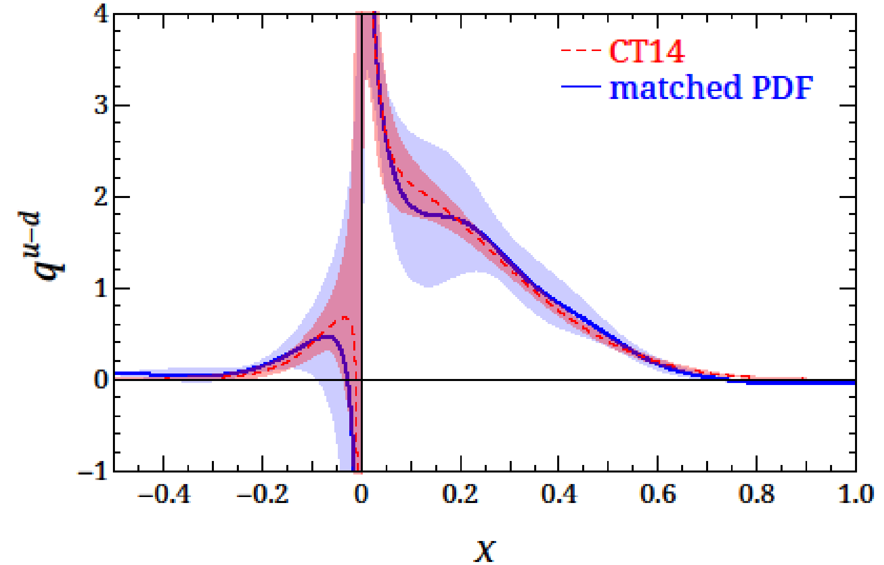
  - Such a ratio can also be factorized into the PDF and a hard kernel [Radyushkin 18', JHZ, Chen and Monahan, 18', Izubuchi, Ji, Jin, Stewart and Zhao, 18']

  - and related either to quasi- or to pseudo-PDF

Result with complete nonperturb. renorm.

- RI/MOM implementation (unpol. isovector quark PDF) [LP3, 17′&18′]





- Exponential increase of renormalization factor at large distance
- Agreement with global analysis within errors

Summary and outlook

- **Large momentum effective theory** opens a new door for *ab initio* studies of hadron structure

- It has been applied to computing dynamical properties of hadrons like PDFs, DAs, and yields encouraging results
  - Renormalization as well as factorization of quasi-PDF to all-orders

- Much more to explore
  - GPDs, TMDs, Wigner distribution, spin structure of nucleon ……

- Future improvement
  - Finer lattice spacing, larger momentum and volume
  - Higher-order matching kernel
  - Higher-twist contribution

Backup Slides