Bayesian analysis for heavy quark studies

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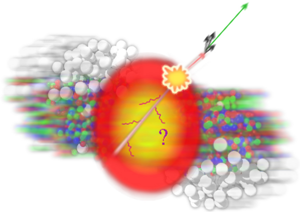


www2.lbl.gov/Science-Articles/Archive/sabl/2008/Feb/jets.html

A modeling problem

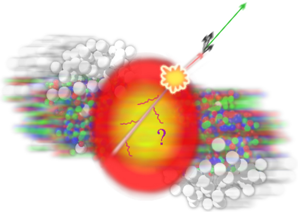
input (parameters)
e.g. diffusion coefficients
$D_s 2\pi T(\alpha), \hat{q}(\alpha_s), \cdots$

model
HQ transport model
Langevin, Boltzmann ...

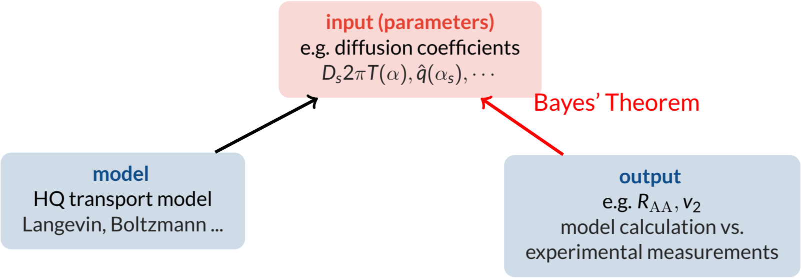
outputs
e.g. R_{AA}, v_2
model calculation vs.
experimental measurements

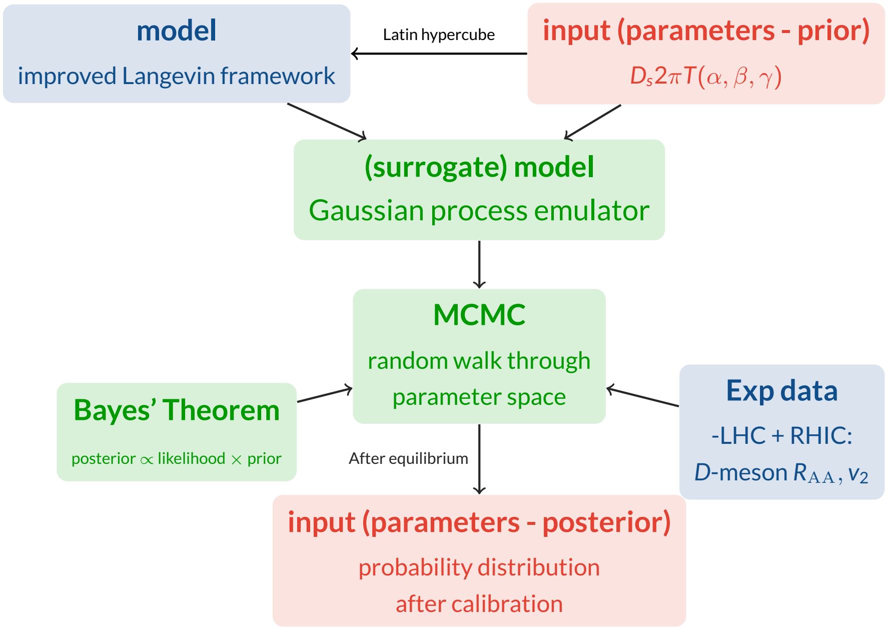
Heavy quark evolution in heavy-ion collisions

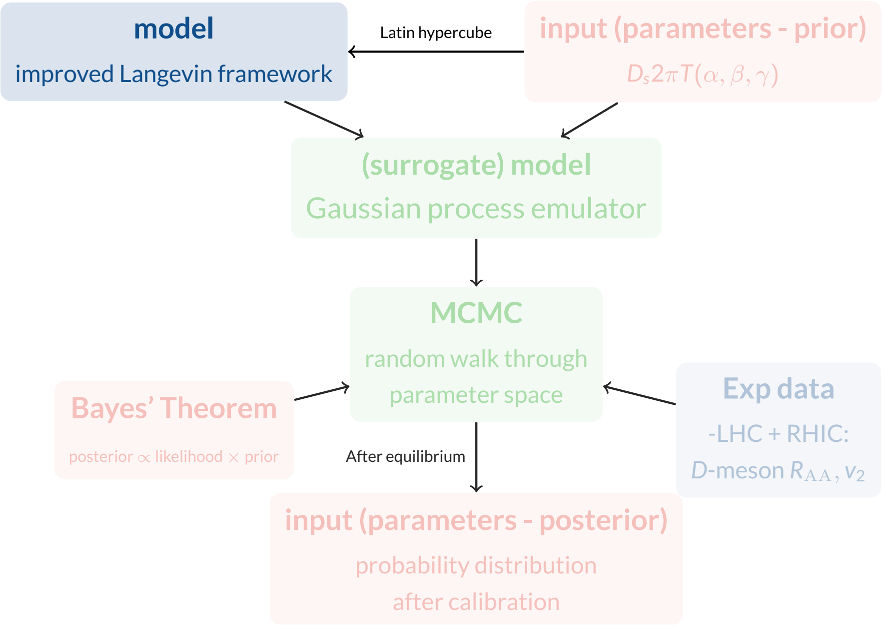
www2.lbl.gov/Science-Articles/Archive/sabl/2008/Feb/jets.html



A modeling problem (data-driven)




model
improved Langevin framework
Latin hypercube
input (parameters - prior)
$D_s 2\pi T(\alpha, \beta, \gamma)$
(surrogate) model
Gaussian process emulator
MCMC
random walk through
parameter space
Bayes' Theorem
posterior $\propto$ likelihood $\times$ prior
Exp data
-LHC + RHIC:
D-meson R_{AA}, v_2
After equilibrium
input (parameters - posterior)
probability distribution
after calibration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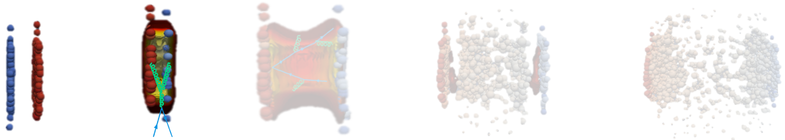
Model: initial condition

figure credit: Hannah Petersen



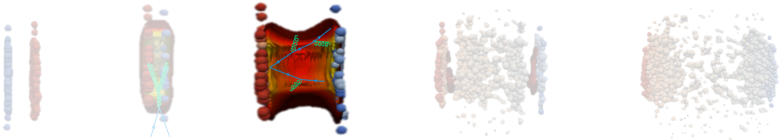
- **Heavy quark initial positions**:
  - scaled by the distribution of binary collisions
- **Heavy quark initial momentum**:
  - FONLL
  - nuclear shadowing effect: EPS09 NLO

M.Cacciari,S.Frixione, and P.Nason, arxiv:hep-ph/0102134

- **Soft matter**: $\mathrm{T_RENTo}$
  - parametric model of initial entropy deposition
  - consistent with a saturation-based picture: e.g. IP-Glasma

J.S.Moreland,J.Bernhard, and S.A.Bass, Phys.Rev.C 92, 011901(2015)

Model: in-medium evolution

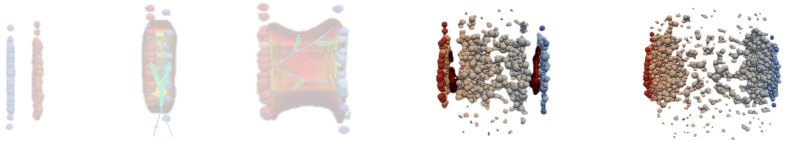


- **Improved Langevin transport model:** S.Cao, G.Qin, and S.A.Bass, Phys.Rev.C 92, 024907(2015)

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f_g}$$

  - Drag force: $\eta_D(p) = \hat{q}/(4TE)$
  - Thermal random force: $\langle \xi^i(t)\xi^j(t')\rangle = \frac{1}{2}\hat{q}\delta^{ij}\delta(t-t')$
  - Recoil force from gluon radiation: $\vec{f_g}(\hat{q}) = -d\vec{p_g}/dt$
  - Diffusion coefficient $D_s = 4T^2/\hat{q} \Rightarrow ?$

Model: hadronization + afterburner



- **Hadronization:**

  Fragmentation (PYTHIA) + recombination

- **Late stage hadronic interactions:**

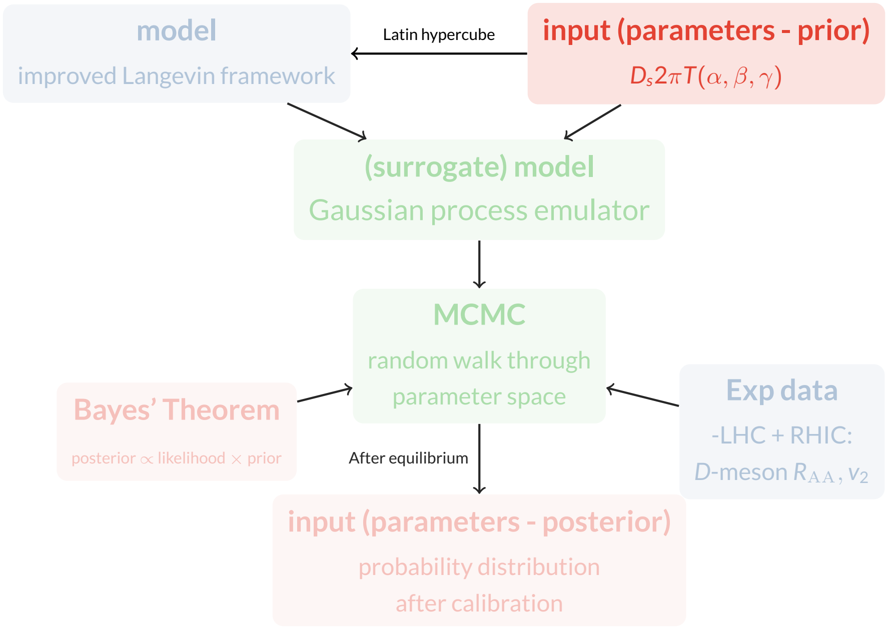
  UrQMD

- **QGP medium evolution:**

  H.Song and U.W.Heinz, Phys.Rev.C 77, 064901(2008)

  - (2+1)D viscous hydrodynamics model: iEbE-VISHNU
  - Shear and bulk viscosities: $\eta/s(T), \zeta/s(T)$
  - All the soft medium related parameters $(\mathrm{normalization}, \eta/s, \zeta/s$ etc.) are calibrated on soft hadronic observables

  J.Bernhard,J.S.Moreland,S.A.Bass,J.Liu, and U.Heinz Phys.Rev.C 94, 024907(2015)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(surrogate) model
Gaussian process emulator
MCMC
random walk through
parameter space
Bayes' Theorem
posterior $\propto$ likelihood $\times$ prior
Exp data
-LHC + RHIC:
D-meson R_{AA}, v_2
After equilibrium
input (parameters - posterior)
probability distribution
after calibration

Input: HQ diffusion coefficient

Parametrization of $D_s 2\pi T(T, p; \alpha, \beta, \gamma)$:

- A combination of linear T-dependent and pQCD calculation

$$D_s 2\pi T(T,p) = \frac{1}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{\mathrm{soft}} + \frac{(\gamma^2 p)^2}{1+(\gamma^2 p)^2}(D_s 2\pi T)^{\mathrm{pQCD}}$$

- $(D_s 2\pi T)^{\mathrm{soft}}$: the linear-$T$ component

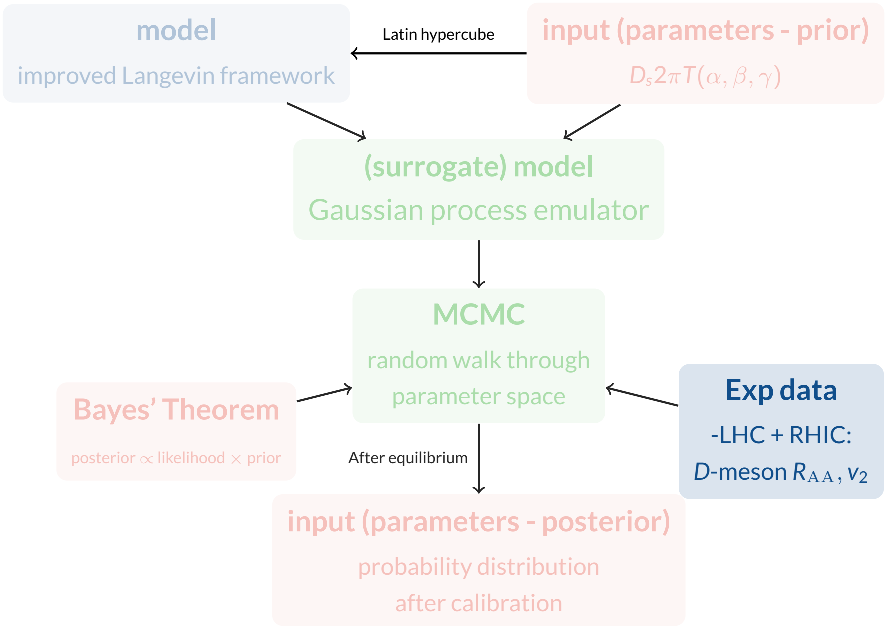
$$(D_s 2\pi T)^{\mathrm{soft}} = \alpha \cdot \left[1 + \beta \cdot \left(\frac{T}{T_c} - 1\right)\right]$$

- $(D_s 2\pi T)^{\mathrm{pQCD}} = 8\pi T^3 / \hat{q}^{\mathrm{pQCD}}$: the pQCD component

$$\hat{q}^{\mathrm{pQCD}} = \left\langle (\vec{p}_{Q'})^2 - (\hat{p}_Q \cdot \vec{p}_{Q'})^2 \right\rangle \propto T^3 \ln\left(\frac{ET}{m_D^2}\right)$$

where $\langle X \rangle = \frac{\gamma_q}{2E_Q} \int f_q(p_q) |\mathcal{M}|^2_{2\to 2} \cdot X$

What is $D_s 2\pi T \Rightarrow$ what is (α, β, γ)?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After equilibrium
input (parameters - posterior)
probability distribution
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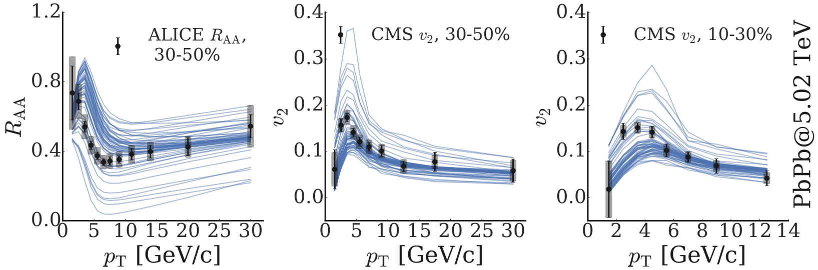
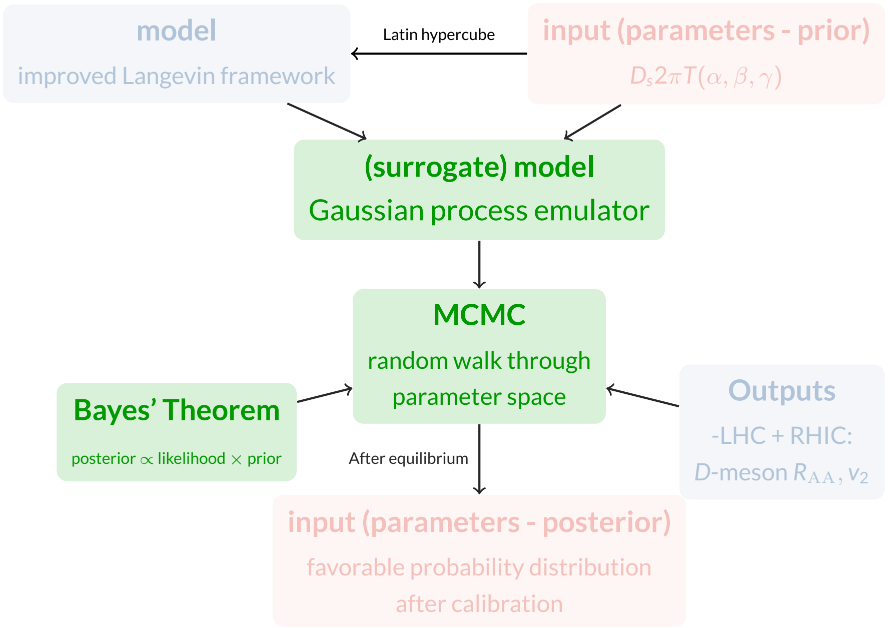
Output: experimental observables

D-meson R_{AA}, v_2 at RHIC and the LHC:

Experiment	variables	centrality/p_T cut
PbPb@5.02 TeV	$R_{\mathrm{AA}}(p_T)$	30-50%
	$v_2(p_T)$	10-30%, 30-50%
PbPb@2.76 TeV	$R_{\mathrm{AA}}(n_{part})$	$p_T \sim$ 5-8, 8-16 GeV/c
	$v_2(p_T)$	30-50%
AuAu@200 GeV	$R_{\mathrm{AA}}(p_T)$	0-10%
	$v_2(p_T)$	0-80%,10-40%



STAR: arxiv 1701.06060, 1601.00695
ALICE: arxiv 1506.06604, PRC **90**, 034904 (2014)
ALICE: prelim at QM'17
CMS: prelim CMS-PAS-HIN-16-007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Outputs
-LHC + RHIC:
D-meson R_{AA}, v_2
After equilibrium
input (parameters - posterior)
favorable probability distribution
after calibration

Baye's Theorem & MCMC

Bayes' Theorem

posterior $\propto$ prior $\times$ likelihood

- posterior: distribution of input parameters favored by experimental observations
- prior: initial distribution of input parameters – uniform
- likelihood: $\propto \exp\left[(\boldsymbol{y} - \boldsymbol{y}_{\rm exp})^T \Sigma^{-1} (\boldsymbol{y} - \boldsymbol{y}_{\rm exp})\right]$
  - $\boldsymbol{y}$ is the model's output at given input parameter $\boldsymbol{x} = (\alpha, \beta, \gamma)$
  - $\Sigma = {\rm diag}(\sigma^2_{\rm stat}) + {\rm diag}(\sigma^2_{\rm sys}) + \sigma^2_{\rm model}$, covariance matrix

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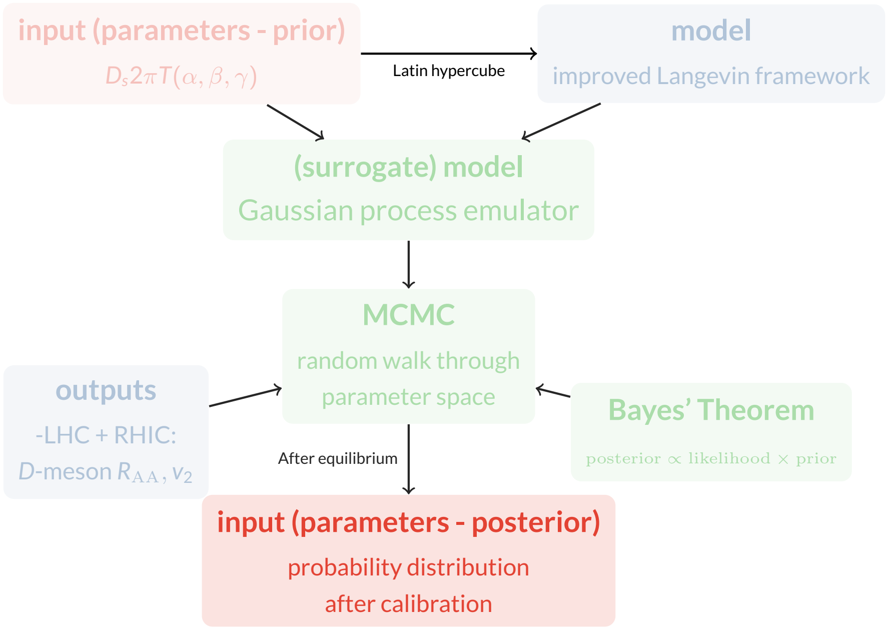
Markov chain Monte Carlo (MCMC)

- Random walk weighted by likelihood
- When reach equilibrium $\rightarrow$ posterior distribution of parameters

Gaussian process emulator

- Surrogate model to real Langevin model $\Leftarrow$ necessary for computationally expensive model
- Returns not only prediction, also the uncertainty $\sigma_{\rm model}$
- We make sure it works!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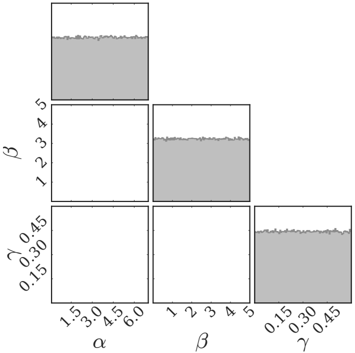
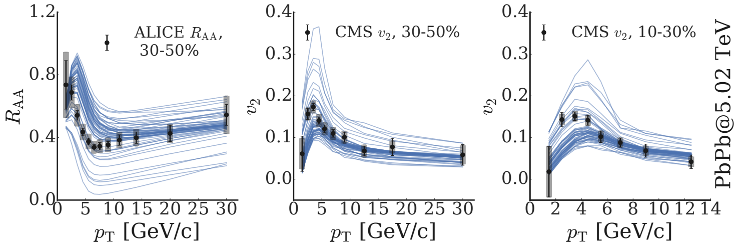
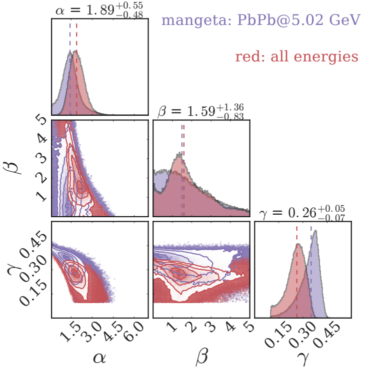
Results: before calibration (prior)



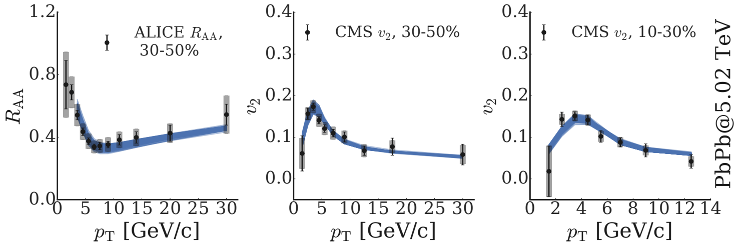
- $\alpha \in (0.1, 7.0), \beta \in (0, 5.0),$
  $\gamma \in (0, 0.6)$
- Uniform prior distributions
- Model outputs: widely spread



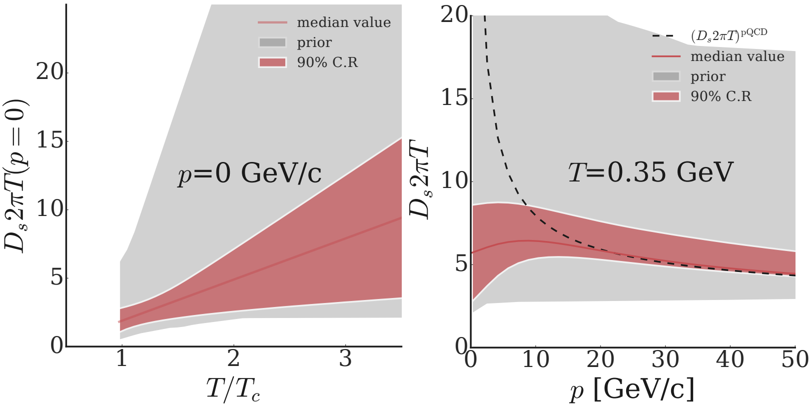
Results: after calibration (posterior)



- (5-95)% percentile: $\alpha \in (1.14, 2.84)$, $\beta \in (0.24, 4.04)$, $\gamma \in (0.14, 0.34)$
- Distinguish peak for α, γ; negative correlation between α, β
- Model output: calibrated on experimental data
- Distribution width $\Leftarrow$ uncertainties: how to improve it?

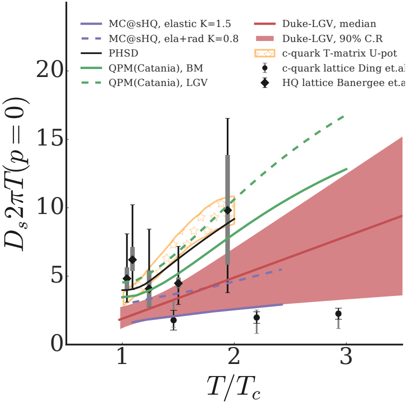


Results: after calibration $D_s 2\pi T$



- Zero momentum $D_s 2\pi T$ has positive temperature dependence, mostly constrained between 1-2 T_c
- Different behavior from $(D_s 2\pi T)^{\mathrm{pQCD}}$ until mid-p_T $\Leftarrow$ significant non-perturbative effects

Results: this work vs. other models

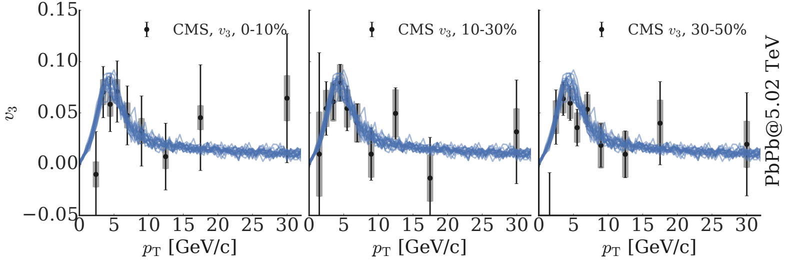


T. Song, H. Berrehrah, D. Cabrera, W. Cassing and E. Bratkovskaya, Phys.Rev.**C93**,034906
F. Scardina, S. K.Das, V. Minissale, S. Plumari and V. Greco, arXiv:1707.05452.
F. Riek and R.Rapp, Phys.Rev.**C82**,035201
P. Gossiaux and J. Aichelin, Phys.Rev.**C78**, 014904
H. T. Ding *et al.* Phys. Rev.**D86**,014509
D. Banerjee, S. Datta, R. Gavai, and P. Majumdar,Phys.Rev.**D85**,014510

- Comparing with other model's calculation, minimum of (1-3) around T_c
- Most models are converging with a minimum of (1-7) near T_c

What's next?

- **Observables beyond R_{AA}, v_2:**
  D-meson v_3 in PbPb collisions at 5.02 TeV:



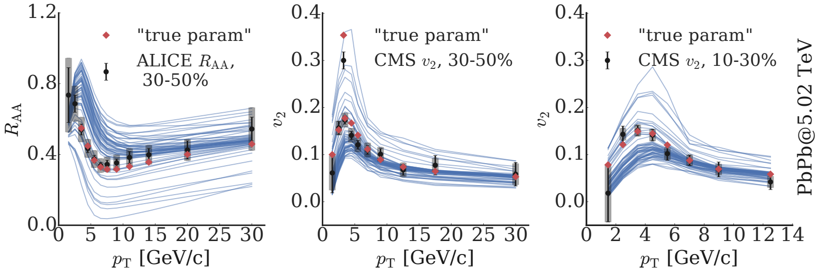
What's next?

- **Observables beyond R_{AA}, v_2**
- **Improvement on constraints on transport coefficients:**
  - Pursue higher precision on current observables
  - Explore other observables

Higher precision?

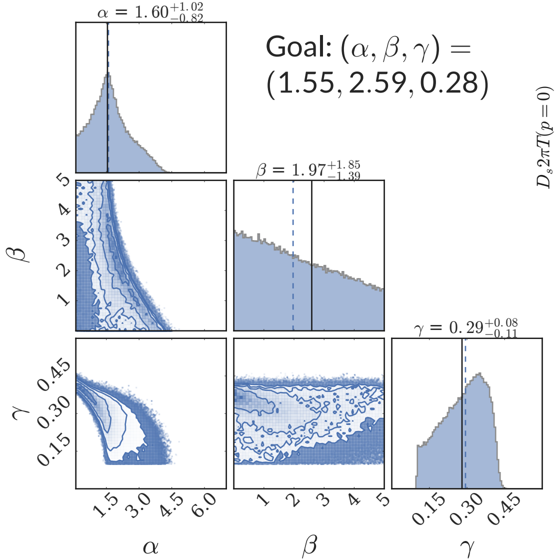
Question: To which degree do we mean high precision?

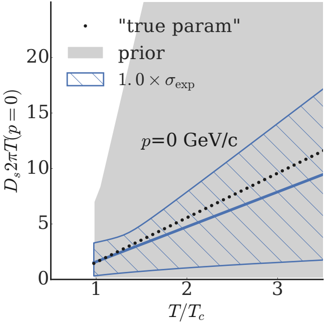
- **Test**: suppose our model describes perfectly the physics of heavy quarks; suppose a true parameter set exists, how accurate the data need to be to recover it from just R_{AA} and v_2:
- A set of parameter: $(\alpha, \beta, \gamma) = (1.55, 2.59, 0.28)$



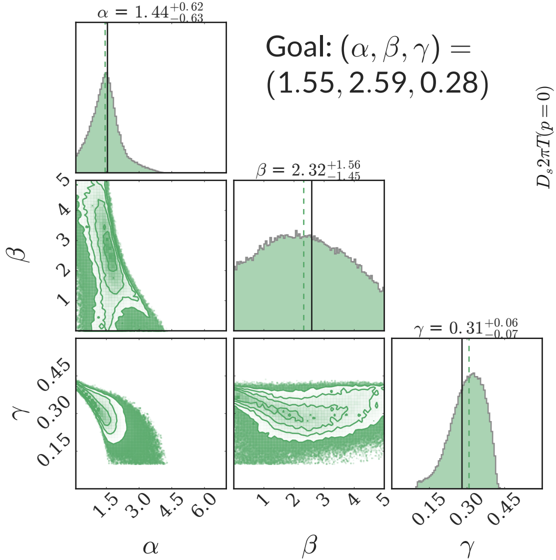
Higher precision: current σ_{exp}

Goal: $(\alpha, \beta, \gamma) = (1.55, 2.59, 0.28)$

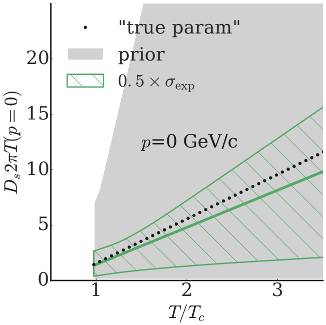




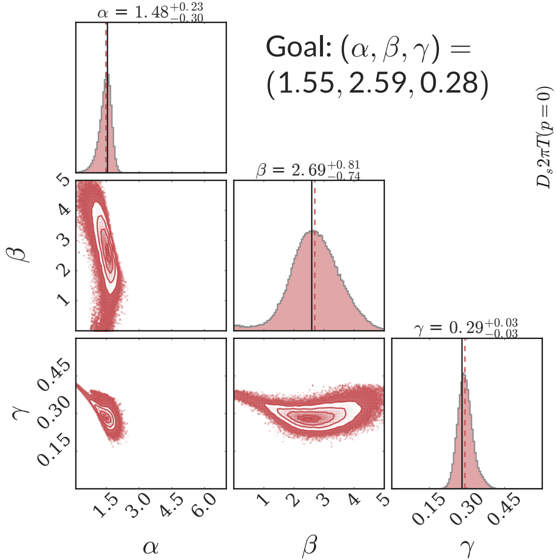
Higher precision: $0.5\sigma_{\mathrm{exp}}$



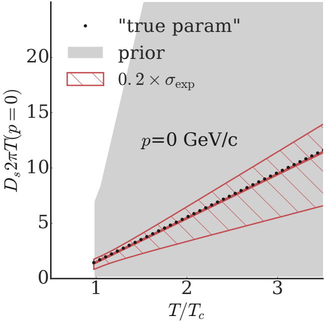
Goal: $(\alpha, \beta, \gamma) =$ $(1.55, 2.59, 0.28)$



Higher precision: $0.2\sigma_{\rm exp}$



Goal: $(\alpha, \beta, \gamma) =$ (1.55, 2.59, 0.28)

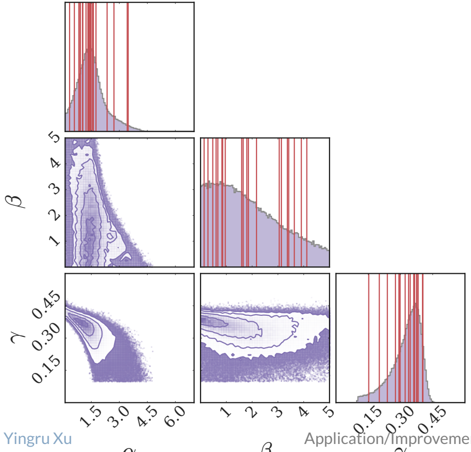


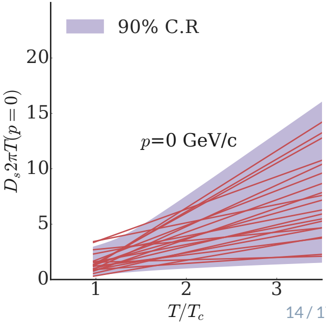
0.2 of current $\sigma_{\rm exp}$!
(considering PbPb @ 5.02 TeV only)

A different approach: more observables

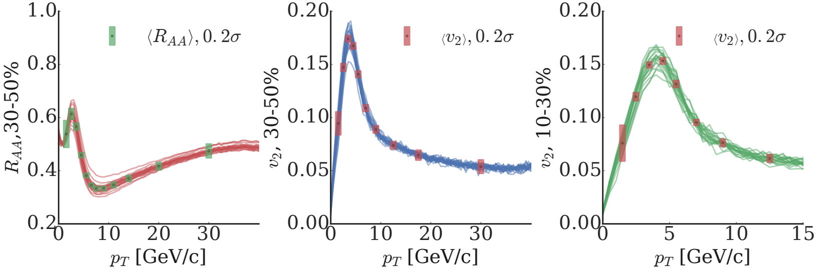
Question: what feature we expect from the new observables?
Wanted: observables that have larger variance/more sensitive (than R_{AA} and v_2) $\rightarrow$ to obtain more information

- Take 20 sets of parameters from the posterior distribution of (α, β, γ)





- *D*-meson differential R_{AA} and v_2



coefficient of variant (ratio of std and mean): $CV = \frac{\sigma}{\mu}$

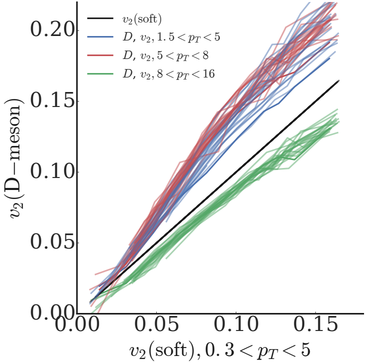
Average over p_T bins:

- $CV[R_{AA}(30-50\%)] = 0.030$
- $CV[v_2(30-50\%)] = 0.051$
- $CV[v_2(10-30\%)] = 0.0528$

soft-hard correlation

- light(soft) hadron and D-meson v_2

M.Nahrgang PhysRevC.91.014904
Prado et al. arXiv:1611.02965v1
P.B. Gossiaux, talk at QM2017



Average over centrality bins:

- $CV[v_2(1.5 < p_T < 5)]$ = 0.2451
- $CV[v_2(5 < p_T < 8)]$ = 0.2434
- $CV[v_2(8 < p_T < 16)]$ = 0.2447

- Minimum bias events, centrality bined by $q^2 = \frac{Q^2}{\sqrt{M}}$
- soft-hard correlation? **Need more investigation (on σ_{model}). But Bayesian analysis is a unique technique!**

Summary

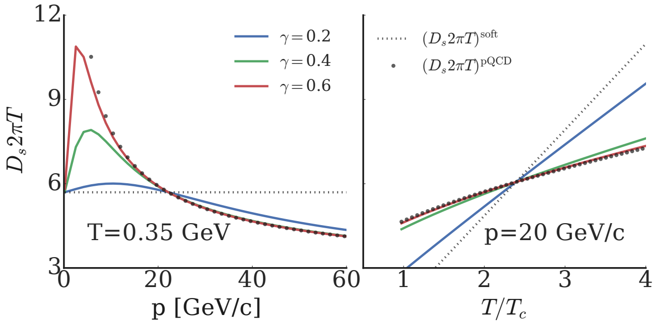
- Quantified charm quark diffusion coefficient $D_s 2\pi T$ using systematic Bayesian analysis
  - $D_s 2\pi T(p=0)$ minimum between (1-3) around T_c
  - non-perturbative effects play a role for $p_T \sim$ (10-20) GeV/c
- Simultaneously described D-meson observables at RHIC and the LHC
- Improved experimental uncertainties is required

- Outlook:
  - Explore other potential observables using Bayesian analysis
  - Higher p_T D-meson, B-meson calibration
  - Applying on Boltzmann transport framework

backups

Input: prior parameter

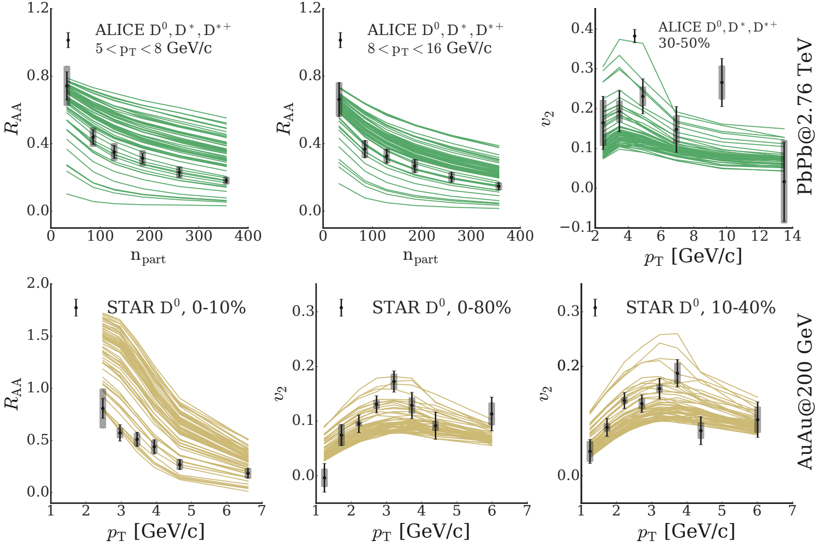
- An example of $D_s 2\pi T$ dependence on γ, at fixed $(\alpha, \beta) = (1.8, 1.7)$:
  - Larger γ, quicker conversion to $(D_s 2\pi T)^{\mathrm{pQCD}}$
  - $p = 1/\gamma^2$, $(D_s 2\pi T)^{\mathrm{soft}} = (D_s 2\pi T)^{\mathrm{pQCD}}$



- Initially: no knowledge/constraint for the parameters $\rightarrow$ **uniform prior distribution**
- $\alpha \in (0, 7)$, $\beta \in (0, 5)$, $\gamma \in (0, 0.6)$

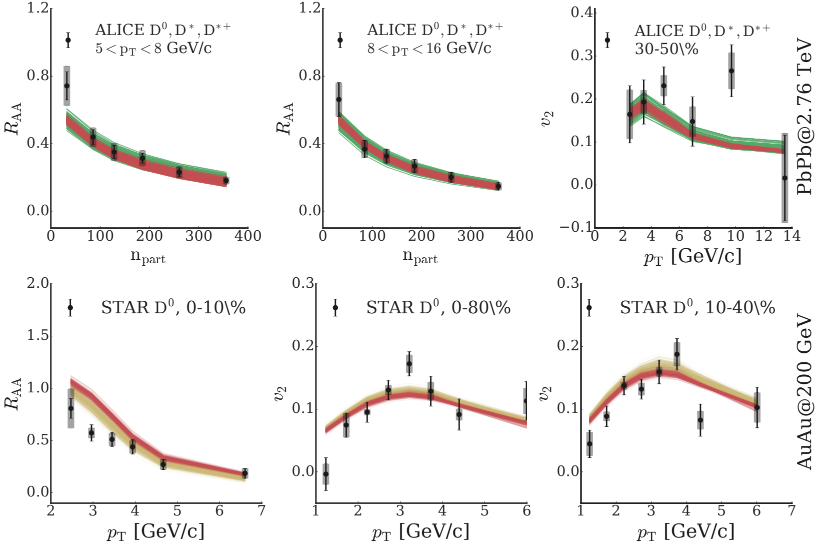
Output: experimental observables

D-meson R_{AA}, v_2 at RHIC and the LHC:

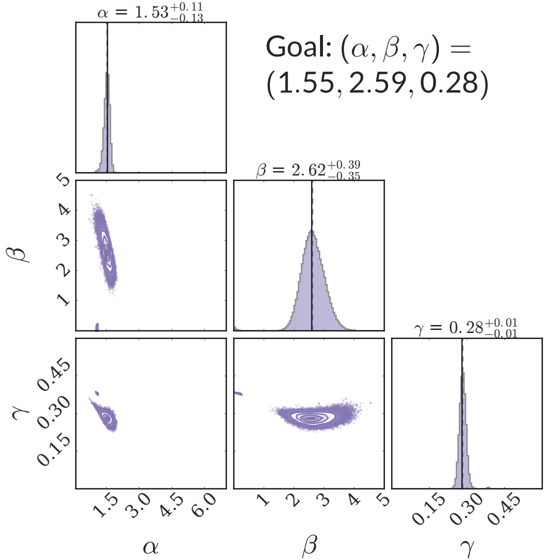


Output: experimental observables

D-meson R_{AA}, v_2 at RHIC and the LHC:



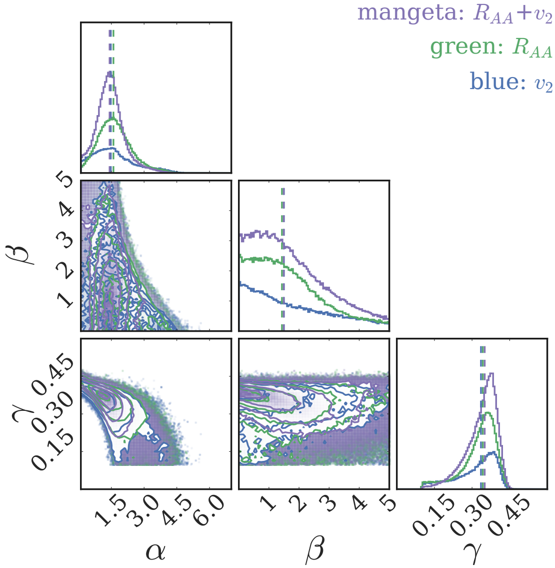
Higher precision: $0.1\sigma_{\text{exp}}$



Goal: $(\alpha, \beta, \gamma) =$ (1.55, 2.59, 0.28)

for higher precision, even better consistency between desired value and resulted value

R_{AA} vs. v_2

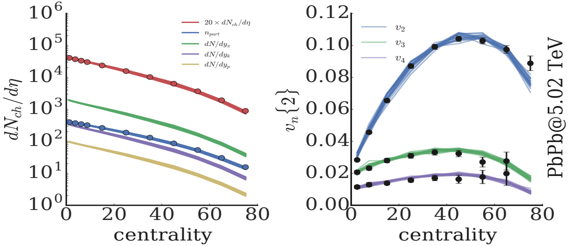


The ability to constrain parameters:

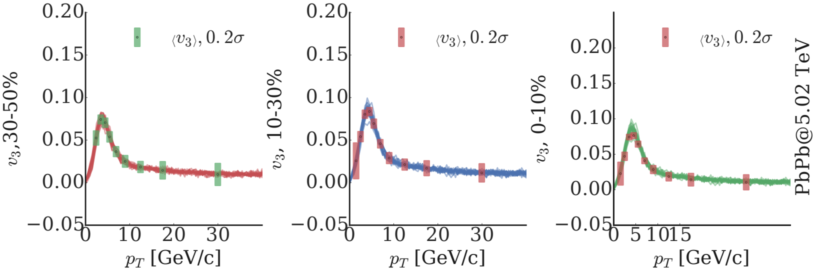
- its uncertainty
- the information it constains

Maybe slightly better for R_{AA} in terms of (α, β, γ) posterior distribution, though not much of the difference from the posterior range of $D_s 2\pi T$; but **more is better**!

- Medium background

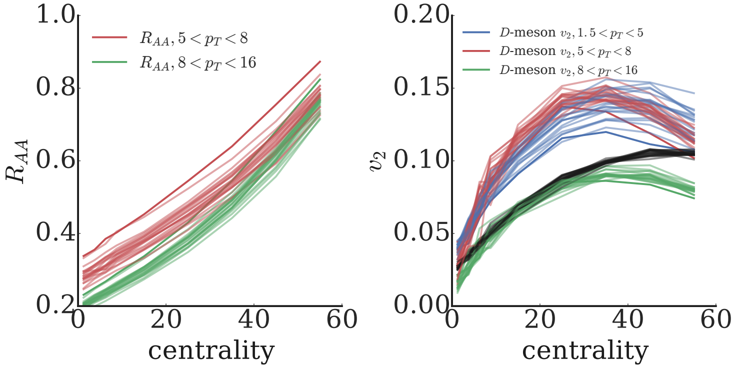


- *D*-meson v_3:



Direction 2: integrated observables

- Integrated R_{AA} and v_2



  - Integrated D-meson R_{AA}, v_2 tend to have larger variance than differential one. $\rightarrow$ models' uncertainty?
  - Results from different centrality classes! $\rightarrow$ probing different average temperature