BFKL challenges from nonperturbative QCD

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Based on:

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Dedicated to memory of Lev Lipatov

Outline[1]

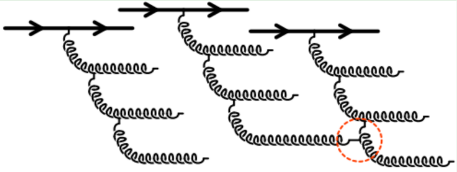
- BFKL in QCD with massive gluons
- EW pomeron in Standard Model (SM)

[1] Disclaimer: This is unfinished research

BFKL short overview

- BFKL validity: high energies ($\sqrt{s} \gg m_N$), large parton transverse momenta $k_T \gg m_N$

$$\partial_Y N(Y, k) = K_{\mathrm{BFKL}}(k, k') \otimes N(Y, k')$$



At small k_T enter the nonperturbative saturation regime, formidable theoretical problem.

Challenges of BFKL

- Diffusion into infrared
- Unitarity violation, $\sim s^{\alpha_{I\!P}-1}$
- Large NLO corrections, resummation is needed

Still BFKL kernel enters BK, JIMWLK, ...

BFKL spectrum in LO

- $\phi(k, \omega) = \int dY\, e^{-i\omega Y} \phi(k, Y)$
- Spectrum is continuous, $\omega \lesssim 4\bar{\alpha}_s \ln 2$.

Conformal invariance$\Rightarrow \phi_\nu(k) \propto k^{\frac{1}{2}+i\nu}$

$$\omega_{\mathrm{LO}}(\nu) = \bar{\alpha}_s \left[2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) + \psi\left(\frac{1}{2} - i\nu\right)\right]$$

BFKL spectrum in NLO

- Full kernel is known, large part is due to running coupling. $K_{\mathrm{BFKL}}^{(\mathrm{NLO})} \in L_2$
- Spectrum is discrete for $\omega > 0$.
- Approximate methods (IR cutoff $k_{\min}$, semiclassical, diffusion) suffer from their own uncertainties.
- Nonperturbative regime$\Rightarrow$dependence on nonperturbative phase $\varphi_{\mathrm{np}} \in [0, 2\pi)$.

BFKL results

Nonperturbative regime is understood much better recently, parametrizations of various gluon distributions available
Our purpose: Understand BFKL in presence of massive terms which break conformal invariance

Nonperturbative model

QCD with massive gluon via Higgs-like mechanism, $M_H \approx 500 - 700\,\mathrm{MeV}$

It is <u>not</u> QCD, but:

- (A. Kovner, and U.A. Wiedemann, PRD 66 (2002) 034031): mass of gluons is indispensable to avoid factorization breaking by powerlike growth of soft pomerons

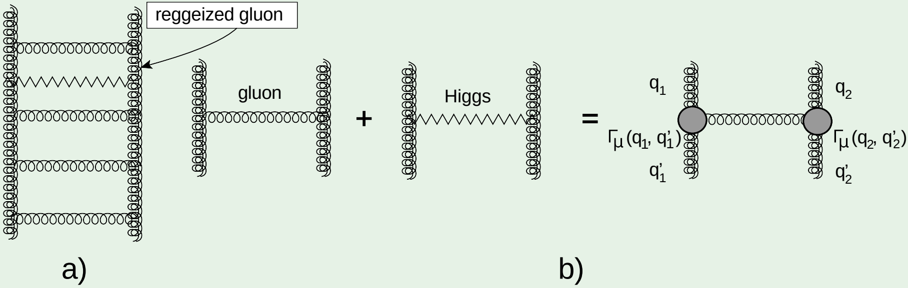
We show that perturbation theory provides two distinct mechanisms for the power like growth of hadronic cross sections at high energy . One, the leading BFKL effect is due to the growth of the parton density, and is characterized by the leading BFKL exponent ω. The other mechanism is due to the infrared diffusion, or the long range nature of the Coulomb field of perturbatively massless gluons. When perturbative saturation effects are taken into account, the first mechanism is rendered ineffective but the second one persists. We suggest

- has correct exponential suppression at large b, is good model to study large-b influence in physical amplitudes
- is gauge invariant, has the same color structure as QCD
- reproduces QCD at large momenta $p_T \gg m$
- has physical realization: SM with $\theta_W = 0$ (we'll see below)

BFKL with massive gluons

BFKL kernel

- (Fadin, Kuraev, Lipatov, 1975-1977)



$$K\left(q_1, q_2 | k_1, k_2\right) = \frac{\bar{\alpha}_s N_c}{2\pi^2} \left\{ \frac{1}{k^2 + m^2} \left(\frac{q_1^2 + m^2}{k_2^2 + m^2} + \frac{q_2^2 + m^2}{k_2^2 + m^2} \right) - \frac{q^2 + \frac{N_c^2 + 1}{N_c^2} m^2}{\left(k_1^2 + m^2\right)\left(k_2^2 + m^2\right)} \right\}$$

BFKL with massive gluons

BFKL kernel

- (Fadin, Kuraev, Lipatov, 1975-1977)

$$K(q_1, q_2 | k_1, k_2) = \frac{\bar{\alpha}_s N_c}{2\pi^2} \left\{ \frac{1}{k^2+m^2} \left(\frac{q_1^2+m^2}{k_2^2+m^2} + \frac{q_2^2+m^2}{k_2^2+m^2} \right) - \frac{q^2 + \frac{N_c^2+1}{N_c^2} m^2}{(k_1^2+m^2)(k_2^2+m^2)} \right\}$$

Forward case $(k_1 - k_2 = q_2 - q_1 = 0)$

$$E\,\phi(\kappa) = \underbrace{\frac{\kappa+1}{\sqrt{\kappa}\sqrt{\kappa+4}} \ln \frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}}_{T(\kappa)-\text{"kinetic" term}} \phi(\kappa) \underbrace{- \int_0^\infty \frac{d\kappa'\,\phi(\kappa')}{\sqrt{(\kappa-\kappa')^2 + 2(\kappa+\kappa')+1}} + \frac{N_c^2+1}{2N_c^2} \frac{1}{\kappa+1} \int_0^\infty \frac{\phi(\kappa')\,d\kappa'}{\kappa'+1}}_{\text{"potential" energy}}$$

$$\kappa = \frac{k^2}{m^2}, \quad \kappa' = \frac{q^2}{m^2}, \quad E = -\frac{\pi\omega}{\alpha_s N_c}$$

- Coincides with BFKL for $\kappa,\ \kappa' \gg 1$
- Additional contact term

Massive BFKL - asymptotic behaviour

Forward case $(k_1 - k_2 = q_2 - q_1 = 0)$

$$E\,\phi(\kappa) = \underbrace{\frac{\kappa+1}{\sqrt{\kappa}\sqrt{\kappa+4}} \ln \frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}}_{T(\kappa)-\text{"kinetic" term}} \phi(\kappa) \underbrace{- \int_0^\infty \frac{d\kappa'\,\phi(\kappa')}{\sqrt{(\kappa-\kappa')^2 + 2(\kappa+\kappa')+1}} + \frac{N_c^2+1}{2N_c^2}\frac{1}{\kappa+1}\int_0^\infty \frac{\phi(\kappa')\,d\kappa'}{\kappa'+1}}_{\text{"potential" energy}}$$

$$\kappa = \frac{k^2}{m^2}, \quad \kappa' = \frac{q^2}{m^2}, \quad E = -\frac{\pi\omega}{\alpha_s N_c}$$

Coordinate space

$$E\,\phi(r) = \mathcal{H}\phi(r)\,, \qquad E = -\omega/\bar{\alpha}_s$$

$$\mathcal{H}\,\phi(r) = T\left(-\nabla^2\right)\phi(r) + \left(-2K_0(m\,r) + \frac{N_c^2+1}{2N_c^2} K_0(m\,r)\int_0^\infty K_0\left(m\,r'\right)\phi\left(r'\right)\,dr'\right)$$

- $mr \gg 1$:

  $$\mathcal{H}\,\phi(r) \approx T\left(-\nabla^2\right)\phi(r)\,, \quad \phi(r) \approx e^{-a\,r}, \quad E = T\left(-a^2\right)$$

  ⋆ Change of asymptotic behaviour near $a = 0$ ($E = 1/2$, $\omega = -\bar{\alpha}_s/2$)

  ⋆ For $a \in \mathbb{R}$, $\phi_{\kappa\ll 1}(\kappa) \sim \left(\kappa + a^2\right)^{-1}$. Analytical continuation for $ia \in \mathbb{R}$: $\phi(\kappa)$ might contain a pole.
- BFKL correspondence: $\phi_{\kappa\gg 1}(\kappa) \sim \kappa^{-1/2+i\nu}$.

Finite difference method

Lattice choice

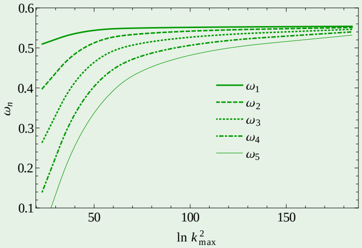
- Use logarithmic grid, with $\kappa_{\min} = 10^{-40}$, $\kappa_{\max} = 10^{80}$, $N_{\text{nodes}} = 2048$

$$\kappa_n \sim \kappa_{\min} \exp\left(\frac{n}{N} \ln\left(\frac{\kappa_{\max}}{\kappa_{\min}}\right)\right), \quad n = 0, \ldots, N$$

Integral equation⇒linear eigenvalue problem

$$\omega\,\phi_n \;=\; \bar{\alpha}_S \sum_{m=0}^{N} \mathcal{K}_{nm}\,\phi_m$$

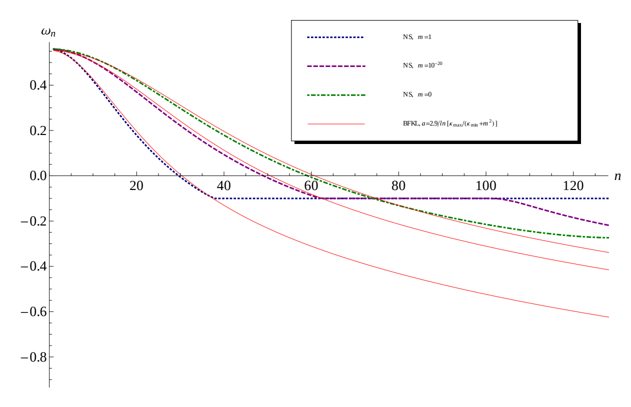
Results for spectrum



- Spectrum is bound from above by $\omega = 4\ln 2\,\bar{\alpha}_s$
- Spectrum is discrete in the lattice, continuous in the limit $\kappa_{\max} \to \infty$(only 5 first roots are shown)
- Dependence on $\kappa_{\min}$ scales as $\kappa_{\max}/\kappa_{\min}$ in massless case, no dependence in massive case

Finite difference method

Results for spectrum



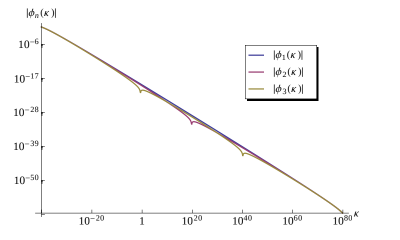
- For small n, both in massive and massless case the spectrum is described by

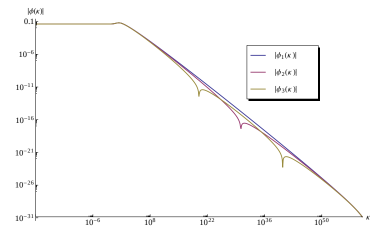
$$\omega_n = \bar{\alpha}_s \left(2\psi(1) - \psi\left(\frac{1}{2} + i\nu_n\right) - \psi\left(\frac{1}{2} + i\nu_n\right)\right), \quad \nu_n = \frac{n\,\pi}{\ln\left(\kappa_{\max}/\left(\kappa_{\min} + m^2\right)\right)}$$

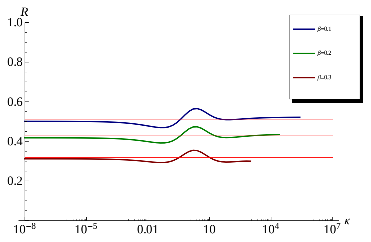
- At larger n, the spectrum ω_n has a degeneration related to the singular behaviour of the wave function, will discuss later.

Finite difference method

Results for eigenfunctions with $\omega_n > 0$







$$R = \left(\frac{K_{\mathrm{BFKL}} \otimes \phi_n^{(\mathrm{approx})}}{\phi_n^{(\mathrm{approx})}} \right) \approx \omega_n$$

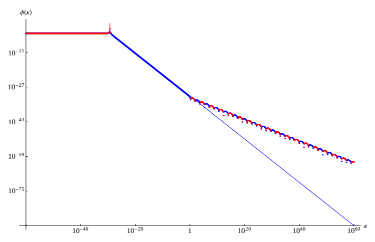
- In both cases, with very good precision is described by

$$\phi_n^{(\mathrm{approx})}(\kappa) = \frac{\mathrm{const}}{\sqrt{\kappa + 4}} \quad (1)$$
$$\times \sin\left(\nu_n \ln\left(\frac{\sqrt{\kappa+4} + \sqrt{\kappa}}{\sqrt{\kappa+4} - \sqrt{\kappa}} \right) + \varphi_n \right)$$

$\kappa \gg 1$: linear combinations of $\phi_{\pm\nu_n}(\kappa) \sim \kappa^{-1/2 \pm i\nu_n}$, for massive case it is regularized by mass

Finite difference method

Results for eigenfunctions with $\omega_n \lesssim -0.5\,\bar{\alpha}_s$



Solid: Absolute value of the WF $|\phi|$. Red=positive WF, blue=negative

Thin: approximation $\left|\frac{1}{\kappa-\kappa_0}\right|$.

- The wave function develops real singularity $\sim (\kappa-\kappa_0)^{-1}$

$$\phi(\kappa) = -\left(\int_0^\infty \frac{d\kappa'\,\phi(\kappa')}{\sqrt{(\kappa-\kappa')^2 + 2(\kappa+\kappa')+1}} + \frac{N_c^2+1}{2N_c^2}\,\frac{1}{\kappa+1}\int_0^\infty \frac{\phi(\kappa')\,d\kappa'}{\kappa'+1}\right) \times \left(\frac{\omega}{\bar{\alpha}_s} - \frac{\kappa+1}{\sqrt{\kappa}\sqrt{\kappa+4}}\ln\frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}\right)^{-1}$$

For $\omega_n \lesssim -0.5\,\bar{\alpha}_s$ the term in red has a pole at $\kappa_0(\omega) = -\frac{12}{5}\left(\frac{\omega}{\bar{\alpha}_s}+\frac{1}{2}\right)$

Finite difference method

Off-forward limit ($q = k_1 - k_2 = q_2 - q_1 \neq 0$)

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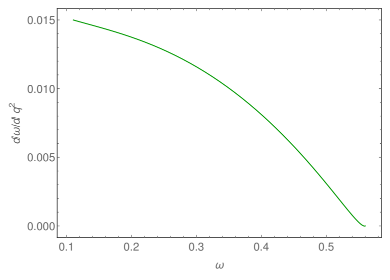


- Practical interest: study of off-forward gluon distributions
- Cannot solve in general case in the lattice (curse of dimensionality)
- Can develop a perturbation theory for small $q^2 \ll k^2$

$$K_{\mathrm{BFKL}} \approx K_{\mathrm{BFKL}}(q=0) + q^2 K^{(1)}_{\mathrm{BFKL}} + \mathcal{O}\left(q^4\right)$$

(omit for brevity explicit expression for $K^{(1)}_{\mathrm{BFKL}}$)

- The slope

$$\frac{d\omega}{dq^2} = \frac{\left\langle \phi_\omega \left| K^{(1)}_{\mathrm{BFKL}} \right| \phi_\omega \right\rangle}{\langle \phi_\omega || \phi_\omega \rangle}$$

is small and vanishes for the leading pole, yet $\mathcal{O}\left(q^2\right)$ corrections change the wave functions (and the Green function).

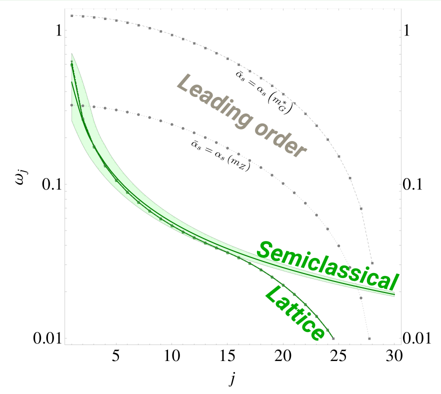
Massive NLO BFKL

Spectrum

- Full kernel is known, large part is due to running coupling $\alpha_s(k) \sim \mathrm{const}/\ln\left(k^2/\Lambda^2_{\mathrm{QCD}}\right)$. $K_{\mathrm{BFKL}} \in L_2$
- Spectrum is discrete for $\omega > 0$, continuous for $\omega < 0$.
  - ⋆ Challenge for lattice: true discrete spectrum vs. discrete due to finite lattice size (depends on $\kappa_{\max}$, much smaller than lattice size)
- Semiclassical methods:

$$\omega_j \approx \frac{\mathrm{const}}{\left(j - \frac{1}{4}\right) + \varphi_{\mathrm{np}}/\pi - \nu \ln\left(k^2_{\mathrm{np}}/\Lambda^2_{\mathrm{QCD}}\right)/\pi}$$

  - ⋆ nonperturbative regime⇒phase $\varphi_{\mathrm{np}} \in [0, 2\pi)$; also depends on "matching point" k_{np}.
  - ⋆ The first root has the largest uncertainty (IR diffusion)



For $j \approx 20$ distance between roots smaller than lattice can resolve; lattice spectrum jumps to continuum spectrum

Massive NLO BFKL

Wave functions

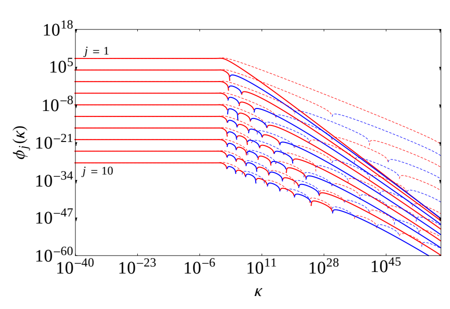
- NLO solution:

$$\phi_\omega(\kappa) \approx \frac{1}{\sqrt{2\pi\omega}} \int_{-\infty}^{+\infty} d\nu\, e^{i\nu t} \times \left[\frac{\Gamma\left(\frac{1}{2}+i\nu\right)}{\Gamma\left(\frac{1}{2}-i\nu\right)} e^{-2i\nu\psi(1)} \right]^{1/\bar{\beta}_0\omega},$$

$$t = \ln\left(\frac{\kappa}{\Lambda^2_{\rm QCD}/m_H^2}\right)$$

$\kappa \gg 1$ limit, s.p. approx.:

$$\phi_\omega(\kappa) \sim \frac{\sin\left(S(\kappa,\omega)+\phi\right)}{\kappa}$$



Solid: NLO eigenfunctions, dashed: Leading order. Normalization adjusted from matching $\phi_\omega^{(\rm LO)}(0) = \phi_\omega^{(\rm NLO)}(0)$

Massive NLO BFKL

Spectrum with triumvirate coupling

- "Triumvirate coupling"

$$\alpha_s\left(\vec{k},\,\vec{k}'\right) = \alpha_s\left(\vec{k}-\vec{k}'\right)\frac{\alpha_s\left(k'\right)}{\alpha_s\left(k\right)}$$

Preferred for the following reasons (NPA 784, 188; PRD 75, 014001; NPA 789, 260):

⋆ Satisfies the bootstrap equation, *i.e.* leads to gluon reggeization

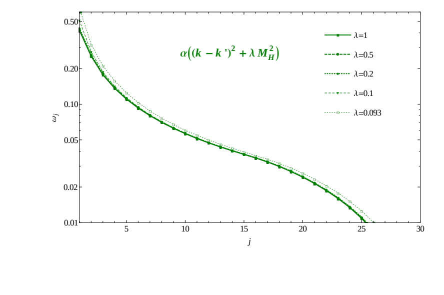
⋆ Summation of Feynman diagrams for $N_f \gg 1$

⋆ Coincides with $\sim \beta_0$ term in NLO BFKL kernel.

Technical challenges

◇ Cannot be studied semiclassically

◇ $\forall k,\, k'$ sensitive to IR ($|\vec{k}-\vec{k}'| \sim \Lambda_{\rm QCD}$), need cutoff



Agrees with results of simple $\alpha_s\left(k\right)$ coupling
Enhanced sensitivity to choice of regularizer, especially for the leading pole

Electroweak BFKL

BFKL equations

- $\alpha_{\rm EW} \ll 1$, require $\sqrt{s} \sim 10^2 - 10^3$ TeV to resolve the power behaviour
- Spontaneous symmetry breaking and admixture of abelian field ($\theta_W \neq 0$).
- Focus on bosonic sector for simplicity $(W^{\pm}, Z, \gamma, H)$, BFKL: system of 10 coupled integral equations (NPB 772, 103)

$$\left(\omega - \omega_i(k) - \omega_j(k)\right)\Phi_{ij}\left(k\right) = \int \frac{d^2k'}{(2\pi)^3} \sum_{i'j' \neq \gamma} K_{ij,j'j'} \frac{(-1)^{N_3(i',j')} c_W^{2N_Z(i',j')} \Phi_{i'j'}\left(k'\right)}{D\left(k', M_i\right) D\left(k', M_j\right)} + \sqrt{2}\int \frac{d^2k'}{(2\pi)^3}{}' K_{ij,cc}\left(k, k'\right) \frac{\Phi_{cc}\left(k'\right)}{D\left(k', M_W\right)^2}$$

$$\left(\omega - 2\omega_c(k)\right)\Phi_{cc}\left(k\right) = \sqrt{2}\int \frac{d^2k'}{(2\pi)^3} \sum_{i'j'} K_{cc,ij} \frac{(-1)^{N_3(i,j)} c_W^{2N_Z(i,j)} s_W^{2N_\gamma(i,j)} \Phi_{ij}\left(k'\right)}{D\left(k', M_i\right) D\left(k', M_j\right)} + \int \frac{d^2k'}{(2\pi)^3} K_{cc,cc}\left(k, k'\right) \frac{\Phi_{cc}\left(k'\right)}{D\left(k', M_W\right)^2},$$

$D\left(k,\ M\right) = k^2 + M^2$,

$c_W \equiv \cos\theta_W$, $s_W \equiv \sin\theta_W$

$N_i\left(a, b\right)$-number of indices of species i in $\left(a,\ b\right)$

$$K_{ij,j'j'} = \frac{g^2 M_W^2}{2c_W^{2N_Z(i,j,i',j')}}\theta\left(i, j, i', j' \neq \gamma\right),$$

$$K_{ij,cc}\left(k,\ k'\right) = g^2\left(-M_{ij}^2 + \frac{\left(D\left(k,\ M_i\right) + D\left(k,\ M_j\right)\right) D\left(k'\right)}{D(k-k')}\right) = g^2\left(-M_{ij}^2 + 2K_{em}\left(k, k'\right) + \frac{\left(M_i^2 + M_j^2 - 2M_W^2\right) D\left(k'\right)}{D(k-k')}\right);$$

$$K_{cc,cc} = g^2\left(-M_W^2 + D\left(k\right) D\left(k'\right)\left(\frac{c_W^2}{D\left(k-k',\ M_Z\right)} + \frac{s_W^2}{D\left(k-k',\ 0\right)}\right)\right)$$

Electroweak BFKL

BFKL equations: case $\theta_W = 0$

- γ decouples, all other neutral channels coincide

$$(\omega - 2\omega(k))\Phi_{nn}(k) = \frac{g^2}{2}\int \frac{d^2k'}{(2\pi)^3}\frac{1}{D^2(k')}\left(\Phi_{nn}(k') - 3\sqrt{2}\Phi_{cc}(k')\right) + 2\sqrt{2}\int \frac{d^2k'}{(2\pi)^3}\frac{K_{em}(k,k')\Phi_{cc}(k')}{D^2(k')},$$

$$(\omega - 2\omega(k))\Phi_{cc}(k) = -g^2\int \frac{d^2k'}{(2\pi)^3}\frac{1}{D^2(k')}\left(\frac{3\sqrt{2}}{2}\Phi_{nn}(\kappa') + \Phi_{cc}(\kappa')\right) + 2\sqrt{2}g^2\int \frac{d^2k'}{(2\pi)^3}\frac{K_{em}(k,k')\Phi_{nn}(k')}{D^2(k')} + 2g^2\int \frac{d^2k'}{(2\pi)^3}\frac{K_{em}(k,k')\Phi_{cc}(k')}{D^2(k')}$$

- Seek for solutions

$$\begin{pmatrix}\Phi_{nn}(k)\\ \Phi_{cc}(k)\end{pmatrix} = \begin{pmatrix}a\\ b\end{pmatrix}\Phi(k)$$

$b/a = \sqrt{2}$ or $b/a = -1/\sqrt{2}$

$$(\omega - 2\omega(k))\Phi(k) = -g^2\frac{5M_W^2}{2}\int \frac{d^2k'}{(2\pi)^3}\frac{\Phi(k')}{D^2(k')} + 4\int \frac{d^2k'}{(2\pi)^3}\frac{K_{em}(k,k')\Phi(k')}{D^2(k')}.$$

- Single equation, coincides with massive BFKL we've studied earlier

Electroweak BFKL

BFKL equations: case $\theta_W \neq 0$

- After several nontrivial algebraic transformations we can rewrite it as a single equation

$$(\omega - 2\omega_c(k))\,\Phi_{cc}(k) = \int \frac{d^2k'}{(2\pi)^3}\,\frac{K_{cc,cc}(k,k')\,\Phi_{cc}(k')}{D(k',\,M_W)^2}$$

$$+ g^2\frac{M_W^2\sqrt{2}}{2}N_\Sigma[\Phi_{cc}]\int\frac{d^2k'}{(2\pi)^3}\sum_{ij\neq\gamma}\frac{K_{cc,ij}(k,k')(-1)^{N_3(i,j)}}{(\omega - N_n(i,j)\omega_n(k'))\,D(k',\,M_i)\,D(k',\,M_j)}$$

$$+ 2\int\frac{d^2k'}{(2\pi)^3}\int\frac{d^2k''}{(2\pi)^3}\sum_{ij}\frac{K_{cc,ij}(k,k')(-1)^{N_3(i,j)}c_W^{2N_Z(i,j)}s_W^{2N_\gamma(i,j)}K_{ij,cc}(k',k'')\,\Phi_{cc}(k'')}{(\omega - N_n(i,j)\omega_n(k'))\,D(k',\,M_i)\,D(k',\,M_j)\,D(k'',\,M_W)^2}.$$

where

$$N_\Sigma[\Phi_{cc}] \equiv \int\frac{d^2k}{(2\pi)^3}\,\varphi_\Sigma(k) =$$

$$= \left[\sqrt{2}\int\frac{d^2k}{(2\pi)^3}\int\frac{d^2k'}{(2\pi)^3}\sum_{ij\neq\gamma}\frac{(-1)^{N_3(i,j)}K_{ij,cc}(k,k')\,\Phi_{cc}(k')}{(\omega - N_n(i,j)\omega_n(k))\,D(k,\,M_i)\,D(k,\,M_j)\,D(k',\,M)^2}\right]\times$$

$$\times\left[1 - g^2\sum_{ij\neq\gamma}\int\frac{d^2k}{(2\pi)^3}\frac{(-1)^{N_3(i,j)}}{(\omega - N_n(i,j)\omega_n(k))\,D(k,\,M_i)\,D(k,\,M_j)}\frac{M_i^2M_j^2}{2M_W^2}\right]^{-1}.$$

-linear integral equation w.r.t. Φ_{cc}, but ω also appears in the r.h.s. denominators.

Electroweak BFKL

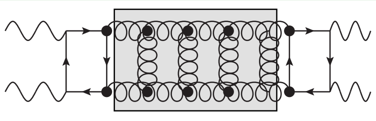
NLO spectrum

- Formally, previous result might be solved algebraically, rewriting it

$$\omega^{(l)}\Phi_{cc}(k) = \int \frac{d^2k'}{(2\pi)^2} K_{\mathrm{BFKL}}\left(k,\ k',\ \omega^{(r)}\right)\Phi_{cc}\left(k'\right)$$

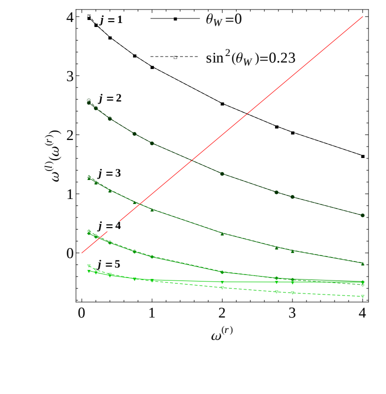
$$\omega^{(l)} = \omega^{(r)}$$

- The leading order intercept $\Delta_{\mathrm{e.w.}} = \alpha_{\mathrm{ew}} 4\log 2 \approx 0.176$.
- The result does not depend on the choice of the mixing angle θ_W.
- Experimental study is challenging due to contributions of strong pomeron:



(larger intercept due to $\alpha_s > \alpha_{\mathrm{e.w.}}$)

Clean channel: $pp \to nn + \mathrm{lepton\ \ pairs}$



Eigenvalues of (1) as a function of $\omega^{(r)}$. Intersections with red line gives the intercepts. For legibility $\kappa_{\max} \sim 10^{10}$, spectrum is continuous in the limit $\kappa_{\max} \to \infty$.

Summary

- The infrared physics essentially influences the intercepts of the BFKL
  - $\star$ For the coupling $\alpha_s(k) \sim \mathrm{const}/\ln\left(k^2/\Lambda_{\mathrm{QCD}}^2\right)$ this is limited to the leading (the most important) pole
  - $\star$ For the coupling $\alpha_s\left(\vec{k},\, \vec{k}'\right) = \alpha_s\left(\vec{k} - \vec{k}'\right)\frac{\alpha_s(k')}{\alpha_s(k)}$ the infrared regularization $\left|\vec{k} - \vec{k}'\right| \sim \Lambda_{\mathrm{QCD}}$ affects all poles
  - $\star$ Manifestation of diffusion into infrared property
  - $\star$ Presents one of the major challenges for studies of the nonperturbative gluonic contents
- We solved Electroweak BFKL
  - $\star$ Dependence on mixing angle θ_W is small, spectrum coincides with expectations of massless BFKL
  - $\star$ Mixes with strong pomeron, requires specially designed observables to see power law behaviour

THANK YOU FOR YOUR ATTENTION!