Magnetized Effective QCD Phase Diagram

M. Loewe

Pontificia Universidad Católica de Chile (PUC) and CCTVAL

6th International Workshop on High Energy Physics in the LHC Era, Valparaíso, January 2016

This talk is based on the following article:

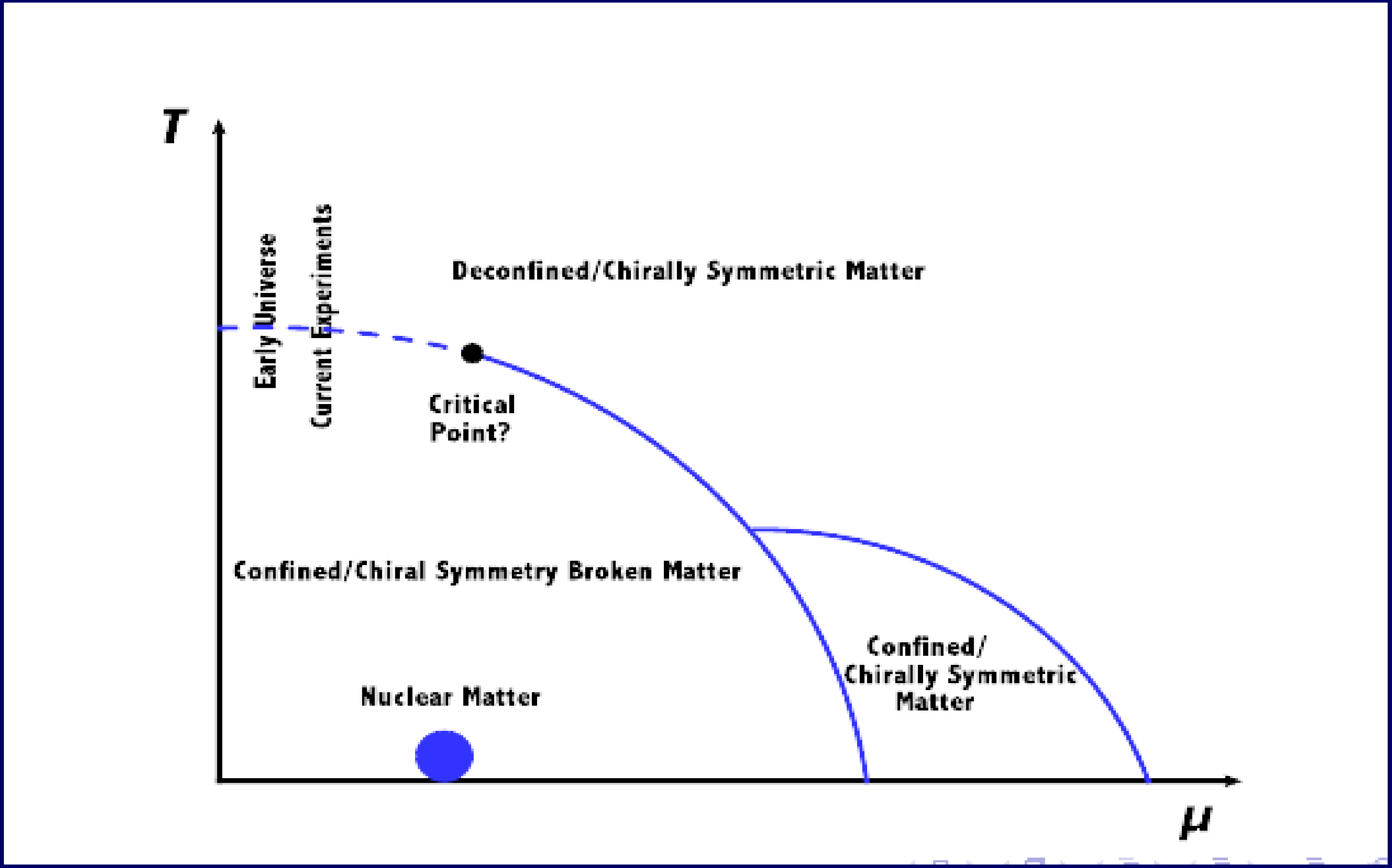
Magnetized Effective QCD Phase Diagram

A. Ayala,C. A. Dominguez, L. A. Hernández, M. Loewe, and R. Zamora

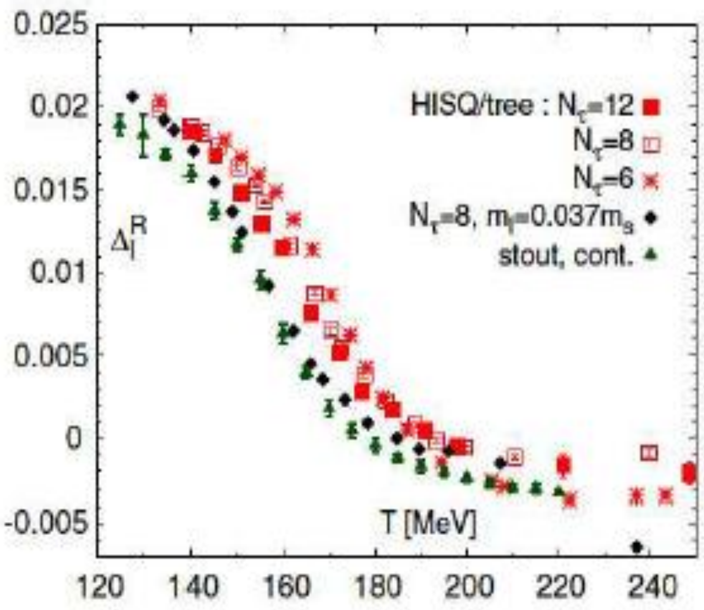
Physical Review D 92, 096011 (2015)

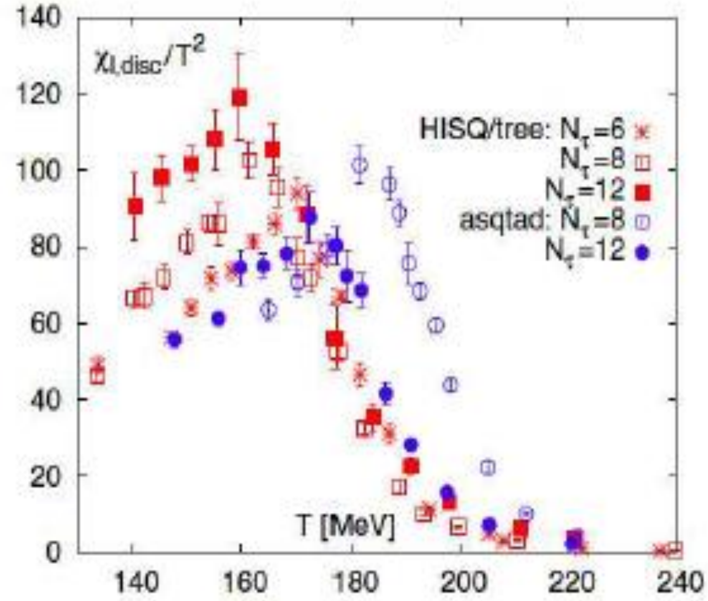
I acknowledge support form : FONDECYT 1130056 and 1120770 (CHILE)

Phase Diagram: Which will be the influence of an external Magnetic Field B?



Quark Condensate





A. Bazavov *et al.*, Phys. Rev. D 85, 054503 (2012).

Linear σ-Model with Quarks

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(D_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$
$$+ i\bar{\psi}\gamma^\mu D_\mu \psi - g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi$$

The idea is to explore the chiral symmetry restoration phase transition under the influence of Temperature (T), Baryonic Chemical Potential (μ) and an external Magnetic Field.

The crucial tool for this purpose is to compute the Effective Potential

- A_μ is the external electromagnetic potential

In the symmetric gauge, it is given by

$$A^\mu = \frac{B}{2}(0, -y, x, 0),$$

The gauge field couples only to the charged pion combinations

$$\pi_\pm = \frac{1}{\sqrt{2}}(\pi_1 \mp i\pi_2)$$

To allow for spontaneous symmetry breaking, we let the σ field to develop a vacuum expectation value v,

$$\sigma \to \sigma + v.$$

This vacuum expectation value can later be identified as the order parameter of the theory. After this shift, the Lagrangian can be rewritten as

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}[\sigma(\partial_\mu + iqA_\mu)^2\sigma] - \frac{1}{2}(3\lambda v^2 - a^2)\sigma^2 \\
& -\frac{1}{2}[\vec{\pi}(\partial_\mu + iqA_\mu)^2\vec{\pi}] - \frac{1}{2}(\lambda v^2 - a^2)\vec{\pi}^2 + \frac{a^2}{2}v^2 \\
& -\frac{\lambda}{4}v^4 + i\bar{\psi}\gamma^\mu D_\mu\psi - gv\bar{\psi}\psi + \mathcal{L}_I^b + \mathcal{L}_I^f,
\end{aligned}
$$

Where the interacting terms are given by

$$\mathcal{L}_I^b = -\frac{\lambda}{4}[(\sigma^2 + (\pi^0)^2)^2 + 4\pi^+\pi^-(\sigma^2 + (\pi^0)^2 + \pi^+\pi^-)],$$

$$\mathcal{L}_I^f = -g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi$$

The σ, the three pions and the quark masses satisfy

$$m_\sigma^2 = 3\lambda v^2 - a^2,$$

$$m_\pi^2 = \lambda v^2 - a^2,$$

$$m_f = gv.$$

In our Lagrangian Ψ is an SU(2) isospin Doublet

$$\vec{\pi} = (\pi_1, \pi_2, \pi_3)$$

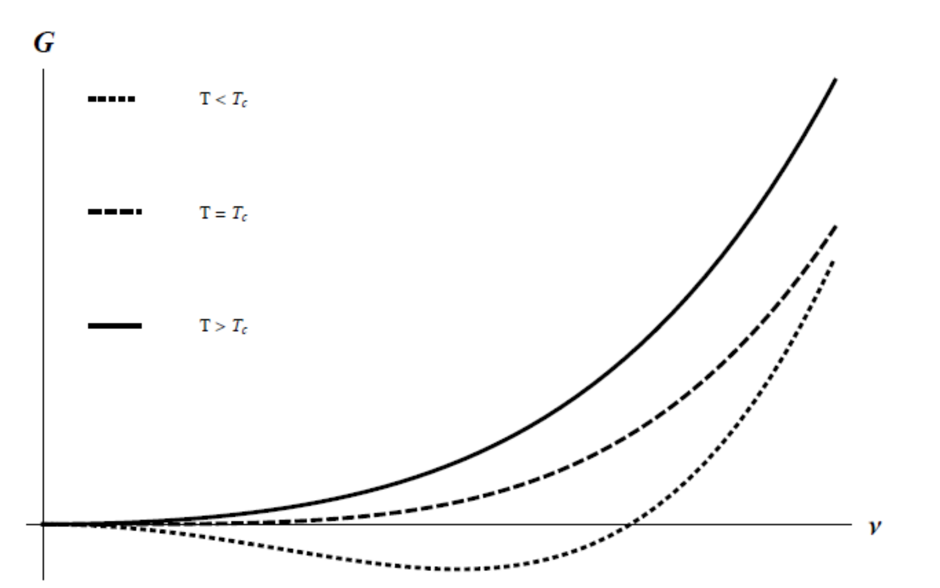
Isospin Triplet

σ is a scalar field correponding to an isospin singlet

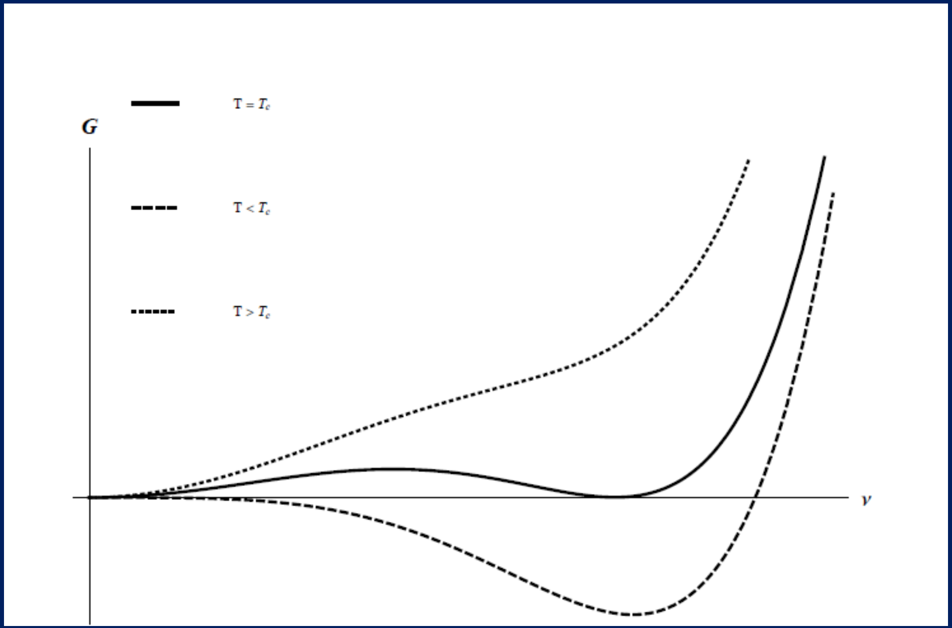
We have a U(1) covariant derivative

$$D_\mu = \partial_\mu + iqA_\mu$$

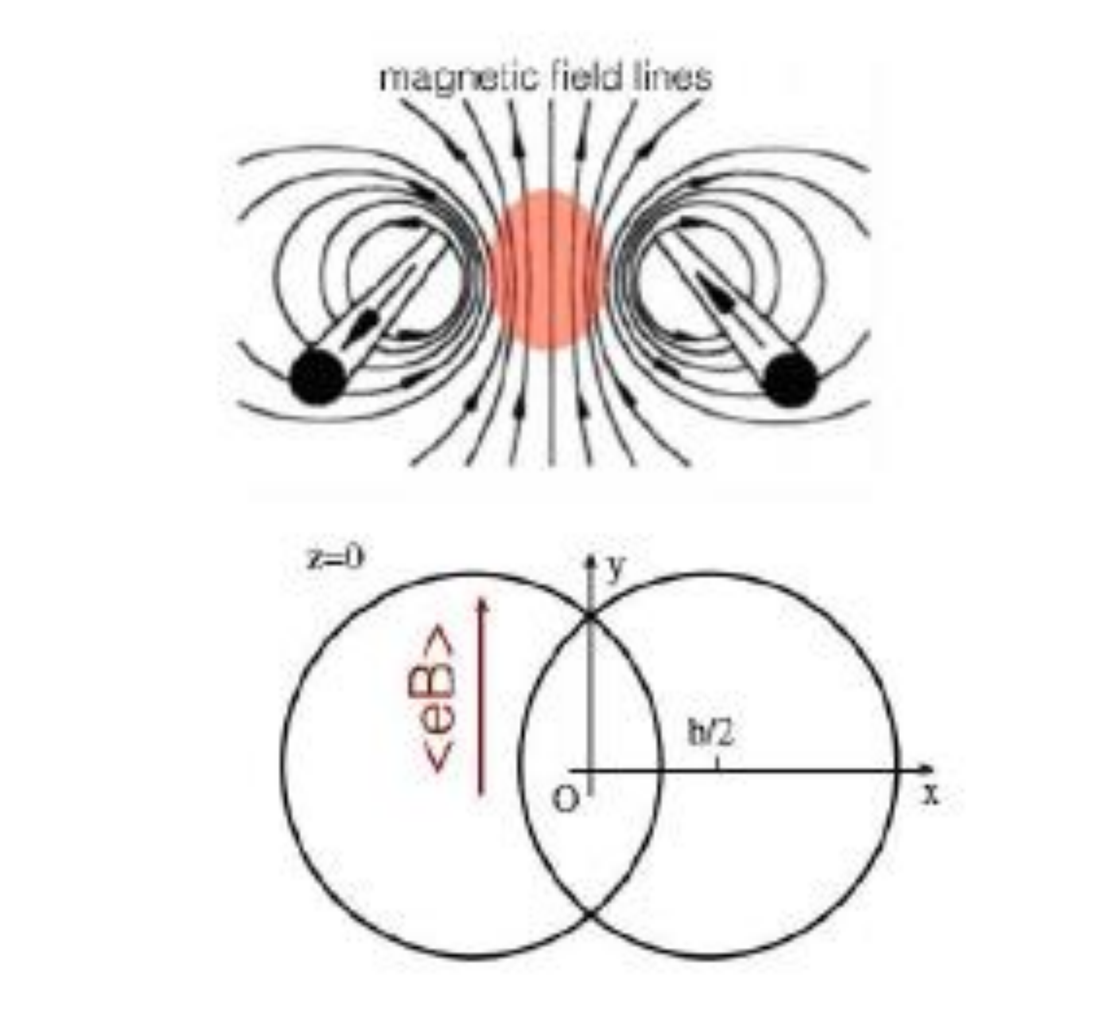
For second order phase transitions we expect



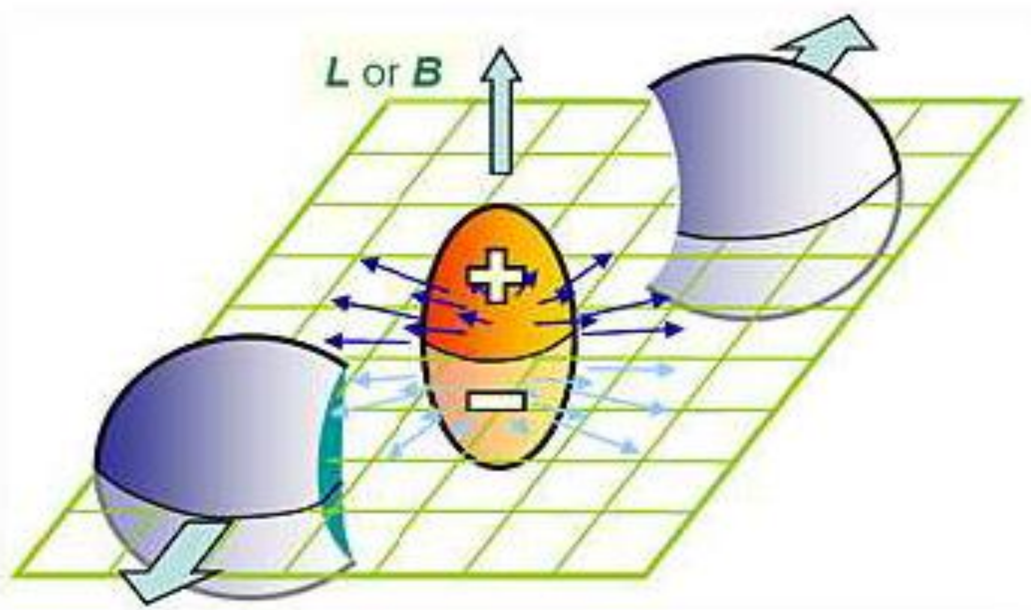
For first order phase transitions



Huge Magnetic fields are produced in perhipheral heavy-ion collisions

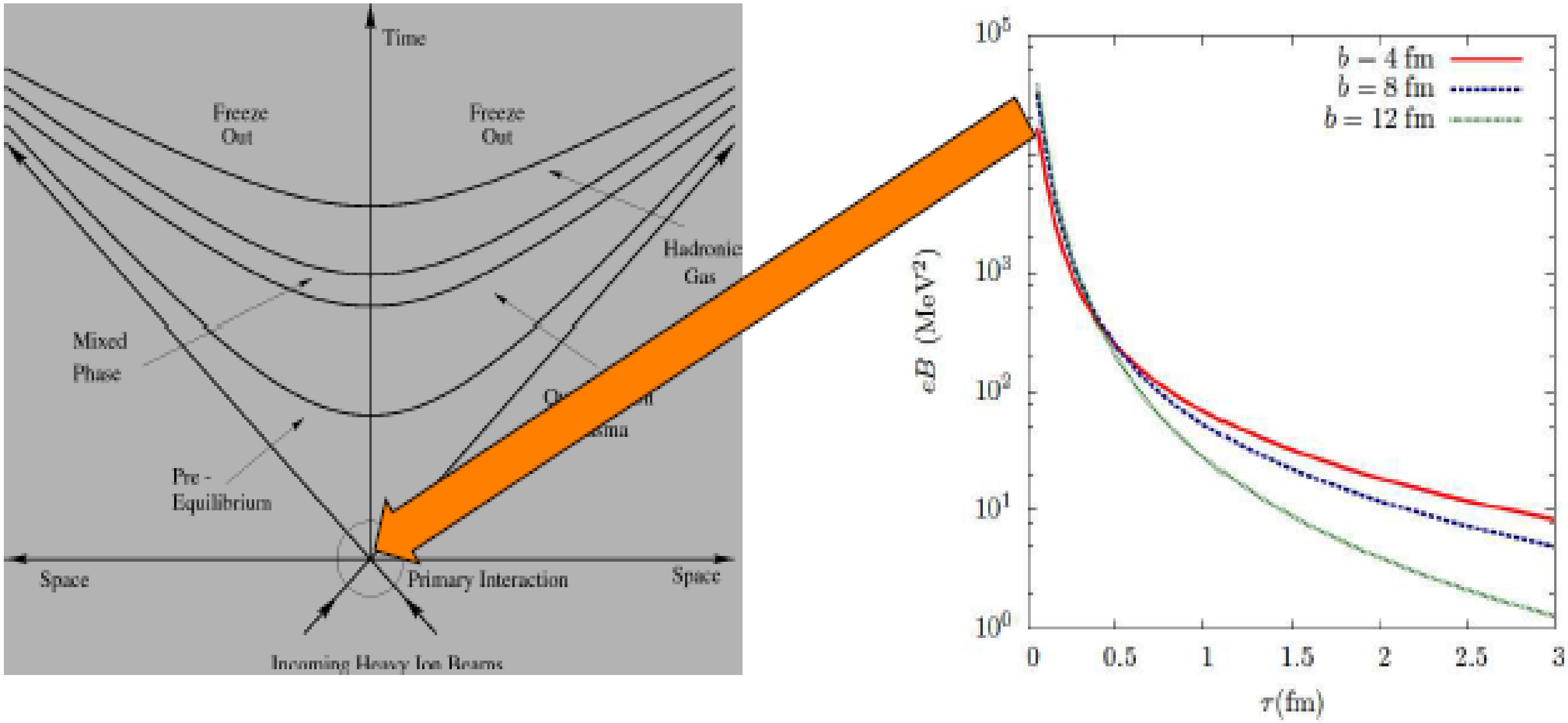


Peripheral Heavy ion collisions



Time evolution of a uniform magnetic field in a heavy-ion collision

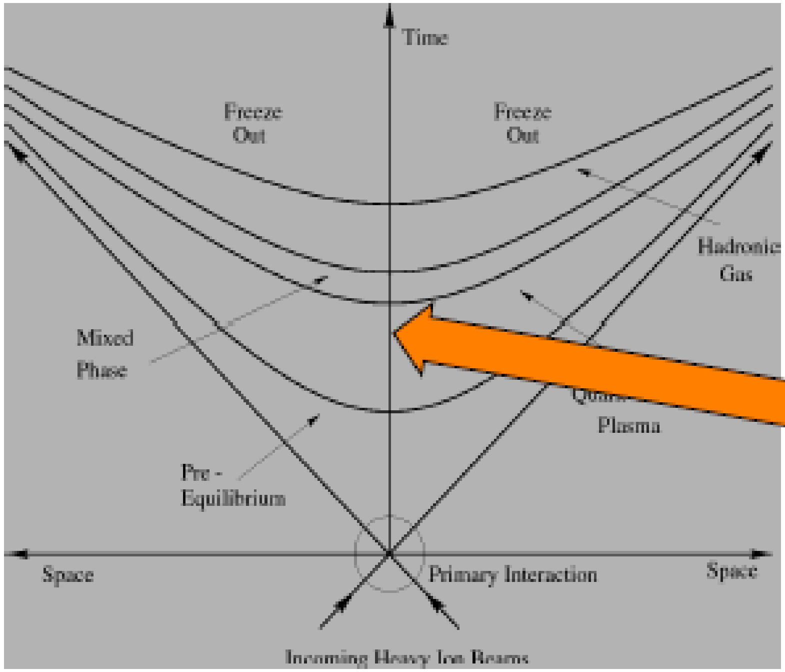
- Very intense field at early collision times

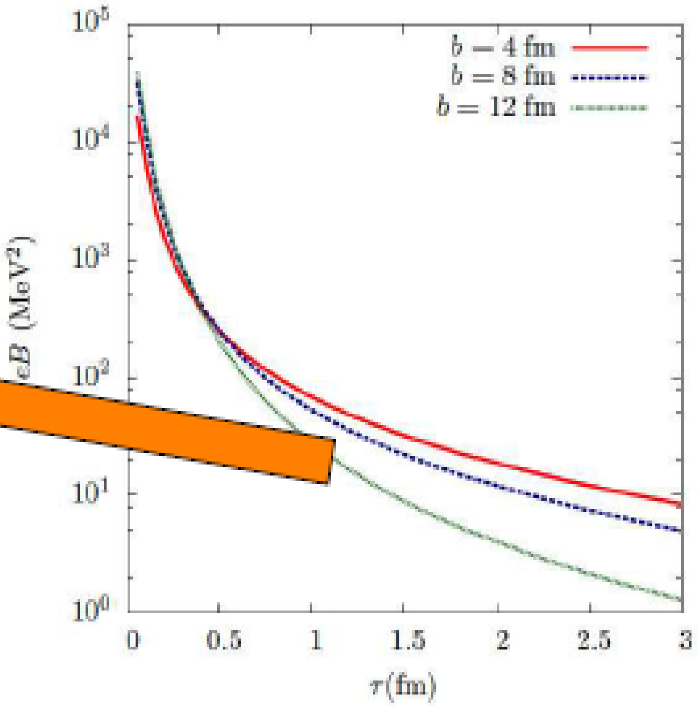


D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

Time evolution of a constant B field in a heavy-ion collision

- Magnetic field rapidly decreasing function of collision time





D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

- By the time the quarks and gluon thermalize the temperature becomes the largest of the energy scales. This means that the weak magnetic field approximation seems to be appropriate

  ($eB << T^2$).

Recently, using the linear σ-model with quarks, we have shown the occurence of magnetic anti-catalysis (in agreement with results from the lattice). T_c (for chiral restoration) diminishes as function of B.

A. Ayala, M. Loewe, and R. Zamora, Phys. Rev. D **91**, 016002 (2015).

A. Ayala, M. Loewe, A. J. Mizher, and R. Zamora, Phys. Rev. D **90**, 036001 (2014).

For a boson field, square mass m_b^2, charge (absolute value) q_b, at finite temperature T and with an external magnetic field B, the effective potential is given by

$$V_b^{(1)} = \frac{T}{2}\sum_n \int dm_b^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{ds}{\cosh(q_b B s)} \times e^{-s\left(\omega_n^2 + k_3^2 + k_\perp^2 \frac{\tanh(q_b B s)}{q_b B s} + m_b^2\right)},$$

where $\omega_n = 2n\pi T$ are the boson Matsubara frequencies

In a similar way, for a fermion field with mass m_f, charge q_f (absolute value), chemical potential μ, temperature T and external magnetic field B, the effective potential is given by

$$V_f^{(1)} = -\sum_{r=\pm 1} T \sum_n \int dm_f^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{ds}{\cosh(q_f B s)} \times e^{-s[(\tilde{\omega}_n - i\mu)^2 + k_3^2 + k_\perp^2 \frac{\tanh(q_f B s)}{q_f B s} + m_f^2 + r q_f B]},$$

Where $\tilde{\omega}_n = (2n+1)\pi T$ are the fermionic Matsubara frequencies

- After a long calculation:
- Sum over Matsubara frequencies,

- Sum over Landau Levels,

- Carrying on mass and charge renormalization, (we choose the renormalization scale as

- $\tilde{\mu} = e^{-1/2} a$)

The thermo-magnetic effective potential, for small up to intermediate field intensities, in a high temperature expansion can be written as

$$
\begin{aligned}
V^{(\mathrm{eff})} = & -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 + \sum_{i=\sigma,\pi^0}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right] - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} - \frac{T}{12\pi}(m_i^2+\Pi)^{3/2}\right\} \\
& + \sum_{i=\pi_+,\pi_-}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1\right] - \frac{\pi^2 T^4}{90} + \frac{m_i^2 T^2}{24} + \frac{T(2qB)^{3/2}}{8\pi}\zeta\left(-\frac{1}{2},\frac{1}{2}+\frac{m_i^2+\Pi}{2qB}\right)\right. \\
& \left. - \frac{(qB)^2}{192\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) - 2\gamma_E + 1 + \zeta(3)\left(\frac{m_i}{2\pi T}\right)^2 - \frac{3}{4}\zeta(5)\left(\frac{m_i}{2\pi T}\right)^4\right]\right\} \\
& - N_c\sum_{f=u,d}\left\{\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{(4\pi T)^2}{2a^2}\right) + \psi^0\left(\frac{1}{2}+\frac{i\mu}{2\pi T}\right) + \psi^0\left(\frac{1}{2}-\frac{i\mu}{2\pi T}\right)\right] + 8m_f^2T^2[Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]\right. \\
& - 32T^4[Li_4(-e^{\mu/T}) + Li_4(-e^{-\mu/T})] + \frac{(q_f B)^2}{24\pi^2}\left[\ln\left(\frac{(\pi T)^2}{2a^2}\right) - 2\gamma_E + 1 - \psi^0\left(\frac{1}{2}+\frac{i\mu}{2\pi T}\right) - \psi^0\left(\frac{1}{2}-\frac{i\mu}{2\pi T}\right)\right. \\
& \left.\left. + \frac{2\pi}{((\pi+i\mu/T)^2+m_f^2/T^2)^{1/2}} + \frac{2\pi}{((\pi-i\mu/T)^2+m_f^2/T^2)^{1/2}} - \frac{4\pi}{(\pi^2+m_f^2/T^2)^{1/2}}\right]\right\},
\end{aligned}
$$

In this long and cumbersome expression:

1) Li_n is the polylogarithm function of order n
2) $\Psi^0(x)$ is the digamma function

The leading temperature plasma screening effects for the boson mass squared (resuma-Tion of ring diagrams) is encoded in the boson self energy

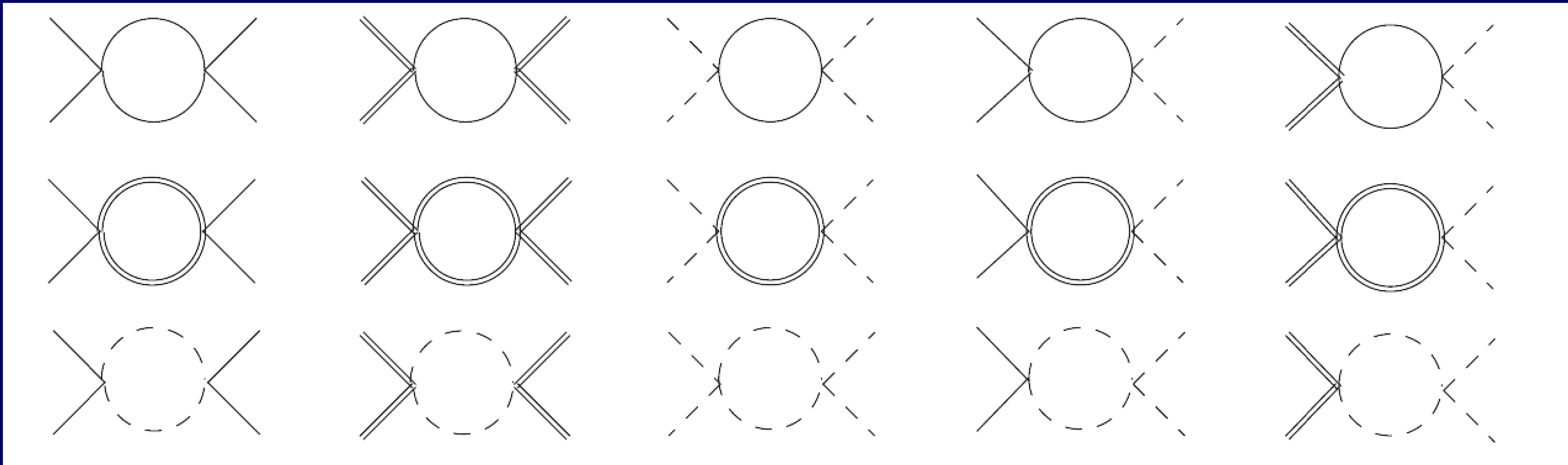
$$\Pi = \lambda \frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2} [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

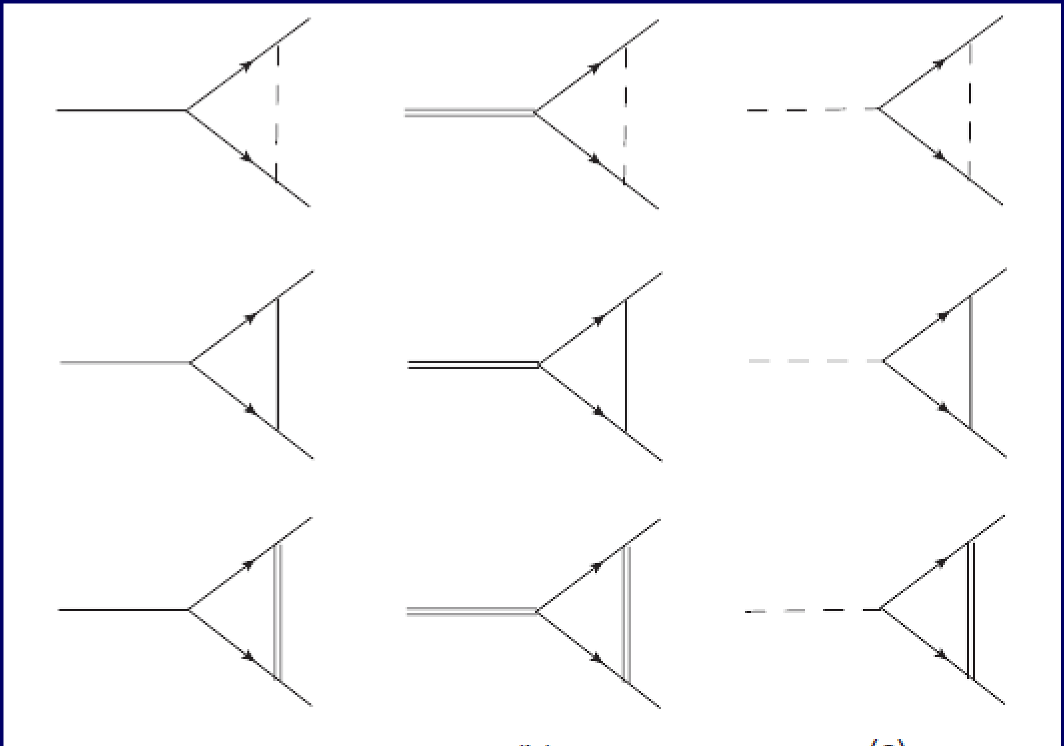
- We require

$$qB/T^2 < 1.$$

Thermo Magnetic Couplings:

We calculate the one loop corrections to the boson coupling λ and the coupling g





(a) (b) (c)

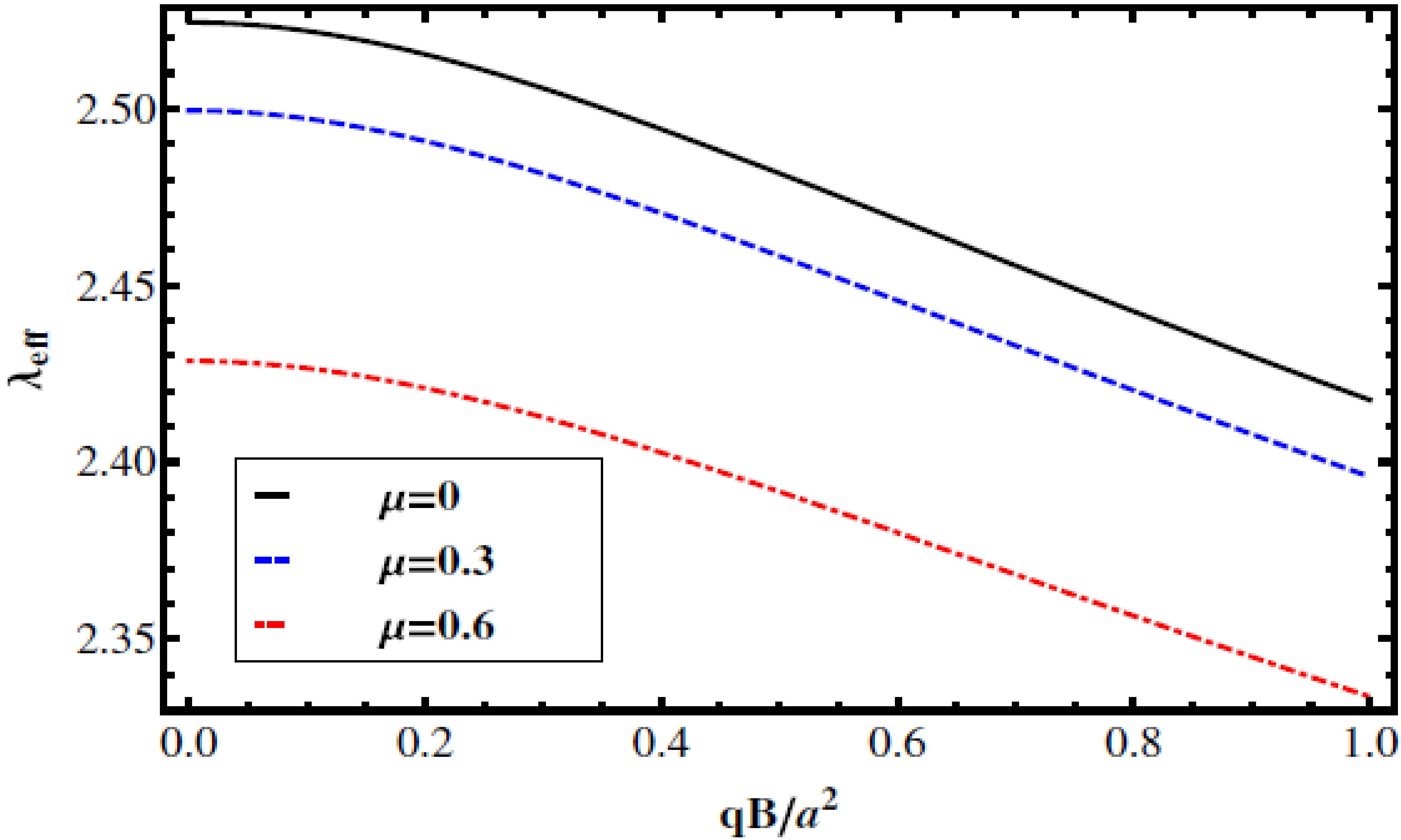


FIG. 3 (color online). Effective boson coupling λ_{eff} evaluated at the temperature $T = 180$ MeV with $\lambda = 0.4$, $g = 0.63$ as a function of the magnetic field strength for different values of μ.

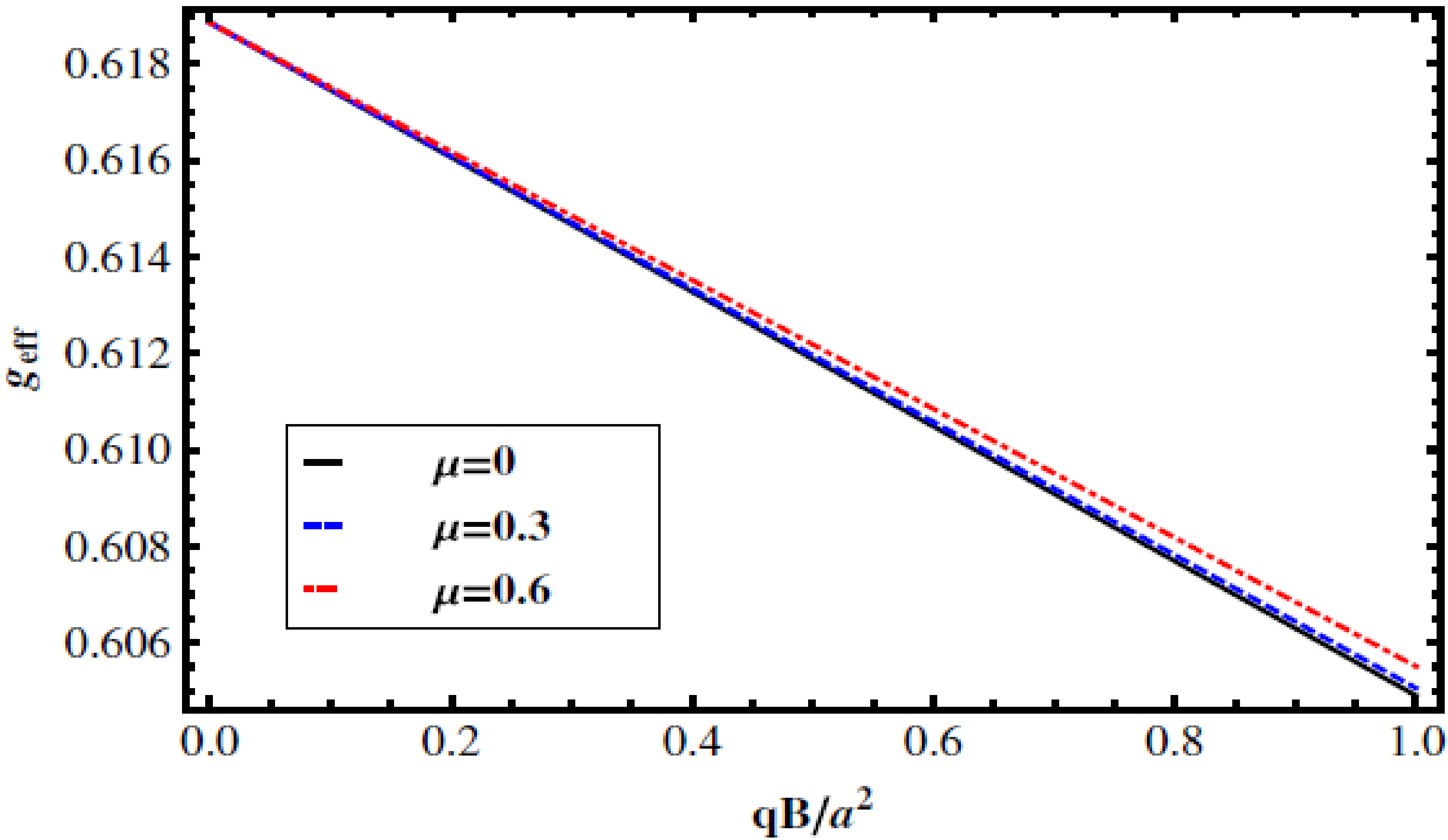


FIG. 4 (color online). Effective boson-fermion coupling $g_{\rm eff}$ evaluated at the temperature $T = 180$ MeV with $\lambda = 0.4$, $g = 0.63$ as a function of the magnetic field strength for different values of μ.

Magnetic Anicatalysis

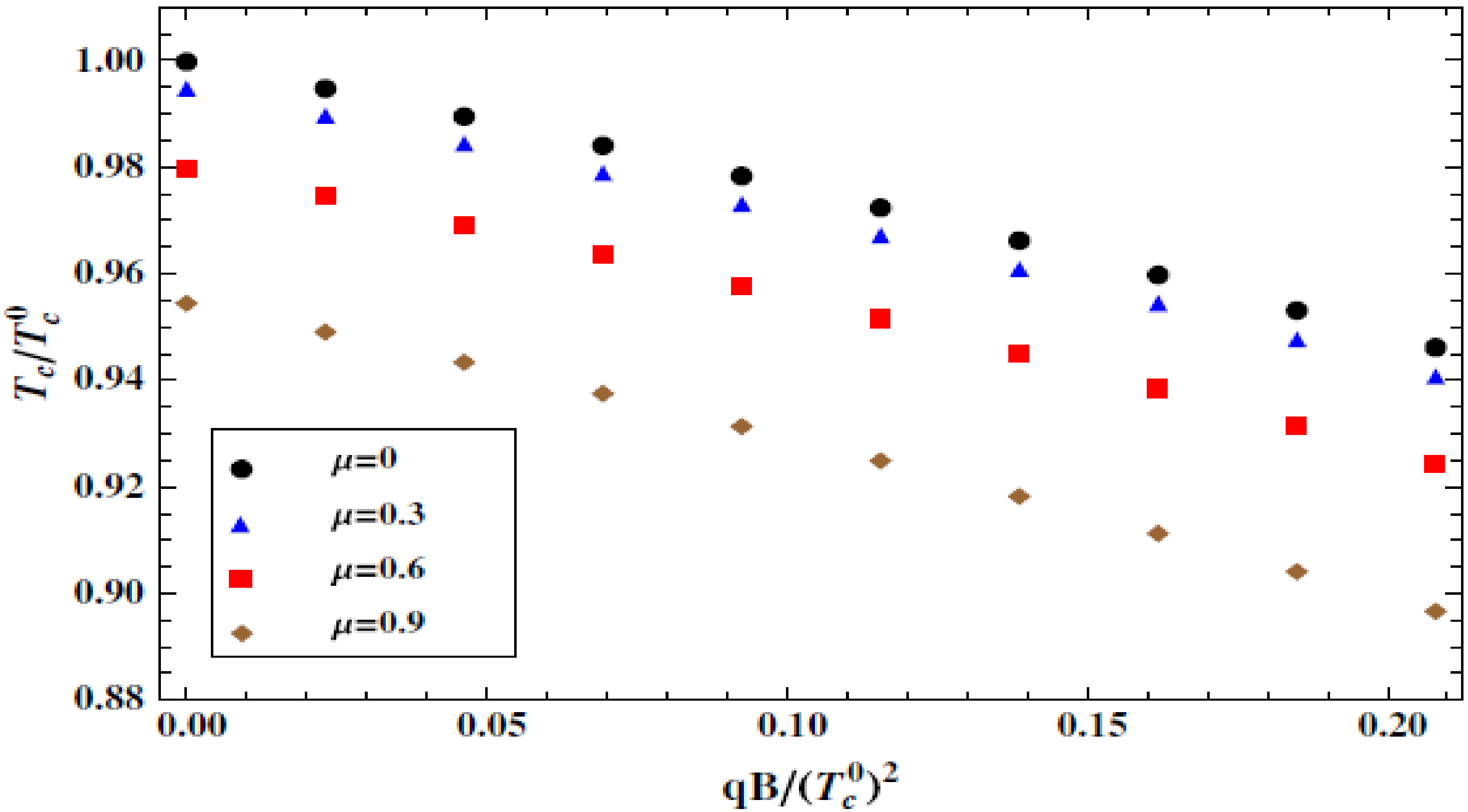


FIG. 5 (color online). Critical temperature as a function of the magnetic field strength evaluated using effective couplings including thermo-magnetic corrections with the bare values of the couplings $\lambda = 0.4$, $g = 0.63$ for different values of μ. Note that in all cases the critical temperature is a monotonically decreasing function of the magnetic field strength.

- Inserting the thermo-magnetic dependence of the couplings in the effective potential we get our main reult:

The evolution of the phase diagram (in the T-μ plane) as function of the external magnetic field.

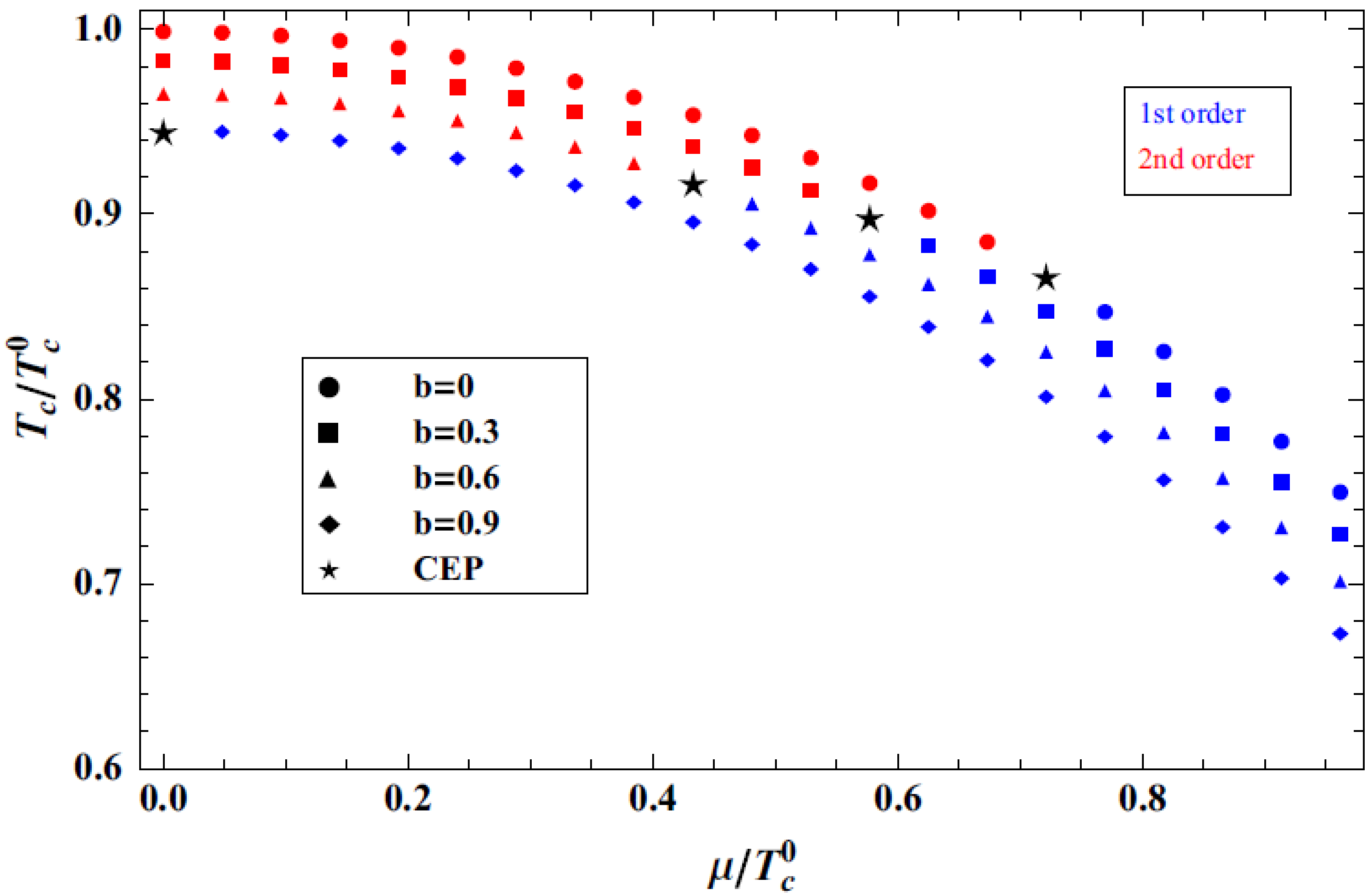


FIG. 9 (color online). Phase diagram in the temperature quark chemical potential plane computed with thermo-magnetic corrections to the couplings using the bare values $\lambda = 0.4$, $g = 0.63$ corresponding to $a/T_c^0 = 0.77$ for different values of the magnetic field strength. The phase transitions to the left (right) of the CEP in each case are of second (first) order.

$b = qB/T_{c0}^2$

Some Conclusions:

External magnetic fields seem to play an important role in the structure of the QCD phase diagram

The CEP moves to the left in the T-μ plane, i.e toward lower values of the critical quark chemical potential, and also to larger values of the critical temperature, as the intensity of the external magnetic field increases.

This fact is associated to the dimensional reduction associated to the formation of the Landau levels.

For this behavior, a proper treatment of the resumation of ring diagrams, including thermo-magnetic effective constants, was essential.