Deuteron Electromagnetic Form Factors in Holographic QCD

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Introduction

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD:
  attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space

- Motivation: AdS/CFT correspondence 1998 (Maldacena, Polyakov, Witten et al)
  Extra-Dimensional (ED) theories including gravity are holographically equivalent to gauge theories living on boundary of ED space

- Symmetry arguments: Conformal group acting in boundary theory isomorphic to $SO(4,2)$ – the isometry group of AdS$_5$ space

- HQCD models reproduce main features of QCD at low and high energies:
  chiral symmetry, confinement, power scaling of hadron form factors,
  Regge trajectories for mass spectrum
  see papers by Brodsky-Teramond, our group, ...

- PDFs, GPDs, Structure functions

- LFWFs from matching AdS/QCD and LF QCD

Deuteron in a soft-wall AdS/QCD approach

- We first propose an effective action describing the dynamics of the deuteron in the presence of an external vector field
- Based on this action the deuteron electromagnetic form factors are calculated, displaying the correct $1/Q^{10}$ power scaling for large Q^2 values.
- This finding is consistent with quark counting rules and the earlier observation that this result holds in confining gauge/gravity duals.
- The Q^2 dependence of the deuteron form factors is defined by a single and universal scale parameter κ, which is fixed from data.

Deuteron in a soft-wall AdS/QCD approach

- Effective action in terms of AdS fields $d^M(x,z)$ and $V^M(x,z)$
- $d^M(x,z)$ – dual to Fock component contributing to deuteron with twist $\tau = 6$
- $V^M(x,z)$ – dual to the electromagnetic field

$$
\begin{aligned}
S &= \int d^4x dz\, e^{-\varphi(z)} \Big[-\frac{1}{4} F_{MN}(x,z) F^{MN}(x,z) - D^M d^\dagger_N(x,z) D_M d^N(x,z) \\
&- i c_2 F^{MN}(x,z) d^\dagger_M(x,z) d_N(x,z) \\
&+ \frac{c_3}{4M_d^2}\, e^{2A(z)}\, \partial^M F^{NK}(x,z) \Big(-d^\dagger_M(x,z) \overleftrightarrow{D}_K\, d_N(x,z) + \mathrm{H.c.} \Big) \\
&+ d^\dagger_M(x,z) \Big(\mu^2 + U(z) \Big) d^M(x,z) \Big]
\end{aligned}
$$

Deuteron in a soft-wall AdS/QCD approach

- $A(z) = \log(R/z)$, R is the AdS radius

  $F^{MN} = \partial^M V^N - \partial^N V^M$ - stress tensor of vector field

  $D^M = \partial^M - ieV^M(x, z)$ - covariant derivative

  $\mu^2 R^2 = (\Delta - 1)(\Delta - 3)$ - five-dimensional mass

  $\varphi(z) = \kappa^2 z^2$ is the background dilaton field

  $\Delta = 6 + L$ is the dimension of $d^M(x, z)$

  L is the maximal value of orbital angular momentum

  $U(z) = U_0\, \varphi(z)/R^2$ is the confinement potential

  U_0 is constant fixed the deuteron mass.

  Use axial gauge for both vector fields $d^z(x, z) = 0$ and $V^z(x, z) = 0$

Deuteron in a soft-wall AdS/QCD approach

- First perform Kaluza-Klein (KK) decomposition for vector AdS field dual to deuteron

$$d^\mu(x,z) = \exp\left[\frac{\varphi(z) - A(z)}{2}\right] \sum_n d_n^\mu(x)\Phi_n(z) \,,$$

$d_n^\mu(x)$ is the tower of the KK fields dual to the deuteron fields with radial quantum number n and twist-dimension $\tau = 6$, and $\Phi_n(z)$ are their bulk profiles.
Then we derive the Schrödinger-type equation of motion for the bulk profile

$$\left[-\frac{d^2}{dz^2} + \frac{4(L+4)^2 - 1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0\right]\Phi_n(z) = M_{d,n}^2 \Phi_n(z) \ .$$

Deuteron in a soft-wall AdS/QCD approach

- The analytical solutions of this EOM read

$$\Phi_n(z) = \sqrt{\frac{2n!}{(n+L+4)!}}\,\kappa^{L+5}\,z^{L+9/2}\,e^{-\kappa^2 z^2/2}\,L_n^{L+4}(\kappa^2 z^2)\,,$$
$$M_{d,n}^2 = 4\kappa^2\left[n + \frac{L+5}{2} + \frac{U_0}{4}\right],$$

  where $L_n^m(x)$ are the generalized Laguerre polynomials

- Restricting to the ground state $(n = 0,\ L = 0)$ we get $M_d = 2\kappa\sqrt{\frac{5}{2} + \frac{U_0}{4}}$
- $\kappa = 190$ MeV (fitted from data on electromagnetic deuteron form factors)
- Using central value of deuteron mass $M_d = 1.875613$ GeV we fix $U_0 = 87.4494$

Deuteron in a soft-wall AdS/QCD approach

- We can compare this value for the deuteron scale parameter to the analogous one of κ_N defining the nucleon properties - mass and electromagnetic form factors. In description of nucleon case we fixed the value to $\kappa_N \simeq 380$ MeV, which is 2 times bigger than the deuteron scale parameter κ.

- Difference between the nucleon and deuteron scale parameters can be related to the change of size of the hadronic systems - the deuteron as a two-nucleon bound state is 2 times larger than the nucleon.

Deuteron in a soft-wall AdS/QCD approach

- In case of the vector field dual to the electromagnetic field we perform a Fourier transform with respect to the Minkowski coordinate

$$V_\mu(x,z) = \int \frac{d^4q}{(2\pi)^4} e^{-iqx} V_\mu(q) V(q,z)$$

where $V(q,z)$ is its bulk profile obeying the following EOM:

$$\partial_z \left(\frac{e^{-\varphi(z)}}{z} \partial_z V(q,z) \right) + q^2 \frac{e^{-\varphi(z)}}{z} V(q,z) = 0 \, .$$

- Its analytical solution can be written in the form of

$$V(Q,z) = \kappa^2 z^2 \int\limits_0^1 \frac{dx}{(1-x)^2} \, e^{-\kappa^2 z^2 x/(1-x)} \, x^a \, , \quad a = \frac{Q^2}{4\kappa^2} \, , \quad Q^2 = -q^2 \, .$$

Deuteron in a soft-wall AdS/QCD approach

- The gauge-invariant matrix element describing the interaction of the deuteron with the external vector field (dual to the electromagnetic field) reads

$$\begin{aligned} M^\mu_{\rm inv}(p,p') &= -\Big(G_1(Q^2)\epsilon^*(p')\cdot\epsilon(p) - \frac{G_3(Q^2)}{2M_d^2}\,\epsilon^*(p')\cdot q\,\epsilon(p)\cdot q\Big)\,(p+p')^\mu \\ &- G_2(Q^2)\,\Big(\epsilon^\mu(p)\,\epsilon^*(p')\cdot q - \epsilon^{*\mu}(p')\,\epsilon(p)\cdot q\Big) \end{aligned}$$

where $\epsilon(\epsilon^*)$ and $p(p')$ are the polarization and four-momentum of the initial (final) deuteron, and $q = p' - p$ is the momentum transfer.

Deuteron in a soft-wall AdS/QCD approach

- Three EM form factors $G_{1,2,3}$ of the deuteron are related to the charge G_C, quadrupole G_Q and magnetic G_M form factors by
- Expressions for the form factors

$$G_C = G_1 + \frac{2}{3}\tau_d G_Q\,, \quad G_M \;=\; G_2\,, \quad G_Q = G_1 - G_2 + (1+\tau_d)G_3, \quad \tau_d = \frac{Q^2}{4M_d^2}\,.$$

These form factors are normalized at zero recoil as

$$G_C(0) = 1\,, \quad G_Q(0) = M_d^2 \mathcal{Q}_d = 25.83\,, \quad G_M(0) = \frac{M_d}{M_N}\mu_d = 1.714\,,$$

- $\mathcal{Q}_d = 7.3424$ GeV^{-2} and $\mu_d = 0.8574$ – quadrupole and magnetic moments of the deuteron.

Deuteron in a soft-wall AdS/QCD approach

- In our approach the deuteron form factor $G_1(Q^2) = F(Q^2)$, where $F(Q^2)$ is the twist-6 hadronic form factor, which is given by the overlap of the square of the bulk profile dual to the deuteron wave function (twist-6 hadronic wave function) and the confined electromagnetic current

$$F(Q^2) = \int\limits_0^\infty dz\, \Phi_0^2(z)\, V(Q,z) = \frac{\Gamma(6)\,\Gamma(a+1)}{\Gamma(a+6)}$$

where $a = Q^2/(4\kappa^2)$.

- This formula follows from the result of Brodsky and Teramond – general and universal formula for hadronic FF with twist τ in terms of bulk profile $\phi_\tau(z) = \sqrt{\frac{2}{(\tau-2)!}}\,\kappa^{\tau-1} z^{\tau-3/2} e^{-\kappa^2 z^2/2}$ dual to the hadronic wave function with twist τ:

$$F_\tau(Q^2) = \int\limits_0^\infty dz\, \phi_\tau^2(z)\, V(Q,z) = \frac{\Gamma(\tau)\,\Gamma(a+1)}{\Gamma(a+\tau)}$$

Deuteron in a soft-wall AdS/QCD approach

- By analogy we calculate the other two deuteron FF G_2 and G_3, expressed in terms of the same universal factor $F(Q^2)$

$$G_i(Q^2) = c_i F(Q^2)\,, \quad i = 2, 3\,.$$

The parameters c_2 and c_3 are defined by normalization of the deuteron form factors as

$$c_2 = G_M(0) = 1.714\,, \quad c_3 = G_M(0) + G_Q(0) - 1 = 26.544\,.$$

- $F(Q^2)$ has the correct power-scaling $F(Q^2) \sim 1/(Q^2)^5$ at large $Q^2 \to \infty$
- It can also be written in the Brodsky-Ji-Lepage form derived within pQCD
- Deuteron FF is factorized in terms of the nucleon FF $F_N(Q^2/4)$ and the so-called "reduced" nuclear form factor $f_d(Q^2)$ (Brodsky-Ji-Lepage):

$$F_d(Q^2) = f_d(Q^2)\, F_N^2(Q^2/4)$$

Deuteron in a soft-wall AdS/QCD approach

- Our result reads

$$F_d(Q^2) \equiv F(Q^2) = \frac{\Gamma(6)\,\Gamma(a+1)}{\Gamma(a+6)} = \frac{5!}{(a+1)\ldots(a+5)} = f_d(Q^2)\,F_N^2(Q^2/4)$$

where predictions for $f_d(Q^2)$ and $F_N(Q^2/4)$ are

$$f_d(Q^2) = \frac{30(a+1)(a+2)}{(a+3)(a+4)(a+5)}\,, \quad F_N(Q^2/4) = \frac{2}{(a+1)(a+2)}$$

where $a = Q^2/(4\kappa^2)$.

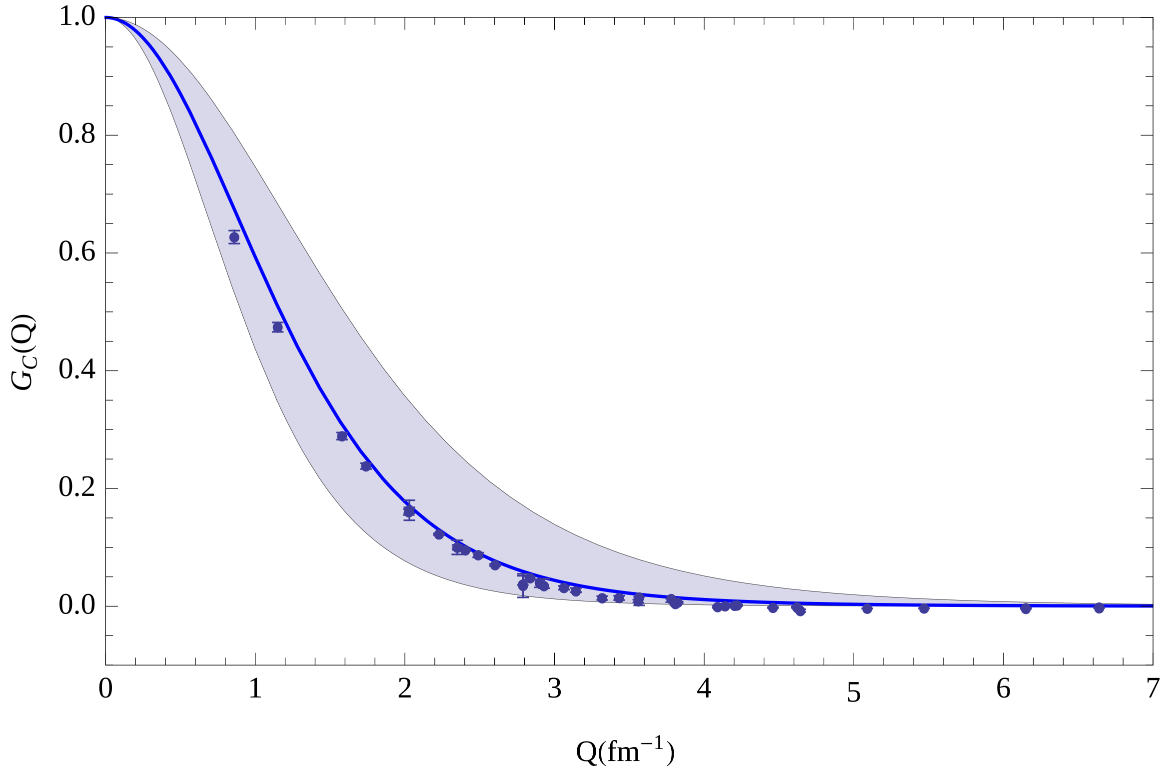
Deuteron in a soft-wall AdS/QCD approach

- Numerical results for deuteron FF
- Shaded band corresponds to values of κ in range of 150 MeV $< \kappa <$ 250 MeV.
- Increase of the parameter κ leads to an enhancement of the form factors.
- The best description of the data on the deuteron form factors is obtained for $\kappa = 190$ MeV and is shown by the solid line.
- With $\kappa = 190$ MeV our result for the deuteron charge radius

  $r_C = \sqrt{-6G'_C(0)} = \sqrt{\frac{137}{40\kappa^2} - \mathcal{Q}_d} = 1.846$ fm close to data

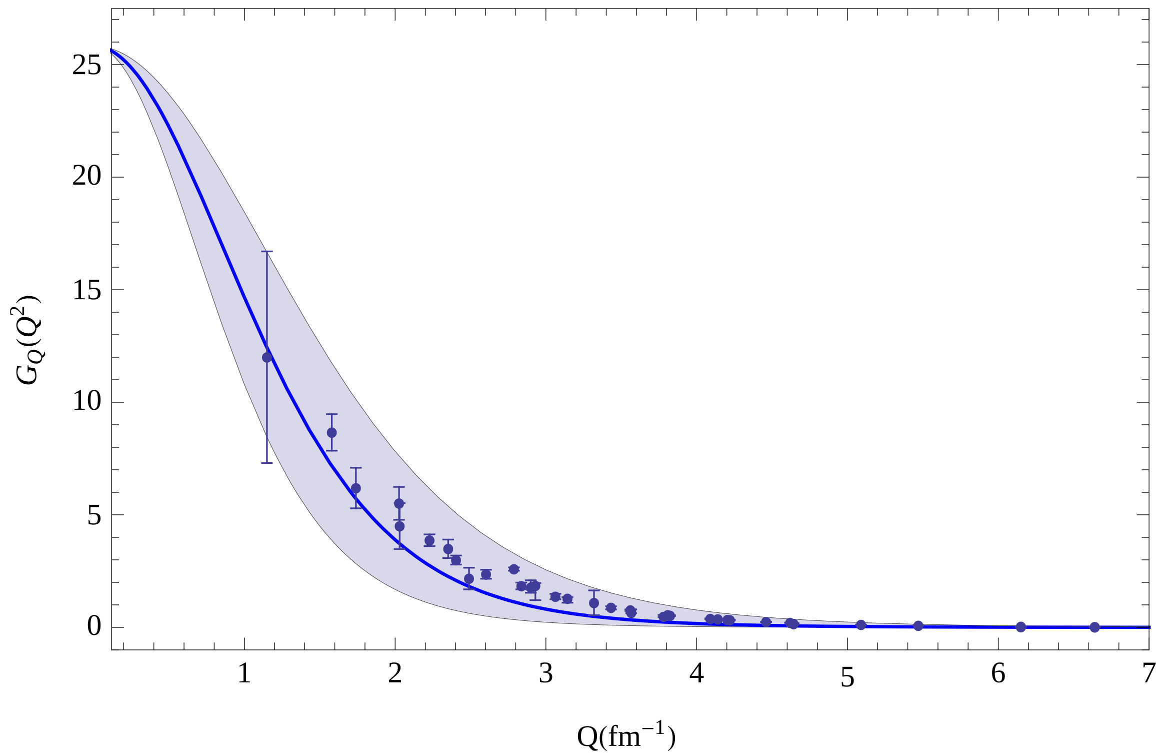
  $r_C = 2.130 \pm 0.010$ fm.

Deuteron in a soft-wall AdS/QCD approach



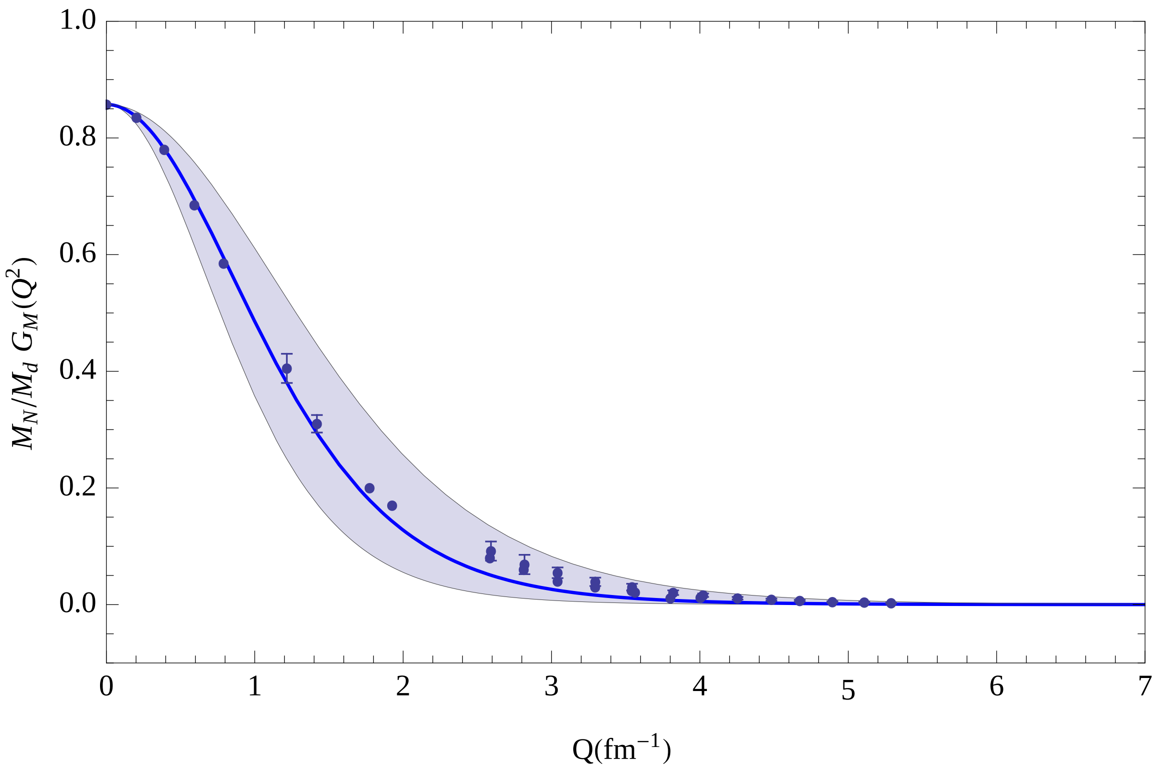
Charge deuteron form factor

Deuteron in a soft-wall AdS/QCD approach



Quadrupole deuteron form factor

Deuteron in a soft-wall AdS/QCD approach



Magnetic deuteron form factor

Summary

- AdS/QCD $\equiv$ Holographic QCD (HQCD) – approximation to QCD: attempt to model Hadronic Physics in terms of fields/strings living in extra dimensions – anti-de Sitter (AdS) space
- HQCD models reproduce main features of QCD at low and high energies
- Soft–wall holographic approach – covariant and analytic model for hadron structure with confinement at large distances and conformal behavior at short distances
- Mesons, baryons, exotics from unified point view and including high Fock states
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