Adjoint $SU(5)$ with T_7 flavour symmetry

Based on:
C. Arbeláez, A. E. Cárcamo, S. Kovalenko, I. Schmidt.
Phys. Rev. D92 (2015) 11, 115015

Carolina Arbeláez R.
Universidad Técnica Federico Santa María.
Valparaiso, Chile.

High Energy Physics
in the LHC era.
6th International Workshop

January 7, 2016.

Overview

- Introduction.
- The model.
- Fermion masses and mixings.
- SU(5) unification.
- Conclusions.

Introduction

* The origin of fermion masses and mixings cannot be understood within the SM.

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	(0–0.13)×10^{-9}	0
e electron	0.000511	–1
ν_M middle neutrino*	(0.009–0.13)×10^{-9}	0
μ muon	0.106	–1
ν_H heaviest neutrino*	(0.04–0.14)×10^{-9}	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

* Neutrinos are puzzling:

- Neutrinos are massive.
- Experimental confirmation of neutrinos oscillations.
- Smallness of the neutrino masses: RHN.
- Dirac or Majorana nature is unsolved.

* Neutrino parameters:

Parameter	$\Delta m_{21}^2 (10^{-5} \mathrm{eV}^2)$	$\Delta m_{31}^2 (10^{-3} \mathrm{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\mathrm{exp}}$	$\left(\sin^2\theta_{23}\right)_{\mathrm{exp}}$	$\left(\sin^2\theta_{13}\right)_{\mathrm{exp}}$
Best fit	7.60	2.48	0.323	0.567	0.0234
1σ range	7.42 – 7.79	2.41 – 2.53	0.307 – 0.339	0.439 – 0.599	0.0214 – 0.0254
2σ range	7.26 – 7.99	2.35 – 2.59	0.292 – 0.357	0.413 – 0.623	0.0195 – 0.0274
3σ range	7.11 – 8.11	2.30 – 2.65	0.278 – 0.375	0.392 – 0.643	0.0183 – 0.0297

→ ~~TBM~~

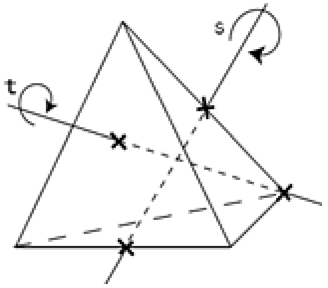
D.V.Forero, M. Tortola and J.W.F. Valle, Phys. Rev. D 90, 093006 (2014)

Additional flavour symmetries

Discrete subgroups of **SU(3)** (… 3 because of the 3 families)

* A_4 → Isomorphic group of the 12 thetraedrom rotations

3-dim irreps.



H. Fritzsch and Z. -z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000) [hep-ph/9912358]

G. Altarelli and F. Feruglio, Springer Tracts Mod. Phys. 190, 169 (2003) [hep-ph/0206077].

* T_7

Is the smallest SU(3) subgroup with two nonequivalent 3-dim irreps. and having a complex triplet. Very useful to generate a viable deviation from the TBM pattern.

Luhn, Nasri and Ramond , PLB (2007) and JMP (2007)

T_7 → Model with a more predictive lepton sector

* T_7 has 21 elements and two generators:

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \rho = e^{\frac{2i\pi}{7}}$$

* Product rules:

$\underline{1}_p \otimes \underline{1}_q \cong \underline{1}_{(p+q) mod(3)}$
$\underline{1}_p \otimes \underline{3} \cong \underline{3}$
$\underline{1}_p \otimes \underline{3}^* \cong \underline{3}^*$
$\underline{3} \otimes \underline{3} \cong \underline{3} \oplus \underline{3}^* \oplus \underline{3}^*$
$\underline{3}^* \otimes \underline{3}^* \cong \underline{3}^* \oplus \underline{3} \oplus \underline{3}$
$\underline{3} \otimes \underline{3}^* \cong \underline{3}^* \oplus \underline{3} \oplus \underline{1}_1 \oplus \underline{1}_2 \oplus \underline{1}_3$

Ishimori, et al, Prog. Theor. Phys. Suppl 2010

The Model

Multi-Higgs extention of the next to minimal SU(5) GUT

$$G = SU(5) \times T_7 \times Z_2 \times Z_3 \times Z_4 \times Z_4' \times Z_{12}$$

$$\Downarrow \;\; \Lambda_{GUT}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\Downarrow \;\; \Lambda_{EW}$$

$$SU(3)_C \times U(1)_{EW}$$

* Scalar sector: **24, 45**, **5**, **1**.

* The fermion assignments:

$$G = SU(5) \times T_7 \times Z_2 \times Z_3 \times Z_4 \times Z_4' \times Z_{12}$$

$$\begin{aligned}
\Psi_{ij}^{(1)} &\sim \left(\mathbf{10}, \mathbf{1}_0, 1, 1, \omega, 1, i\right), \quad \Psi_{ij}^{(2)} \sim \left(\mathbf{10}, \mathbf{1}_1, 1, 1, \omega^2, 1, e^{\frac{\pi i}{3}}\right) \\
\Psi_{ij}^{(3)} &\sim \left(\mathbf{10}, \mathbf{1}_2, 1, 1, 1, 1, 1\right), \quad i, j = 1, 2, 3, 4, 5 \\
\psi^i &= \left(\psi^{i(1)}, \psi^{i(2)}, \psi^{i(3)}\right) \sim \left(\overline{\mathbf{5}}, \overline{\mathbf{3}}, 1, 1, 1, 1, 1\right) \\
N_R &\sim \left(\mathbf{1}, \mathbf{1}_0, -1, 1, 1, 1, 1\right), \qquad \rho \sim \left(\mathbf{24}, \mathbf{1}_0, -1, -1, 1, 1, 1\right)
\end{aligned}$$

* The scalar assignments:

$$\begin{aligned}
\sigma &\sim \left(\mathbf{1}, \mathbf{1}_0, 1, 1, 1, 1, e^{-\frac{i\pi}{6}}\right), \quad \tau \sim \left(\mathbf{1}, \mathbf{1}_0, 1, 1, \omega, i, e^{-\frac{i\pi}{6}}\right) \\
\varphi &\sim \left(\mathbf{1}, \mathbf{1}_0, 1, 1, \omega, -1, 1\right), \\
\xi &= \left(\xi_1, \xi_2, \xi_3\right) \sim \left(\mathbf{1}, \mathbf{3}, -1, 1, 1, 1, 1\right) \\
\eta &= \left(\eta_1, \eta_2, \eta_3\right) \sim \left(\mathbf{1}, 1, 1, 1, 1, 1, 1\right) \\
\chi &= \left(\chi_1, \chi_2, \chi_3\right) \sim \left(\mathbf{1}, \mathbf{3}, 1, -1, 1, 1, 1\right) \\
H_i^{(1)} &\sim \left(\mathbf{5}, \mathbf{1}_0, -1, 1, 1, 1, e^{-\frac{i\pi}{3}}\right), \quad H_i^{(2)} \sim \left(\mathbf{5}, \mathbf{1}_0, 1, 1, \omega, 1, 1\right) \\
H_i^{(3)} &\sim \left(\mathbf{5}, \mathbf{1}_1, 1, 1, \omega^2, 1, 1\right), \quad H_i^{(4)} \sim \left(\mathbf{5}, \mathbf{1}_2, 1, 1, 1, 1, 1\right) \\
\Xi_j^i &\sim \left(\mathbf{24}, \mathbf{1}_0, 1, 1, 1, 1, 1\right), \quad \Phi_{jk}^i \sim \left(\mathbf{45}, \mathbf{1}_0, -1, 1, 1, 1, e^{-\frac{i\pi}{3}}\right)
\end{aligned}$$

* Fermions:

$$G = SU(5) \times T_7 \times Z_2 \times Z_3 \times Z_4 \times Z_4' \times Z_{12}$$

	$\Psi_{ij}^{(1)}$	$\Psi_{ij}^{(2)}$	$\Psi_{ij}^{(3)}$	ψ^i	N_R	ρ
$SU(5)$	10	10	10	$\bar{5}$	1	24
T_7	1	1	1	$\bar{3}$	1	1

* Scalars:

	σ	τ	φ	ξ	η	χ
$SU(5)$	1	1	1	1	1	1
T_7	1	1	1	3	1	3

	$H_i^{(1)}$	$H_i^{(2)}$	$H_i^{(3)}$	$H_i^{(4)}$	Σ_j^i	Φ_{jk}^i
$SU(5)$	5	5	5	5	24	45
T_7	1	1	1	1	1	1

* Family symmetries

T_7 : Unifies in antitriplets the 3 families of the $\bar{5}$ fermionic irreps. of SU(5).

Z_2 : Separates the SU(5) charged lepton and down type Yukawa interactions:

$$L_Y^{(1)} = \alpha_1 (\psi^i \xi)_{1_0} H^{j(1)} \Psi_{ij}^{(1)} \frac{\sigma^5 \varphi^2 + k\sigma\tau^4 \varphi^{*2}}{\Lambda^8} + \cdots$$

$$\ldots + \beta_1 (\psi^i \xi)_{1_0} \Phi_i^{jk} \Psi_{jk}^{(1)} \frac{\sigma^5 \varphi^2 + k\sigma\tau^4 \varphi^{*2}}{\Lambda^8} + \cdots$$

from the up type Yukawa interactions:

$$L_Y^{(2)} = \varepsilon^{ijklp} (\gamma_{11} \Psi_{ij}^{(1)} H_p^{(2)} \Psi_{kl}^{(1)} \frac{\sigma^6}{\Lambda^6} + \cdots)$$

Since the charged fermion mass and quark mixing pattern arises from the $Z_3 \otimes Z_4 \otimes Z_{12}$ symmetry breaking, we set

$$v_\chi \ll v_\eta \sim v_\varphi = v_\tau = v_\zeta = v_\sigma = \Lambda_{GUT} = \lambda\Lambda$$

where $\lambda = 0.225$.

Z_4' : Crucial to get predictive neutrino mass matrix texture.

$$L_Y^{(3)} = \dots + \frac{\lambda_{4\nu}}{\Lambda}\left(\psi^i\chi\right)_{\mathbf{1}_0}\Phi_{ij}^k\rho_k^j + m_N\overline{N}_R N_R^c + y_1\overline{N}_R N_R^c\frac{\sigma^*\sigma + x_1\tau^*\tau + x_2\varphi^*\varphi}{\Lambda} + y_2 Tr\left(\rho^2\right)\eta + y_3 Tr\left(\rho^2\Xi\right)\frac{\eta}{\Lambda}$$

Lepton masses and mixings

The charged lepton mass matrix is described by:

$$M_l = \frac{v}{\sqrt{2}} V_{lL}^{\dagger} \begin{pmatrix} a_1^{(l)}\lambda^8 & 0 & 0 \\ 0 & a_2^{(l)}\lambda^5 & 0 \\ 0 & 0 & a_3^{(l)}\lambda^3 \end{pmatrix} = V_{lL}^{\dagger} \mathrm{diag}\left(m_e, m_\mu, m_\tau\right)$$

$$a_1^{(l)} = \frac{1}{v}\left(\alpha_1 v_H^{(1)} - 6\beta_1 v_\Phi\right), \quad a_2^{(l)} = \frac{1}{v}\left(\alpha_2 v_H^{(1)} - 6\beta_2 v_\Phi\right)$$

$$a_3^{(l)} = \frac{1}{v}\left(\alpha_3 v_H^{(1)} - 6\beta_3 v_\Phi\right)$$

$$V_{lL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \qquad \omega = e^{\frac{2\pi i}{3}}$$

Here $\lambda = 0.225$ is the Wolfenstein parameter.

The model has 16 effective free parameters

18 observables: 9 charged fermions masses, 2 neutrino mass squared splitting, 3 lepton mixing parameters and 4 parameters of the Wolfenstein CKM quark mixing matrix parameterization.

Quark sector

Observable	Model value	Experimental value
$m_u(MeV)$	0.86	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	673	635 ± 86
$m_t(GeV)$	174.2	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	57.7	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.82	$2.82^{+0.09}_{-0.04}$
$\lvert V_{ud} \rvert$	0.974	0.97427 ± 0.00015
$\lvert V_{us} \rvert$	0.2257	0.22534 ± 0.00065
$\lvert V_{ub} \rvert$	0.00305	$0.00351^{+0.00015}_{-0.00014}$
$\lvert V_{cd} \rvert$	0.2256	0.22520 ± 0.00065
$\lvert V_{cs} \rvert$	0.97347	0.97344 ± 0.00016
$\lvert V_{cb} \rvert$	0.0384	$0.0412^{+0.0011}_{-0.0005}$
$\lvert V_{td} \rvert$	0.00785	$0.00867^{+0.00029}_{-0.00031}$
$\lvert V_{ts} \rvert$	0.0377	$0.0404^{+0.0011}_{-0.0005}$
$\lvert V_{tb} \rvert$	0.999145	$0.999146^{+0.000021}_{-0.000046}$

Gauge Coupling SU(5) Unification

* Scalar sector

$$\begin{aligned}
\mathbf{5}_s &= (1,2)_{1/2} \oplus (3,1)_{-1/3}, \\
&= H_1 \oplus H_2, \\
\mathbf{24}_s &= (1,1)_0 \oplus (1,3)_0 \oplus (8,1)_0 \oplus (3,2)_{-5/6} \oplus (\overline{3},2)_{5/6}, \\
&= \Xi_1 \oplus \Xi_3 \oplus \Xi_8 \oplus \Xi_{(3,2)} \oplus \Xi_{(\overline{3},2)}, \\
\mathbf{45}_s &= (1,2)_{1/2} \oplus (3,1)_{-1/3} \oplus (3,3)_{-1/3} \oplus (\overline{3},1)_{4/3} \oplus (\overline{3},2)_{-7/6} \oplus (6,1)_{-1/3} \oplus (8,2)_{1/2}, \\
&= \Phi_1 \oplus \Phi_2 \oplus \Phi_3 \oplus \Phi_4 \oplus \Phi_5 \oplus \Phi_6 \oplus \Phi_7.
\end{aligned}$$

* Fermionic sector :

$$\begin{aligned}
\mathbf{24}_f &= (1,1)_0 \oplus (1,3)_0 \oplus (8,1)_0 \oplus (3,2)_{-5/6} \oplus (\overline{3},2)_{5/6}, \\
&= \rho_0 \oplus \rho_3 \oplus \rho_8 \oplus \rho_{(3,2)} \oplus \rho_{(\overline{3},2)}.
\end{aligned}$$

⬇

Type I-III seesaw realization: N_R, ρ_0, ρ_3.

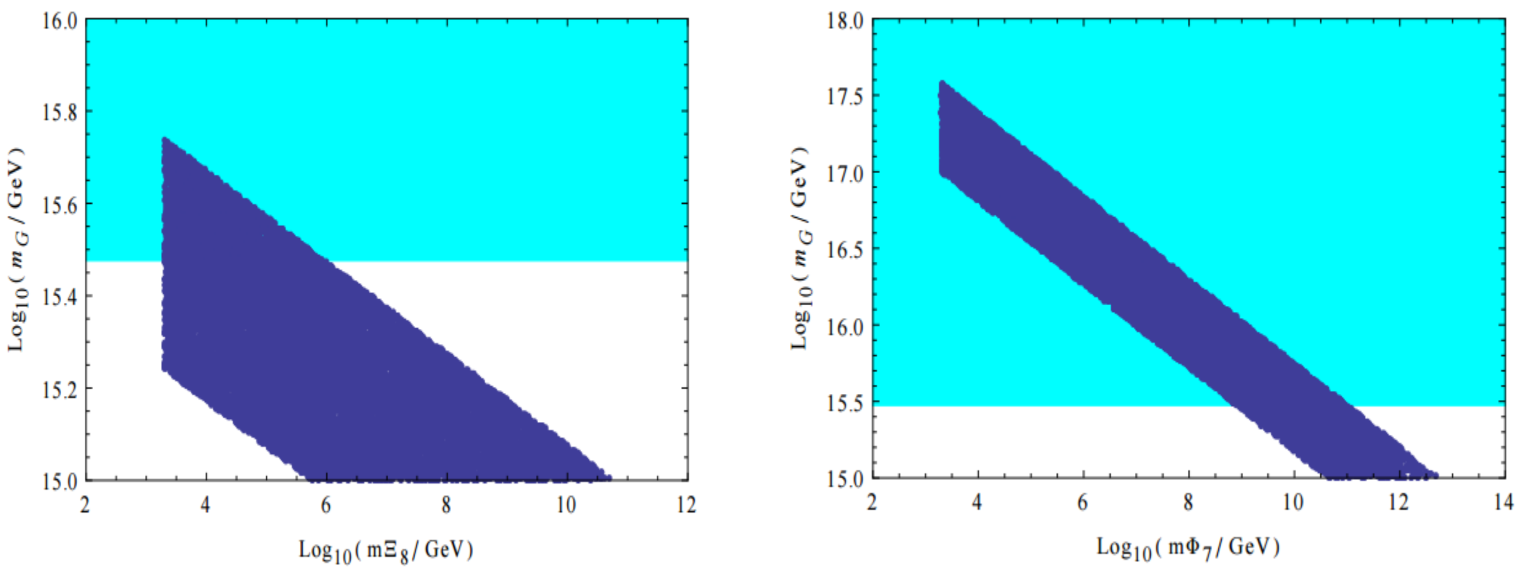
* Model a: X: $\Xi_8 + 2\Xi_3 + N_R + \rho_0 + \rho_3$, $m_G = 2.82 \times 10^{15}$ GeV.

  with: $\Xi_{3,8}$ at $m_{NP} = 2\ TeV$ and $m_S = 10^{14}\ GeV$.

* Model b: X: $2\Phi_7 + \Phi_1 + \Xi_3 + N_R + \rho_0 + \rho_3$, $m_G = 4.17 \times 10^{17}$ GeV.

  with: $\Phi_{1,7}$ and Ξ_3 at $m_{NP} = 2\ TeV$ and $m_S = 10^{14}\ GeV$.

Parameter space running: 2 TeV $\leq m\Xi_{3,8}, m\Phi_7 \leq m_S < m_G$



The region compatible with the proton decay constraints is explicitly shown in light blue.

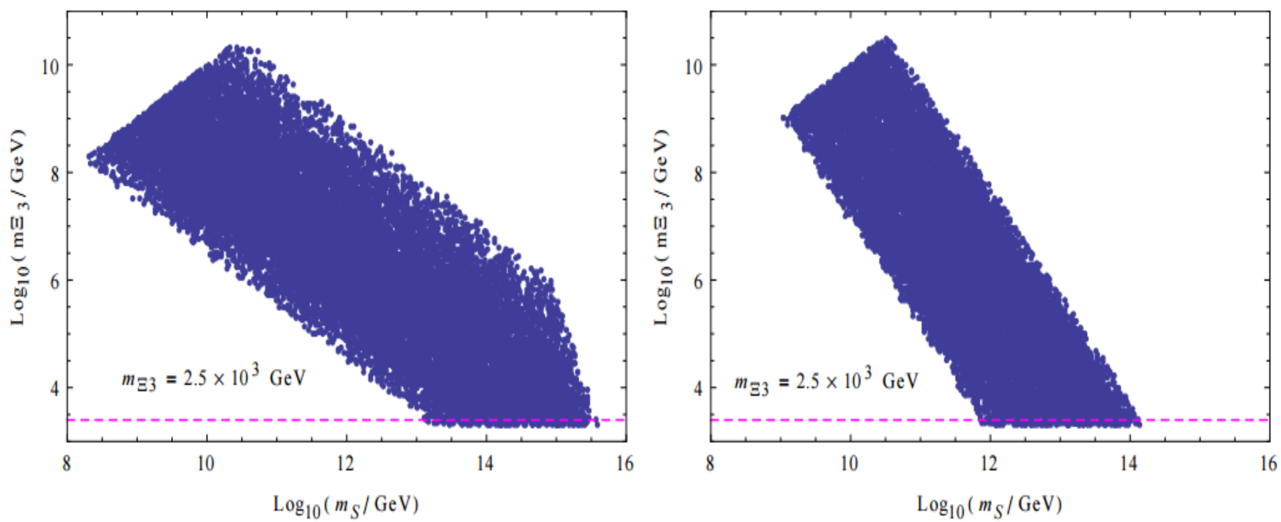
Model a: 2 TeV $\leq m\Xi_8 \leq 10^6$ GeV $\Rightarrow$ LHC $\Rightarrow \Xi_8 \rightarrow gg$

Model b: 2 TeV $\leq m\Phi_7 \leq 10^{11}$ GeV $\Rightarrow$ LHC $\Rightarrow q\bar{q} \rightarrow S^0, S^{\pm}$

Dark Matter

* The neutral component of $\Xi_3 \sim (1,3)_0 \subset 24_s$ as a Cold Dark Matter candidate.
* Vanishing of the trilinear coupling of $H^\dagger \Xi_3 H$ $\Rightarrow$ Stability of DM.
* The termal relic abundance could be compatible with the observed DM abundance: $\Omega_{DM} = 0.110 \pm 0.005$ if $m\Xi_3 \sim 2.5$ TeV.

Parameter space: Lower bounds for the seesaw mediator masses m_s



Allowed parameter space in m_S and $m\Xi_3$. Left and right for the Model I and Model II respectively.

Model I: $m_S > 10^{13}$ GeV.
Model II: $m_S > 10^{12}$ GeV.

Conclusions

* Fermion masses and mixings are successfully accommodated.

* The observed charged fermion mass and quark mixing hierarchy arises from the breaking of the Z_3, Z_4, Z_{12} discrete group at a very high energy.

* The model has in total 16 effective free parameters in the Yukawa sector, from which 2 are fixed and 14 are fitted to reproduce the experimental values of 18 observables of the fermion sector.

* Constraints arising from unification, proton decay and CDM:
  2 TeV $\leq m\Xi_8 \leq 10^6$ GeV , $m_\varsigma > 10^{13}$ GeV for Model I.
  2 TeV $\leq m\Phi_7 \leq 10^{11}$ GeV, $m_\varsigma > 10^{12}$ GeV for Model II.

* Testable models at the LHC.

Thanks for your kind attention!

Backup

21 elements, two generators:

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\rho = e^{\frac{2i\pi}{7}}$$

Important identities:

$$a^7 = e, \; b^3 = e, \; ba = a^2 b$$

$$e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad b^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix} \quad ab = \begin{pmatrix} 0 & \rho & 0 \\ 0 & 0 & \rho^2 \\ \rho^4 & 0 & 0 \end{pmatrix} \quad ab^2 = \begin{pmatrix} 0 & 0 & \rho \\ \rho^2 & 0 & 0 \\ 0 & \rho^4 & 0 \end{pmatrix}$$

$$a^2 = \begin{pmatrix} \rho^2 & 0 & 0 \\ 0 & \rho^4 & 0 \\ 0 & 0 & \rho \end{pmatrix} \quad a^2b = \begin{pmatrix} 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \\ \rho & 0 & 0 \end{pmatrix} \quad a^2b^2 = \begin{pmatrix} 0 & 0 & \rho^2 \\ \rho^4 & 0 & 0 \\ 0 & \rho & 0 \end{pmatrix}$$

$$a^3 = \begin{pmatrix} \rho^3 & 0 & 0 \\ 0 & \rho^6 & 0 \\ 0 & 0 & \rho^5 \end{pmatrix} \quad a^3b = \begin{pmatrix} 0 & \rho^3 & 0 \\ 0 & 0 & \rho^6 \\ \rho^5 & 0 & 0 \end{pmatrix} \quad a^3b^2 = \begin{pmatrix} 0 & 0 & \rho^3 \\ \rho^6 & 0 & 0 \\ 0 & \rho^5 & 0 \end{pmatrix}$$

$$a^4 = \begin{pmatrix} \rho^4 & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho^2 \end{pmatrix} \quad a^4b = \begin{pmatrix} 0 & \rho^4 & 0 \\ 0 & 0 & \rho \\ \rho^2 & 0 & 0 \end{pmatrix} \quad a^4b^2 = \begin{pmatrix} 0 & 0 & \rho^4 \\ \rho & 0 & 0 \\ 0 & \rho^2 & 0 \end{pmatrix}$$

$$a^5 = \begin{pmatrix} \rho^5 & 0 & 0 \\ 0 & \rho^3 & 0 \\ 0 & 0 & \rho^6 \end{pmatrix} \quad a^5b = \begin{pmatrix} 0 & \rho^5 & 0 \\ 0 & 0 & \rho^3 \\ \rho^6 & 0 & 0 \end{pmatrix} \quad a^5b^2 = \begin{pmatrix} 0 & 0 & \rho^5 \\ \rho^3 & 0 & 0 \\ 0 & \rho^6 & 0 \end{pmatrix}$$

$$a^6 = \begin{pmatrix} \rho^6 & 0 & 0 \\ 0 & \rho^5 & 0 \\ 0 & 0 & \rho^3 \end{pmatrix} \quad a^6b = \begin{pmatrix} 0 & \rho^6 & 0 \\ 0 & 0 & \rho^5 \\ \rho^3 & 0 & 0 \end{pmatrix} \quad a^6b^2 = \begin{pmatrix} 0 & 0 & \rho^6 \\ \rho^5 & 0 & 0 \\ 0 & \rho^3 & 0 \end{pmatrix}$$

The PMNS leptonic mixing matrix is:

$$U = V_{lL}^{\dagger} V_{\nu} = \begin{pmatrix} \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\phi}\sin\theta}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{3}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{3}} \\ \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\phi+\frac{2i\pi}{3}}\sin\theta}{\sqrt{3}} & \frac{e^{-\frac{2i\pi}{3}}}{\sqrt{3}} & \frac{e^{\frac{2i\pi}{3}}\cos\theta}{\sqrt{3}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{3}} \\ \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\phi-\frac{2i\pi}{3}}\sin\theta}{\sqrt{3}} & \frac{e^{\frac{2i\pi}{3}}}{\sqrt{3}} & \frac{e^{-\frac{2i\pi}{3}}\cos\theta}{\sqrt{3}} + \frac{e^{-i\phi}\sin\theta}{\sqrt{3}} \end{pmatrix} P_{\nu}.$$

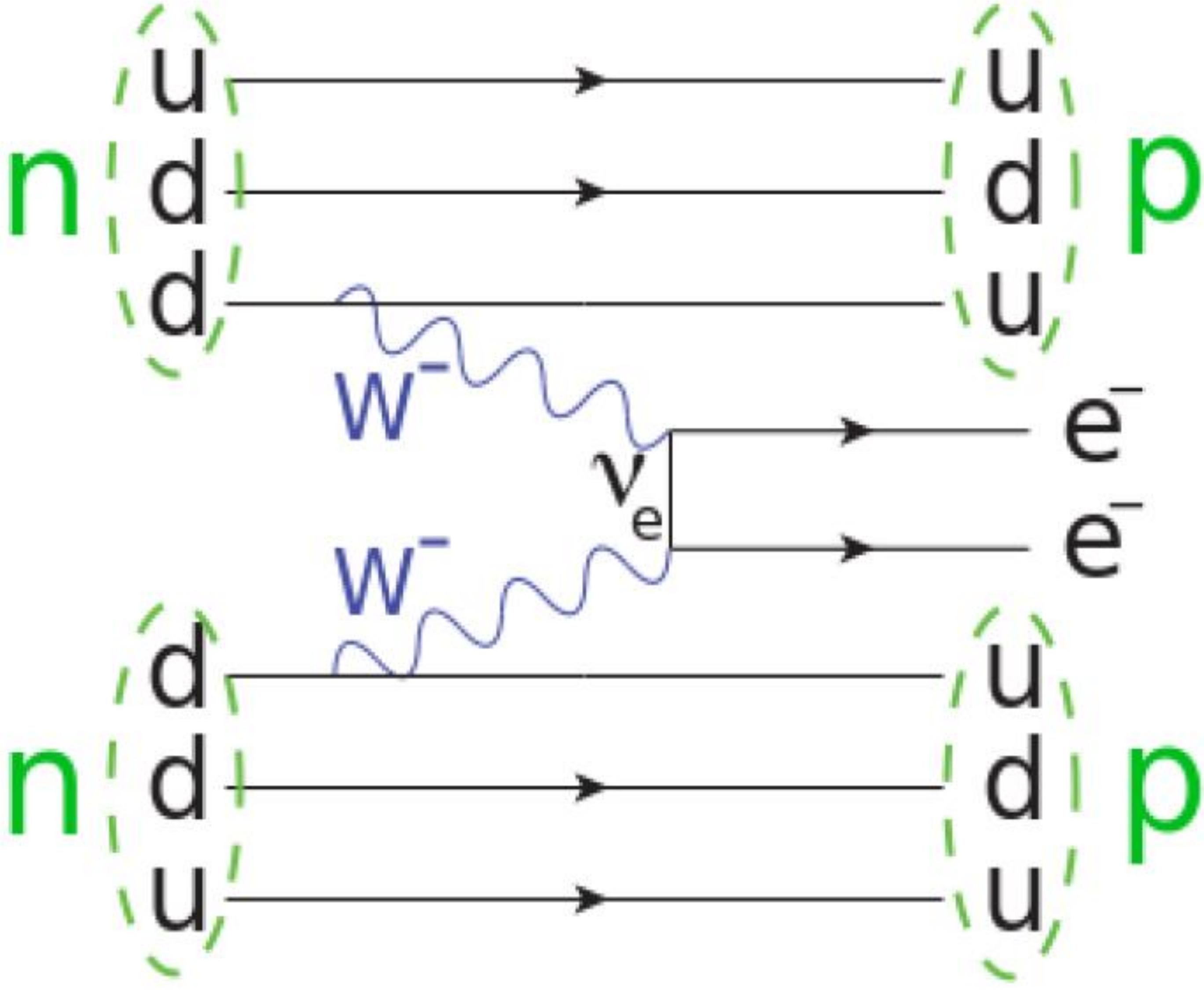
The lepton mixing angles are:

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1-|U_{e3}|^2} = \frac{1}{2 - z\cdot\cos\phi},$$

$$\sin^2\theta_{13} = |U_{e3}|^2 = \frac{1}{3}(1 + z\cdot\cos\phi),$$

$$\sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1-|U_{e3}|^2} = \frac{2 - z\cdot(\cos\phi + \sqrt{3}\sin\phi)}{4 - 2z\cdot\cos\phi}, \qquad (25)$$

with $z = 1$ and $z = -1$ for NH and IH, respectively. From the relation $\theta = \pm\frac{\pi}{4}$, we predict $J = 0$ and $\delta = 0$.


n
u
d
d
W⁻
νe
e⁻
e⁻
W⁻
d
d
u
n
u
d
u
p
u
d
u
p

Furthermore, it is well known that the amplitude for neutrinoless double beta decay is proportional to the combination

$$m_{ee} = \sum_k U_{ek}^2 m_{\nu_k} \tag{26}$$

We predict the following effective neutrino mass for both hierarchies:

$$m_{ee} = \frac{1}{3}\left(B + 4A\cos^2\frac{\phi}{2}\right) \tag{27}$$

Using our numerical results obtained fron the fit we get $m_{ee} \approx 4$ meV and $m_{ee} \approx 50$ meV for normal and inverted hierarchy, respectively. Therefore our predicted effective Majorana neutrino mass is consistent with its current experimental bound $|m_{ee}| < 0.3$ eV.

Lepton masses and mixings

The charged lepton mass matrix is described by:

$$M_l = \frac{v}{\sqrt{2}} V_{lL}^{\dagger} \begin{pmatrix} a_1^{(l)}\lambda^8 & 0 & 0 \\ 0 & a_2^{(l)}\lambda^5 & 0 \\ 0 & 0 & a_3^{(l)}\lambda^3 \end{pmatrix} = V_{lL}^{\dagger} \mathrm{diag}\left(m_e, m_\mu, m_\tau\right)$$

with:

$$a_1^{(l)} = \frac{1}{v}\left(\alpha_1 v_H^{(1)} - 6\beta_1 v_\Phi\right), \quad a_2^{(l)} = \frac{1}{v}\left(\alpha_2 v_H^{(1)} - 6\beta_2 v_\Phi\right)$$

$$a_3^{(l)} = \frac{1}{v}\left(\alpha_3 v_H^{(1)} - 6\beta_3 v_\Phi\right)$$

$$V_{lL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \qquad \omega = e^{\frac{2\pi i}{3}}$$

Here $\lambda = 0.225$ is the Wolfenstein parameter.

$$\mathrm{NH}: m_{\nu_1} = 0, \quad m_{\nu_2} = B = \sqrt{\Delta m^2_{21}} \approx 9\mathrm{meV},$$
$$m_{\nu_3} = 2A = \sqrt{\Delta m^2_{31}} \approx 50\mathrm{meV};$$

$$\mathrm{IH}: m_{\nu_2} = B = \sqrt{\Delta m^2_{21} + \Delta m^2_{13}} \approx 50\mathrm{meV},$$
$$m_{\nu_1} = 2A = \sqrt{\Delta m^2_{13}} \approx 49\mathrm{meV}, \quad m_{\nu_3} = 0,$$

$$\mathrm{NH}: \phi = -0.88\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0232;$$

$$\mathrm{IH}: \phi = 0.12\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0238.$$

we see that $\sin^2\theta_{13}$ is in excellent agreement with the experimental data, for both NH and IH, with $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ within a 2σ deviation from their best fit values.

The masses of the fermionic fields conitained in the $\mathbf{24}_f$ SU(5) irrep are:

$$
\begin{aligned}
m_{\rho_0} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_3} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{3 y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_8} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} + \frac{2 y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_{(3,2)}} &= m_{\rho_{(\overline{3},2)}} = \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{y_3 v_\Xi}{2\sqrt{30}}.
\end{aligned}
$$

Here m_{ρ_0}, m_{ρ_3} and m_{ρ_8} are the masses of the fermionic singlet ρ_0, triplet ρ_3 and octet ρ_8 contained in the **24** fermionic irrep of $SU(5)$, respectively. We denoted by $m_{\rho_{(3,2)}}$ and $m_{\rho_{(\overline{3},2)}}$ the masses of the $(3,2)$ and $(\overline{3},2)$ fermionic fields corresponding to the $SU(3)$ triplet and $SU(3)$ antitriplet, $SU(2)$ doublet parts of ρ, respectively.

The masses of the fermionic fields conitained in the $\mathbf{24}_f$ SU(5) irrep are:

$$
\begin{aligned}
m_{\rho_0} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_3} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{3 y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_8} &= \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} + \frac{2 y_3 v_\Xi}{\sqrt{30}}, \\
m_{\rho_{(3,2)}} &= m_{\rho_{(\overline{3},2)}} = \lambda y_2 (1 + x_1 + x_2) \Lambda_{GUT} - \frac{y_3 v_\Xi}{2\sqrt{30}}.
\end{aligned}
$$

Here m_{ρ_0}, m_{ρ_3} and m_{ρ_8} are the masses of the fermionic singlet ρ_0, triplet ρ_3 and octet ρ_8 contained in the **24** fermionic irrep of $SU(5)$, respectively. We denoted by $m_{\rho_{(3,2)}}$ and $m_{\rho_{(\overline{3},2)}}$ the masses of the $(3,2)$ and $(\overline{3},2)$ fermionic fields corresponding to the $SU(3)$ triplet and $SU(3)$ antitriplet, $SU(2)$ doublet parts of ρ, respectively.

M_L can be diagonalized as:

$$V_\nu^\dagger M_L (V_\nu^\dagger)^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$V_\nu = \begin{pmatrix} \cos\theta & 0 & \sin\theta e^{-i\phi} \\ 0 & 1 & 0 \\ -\sin\theta e^{i\phi} & 0 & \cos\theta \end{pmatrix} P_\nu \qquad \theta = \pm\frac{\pi}{4}$$

$$P_\nu = diag\left(e^{i\alpha_1/2}, e^{i\alpha_2/2}, e^{i\alpha_3/2}\right)$$

The solutions corresponding to $\theta = +\pi/4$ and $\theta = -\pi/4$ we identify with the normal (NH) and inverted (IH) neutrino mass hierarchies, respectively, so that

$$\text{NH} \;:\; m_{\nu_1} = 0, \quad m_{\nu_2} = B, \quad m_{\nu_3} = 2A, \quad \alpha_1 = \alpha_2 = 0, \quad \alpha_3 = \phi,$$

$$\text{IH} \;:\; m_{\nu_1} = 2A, \quad m_{\nu_2} = B, \quad m_{\nu_3} = 0, \quad \alpha_2 = \alpha_3 = 0, \quad \alpha_1 = -\phi.$$

The quark mass matrices are:

$$M_U = \begin{pmatrix} a_{11}^{(U)}\lambda^6 & a_{12}^{(U)}\lambda^5 & a_{13}^{(U)}\lambda^3 \\ a_{12}^{(U)}\lambda^5 & a_{22}^{(U)}\lambda^4 & a_{23}^{(U)}\lambda^2 \\ a_{13}^{(U)}\lambda^3 & a_{23}^{(U)}\lambda^2 & a_{33}^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{28}$$

$$M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} a_1^{(D)}\lambda^8 & 0 & 0 \\ 0 & a_2^{(D)}\lambda^5 & 0 \\ 0 & 0 & a_3^{(D)}\lambda^3 \end{pmatrix} \left(V_{lL}^{\dagger}\right)^T \tag{29}$$

$$a_{12}^{(U)} = 2\sqrt{2}\left(\gamma_{12}+\gamma_{21}\right)\frac{v_H^{(4)}}{v}, a_{11}^{(U)} = 4\sqrt{2}\gamma_{11}\frac{v_H^{(2)}}{v}, \tag{30}$$

$$a_{23}^{(U)} = 2\sqrt{2}\left(\gamma_{23}+\gamma_{32}\right)\frac{v_H^{(2)}}{v}, a_{22}^{(U)} = 4\sqrt{2}\gamma_{22}\frac{v_H^{(3)}}{v}, a_{33}^{(U)} = 4\sqrt{2}\gamma_{33}\frac{v_H^{(4)}}{v},$$

$$a_{13}^{(U)} = 2\sqrt{2}\left(\gamma_{13}+\gamma_{31}\right)\frac{v_H^{(3)}}{v}, a_l^{(D)} = \frac{1}{v}\left(\alpha_l v_H^{(1)} + 2\beta_l v_\Phi\right), l = 1, 2, 3$$

We fix $a_{33}^{(U)} = 1$ and assume a breaking of universality in the Yukawa couplings:

$$\gamma_{11} = \sqrt{1-\frac{\lambda^2}{2}}\gamma_1 e^{i\phi_1}, \gamma_{12} = \gamma_{21} = -\gamma_1 e^{i\phi_2}, \gamma_{22} = \gamma_1\left(1-\frac{\lambda^2}{2}\right)^{-\frac{1}{2}},$$

$$\gamma_{13} = \gamma_{31} = \gamma_2\left(1-\frac{\lambda^2}{2}\right)^3 e^{i\phi_3}, \gamma_{23} = \gamma_{32} = -\gamma_2, \alpha_i = \beta_i, i = 1,2,3,$$

Fitting $a_1^{(U)}$, $a_2^{(U)}$, $a_1^{(D)}$, $a_2^{(D)}$, $a_3^{(D)}$ and the phases ϕ_l $(l = 1,2,3)$, we get:

$$a_1^{(U)} \simeq 1.96, \quad a_2^{(U)} \simeq 0.74, \quad \phi_1 \simeq 10.94^\circ, \quad \phi_2 \simeq 6.02^\circ,$$
$$a_1^{(D)} \simeq 2.54, \quad a_2^{(D)} \simeq 0.58, \quad a_3^{(D)} \simeq 1.42, \quad \phi_3 \simeq 21.65^\circ.$$

With 8 free parameters, the 10 physical observables of the quark sector can successfully be fitted.

* The role of the different discrete symmetries are:

- T_7: Unifies in antitriplets the 3 families of $\overline{\mathbf{5}}'$ fermionic irreps of SU(5).

- Z_2: Separates the SU(5) scalars multiplets participating in the charged lepton and down type quark Yukawa interactions from those ones participating in the up type quarks Yukawa interactions.

- $Z2'$: Crucial to get a predictive neutrino mass matrix texture that only depends on three effective parameters and that gives rise to the experimentally observed deviation from the tribimaximal mixing pattern.

- Z_3 and Z_4: Crucial to get the right value of the down quark and electron masses.

- Z_{12}: Shapes the charged fermion mass and quark mixing hierarchy.

Since the charged fermion mass and quark mixing pattern arises from the $Z_3 \otimes Z_4 \otimes Z_{12}$ symmetry breaking, we set

$$v_\eta \sim v_\chi \sim v_\varphi = v_\tau = v_\xi = v_\sigma = \Lambda_{GUT} = \lambda\Lambda,$$

where $\lambda = 0.225$.

The relevant down type quark and charged leptons Yukawa terms are

$$\begin{aligned}
\mathcal{L}_Y^{(1)} &= \alpha_1 \left(\psi^i \xi\right)_{1_0} H^{j\,(1)} \Psi_{ij}^{(1)} \frac{\sigma^5 \varphi^2 + \kappa \sigma \tau^4 \varphi^{*2}}{\Lambda^8} \\
&+ \alpha_2 \left(\psi^i \xi\right)_{1_2} H^{j\,(1)} \Psi_{ij}^{(2)} \frac{\tau^4}{\Lambda^5} + \alpha_3 \left(\psi^i \xi\right)_{1_1} H^{j\,(1)} \Psi_{ij}^{(3)} \frac{\sigma^2}{\Lambda^3} \\
&+ \beta_1 \left(\psi^i \xi\right)_{1_0} \Phi_i^{jk} \Psi_{jk}^{(1)} \frac{\sigma^5 \varphi^2 + \kappa \sigma \tau^4 \varphi^{*2}}{\Lambda^8} \\
&+ \beta_2 \left(\psi^i \xi\right)_{1_2} \Phi_i^{jk} \Psi_{jk}^{(2)} \frac{\tau^4}{\Lambda^5} + \beta_3 \left(\psi^i \xi\right)_{1_1} \Phi_i^{jk} \Psi_{jk}^{(3)} \frac{\sigma^2}{\Lambda^3}
\end{aligned}$$

The relevant up type quark Yukawa terms are

$$\begin{aligned}
\mathcal{L}_Y^{(2)} &= \varepsilon^{ijklp}\Big\{\gamma_{11}\Psi_{ij}^{(1)}H_p^{(2)}\Psi_{kl}^{(1)}\frac{\sigma^6}{\Lambda^6} + \gamma_{12}\Psi_{ij}^{(1)}H_p^{(4)}\Psi_{kl}^{(2)}\frac{\sigma^5}{\Lambda^5} \\
&+ \gamma_{21}\Psi_{ij}^{(2)}H_p^{(4)}\Psi_{kl}^{(1)}\frac{\sigma^5}{\Lambda^5} + \gamma_{22}\Psi_{ij}^{(2)}H_p^{(3)}\Psi_{kl}^{(2)}\frac{\sigma^4}{\Lambda^4} \\
&+ \gamma_{13}\Psi_{ij}^{(1)}H_p^{(3)}\Psi_{kl}^{(3)}\frac{\sigma^3}{\Lambda^3} + \gamma_{31}\Psi_{ij}^{(3)}H_p^{(3)}\Psi_{kl}^{(1)}\frac{\sigma^3}{\Lambda^3} \\
&+ \gamma_{23}\Psi_{ij}^{(2)}H_p^{(2)}\Psi_{kl}^{(3)}\frac{\sigma^2}{\Lambda^2} + \gamma_{32}\Psi_{ij}^{(3)}H_p^{(2)}\Psi_{kl}^{(2)}\frac{\sigma^2}{\Lambda^2} + \gamma_{33}\Psi_{ij}^{(3)}H_p^{(4)}\Psi_{kl}^{(3)}\Big\}
\end{aligned}$$

The relevant neutrino Yukawa terms are

$$\begin{aligned}
\mathcal{L}_Y^{(3)} &= \lambda_{1\nu}\left(\psi^i\eta\right)_{1_0} H_i^{(1)} N_R \frac{\sigma^{*2}}{\Lambda^3} \\
&+ \lambda_{2\nu}\left(\psi^i\chi\right)_{1_0} H_j^{(1)}\rho_i^j\frac{\sigma^{*2}}{\Lambda^3} + \lambda_{3\nu}\left(\psi^i\chi\right)_{1_0}\Phi_{ij}^k\rho_k^j\frac{\sigma^{*2}}{\Lambda^3} \\
&+ \left[y_1\overline{N}_R N_R^c + y_2 Tr\left(\rho^2\right)\right]\frac{\sigma^*\sigma + x_1\tau^*\tau + x_2\varphi^*\varphi}{\Lambda} + y_3 Tr\left(\rho^2\Xi\right)
\end{aligned}$$