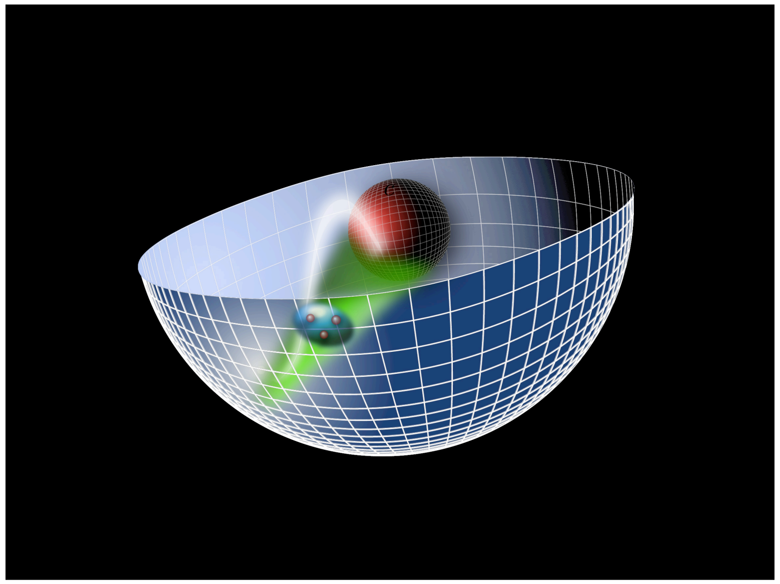
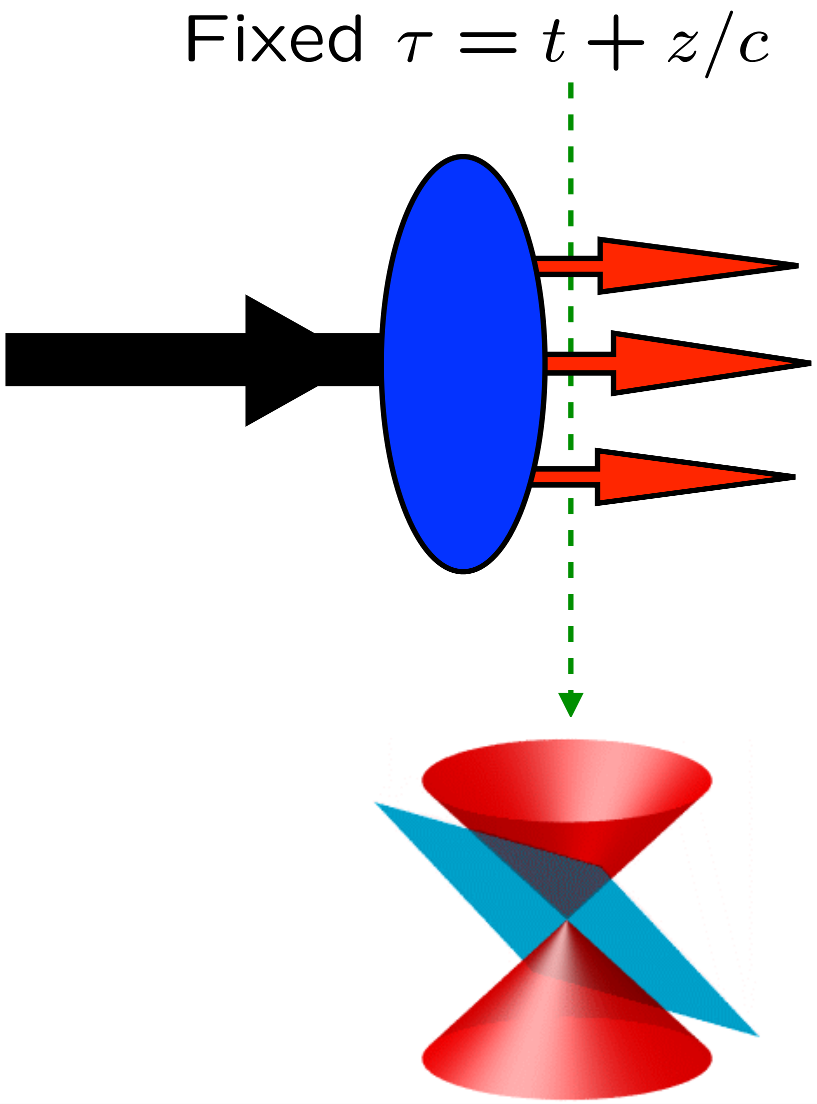
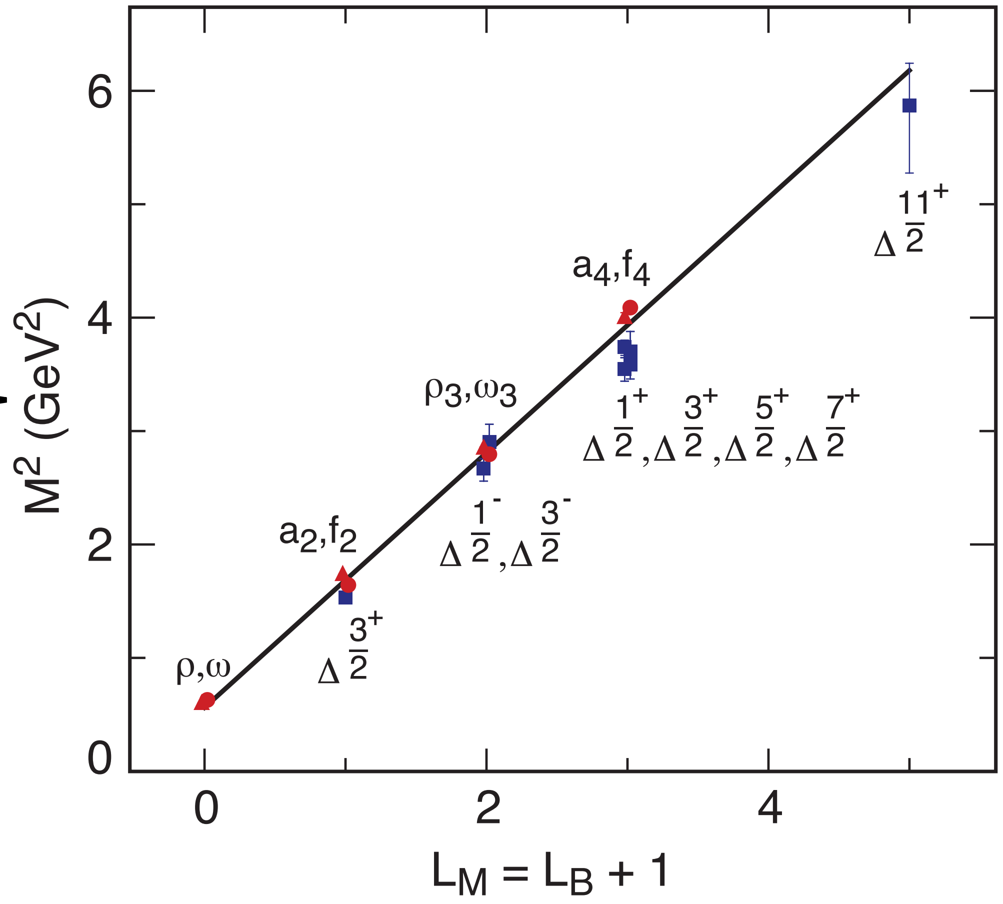
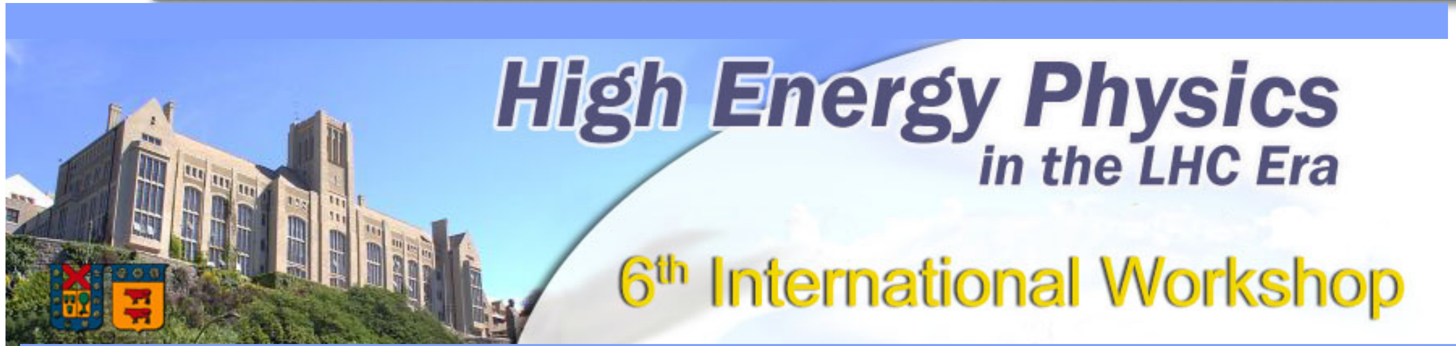
New Insights into Color Confinement and Hadron Dynamics from Light-Front Holography and Superconformal Quantum Mechanics







with Guy de Tèramond, Hans Günter Dosch, and Alexandre Deur



6-12 January 2016
Universidad Técnica Federico Santa María, Valparaiso, Chile

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

Stanford University

Gell Mann, Fritzsch, Leutwyler

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

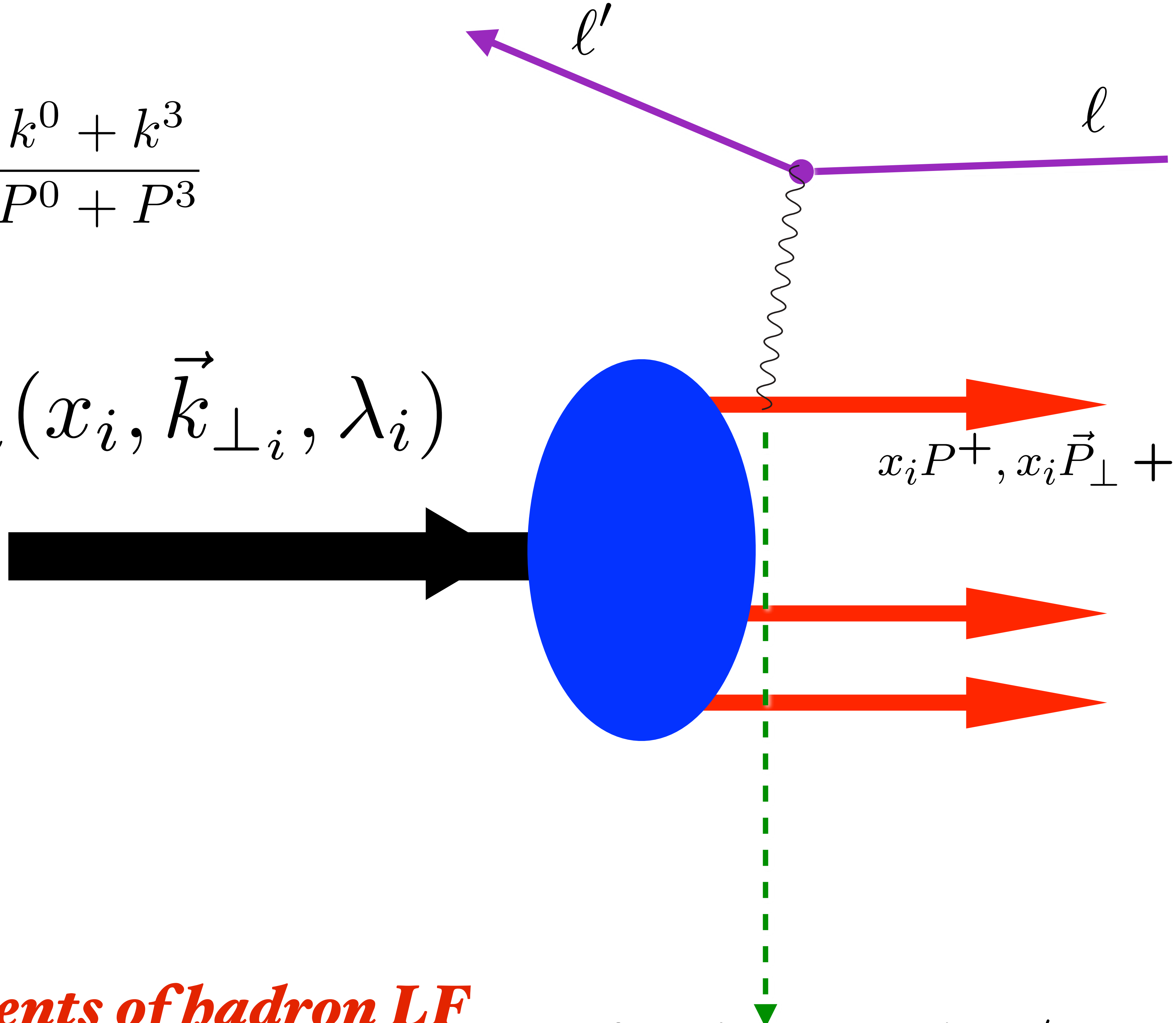
Where does the QCD Mass Scale come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Fixed $\tau = t + z/c$

Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

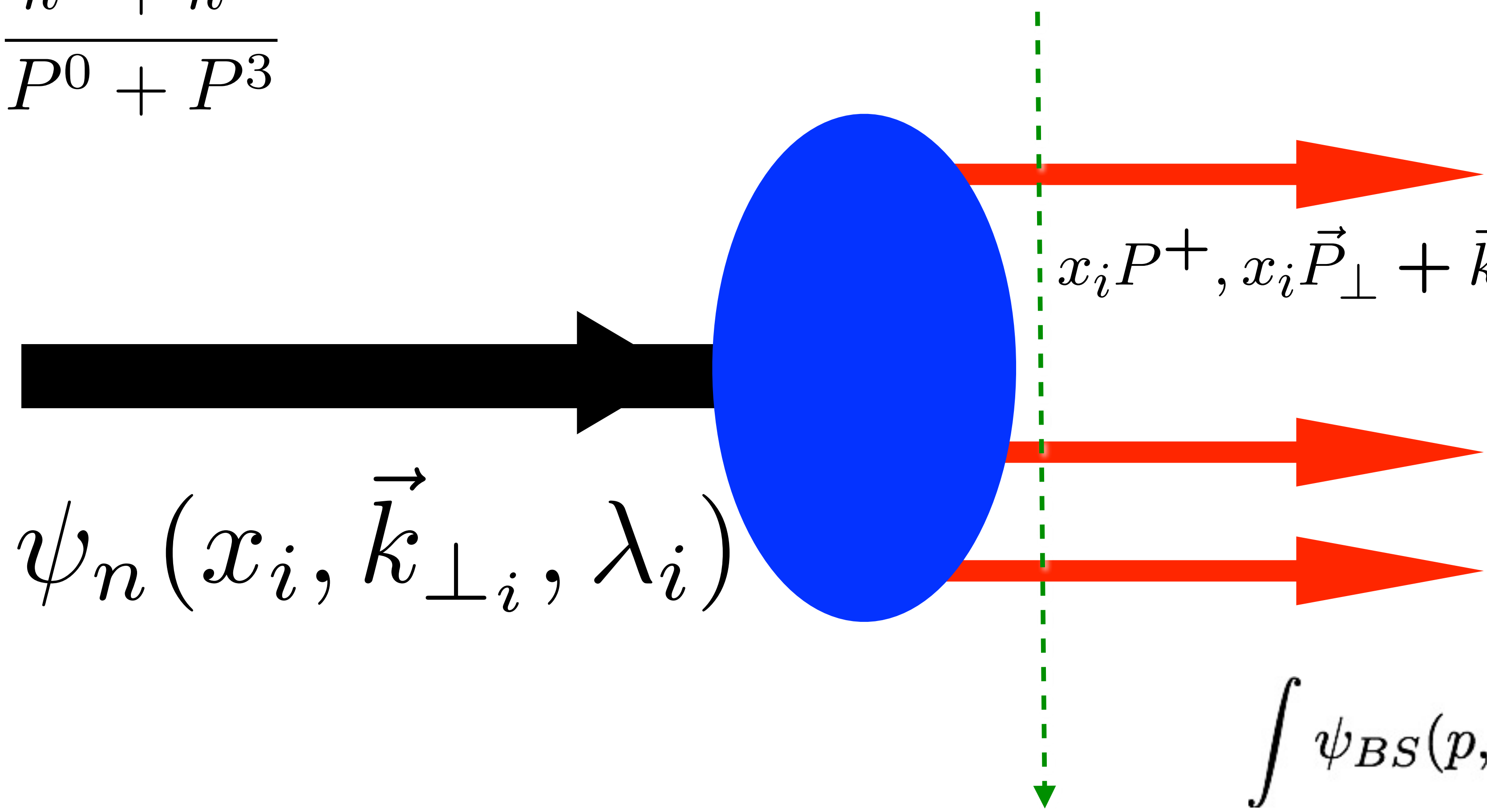
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$\int \psi_{BS}(p,k) dk^- \rightarrow \psi_{LF}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

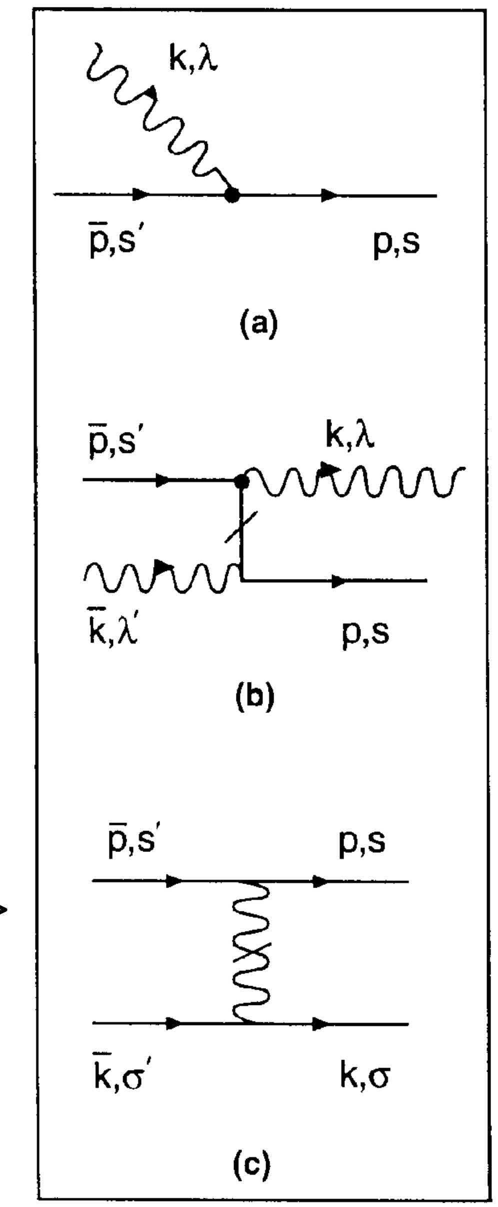
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

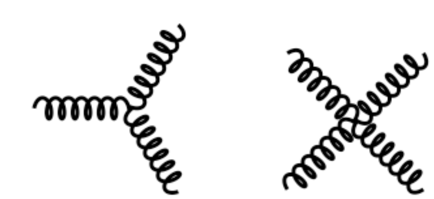
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



H^{int}_{LF}

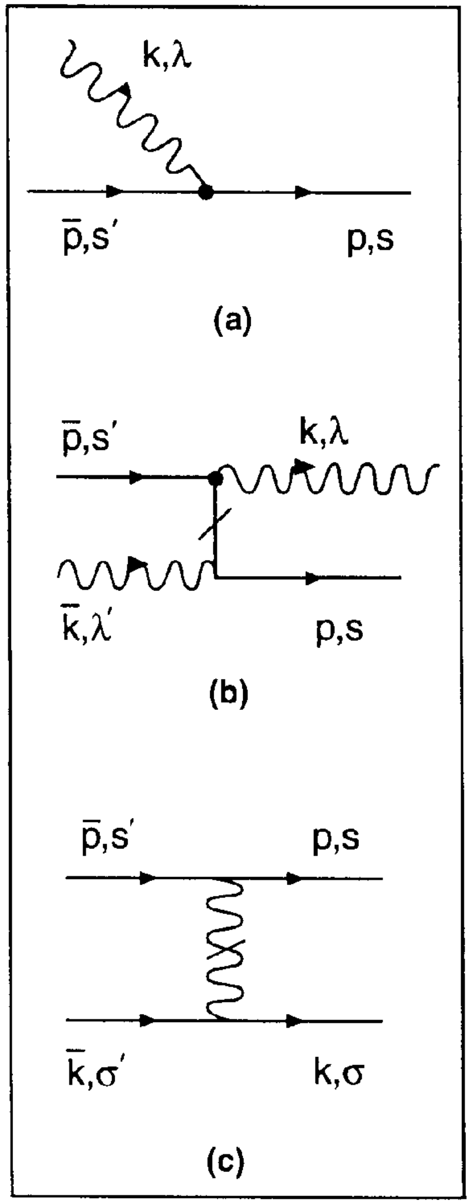
LFWFs: Off-shell in P- and invariant mass

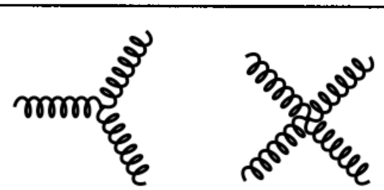
Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\ |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb





n	Sector	1 qq̄	2 gg	3 qq̄ g	4 qq̄ qq̄	5 gg g	6 qq̄ gg	7 qq̄ qq̄ g	8 qq̄ qq̄ qq̄	9 gg gg	10 qq̄ gg g	11 qq̄ qq̄ gg	12 qq̄ qq̄ qq̄ g	13 qq̄ qq̄ qq̄ qq̄
1	qq̄					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	qq̄ g								•	•		•	•	•
4	qq̄ qq̄		•			•				•	•		•	•
5	gg g	•			•			•	•			•	•	•
6	qq̄ gg								•				•	•
7	qq̄ qq̄ g	•	•			•				•				•
8	qq̄ qq̄ qq̄	•	•	•		•	•			•	•			
9	gg gg	•		•	•			•	•			•	•	•
10	qq̄ gg g	•	•		•				•				•	•
11	qq̄ qq̄ gg	•	•	•		•				•				•
12	qq̄ qq̄ qq̄ g	•	•	•	•	•	•			•	•			
13	qq̄ qq̄ qq̄ qq̄	•	•	•	•	•	•	•		•	•	•		

Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

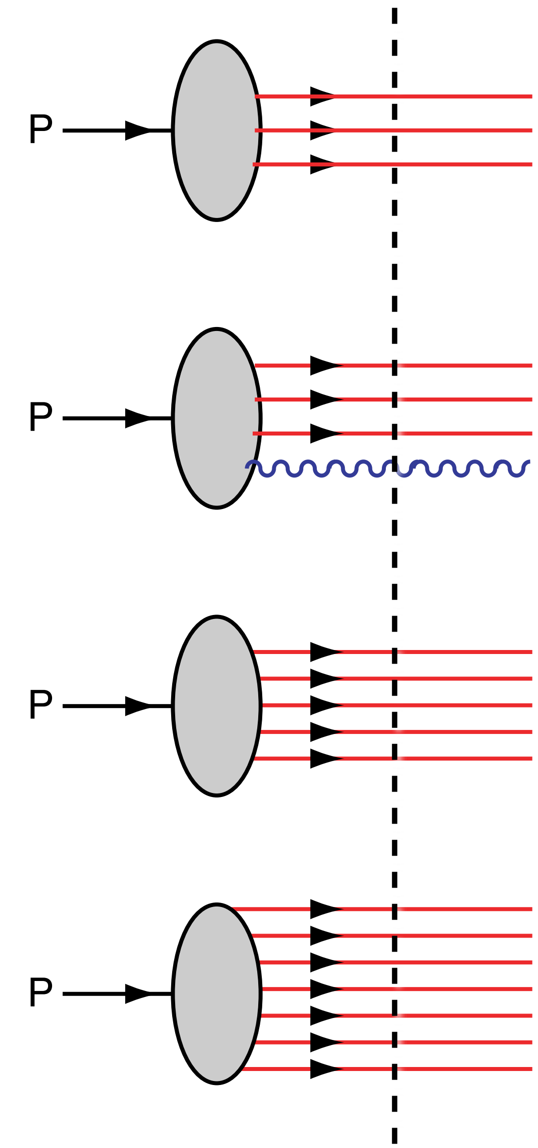
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

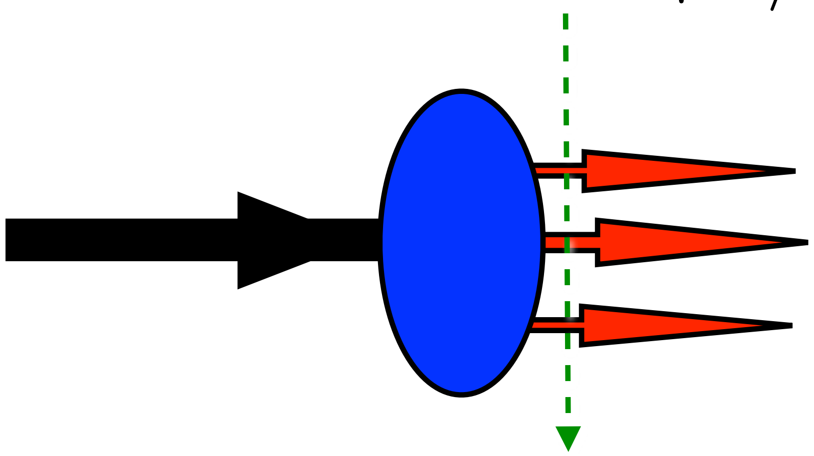
$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Hidden Color

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

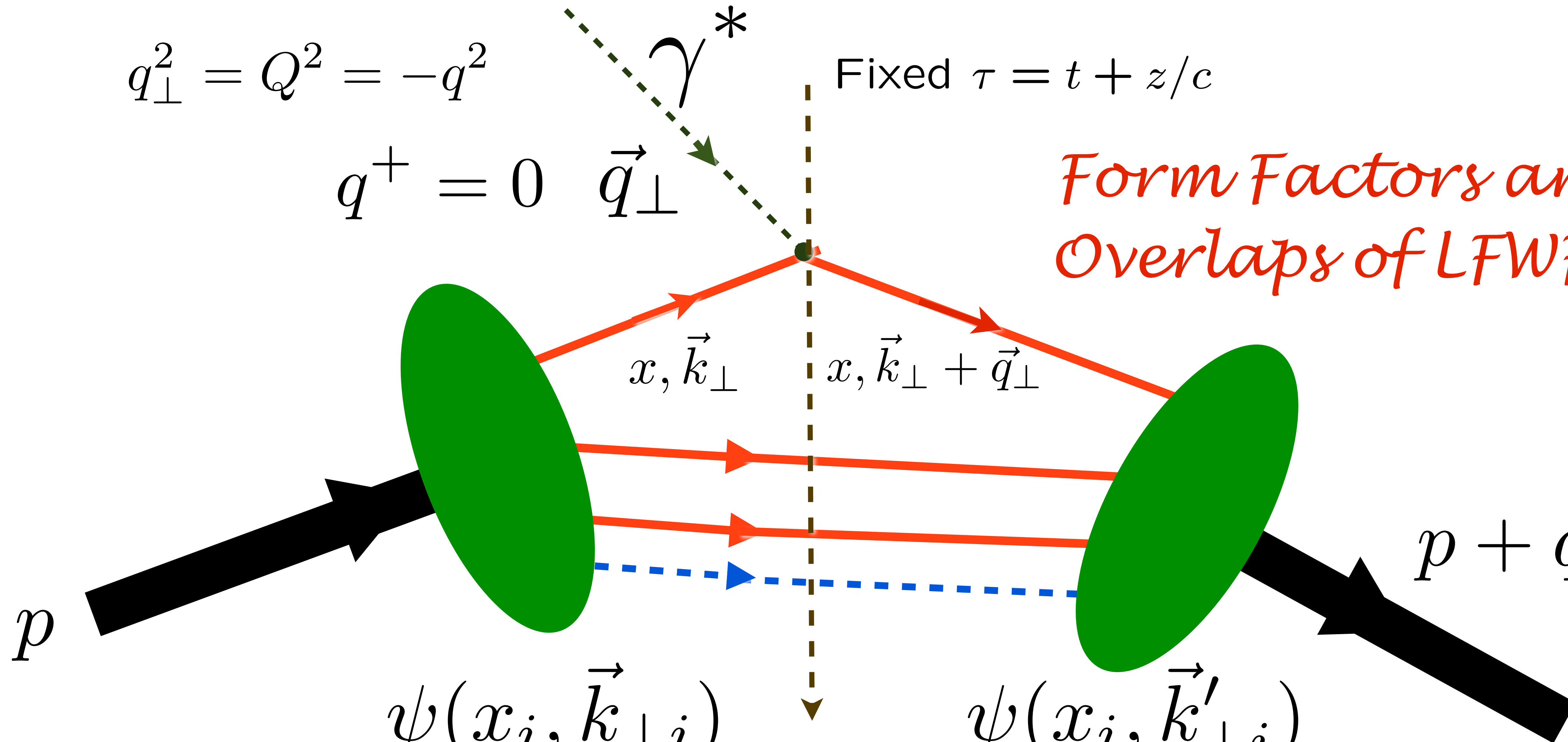
Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



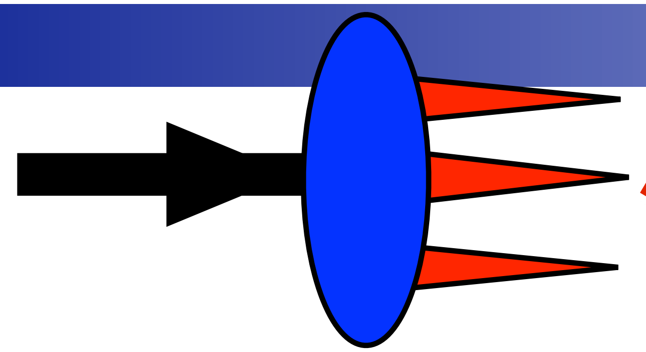
Form Factors are Overlaps of LFWFs

struck $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

- *Light Front Wavefunctions:*



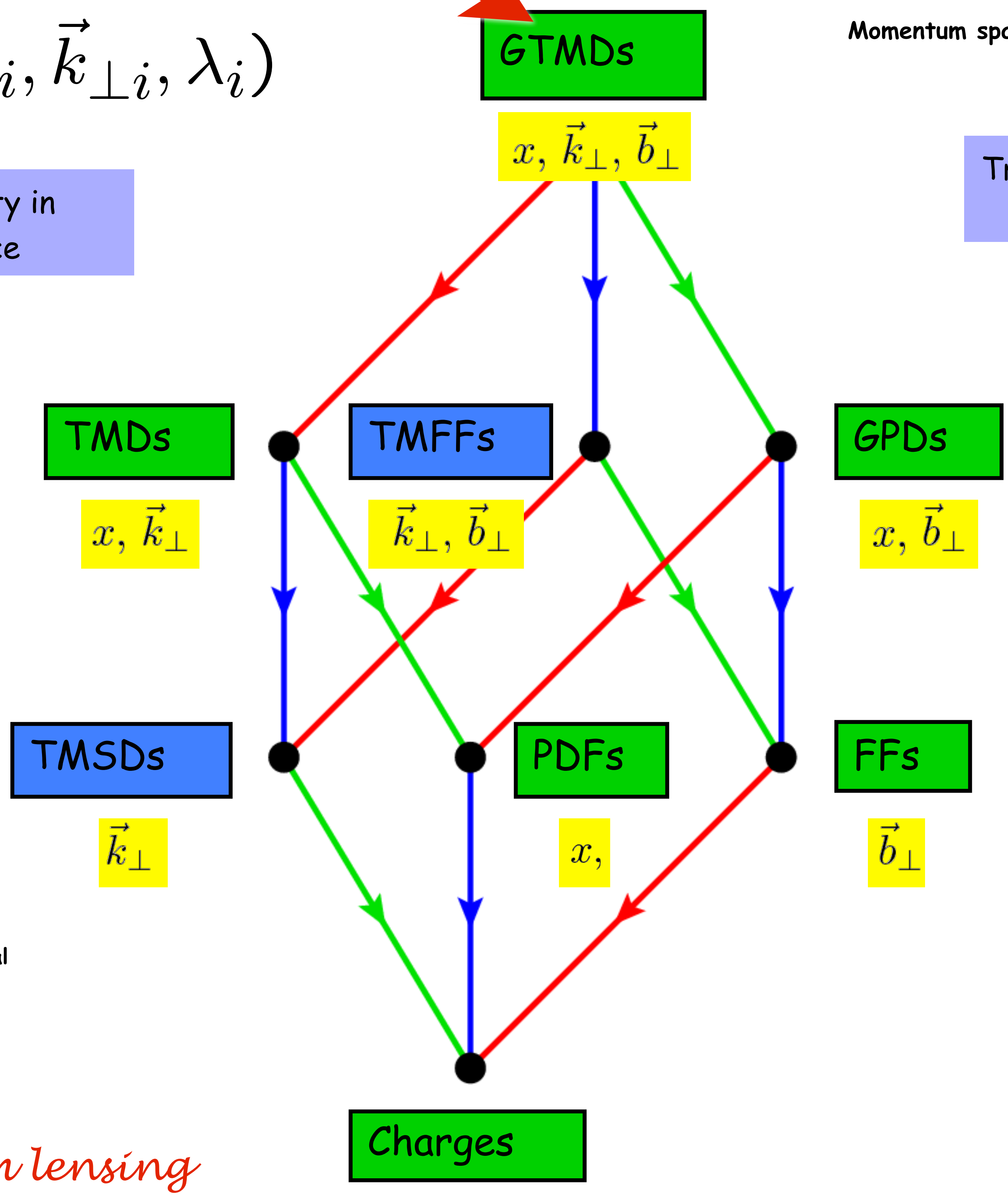
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

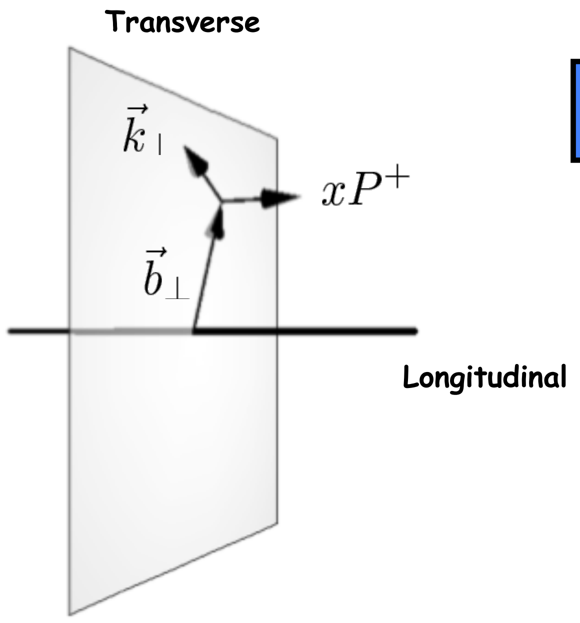
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

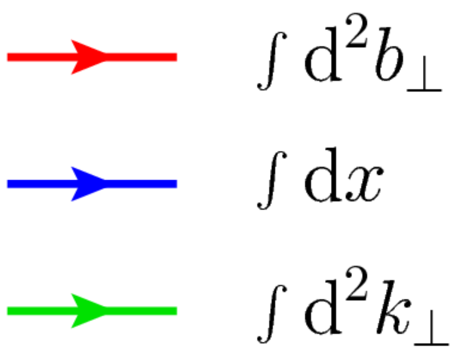
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



Lorce, Pasquini





Sivers, T-odd from lensing

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function in e p collider and p rest frame
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates! *Roberts, Shrock, Tandy, sjb*
- Profound implications for Cosmological Constant

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale if m_q=0

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int}) \; |\Psi >= E \; |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Semiclassical first approximation to QED

Coulomb potential

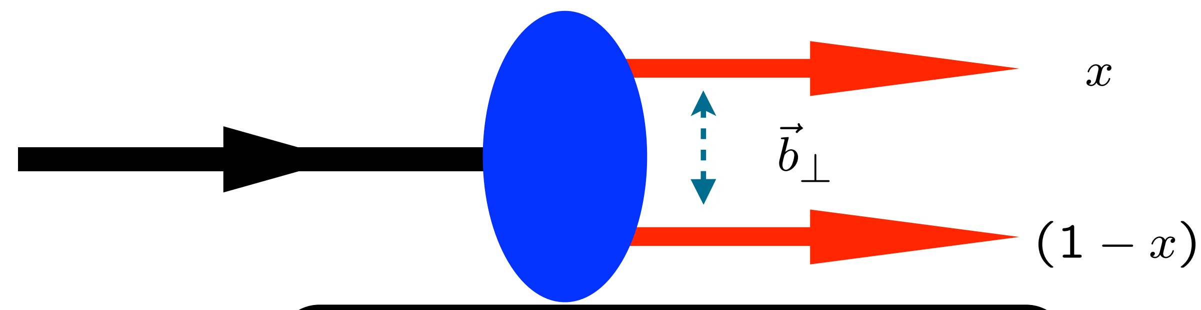
Bohr Spectrum

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis

ζ, ϕ

$m_q = 0$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Confining AdS/QCD potential!

Sums an infinite # diagrams

Semiclassical first approximation to QCD

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

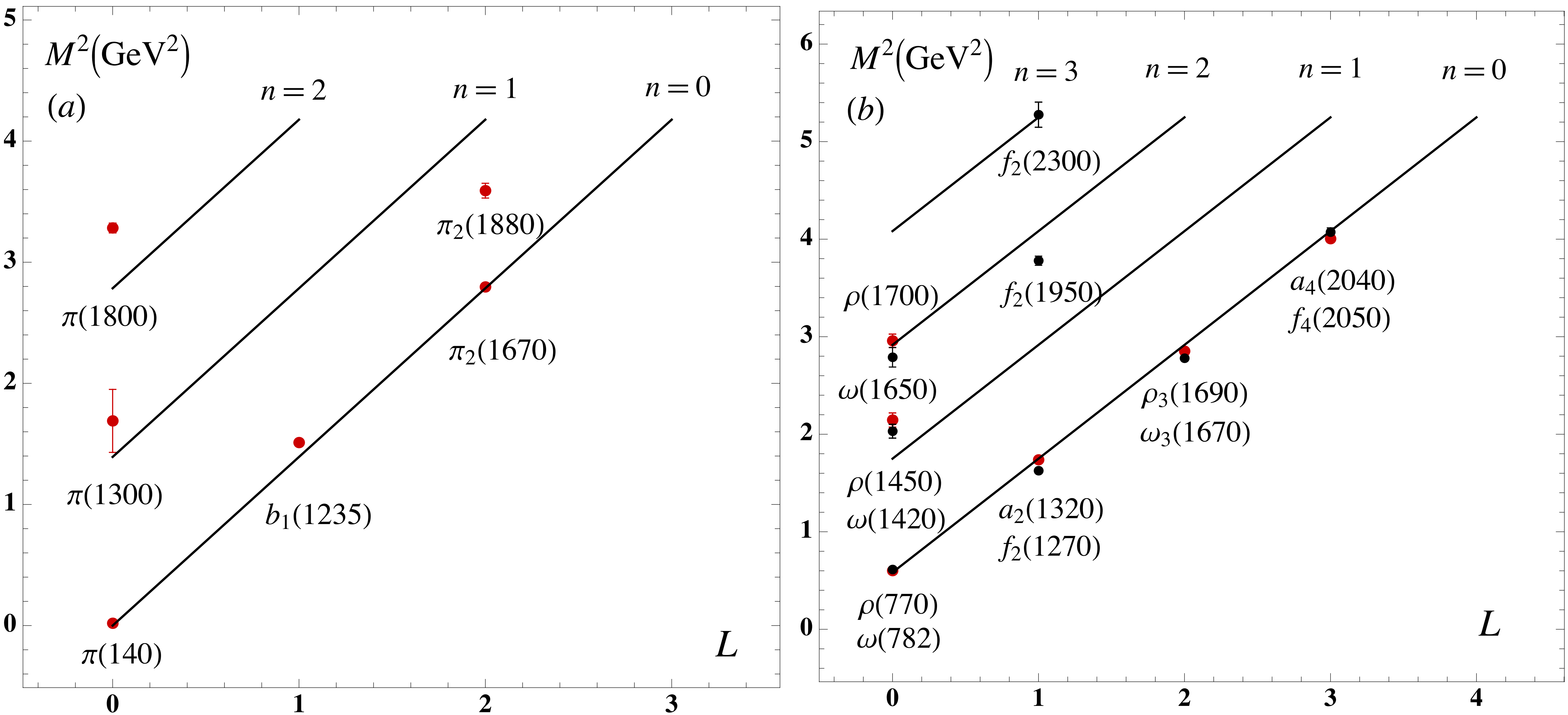
- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

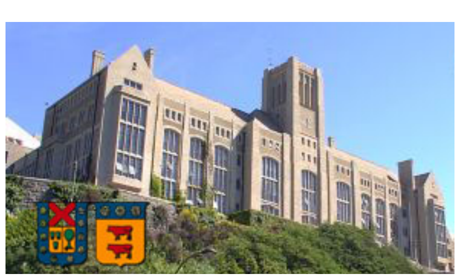
G. de Teramond, H. G. Dosch, sjb

$m_u = m_d = 0$

de Tèramond, Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$



Light-Front Schrödinger Equation

G. de Teramond, sjb

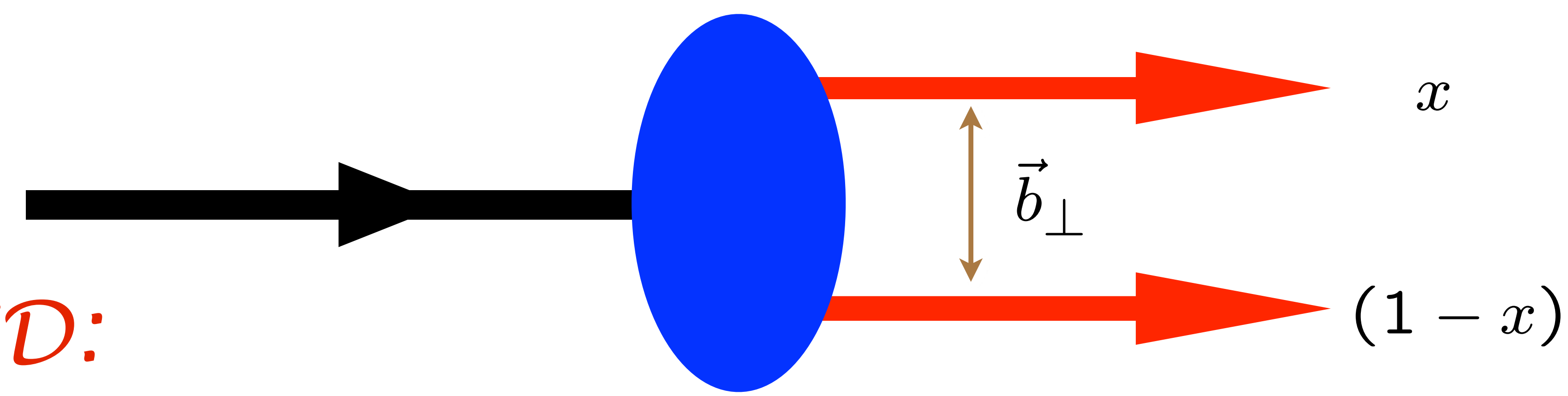
Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$\left[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)\right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

$m_q \sim 0$

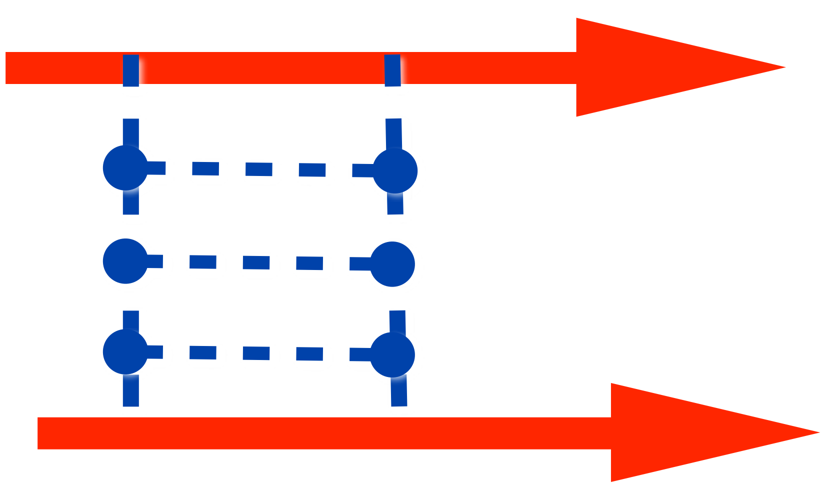
$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



AdS/QCD:

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

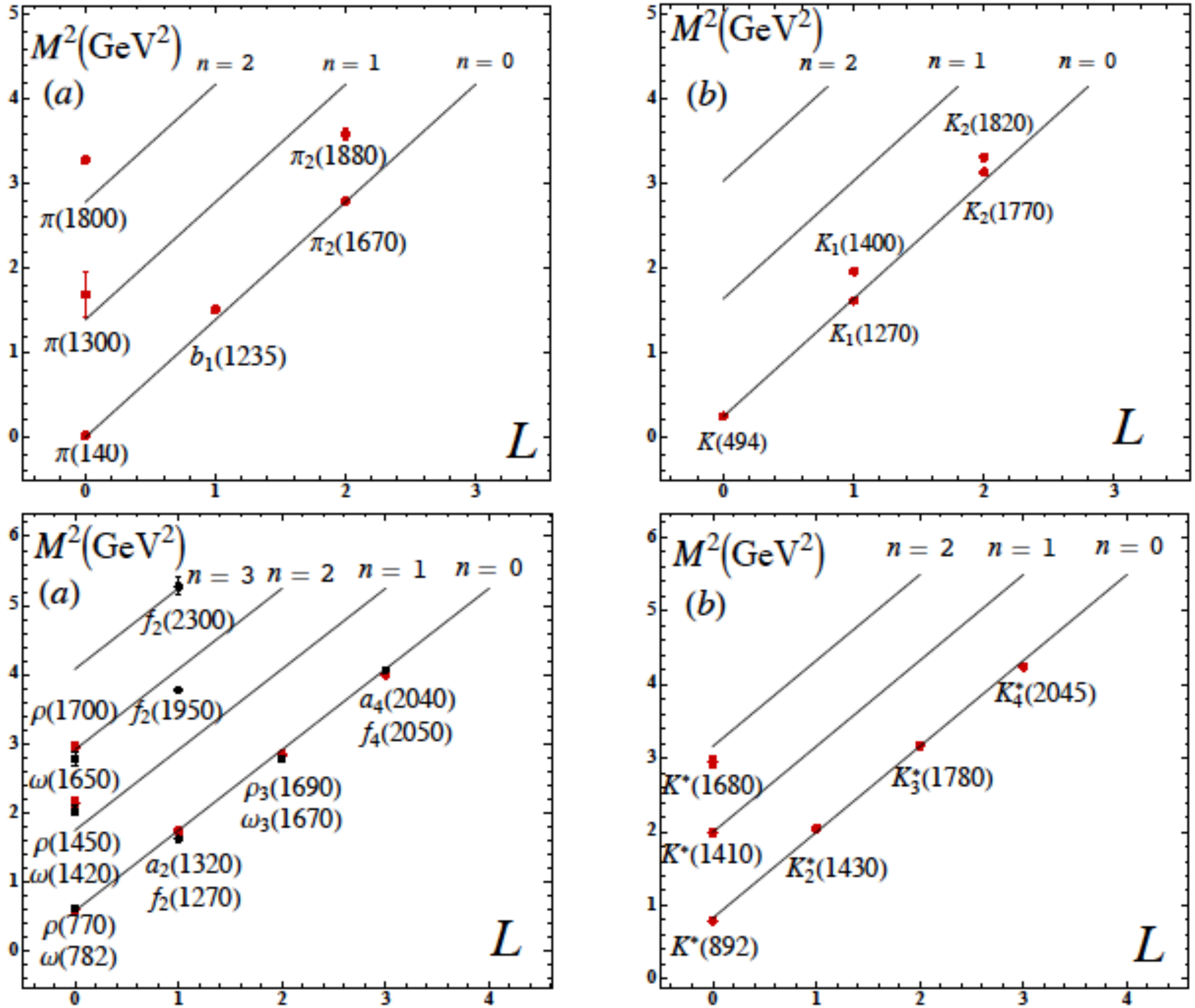
U is the exact QCD potential
Conjecture: 'H'-diagrams generate U?



De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



De Teramond, Dosch, sjb

- Results easily extended to light quarks masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi|\sum_a m_a^2/x_a|\psi\rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda\left(n + \frac{J+L}{2}\right)$$

- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

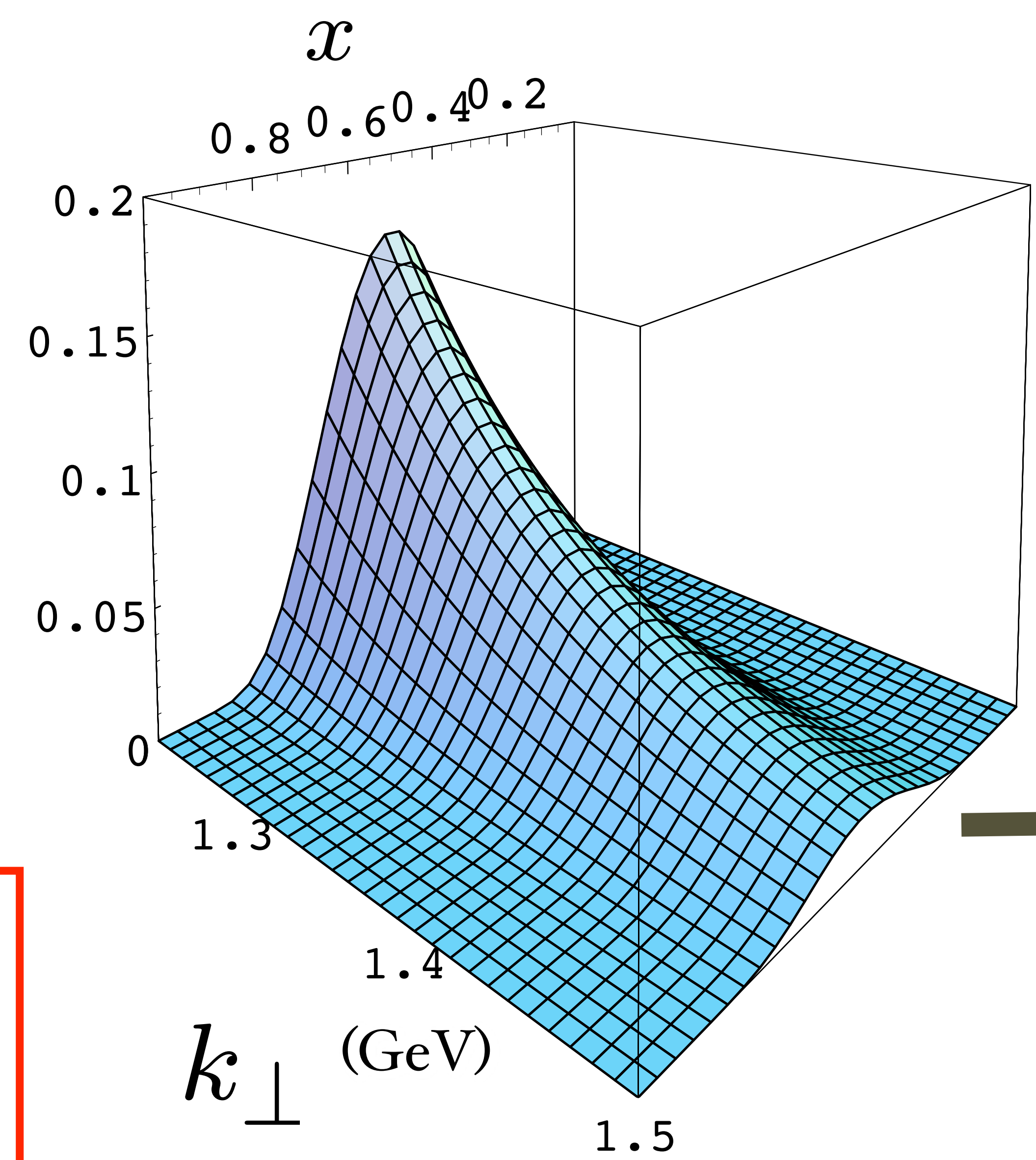
$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

Prediction from AdS/QCD: Meson LFWF

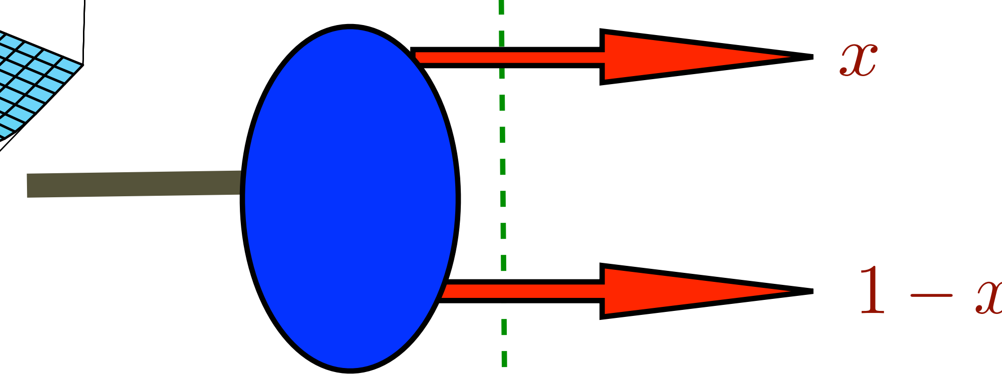
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

de Teramond, Cao, sjb

$$\psi_M(x, k_\perp^2)$$



"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

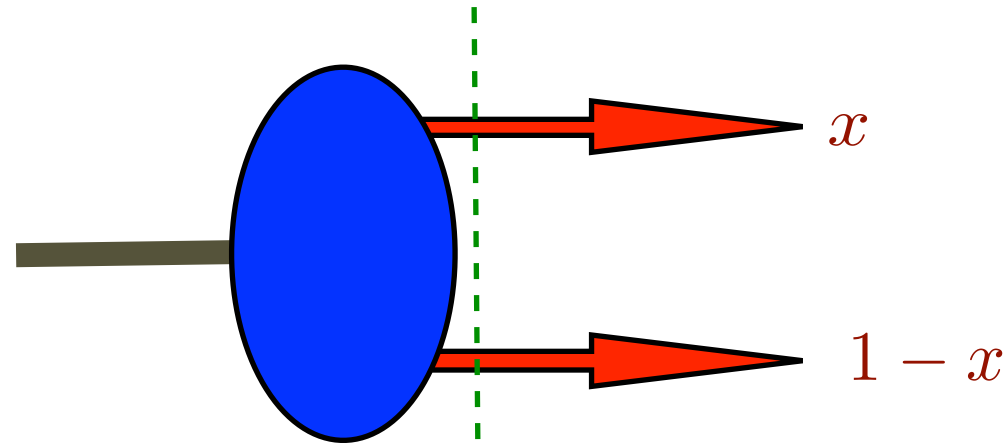
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE!

Provides Connection of Confinement to Hadron Structure

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

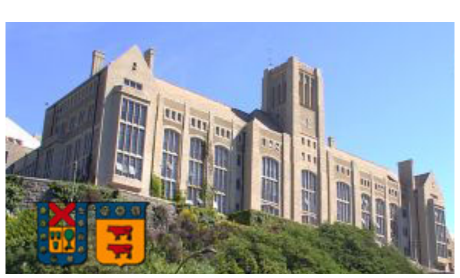
- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge



PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*
Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

R. Sandapen†
Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada

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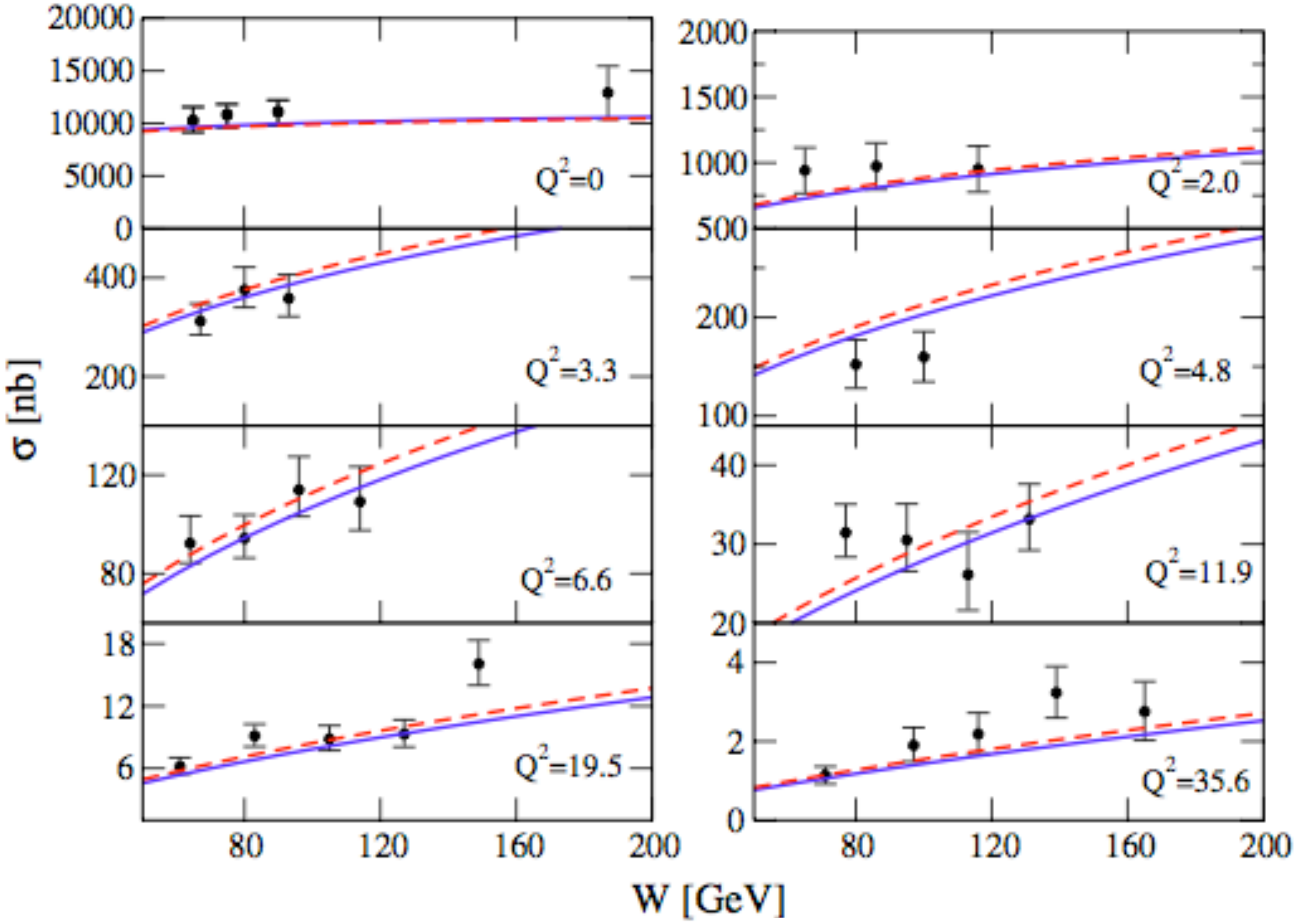
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x,k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

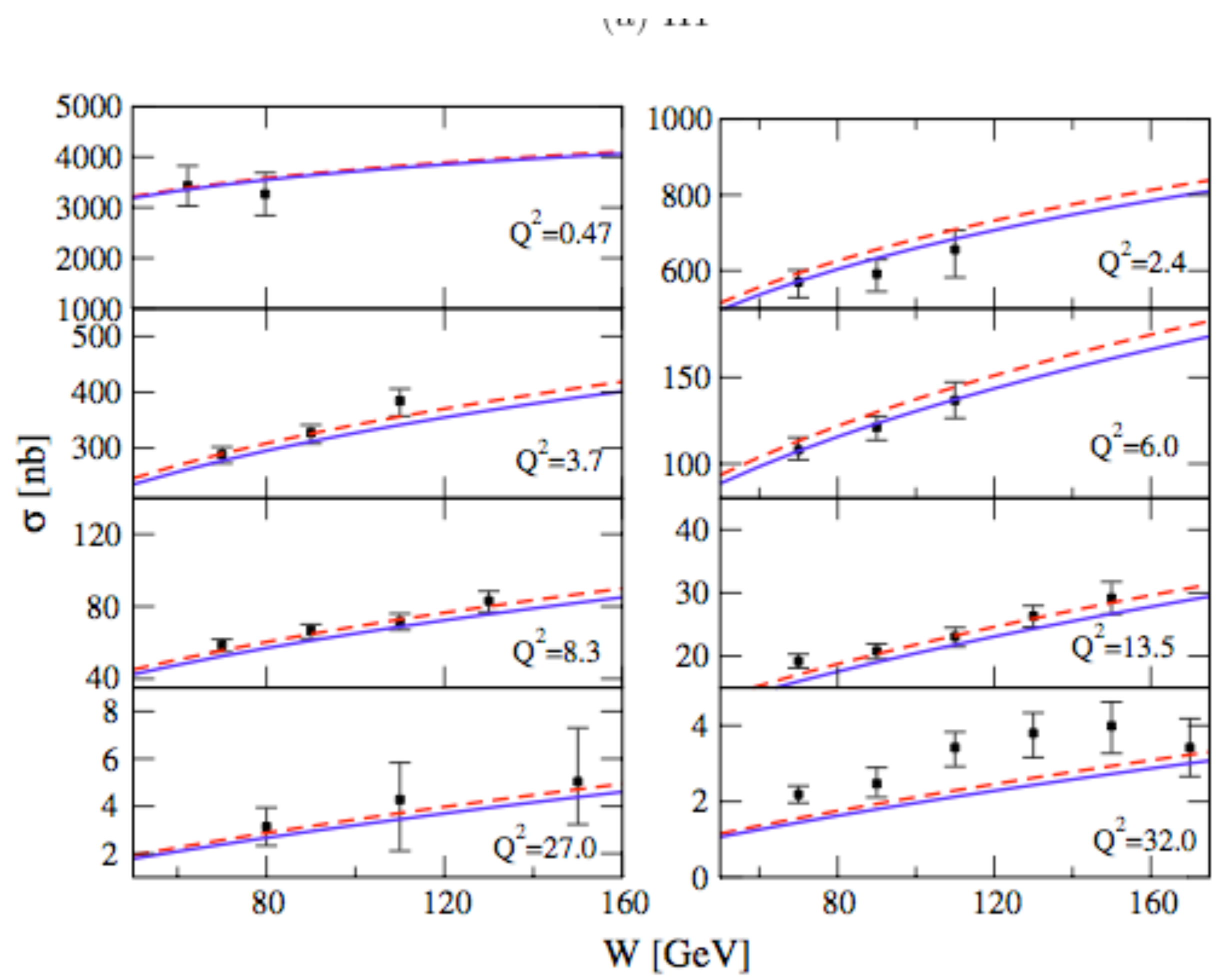
See also Ferreira and Dosch

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

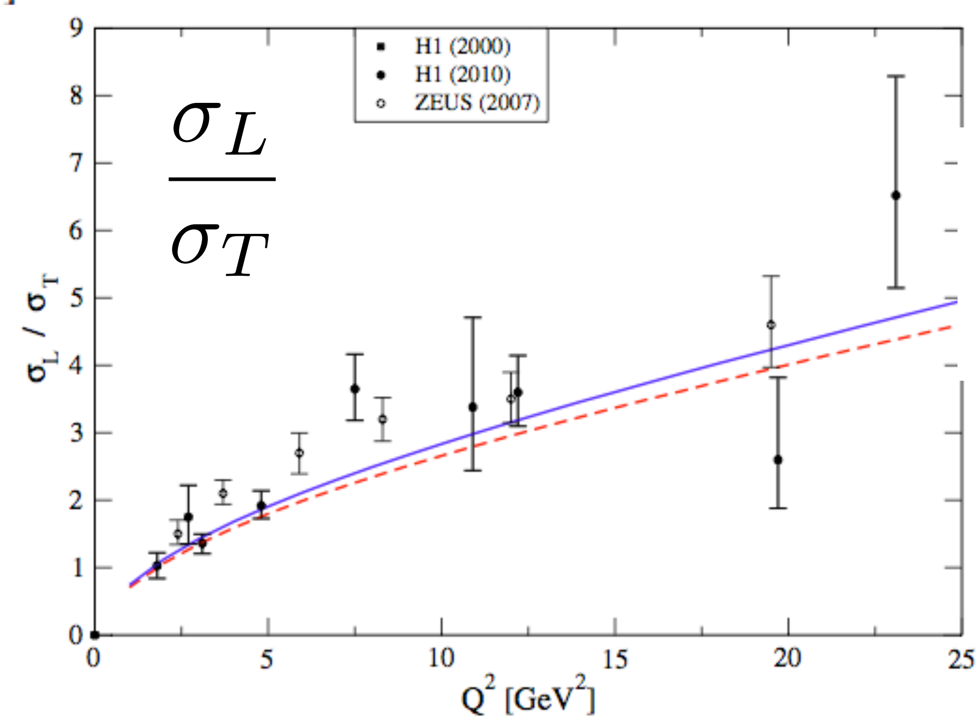
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H



(b) ZEUS



$$\frac{\sigma_L}{\sigma_T}$$

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

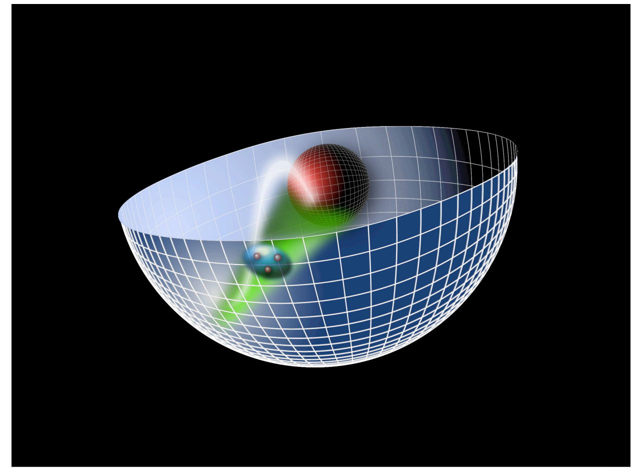
$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

See also Ferreira and Dosch

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

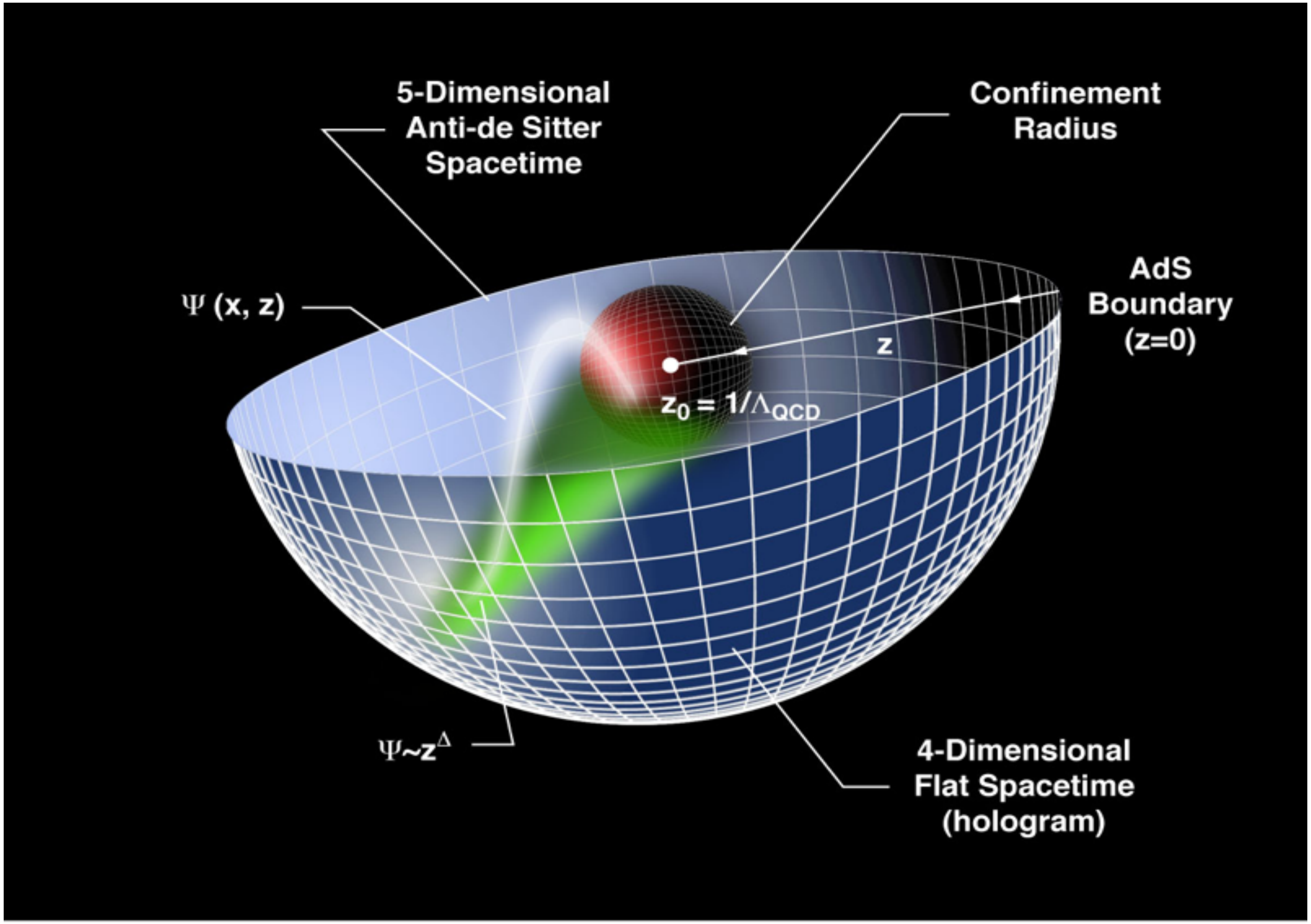
$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Applications of AdS/CFT to QCD



AdS_5

Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond and H. Guenter Dosch

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

AdS$_5$

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

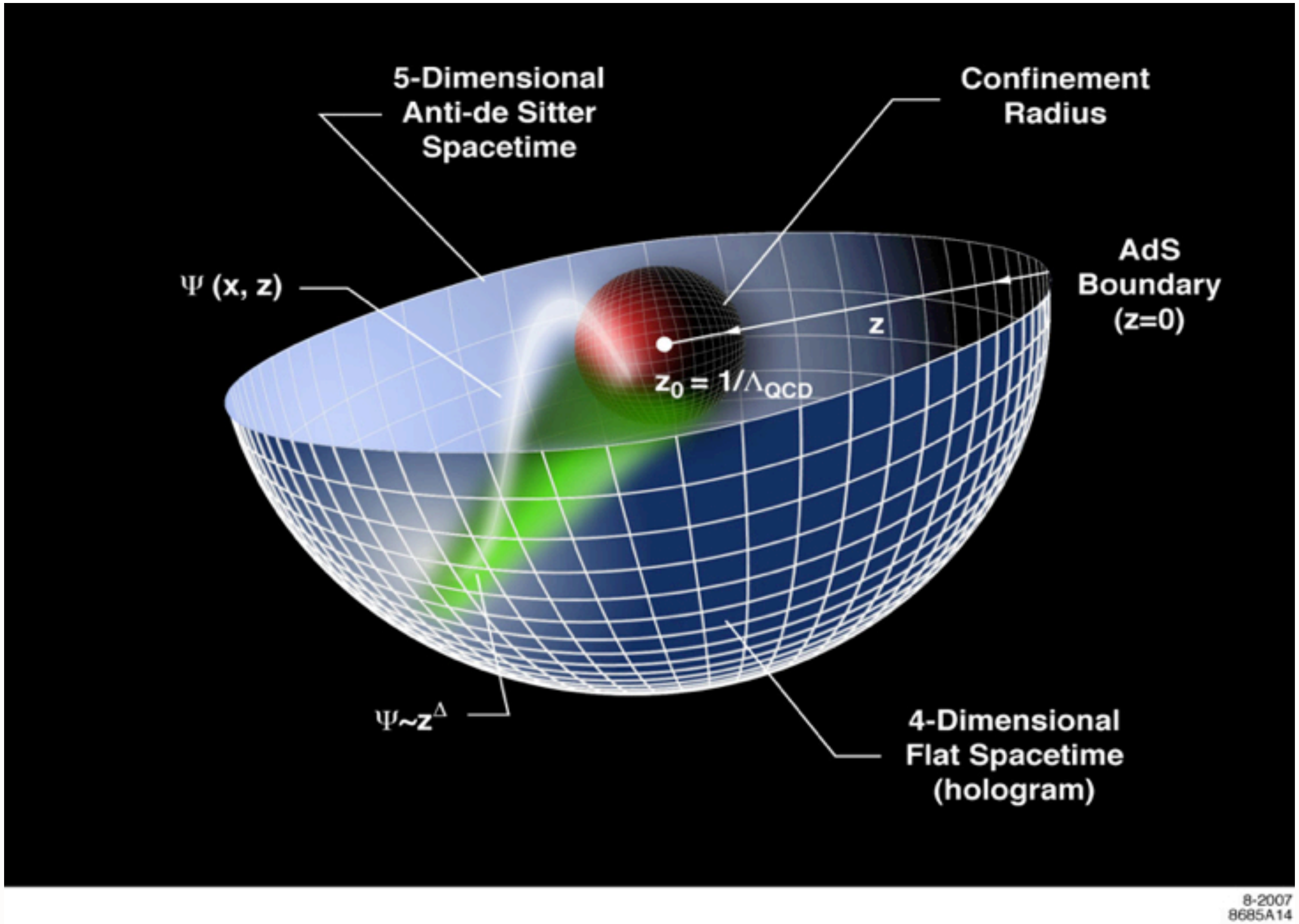
- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.



Changes in physical length scale mapped to evolution in the 5th dimension z

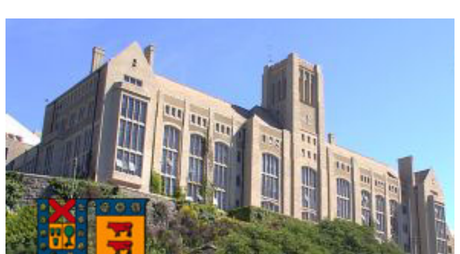
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- ***Soft-wall dilaton profile breaks conformal invariance*** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- ***Color Confinement***
- ***Introduces confinement scale*** κ
- ***Uses AdS_5 as template for conformal theory***



Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

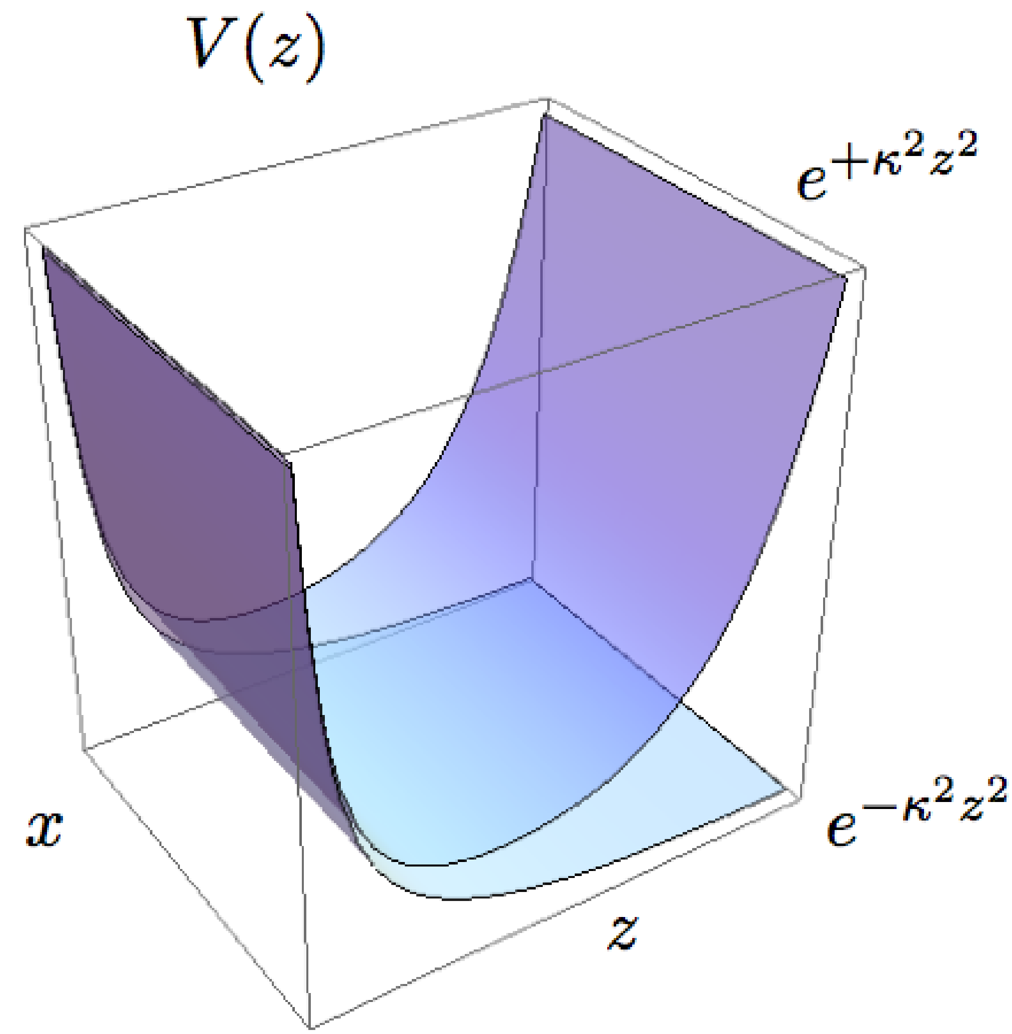
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

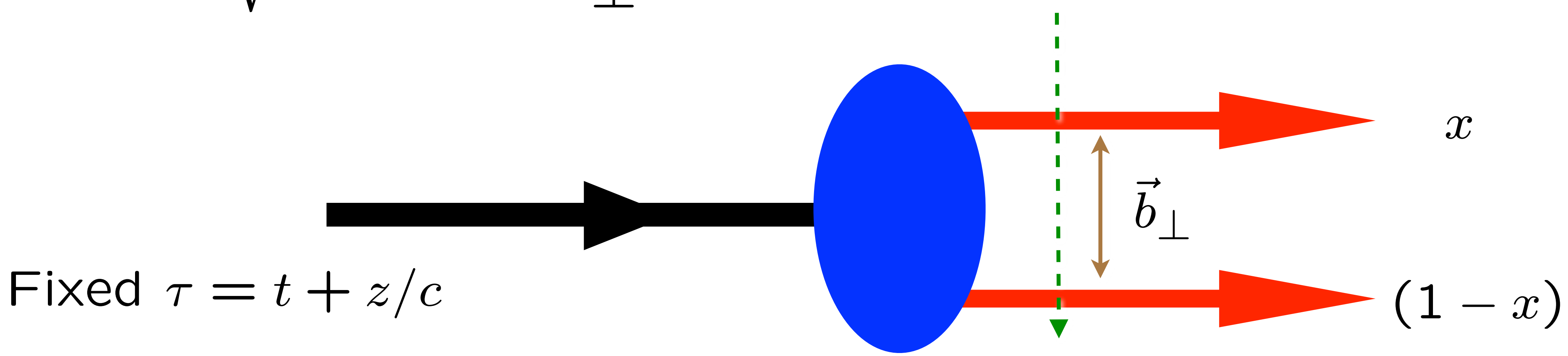
de Teramond, sjb

LF(3+1) ⟷ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$

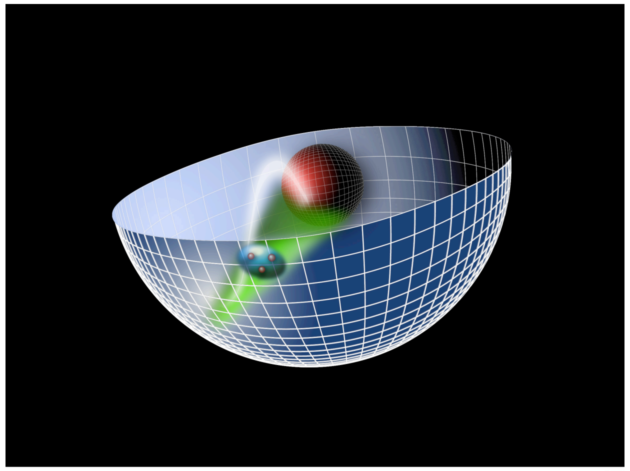


$$\psi(x,\zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb



AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

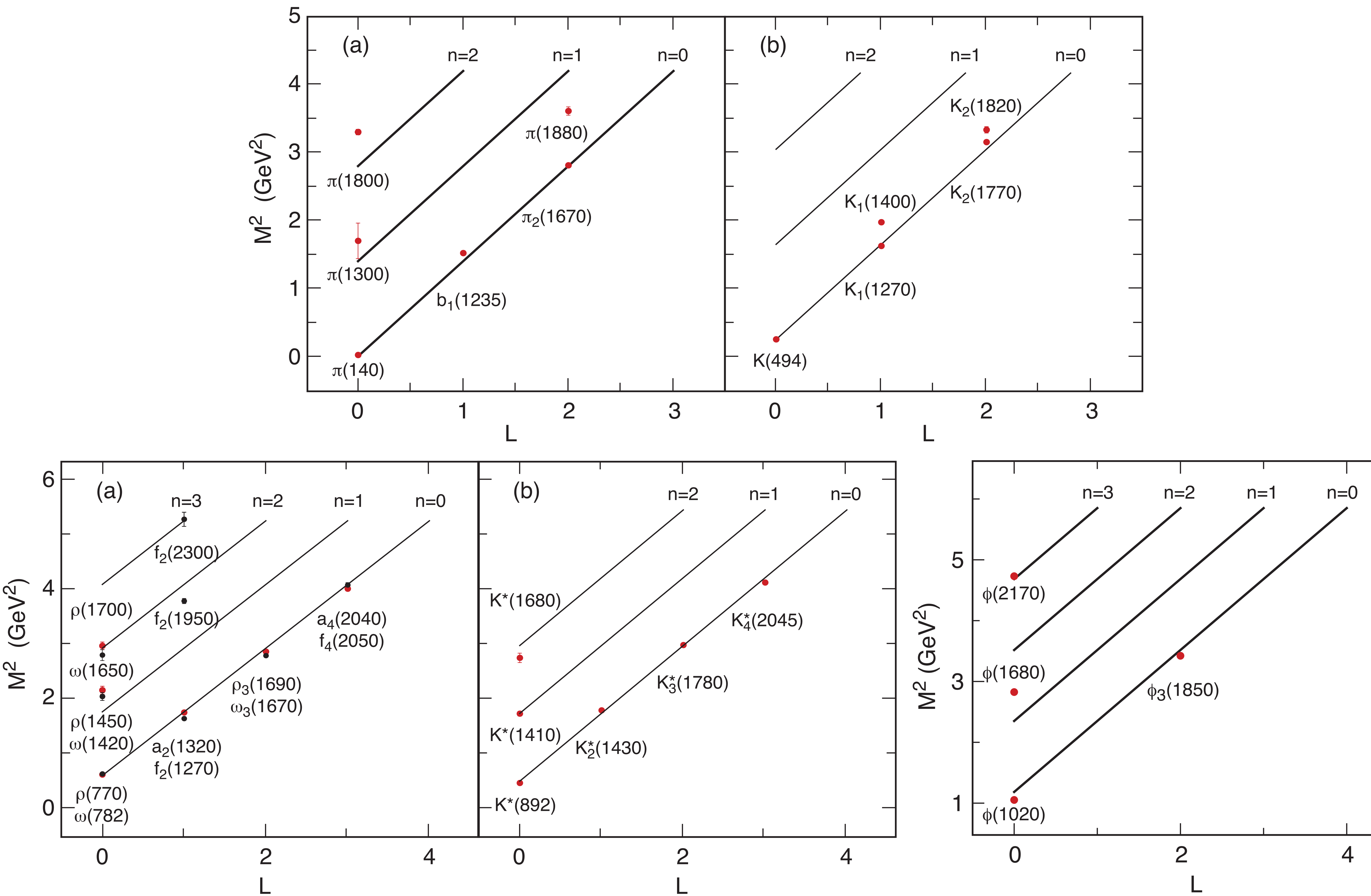
$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- Identify with difference of LF time $\Delta x^+/P^+$ between constituents
- Finite range
- Measure in Double-Parton Processes



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Orbital and Radial Pseudoscalar and Vector Meson Excitations

Haag, Lopuszanski, Sohnius (1974)

Superconformal Algebra

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates the conformal algebra

$$[H,D] = i\,H, \quad [H, K] = 2\,i\,D, \quad [K, D] = -\,i\,K$$

Superconformal Algebra

Baryon Equation

Consider $R_w = Q + wS;$ w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B$, $w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Algebra

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

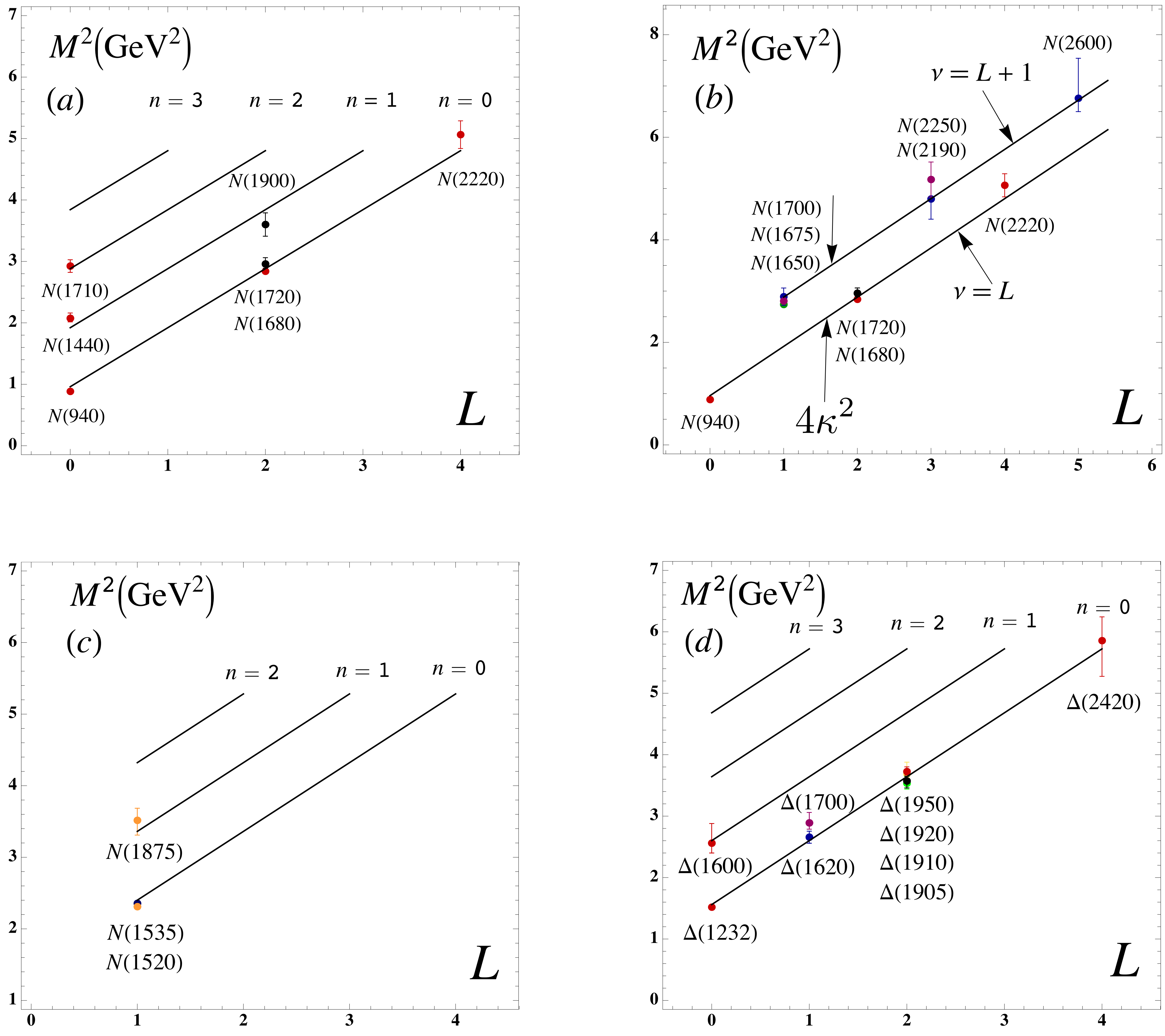
$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ !

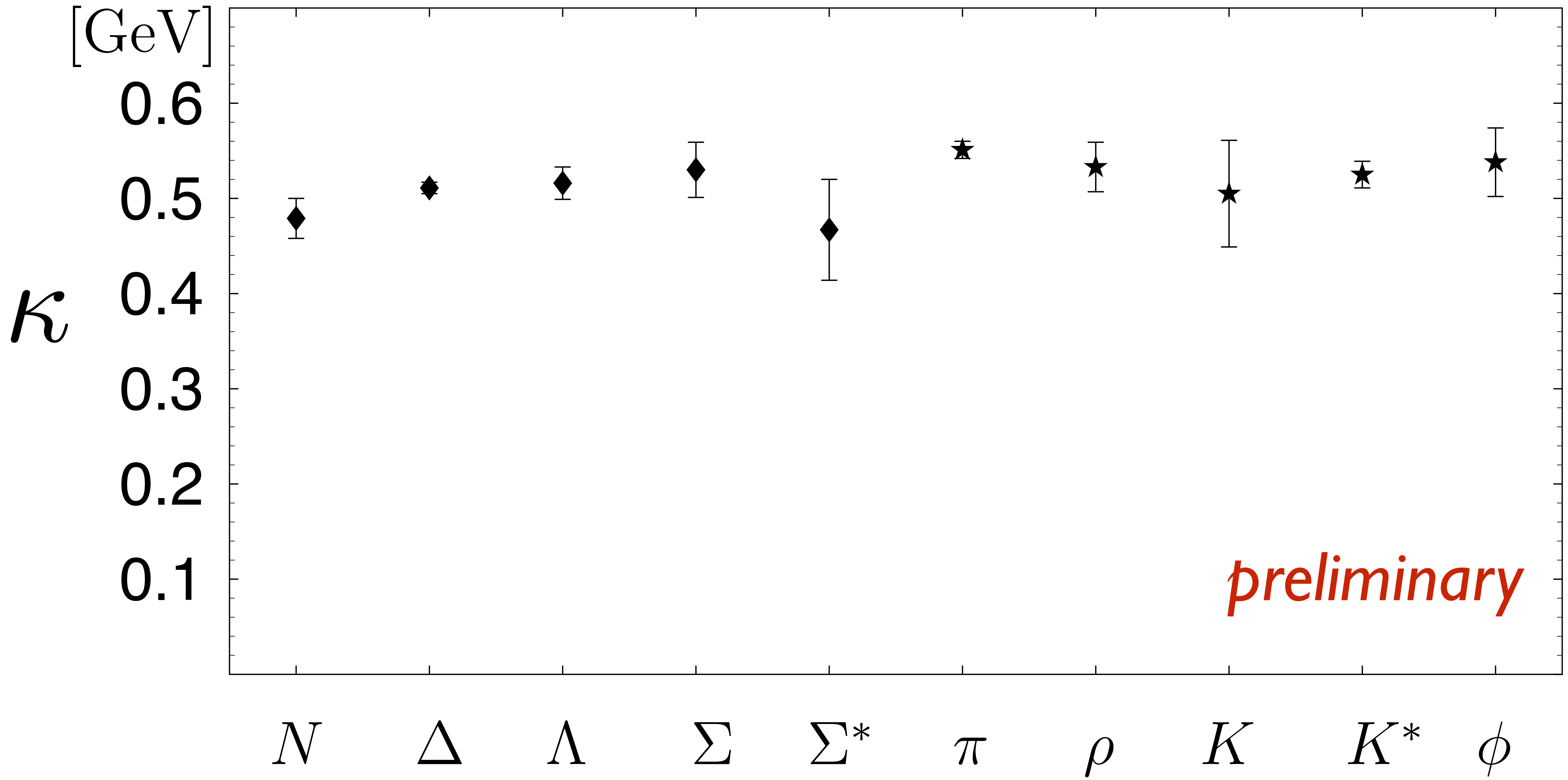
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$



$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
ν = L + 1
N(2250)
N(2190)
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
4κ²
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
L

$m_u = m_d = 46$ MeV, $m_s = 357$ MeV

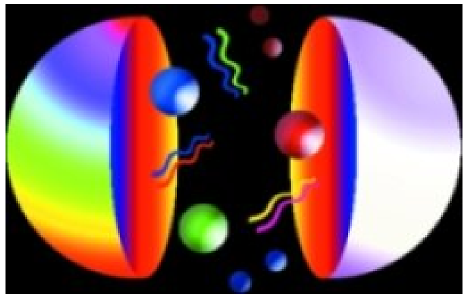


Fit to the slope of Regge trajectories, including radial excitations

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Features of Supersymmetric Equations

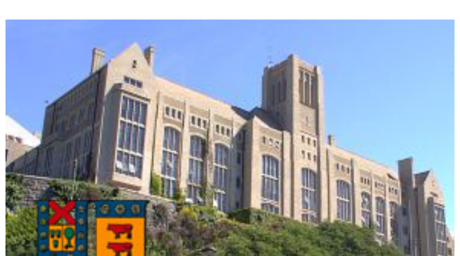
- $J = L+S$ baryon simultaneously satisfies both equations of G with L , L+1 for same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$

- Baryon spin carried by quark orbital angular momentum: $\langle J^z \rangle = L^z + 1/2$

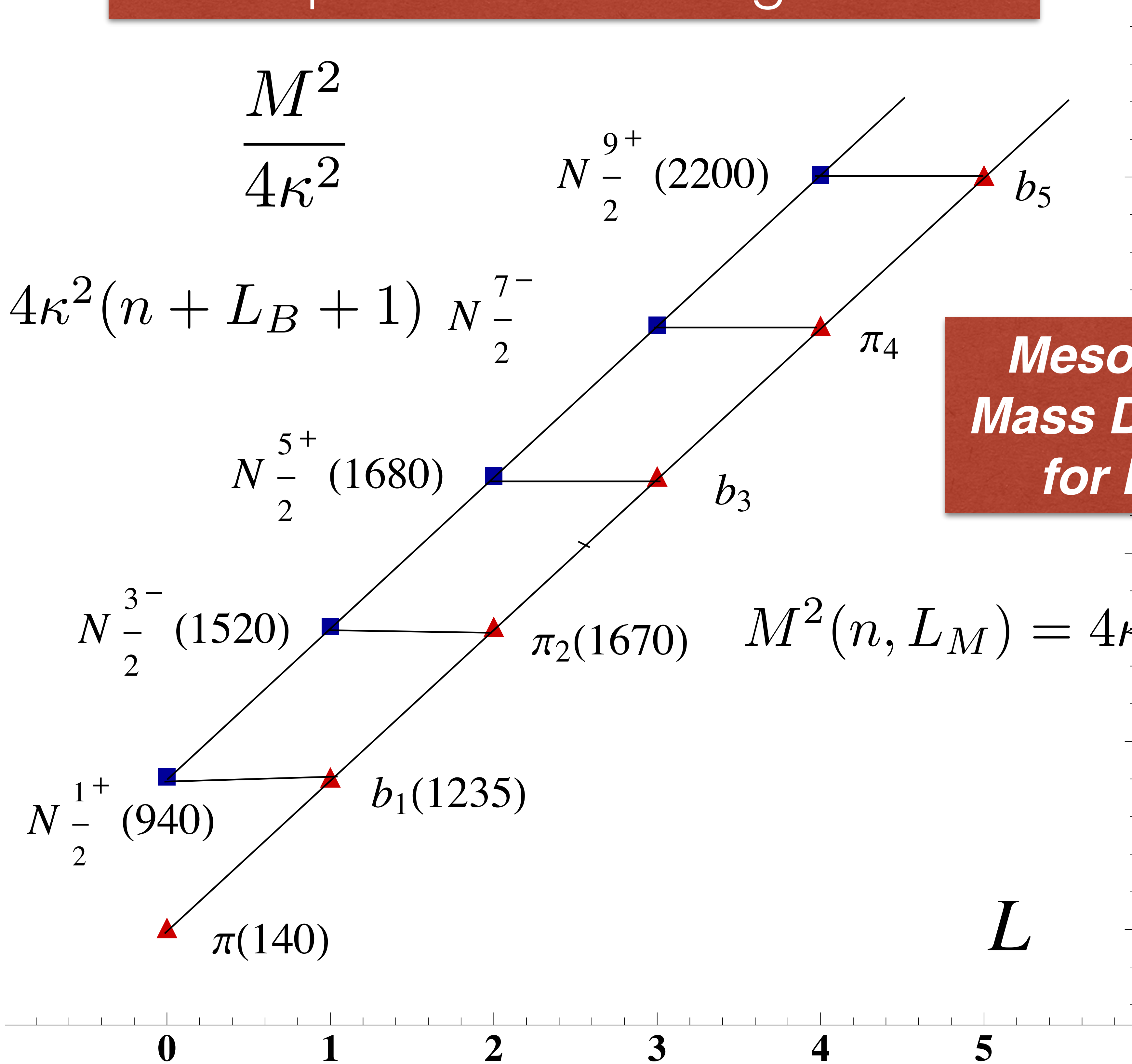
- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

Meson and baryon have same κ !



Superconformal Algebra

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Same κ

$$\lambda = \kappa^2$$

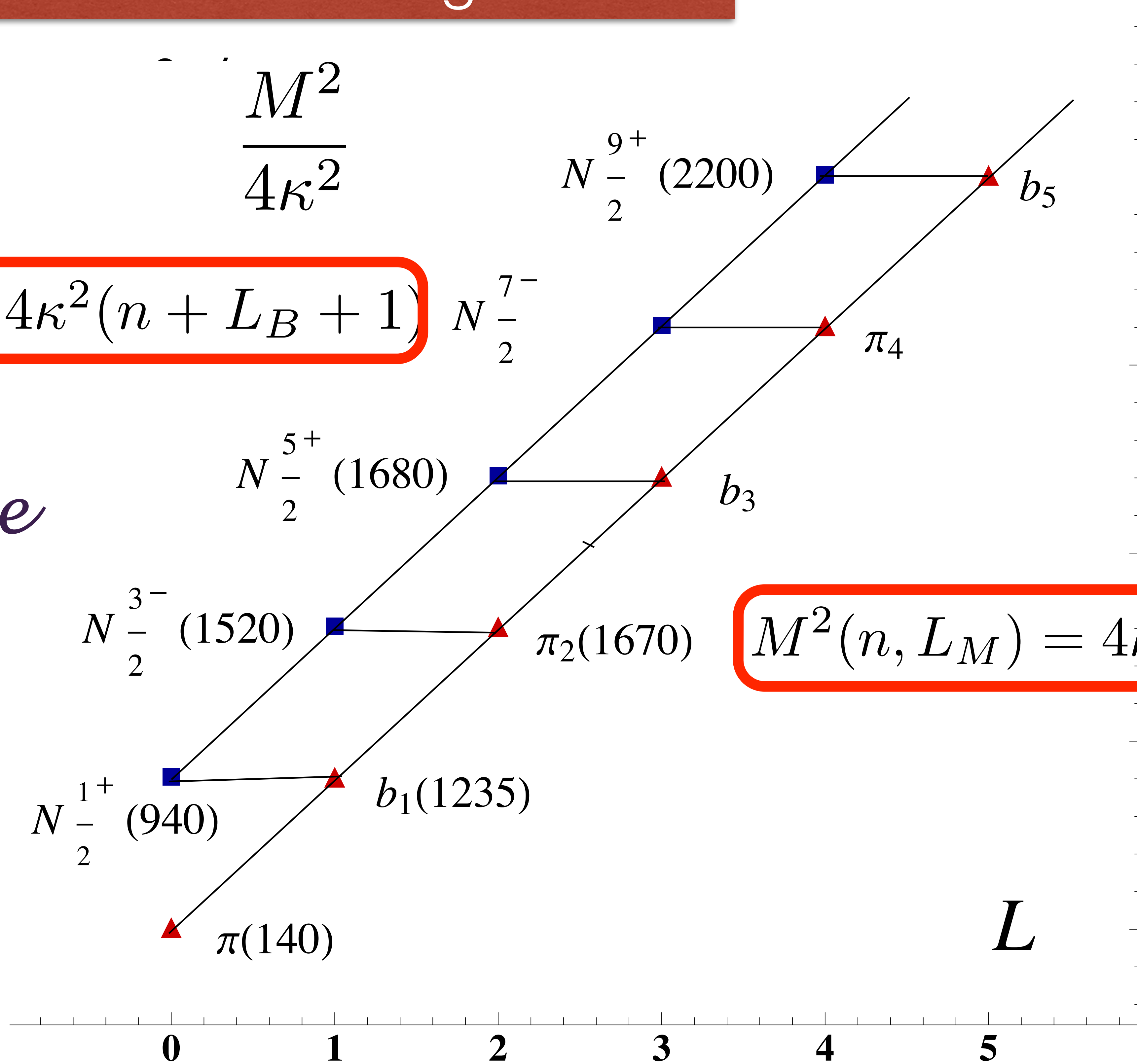
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Superconformal Algebra

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

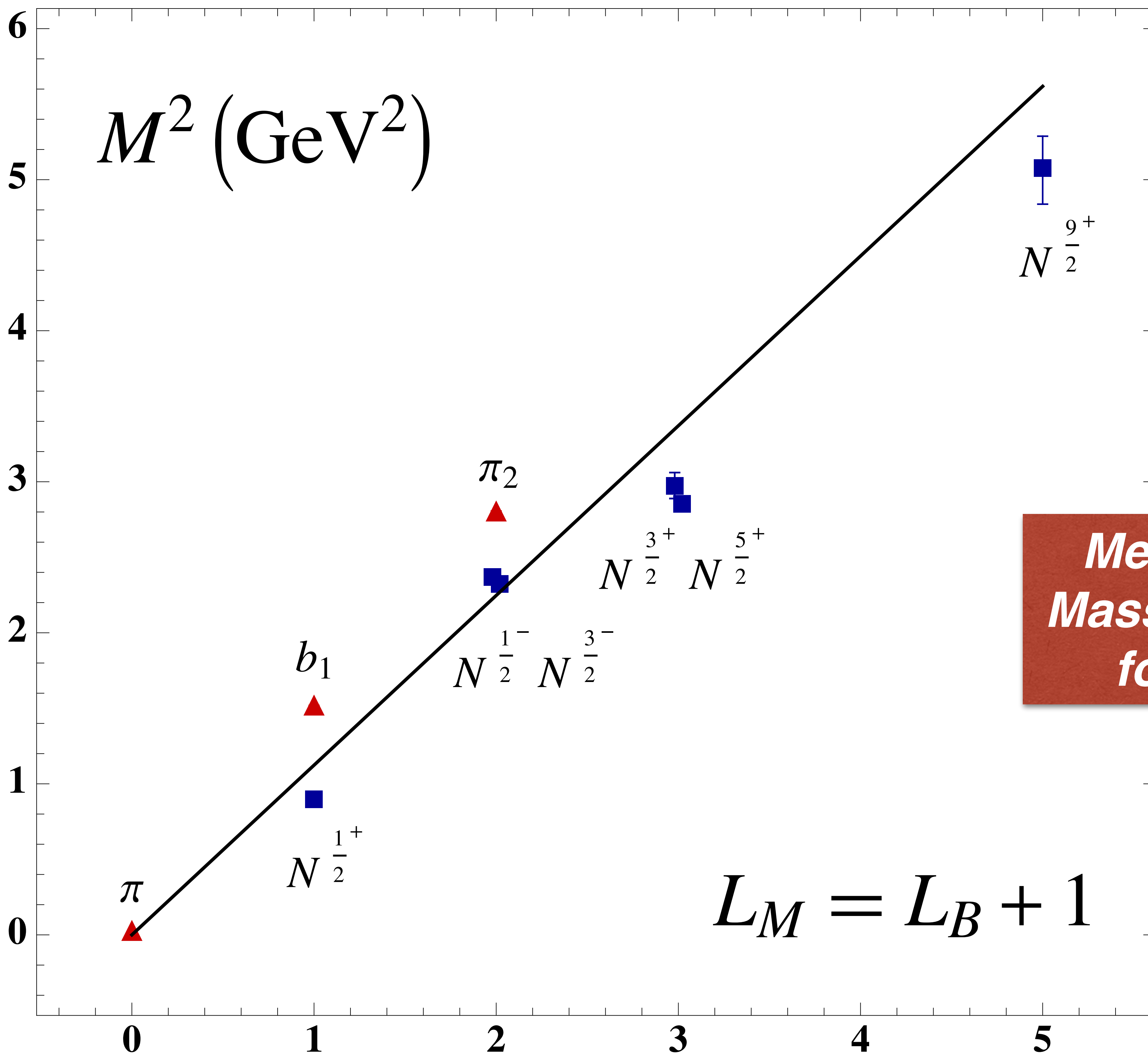


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

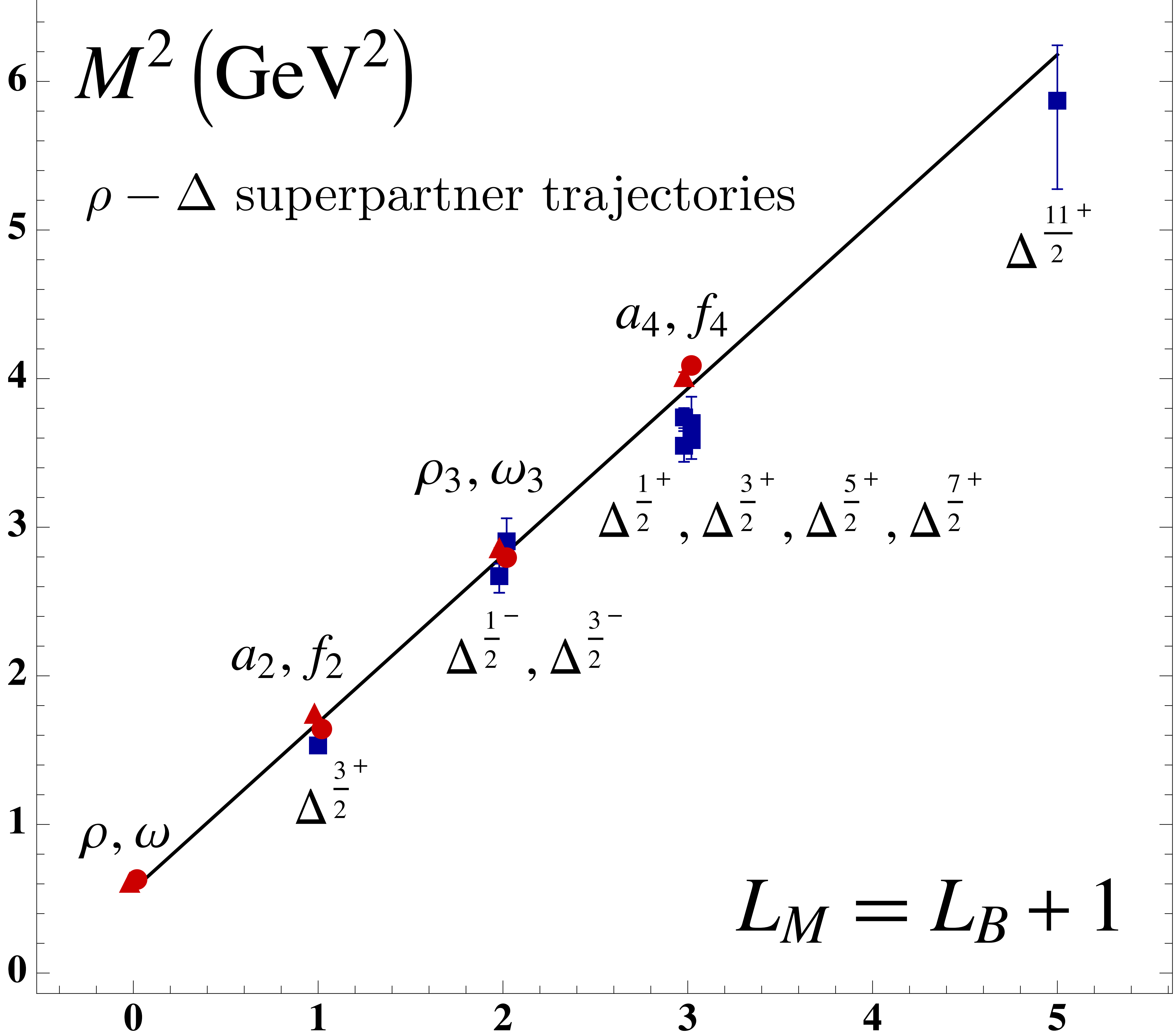
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!

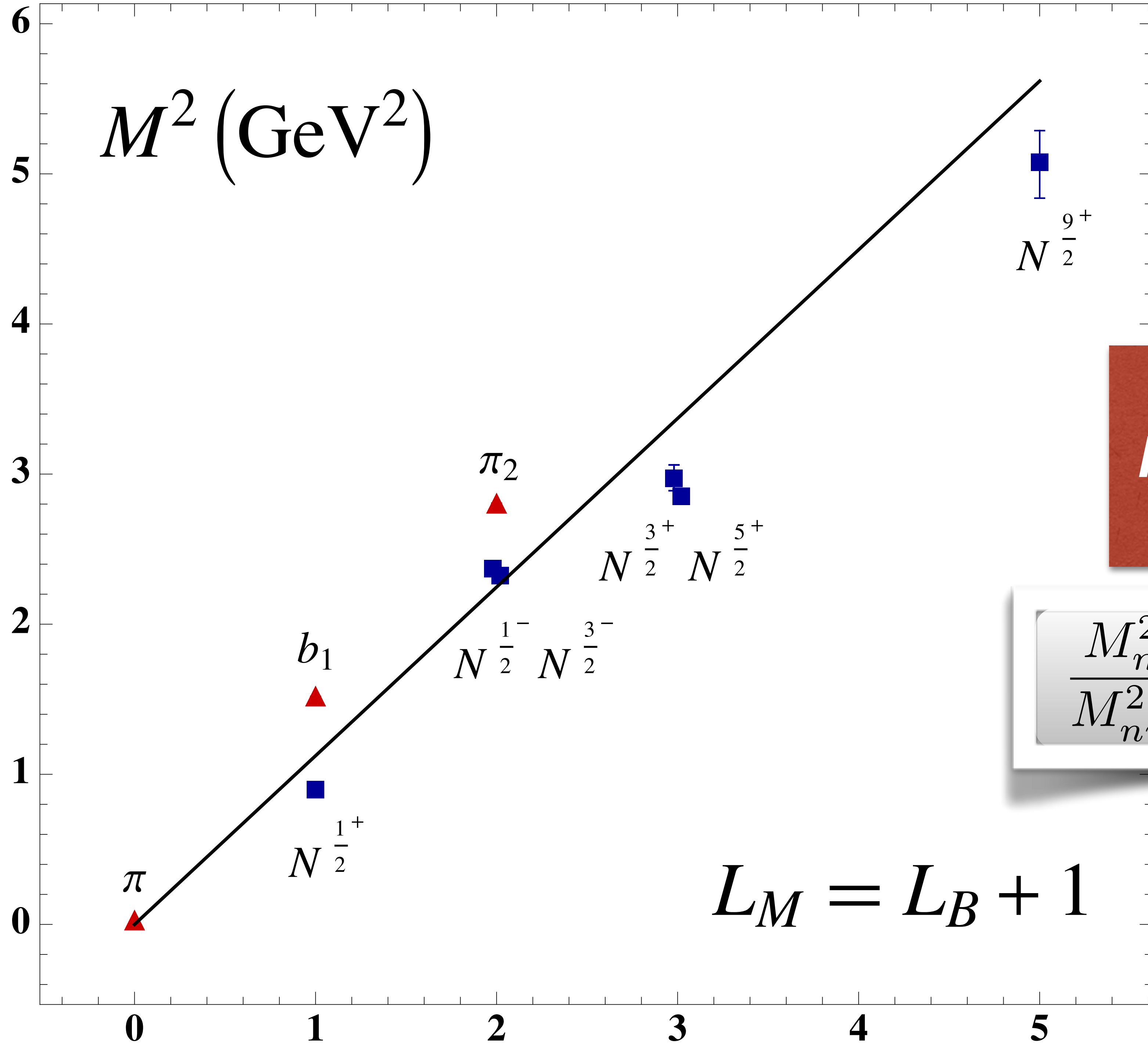


Meson-Baryon Mass Degeneracy for $L_M=L_B+1$



Dosch, de Teramond, sjb

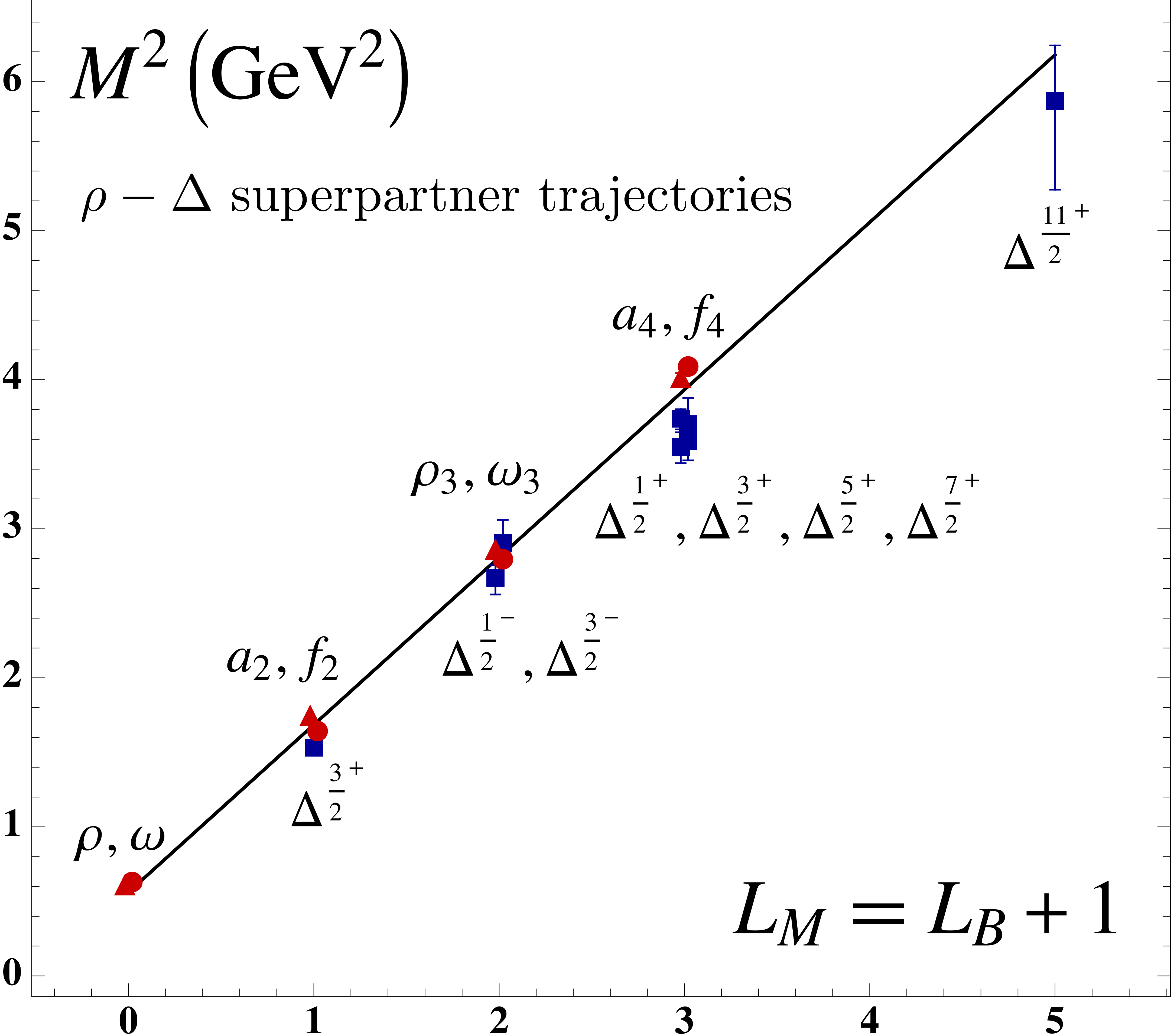
Superconformal AdS Light-Front Holographic QCD (LFHQCD): Identical meson and baryon spectra!



Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

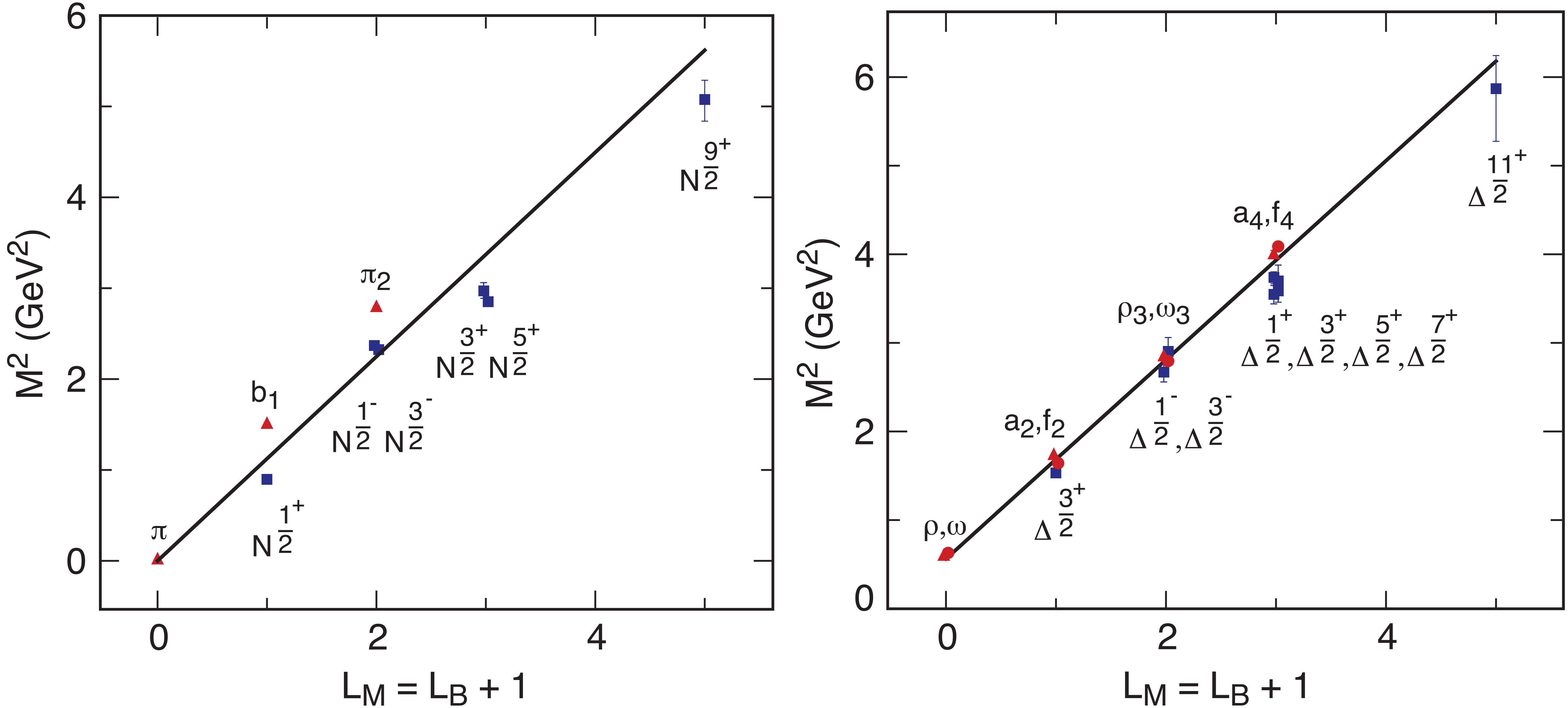
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon



Dosch, de Teramond, sjb

Solid line: $\kappa = 0.53$ GeV



de Tèramond, Dosch, sib

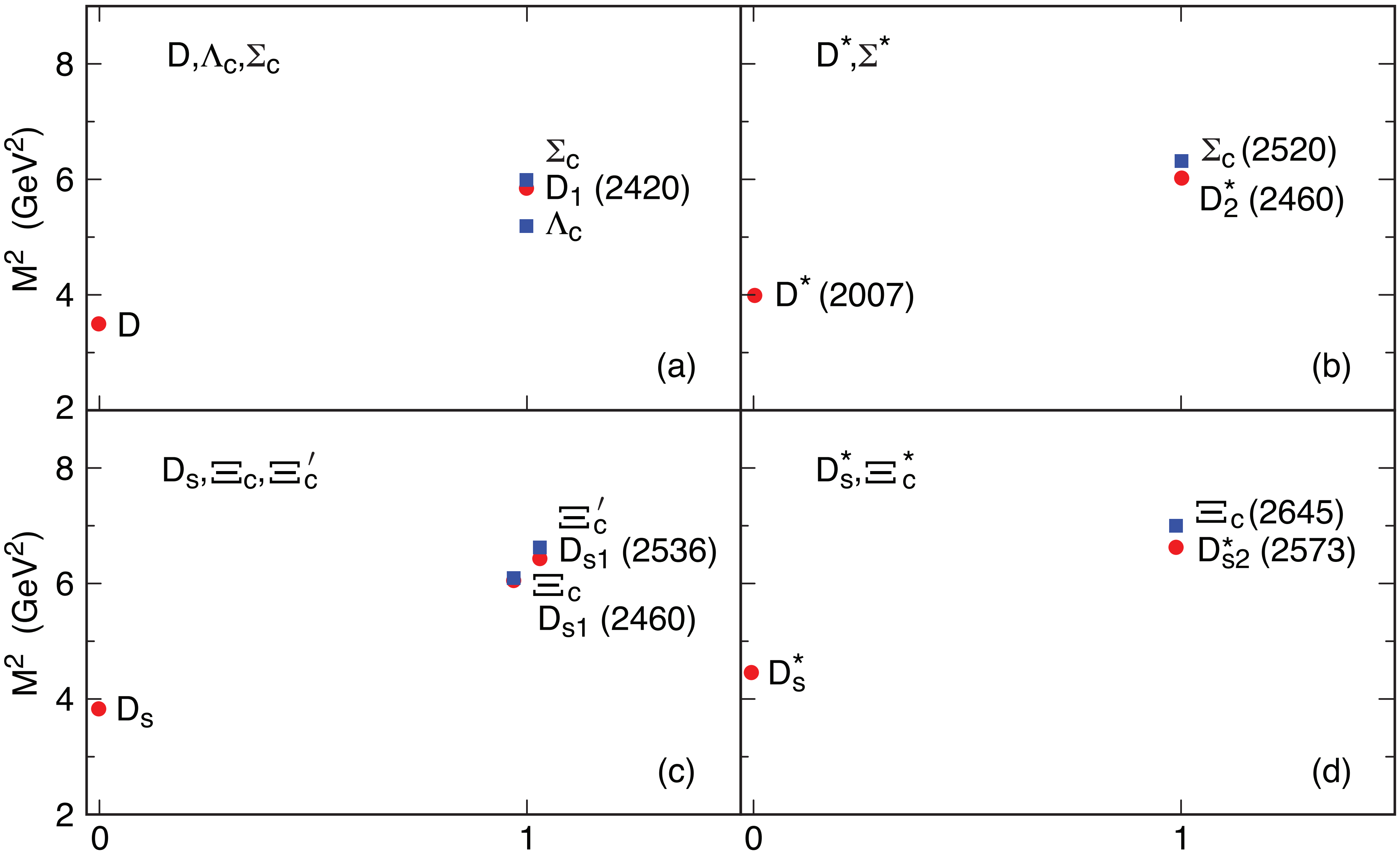
Superconformal meson-nucleon partners

Identical LFWFs, form factors! Test in $e^+e^- \to \bar{H}H'$

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum

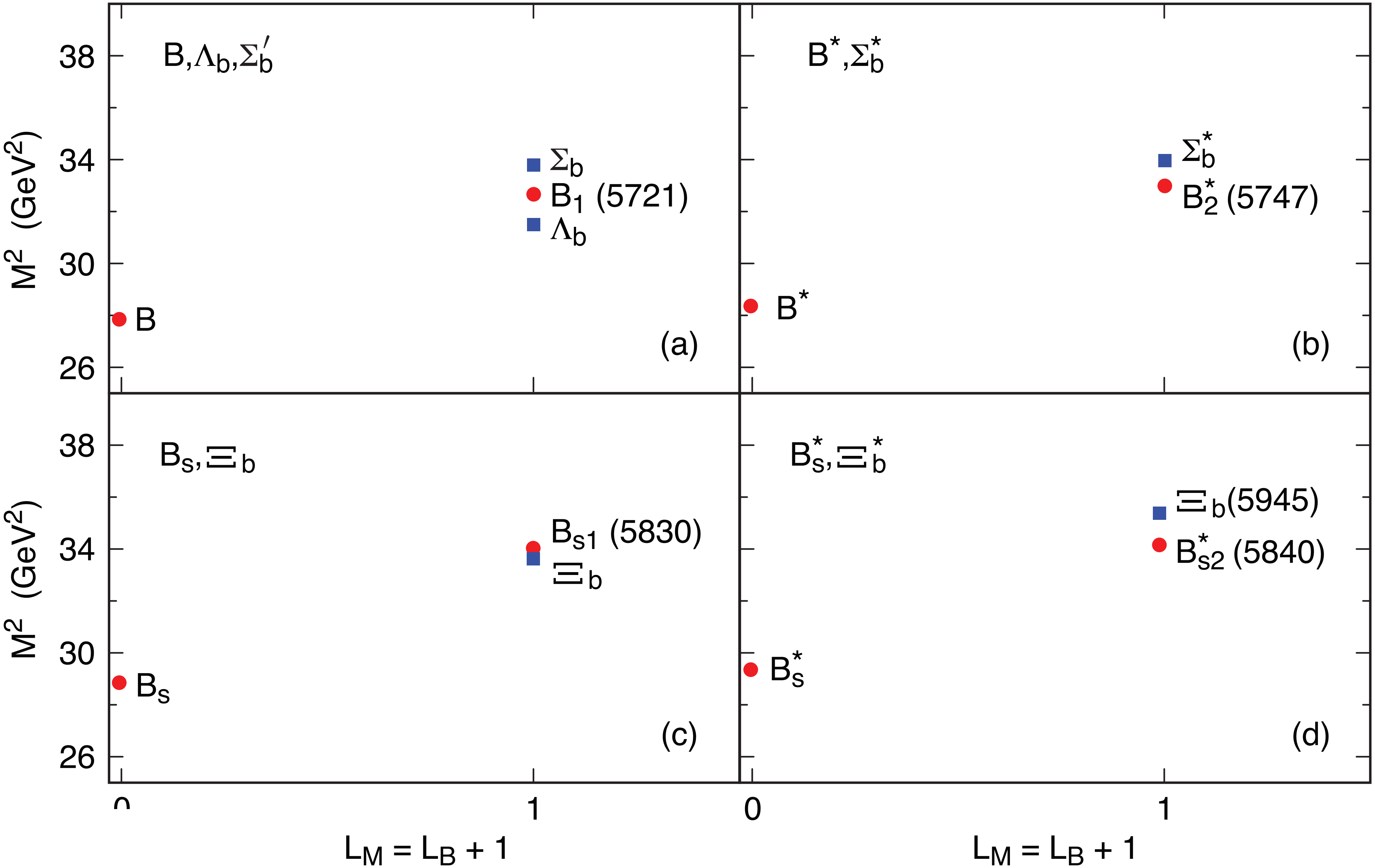
- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Supersymmetric relations for mesons and baryons with c quarks

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Supersymmetric relations for mesons and baryons with b quarks

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z >= 0$$

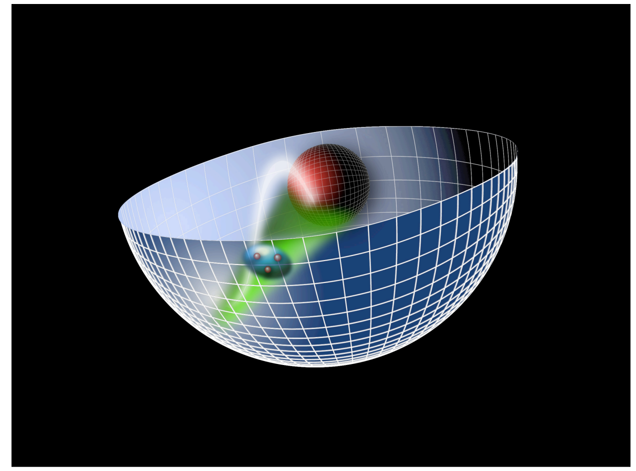
- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Preserves Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

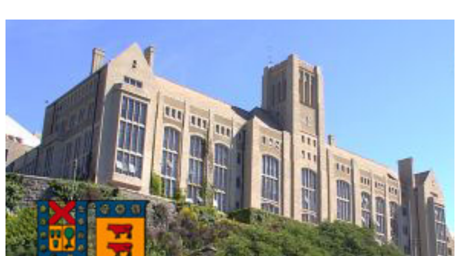
Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$



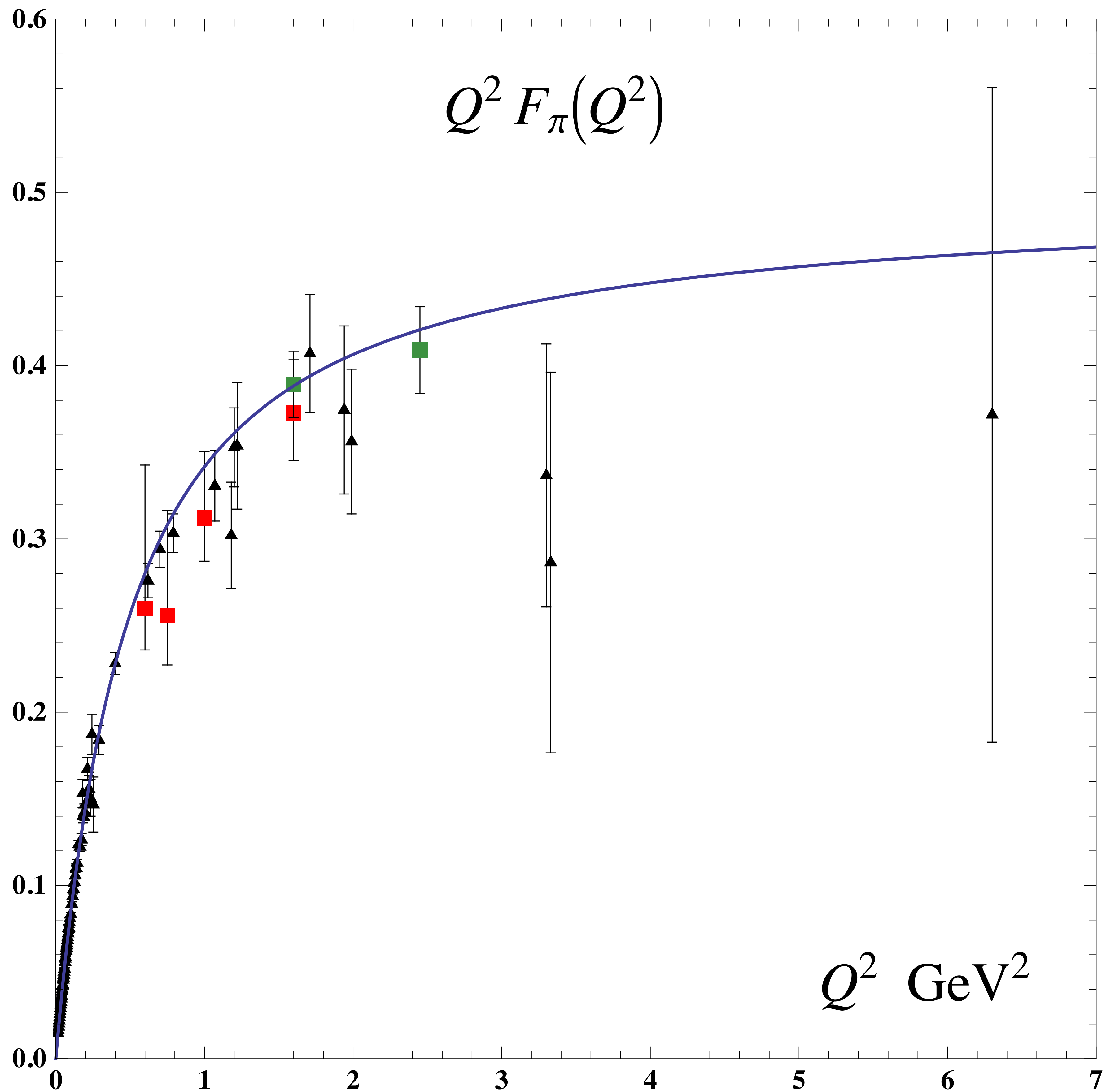
Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

↕

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb



$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

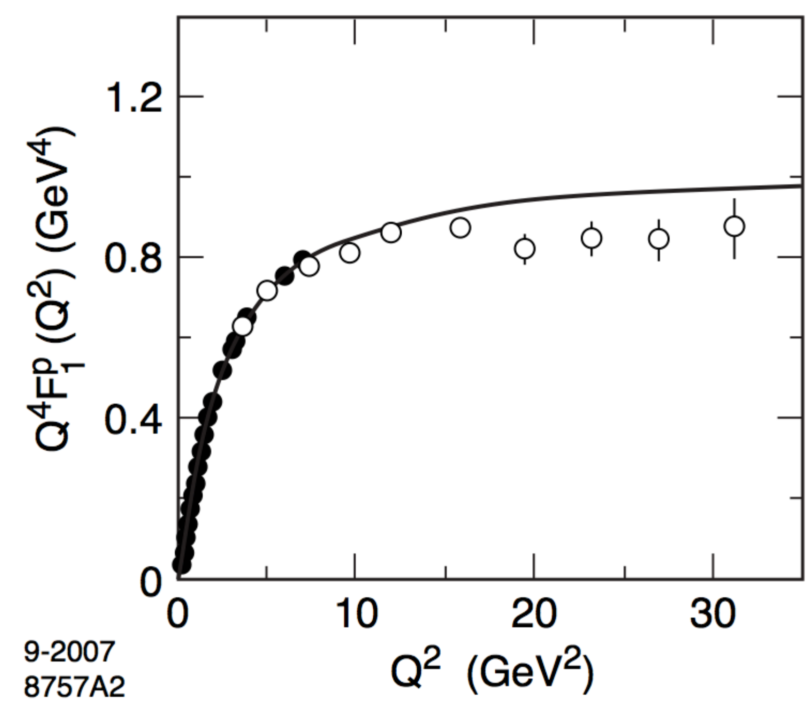
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$

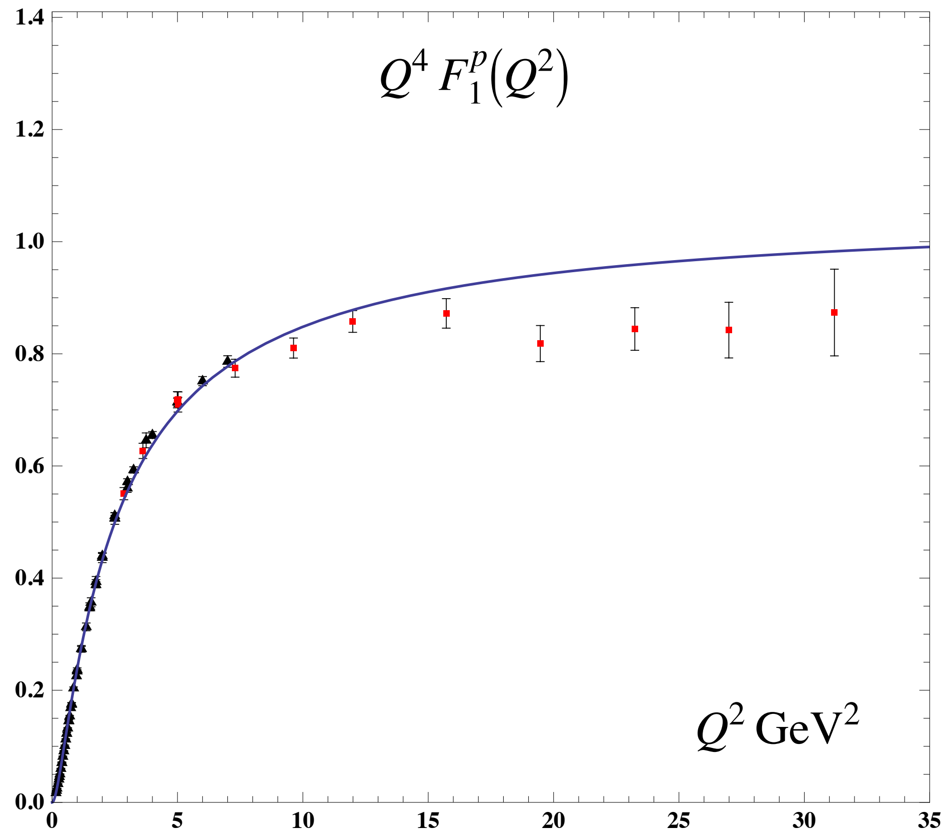
de Teramond, sjb

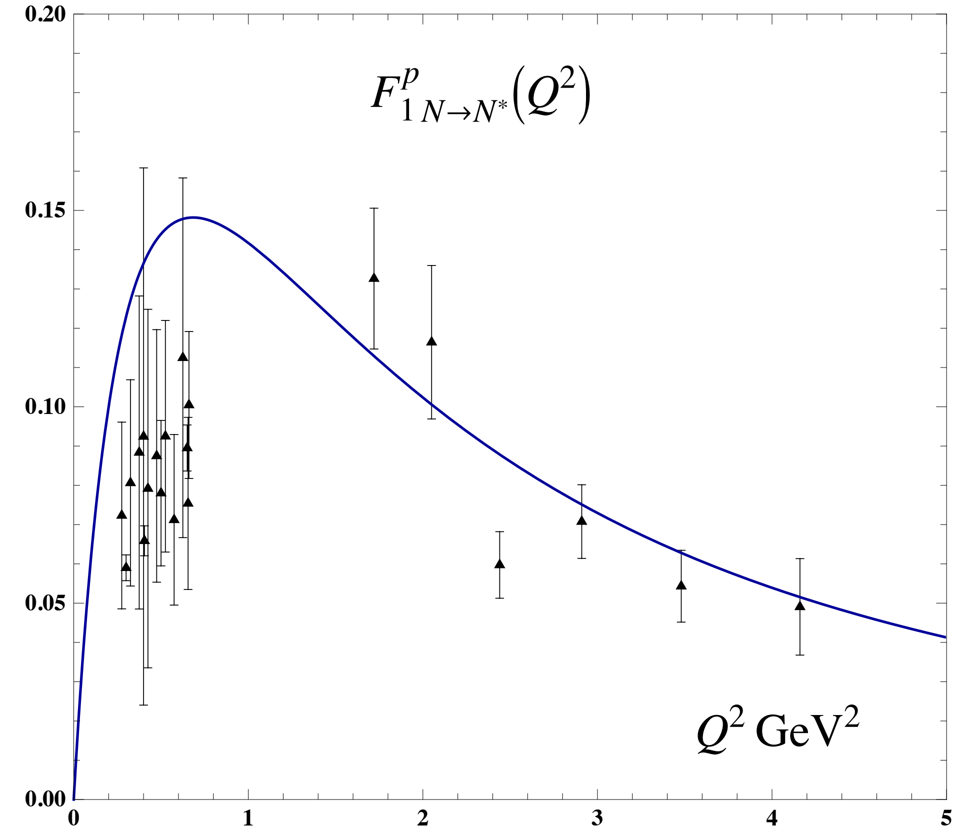
Consistent with counting rule, twist 3

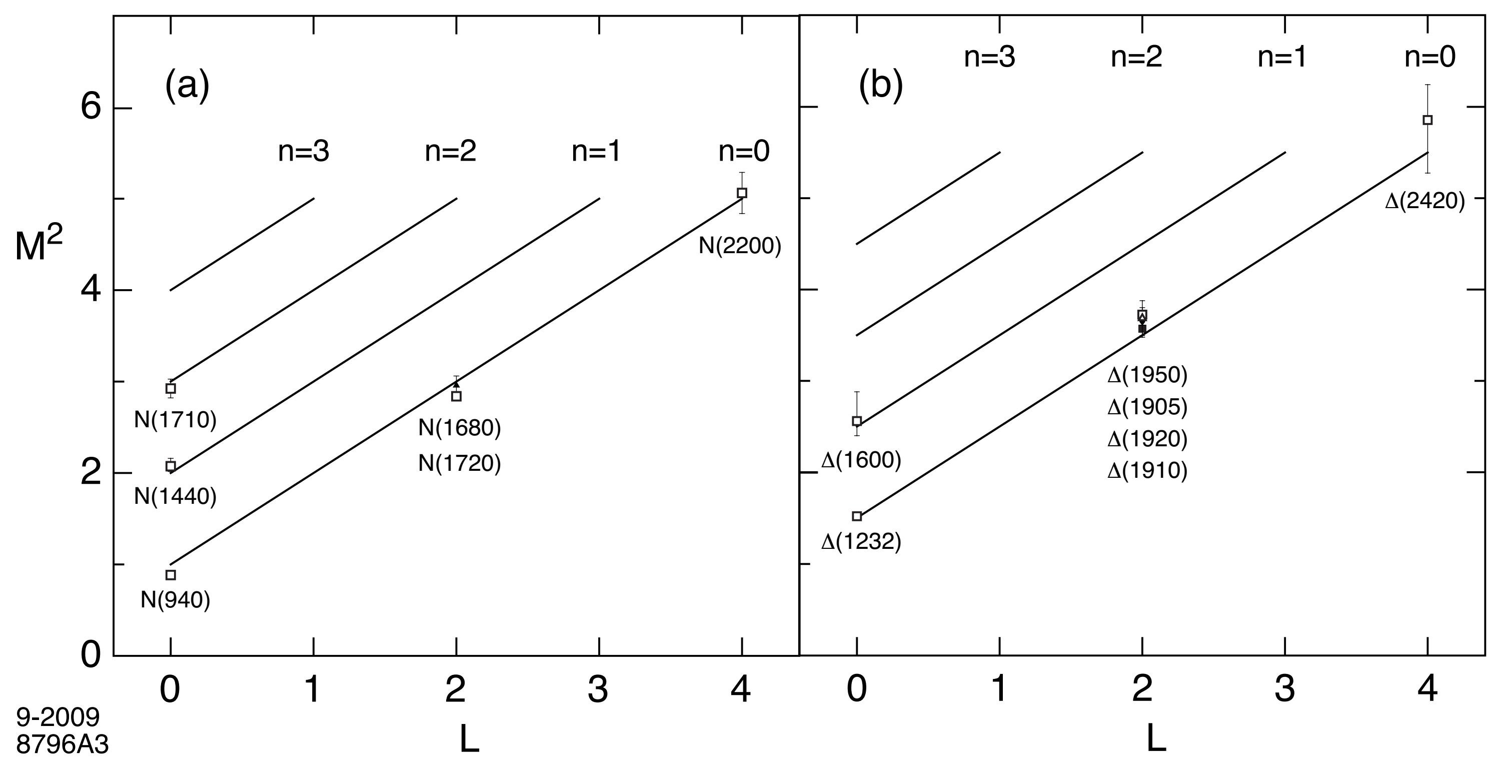
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

G. de Teramond & sjb

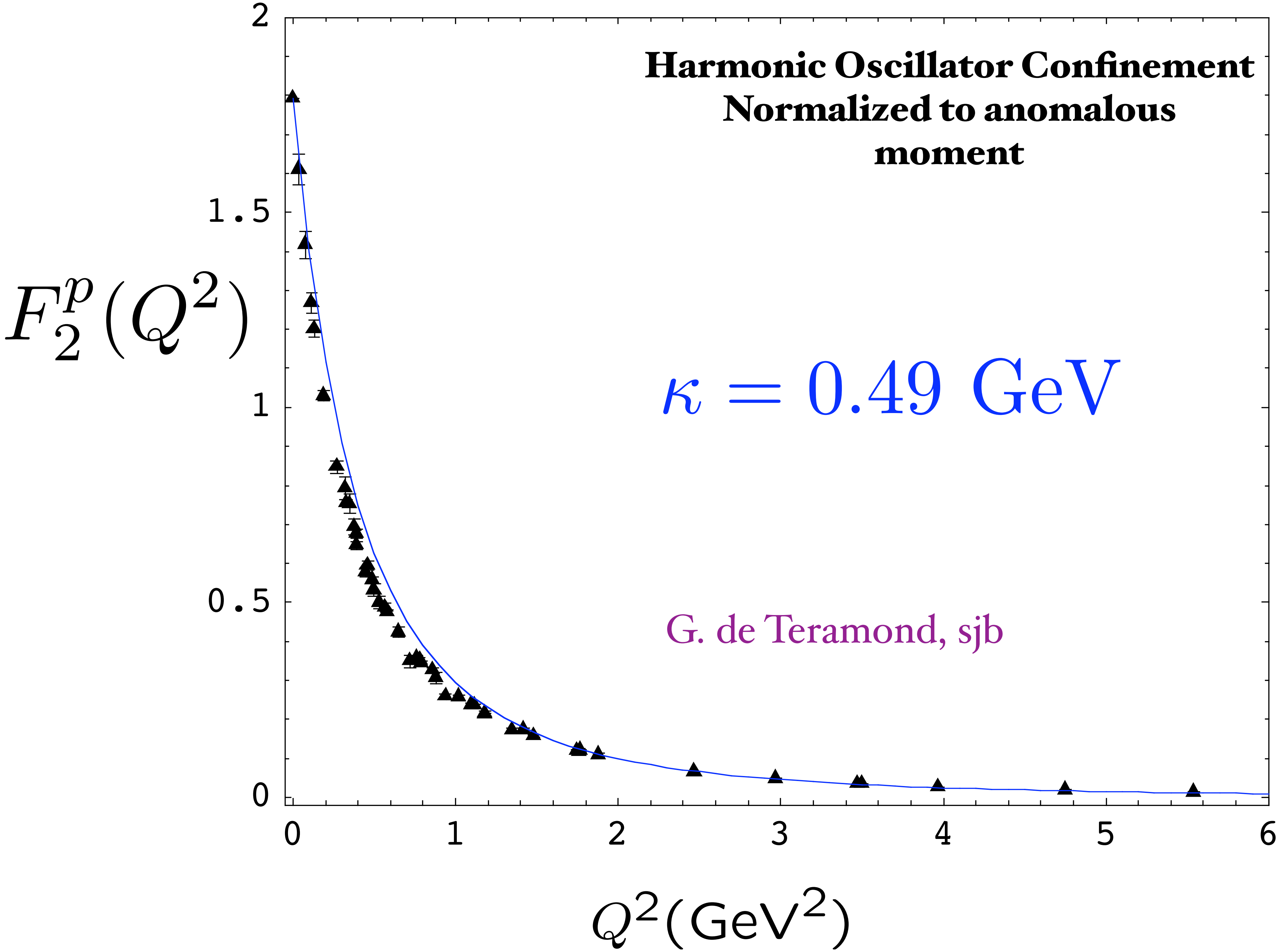




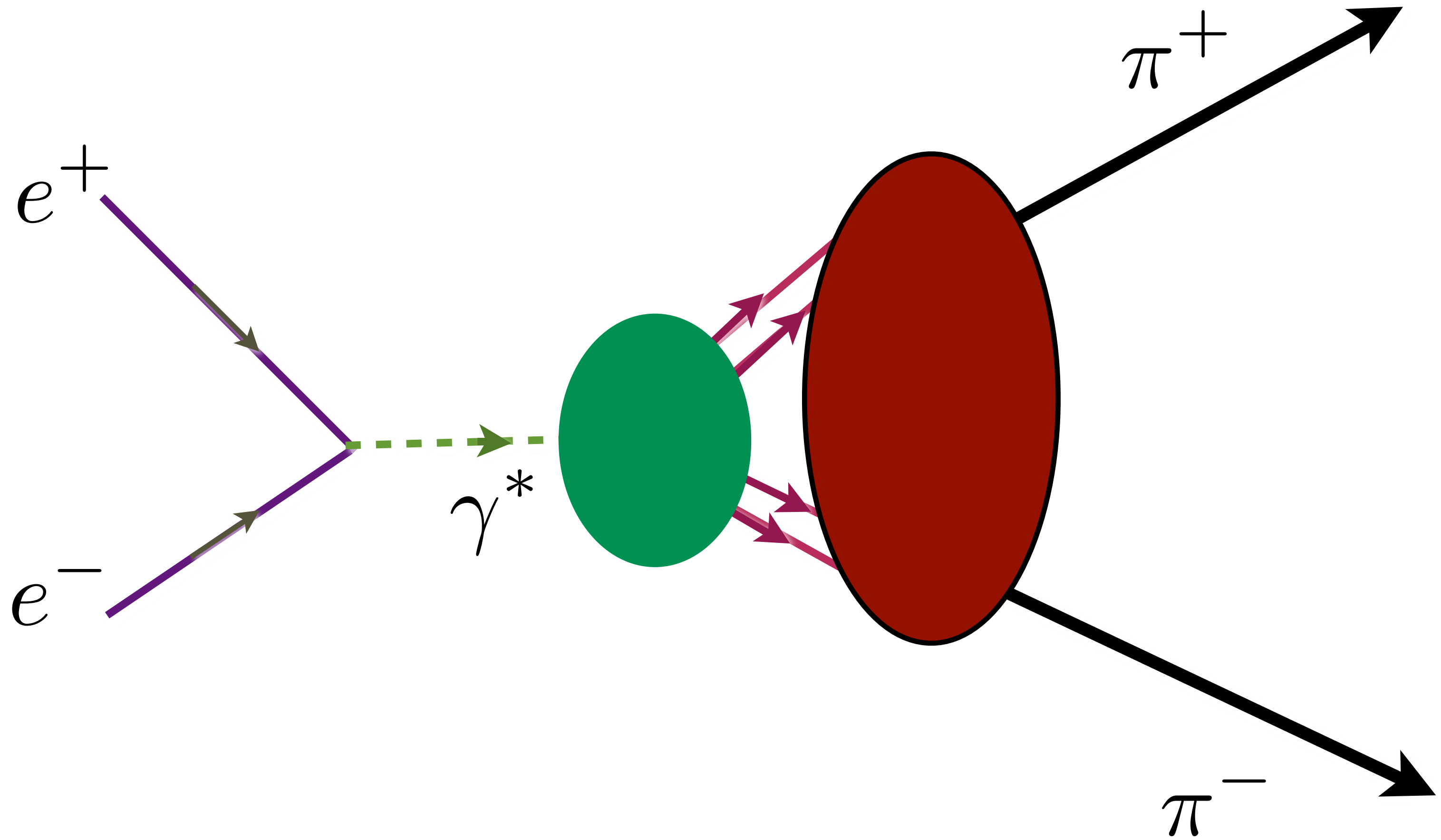


Spacelike Pauli Form Factor

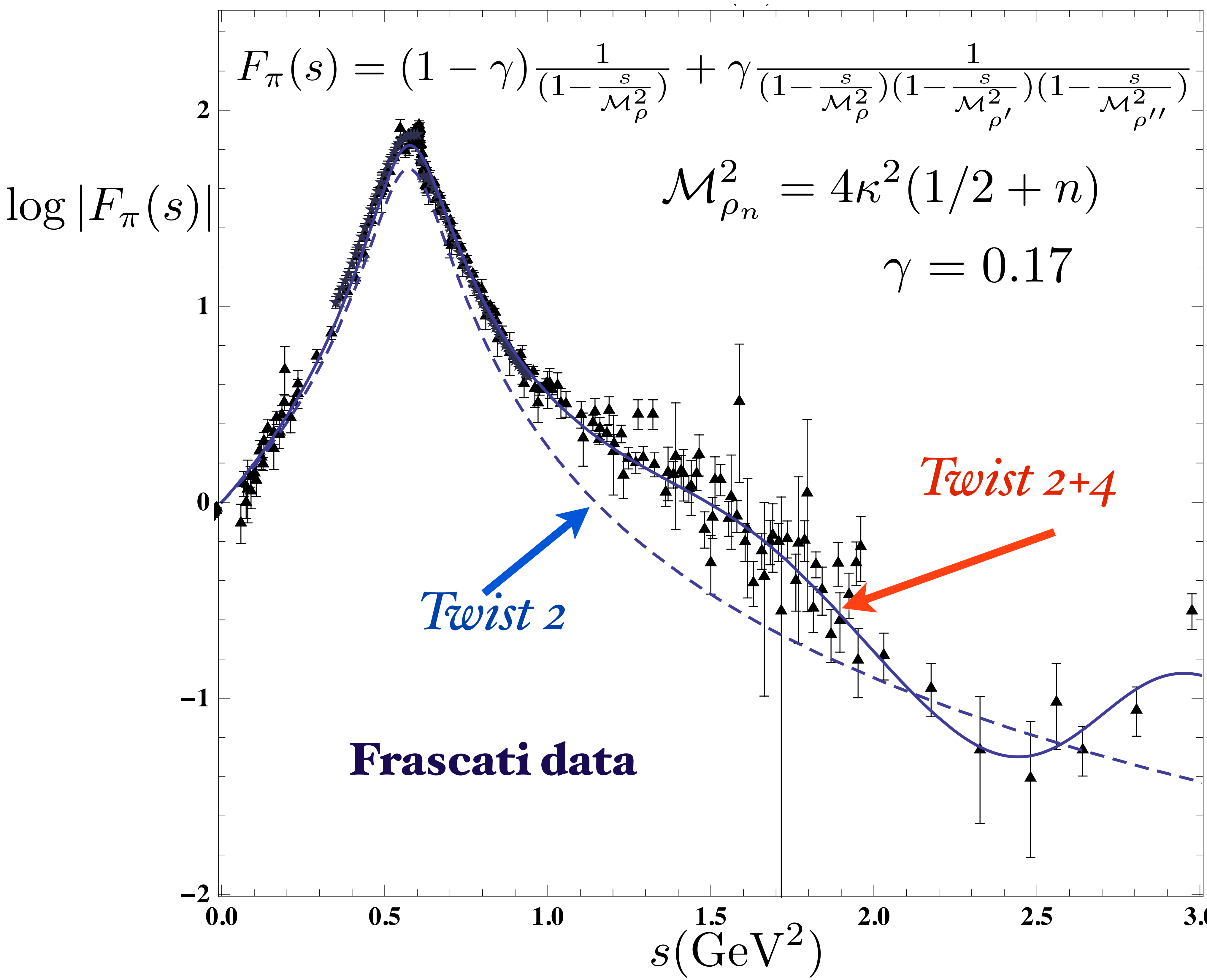
From overlap of $L = 1$ and $L = 0$ LFWFs



Dressed soft-wall current brings in higher Fock states and more vector meson poles



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



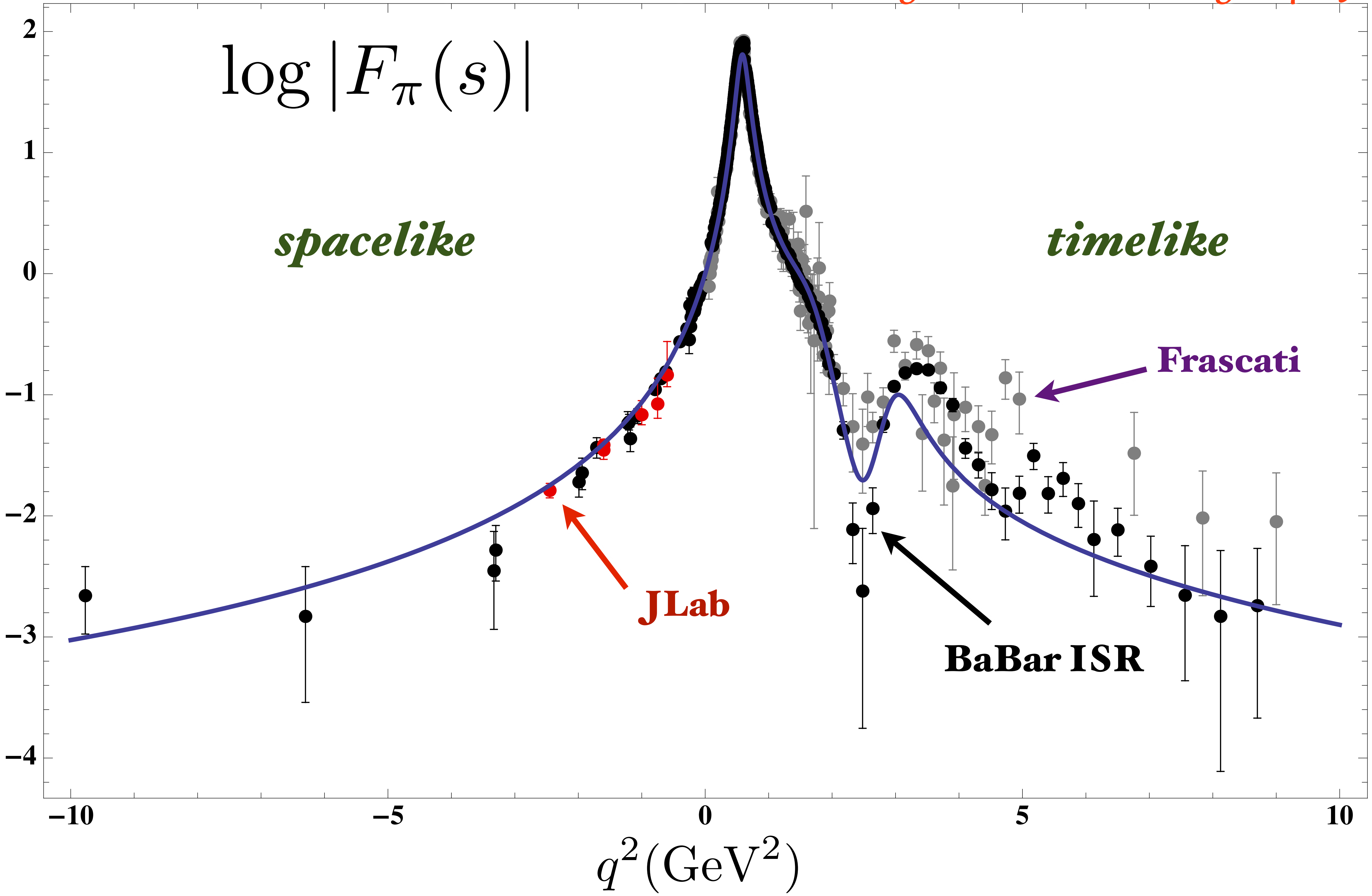
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Nuclear physics in soft-wall AdS/QCD: deuteron electromagnetic form factors

Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt, and Alfredo Vega

$$F_D(Q^2) \equiv f_D(Q^2) F_p(\frac{Q^2}{4}) F_n(\frac{Q^2}{4})$$

Chertok, sjb

Ji, Lepage, sjb

$$\left[-\frac{d^2}{dz^2} + \frac{4(L+4)^2-1}{4z^2} + \kappa^4 z^2 + \kappa^2 U_0\right]\Phi_n(z) = M^2_{d,n}\Phi_n(z)$$

AdS/QCD, LF Holography

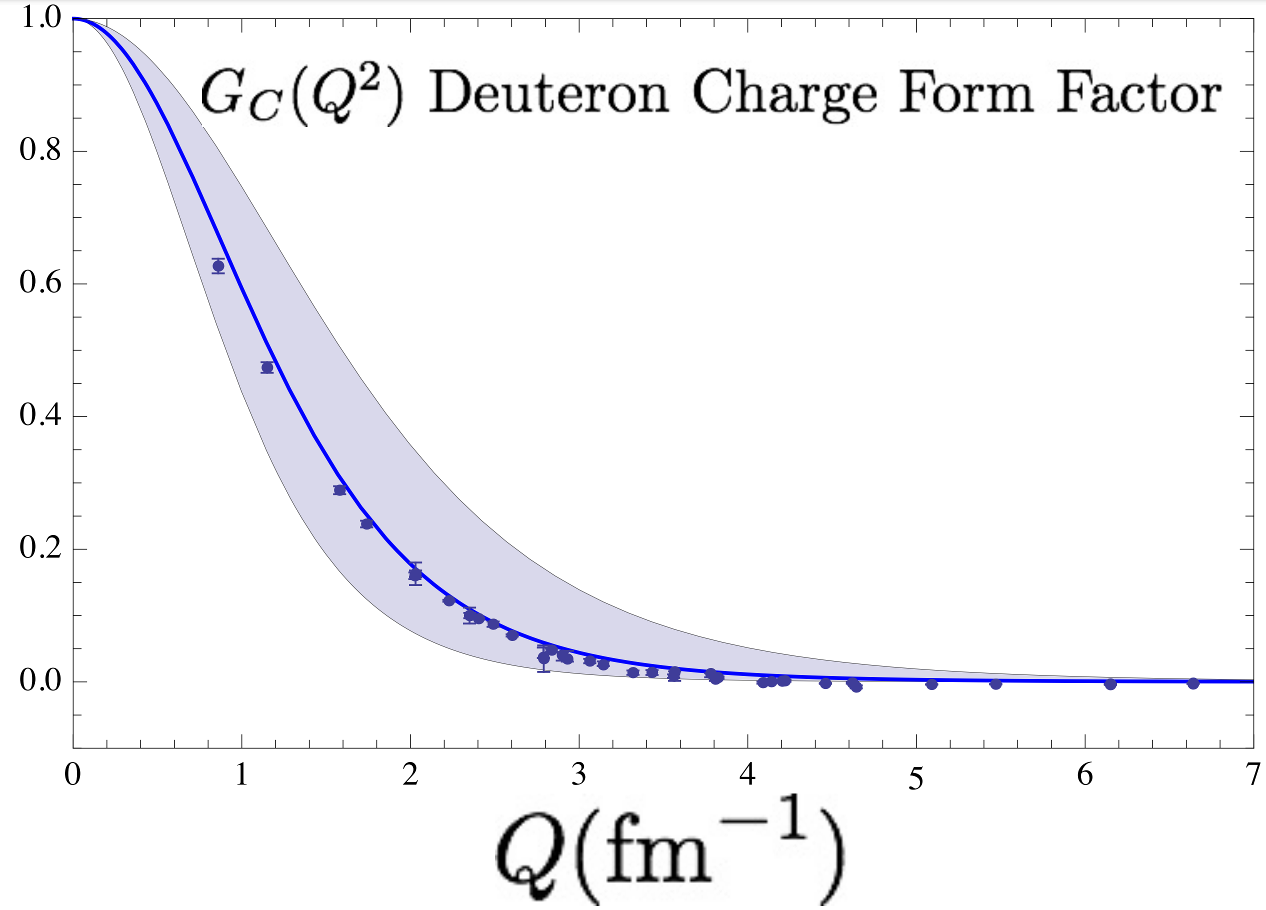
Katz, et al

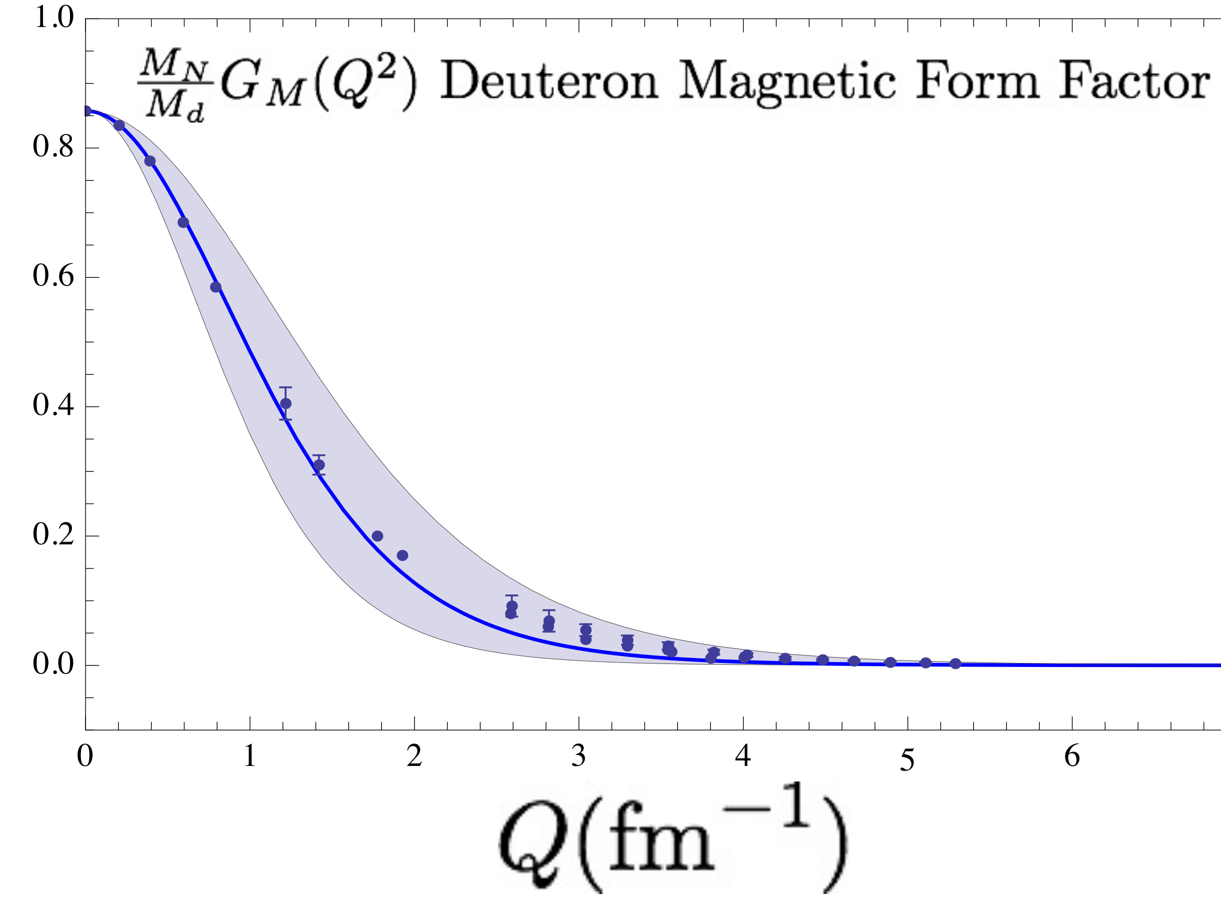
de Tèramond, sjb

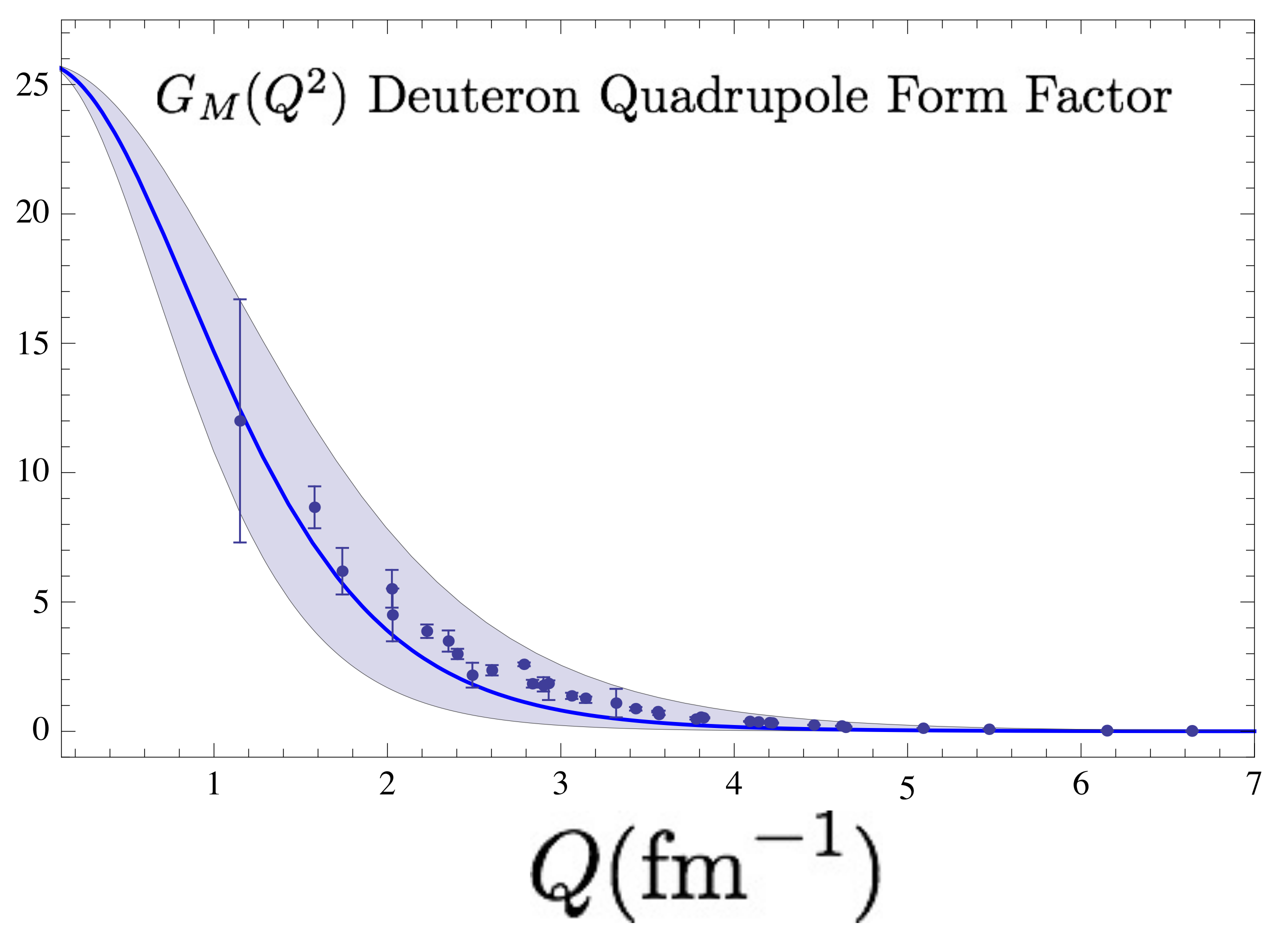
$$f_d(Q^2) = \frac{30(a+1)(a+2)}{(a+3)(a+4)(a+5)}, \quad F_N(Q^2/4) = \frac{2}{(a+1)(a+2)}$$

$$a = Q^2/4\kappa^2 = Q^2/m_\rho^2 \qquad Q^2 f_d(Q^2) \to const$$

Application of Light-Front Holography to the Deuteron Form Factors







Thomas Gutsche, Valery E. Lyubovitskij, Ivan Schmidt, and Alfredo Vega

http://arxiv.org/abs/1501.02738v3

Consistent with quark counting rules

Ji, Lepage, sjb

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Deur, de Teramond, sjb

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

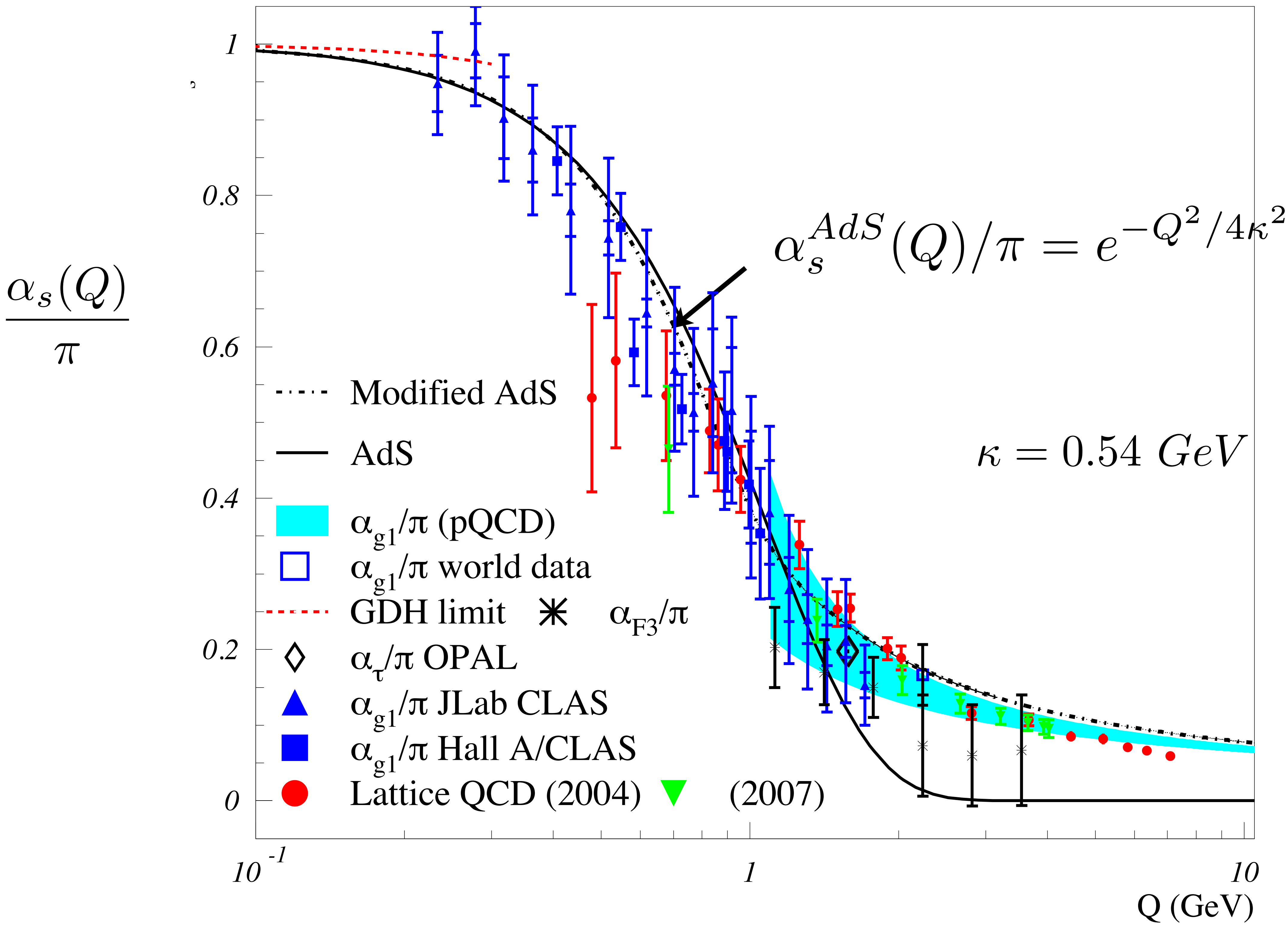
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Deur, de Tèramond, sjb

All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

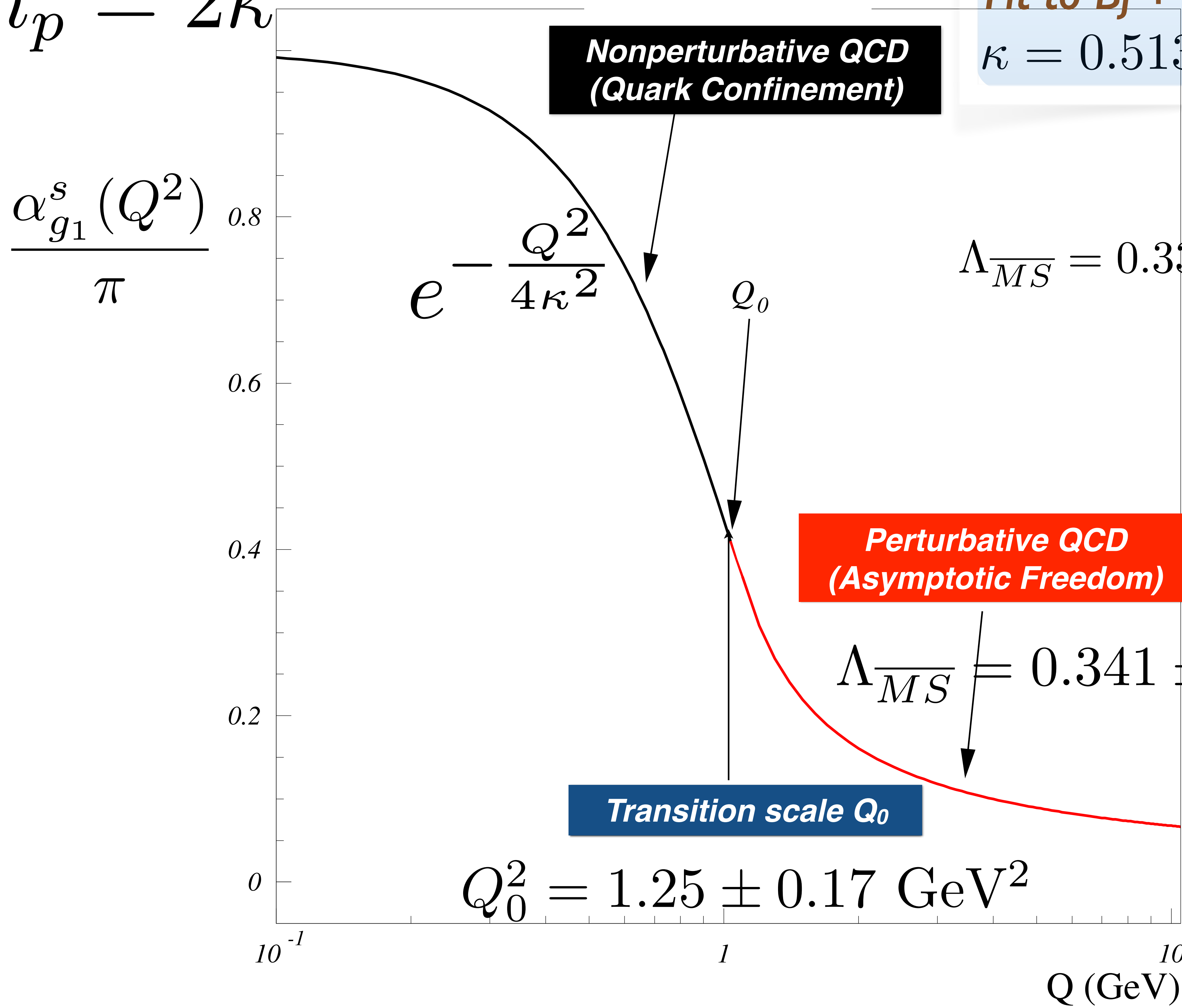
$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

Expt:

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.016\ GeV$$

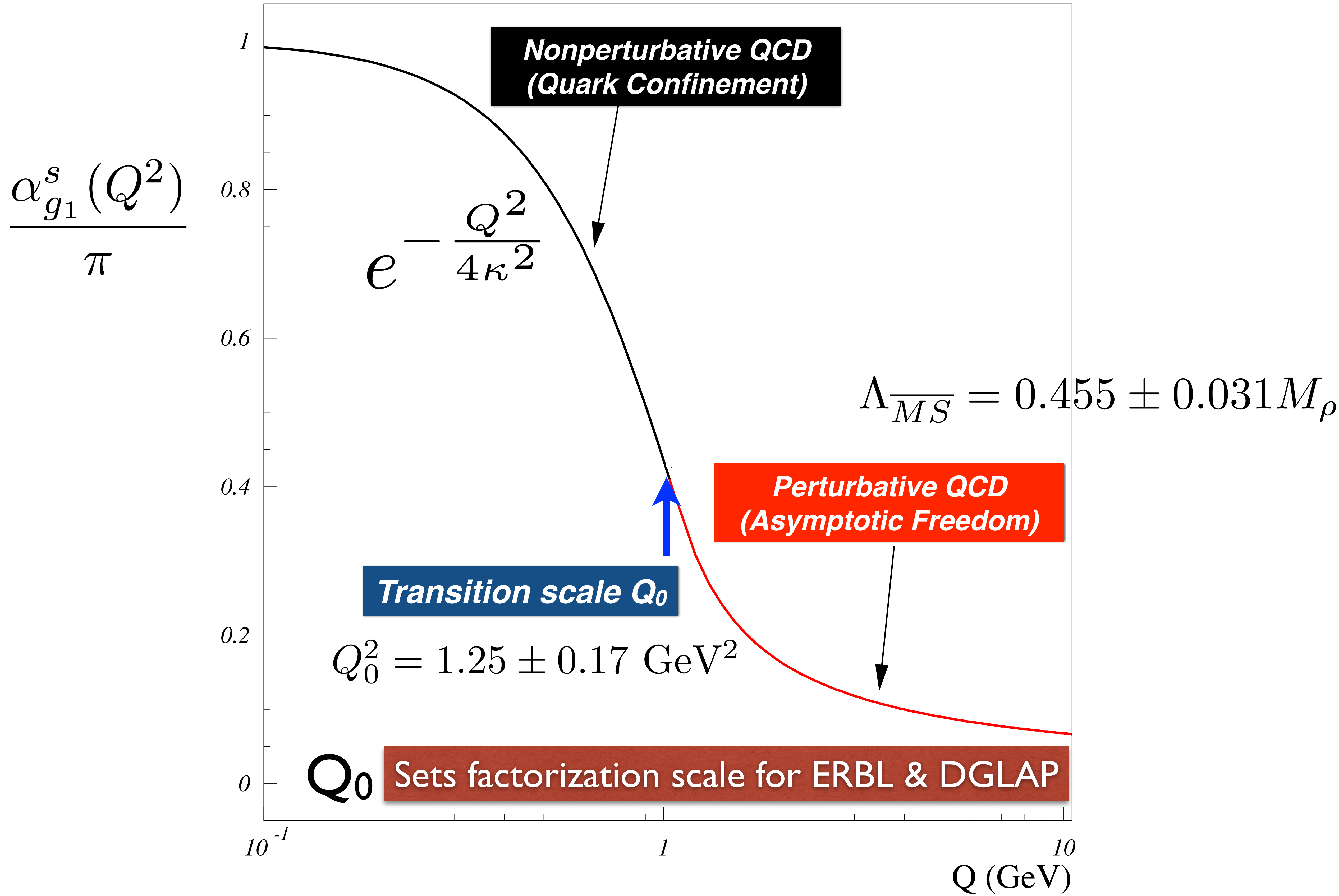


$$m_\rho = \sqrt{2}\kappa$$

All-Scale QCD Coupling

Deur, de Teramond, sjb

Prediction from AdS/QCD:



de Tèramond, Dosch, sjb

Tony Zee

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21 \, \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2 \, \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$(m_q = 0)$$

$$m_\pi = 0$$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Goal: An analytic first approximation to QCD

- **As Simple as Schrödinger Theory in Atomic Physics**
- **Relativistic, Frame-Independent, Color-Confining**
- **Confinement in QCD -- What is the analytic form of the confining interaction?**
- **What sets the QCD mass scale?**
- **QCD Running Coupling at all scales**
- **Hadron Spectroscopy-Regge Trajectories**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Insights into QCD Condensates**
- **Chiral Symmetry**
- **Systematically improvable**

Goals

- **Test QCD to maximum precision at the LHC**
- **Maximize sensitivity to new physics**
- **High precision determination of fundamental parameters**
- **Determine renormalizations scales without ambiguity**
- **Eliminate scheme dependence**

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

Electron-Electron Scattering in QED

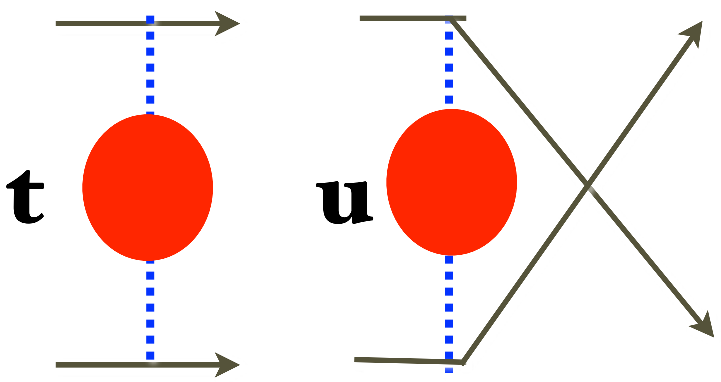
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- **Two separate physical scales: t, u = photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza*

CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Stanley J. Brodsky†

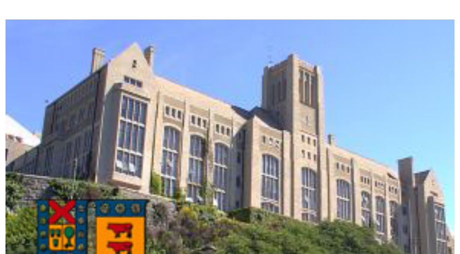
SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA

Xing-Gang Wu‡

Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China

(Received 13 January 2013; published 10 May 2013)

We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.



δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\rm MS}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu^2_\delta = \mu^2_{\overline{\rm MS}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

M. Mojaza, Xing-Gang Wu, sjb

M. Mojaza, Xing-Gang Wu, sjb

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\mathrm{MS}} \,, \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \mathrm{MS} \qquad \mu^2 = \mu_{\overline{\mathrm{MS}}}^2 \exp(\ln 4\pi - \gamma_E) \,, \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \exp(\delta_2 - \delta_1)$$

Note the divergent 'renormalon series' $n! \beta^n \alpha_s^n$

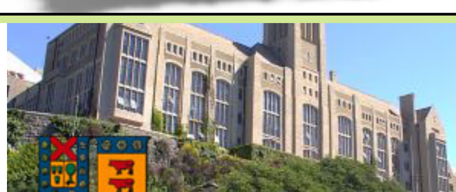
Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \mathrm{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

y



Special Degeneracy in PQCD

There is nothing special about a particular value for δ, thus for any δ

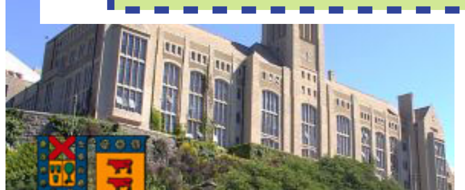
$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 \underline{r_{2,1}}]a(Q)^2 + [r_{3,0} + \beta_1 \underline{r_{2,1}} + 2\beta_0 \underline{r_{3,1}} + \beta_0^2 \underline{r_{3,2}}]a(Q)^3$$
$$+ [r_{4,0} + \beta_2 \underline{r_{2,1}} + 2\beta_1 \underline{r_{3,1}} + \frac{5}{2}\beta_1\beta_0 \underline{r_{3,2}} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4$$

According to the principal of maximum conformality we must set the scales such to absorb all 'renormalon-terms', i.e. non-conformal terms

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + (\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \beta_2 a(Q)^4 + \cdots)r_{2,1}$$
$$+ (\beta_0^2 a(Q)^3 + \frac{5}{2}\beta_1\beta_0 a(Q)^4 + \cdots)r_{3,2} + (\beta_0^3 + \cdots)r_{4,3}$$
$$+ r_{2,0}a(Q)^2 + 2a(Q)(\beta_0 a(Q)^2 + \beta_1 a(Q)^3 + \cdots)r_{3,1}$$
$$+ \cdots$$

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1} + \frac{1}{2}\beta(a)\frac{\partial\beta}{\partial a}r_{3,2} + \cdots + \frac{(-1)^n}{n!}\frac{d^{n-1}\beta}{(d\ln\mu^2)^{n-1}}r_{n+1,n}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1} + \cdots$$



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General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\begin{aligned}\rho(Q^2) =& r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 \\ &+ [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 \\ &+ [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} \\ &+ 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5)\end{aligned}$$

PMC scales thus satisfy

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_3)^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

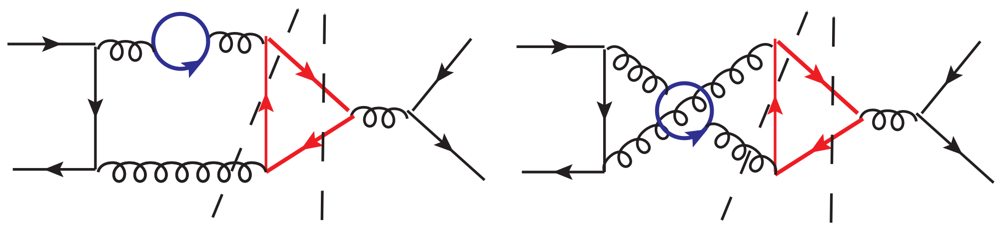
$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

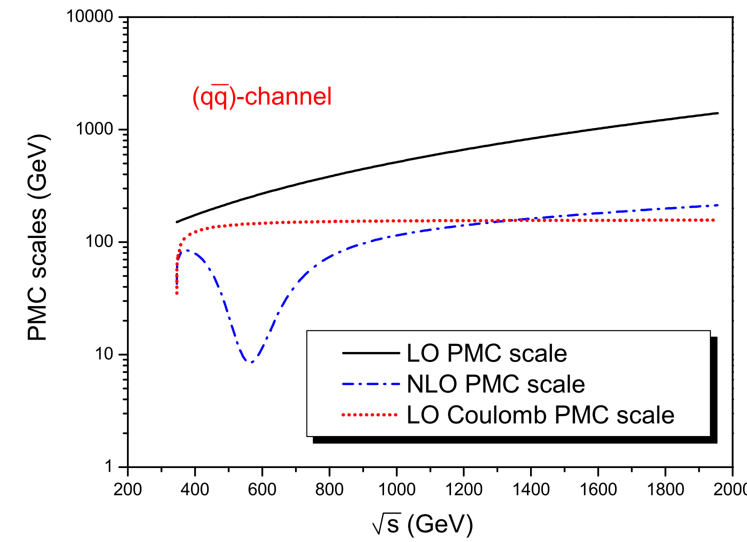
Important Example: Top-Quark FB Asymmetry

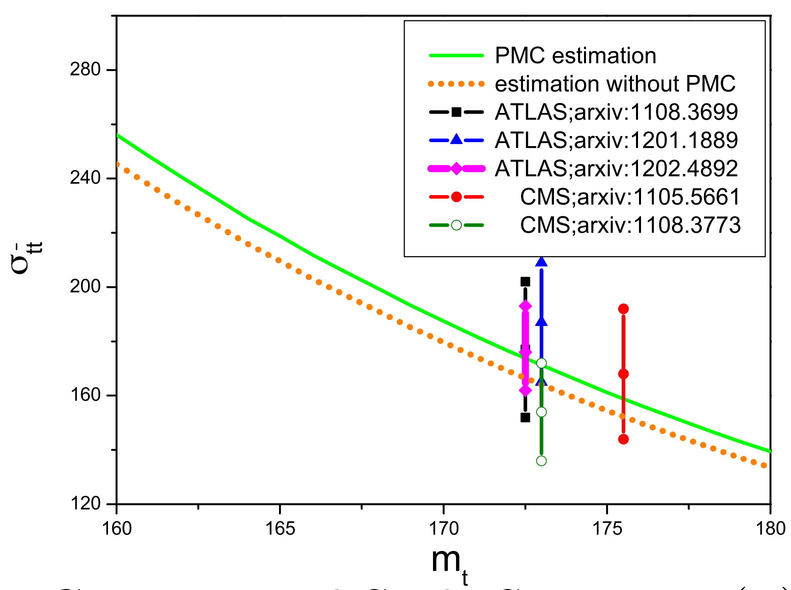
Brodsky, Wu, **Phys.Rev.Lett. 109**, [arXiv:1203.5312]

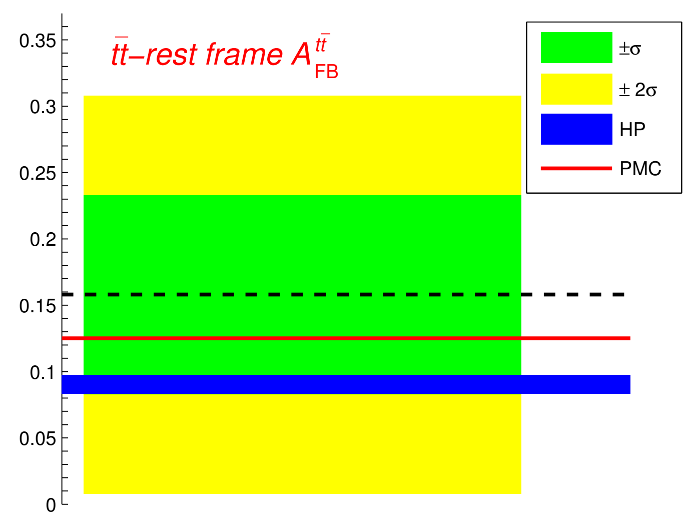
$$A_{FB}^{t\bar{t}} = \frac{\sigma(y_t^{t\bar{t}} > 0) - \sigma(y_t^{t\bar{t}} < 0)}{\sigma(y_t^{t\bar{t}} > 0) + \sigma(y_t^{t\bar{t}} < 0)}$$

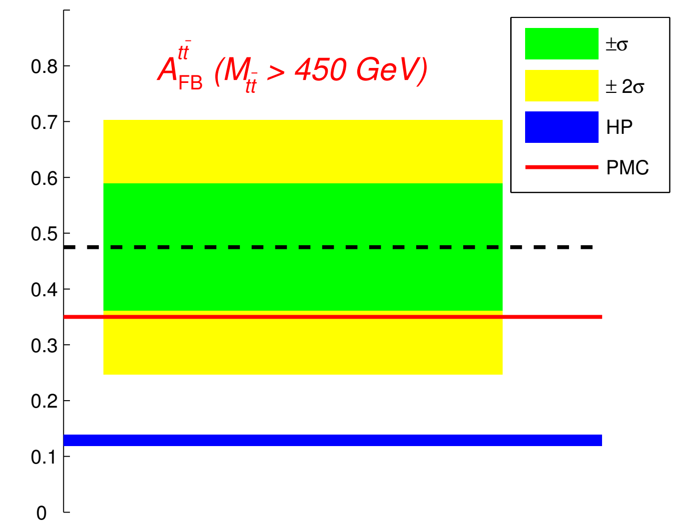


$$\mu_r \neq \mu_f \ (!)$$









Conventional Scale Setting: $\alpha(\mu) = \alpha_{\overline{MS}}(\mu)$ and $\mu = [\frac{1}{2}Q, 2Q]$

HP: Hollik, Pagani, **Phys.Rev. D84**(2011)

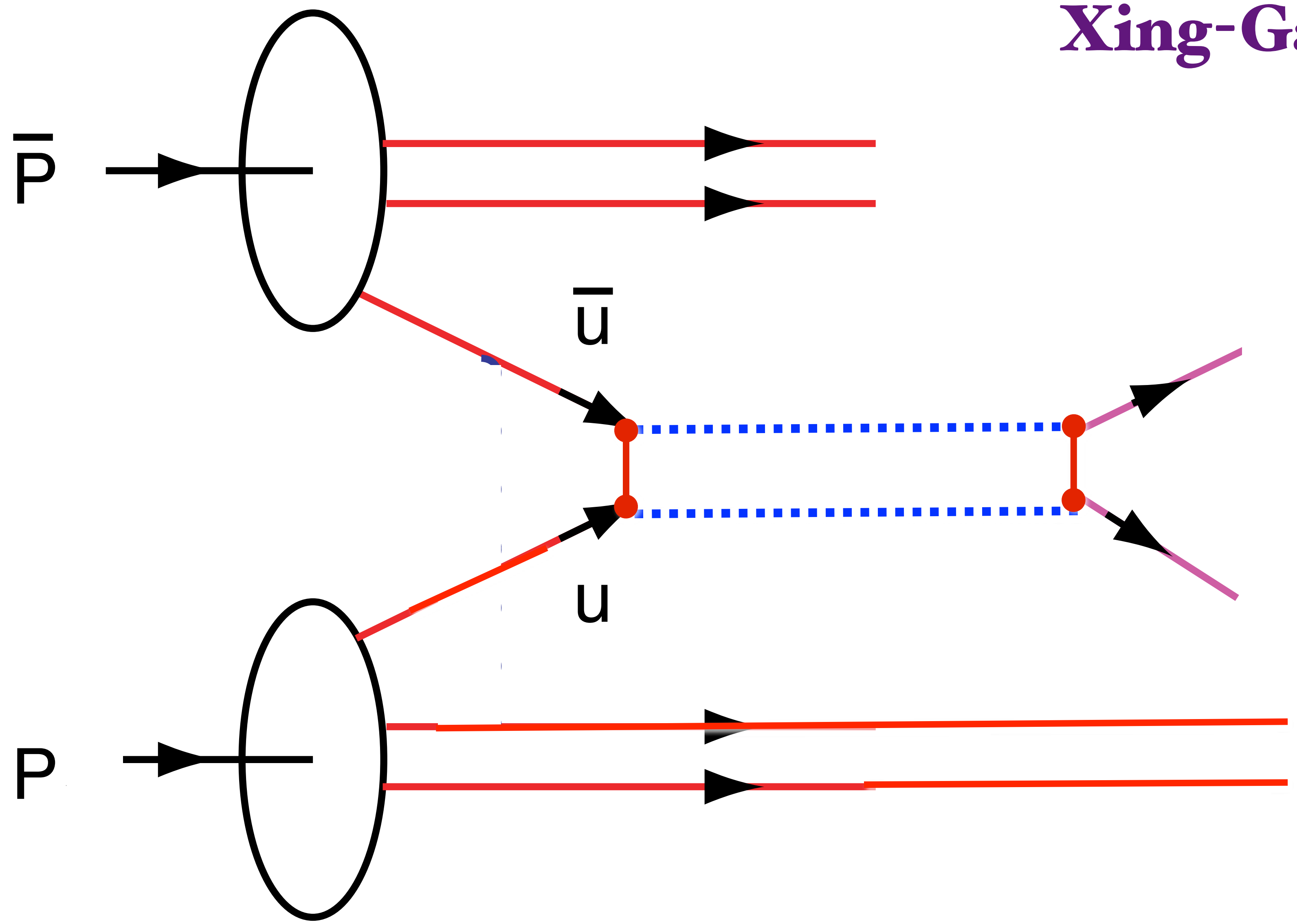
Conventional ‘uncertainty estimate’ can be misleading

(see also Blumlein & van Neerven, Phys.Lett. B450, 417[1999])

Improving pQCD precision important for exposing new physics correctly!

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron

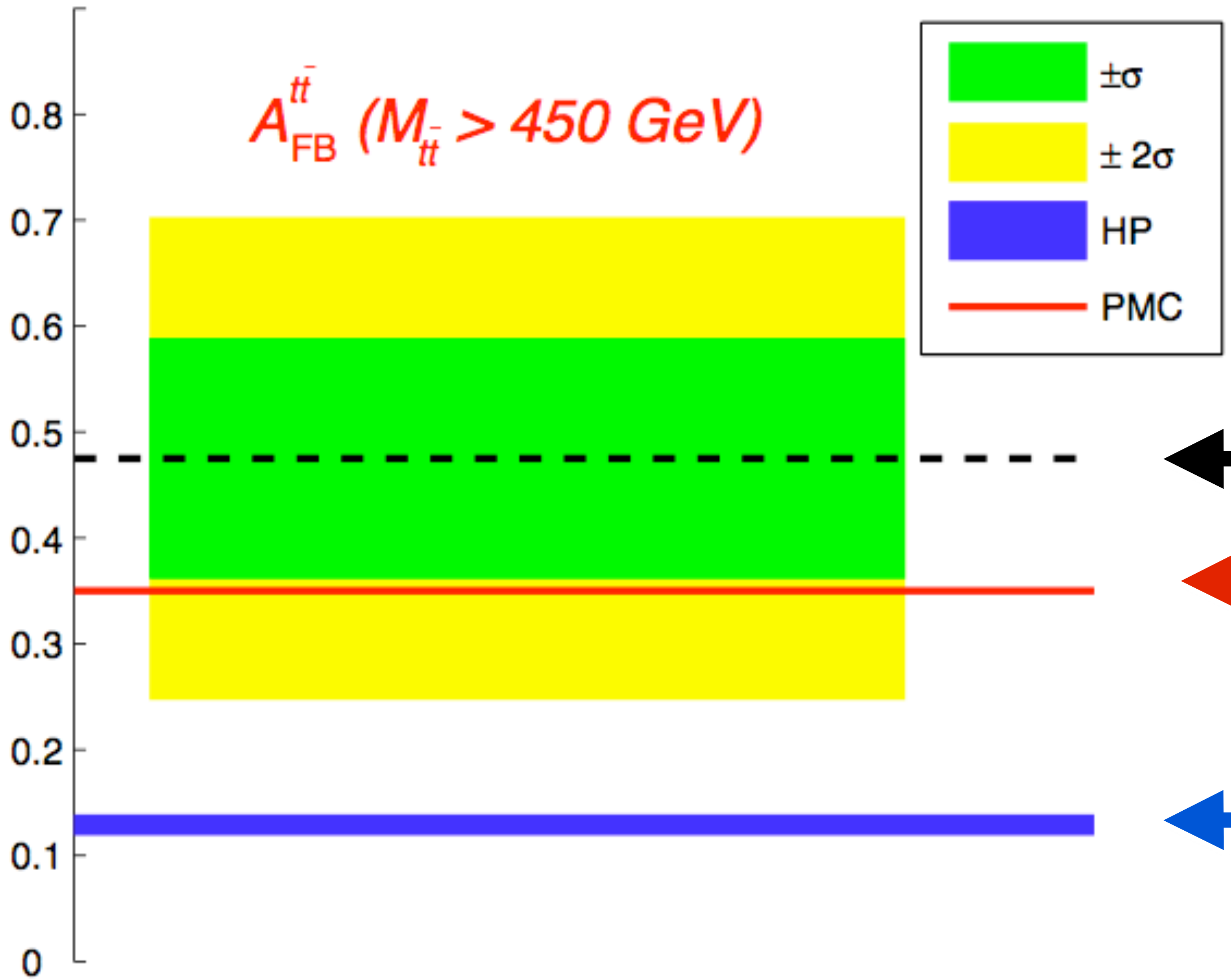
Xing-Gang Wu, sjb



Interferes with Born term.

Small value of renormalization scale increases asymmetry, just as in QED

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

Experimental asymmetry

PMC Prediction

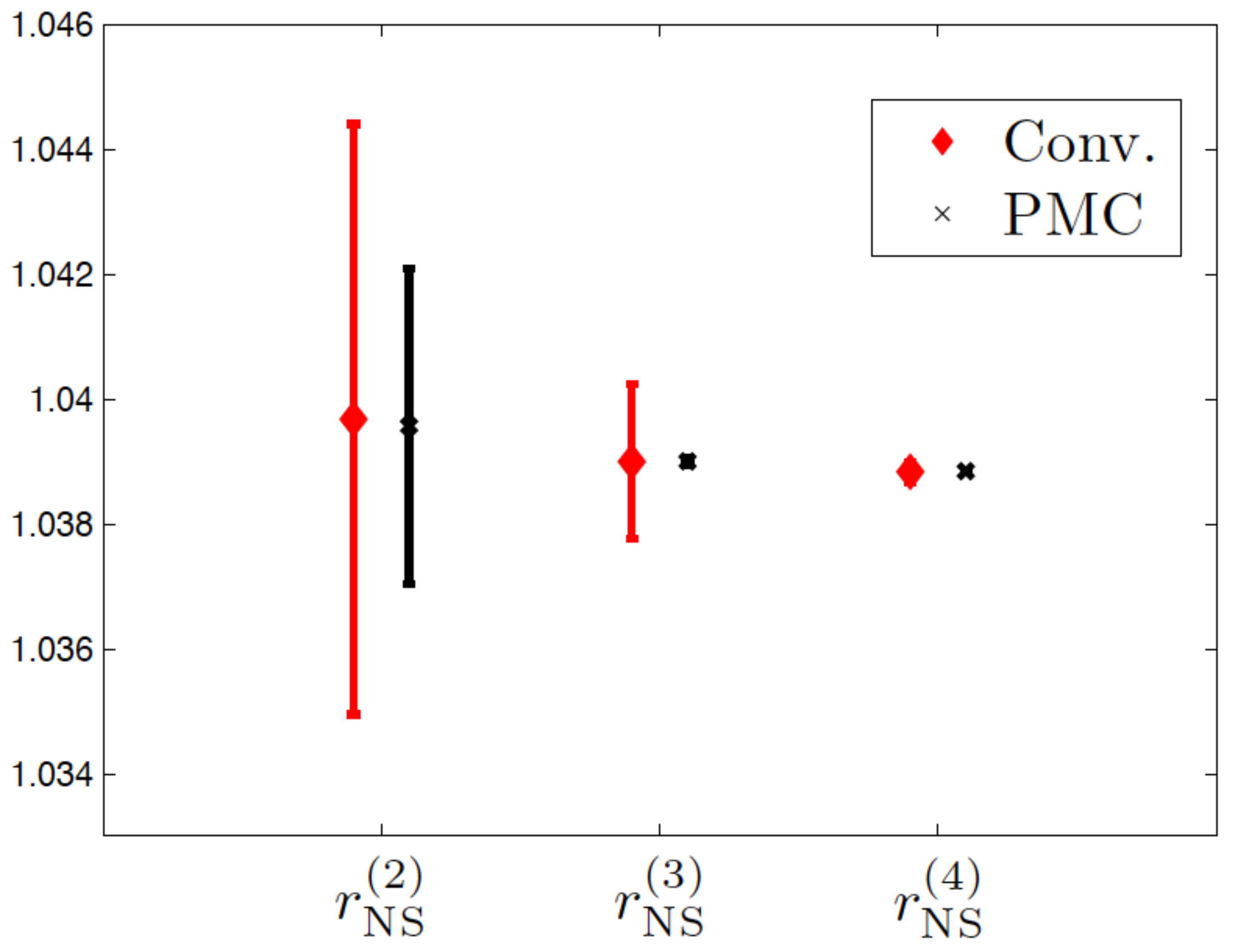
Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Reanalysis of the Higher Order Perturbative QCD corrections to Hadronic Z Decays using the Principle of Maximum Conformality

S-Q Wang, X-G Wu, sjb

P.A. Baikov, K.G. Chetyrkin, J.H. Kuhn, and J. Rittinger, Phys. Rev. Lett. 108, 222003 (2012).



The values of $r_{\mathrm{NS}}^{(n)} = 1 + \sum_{i=1}^{n} C_i^{\mathrm{NS}} a_s^i$ and their errors $\pm|C_n^{\mathrm{NS}} a_s^n|_{\mathrm{MAX}}$. The diamonds and the crosses are for conventional (Conv.) and PMC scale settings, respectively. The central values assume the initial scale choice $\mu_r^{\mathrm{init}} = M_Z$.

What is PMC ?

Principle of Maximum Conformality

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

↓

Choose $\mu_R^{\rm init}$; arbitrary initial renormalization scale

↓

Identify $\{\beta_i^R\}$ − terms using n_f − terms

through the PMC − BLM correspondence principle

↓ order-by-order

Shift scale of α_s to $\mu_R^{\rm PMC}$ to eliminate $\{\beta_i^R\}$ − terms

↓

Conformal Series

Result is independent of $\mu_R^{\rm init}$ and scheme at fixed order

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

PMC–BLM – one

Phys. Rev. Lett. 109, 042002 (2012)

R_δ–scheme – two

Phys. Rev. Lett. 110, 192001 (2013)

Eliminate β–terms

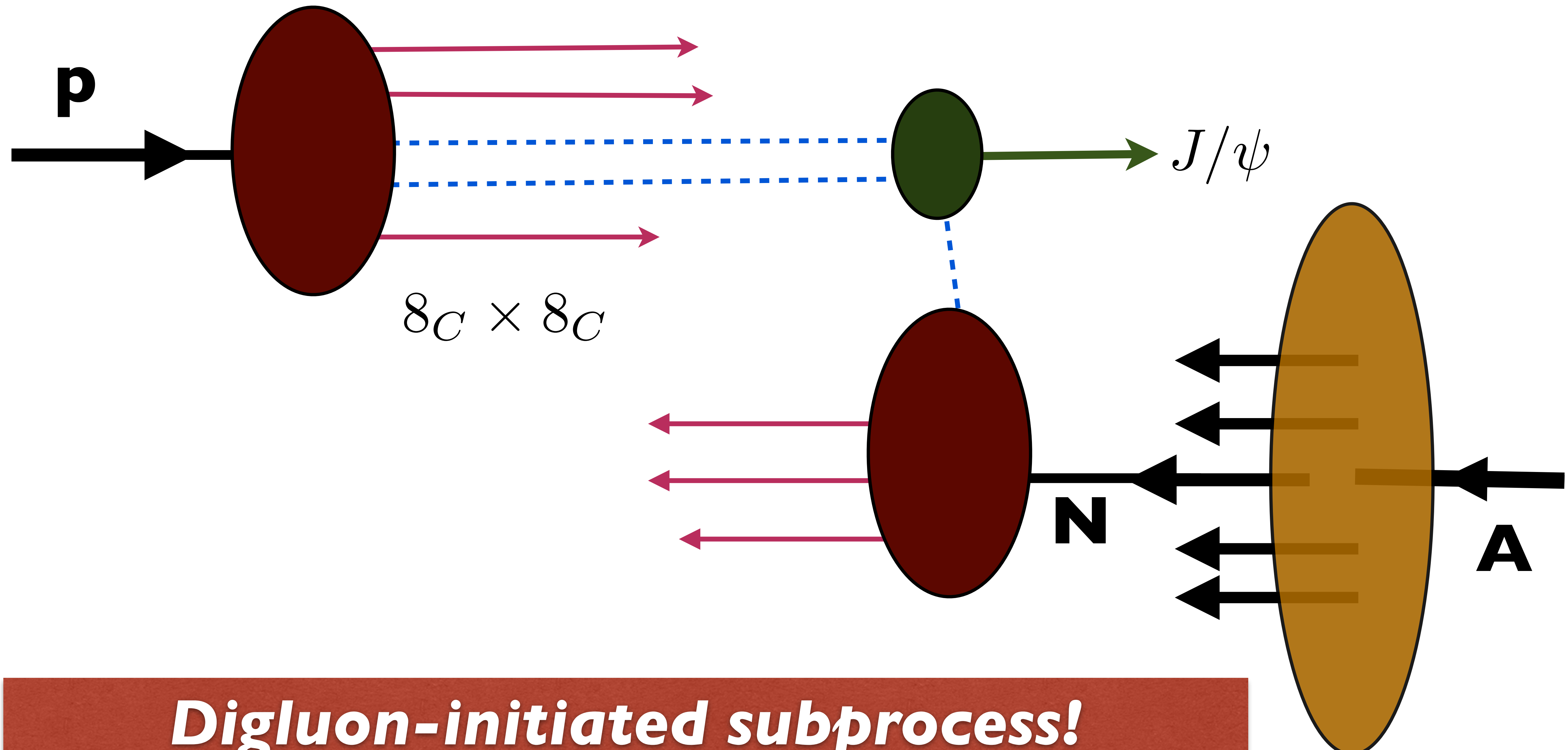
n_f dependence of pQCD series does not uniquely identify the β terms

Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (*Kataev, Lu, Rathsman, sjb*)***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Eliminates unnecessary theory error***

$$pA \to J/\psi X$$

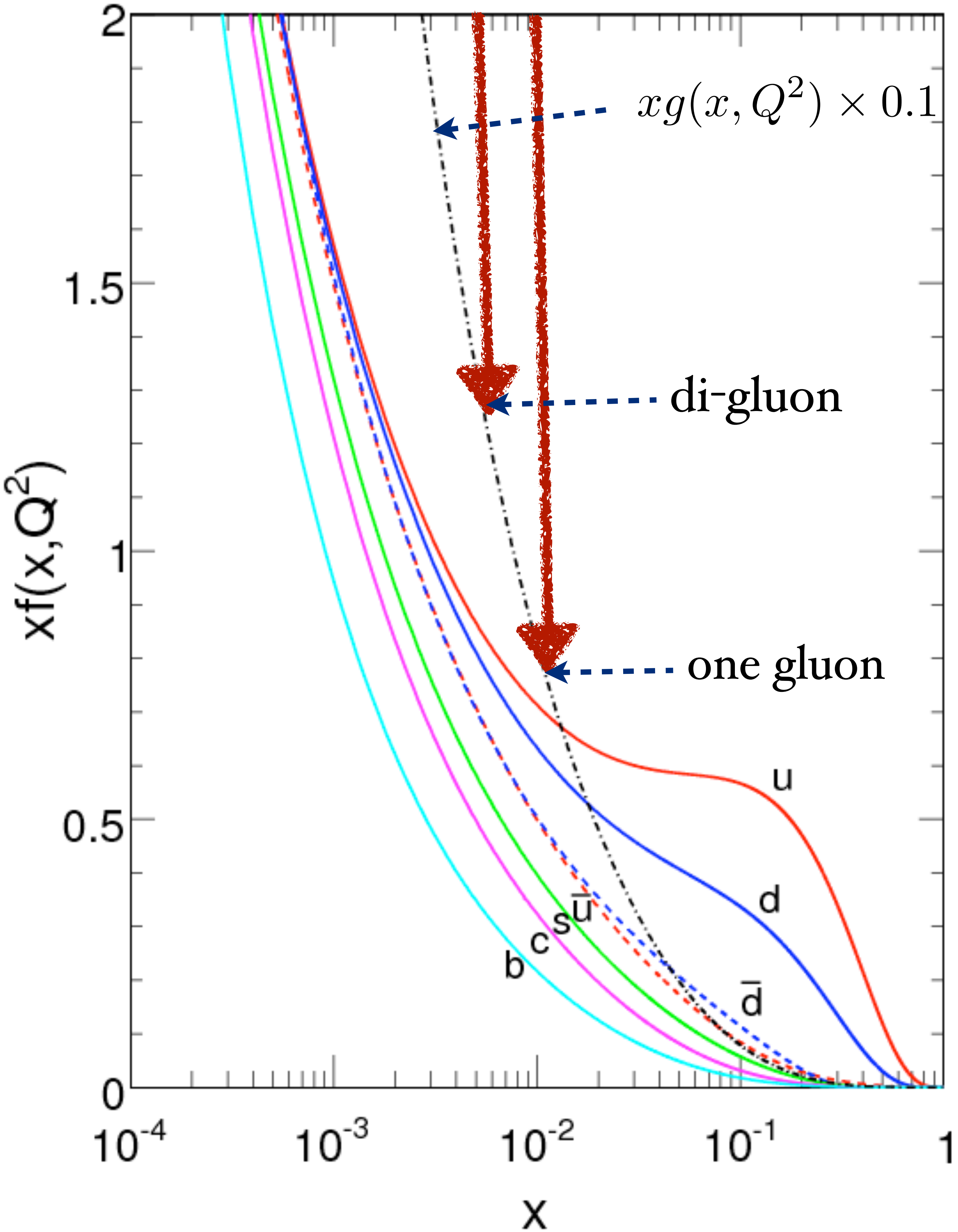
$$(gg)_{8_C} + g_{8_C} \to J/\psi$$



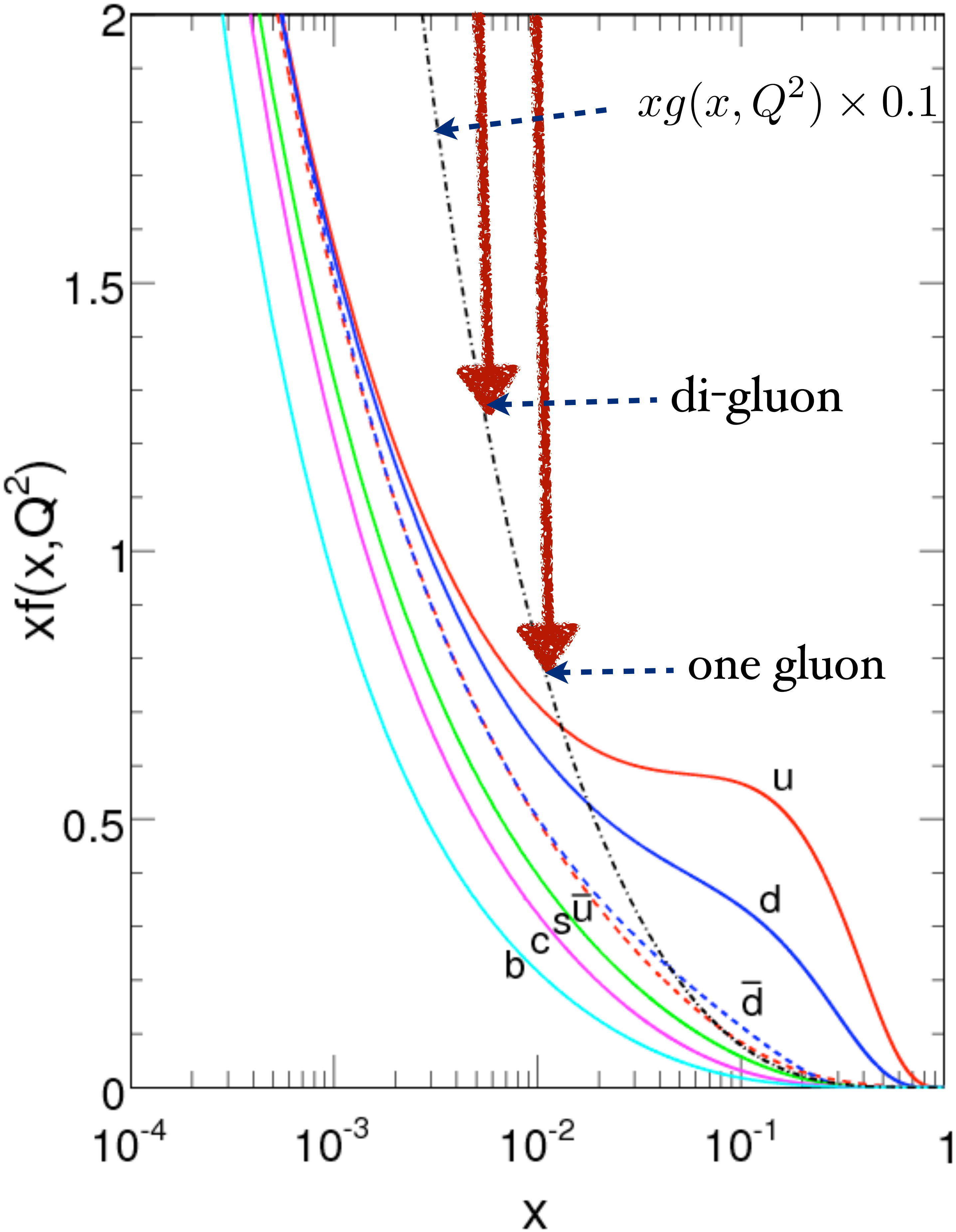
Digluon-initiated subprocess!

Higher-Twist but can dominate at forward rapidity, small p_T

Two gluons at $g(0.005) \sim \frac{13}{0.005} = 2600$ vs. one gluon at $g(0.01) \sim \frac{8}{0.01} = 800$



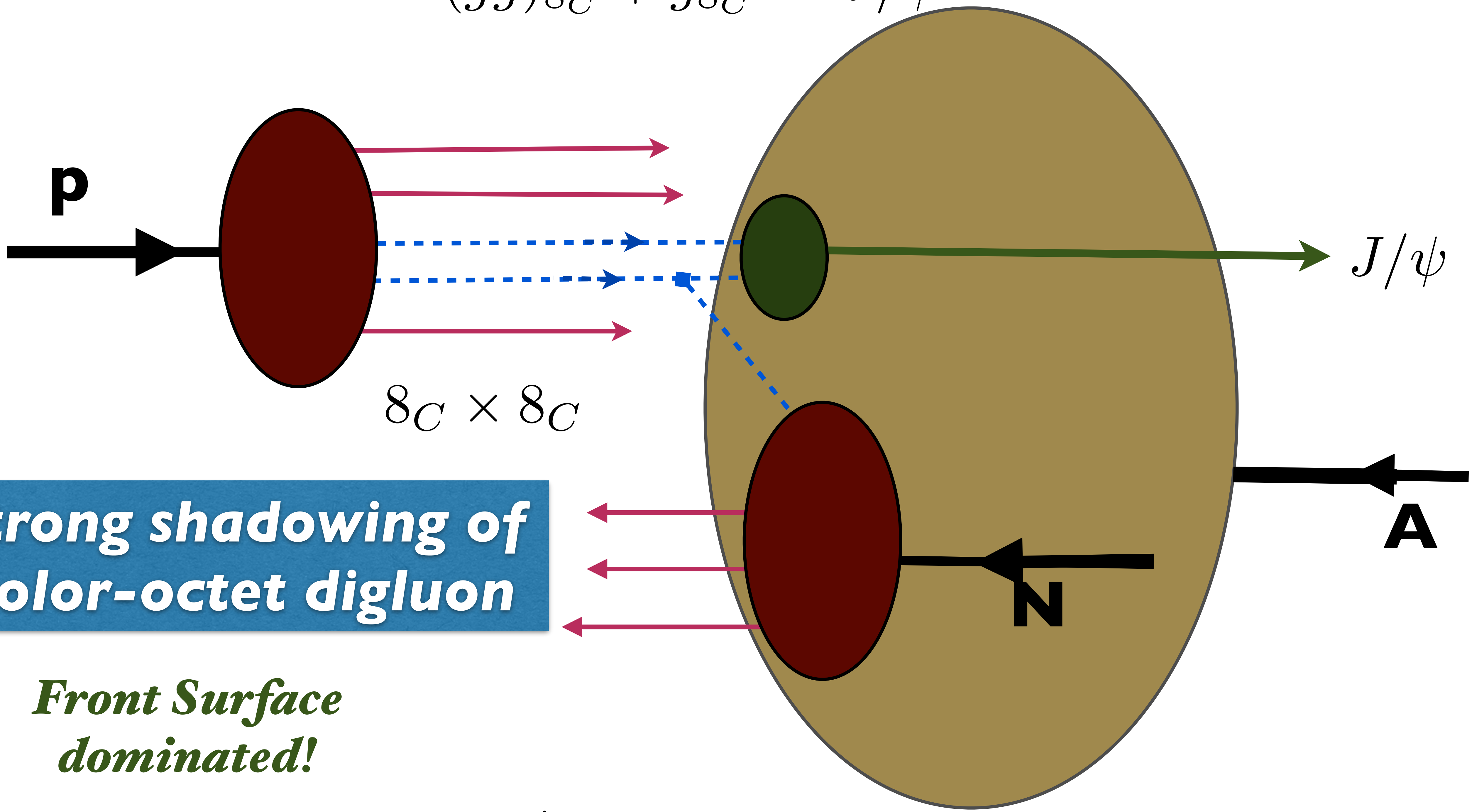
Two gluons at $g(0.005) \sim \frac{13}{0.005} = 2600$ vs. one gluon at $g(0.01) \sim \frac{8}{0.01} = 800$



Forward rapidity y ~4

$$pA \to J/\psi X$$

$$(gg)_{8_C} + g_{8_C} \to J/\psi$$



Strong shadowing of color-octet digluon

Front Surface dominated!

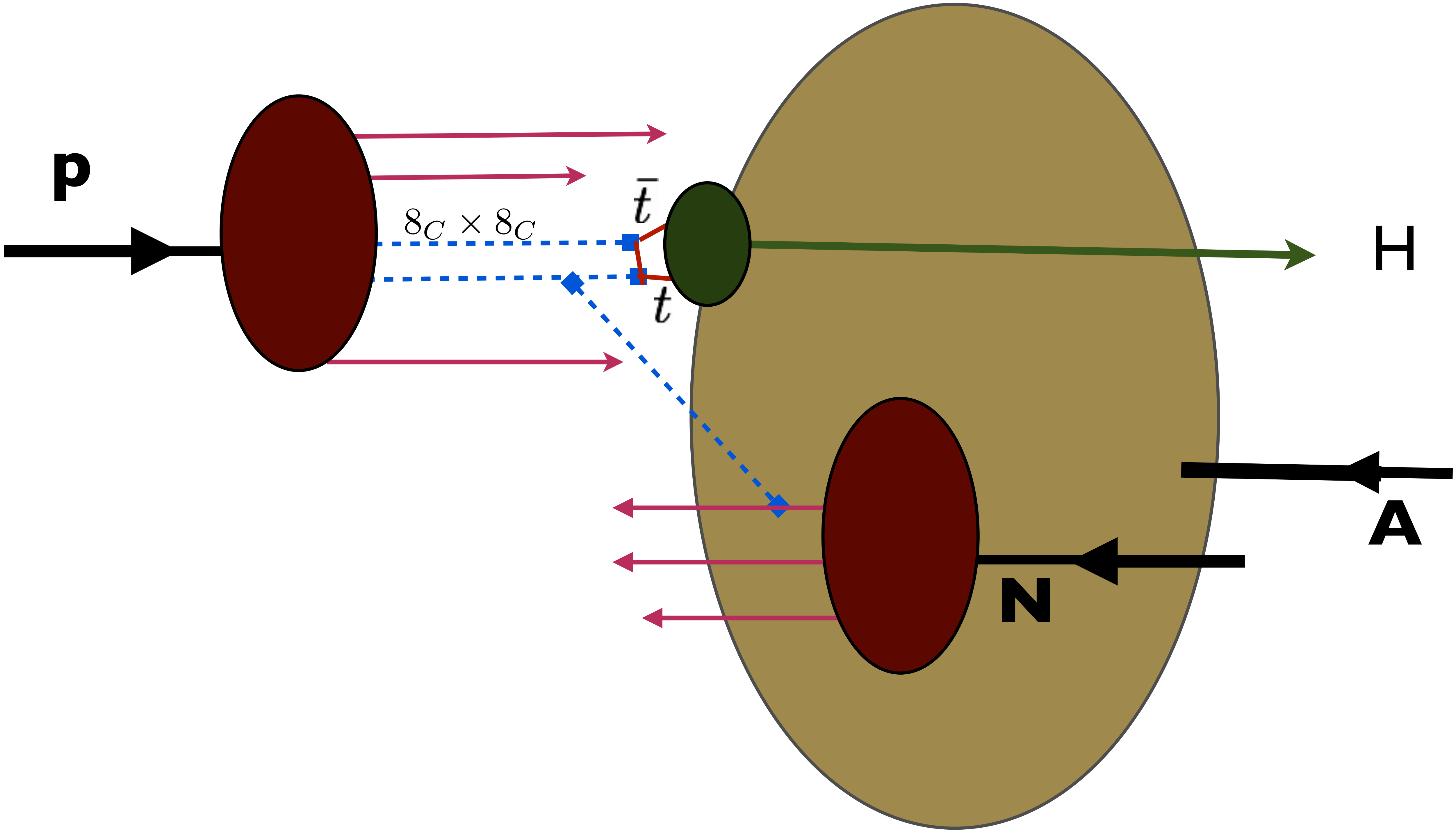
ψ' suppressed as it propagates through the nucleus

Crossing: Diffractive & pomeron exchange

Digluon-initiated subprocess!

$$pA \to HX$$

$$(gg)_{8_C} + g_{8_C} \to H$$



Double-gluon subprocess for Higgs production at forward rapidity

Angular Momentum on the Light-Front

LC gauge A⁺=0

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

pQED: Ma, Hwang, Schmidt, sjb

Single-spin asymmetries

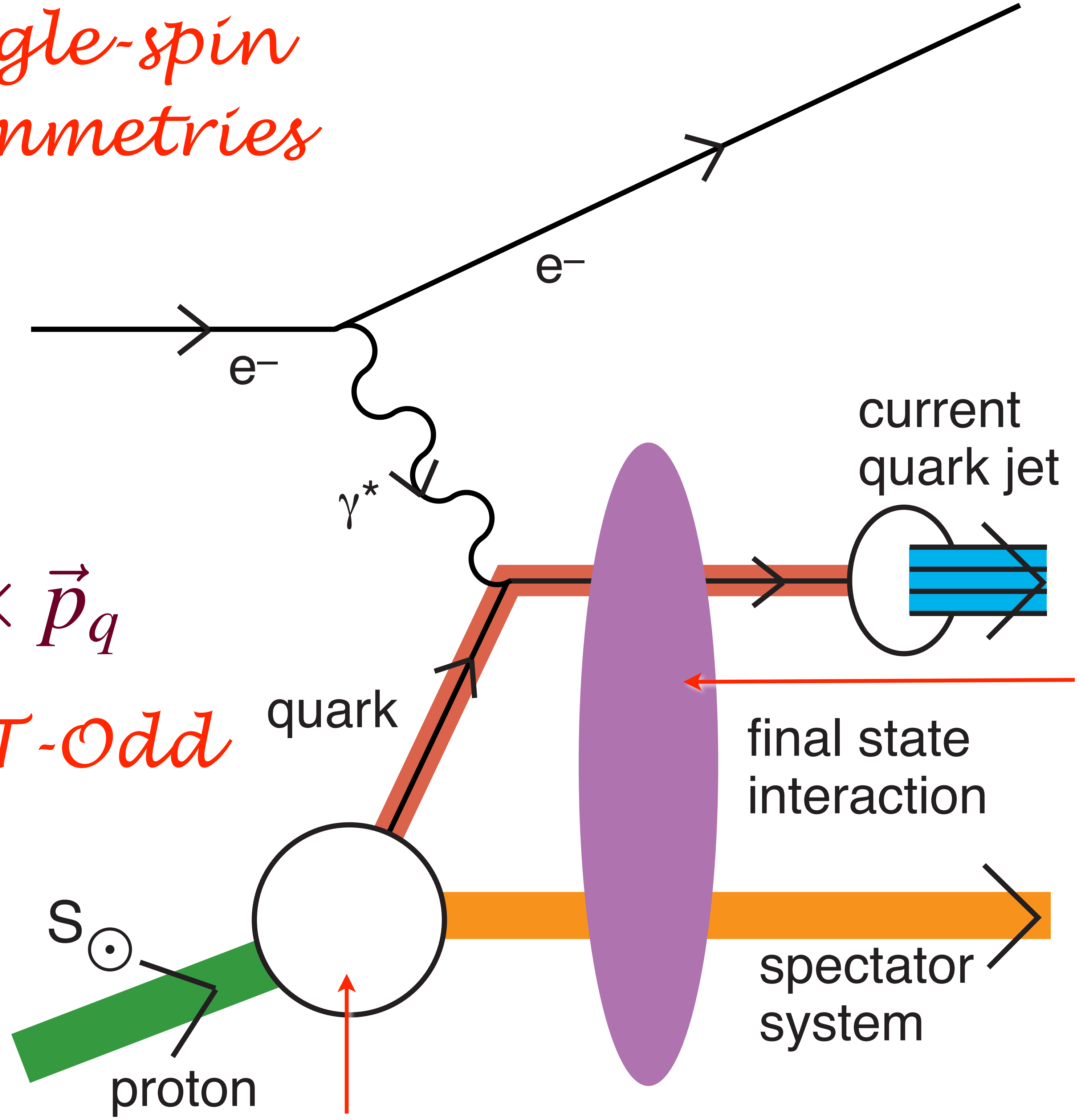
Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



Light-Front Wavefunction S and P- Waves!

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

"Lensing" involves soft scales

Sign reversal in DY!

Leading-Twist Rescattering Violates pQCD Factorization!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

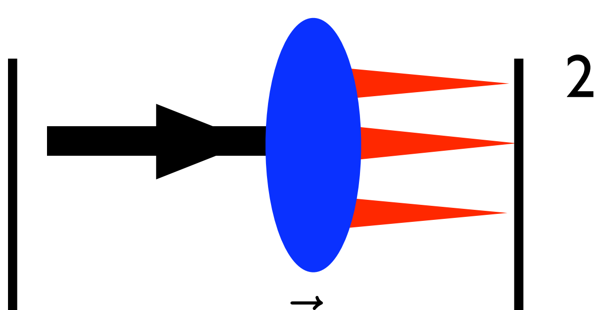
Hwang, Schmidt, sjb,

Mulders, Boer

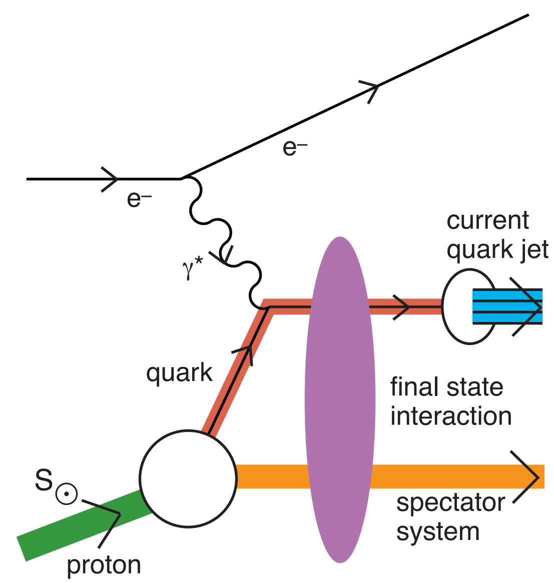
Qiu, Sterman

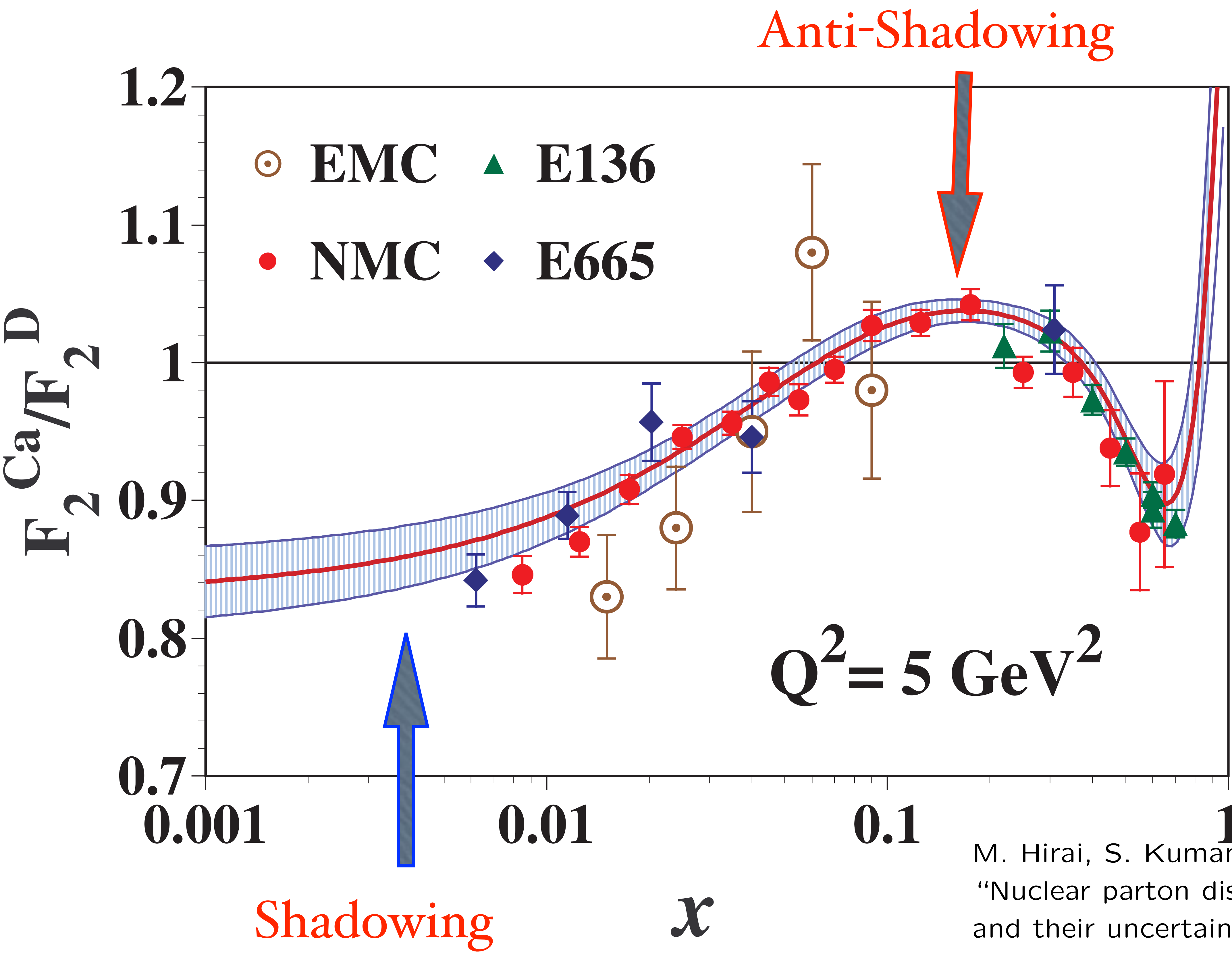
Collins, Qiu

Pasquini, Xiao, Yuan, sjb

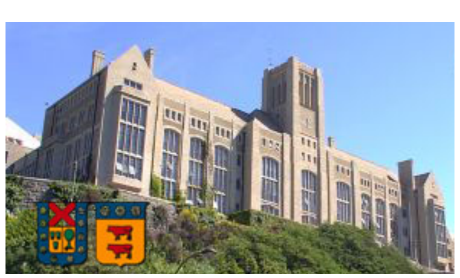


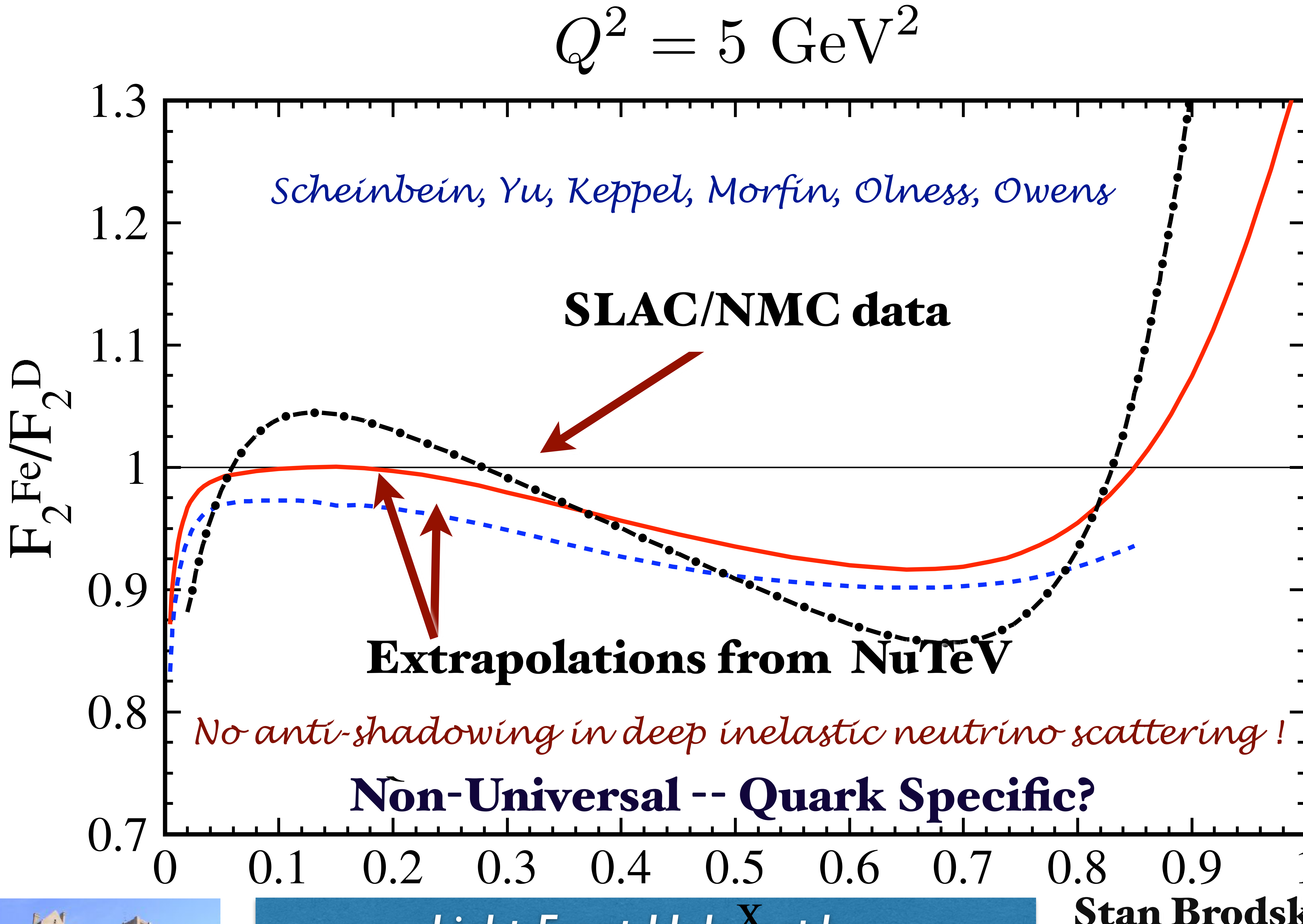
$$\left|\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$

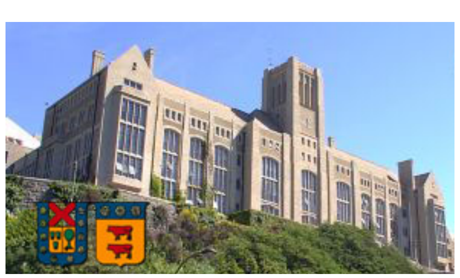




M. Hirai, S. Kumano and T. H. Nagai, "Nuclear parton distribution functions and their uncertainties," Phys. Rev. C **70**, 044905 (2004) [arXiv:hep-ph/0404093].

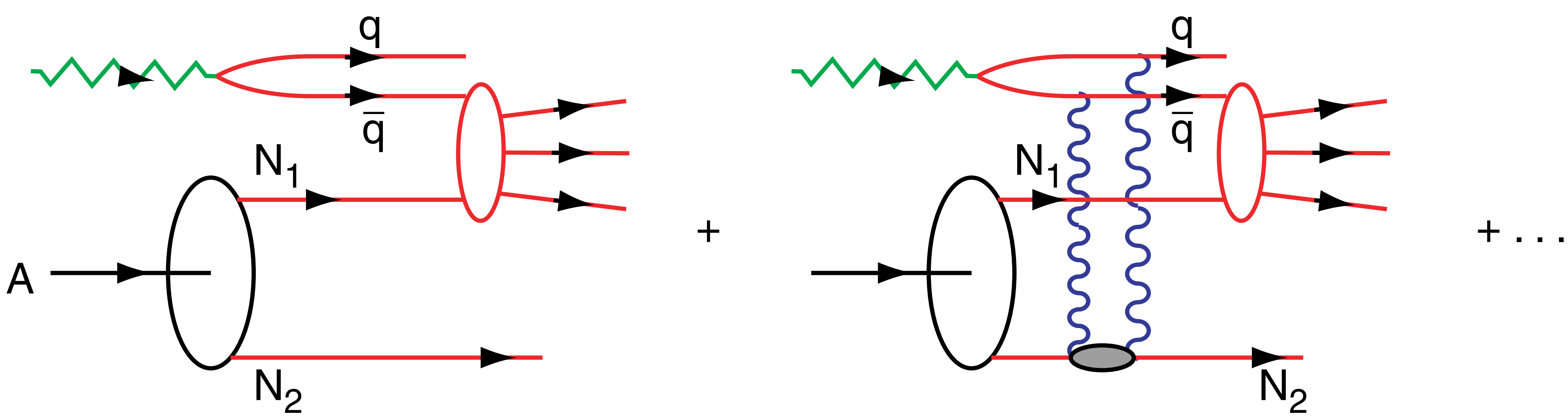



$Q^2 = 5\ \mathrm{GeV}^2$
1.3
1.2
1.1
1
0.9
0.8
0.7
F_2^{Fe}/F_2^{D}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
Scheinbein, Yu, Keppel, Morfin, Olness, Owens
SLAC/NMC data
Extrapolations from NuTeV
No anti-shadowing in deep inelastic neutrino scattering !
Non-Universal -- Quark Specific?



Stodolsky
Pumplin, sjb
Gribov

Nuclear Shadowing in QCD



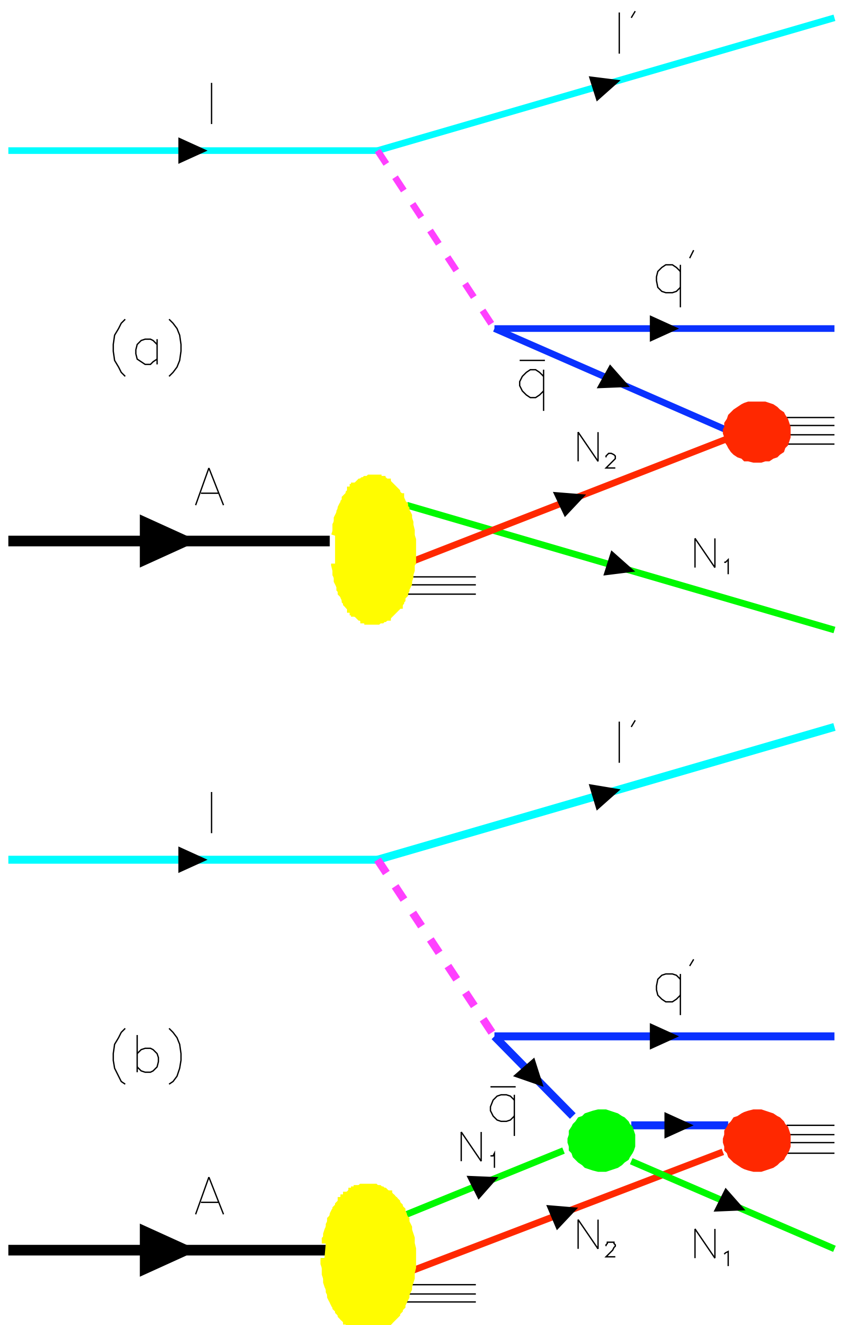
Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF !

Dynamical effect due to virtual photon interacting in nucleus

Diffraction via Reggeon gives constructive interference!

Anti-shadowing not universal



The one-step and two-step processes in DIS on a nucleus.

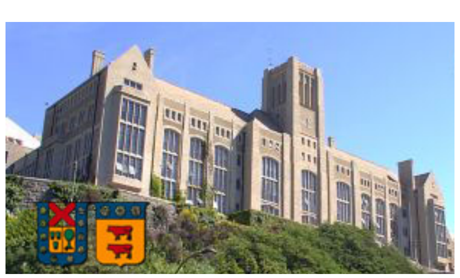
Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

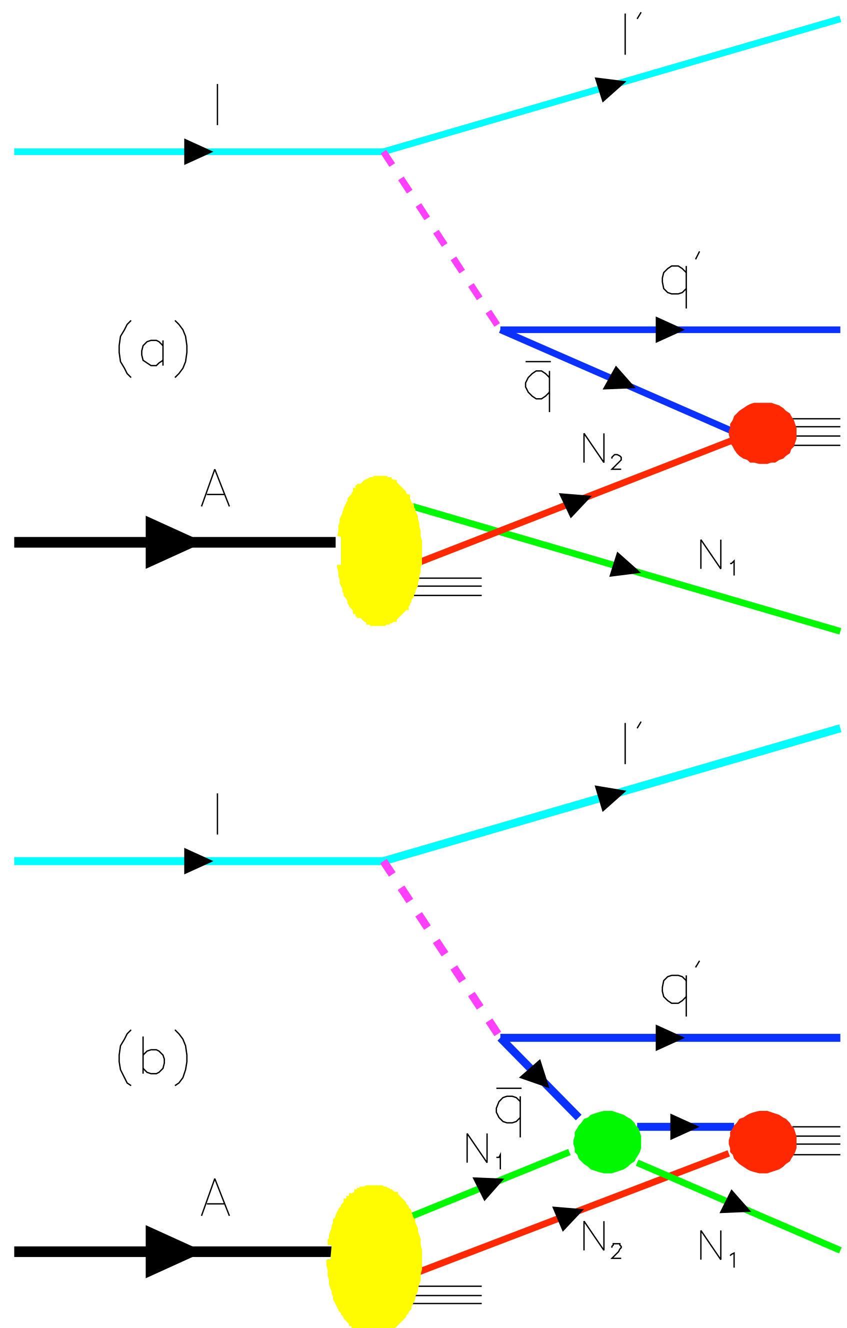
If the scattering on nucleon N_1 is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.

$\rightarrow$ Shadowing of the DIS nuclear structure functions.

Diffraction via Pomeron gives destructive interference!

Shadowing





The one-step and two-step processes in DIS on a nucleus.

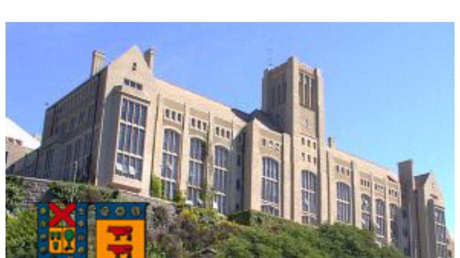
Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

Regge

If the scattering on nucleon N_1 is via ~~pomeron~~ exchange, the one-step and two-step amplitudes are ~~opposite in phase, thus diminishing the $\bar{q}$ flux reaching N_2.~~

constructive in phase
thus increasing the flux reaching N_2

Reggeon DDIS produces nuclear flavor-dependent anti-shadowing



Light-Front Holography and Supersymmetric Features of QCD

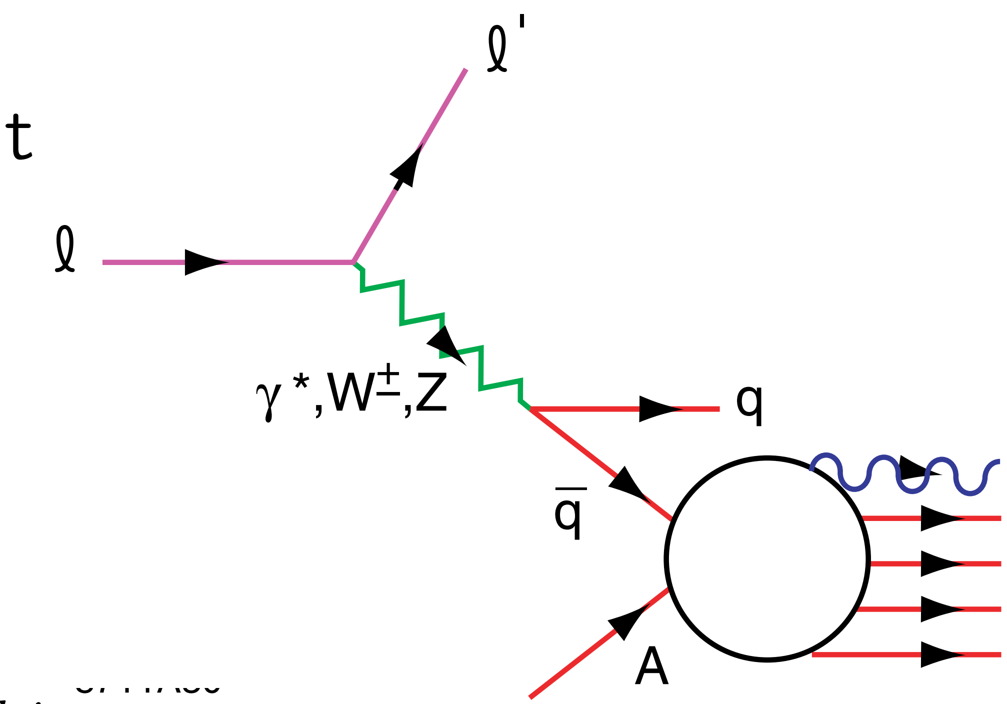
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

UTSM Jan. 7, 2016

$$F_{2p}(x) - F_{2n}(x) \propto x^{1/2}$$

Antiquark interacts with target nucleus at energy $\widehat{s} \propto \frac{1}{x_{bj}}$



Regge contribution: $\sigma_{\bar{q}N} \sim \widehat{s}^{\alpha_R - 1}$

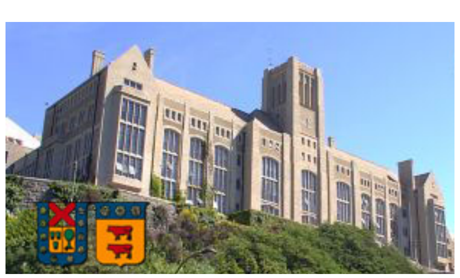
Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x}_{bj}$ at small x_{bj}.

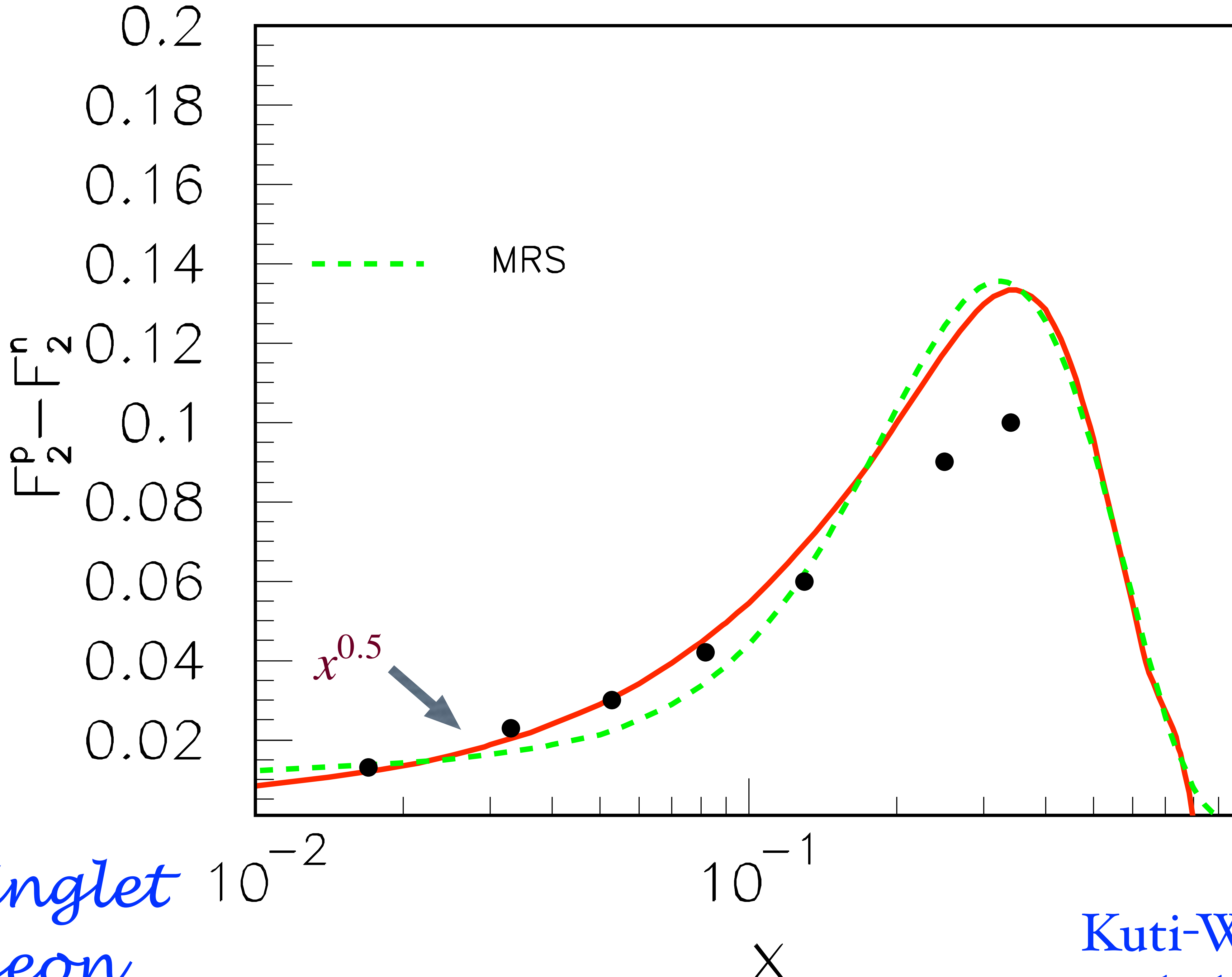
Landshoff, Polkinghorne, Short

Close, Gunion, sjb

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.

Schmidt, Yang, Lu, sjb





Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

Light-Front Holography and Supersymmetric Features of QCD

Stan Brodsky
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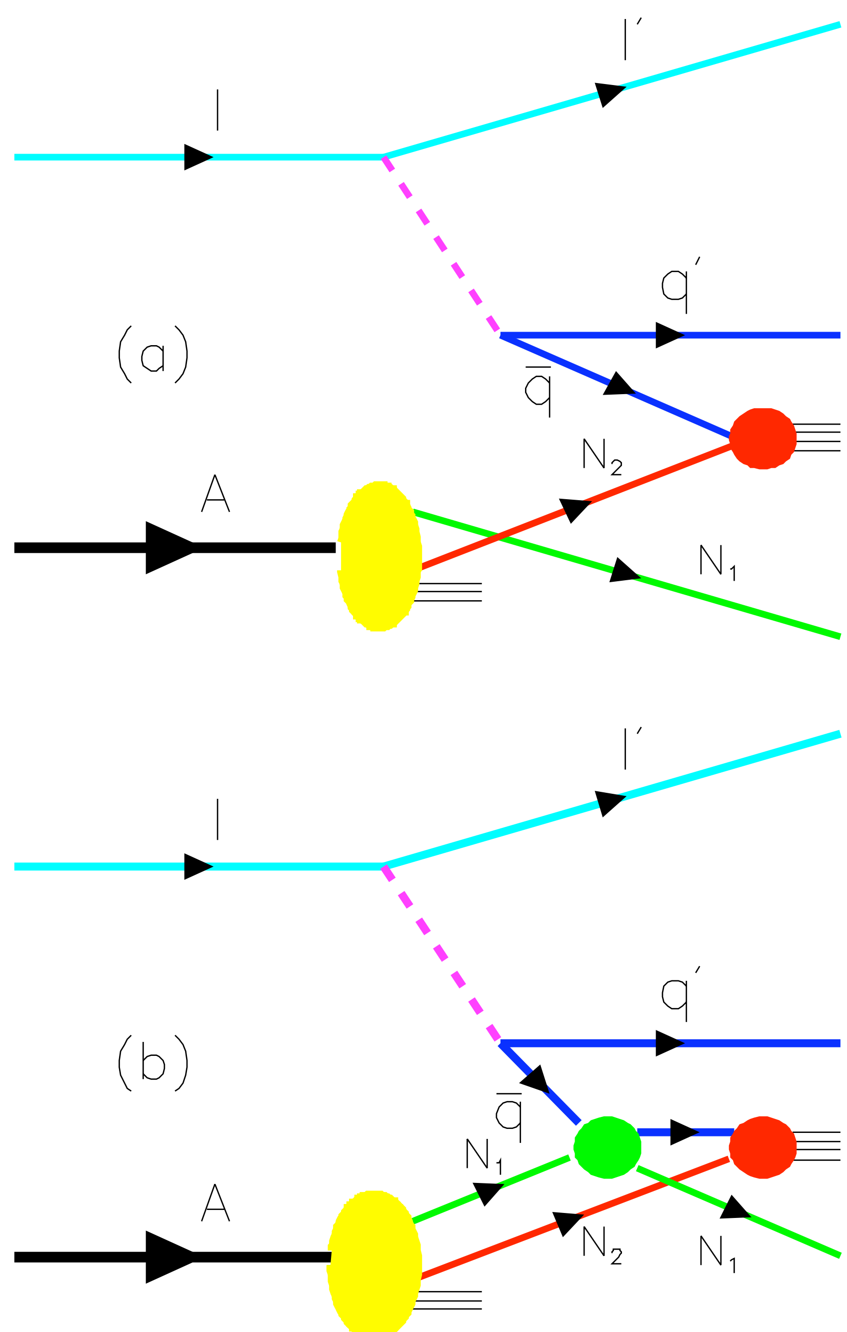


The one-step and two-step processes in DIS on a nucleus.

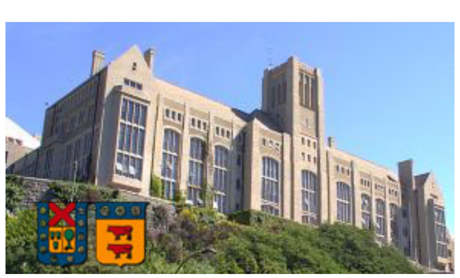
Coherence at small Bjorken x_B :
$1/Mx_B = 2\nu/Q^2 \geq L_A$.

Reggeon

If the scattering on nucleon N_1 is via ~~pomeron~~ exchange, the one-step and two-step amplitudes are ~~opposite~~ in phase, thus diminishing the $\overline{q}$ flux reaching N_2.

Diffraction via Reggeon gives constructive interference!

Anti-shadowing



Reggeon Exchange

Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1-i)\times i=\frac{1}{\sqrt{2}}(i+1)$$

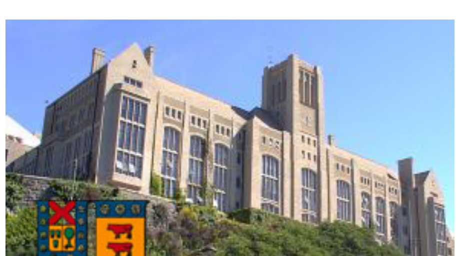
Constructive Interference

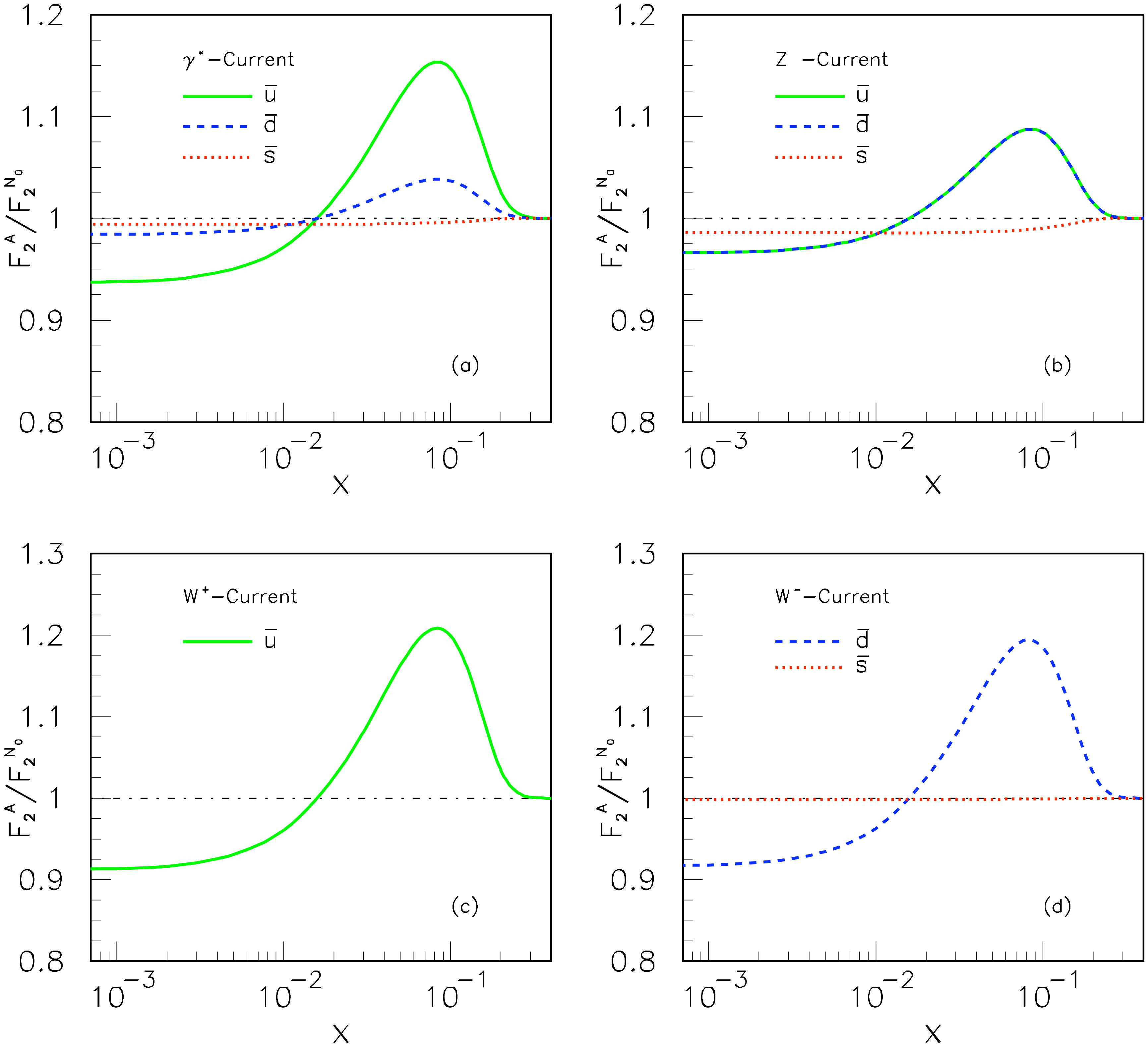
Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

Critical test: Tagged Drell-Yan





Schmidt, Yang; sjb

Modifies NuTeV extraction of

$$\sin^2 \theta_W$$

Test in flavor-tagged DIS at the EIC

Nuclear Antishadowing not universal !

Light-Front Holography and Supersymmetric Features of QCD

...dsky

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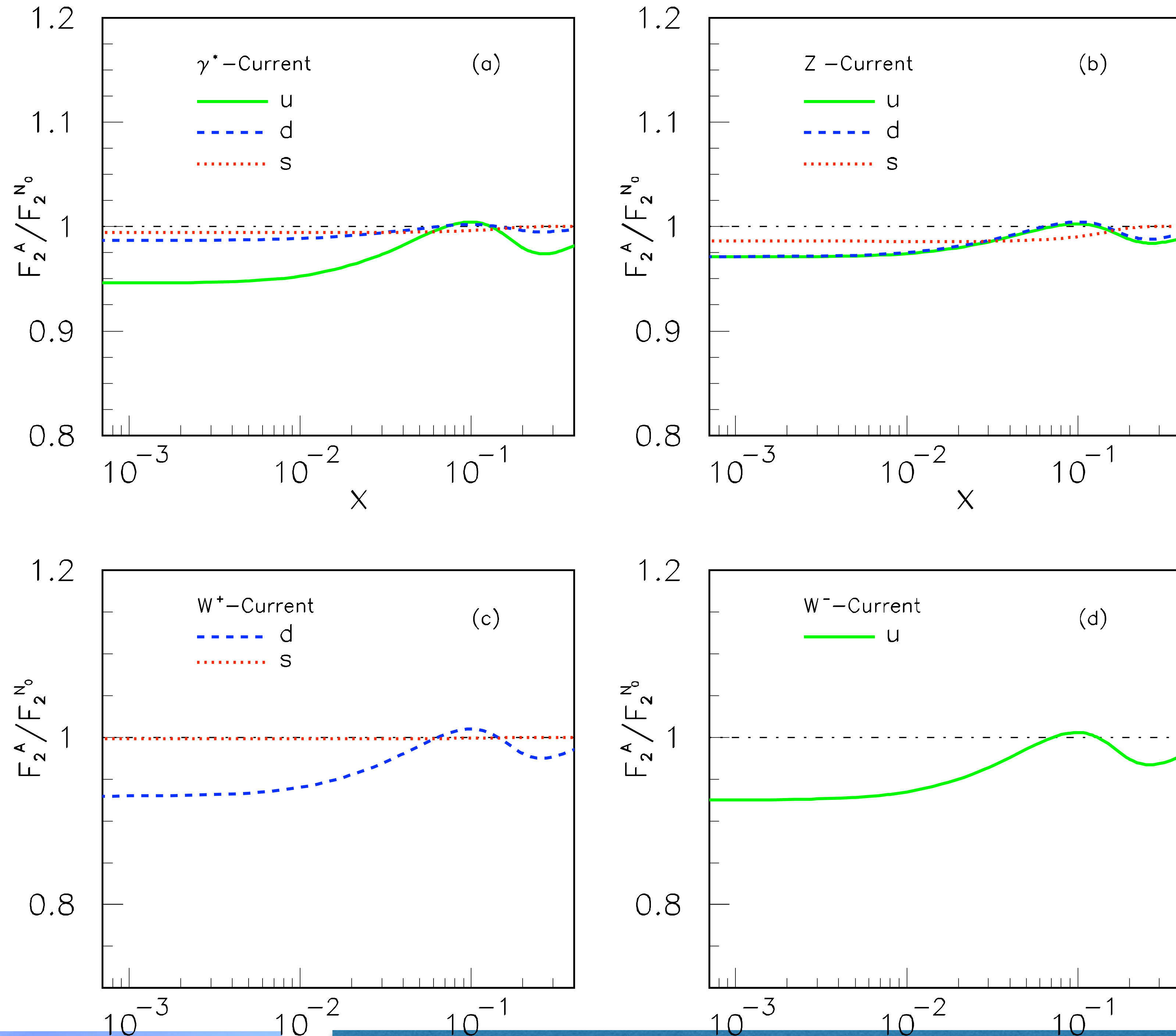
Shadowing and Antishadowing of DIS Structure Functions



S. J. Brodsky, I. Schmidt and J. J. Yang, "Nuclear Antishadowing in Neutrino Deep Inelastic Scattering," Phys. Rev. D 70, 116003 (2004) [arXiv:hep-ph/0409279].

Modifies NuTeV extraction of

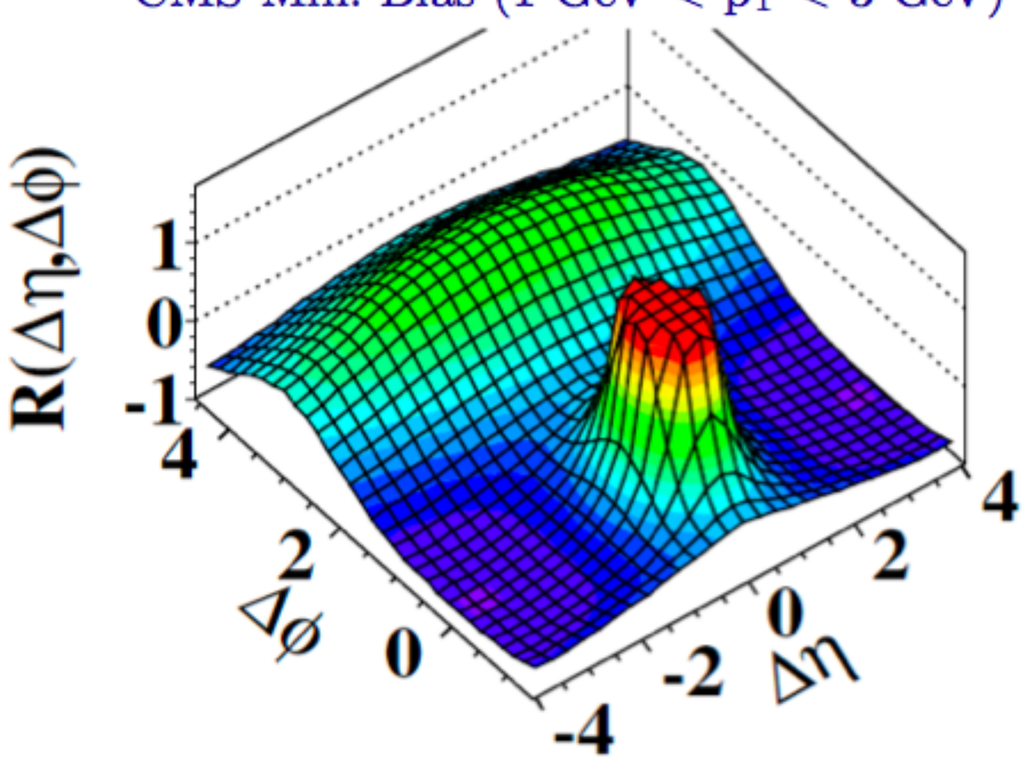
$$\sin^2\theta_W$$

Test in flavor-tagged lepton-nucleus collisions

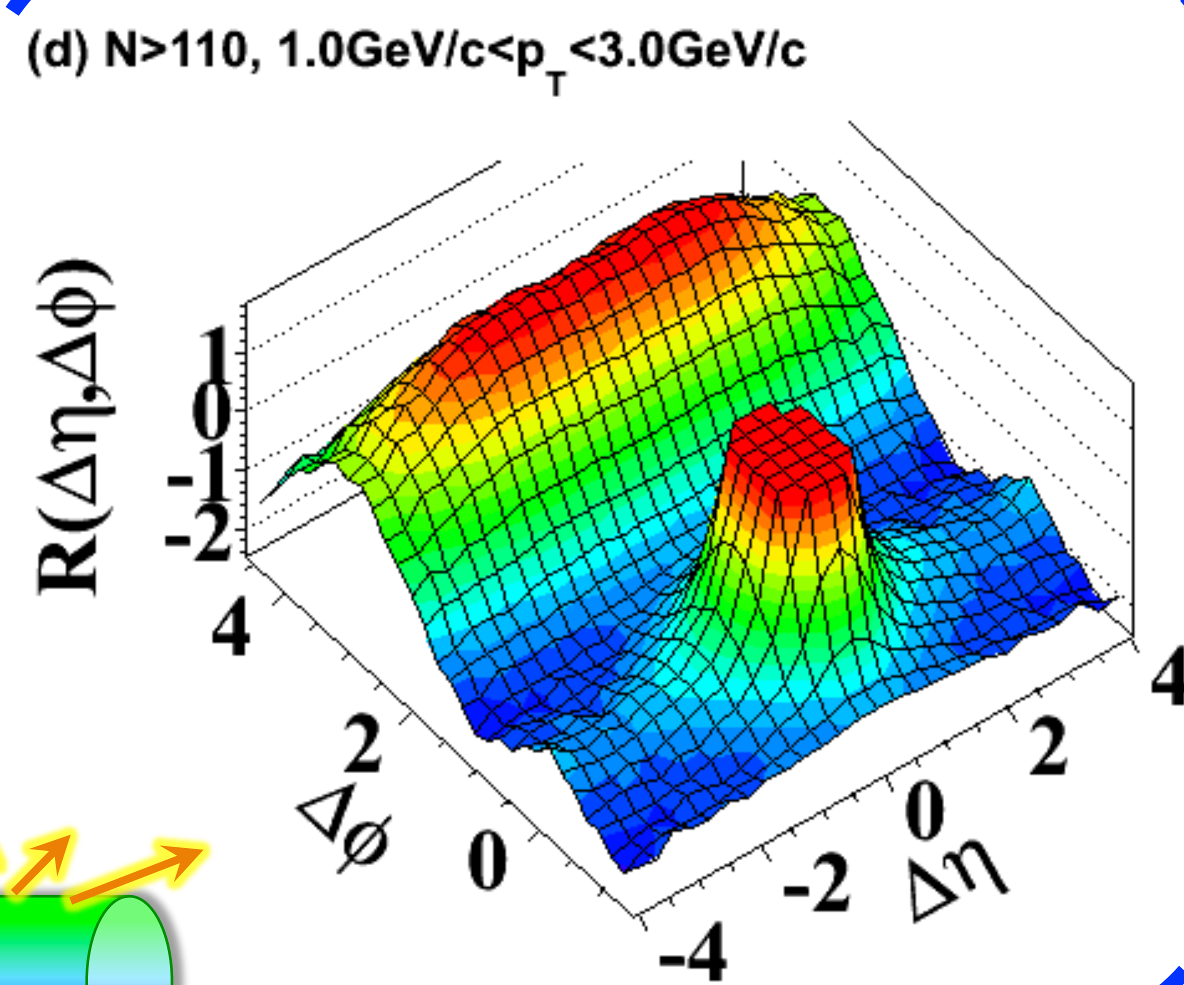
Light-Front Holography and Supersymmetric Features of QCD

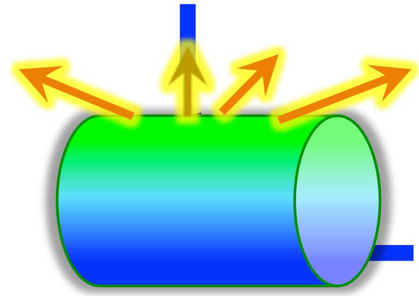
Ridge in high-multiplicity p p collisions

Two-particle correlations: CMS results



"Discovery"

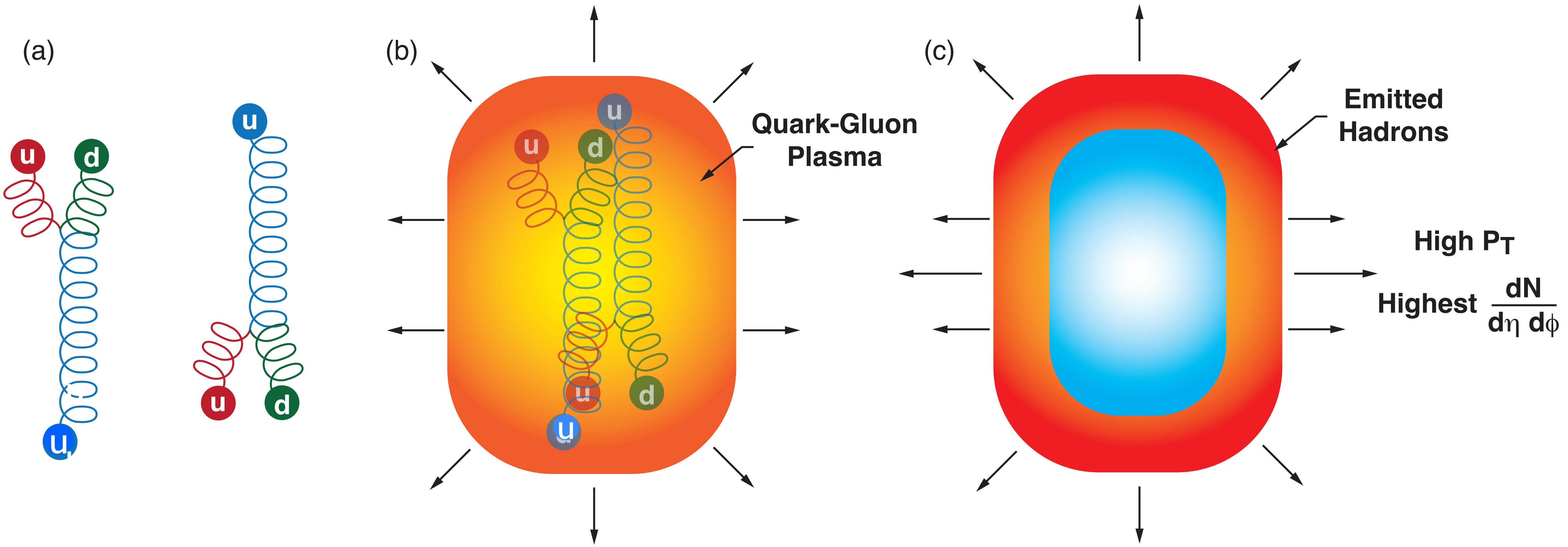




- **Ridge: Distinct long range correlation in η collimated around ΔΦ≈ 0 for two hadrons in the intermediate 1 < p_T, q_T < 3 GeV**

Raju Venugopalan

Ridge may reflect collision of aligned flux tubes



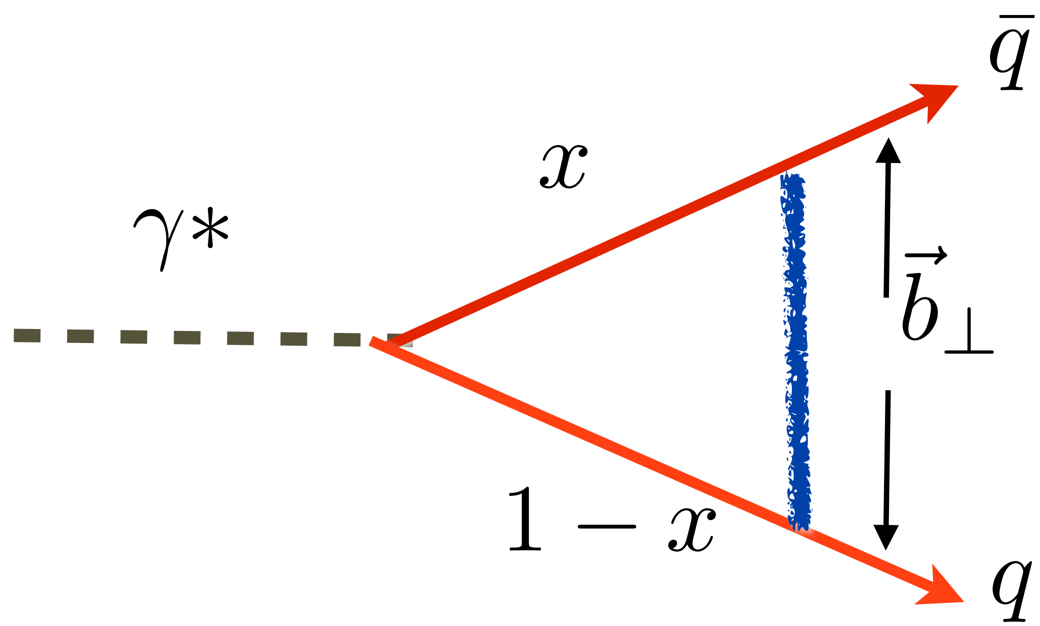
Bjorken, Goldhaber, sjb

Two-Dimensional Confinement

Interesting feature from AdS/QCD

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

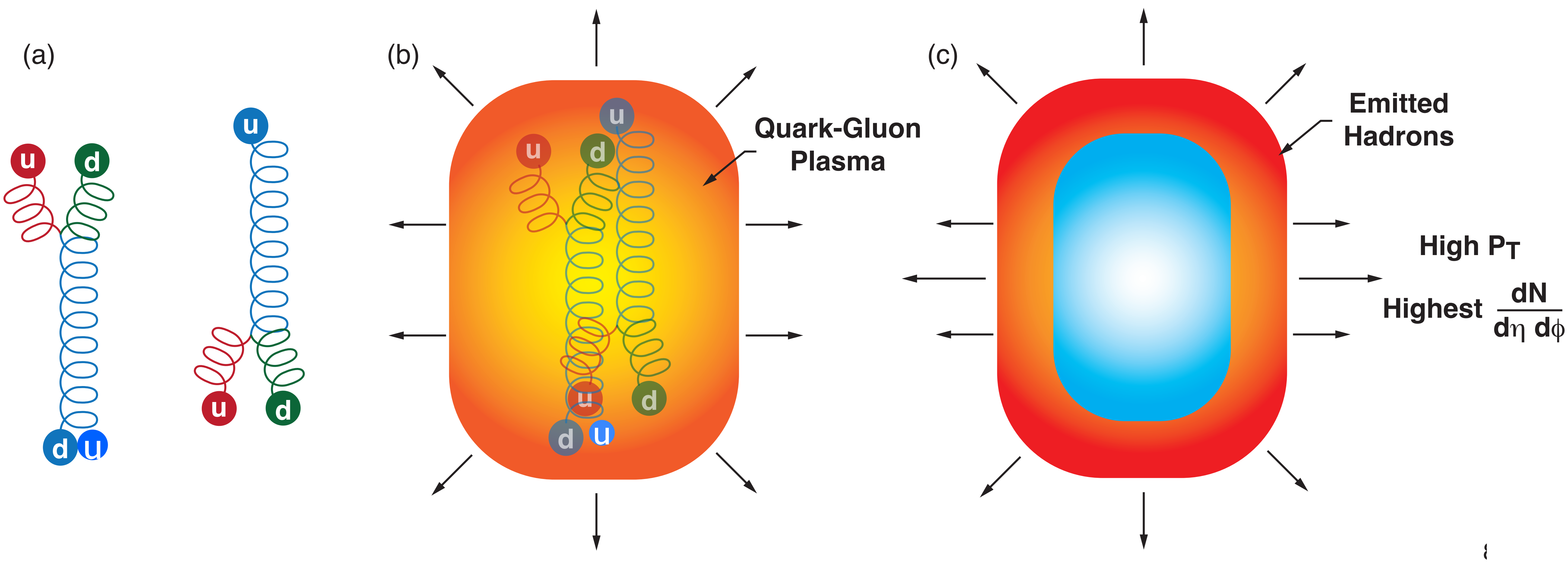
$$\vec{\zeta}_\perp = \vec{b}_\perp \sqrt{x(1-x)}$$



confinement in plane of pair

Possible origin of same-side CMS ridge in p p collisions

Bjorken, Goldhaber, sjb



$$\vec{V} = \sum_{i=1}^{N} [\cos 2\phi_i \hat{x} + \sin 2\phi_i \hat{y}]$$

v_3 from collisions of Y junctions

Multiparticle ridge-like correlations in very high multiplicity proton-proton collisions

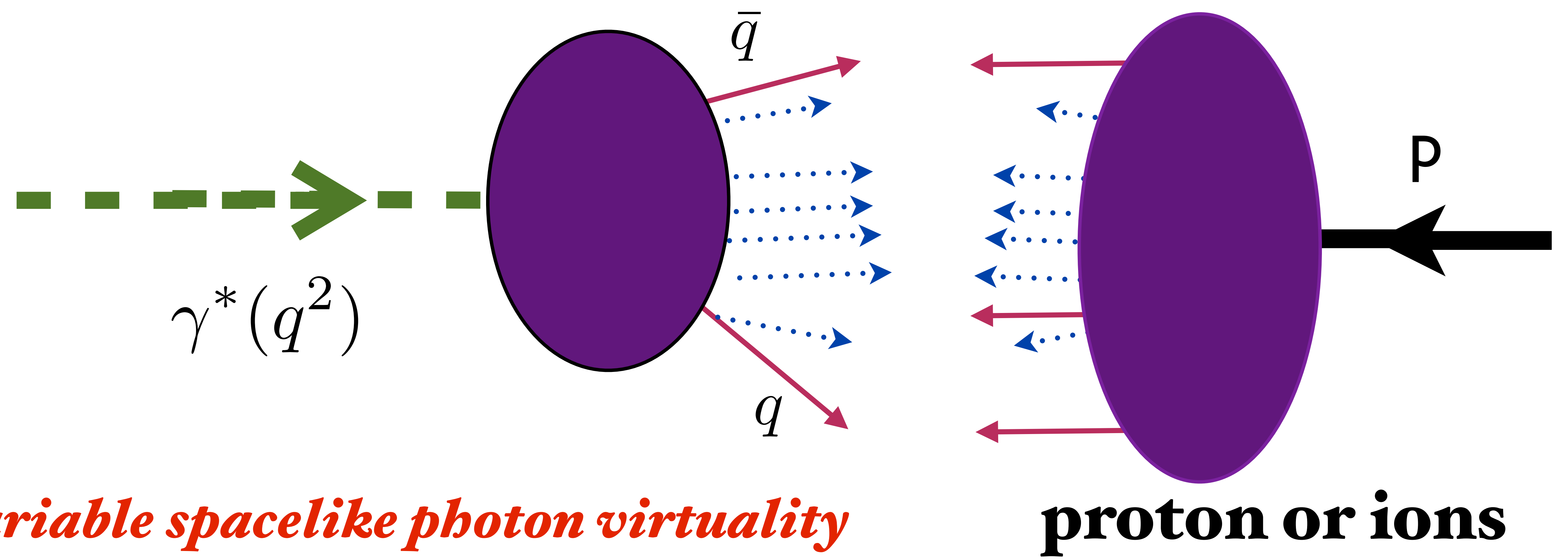
Bjorken, Goldhaber, sjb

We suggest that this "ridge"-like correlation may be a reflection of the rare events generated by the collision of aligned flux tubes connecting the valence quarks in the wave functions of the colliding protons.

The "spray" of particles resulting from the approximate line source produced in such inelastic collisions then gives rise to events with a strong correlation between particles produced over a large range of both positive and negative rapidity.

EIC: Virtual Photon-Proton Collider

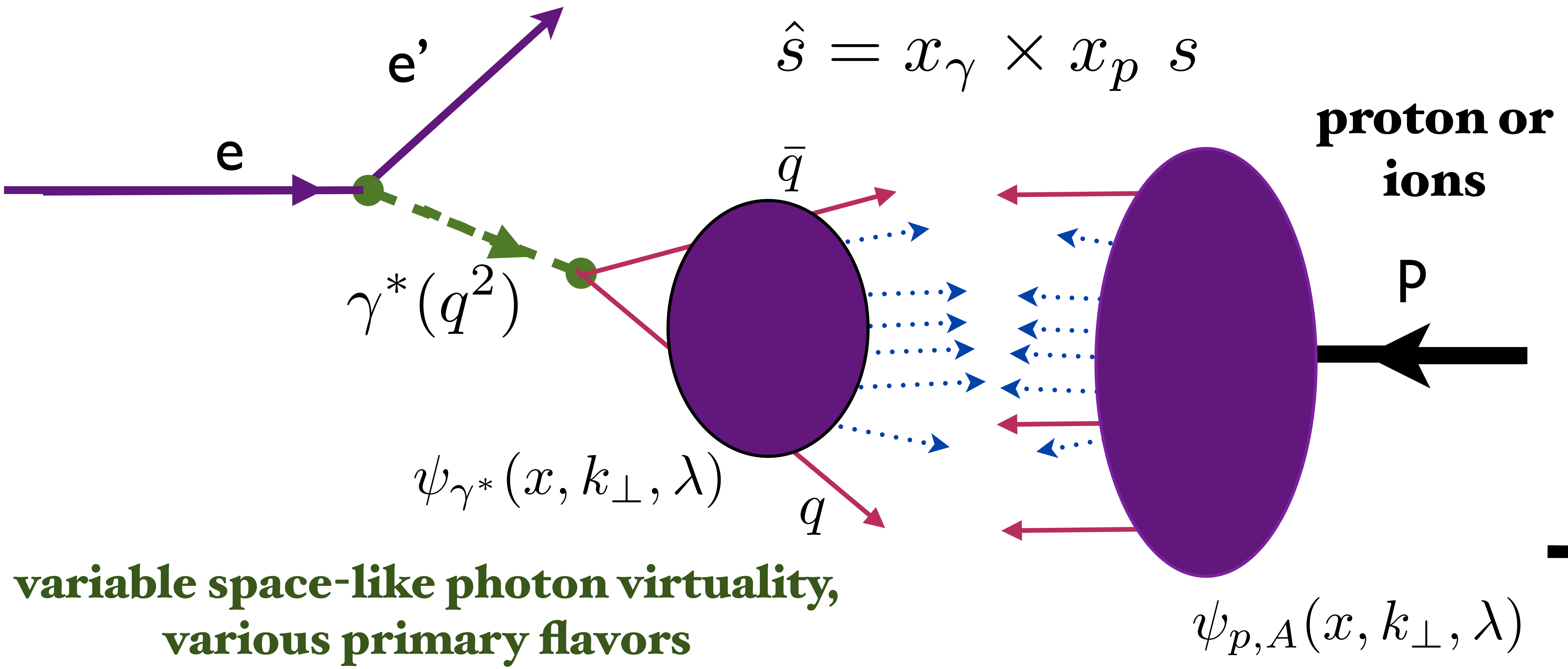
Perspective from the photon-proton collider frame



variable spacelike photon virtuality
various primary flavors
Study Ridge Phenomena with Controlled source

Electron-Ion Colliders: Virtual Photon-Ion Collider

Perspective from the e-p collider frame



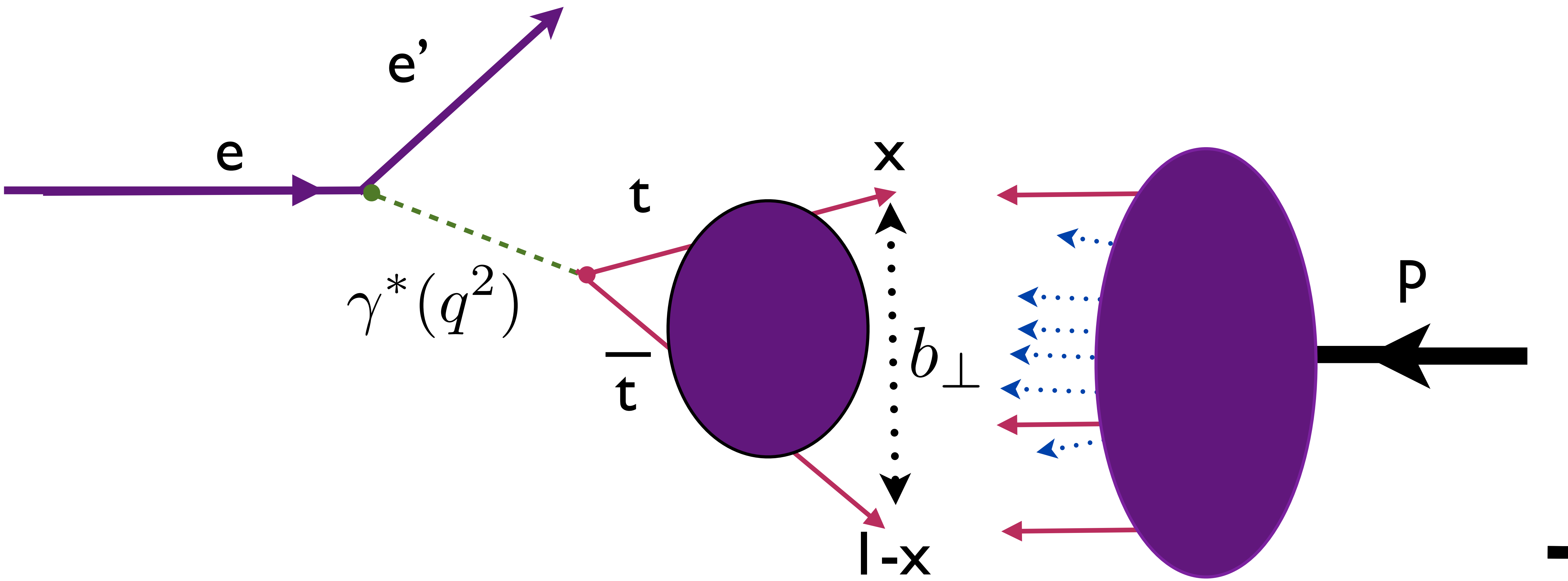
variable space-like photon virtuality, various primary flavors

$\bar{q}\,q$ plane aligned with lepton scattering plane ~ $\cos^2\phi$

Front-surface dynamics: shadowing/antishadowing

$\bar{t}\,t$ acts as a 'drill'

$$\langle b_\perp^2 \rangle \sim \frac{1}{Q^2 x(1-x) + M_t^2}$$

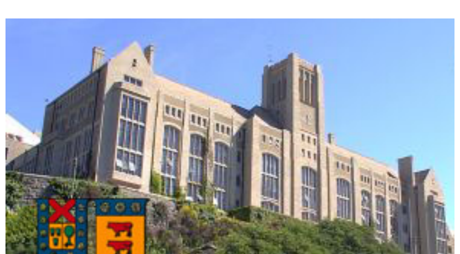


High Q^2, high M^2_Q virtual photon at LHeC acts as a precision, small bore, linearly oriented, flavor-dependent probe acting on a proton or nuclear target.

Study final-state hadron multiplicity distributions, ridges, nuclear dependence, etc.

Novel QCD Physics at an Electron-Ion Collider

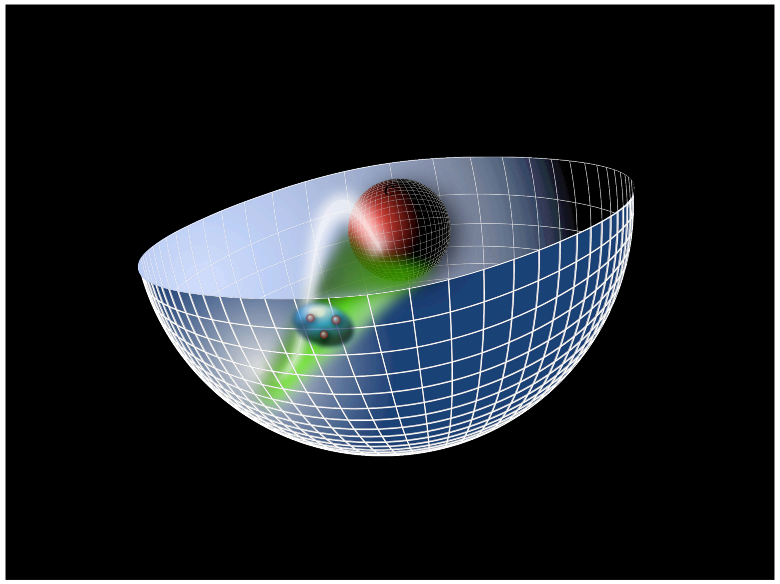
- Control Collisions of Flux Tubes and Ridge Phenomena
- Study Flavor-Dependence of Anti-Shadowing
- Heavy Quarks at Large x; Exotic States
- Direct, color-transparent hard subprocesses and the baryon anomaly
- Tri-Jet Production and the proton's LFWF
- Odderon-Pomeron Interference
- Digluon-initiated subprocesses and anomalous nuclear dependence of quarkonium production
- Factorization-Breaking Lensing Corrections

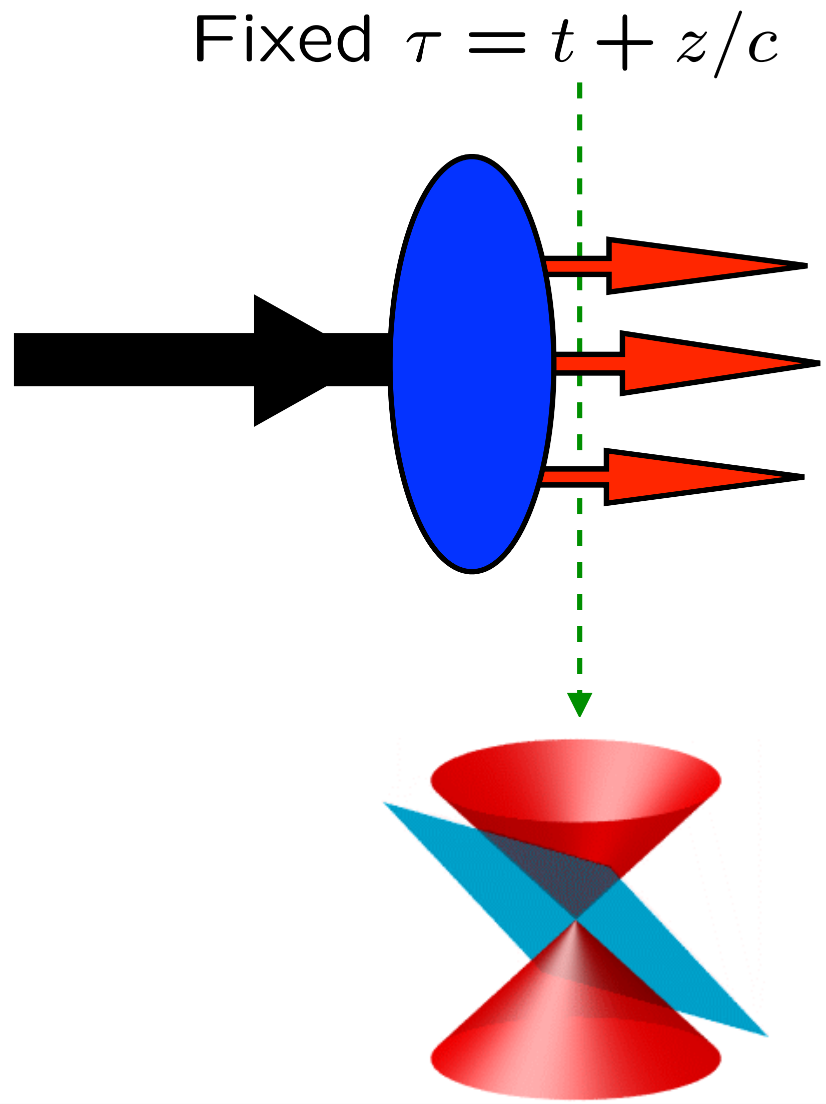


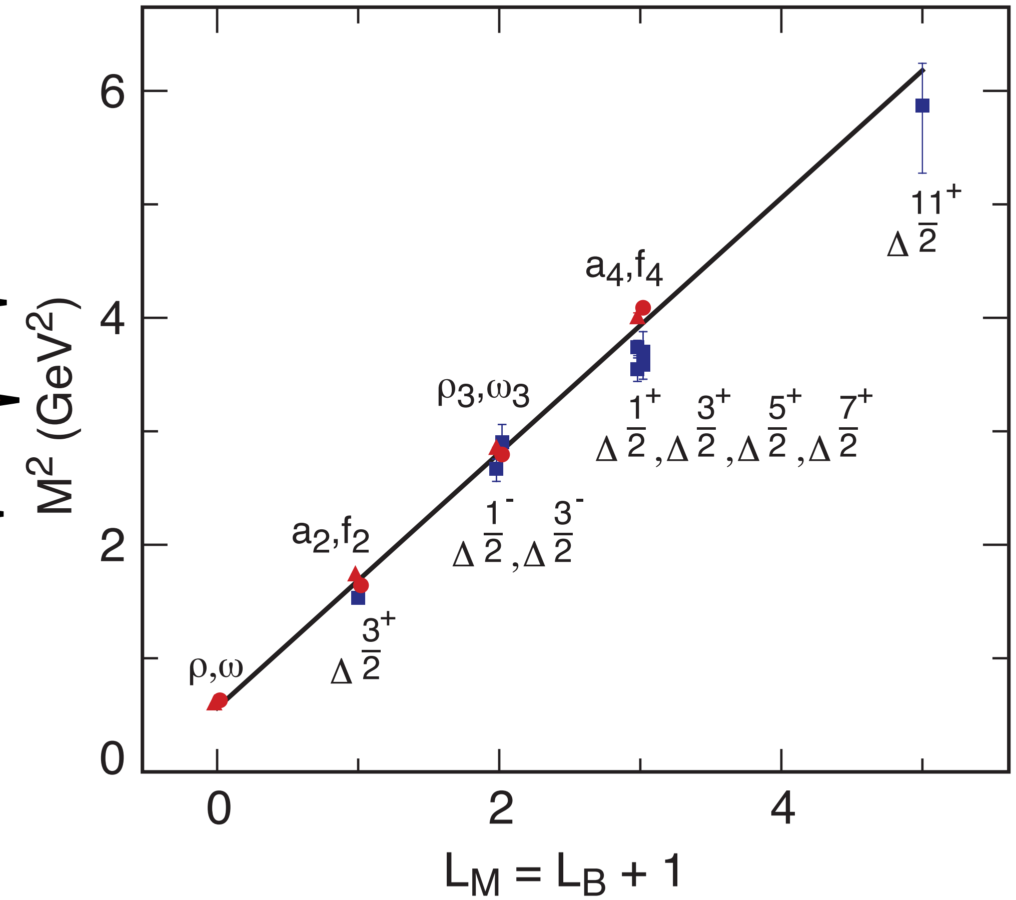
QCD Myths

- **ISI and FSI are higher twist effects - only a phase**
- **Momentum and Spin Sum Rules valid for nuclei - in fact not proven!**
- **Anti-Shadowing is Universal -**
  In fact, anti-shadowing is Flavor Dependent!
- **High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!**
- **Heavy quarks arise only from gluon splitting — Intrinsic Strange, Charm, and Bottom**
- **Renormalization scale cannot be fixed — PMC**
- **QCD condensates are vacuum effects**
- **QCD gives 10^{42} to the cosmological constant**

New Insights into Color Confinement and Hadron Dynamics from Light-Front Holography and Superconformal Quantum Mechanics







with Guy de Tèramond, Hans Günter Dosch, and Alexandre Deur

6-12 January 2016
Universidad Técnica Federico Santa María, Valparaiso, Chile

Stan Brodsky
SLAC NATIONAL ACCELERATOR LABORATORY