FROGGATT-NIELSEN-NELSON-BARR-OGENESIS

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In collaboration with:

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THE PUZZLE OF CP VIOLATION

Unification?

Quark sector
flavor structure

Lepton sector
flavor structure
Neutrino masses?

CP violation

Baryogenesis:
Sakharov
conditions

Strong CP problem

$$\mathcal{L} \supset \frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\bar{\theta} = \theta - \mathrm{Arg\ Det}\ M_u M_d$$

A COMPLETE MODEL

We will build a *complete, calculable, predictive and testable* model for all known and required CP violation

THE NELSON-BARR MECHANISM

- Impose CP to be a *symmetry of the following Lagrangian,*

$$\underline{M} D\bar{D} + \underline{\kappa_{aj}} S_a \bar{d}_j D + \underline{\tilde{\lambda}^d_{ij}} Q_i H_d \bar{d}_j + \underline{\tilde{\lambda}^u_{ij}} Q_i H_u \bar{u}_j$$

$$a = 1, 2$$ *all couplings here are real*

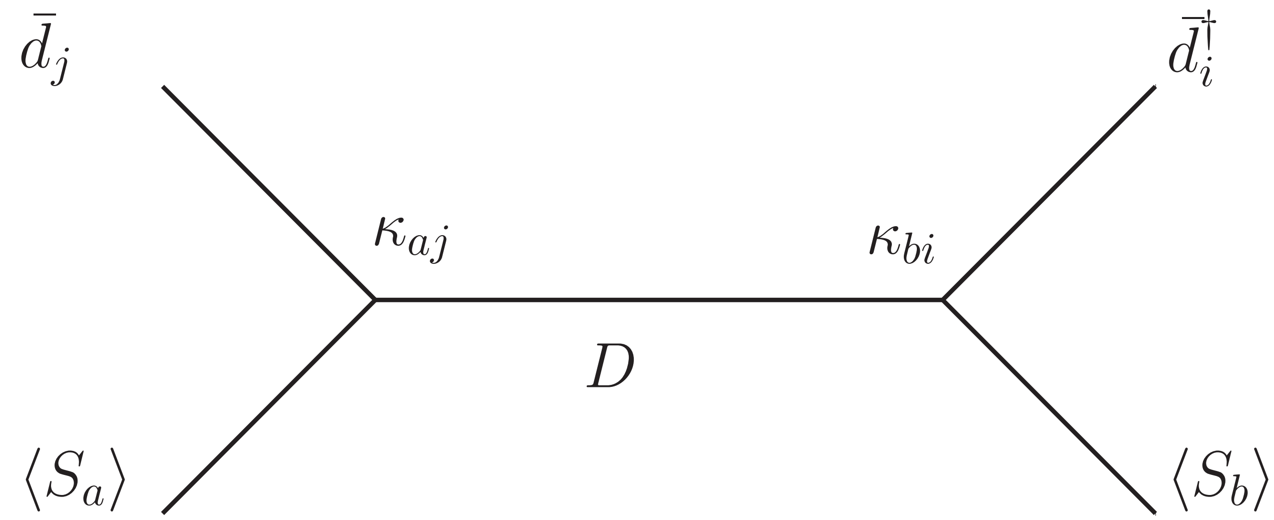
- Now allow BN fields to break CP spontaneously.

$$\Theta \equiv \mathrm{Arg} \langle S_1 S_2^* \rangle$$ *Unique source of CP violation*

- *For simplicity,* consider diagonal Yukawas.

IMPLICATIONS OF THE NB MECHANISM

- *Treat the Nelson-Barr fields as spurions*. Integrate out the heavy vector like quarks



$$\mathcal{Z}_{ij}\bar{d}_i^\dagger \bar{d}_j + \dots \qquad \mathcal{Z}_{ij} = \delta_{ij} + \frac{\kappa_{aj}\kappa_{bi}\langle S_a \rangle \langle S_b \rangle}{M^2} \qquad \mathcal{Z}^{-1} = AA^\dagger$$

IMPLICATIONS OF NB MECHANISM

- Renormalize the RH quarks

$$\lambda^d = \tilde{\lambda}^d A \qquad \lambda^u = \tilde{\lambda}^u \qquad\qquad (1)$$

- The strong CP phase vanishes

No explicit $\not{CP}$ *A is hermitian*

$$\bar{\theta} = \cancel{\theta - \mathrm{Arg\,Det}}\big(\tilde{\lambda}^u \tilde{\lambda}^d\big) \cancel{- \mathrm{Arg\,Det}\big(A\big)}$$

- There is a CKM phase: a basis always exists in which (1) holds.

THE COMPLETE MODEL

- Our complete model is

$$
\begin{aligned}
&\tilde{\lambda}^{d}_{ij}\ Q_i H_d \bar{d}_j + \tilde{\lambda}^{u}_{ij}\ Q_i H_u \bar{u}_j + \tilde{\lambda}^{\ell}_{ij}\ L_i H_d \bar{\ell}_j \\
&+ \tilde{\lambda}^{N}_{ij}\ L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j \\
&+ M_5 \left[D_4 \bar{D}_4 + L_4 \bar{L}_4 \right] + \kappa_{aj}\, S_a \left[\bar{d}_j D_4 + L_j \bar{L}_4 \right]
\end{aligned}
$$

ORGANIZE THE MODEL IN EFT

A model of spontaneous CP violation and flavor mixing

$$\Theta \equiv \mathrm{Arg}\left\langle S_1 S_2^* \right\rangle$$

$\Lambda = \zeta \sim M_5$

$M_i < \Lambda < M_5$

MSSM + RH neutrinos
with all *CP violation and flavor mixing* coming from unified wave function renormalization

$\Lambda < M_i$

MSSM+Majorana neutrino masses with some conditions

Observables: $\mathcal{O}_{IR}$

THE EFFECTIVE THEORY BELOW M_1, M_2, M_3

- Now integrate out the RH neutrinos,

$$W_{MSSM} + \lambda^{\nu}_{ij}(L_i H_u)(L_j H_u) \qquad (m^{\nu} = v_u^2 \lambda^{\nu})$$

Projective unification conditions

$$\boxed{\lambda^{\ell} = \left(\lambda^d\right)^T \Gamma^{\ell} \quad \lambda^{\nu} = \left(\lambda^d\right)^T \Gamma^m \lambda^d \qquad \bar{\theta} = 0}$$

$$\Gamma^{\ell} = \left(\tilde{\lambda}^d\right)^{-1} \tilde{\lambda}^{\ell}\ ,\ \Gamma^m = \frac{v_u^2}{v_d^2}\left[(\tilde{\lambda}^d)^{-1}\tilde{\lambda}^N M^{-1} \tilde{\lambda}^N (\tilde{\lambda}^d)^{-1}\right]$$ are *real* and *diagonal*

Daniel Egana-Ugrinovic, Rutgers University

WHAT DO OUR CONDITIONS MEAN?

- How to relate the Yukawas with known physical quantities?
- The only thing we can get with no effort is (in our basis)

$$
\begin{aligned}
v_u^2 \lambda^u \lambda^{u\dagger} &= \mathrm{diag}(m_u^2, m_c^2, m_t^2) \\
v_d^2 \lambda^d \lambda^{d\dagger} &= \mathrm{diag}(1, e^{i\gamma_1}, e^{i\gamma_2})\, V_{CKM}^* \,\mathrm{diag}(m_d^2, m_s^2, m_b^2)\, V_{CKM}^T \,\mathrm{diag}(1, e^{-i\gamma_1}, e^{-i\gamma_2})
\end{aligned}
$$

- This imposes the physical constraints on the quark sector. What are the constraints from lepton sector data?

We have to find simple *physical combinations of Lagrangian parameters*

(LEPTONIC FLAVOR INVARIANTS

Concentrate on *leptonic sector:*
12 observables

↓

12 *independent invariants* under all BG symmetries

- The background flavor symmetries are

	$U(3)_L$	$U(3)_\ell$
L	$\mathbf{3}_1$	
$\bar{\ell}$		$\overline{\mathbf{3}}_1$
λ_ℓ	$\overline{\mathbf{3}}_{-1}$	$\mathbf{3}_{-1}$
m_ν	$\overline{\mathbf{6}}_{-2}$	

* I omit the weak Θ angle:
there is no invariant for it so it is not physical.

LEPTONIC FLAVOR INVARIANTS

- The simplest invariants are

$$
\begin{aligned}
v_d^{2n}\,\mathrm{Tr}\left[\left(\lambda_\ell \lambda_\ell^\dagger\right)^n\right] &= m_e^{2n} + m_\mu^{2n} + m_\tau^{2n} \\
\mathrm{Tr}\left[\left(m_\nu m_\nu^\dagger\right)^n\right] &= m_{\nu_e}^{2n} + m_{\nu_\mu}^{2n} + m_{\nu_\tau}^{2n}
\end{aligned}
$$

- For n>3, the invariants are not independent from the ones with n=1,2,3.

- This gives six invariants related to the *masses*.

LEPTONIC FLAVOR INVARIANTS)

- There are three more related to the *mixing angles*

$$\mathrm{Tr}\left(\left[\,\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\,\right]^2\right)$$

$$\mathrm{Tr}\left(\left[\,\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\,\right]^2\left(\lambda_\ell\lambda_\ell^\dagger\right)^2\right)$$

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\,\right]^2\left(m_\nu m_\nu^\dagger\right)^2\right)$$

They have to involve *commutators*

- And finally three related to the *CP violating phases in the PMNS matrix* (Dreiner et.al. 0703074)

You know this one! It's a Jarlskob like invariant (C. Jarlskob PRL 55 (1985) 1039)

$$\mathrm{Tr}\left(\left[\,\lambda_\ell\lambda_\ell^\dagger\,, m_\nu m_\nu^\dagger\,\right]^3\right)$$

$$\mathrm{Tr}\left(\left[\,\lambda_\ell\lambda_\ell^\dagger\,, m^\nu\left(\lambda^\ell\lambda^{\ell\dagger}\right)^* m^{\nu\dagger}\,\right]^3\right)$$

$$\mathrm{Tr}\left(\left[\,\lambda_\ell\lambda_\ell^\dagger\,, m_\nu m_\nu^\dagger\,\right]\left(m^\nu\left(\lambda^\ell\lambda^{\ell\dagger}\right)^* m^{\nu\dagger}\right)\right)$$

III. CONSTRAINING THE MODEL WITH DATA

- We are ready to understand the non-trivial constraints of the theory.

Statement:
Using our EFT conditions, all leptonic invariants can be written in terms of traces involving

$$\lambda^d \lambda^{d\dagger}, \Gamma^\ell, \Gamma^m$$

- As an example, consider the simplest invariant,

$$\mathrm{Tr}[\lambda_\ell \lambda_\ell^\dagger] = \mathrm{Tr}\Big[\lambda^{d\,T}\,\Gamma^\ell\,\Gamma^{\ell T}\,\lambda^{d*}\Big] = \mathrm{Tr}\Big[\Gamma^\ell \Gamma^{\ell T}\big(\lambda^d \lambda^{d\dagger}\big)^*\Big]$$

Use projective conditions

PARAMETER COUNTING

- The matrix $\lambda^d \lambda^{d\dagger}$ is specified by quark sector data and two model parameters γ_1, γ_2
- The matrices Γ^ℓ, Γ^m contain 6 more model parameters.

The leptonic sector is completely specified
by 8 model parameters…

… but there are exactly 8 measured quantities
in the leptonic sector!

We can predict the remaining unmeasured parameters.

PREDICTIONS : THE LIGHTEST NEUTRINO MASS

- *The hierarchy is predicted to be normal*, since the mass hierarchy of the quarks tends to be inherited to the lepton sector (*unification Ansatz*)

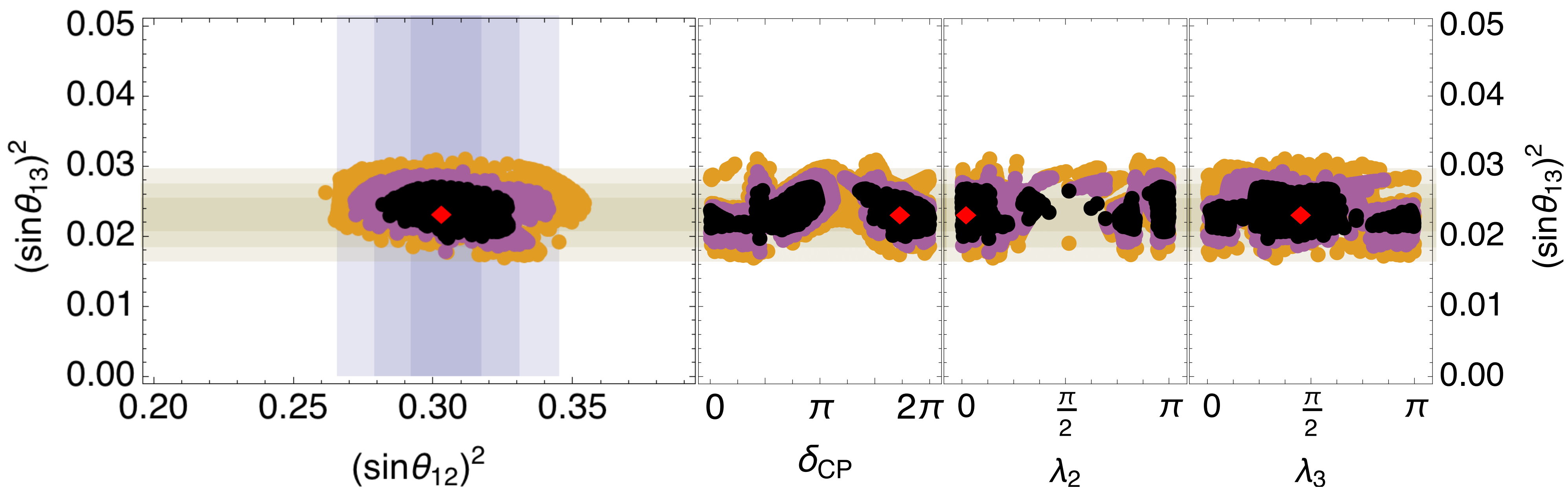
- The only allowed mass window for the lightest neutrino mass is

$$\boxed{10^{-3}\,\mathrm{eV} \lesssim m_{\nu_1} \lesssim 10^{-2}\,\mathrm{eV}}$$

(one-two orders of magnitude below sensitivity of current neutrinoless double beta decay exp.)

III. PREDICTIONS: IR CP VIOLATING PHASES

- The CP violating phases cannot be predicted with precision with the current uncertainties in mixing angles. *They are generic*.

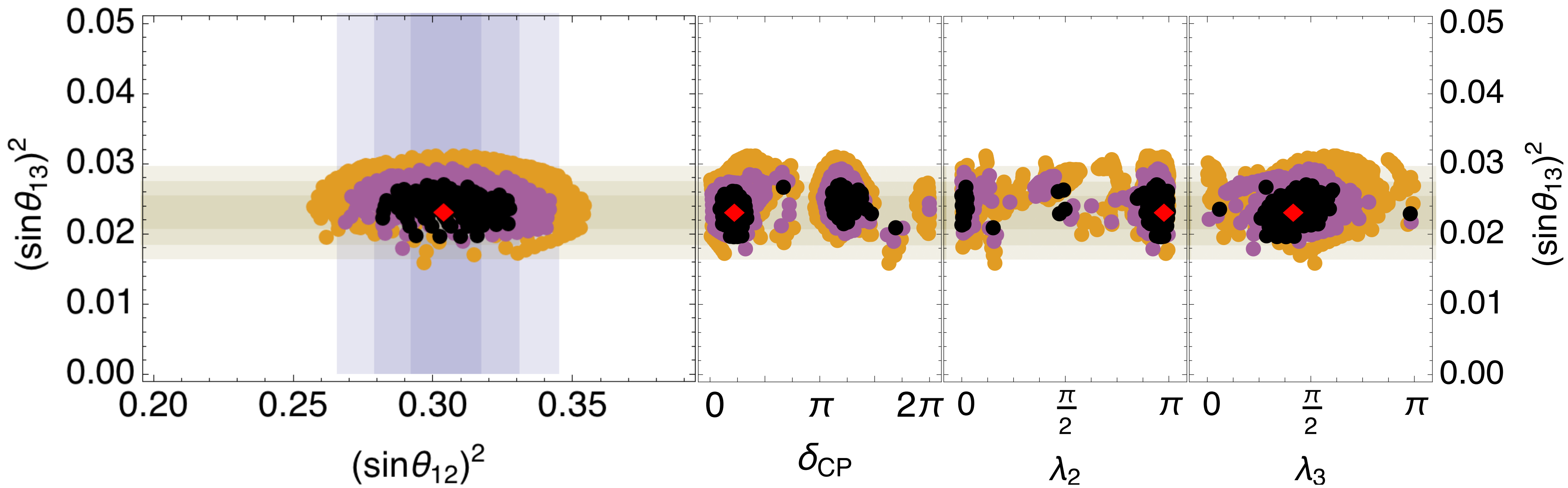


In these plots

$$m_{\nu_1} = 5 \cdot 10^{-3}\,\text{eV}$$

CP VIOLATING PHASES

- Now impose the constrain that we actually live in a universe with matter.



$$m_{\nu 1} = 5 \cdot 10^{-3}\,\mathrm{eV}$$

CONCLUSIONS

We built a ***complete, calculable and predictive*** model for ***all*** known and required CP violation.

- ✓ Solves the strong CP problem.
- ✓ Accommodates all measured masses and mixing angles.
- ✓ Predicts the lightest neutrino mass.
- ✓ Predicts the CP violating phases of the PMNS matrix (limited only by experimental uncertainties on measured mixing angles).
- ✓ Predicts a normal neutrino hierarchy.
- ★ Partially explains the flavor structure of the lepton sector.
- ★ For given RH neutrino masses and $tan\beta$, predicts the UV CP violating phases and mixing angles of a type I seesaw (*crucial* for leptogenesis)
- ★ Gives the correct baryon asymmetry for $M_1 \gtrsim 10^9\,\mathrm{GeV}$

Full presentation

FROGGATT-NIELSEN-NELSON BARR-OGENESIS (OR *A COMPLETE CALCULABLE MODEL FOR ALL KNOWN AND REQUIRED CP VIOLATION*)

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THE PUZZLE OF CP VIOLATION

Unification?

Quark sector
flavor structure

Lepton sector
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Neutrino masses?

CP violation

Baryogenesis:
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Strong CP problem

$$\mathcal{L} \supset \frac{\theta}{32\pi^2} F_{\mu\nu}\tilde{F^{\mu\nu}}$$

$$\bar{\theta} = \theta - \mathrm{Arg}\ \mathrm{Det}\ M_u M_d$$

A COMMON ORIGIN FOR ALL CP VIOLATION

In this talk, we will build a
complete, calculable, testable and predictive model
for *all* known and required CP violation.

… we will have to think about
the flavor structure of the SM

OUTLINE

I. Building the model: the Nelson-Barr mechanism as a guiding principle.

II. The effective theories at different mass scales.

Intermezzo: flavor invariants.

III. Constraining the model with IR data and predictions for IR physics.

IV. Predictions for UV physics.

V. Leptogenesis.

1. THE NELSON-BARR MECHANISM

- It falls in the class of UV solutions to the strong CP problem (A. Nelson Phys. Lett. B136 (1984) 387, S. Barr PRL 53 (1984) 329).

- Impose CP to be a *symmetry of the following Lagrangian,*

$$\underline{M}\, D\bar{D} + \underline{\kappa_{aj}}\, S_a \bar{d}_j D + \underline{\tilde{\lambda}^d_{ij}}\, Q_i H^c \bar{d}_j + \underline{\tilde{\lambda}^u_{ij}}\, Q_i H \bar{u}_j$$

all couplings here are real

think $M > 10^{11}\ \mathrm{GeV}$

$$a = 1, 2$$

1. CP VIOLATION IN THE NB MECHANISM

- Now allow BN fields to break CP spontaneously. There is a background symmetry

	$U(1)_S$
All SM fields	0
D	1
$\bar{D}$	-1
$\langle S_a \rangle$	-1

- Unique BG invariant phase

$$\Theta \equiv \mathrm{Arg}\,\langle S_1 S_2^* \rangle$$

Unique source of CP violation

1. IMPLICATIONS OF THE NB SETUP

1. **The theta angle vanishes in the tree level calculation**

$$M_u = \tilde{\lambda}^u v_u \qquad M_d = \begin{pmatrix} \tilde{\lambda}^d v_d & \zeta \\ 0 & M \end{pmatrix} \qquad \zeta_i \equiv \langle S_a \rangle \kappa_{ai}$$

No explicit $\cancel{CP}$ *Direct calculation*

$$\bar{\theta} = \theta - \text{Arg Det } M_u - \text{Arg Det } M_d$$

Add SUSY to protect the result.

1. IMPLICATIONS OF THE NB SETUP

2. The low energy theory contains a CKM phase

- Mixing in the EFT is encoded in WF renormalization

$$\mathcal{Z}_{ij}\bar{d}_i^\dagger \bar{d}_j + \ldots \qquad \mathcal{Z}_{ij} = \delta_{ij} + \frac{\zeta_i \zeta_j^*}{M^2} \qquad \mathcal{Z}^{-1} = AA^\dagger \qquad \zeta_i \approx M$$

- Renormalize the RH quarks. The EFT is

$$\lambda^d_{ij}\ Q_i H_d \bar{d}_j + \lambda^u_{ij}\ Q_i H_u \bar{u}_j$$

$$\lambda^d = \tilde{\lambda}^d A \qquad \lambda^u = \tilde{\lambda}^u \qquad \boxed{\bar{\theta} = 0}$$

*Only „non-trivial" condition**

We interpret this as our choice of basis

** If we take 3 vector like pairs of quarks.*

1. ALL $\not{C}\!\!P$ AND MIXING FROM $\mathcal{Z}$

- We will also take the Ansatz that all *flavor mixing* comes from WF renormalization,

$\tilde{\lambda}^u, \tilde{\lambda}^d$ are *real*, *diagonal* matrices in the basis we work on

- This Ansatz is protected by the non-renormalization theorem.

$$\boxed{\text{Our guiding principle: } \textit{All } \not{C}\!\!P \textit{ and flavor mixing comes from the NB sector.}}$$

1. THE LEPTON SECTOR

- We follow the consequences of our guiding principle to the lepton sector. Assume unification for simplicity

$$\tilde{\lambda}^{\ell}_{ij}\; L_i H_d \bar{\ell}_j + \; M_5 \left[D_4 \bar{D}_4 + L_4 \bar{L}_4 \right]$$

$$+\kappa_{aj}\, S_a \left[\bar{d}_j D_4 + L_j \bar{L}_4 \right] + \dots$$

- A complete model should also include neutrino masses and a mechanism for baryogenesis, so include

$$\tilde{\lambda}^{N}_{ij}\; L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j$$

$$M_1, M_2, M_3 < M_5$$

1. THE COMPLETE MODEL

- Our complete model is

$$
\begin{aligned}
&\tilde{\lambda}^{d}_{ij}\ Q_i H_d \bar{d}_j + \tilde{\lambda}^{u}_{ij}\ Q_i H_u \bar{u}_j + \tilde{\lambda}^{\ell}_{ij}\ L_i H_d \bar{\ell}_j \\
&+ \tilde{\lambda}^{N}_{ij}\ L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j \\
&+ M_5 \left[D_4 \bar{D}_4 + L_4 \bar{L}_4 \right] + \kappa_{aj}\, S_a \left[\bar{d}_j D_4 + L_j \bar{L}_4 \right]
\end{aligned}
$$

where all couplings are real
and diagonal in flavor space
<u>in the basis we work on</u>

II. THE EFFECTIVE THEORIES

All $\cancel{CP}$ in a unique phase

$$\Theta \equiv \mathrm{Arg}\left\langle S_1 S_2^* \right\rangle$$

$\Lambda = \zeta \sim M_5$ Nelson-Barr scale

$M_i < \Lambda < M_5$

RH neutrino scale

$\Lambda < M_i$ *Observables:* $\mathcal{O}_{IR}$

II. THE EFFECTIVE THEORY BELOW M_5

- *Treat the NB fields as spurions*. Integrate out the vector like heavy fields.

- The resulting superpotential is

$$W_{MSSM} + \lambda^N_{ij}\, L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j$$

$$\lambda^u = \tilde{\lambda}^u \quad \lambda^d = \tilde{\lambda}^d A \quad \lambda^\ell = A^T \tilde{\lambda}^\ell \quad \lambda^N = A^T \tilde{\lambda}^N \quad \bar{\theta} = 0$$

$\tilde{\lambda}^u, \tilde{\lambda}^d, \tilde{\lambda}^\ell, \tilde{\lambda}^N$ *real* and *diagonal*

$\longrightarrow$ SUSY Type I seesaw model, with all $\cancel{CP}$ and flavor mixing coming from WF renormalization

II. THE EFFECTIVE THEORY BELOW M_5

$$\lambda^u = \tilde{\lambda}^u \quad \lambda^d = \tilde{\lambda}^d A \quad \lambda^\ell = A^T \tilde{\lambda}^\ell \quad \lambda^N = A^T \tilde{\lambda}^N \quad \bar{\theta} = 0$$

choice of basis

using $A = (\tilde{\lambda}^d)^{-1} \lambda^d$

Complete set of non trivial conditions

$$\boxed{\begin{gathered} \bar{\theta} = 0 \\ \lambda^\ell = \left(\lambda^d\right)^T \Gamma^\ell \\ \lambda^N = \lambda^{dT} \Gamma^\nu \end{gathered}}$$

Unification Ansatz: Hierarchies will be inherited from quark sector

where $\Gamma^\ell = \left(\tilde{\lambda^d}\right)^{-1} \tilde{\lambda}^\ell$ and $\Gamma^\nu = \left(\tilde{\lambda^d}\right)^{-1} \tilde{\lambda}^N$ are *real and diagonal*

II. THE EFFECTIVE THEORY BELOW M_1, M_2, M_3

- Integrating out the RH neutrinos, the EFT at the EW scale can be summarized by

$$W_{MSSM} + \lambda^{\nu}_{ij}(L_i H_u)(L_j H_u) \qquad\qquad (m^{\nu} = v_u^2 \lambda^{\nu})$$

Complete set of non trivial conditions

$$\boxed{\lambda^{\ell} = \left(\lambda^d\right)^T \Gamma^{\ell} \qquad \lambda^{\nu} = \left(\lambda^d\right)^T \Gamma^m \lambda^d \qquad\qquad \bar{\theta} = 0}$$

$$\Gamma^{\ell} = \left(\tilde{\lambda}^d\right)^{-1} \tilde{\lambda}^{\ell}\ ,\ \Gamma^m = \frac{v_u^2}{v_d^2}\left[(\tilde{\lambda}^d)^{-1}\tilde{\lambda}^N M^{-1} \tilde{\lambda}^N \left(\tilde{\lambda}^d\right)^{-1}\right]$$ are *real* and *diagonal*

II. THE EFFECTIVE THEORIES

All $\not{CP}$ in a unique phase

$$\Theta \equiv \mathrm{Arg}\left\langle S_1 S_2^* \right\rangle$$

$\Lambda = \zeta \sim M_5$

SUSY Type I seesaw with all
CP and flavor mixing coming from $\mathcal{Z}$

$M_i < \Lambda < M_5$

Observables: $\mathcal{O}_{UV} \supset \mathcal{O}_{IR}$

MSSM+Majorana neutrino masses
with some **EFT conditions**

$\Lambda < M_i$

Observables: $\mathcal{O}_{IR}$

II. THE EFFECTIVE THEORIES

All $\cancel{CP}$ in a unique phase

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SUSY Type I seesaw with all
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Observables: $\mathcal{O}_{UV} \supset \mathcal{O}_{IR}$

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MSSM+Majorana neutrino masses
with some **EFT conditions**

Observables: $\mathcal{O}_{IR}$

II. WHAT DO OUR CONDITIONS MEAN?

- How to relate the Yukawas with known physical quantities?
- The only thing we can get with no effort is (in our basis)

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\begin{aligned}
v_u^2 \lambda^u \lambda^{u\dagger} &= \mathrm{diag}(m_u^2, m_c^2, m_t^2) \\
v_d^2 \lambda^d \lambda^{d\dagger} &= \mathrm{diag}(1, e^{i\gamma_1}, e^{i\gamma_2})\, V_{CKM}^* \,\mathrm{diag}(m_d^2, m_s^2, m_b^2)\, V_{CKM}^T \,\mathrm{diag}(1, e^{-i\gamma_1}, e^{-i\gamma_2})
\end{aligned}
$$

- This imposes the physical constraints on the quark sector. What are the constraints from lepton sector data?

We have to find the *physical combinations of Lagrangian parameters*

(LEPTONIC FLAVOR INVARIANTS

- Our conditions involve the lepton Yukawas

Concentrate on *leptonic sector:*

12 observables

↓

12 *independent invariants* under all BG symmetries

- The BG symmetries are

	$U(3)_L$	$U(3)_\ell$
L	$\mathbf{3}_1$	
$\bar{\ell}$		$\overline{\mathbf{3}}_1$
λ_ℓ	$\overline{\mathbf{3}}_{-1}$	$\mathbf{3}_{-1}$
m_ν	$\overline{\mathbf{6}}_{-2}$	

* I omit the weak Θ angle:

there is no invariant for it
so it is not physical.

LEPTONIC FLAVOR INVARIANTS

- The simplest invariants are

$$\begin{aligned} v_d^{2n}\,\mathrm{Tr}\left[\left(\lambda_\ell \lambda_\ell^\dagger\right)^n\right] &= m_e^{2n} + m_\mu^{2n} + m_\tau^{2n} \\ \mathrm{Tr}\left[\left(m_\nu m_\nu^\dagger\right)^n\right] &= m_{\nu_e}^{2n} + m_{\nu_\mu}^{2n} + m_{\nu_\tau}^{2n} \end{aligned}$$

- For n>3, the invariants are not independent from the ones with n=1,2,3.

- This gives six invariants related to the *masses*.

LEPTONIC FLAVOR INVARIANTS)

- There are three more related to the *mixing angles*

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\right]^2\right)$$

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\right]^2\left(\lambda_\ell\lambda_\ell^\dagger\right)^2\right)$$

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\right]^2\left(m_\nu m_\nu^\dagger\right)^2\right)$$

They have to involve *commutators*

- And finally three related to the *CP violating phases in the PMNS matrix* (Dreiner et.al. 0703074)

You know this one! It's a Jarlskob like invariant (C. Jarlskob PRL 55 (1985) 1039)

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\right]^3\right)$$

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m^\nu(\lambda^\ell\lambda^{\ell\dagger})^* m^{\nu\dagger}\right]^3\right)$$

$$\mathrm{Tr}\left(\left[\lambda_\ell\lambda_\ell^\dagger, m_\nu m_\nu^\dagger\right]\left(m^\nu(\lambda^\ell\lambda^{\ell\dagger})^* m^{\nu\dagger}\right)\right)$$

III. CONSTRAINING THE MODEL WITH DATA

- We are ready to understand the non-trivial constraints of the theory.

Statement:
Using our EFT conditions, all leptonic invariants can be written in terms of traces involving

$$\lambda^d \lambda^{d\dagger}, \Gamma^\ell, \Gamma^m$$

- As an example, consider the simplest invariant,

$$\mathrm{Tr}[\lambda_\ell \lambda_\ell^\dagger] = \mathrm{Tr}\Big[\lambda^{d\,T}\,\Gamma^\ell\,\Gamma^{\ell T}\,\lambda^{d*}\Big] = \mathrm{Tr}\Big[\Gamma^\ell\Gamma^{\ell T}\big(\lambda^d\lambda^{d\dagger}\big)^*\Big]$$

Use EFT conditions

III. PARAMETER COUNTING

- The matrix $\lambda^d \lambda^{d\dagger}$ is specified by quark sector data and two model parameters γ_1, γ_2

- The matrices Γ^ℓ, Γ^m contain 6 more model parameters.

The leptonic sector is completely specified by 8 model parameters.

III. THE EFT IS COMPLETELY SPECIFIED

- But there are also exactly 8 *experimentally measured* quantities in the leptonic sector! (3 charged lepton masses, 2 neutrino mass splittings, 3 mixing angles)

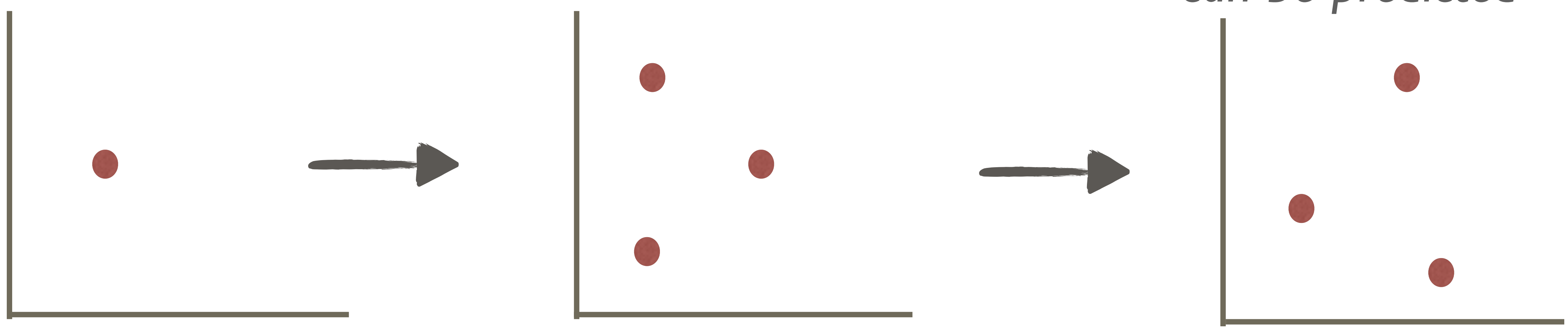
Number (yet) unkown EFT parameters at the EW scale	=	Number of known physical observables

- The remaining 4 physical quantities *are predicted:*

> The lightest neutrino mass, the hierarchy, and the 3 CP violating phases of the PMNS matrix are *predictions*

III. HOW DOES THIS WORK IN PRACTICE

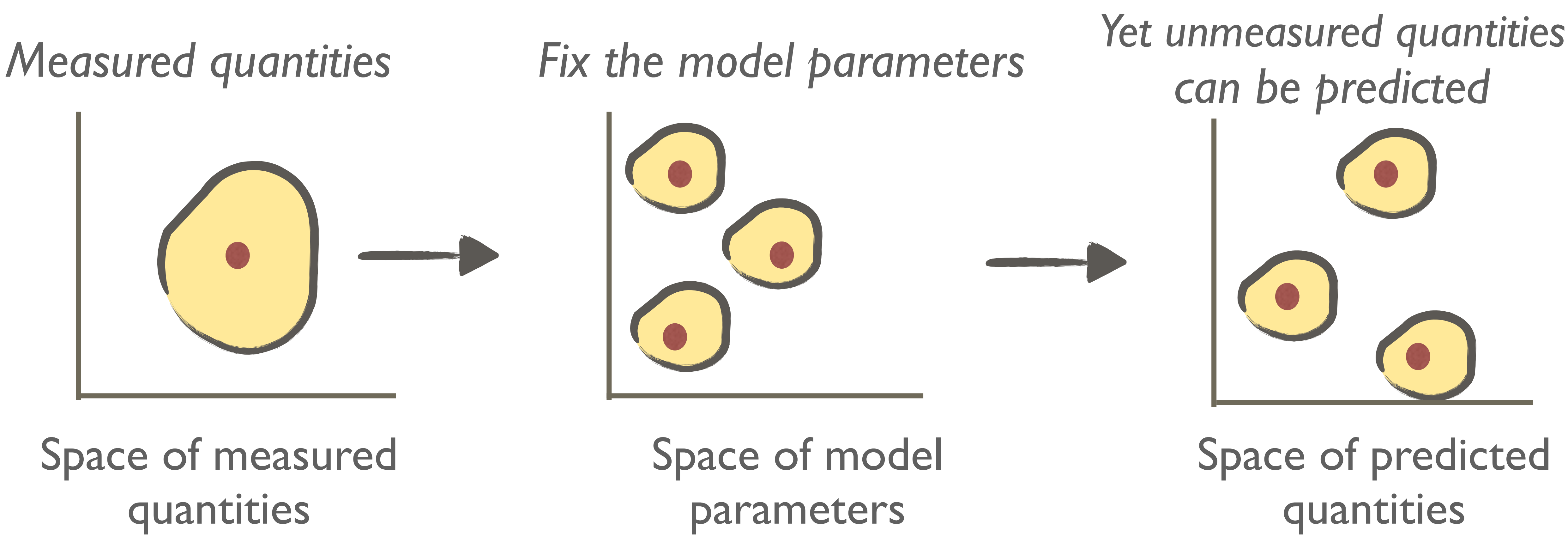
Measured quantities | *Fix the model parameters* | *Yet unmeasured quantities can be predicted*



Space of measured quantities | Space of model parameters | Space of predicted quantities

III. HOW DOES THIS WORK IN PRACTICE

- In practice, the neutrino mixing angles are only known up to large 3σ bands, so the predictions only lie within bands.



III. PREDICTIONS : THE LIGHTEST NEUTRINO MASS

- *The hierarchy is predicted to be normal*, since the mass hierarchy of the quarks tends to be inherited to the lepton sector (*unification Ansatz*)

- The only allowed mass window for the lightest neutrino mass is

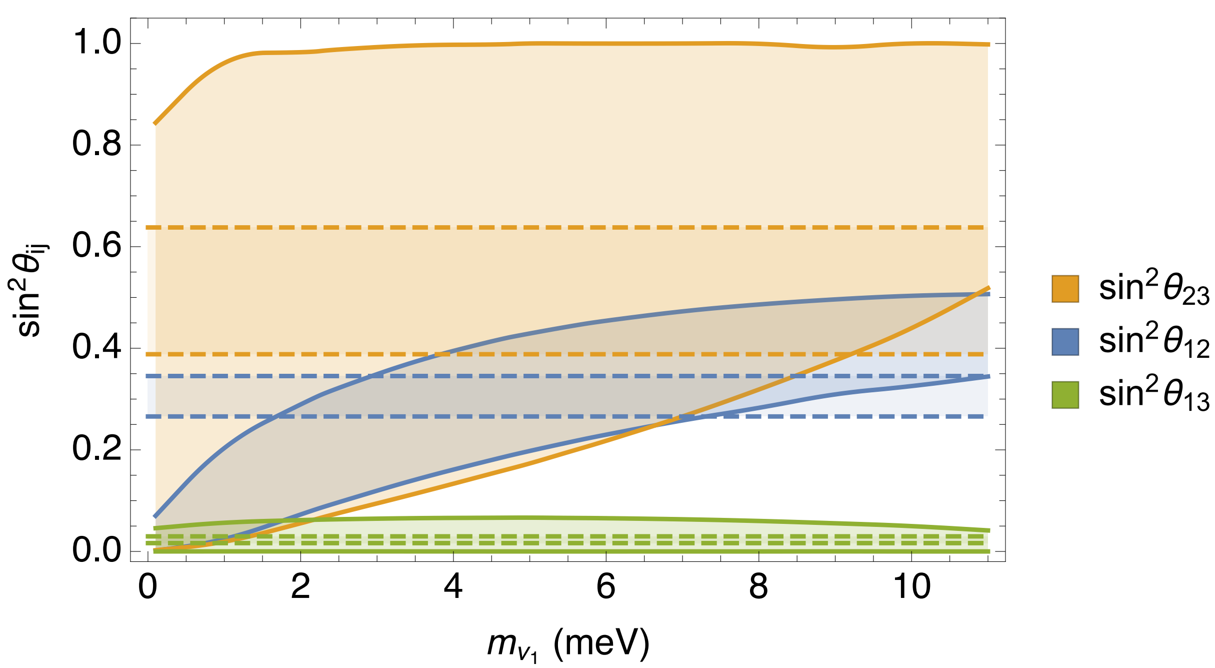
$$\boxed{10^{-3}\,\mathrm{eV} \lesssim m_{\nu_1} \lesssim 10^{-2}\,\mathrm{eV}}$$

(one-two orders of magnitude below sensitivity of current neutrinoless double beta decay exp.)

III. WHY ARE WE EVEN GETTING SOLUTIONS?

The objective of the plot below is
to see what does our model *prefer to do.*

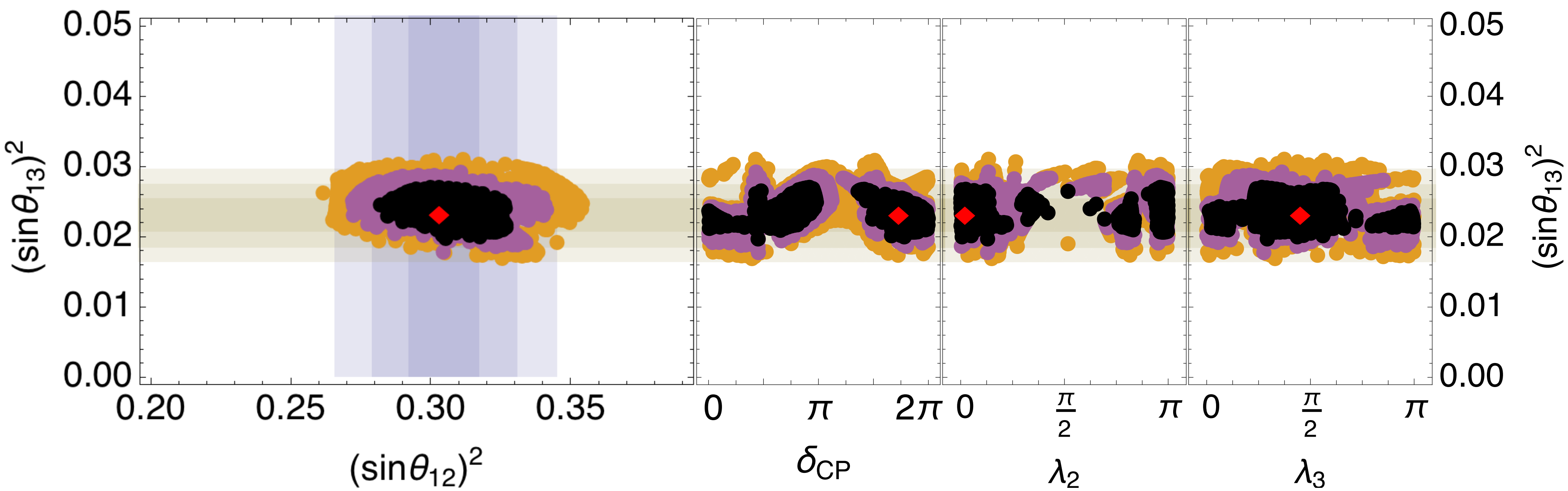
- Fix just the known masses (do not fix the mixing angles)
- Scan over the rest of parameter space



Note hierarchies
of angles tends to be
the right one (*unification Ansatz*)

III. PREDICTIONS: IR CP VIOLATING PHASES

- The CP violating phases cannot be predicted with precision with the current uncertainties in mixing angles. *They are generic.*



In these plots $m_{\nu_1} = 5 \cdot 10^{-3}\,\mathrm{eV}$

IV. MOVING UP IN ENERGY AND LEPTOGENESIS

All $\cancel{CP}$ in a unique phase

$$\Theta \equiv \mathrm{Arg}\left\langle S_1 S_2^* \right\rangle$$

$\Lambda = \zeta \sim M_5$

$M_i < \Lambda < M_5$

SUSY Type I seesaw with all
CP and flavor mixing coming from $\mathcal{Z}$

Observables: $\mathcal{O}_{UV} \supset \mathcal{O}_{IR}$

$\Lambda < M_i$

MSSM+Majorana neutrino masses

Observables: $\mathcal{O}_{IR}$

IV. MOVING UP IN ENERGY: LEPTOGENESIS

- The effective theory is

$$W_{MSSM} + \lambda_{ij}^{N}\ L_i H_u N_j + \frac{1}{2} M_{ij} N_i N_j$$

$$\lambda^{\ell} = \left(\lambda^{d}\right)^{T} \Gamma^{\ell}$$

$$\lambda^{N} = \lambda^{dT}\, \Gamma^{\nu}$$

$$\bar{\theta} = 0$$

- The type I seesaw model contains 9 observables *in addition* to the low energy leptonic observables:
    - 3 RH neutrino masses
    - 3 „UV mixing angles“
    - 3 „UV CP violating phases“

IV. PARAMETER COUNTING IN THE UV

Statement:
The 9 UV invariants can be computed from IR data and the three RH neutrino masses

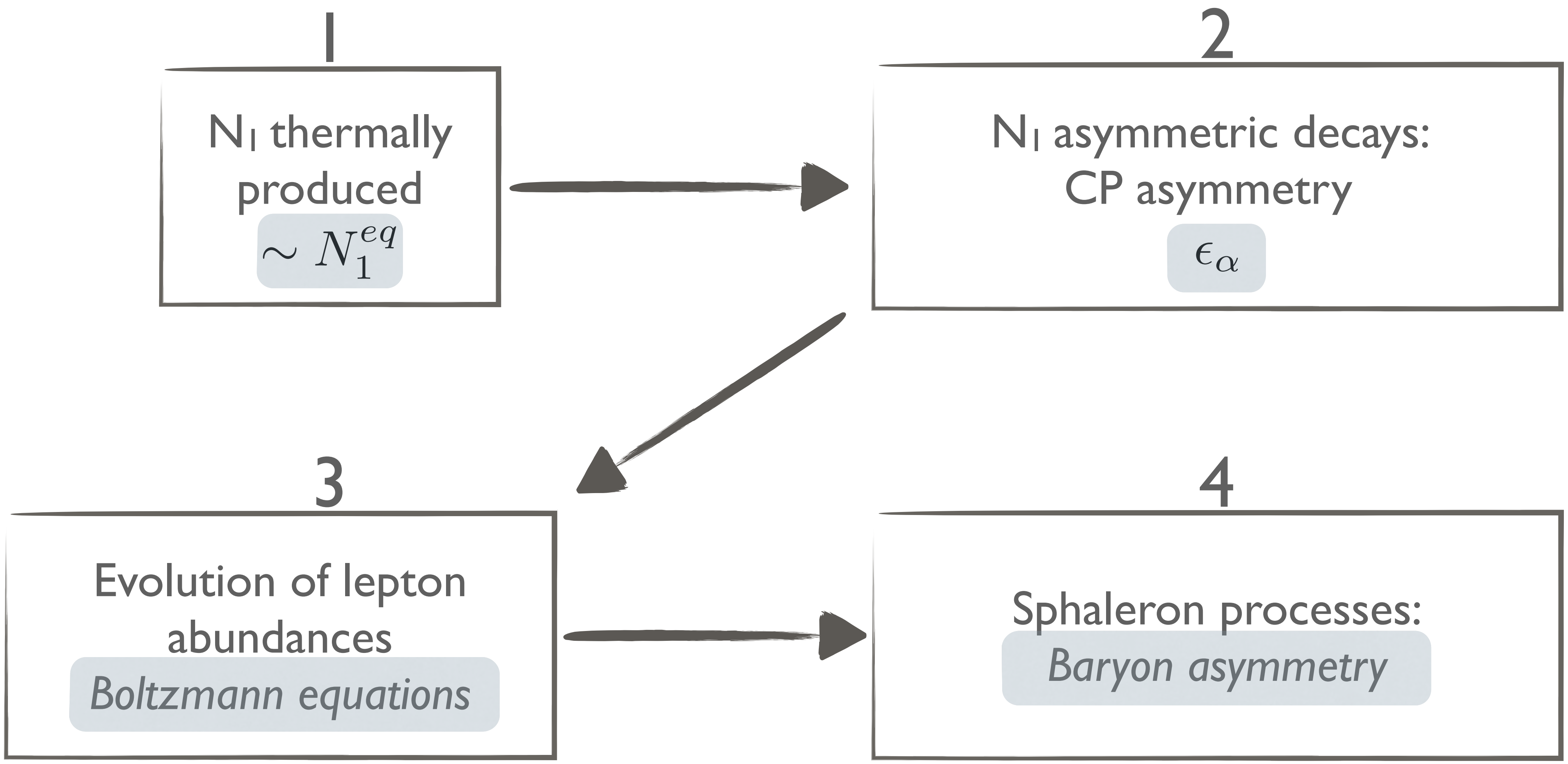
Given the RH neutrino masses and $\tan\beta$ the UV mixing angles and CP phases can be *predicted*

- C*rucial for leptogenesis*. Thermal leptogenesis only depends on UV CP violating phases (Branco et.al. 0107164).

Our model provides a *connection* between *IR data* and *UV CP violating phases*

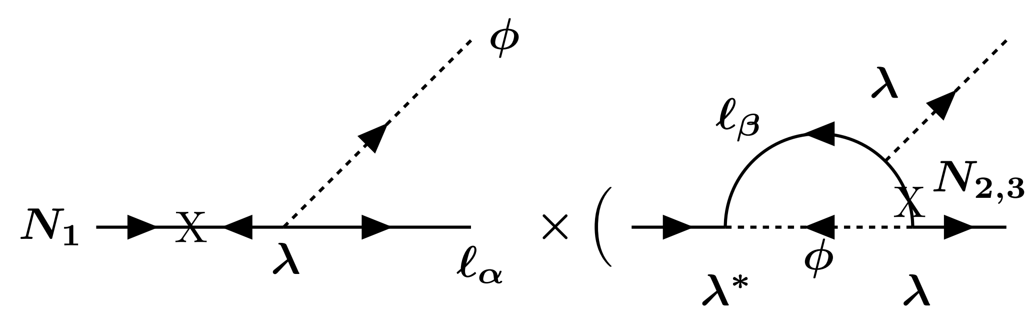
V. LEPTOGENESIS: A SHORT SUMMARY

- Thermal, hierarchical leptogenesis (Fukugita, Yanagida, PLB 174 (1986)45) („vanilla leptogenesis“).



V. THE CP ASYMMETRY

- The RH neutrino decay diagrams are of the form



plots taken from Davidson et.al 0802.2962v3

- The CP asymmetry is

$$\begin{aligned}\epsilon_\alpha &= \frac{\Gamma_{N_a\to l_\alpha h_u} - \Gamma_{N_a \to \bar{l}_\alpha h_u}}{\Gamma_{N_a}} \\ &= \frac{3M_a}{16\pi v_u^2\, \lambda^{N*}_{\xi a}\lambda^N_{\xi a}} \mathrm{Im}\left[\lambda^N_{\alpha a}(m^{\nu *}_{\alpha\xi}\lambda^N_{\xi a})\right]\end{aligned}$$

Do not despair:
trust that the invariants said we can calculate this

V. THE FINAL BARYON ASYMMETRY

- We find that the asymmetry is linear in $|M_1|$

$$\epsilon_\alpha = \xi_\alpha |M_1|$$

where ξ_α are numbers completely specified by IR data

- Solving the Boltzmann equations and using the sphaleron factor

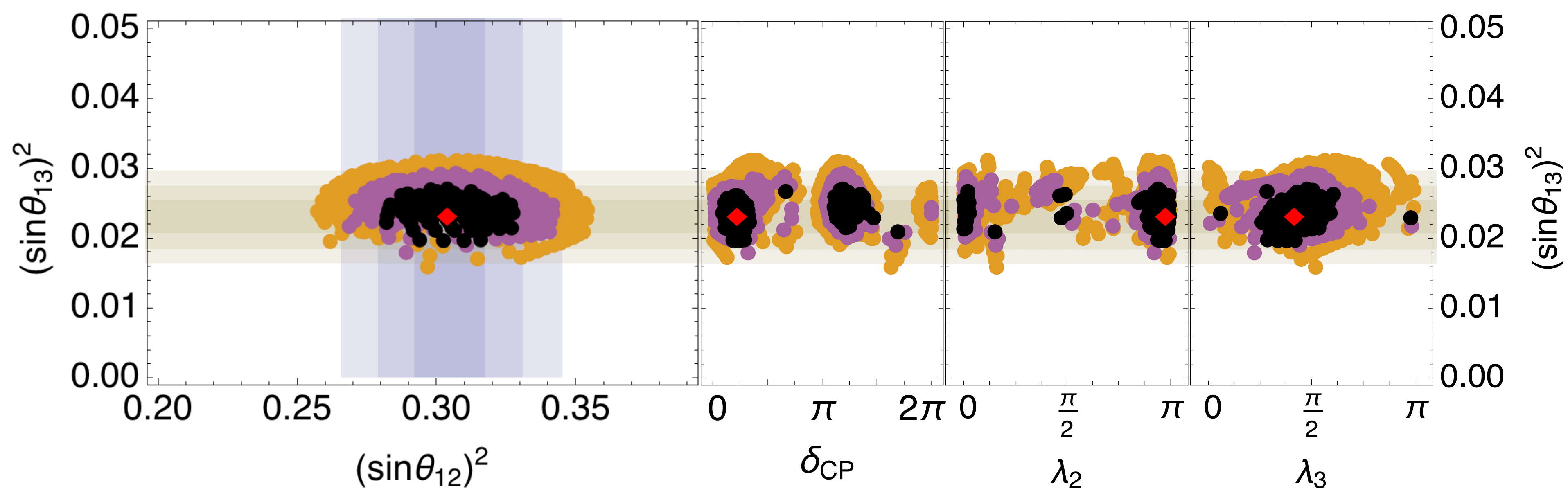
$$\frac{n_{\Delta B}}{s} = \eta |M_1|$$

the baryon asymmetry is also linear in $|M_1|$. η *is completely specified by IR data*

V. LEPTOGENESIS: RESULTS

- We find that the observed baryon asymmetry can be obtained with

$$10^9 \text{ GeV} \lesssim M_1 \lesssim 10^{11} \text{ GeV}$$



CONCLUSIONS

- We built a calculable model for all observed and required CP violation (which includes a solution to the strong CP problem).
- All CP violation is encoded in a single primordial phase.
- The model is constrained by known IR data. It predicts:
  - Generic CP violating phases in the PMNS matrix (with the current mixing angles uncertanties)
  - A normal hierarchy.
  - The lightest neutrino mass $10^{-3}\,\mathrm{eV} \lesssim m_{\nu_1} \lesssim 10^{-2}\,\mathrm{eV}$
  - The UV CP violating phases and mixing angles (for given RH neutrino masses and $\tan\beta$)
- Vanilla, thermal leptogenesis can be obtained for $10^{9}\ \mathrm{GeV} \lesssim M_1 \lesssim 10^{11}\ \mathrm{GeV}$

IV. BACKUP. FLAVOR INVARIANTS FOR A SEESAW

- The BG symmetries are

	$U(3)_\nu$	$U(3)_L$	$U(3)_\ell$
N	$\mathbf{3}_1$		
L		$\mathbf{3}_1$	
$\bar{\ell}$			$\bar{\mathbf{3}}_1$
λ_ν	$\mathbf{3}_1$	$\bar{\mathbf{3}}_{-1}$	
M	$\bar{\mathbf{6}}_{-2}$		
λ_ℓ		$\bar{\mathbf{3}}_{-1}$	$\mathbf{3}_{-1}$

- We follow our recipe and build the 9 missing invariants.

BACKUP FLAVOR INVARIANTS FOR A SEESAW

- The invariants related to the RH neutrino masses are

$$\mathrm{Tr}\Big[\big(M^* M\big)^n\Big] \qquad n = 1, 2, 3$$

- The invariants measuring „UV mixing“ and „UV CP violation“ are

$$\mathrm{Tr}\big[\lambda^{N\dagger}\lambda^N, M^* M\big]^2 \qquad \mathrm{Tr}\big[\lambda^{N\dagger}\lambda^N, M^* M\big]^3$$

$$\mathrm{Tr}\big[\lambda^{N\dagger}\lambda^N, M^* \lambda^{N\dagger}\lambda^N M\big]^2 \qquad \mathrm{Tr}\big[\lambda^{N\dagger}\lambda^N, M^* \lambda^{N\dagger}\lambda^N M\big]^3$$

$$\mathrm{Tr}\Big(\big[\lambda^{N\dagger}\lambda^N, M^* M\big]^2 \big(M^* \lambda^{N\dagger}\lambda^N M\big)^2\Big) \quad \mathrm{Tr}\Big(\big[\lambda^{N\dagger}\lambda^N, M^* M\big] \big(M^* \lambda^{N\dagger}\lambda^N M\big)^2\Big)$$

$\longrightarrow$ we need to know M and $\lambda^{N\dagger}\lambda^N$

BACKUP CONSTRAINING UV PHYSICS

- We work in a basis in which M is diagonal so it is completely specified by the RH neutrino masses.

- On the other hand, using the EFT conditions we can write

$$\lambda^{N\dagger}\lambda^{N} = \boxed{\Gamma^{\nu}}\boxed{(\lambda^{d}\lambda^{d\dagger})^{*}}\boxed{\Gamma^{\nu}}$$

specified by IR data and the RH neutrino masses

$$\Gamma^{\nu} = \frac{v_d}{v_u}\sqrt{M}\sqrt{\Gamma^{m}}$$

Already solved for by using IR data

BACKUP CALCULATING THE ASYMMETRY

- In fact, we can choose

$$\lambda^N = \frac{1}{v_u}\Big[\mathrm{diag}(m_d, m_s, m_b)\, V_{CKM}^T\, \mathrm{diag}(1, e^{-i\gamma_1}, e^{-i\gamma_2})\Big]^* \sqrt{M}\sqrt{\Gamma^m}$$

$\tan\beta$ is unknown

RH neutrino masses are unknown

- While for the neutrino masses we can choose

$$m^\nu_{\alpha\xi} = \Big[\mathrm{diag}(m_d, m_s, m_b)\, V_{CKM}^\dagger$$
$$\mathrm{diag}(1, e^{i\gamma_1}, e^{i\gamma_2})\, \Gamma^m\, \mathrm{diag}(1, e^{i\gamma_1}, e^{i\gamma_2})\, V_{CKM}^*\, \mathrm{diag}(m_d, m_s, m_b)\Big]_{\alpha\xi}$$

where all is known in this expression from IR data